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Research Article

Stabilization of Uncertain Switched Systems with Frequent Asynchronism via Event-Triggered Dynamic Output-Feedback Control

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This article investigates stabilization for a group of uncertain switched systems with frequent asynchronism. Without the limitation of minimum residence time, the average dwell-time strategy makes it possible for switched systems with uncertain parameters to switch frequently over successive event intervals. Since it is highbrow and expensive to obtain the whole state information in practice, the dynamic output-feedback controller is applied. With the aid of a controller-pattern-related Lyapunov functional and an event-triggered dynamic output-feedback controller, sufficient conditions are established to ensure the stability of the resulting uncertain closed-loop system. To appropriately deal with the uncertain parameters, some inequalities of the linear matrix are tactfully utilized together with the Lyapunov functional and controller gains are constructed by the strategy of the block matrix. Furthermore, the presence of the lower boundary on adjacent event intervals is earnestly discussed to eliminate the Zeno behavior. Eventually, the feasibility and availability of the theoretical results are illuminated by a numerical simulation.

1. Introduction

As the control objects become more and more complex, the requirements for the control performance index become higher and higher. At the same time, the system's operation mechanism is restricted by many factors. Many practical control problems can be better solved through the switched system theory. The analysis and integration of switched systems have become a hot issue in the academic and engineering research fields. The concept of switched systems has been formally put forward in the earlier literature [1, 2]. Switched systems, in recent decades, have gained widespread attention in the field of control and have achieved plenteous accomplishments, such as [3–6] and its references. There are two leading reasons for the wide attention paid to switched system theory. On the one hand, switched systems have an extensive range of practical applications in numerous domains, such as transportation systems, robot control systems, power systems, and communication systems. On the other hand, the switched systems have a certain complicacy and idiosyncrasy in comparison with the traditional unitary mode control systems. Nonetheless, it enables the physical procedure and the considered control issues to be more accurate. Over the past decades, a number of achievements for multifarious switched systems have been made in stability analysis as well as control synthesis. The dynamic behavior of switched systems depends not only on each switching subsystem but also on switching rules. An average dwell-time-based switching rule is an effective tool for switched system analysis and synthesis. To name a few, a group of convex lifted conditions about the robust l_2 -stability of discrete-time switched linear systems is proposed by utilizing the switching characteristic of minimum residence time [7]. For the sake of stabilizing the switched systems that incorporate bounded additional perturbations, a quasi-time-varying Lyapunov function is constructed in [8]. And to guarantee the exponential stabilization of switched systems in the presence of slow and fast switching restricted to unstable and stable subsystems severally, a multiple discontinuous Lyapunov functional technique is proposed in [9].

In the wake of the rapid development of the microelectronic technique, it has become feasible to implement wide-scale calculation in postmodern control theory, which boosts the dynamic analysis of control systems with information sampling [10, 11]. For time-triggered control and the traditional periodic sampling, the sensor sampling systems carry out states or measured outputs at a regular time interval. In spite of this sampling method being conducive to theoretical analysis and implementation, it generally produces vast data packets and results in the sending of redundant data. At this moment, if a state-feedback controller is utilized to feedback the system state, more resources will be wasted. An event-triggered mechanism has been proved to maintain the performance of a networked control system and reduce the number of data packets. However, there is still room for progress in reducing the number of data packets because most of the trigger parameters are static. Then how to further reduce the transmission rate and how to design an adaptive event-triggered mechanism are worth studying. Add in the fact that not all states are measurable in practice; therefore, event-triggered strategy and dynamic output-feedback controller attract considerable attention. For example, on the basis of introducing internal dynamic variables, the dynamic event-triggered approach for linear and nonlinear systems is proposed in [12]. It is testified that the method of dynamic event triggering has less data transmission than the conventional static one. Nevertheless, on account of the characteristics of switched systems, it is a great challenge to set up event-triggered control for system analysis and synthesis. The minimum commutative law is used to design the controller of switched systems so that system switching merely occurs at the triggered moments [13]. In the case of ignoring asynchrony between subsystems and their controllers and guaranteeing the Zeno behavior is eliminated during the event-triggered course, the event-triggered strategy-approved observer-based design of switched linear systems is investigated [14]. Equally, by supposing fully synchronism, the sampling time finiteness of the switched linear systems that incorporate switching signals of average dwell time is guaranteed by processing the event-triggered controller [15]. It is obvious that the assumption in [14, 15] is fairly strict. For the purpose of overcoming this weakness, asynchrony can be considered and event-triggered stability of switched linear neutral systems may be realized in virtue of introducing a maximum asynchrony interval and a minimum dwell time. Subsequently, a switched system with frequent switching between events is investigated in [16].

There are many key structured subsystems in mechanical systems, electronic circuits, robots, and other engineering fields. Meanwhile, these subsystems are mainly composed of core components with uncertainty. It is of great practical significance to study the control problem of complex uncertain dynamic systems and ensure the performance of the system under uncertain disturbances. As is known to all, uncertainty, one of the factors causing system instability, exists in almost all systems. It is inevitable for the existence of uncertainty in the model due to environmental noise, uncertain or slowly changing parameters, and other reasons. Without considering the uncertainty of the model, in practice, it seems preposterous to analyze the performance of a system like estimating the performance indexes in dynamic and steady state. Furthermore, there is usually uncertainty due to random disturbance, inherent variations, missing information, human error, or measurement inaccuracies for nonlinear and linear systems. The source of this uncertainty is known as parametric uncertainty, and parametric uncertainty is probably the most crucial source of model uncertainty [17]. Under the effect of certain factors, in other words, the switched systems with uncertain parameters can depict a broader range of the linear systems. Actually, it is still an unsolved problem to study uncertain switched systems with frequent asynchrony in event-triggered dynamic output-feedback control, and to our astonishment, the uncertainty of output parameters for switched linear systems has not yet received much attention from scholars. After all, in comparison with the traditional processing strategies, the robust treatment of many uncertain parameters in switched systems becomes exceedingly tricky. So far, there is no miraculous solution for detecting the error effects of parameter uncertainty.

In the light of the foregoing discussion, this article is devoted to investigating the exponential stabilization of a frequently switched system which is equipped with the uncertainty of state parameters, input parameters, and measured output parameters. Primarily, some uncertain parameters are inserted in the appropriate position according to the necessity of the model, and some ingenious ways of dealing with them are introduced. In addition, based on the logic mechanism triggered by events and the switching rules of the system and its controller, the matching situation of subsystems and controller could be categorized into synchronous and asynchronous to study the dynamic and steady state performance of the uncertain closed-loop system. The solution of the uncertain closed-loop system is globally exponential stable and no Zeno behavior occurs during the data sampling process, which proves that the selection of controller and problem-resolving method is valid and reasonable. Generally speaking, the primary contributions of this article could be summarized into three points:

(1) Switching linear system model with uncertain parameters is established. By making use of a few inequalities of linear matrix and the strategy of partitioned matrices, the values of parametric uncertainties are legitimately estimated.

(2) Based on the approach of the average dwell time, the co-designing of the event-triggered strategy and the dynamic output-feedback controller, and the construction of the controller-pattern-related Lyapunov functional by adopting block matrix method, the stability criterion of uncertain linear switching system with frequent asynchronism is guaranteed.

(3) In the uncertain switched system, the lower bound criterion of successive event intervals is established to avoid the Zeno behavior, which simultaneously suggests that it can be researched by more extensible frameworks with the methods exploited.

The remaining layout of this article is included as follows: Section 2 furnishes the uncertain system model and preparatory works. Section 3 puts forward the primary theorems for exponential stabilization analysis, controller device, and Zeno behavior elimination. Furthermore, in Section 4, a numerical simulation is added to corroborate the effectiveness of the derived results. Lastly, Section 5 draws a conclusion.

Notations: throughout this article, $\mathbb{R}^{p\times m}$ and $\mathbb{R}^p$ are severally the set of all $p\times m$ real matrices and p-dimensional Euclidean space. $\mathbb{N}$ means nonnegative set. $\mathbb{N}^+$ stands for the set of positive integers. I in matrices or matrix inequalities represents an identity matrix with matched dimensionality. 0 in matrices is a zero matrix of appropriate dimensions. The superscripts -1 and T of matrix $\mathfrak{G}$ represent the inverse and transposition of $\mathfrak{G}$, respectively. $He(\mathfrak{G})$ is defined as $He(\mathfrak{G}) = \mathfrak{G} + \mathfrak{G}^T$. $\mathfrak{G} > 0$ represents that matrix $\mathfrak{G}$ is positive definite and symmetric. $\lambda_{\max}(\mathfrak{G})$ and $\lambda_{\min}(\mathfrak{G})$ represent the maximum and minimum eigenvalue of matrix $\mathfrak{G}$ separately. We utilize $\star$ in a matrix to denote symmetry term. $\|\mathfrak{G}\|$ is defined as the 2-norm of matrix $\mathfrak{G}$.

2. Preliminaries

The switched linear system with parametric uncertainty is of the following form:

$$\begin{aligned}\dot{x}(t) &= \left(A_{\sigma(t)} + \Delta A_{\sigma(t)}\right)x(t) + \left(B_{\sigma(t)} + \Delta B_{\sigma(t)}\right)u(t),\\ y(t) &= \left(C_{\sigma(t)} + \Delta C_{\sigma(t)}\right)x(t),\end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^p$, and $y(t) \in \mathbb{R}^q$ are the system state, control input, and measured output, respectively. $A_{\sigma(t)}$, $B_{\sigma(t)}$, and $C_{\sigma(t)}$ are all known constant matrices, and $\Delta A_{\sigma(t)}$, $\Delta B_{\sigma(t)}$, and $\Delta C_{\sigma(t)}$ are the uncertain parameters. $\sigma(t)$: $[0,+\infty) \longrightarrow \mathscr{L} = \{1,2,\ldots,\mathfrak{n}\}$, signifying the switching signal, is a piecewise constant functional, where $\mathfrak{n} \in \mathbb{N}^+$ represents the quantity of subsystems. $\sigma(t) = j, j \in \mathscr{L}$ indicates the j-th subsystem is active. A chronological sequence $\{t_q, q \in \mathbb{N}\}$ is constructed to indicate that the switching instant is t_q.

For system (1) with parameter uncertainty and unknown nonlinearity, how to introduce an effective control mechanism and this control mechanism will ensure the stability of the closed-loop system and make the system state quickly converge to the ideal state is a difficult research problem.

An adaptive event-triggered precept is adopted to transmit the corresponding activated mode information and measurement output for the continuously updated controller at the instants $\{s_k, k \in \mathbb{N}\}$, which is governed as

$$s_{k+1} = \inf\left\{s_k < t \le s_k + T | e_{s_k}^T(t)\Omega_{\sigma(s_k)}e_{s_k}(t) \ge \epsilon y^T(t)\Omega_{\sigma(s_k)}y(t)\right\}, \tag{2}$$

where $e_{s_k}(t) = y(s_k) - y(t)$, $\epsilon > 0, T > 0$ are prescribed constants and $\Omega_{\sigma(s_k)}$ is a known positive definite matrix.

Remark 1. The argument T restricts the upper bound on each successive event interval and the overall asynchronous time. Not only is it beneficial to analyze and synthesize the problem, but it can also prevent the controller from not being updated for ages. One more point needs to be noted that when the foregoing mechanism is not triggered, according to its conditions, the inequation $e_{s_k}^T(t)\Omega_{\sigma(s_k)}e_{s_k}(t) < \epsilon y^T(t)\Omega_{\sigma(s_k)}y(t)$ holds. And we will employ it later in the proof of theorems.

For uncertain system (1), construct the following dynamic output-feedback control scheme:

$$\begin{aligned}\dot{\tilde{x}}(t) &= \tilde{A}_{\sigma(s_k)}\tilde{x}(t) + \tilde{B}_{\sigma(s_k)}y(s_k),\\ u(t) &= \tilde{C}_{\sigma(s_k)}\tilde{x}(t) + \tilde{D}_{\sigma(s_k)}y(s_k).\end{aligned} \tag{3}$$

For $t \in [s_k, s_{k+1})$, in which $\tilde{x}(t) \in \mathbb{R}^n$ is a controller state, $\tilde{A}_{\sigma(s_k)}$, $\tilde{B}_{\sigma(s_k)}$, $\tilde{C}_{\sigma(s_k)}$, and $\tilde{D}_{\sigma(s_k)}$, to be decided, are all constant matrices.

Substituting (3) into (1), the following augmented system is deduced:

$$\dot{\mathfrak{I}}(t) = \overline{A}_{\sigma(t)\sigma(s_k)}\mathfrak{I}(t) + \overline{B}_{\sigma(t)\sigma(s_k)}e_{s_k}(t), \tag{4}$$

for $t \in [s_k, s_{k+1})$, where $\mathfrak{I}(t) = [x^T(t), \tilde{x}^T(t)]^T$,

$$\begin{aligned}\overline{A}_{\sigma(t)\sigma(s_k)} &= \begin{bmatrix}\mathfrak{A}_{\sigma(t)} + \mathfrak{B}_{\sigma(t)}\tilde{D}_{\sigma(s_k)}\mathfrak{C}_{\sigma(t)} & \mathfrak{B}_{\sigma(t)}\tilde{C}_{\sigma(s_k)}\\ \tilde{B}_{\sigma(s_k)}\mathfrak{C}_{\sigma(t)} & \tilde{A}_{\sigma(s_k)}\end{bmatrix},\\ \overline{B}_{\sigma(t)\sigma(s_k)} &= \begin{bmatrix}\mathfrak{B}_{\sigma(t)}\tilde{D}_{\sigma(s_k)}\\ \tilde{B}_{\sigma(s_k)}\end{bmatrix},\\ \mathfrak{A}_{\sigma(t)} &= A_{\sigma(t)} + \Delta A_{\sigma(t)},\\ \mathfrak{B}_{\sigma(t)} &= B_{\sigma(t)} + \Delta B_{\sigma(t)},\\ \mathfrak{C}_{\sigma(\sigma(t)t)} &= C_{\sigma(t)} + \Delta C_{\sigma(t)}.\end{aligned} \tag{5}$$

In this article, the condition of the aforementioned adaptive event-triggered strategy relies on the error $e_{s_k}(t)$ between the recently updated sampling output and the current sampling output for transmission. When the condition of (2) holds $[s_k, s_k + T)$, the next event will be triggered at $t = s_{k+1}$; subsequently, the controller will update its mode based on the data collected and control system (1). The architecture diagram of the uncertain closed-loop system under event-triggered dynamic output-feedback control is presented in Figure 1.

Now, we hypothesize $\sigma(s_k) = i, \sigma(t_q) = j$ in the chronological sequence $s_k < t_q$ and focus attention on a running

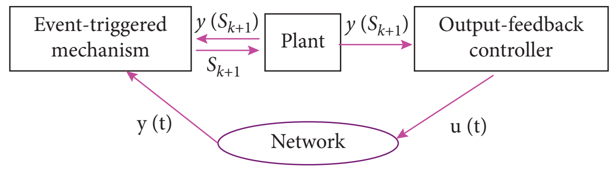

Figure 1: Diagram of the uncertain closed-loop system.

interval $[t_q, t_{q+1})$, which indicates $\sigma(t) = j$ for $\forall t \in [t_q, t_{q+1})$. Afterward, the system dynamics of (4) can be summarized as the following two cases:

Case (i): when mechanism (2) is not triggered in $[t_q, t_{q+1})$, i.e., $s_k < t_q < t_{q+1} \leq s_{k+1}$, uncertain closed-loop system (4) is transformed into

$$\dot{\mathfrak{I}}(t) = \overline{A}_{ji}\mathfrak{I}(t) + \overline{B}_{ji}e(t), \tag{6}$$

where $e(t) \triangleq e_{s_k}(t), t \in [t_q, t_{q+1})$.

Case (ii): when mechanism (2) is triggered $m (\in \mathbb{N}^+)$ times in $[t_q, t_{q+1})$, i.e., $s_k < t_q \leq s_{k+1} < \cdots < s_{k+m} \leq t_{q+1} < s_{k+m+1}$, uncertain closed-loop system (4) is converted into

$$\dot{\mathfrak{I}}(t) = \begin{cases} \overline{A}_{ji}\mathfrak{I}(t) + \overline{B}_{ji}e(t), & t \in [t_q, s_{k+1}), \\ \overline{A}_{jj}\mathfrak{I}(t) + \overline{B}_{jj}e(t), & t \in [s_{k+1}, s_{k+2}), \\ \vdots \\ \overline{A}_{jj}\mathfrak{I}(t) + \overline{B}_{jj}e(t), & t \in [s_{k+m}, t_{q+1}), \end{cases} \tag{7}$$

where

$$e(t) \triangleq \begin{cases} e_{s_k}(t), & t \in [t_q, s_{k+1}), \\ e_{s_{k+1}}(t), & t \in [s_{k+1}, s_{k+2}), \\ \vdots \\ e_{s_{k+m}}(t), & t \in [s_{k+m}, t_{q+1}). \end{cases} \tag{8}$$

It is notable that the controller merely updates its state in contrast to the subsystem, in which both modes and states change in $[t_q, s_{k+1})$. Once (2) is triggered, (3) will receive both the activated mode information and the measured output, which will generate the synchronism with the corresponding subsystem for $t \in [s_{k+1}, s_{k+2}), \ldots, [s_{k+m}, t_{q+1})$ in Case (ii). Notations $T_\uparrow[t_q, t_{q+1})$ and $T_\downarrow[t_q, t_{q+1})$ are introduced to stand for the asynchronous and synchronous interval of $[t_q, t_{q+1})$ severally, where each subsystem does not match with their controller in interval $T_\uparrow[t_q, t_{q+1})$. After that, the aforementioned uncertain closed-loop system could be rewritten as

$$\dot{\mathfrak{I}}(t) = \begin{cases} \overline{A}_{ji}\mathfrak{I}(t) + \overline{B}_{ji}e(t), & t \in T_\uparrow[t_q, t_{q+1}), \\ \overline{A}_{jj}\mathfrak{I}(t) + \overline{B}_{jj}e(t), & t \in T_\downarrow[t_q, t_{q+1}). \end{cases} \tag{9}$$

As an assumption, several definitions and lemmas are given for employing in the sequel.

Assumption 1.

$$\left[\Delta A_{\sigma(t)} \ \ \Delta B_{\sigma(t)} \ \ \Delta C_{\sigma(t)}\right] = M_{\sigma(t)}E_{\sigma(t)}(t)\left[F_{\sigma(t)} \ \ F'_{\sigma(t)} \ \ F''_{\sigma(t)}\right], \tag{10}$$

where $E_{\sigma(t)}(t)$ is the unknown time-varying matrix with $E^T_{\sigma(t)}(t)E_{\sigma(t)}(t) \leq I$, $F_{\sigma(t)}$ and $F''_{\sigma(t)}$ are constant matrices with applicable dimensionality and $M_{\sigma(t)}$ and $F'_{\sigma(t)}$ are nonsingular real matrices with appropriate dimensionality.

Remark 2. The uncertainties have a direct influence on the structure and stability of switched systems due to their own uncertainty. In order to avoid needlessly sophisticated notations and suppress parameter uncertainty, we only consider norm-bounded uncertainties. By utilizing subsequent Lemma 1, the effect of the uncertainty time-varying matrix $E_{\sigma(t)}(t)$ could be reasonably eliminated so that the uncertainties $\Delta A_{\sigma(t)}$, $\Delta B_{\sigma(t)}$, and $\Delta C_{\sigma(t)}$ could be tactfully estimated. However, when the uncertain terms also appear in other singular structures, the results obtained by the lemma could be extended to this case in parallel.

Definition 1. For any initial conditions and constants $k > 0$ and $\lambda > 0$, system (9) is said to be globally exponentially stable under some switching signals $\sigma(t)$, if the solution of system (9) satisfies $\|\mathfrak{I}(t)\| \leq ke^{-\lambda(t-t_0)}\|\mathfrak{I}(t_0)\|$, $\forall t \geq t_0$.

Definition 2. For the switching signal $\sigma(t)$ and any $t_2 \geq t_1 \geq 0$, the switching number of $\sigma(t)$ labelled as $\mathfrak{n}_\sigma(t_1, t_2)$ during the interval (t_1, t_2). If there exist constants $\mathfrak{n}_0 > 0$ and $\tau_a > 0$ such that $\mathfrak{n}_\sigma(t_1, t_2) \leq \mathfrak{n}_0 + t_2 - t_1/\tau_a$, then τ_a is called the average dwell time of $\sigma(t)$ and $\mathfrak{n}_0$ is called the chatter bound.

Lemma 1 (see [18]). *Let $\mathcal{G}$, $\mathcal{H}$, and $\mathcal{J}$ be constant matrices of suitable dimensionality and $\mathcal{F}(t)$ be a matrix function.*

(1) For any $\varepsilon > 0$ and $\mathcal{F}^T(t)\mathcal{F}(t) \leq I$, then

$$\mathscr{H}\mathscr{F}(t)\mathscr{J}+\mathscr{J}^T\mathscr{F}^T(t)\mathscr{H}^T\leq\frac{1}{\varepsilon}\mathscr{H}\mathscr{H}^T+\varepsilon\mathscr{J}^T\mathscr{J}. \tag{11}$$

(2) *For any $\varepsilon>0$ such that $\varepsilon\mathscr{J}^T\mathscr{J}<I$ and $\mathscr{F}^T(t)\mathscr{F}(t)\leq I$, then*

$$(\mathscr{G}+\mathscr{H}\mathscr{F}(t)\mathscr{J})(\mathscr{G}+\mathscr{H}\mathscr{F}(t)\mathscr{J})^T\leq\mathscr{G}\left(I-\varepsilon\mathscr{J}^T\mathscr{J}\right)^{-1}\mathscr{G}^T+\frac{1}{\varepsilon}\mathscr{H}\mathscr{H}^T. \tag{12}$$

Especially, when $\mathscr{G}\equiv 0$, we get

$$\mathscr{H}\mathscr{F}(t)\mathscr{J}(\mathscr{H}\mathscr{F}(t)\mathscr{J})^T\leq\frac{1}{\varepsilon}\mathscr{H}\mathscr{H}^T. \tag{13}$$

Lemma 2. *Given constant matrices $\mathscr{M}_1,\mathscr{M}_2,\mathscr{M}_3$, where $\mathscr{M}_1=\mathscr{M}_1^T$ and $\mathscr{M}_2>0$, then*

$$\mathscr{M}_1+\mathscr{M}_3^T\mathscr{M}_2^{-1}\mathscr{M}_3<0, \tag{14}$$

if

$$\begin{bmatrix}\mathscr{M}_1 & \mathscr{M}_3^T\\ \mathscr{M}_3 & -\mathscr{M}_2\end{bmatrix}<0 \quad\text{or}\quad \begin{bmatrix}-\mathscr{M}_2 & \mathscr{M}_3\\ \mathscr{M}_3^T & \mathscr{M}_1\end{bmatrix}<0. \tag{15}$$

Lemma 3 (see [19]). *The following representations are equivalent:*

(1) *There exist constant matrices $\mathscr{N}$, $\mathscr{V}$, $\mathscr{W}$, and $\mathscr{Z}$ and real scalar ς such that*

$$\begin{bmatrix}\mathscr{N} & \star\\ \mathscr{W}-\varsigma & \varsigma\mathscr{Z}+\varsigma\mathscr{Z}^T\end{bmatrix}<0. \tag{16}$$

(2) *There exist constant matrices $\mathscr{N}$, $\mathscr{V}$, and $\mathscr{W}$ such that*

$$\begin{aligned}&\mathscr{N}<0\\ &\mathscr{N}+\mathscr{V}^T\mathscr{W}+\mathscr{W}^T\mathscr{V}<0.\end{aligned} \tag{17}$$

Lemma 4 (see [16]). *Define $\mathscr{S}_1(t)$, $\mathscr{S}_2(t)$, and $\mathscr{G}(t,\cdot)$ as continuous functions for any $t\geq 0$; in particular, $\mathscr{G}(t,\cdot)$ is differentiable. If $\mathscr{S}_2(t_0)\leq\mathscr{S}_1(t_0)$ and*

$$\begin{aligned}\dot{\mathscr{S}}_1(t)&=\mathscr{G}(t,\mathscr{S}_1(t)),\\ \dot{\mathscr{S}}_2(t)&\leq\mathscr{G}(t,\mathscr{S}_2(t)),\end{aligned} \tag{18}$$

then $\mathscr{S}_2(t)\leq\mathscr{S}_1(t)$, $\forall t\geq t_0$.

3. Main Results

In this section, the event-triggered mechanism and the dynamic output-feedback controller will be jointly devised to investigate the exponential stability of the switched linear system with uncertainties under the asynchrony phenomenon. In addition, we exclude Zeno's behavior by proving the existence of the lower boundary of the interevent intervals.

3.1. Stability Analysis. This section aims at the stability criterion for uncertain systems (9) with the adhibition of the average dwell-time switching approach and a controller-pattern-dependent Lyapunov functional.

Theorem 1. *With regard to given scalars $\gamma>0$, $\delta>0$, $\eta\geq 1$, $T>0$, and $\epsilon>0$, if Assumption 1 holds and there exist matrices $P_i>0$, $\Omega_i>0$, $\exists h_{iab}>0$ $(a=1,2,\ldots,7;b=1,2)$, $\exists h_{i8}>0$, and $\forall i\in\mathscr{L}$ such that*

$$\Psi_{ji}=\begin{bmatrix}\Psi_{ji}^1 & P_i\overline{\overline{B}}_{ji} & \mathscr{E}_i\end{bmatrix}<0,\quad \forall i\neq j\in\mathscr{L}, \tag{19}$$

$$\Psi_{jj}=\begin{bmatrix}\Psi_{jj}^1 & P_j\overline{\overline{B}}_{jj} & \mathscr{E}_j\end{bmatrix}<0,\quad \forall j\in\mathscr{L}, \tag{20}$$

$$P_j\leq\eta P_i,\quad \forall i\neq j\in\mathscr{L}, \tag{21}$$

$$\begin{bmatrix}I & F_j'\widetilde{D}_iM_j\\ M_j^T\widetilde{D}_i^TF_j'^T & \dfrac{1}{h_{i8}}I\end{bmatrix}>0,\quad \forall i\neq j\in\mathscr{L}, \tag{22}$$

$$\begin{bmatrix}I & F_j'\widetilde{D}_jM_j\\ M_j^T\widetilde{D}_j^TF_j'^T & \dfrac{1}{h_{j8}}I\end{bmatrix}>0,\quad \forall j\in\mathscr{L}, \tag{23}$$

where

$$\begin{aligned}
\Psi_{ji}^{1} &= He\left(P_i\overline{\overline{A}}_{ji}\right) + \mathcal{Q}_i - \delta P_i,\\
\Psi_{jj}^{1} &= He\left(P_j\overline{\overline{A}}_{jj}\right) + \mathcal{Q}_j + \gamma P_j,\\
\widetilde{A}_i &= \begin{bmatrix} A_j + B_j\widetilde{D}_iC_j & B_j\widetilde{C}_i \\ \widetilde{B}_iC_j & \widetilde{A}_i \end{bmatrix},\\
\widetilde{B}_i &= \begin{bmatrix} B_j\widetilde{D}_i \\ \widetilde{B}_i \end{bmatrix},\\
\mathcal{Q}_i &= \begin{bmatrix} \mathcal{Q}_{i1} & \mathcal{Q}_{i2} \\ \mathcal{Q}_{i3} & \mathcal{Q}_{i4} \end{bmatrix},\\
P_i &= \begin{bmatrix} P_{i1} & P_{i2} \\ P_{i2}^T & P_{i3} \end{bmatrix},\\
\mathcal{Q}_{i1} &= (h_{i11} + h_{i21} + h_{i71} + h_{i41})P_{i1}M_jM_j^TP_{i1}\\
&\quad + h_{i31}P_{i1}B_j\widetilde{D}_iM_jM_j^T\widetilde{D}_i^TB_j^TP_{i1} + h_{i51}P_{i2}\widetilde{B}_iM_jM_j^T\widetilde{B}_i^TP_{i2}^T\\
&\quad + \frac{1}{h_{i11}}F_j^TF_j + \left(\frac{1}{h_{i31}} + \frac{1}{h_{i51}} + \frac{1}{h_{i41}h_{i8}}\right)F''^T_jF''_j\\
&\quad + \frac{1}{h_{i21}}C_j^T\widetilde{D}_i^TF'^T_jF'_j\widetilde{D}_iC_j + \epsilon C_j^T\Omega_iC_j,\\
\mathcal{Q}_{i3} &= (h_{i12} + h_{i22} + h_{i42})P_{i2}^TM_jM_j^TP_{i2} + h_{i32}P_{i2}^TB_j\widetilde{D}_iM_jM_j^T\widetilde{D}_i^TB_j^TP_{i2}\\
&\quad + h_{i52}P_{i3}\widetilde{B}_iM_jM_j^T\widetilde{B}_i^TP_{i3} + \frac{1}{h_{i12}}F_j^TF_j + \left(\frac{1}{h_{i32}} + \frac{1}{h_{i52}} + \frac{1}{h_{i42}h_{i8}}\right)F_j''^TF_j''\\
&\quad + \frac{1}{h_{i22}}C_j^T\widetilde{D}_i^TF_j'^TF_j'\widetilde{D}_iC_j,\\
\mathcal{Q}_{i2} &= \frac{1}{h_{i61}}\widetilde{C}_i^TF'^T_jF'_j\widetilde{C}_i + h_{i61}P_{i1}M_jM_j^TP_{i1},\\
\mathcal{Q}_{i4} &= \frac{1}{h_{i62}}\widetilde{C}_i^TF_j'^TF_j'\widetilde{C}_i + (h_{i62} + h_{i72})P_{i2}^TM_jM_j^TP_{i2},\\
\mathcal{X}_i &= \left(\frac{1}{h_{i71}} + \frac{1}{h_{i72}}\right)\widetilde{D}_i^TF_j'^TF_j'\widetilde{D}_i - \Omega_i,
\end{aligned} \tag{24}$$

then uncertain system (9) is globally exponentially stable with the average dwell time τ_a satisfying

$$\tau_a \geq \frac{\ln\eta + (\gamma + \delta)T}{\gamma}. \tag{25}$$

Proof. Construct the controller-pattern-related Lyapunov function $V(\mathfrak{J}(t))\mathfrak{J}^T(t)P_{\sigma(s_k)}\mathfrak{J}(t)$ by the strategy of the block matrix. It is evident that $V(\mathfrak{J}(t))$, except for a limited number of discontinuities, remains continuous. Then, in accordance with the cases in the hereinabove section, the stabilization analysis can be accordingly carried out from the two standpoints as follows:

Case (i): when mechanism (2) is not triggered during $[t_q, t_{q+1})$, the uncertain closed-loop system is $\overline{A}_{ji}\mathfrak{J}(t) + \overline{B}_{ji}e(t)$ with the Lyapunov function $V(\mathfrak{J}(t)) = \mathfrak{J}^T(t)P_i\mathfrak{J}(t)$.

By means of taking the derivative with respect to $V(\mathfrak{J}(t))$ along the system trajectory and exploiting the mechanism (2), we get

$$\begin{aligned}
\dot{V}(\mathfrak{J}(t)) - \delta V(\mathfrak{J}(t)) &= 2\mathfrak{J}^T(t)P_i\dot{\mathfrak{J}}(t) - \delta\mathfrak{J}^T(t)P_i\mathfrak{J}(t)\\
&\leq 2\mathfrak{J}^T(t)P_i\left(\overline{A}_{ji}\mathfrak{J}(t) + \overline{B}_{ji}e(t)\right)\\
&\quad - \delta\mathfrak{J}^T(t)P_i\mathfrak{J}(t) - e(t)^T\Omega_ie(t) + \varepsilon x^T(t)C_j^T\Omega_iC_jx(t).
\end{aligned} \tag{26}$$

On account of Lemma 1 (1), (2), Lemma 2, and (22), we get

$$
\begin{aligned}
&He\left(\begin{bmatrix} P_{i1}\Delta A_j \\ P_{i2}^T\Delta A_j \end{bmatrix}\right) \le \begin{bmatrix} \frac{1}{h_{i11}}F_j^T F_j + h_{i11}P_{i1}M_j M_j^T P_{i1} \\ \frac{1}{h_{i12}}F_j^T F_j + h_{i12}P_{i2}^T M_j M_j^T P_{i2} \end{bmatrix}, \\
&He\left(\begin{bmatrix} P_{i1}\Delta B_j \widetilde{D}_i C_j \\ P_{i2}^T\Delta B_j \widetilde{D}_i C_j \end{bmatrix}\right) \le \begin{bmatrix} \frac{1}{h_{i21}}C_j^T \widetilde{D}_i^T F_j'^T F_j' \widetilde{D}_i C_j + h_{i21}P_{i1}M_j M_j^T P_{i1} \\ \frac{1}{h_{i22}}C_j^T \widetilde{D}_i^T F_j'^T F_j' \widetilde{D}_i C_j + h_{i22}P_{i2}^T M_j M_j^T P_{i2} \end{bmatrix}, \\
&He\left(\begin{bmatrix} P_{i1}B_j \widetilde{D}_i \Delta C_j \\ P_{i2}^T B_j \widetilde{D}_i \Delta C_j \end{bmatrix}\right) \le \begin{bmatrix} \frac{1}{h_{i31}}F_j'^T F_j' h_{i31}P_{i1}B_j \widetilde{D}_i M_j M_j^T \widetilde{D}_i^T B_j^T P_{i1} \\ \frac{1}{h_{i32}}F_j'^T F_j' + h_{i32}P_{i2}^T B_j \widetilde{D}_i M_j M_j^T \widetilde{D}_i^T B_j^T P_{i2} \end{bmatrix}, \\
&He\left(\begin{bmatrix} P_{i1}\Delta B_j \widetilde{D}_i \Delta C_j \\ P_{i2}^T\Delta B_j \widetilde{D}_i \Delta C_j \end{bmatrix}\right) \le \begin{bmatrix} \frac{1}{h_{i41}}F_j''^T E_j^T M_j^T \widetilde{D}_i^T F_j'^T F_j' \widetilde{D}_i M_j E_j F_j'' \\ \frac{1}{h_{i42}}F_j''^T E_j^T M_j^T \widetilde{D}_i^T F_j'^T F_j' \widetilde{D}_i M_j E_j F_j'' \end{bmatrix} \\
&+\begin{bmatrix} h_{i41}P_{i1}M_j M_j^T P_{i1} \\ h_{i42}P_{i2}^T M_j M_j^T P_{i2} \end{bmatrix} \le \begin{bmatrix} \frac{1}{h_{i41}\cdot h_{i8}}F_j''^T F_j'' + h_{i41}P_{i1}M_j M_j^T P_{i1} \\ \frac{1}{h_{i42}\cdot h_{i8}}F_j''^T F_j'' + h_{i42}P_{i2}^T M_j M_j^T P_{i2} \end{bmatrix}, \\
&He\left(\begin{bmatrix} P_{i2}\widetilde{B}_i \Delta C_j \\ P_{i3}\widetilde{B}_i \Delta C_j \end{bmatrix}\right) \le \begin{bmatrix} \frac{1}{h_{i51}}F_j''^T F_j'' + h_{i51}P_{i2}\widetilde{B}_i M_j M_j^T \widetilde{B}_i^T P_{i2}^T \\ \frac{1}{h_{i52}}F_j''^T F_j'' + h_{i52}P_{i3}\widetilde{B}_i M_j M_j^T \widetilde{B}_i^T P_{i3} \end{bmatrix}, \\
&He\left(\begin{bmatrix} P_{i1}\Delta B_j \widetilde{C}_i \\ P_{i2}^T\Delta B_j \widetilde{C}_i \end{bmatrix}\right) \le \begin{bmatrix} \frac{1}{h_{i61}}\widetilde{C}_i^T F_j'^T F_j' \widetilde{C}_i + h_{i61}P_{i1}M_j M_j^T P_{i1} \\ \frac{1}{h_{i62}}\widetilde{C}_i^T F'^T_j F_j' \widetilde{C}_i + h_{i62}P_{i2}^T M_j M_j^T P_{i2} \end{bmatrix}, \\
&He\left(\begin{bmatrix} x^T(t)P_{i1}\Delta B_j \widetilde{D}_i e(t) \\ \widetilde{x}^T(t)P_{i2}^T\Delta B_j \widetilde{D}_{ii} e(t) \end{bmatrix}\right) \le \begin{bmatrix} \frac{1}{h_{i71}}e^T(t)\widetilde{D}_{ii}^T F_j'^T F_j' \widetilde{D}_i e(t) \\ \frac{1}{h_{i72}}e^T(t)\widetilde{D}_{ii}^T F_j'^T F_j' \widetilde{D}_{ii} e(t) \end{bmatrix} \\
&\qquad + \begin{bmatrix} h_{i71}x^T(t)P_{i1}M_j M_j^T P_{i1}x(t) \\ h_{i72}\widetilde{x}^T(t)P_{i2}^T M_j M_j^T P_{i2}\widetilde{x}(t) \end{bmatrix}.
\end{aligned} \tag{27}
$$

Then, it holds that

$$\dot{V}(\mathfrak{J}(t)) - \delta V(\mathfrak{J}(t)) = \mathfrak{J}_1^T(t)\Psi_{ji}\mathfrak{J}_1(t), \tag{28}$$

where $\mathfrak{J}_1(t) = [\mathfrak{J}^T(t), e^T(t)]^T$. It is apparent that $\dot{V}(\mathfrak{J}(t)) \le \delta V(\mathfrak{J}(t))$ from (19) for $\forall t \in [t_q, t_{q+1})$.

Combining the property of $V(\mathfrak{J}(t))$ at the instant $t = t_{q+1}$, it could be calculated that

$$V\left(\mathfrak{J}\left(t_{q+1}\right)\right) = V\left(\mathfrak{J}\left(t_{q+1}^-\right)\right) \le e^{\delta\left(t_{q+1}-t_q\right)} V\left(\mathfrak{J}\left(t_q\right)\right). \tag{29}$$

Case (ii): when mechanism (2) is triggered $m(\in \mathbb{N}^+)$ times during $[t_q, t_{q+1})$, the uncertain closed-loop system is converted into

$$\dot{\mathfrak{I}}(t) = \begin{cases} \overline{A}_{ji}\mathfrak{I}(t) + \overline{B}_{ji}e(t), & t \in [t_q, s_{k+1}), \\ \overline{A}_{jj}\mathfrak{I}(t) + \overline{B}_{jj}e(t), & t \in [s_{k+1}, t_{q+1}), \end{cases} \tag{30}$$

with the Lyapunov functional

$$V(\mathfrak{I}(t)) = \begin{cases} \mathfrak{I}^T(t)P_i\mathfrak{I}(t), & t \in [t_q, s_{k+1}), \\ \mathfrak{I}^T(t)P_j\mathfrak{I}(t), & t \in [t_q, s_{k+1}). \end{cases} \tag{31}$$

Executing the same procedure in Case (i) and employing condition (21), it could be obtained that

$$V(\mathfrak{I}(s_{k+1})) = V(\mathfrak{I}(s_{k+1}^-)) \leq \eta e^{\delta(s_{k+1}-t_q)} V(\mathfrak{I}(t_q)). \tag{32}$$

For $t \in [s_{k+1}, t_{q+1})$, by virtue of Lemma 1 (1), (2), Lemma 2, and (23), we have

$$\begin{aligned}
He\left(\begin{bmatrix} P_{j1}\Delta A_j \\ P_{j2}^T\Delta A_j \end{bmatrix}\right) &\leq \begin{bmatrix} \frac{1}{h_{j11}}F_j^TF_j + h_{j11}P_{j1}M_jM_j^TP_{j1} \\ \frac{1}{h_{j12}}F_j^TF_j + h_{j12}P_{j2}^TM_jM_j^TP_{j2} \end{bmatrix}, \\
He\left(\begin{bmatrix} P_{j1}\Delta B_j\widetilde{D}_jC_j \\ P_{j2}^T\Delta B_j\widetilde{D}_jC_j \end{bmatrix}\right) &\leq \begin{bmatrix} \frac{1}{h_{j21}}C_j^T\widetilde{D}_j^TF_j'^TF_j'\widetilde{D}_jC_j + h_{j21}P_{j1}M_jM_j^TP_{j1} \\ \frac{1}{h_{j22}}C_j^T\widetilde{D}_j^TF_j'^TF_j'\widetilde{D}_jC_j + h_{j22}P_{j2}^TM_jM_j^TP_{j2} \end{bmatrix}, \\
He\left(\begin{bmatrix} P_{j1}B_j\widetilde{D}_j\Delta C_j \\ P_{j2}^TB_j\widetilde{D}_j\Delta C_j \end{bmatrix}\right) &\leq \begin{bmatrix} \frac{1}{h_{j31}}F_j''^TF_j'' + h_{j31}P_{j1}B_j\widetilde{D}_jM_jM_j^T\widetilde{D}_j^TB_j^TP_{j1} \\ \frac{1}{h_{j32}}F_j''^TF_j'' + h_{j32}P_{j2}^TB_j\widetilde{D}_jM_jM_j^T\widetilde{D}_j^TB_j^TP_{j2} \end{bmatrix}, \\
He\left(\begin{bmatrix} P_{j1}\Delta B_j\widetilde{D}_j\Delta C_j \\ P_{j2}^T\Delta B_j\widetilde{D}_j\Delta C_j \end{bmatrix}\right) &\leq \begin{bmatrix} \frac{1}{h_{j41}\cdot h_{j8}}F_j''^TF_j'' + h_{j41}P_{j1}M_jM_j^TP_{j1} \\ \frac{1}{h_{j42}\cdot h_{j8}}F_j''^TF_j'' + h_{j42}P_{j2}^TM_jM_j^TP_{j2} \end{bmatrix}, \\
He\left(\begin{bmatrix} P_{j2}\widetilde{B}_j\Delta C_j \\ P_{j3}\widetilde{B}_j\Delta C_j \end{bmatrix}\right) &\leq \begin{bmatrix} \frac{1}{h_{j51}}F_j''^TF_j'' + h_{j51}P_{j2}\widetilde{B}_jM_jM_j^T\widetilde{B}_j^TP_{j2}^T \\ \frac{1}{h_{j52}}F_j''^TF_j'' + h_{j52}P_{j3}\widetilde{B}_jM_jM_j^T\widetilde{B}_j^TP_{j3} \end{bmatrix}, \\
He\left(\begin{bmatrix} P_{j1}\Delta B_j\widetilde{C}_j \\ P_{j2}^T\Delta B_j\widetilde{C}_j \end{bmatrix}\right) &\leq \begin{bmatrix} \frac{1}{h_{j61}}\widetilde{C}_j^TF_j'^TF_j'\widetilde{C}_j + h_{i61}P_{j1}M_jM_j^TP_{j1} \\ \frac{1}{h_{j62}}\widetilde{C}_j^TF_j'^TF_j'\widetilde{C}_j + h_{j62}P_{j2}^TM_jM_j^TP_{j2} \end{bmatrix}, \\
He\left(\begin{bmatrix} x^T(t)P_{j1}\Delta B_j\widetilde{D}_je(t) \\ \widetilde{x}^T(t)P_{j2}^T\Delta B_j\widetilde{D}_je(t) \end{bmatrix}\right) &\leq \begin{bmatrix} \frac{1}{h_{j71}}e^T(t)\widetilde{D}_j^TF_j'^TF_j'\widetilde{D}_je(t) \\ \frac{1}{h_{j72}}e^T(t)\widetilde{D}_j^TF_j'^TF_j'\widetilde{D}_je(t) \end{bmatrix} \\
&\quad + \begin{bmatrix} h_{j71}x^T(t)P_{j1}M_jM_j^TP_{j1}x(t) \\ h_{j72}\widetilde{x}^T(t)P_{j2}^TM_jM_j^TP_{j2}\widetilde{x}(t) \end{bmatrix}.
\end{aligned} \tag{33}$$

Then, we derive

$$\begin{aligned}&\dot{V}(\mathfrak{J}(t))+\gamma V(\mathfrak{J}(t))=2\mathfrak{J}^{T}(t)P_{j}\dot{\mathfrak{J}}(t)+\gamma\mathfrak{J}^{T}(t)P_{j}\mathfrak{J}(t)\\&\leq 2\mathfrak{J}^{T}(t)P_{j}\left(\overline{A}_{j,j}\mathfrak{J}(t)+\overline{B}_{j,j}e(t)\right)+\gamma\mathfrak{J}^{T}(t)P_{j}\mathfrak{J}(t)\\&\quad-e(t)^{T}\Omega_{j}e(t)+\varepsilon x^{T}(t)C_{j}^{T}\Omega_{j}C_{j}x(t)\\&=\mathfrak{J}_{1}^{T}(t)\Psi_{jj}\mathfrak{J}_{1}(t).\end{aligned}\tag{34}$$

This shadows

$$\dot{V}(\mathfrak{J}(t))\leq-\gamma V(\mathfrak{J}(t)),\quad\forall t\in[s_{k+1},t_{q+1}),\tag{35}$$

because of (20). Then, the following inequation can be guaranteed with the nature of $V(\mathfrak{J}(t))$ at the instant $t=t_{q+1}$:

$$\begin{aligned}V\left(\mathfrak{J}\left(t_{q+1}\right)\right)&=V\left(\mathfrak{J}\left(t_{q+1}^{-}\right)\right)\\&\leq e^{-\gamma\left(t_{q+1}-s_{k+1}\right)}V\left(\mathfrak{J}\left(s_{k+1}\right)\right)\\&\leq\eta e^{-\gamma\left(t_{q+1}-s_{k+1}\right)}V\left(\mathfrak{J}\left(s_{k+1}^{-}\right)\right)\\&\leq\eta e^{-\gamma\left(t_{q+1}-s_{k+1}\right)+\delta\left(s_{k+1}-t_{q}\right)}V\left(\mathfrak{J}\left(t_{q}\right)\right).\end{aligned}\tag{36}$$

As a consequence, it can be summarized from (29) and (36) that

$$V\left(\mathfrak{J}\left(t_{q+1}\right)\right)\leq\eta e^{-\gamma T_{\downarrow}\left(t_{q},t_{q+1}\right)+\delta T_{\uparrow}\left(t_{q},t_{q+1}\right)}V\left(\mathfrak{J}\left(t_{q}\right)\right).\tag{37}$$

Define $\widehat{\mathfrak{n}}_{\sigma}(0,t)$ as the switch time of controller in $[0,t)$, which is not more than the switch time of system $\mathfrak{n}_{\sigma}(0,t)$. Accordingly for $\forall t>0$,

$$\begin{aligned}V(\mathfrak{J}(t))&\leq\eta^{\widehat{\mathfrak{n}}_{\sigma}(0,t)}e^{-\gamma T_{\downarrow}(0,t)+\delta T_{\uparrow}(0,t)}V(\mathfrak{J}(0))\\&\leq\eta^{\mathfrak{n}_{\sigma}(0,t)}e^{-\gamma t}e^{(\gamma+\delta)T\mathfrak{n}_{\sigma}(0,t)}V(\mathfrak{J}(0))\\&\leq\eta^{\left(\mathfrak{n}_{0}+t/\tau_{a}\right)}e^{-\gamma t}e^{(\gamma+\delta)T\left(\mathfrak{n}_{0}+t/\tau_{a}\right)}V(\mathfrak{J}(0))\\&=\left(\eta e^{(\gamma+\delta)T}\right)^{\mathfrak{n}_{0}}e^{\left(\ln\mu+(\gamma+\delta)T/\tau_{a}-\gamma\right)t}V(\mathfrak{J}(0)).\end{aligned}\tag{38}$$

According to condition (25), there must exist a constant $\lambda>0$ such that $V(\mathfrak{J}(t))\leq ce^{-2\lambda(t-0)}V(\mathfrak{J}(0))$. Considering inequalities $V(\mathfrak{J}(t))\geq\lambda_{1}\|\mathfrak{J}(t)\|^{2}$ and $V(\mathfrak{J}(0))\leq\lambda_{2}\|\mathfrak{J}(0)\|^{2}$, we can derive $\|\mathfrak{J}(t)\|\leq\kappa e^{-\lambda(t-0)}\|\mathfrak{J}(0)\|$, where $c=(\eta e^{(\gamma+\delta)T})^{\mathfrak{n}_{0}}$, $\lambda_{1}=\min_{i\in\mathscr{L}}\lambda_{\min}(P_{i})$, $\lambda_{2}=\max_{i\in\mathscr{L}}\lambda_{\max}(P_{i})$, and $\kappa=\sqrt{c\lambda_{2}/\lambda_{1}}$. Combined with Definition 1, system (9) is exponentially stable, which accomplishes the proof.

Remark 3. Motivated by the analysis of Case (ii), apart from a limited number of discontinuities, the error $e(t)$ remains continuous. To be specific, $e(t)$ could be piecewise continuous and bounded over a subinterval $[s_{k+1},t_{q+1})$ provided that the lower bound exists on T, which would be clarified in Theorem 3. Furthermore, the Lyapunov function $V(\mathfrak{J}(t))$, whose structure determines that it has the property of global decay, is also continuous except for a limited number of discontinuities.

Remark 4. For $t\in(t_{q}^{-},t_{q+1}^{+})$, the inequality $\widehat{\mathfrak{n}}_{\sigma}(t_{q}^{-},t_{q+1}^{+})\leq\mathfrak{n}_{\sigma}(t_{q}^{-},t_{q+1}^{+})$ can be illuminated in three cases as indicated in Figure 2. Case (a), Case (b), and Case (c) correspond to the above Case (i), Case (ii) with $m=1$, and Case (ii) with $m>1$, respectively. As for Case (i), it is obvious that $\widehat{\mathfrak{n}}_{\sigma}(t_{q}^{-},t_{q+1}^{+})=0\leq\mathfrak{n}_{\sigma}(t_{q}^{-},t_{q+1}^{+})=2$. Case (b) owns one triggered moment $s_{k+1}\in[t_{q},t_{q+1})$, at which (3) receives the measurement date to change its pattern, meanwhile updating its input. Hence, $\widehat{\mathfrak{n}}_{\sigma}(t_{q}^{-},t_{q+1}^{+})=1\leq\mathfrak{n}_{\sigma}(t_{q}^{-},t_{q+1}^{+})=2$. Although m triggered moments in Case (c) over the interval $[t_{q},t_{q+1})$, the controller pattern at the first triggered instant s_{k+1} is switched only once and the control input of the controller is only updated without switching its pattern at the other $m-1$ triggered moments. Thus, $\widehat{\mathfrak{n}}_{\sigma}(t_{q}^{-},t_{q+1}^{+})=1\leq\mathfrak{n}_{\sigma}(t_{q}^{-},t_{q+1}^{+})=2$ is still valid. In other words, the controller can be perceived as a postponed version of the uncertain system, more or less, in terms of switching. The switch time of the controller, consequently, will not surpass the switch time of the uncertain system.

3.2. Controller Design. In this section, the event-triggered scheme and the dynamic output-feedback controller for uncertain system (9) are codesigned by combining the above stability analysis. To calculate the control gains, we introduce notations as follows:

$$\begin{aligned}\mathscr{A}_{j}&=\begin{bmatrix}A_{j}&0\\0&0\end{bmatrix},\\\mathscr{B}_{j}&=\begin{bmatrix}0&B_{j}\\I&0\end{bmatrix},\\\mathscr{C}_{j}&=\begin{bmatrix}0&I\\C_{j}&0\end{bmatrix},\\\mathrm{E}&=\begin{bmatrix}0\\I\end{bmatrix},\\\mathscr{K}_{i}&=\begin{bmatrix}\widetilde{A}_{i}&\widetilde{B}_{i}\\\widetilde{C}_{i}&\widetilde{D}_{i}\end{bmatrix},\end{aligned}\tag{39}$$

which implies

$$\begin{aligned}\widetilde{A}_{i}&=\mathscr{A}_{j}+\mathscr{B}_{j}\mathscr{K}_{i}\mathscr{C}_{j},\\\widetilde{B}_{i}&=\mathscr{B}_{j}\mathscr{K}_{i}\mathscr{E},\ \forall i,\quad j\in\mathscr{L}.\end{aligned}\tag{40}$$

Theorem 2. *Given scalars $\gamma>0$, $\delta>0$, $\eta\geq1$, $T>0$, $\epsilon>0$, and $\varsigma>0$, uncertain system (9) is globally exponentially stable for any $\sigma(t)$ if Assumption 1 holds and there exist matrices $P_{i}>0$, $\Omega_{i}>0$, R_{i}, S_{i}, $\mathscr{T}_{ic}$, $\exists h_{iab}>0$ $(a=1,2,\ldots\infty,\ 7;b=1,2;c=1,2,3,4)$, $\exists h_{i8}>0$, and $\forall i\in\mathscr{L}$ such that (21)–(25) and*

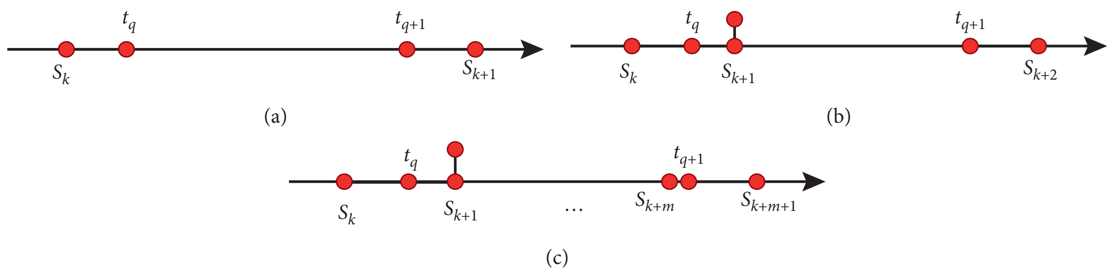

Figure 2: Moments of event triggering and system switching. (a) Case (i), (b) Case (ii), $m = 1$, (c) Case (ii), $m > 1$.

$$\Phi_{ji} = \begin{bmatrix} \Phi_{ji}^{1} & \mathscr{B}_j S_i \mathscr{E} & 0 & P_j \mathscr{B}_j - \mathscr{B}_j R_i + \varsigma \mathscr{C}_j^T S_i^T \\ \star & -\Omega_i & \mathscr{T}_{i3}^T & \varsigma \mathscr{E}^T S_i^T \\ \star & \star & -\left(\dfrac{h_{i71} \cdot h_{i72}}{h_{i71} + h_{i72}}\right) I & 0 \\ \star & \star & \star & -\varsigma R_i - \varsigma R_i^T \end{bmatrix} < 0, \quad \forall i \neq j \in \mathscr{L}, \tag{41}$$

$$\Phi_{jj} = \begin{bmatrix} \Phi_{jj}^{1} & \mathscr{B}_j S_j \mathscr{E} & 0 & P_j \mathscr{B}_j - \mathscr{B}_j R_j + \varsigma \mathscr{C}_j^T S_j^T \\ \star & -\Omega_j & \mathscr{T}_{j3}^T & \varsigma \mathscr{E}^T S_j^T \\ \star & \star & -\left(\dfrac{h_{j71} \cdot h_{j72}}{h_{j71} + h_{j72}}\right) I & 0 \\ \star & \star & \star & -\varsigma R_j - \varsigma R_j^T \end{bmatrix} < 0, \quad \forall j \in \mathscr{L}, \tag{42}$$

where

$$\begin{aligned}
\Phi_{ji}^{1} &= \mathrm{He}\left(P_i \mathscr{A}_j + \mathscr{B}_j S_i \mathscr{C}_j\right) + \overline{\mathscr{Q}}_i - \delta P_i, \\
\Phi_{jj}^{1} &= \mathrm{He}\left(P_i \mathscr{A}_j + \mathscr{B}_j S_i \mathscr{C}_j\right) + \overline{\mathscr{Q}}_j + \gamma P_j, \\
\overline{\mathscr{Q}}_i &= \begin{bmatrix} \overline{\mathscr{Q}}_{i1} & \overline{\mathscr{Q}}_{i2} \\ \overline{\mathscr{Q}}_{i3} & \overline{\mathscr{Q}}_{i4} \end{bmatrix}, \forall i \in \mathscr{L}, \\
\overline{\mathscr{Q}}_{i1} &= \left(h_{i11} + h_{i21} + h_{i71} + h_{i41}\right) P_{i1} M_j M_j^T P_{i1} + h_{i31} P_{i1} B_j \mathscr{T}_{i4} \mathscr{T}_{i4}^T B_j^T P_{i1} + h_{i51} P_{i2} \mathscr{T}_{i1} \mathscr{T}_{i1}^T P_{i2}^T \\
&\quad + \frac{1}{h_{i11}} F_j^T F_j + \left(\frac{1}{h_{i31}} + \frac{1}{h_{i51}} + \frac{1}{h_{i41} h_{i8}}\right) F_j''^T F_j'' + \frac{1}{h_{i21}} C_j^T \mathscr{T}_{i3}^T \mathscr{T}_{i3} C_j + \epsilon C_j^T \Omega_i C_j, \\
\overline{\mathscr{Q}}_{i3} &= \left(h_{i12} + h_{i22} + h_{i42}\right) P_{i2}^T M_j M_j^T P_{i2} + h_{i32} P_{i2}^T B_j \mathscr{T}_{i4} \mathscr{T}_{i4}^T B_j^T P_{i2} + h_{i52} P_{i3} \mathscr{T}_{i1} \mathscr{T}_{i1}^T P_{i3} \\
&\quad + \frac{1}{h_{i12}} F_j^T F_j + \left(\frac{1}{h_{i32}} + \frac{1}{h_{i52}} + \frac{1}{h_{i42} h_{i8}}\right) F_j''^T F_j'' + \frac{1}{h_{i22}} C_j^T \mathscr{T}_{i3}^T \mathscr{T}_{i3} C_j, \\
\overline{\mathscr{Q}}_{i2} &= \frac{1}{h_{i61}} \mathscr{T}_{i2}^T \mathscr{T}_{i2} + h_{i61} P_{i1} M_j M_j^T P_{i1}, \\
\overline{\mathscr{Q}}_{i4} &= \frac{1}{h_{i62}} \mathscr{T}_{i2}^T \mathscr{T}_{i2} + \left(h_{i62} + h_{i72}\right) P_{i2}^T M_j M_j^T P_{i2}.
\end{aligned} \tag{43}$$

Furthermore, the controller gain matrices are devised as $\mathscr{K}_i = R_i^{-1}S_i, \forall i \in \mathscr{L}$.

Proof. On the grounds of Theorem 1 and supposing that $\widetilde{B}_i = \mathscr{T}_{i1}M_j^{-1}, \widetilde{C}_i = F'j_{-1}\mathscr{T}_{i2}, \widetilde{D}_i = F'j_{-1}\mathscr{T}_{i3} = \mathscr{T}_{i4}M_j^{-1}$, such that the controller gains are presented as

$$\mathscr{K}_i = R_i^{-1}S, \quad = \begin{bmatrix} \widetilde{A}_i & \mathscr{T}_{i1}M_j^{-1} \\ F'j_{-1}\mathscr{T}_{i2} & F'j_{-1}\mathscr{T}_{i3} \end{bmatrix}, \quad \forall i \in \mathscr{L}. \tag{44}$$

define

$$X_{ji} = R_i^{-1}\left[S_i\mathscr{C}_j \ \ S_i\mathscr{E} \ \ 0 \right], \quad Y_{ji} = \left[\mathscr{B}_j^T P_i - R_i^T \mathscr{B}_j^T \ \ 0 \ \ 0 \right]. \tag{45}$$

Then,

$$X_{ji}^T Y_{ji} = \begin{bmatrix} \left(R_i^{-1}S_i\mathscr{C}_j\right)^T \mathscr{B}_j^T P_i - \left(S_i\mathscr{C}_j\right)^T \mathscr{B}_j^T & 0 & 0 \\ \left(R_i^{-1}S_i\mathscr{E}\right)^T \mathscr{B}_j^T P_i - \left(S_i\mathscr{E}\right)^T \mathscr{B}_j^T & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$
$$Y_{ji}^T X_{ji} = \begin{bmatrix} P_i\mathscr{B}_j R_i^{-1}S_i\mathscr{C}_j - \mathscr{B}_j S_i\mathscr{C}_j & P_i\mathscr{B}_j R_i^{-1}S_i\mathscr{E} - \mathscr{B}_j S_i\mathscr{E} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{46}$$

Combining with Lemma 3 and (41), one derives

$$\begin{bmatrix} \Phi_{ji}^1 & \mathscr{B}_j S_i \mathscr{E} & 0 \\ \star & -\Omega_i & \mathscr{T}_{i3}^T \\ \star & \star & -\left(\dfrac{h_{i71}\cdot h_{i72}}{h_{i71}+h_{i72}}\right)I \end{bmatrix} + X_{ji}^T Y_{ji} + Y_{ji}^T X_{ji} < 0, \tag{47}$$

which is of equivalence to

$$\begin{bmatrix} He\left(P_i\mathscr{A}_j + \mathscr{B}_j\mathscr{K}_i\mathscr{C}_j\right) + \overline{\mathscr{Q}}_i - \delta P_i & P_i\mathscr{B}_j\mathscr{K}_i\mathscr{E} & 0 \\ \star & -\Omega_i & \mathscr{T}_{i3}^T \\ \star & \star & -\left(\dfrac{h_{i71}\cdot h_{i72}}{h_{i71}+h_{i72}}\right)I \end{bmatrix} < 0. \tag{48}$$

In accordance with (40) and above definition, one obtains

$$\begin{bmatrix} \Psi_{ji}^1 & P_i\overline{\overline{B}}_{ji} & 0 \\ \star & -\Omega_i & \widetilde{D}_i^T F_j'^T \\ \star & \star & -\left(\dfrac{h_{i71}\cdot h_{i72}}{h_{i71}+h_{i72}}\right)I \end{bmatrix} < 0. \tag{49}$$

On the basis of Lemma 2, one realizes that (41) ensures condition (19). Under the same technique as above, (20) is guaranteed by condition (42). In consequence, the conclusion can be obtained by Theorem 1.

Remark 5. By comparison with [14], the average dwell-time switching signals can be used to handle quick switching issues and are more widespread and pragmatic without minimum dwell-time restrictions [16]. And it is necessary to point out that the related works investigate the asynchronous stabilization via the strategy of multiple Lyapunov function. For instance, the Lyapunov function is delegated as $V(x(t)) = x^T(t)P_j x(t)$ for the synchronous course as well as $V(x(t)) = x^T(t)P_{ji}x(t)$ for the asynchronous period. The variable number deduced by the multiple Lyapunov functional matrices is $n(n+1)/2\mathfrak{n} + n(n+1)/2\mathfrak{n}(n-1)$, where $\mathfrak{n}$ represents the quantity of the Lyapunov functional matrix. Difference in these works, a piecewise Lyapunov functional, is constructed in this article, which only rests with the controller pattern. Under this choice, what delighted us is that the variable number is $n(n+1)/2\mathfrak{n}$. It is evident that the computation complexity can be effectually decreased, specifically for systems with a large dimensionality or plenty of modes. Most recently, great achievements have been dedicated to inquiry into the switched systems of state parameters uncertainty and input parameters uncertainty. However, as yet, little attention has been attracted to the stabilization of frequently switched systems that are equipped with state, input, and especially measured output parameter uncertainties, which motivates this work.

In recent years, with the rise of large-scale networks, the calculation of performance criteria described by matrix equations or matrix inequalities has attracted much attention. In fact, in control systems, the investigation of many important characteristics such as stability and controllability can be transformed into the exploration of the constrained solution of the corresponding nonlinear matrix equation. The traditional method of solving matrix equations or matrix inequality is based on an iterative numerical algorithm. This kind of algorithm may meet the requirements of real-time for matrix equations or matrix inequality with small dimensions. Once the matrix dimension reaches a certain order of magnitude, it is generally difficult to find a solution method suitable for real-time large-scale applications. Theorems 1 and 2 in this paper are in the form of matrix inequalities, and how to systematically work out the problem of solving a high-dimensional matrix caused by the huge dimension of the network determines the scope of application of Theorems 1 and 2 to a certain extent.

3.3. Excluding the Zeno Behavior. In this section, the existence of T's lower definite bound is demonstrated, which thus eliminates Zeno behavior. Combining with (1) and (3) under event-triggered strategy (2), the uncertain closed-loop system is inferred as

$$\begin{aligned}\dot{x}(t) = &\left(A_{\sigma(t)} + \Delta A_{\sigma(t)}\right)x(t) + \left(B_{\sigma(t)} + \Delta B_{\sigma(t)}\right)\\ &\widetilde{D}_{\sigma(s_k)}\left(y(t) + e_{s_k}(t)\right)\\ &+\left(B_{\sigma(t)} + \Delta B_{\sigma(t)}\right)\widetilde{C}_{\sigma(s_k)}\widetilde{x}(t), \quad t \in [s_k, s_{k+1}).\end{aligned} \tag{50}$$

Theorem 3. *For system (50), suppose Assumption 1 holds. Given a scalar $\beta(>0)$ satisfying $\max\{\|x(t)\|, \|\widetilde{x}(t)\|\} \le \beta\|y(t)\|$, the adjacent event interval would be lower bounded via constant $T_1(>0)$ conforming to*

$$T_1 = \min\left\{T, \frac{1}{\overline{\Lambda} - \widehat{\Lambda}}\ln\left(1 + \frac{(\overline{\Lambda} - \widehat{\Lambda})\sqrt{\epsilon}}{\overline{\Lambda} + \widehat{\Lambda}\sqrt{\epsilon}}\right)\right\}, \tag{51}$$

where T and ϵ are specified in (2) and

$$\begin{aligned}\overline{\Lambda} = \max_{i,j\in\mathscr{L}} \frac{1}{\lambda_{\min}\sqrt{\Omega_i}} &\left[\begin{pmatrix}\left\|\sqrt{\Omega_i}C_jA_j\right\| + \left\|\sqrt{\Omega_i}C_jM_j\right\|\cdot\left\|F_j\right\| + \left\|\sqrt{\Omega_i}M_j\right\|\cdot\left\|F_j''A_j\right\| \\ +\left\|\sqrt{\Omega_i}M_j\right\|\cdot\left\|F_j''M_j\right\|\cdot\left\|F_j\right\| + \left\|\sqrt{\Omega_i}C_jB_j\widetilde{C}_i\right\| \\ +\left\|\sqrt{\Omega_i}C_jM_j\right\|\cdot\left\|F_j'\widetilde{C}_i\right\| + \left\|\sqrt{\Omega_i}M_j\right\|\cdot\left\|F_j''B_j\widetilde{C}_i\right\| \\ +\left\|\sqrt{\Omega_i}M_j\right\|\cdot\left\|F_j''M_j\right\|\cdot\left\|F_j'\widetilde{C}_i\right\|\end{pmatrix}\beta, \right. \\ &\quad +\left\|\sqrt{\Omega_i}C_jB_j\widetilde{D}_i\right\| + \left\|\sqrt{\Omega_i}C_jM_j\right\|\cdot\left\|F_j'\widetilde{D}_i\right\| + \left\|\sqrt{\Omega_i}M_j\right\|\cdot\left\|F_j''B_j\widetilde{D}_i\right\| \\ &\left.\quad +\left\|\sqrt{\Omega_i}M_j\right\|\cdot\left\|F_j''M_j\right\|\cdot\left\|F_j'D_j\right\|\right] \\ \widehat{\Lambda} = \max_{i,j\in\mathscr{L}} &\left\|\sqrt{\Omega_i}C_jB_j\widetilde{D}_i\sqrt{\Omega_i}^{-1}\right\| + \left\|\sqrt{\Omega_i}C_jM_j\right\|\cdot\left\|F_j'\widetilde{D}_i\sqrt{\Omega_i}^{-1}\right\| \\ &+\left\|\sqrt{\Omega_i}M_j\right\|\cdot\left\|F_j''B_j\widetilde{D}_i\sqrt{\Omega_i}^{-1}\right\| + \left\|\sqrt{\Omega_i}M_j\right\|\cdot\left\|F_j''M_j\right\|\cdot\left\|F_j''B_j\widetilde{D}_i\sqrt{\Omega_i}^{-1}\right\|.\end{aligned} \tag{52}$$

Proof. Suppose that $\sigma(s_k) = i$, and construct an auxiliary function $\mathfrak{R}(t) = \|\sqrt{\Omega_i}e_{s_k}(t)\|/\|\sqrt{\Omega_i}y(t)\|$ for $t \in [s_k, s_{k+1})$. Event scheme (2) would be triggered only if $\mathfrak{R}(t) = \sqrt{\epsilon}$ or $t = s_k + T$. When the latter holds, $s_{k+1} - s_k = T \ge T_1$. Obviously, that is a bromidic case. Next, we will be dedicated to testify that $s_{k+1} - s_k \ge T_1$ holds when s_{k+1} meets regulation $\mathfrak{R}(t) = \sqrt{\epsilon}$. The proof is segmented into two steps as follows.

Step 1. When the system does not switch between the successive triggering moments s_k and s_{k+1}, $\sigma(t) = \sigma(s_k) = i$, $\forall t \in [s_k, s_{k+1})$, correspondingly, uncertain system (50) could be transformed into

$$\begin{aligned}\dot{x}(t) = &(A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)\widetilde{D}_i\left(y(t) + e_{s_k}(t)\right)\\ &+ (B_i + \Delta B_i)\widetilde{C}_i\widetilde{x}(t), \ t \in [s_k, s_{k+1}).\end{aligned} \tag{53}$$

Differentiating the function $\mathfrak{R}(t)$, one gets

$$\begin{aligned}\dot{\mathfrak{R}}(t) &\leq \frac{\left\|\sqrt{\Omega_i}\,\dot{e}_{s_k}(t)\right\|}{\left\|\sqrt{\Omega_i}\,\dot{y}(t)\right\|} + \frac{\left\|\sqrt{\Omega_i}\,\dot{y}(t)\right\| \cdot \left\|\sqrt{\Omega_i}\,\dot{e}_{s_k}(t)\right\|}{\left\|\sqrt{\Omega_i}\,\dot{y}(t)\right\|^2} \\ &= (1+\mathfrak{R}(t))\frac{\left\|\sqrt{\Omega_i}\,(C_i+\Delta C_i)\dot{x}(t)\right\|}{\left\|\sqrt{\Omega_i}\,\dot{y}(t)\right\|} \leq \\ &\frac{1+\mathfrak{R}(t)}{\left\|\sqrt{\Omega_i}\,\dot{y}(t)\right\|}\begin{pmatrix} \left\|\sqrt{\Omega_i}\,(C_i+\Delta C_i)(A_i+\Delta A_i)\right\| \cdot \|x(t)\| \\ + \left\|\sqrt{\Omega_i}\,(C_i+\Delta C_i)(B_i+\Delta B_i)\widetilde{C}_i\right\| \cdot \|\widetilde{x}(t)\| \\ + \left\|\sqrt{\Omega_i}\,(C_i+\Delta C_i)(B_i+\Delta B_i)\widetilde{D}_i\right\| \cdot \|y(t)\| \\ + \left\|\sqrt{\Omega_i}\,(C_i+\Delta C_i)(B_i+\Delta B_i)\widetilde{D}_i e_{s_k}(t)\right\| \end{pmatrix}. \end{aligned} \tag{54}$$

Since

$$\left\|E_i\right\| = \sqrt{\lambda_{\max}\left(E_i^T E_i\right)} \leq \sqrt{\lambda_{\max}(I)} = 1 \tag{55}$$

$$\|x(t)\|, \|\widetilde{x}(t)\| \leq \|\beta y(t)\|, \tag{56}$$

it satisfies

$$\begin{aligned}\dot{\mathfrak{R}}(t) &\leq (1+\mathfrak{R}(t))\left\{\begin{matrix} \dfrac{1}{\lambda_{\min}\sqrt{\Omega_i}}\left[\begin{matrix}\begin{pmatrix} \left\|\sqrt{\Omega_i}\,C_iA_i\right\| + \left\|\sqrt{\Omega_i}\,C_iM_i\right\| \cdot \|F\|_i + \lfloor\sqrt{\Omega_i}\,M_i\rfloor \cdot \left\|F_j'A_i\right\| \\ + \left\|\sqrt{\Omega_i}\,M_i\right\| \cdot \left\|F_j''M_i\right\| \cdot \left\|F_i\right\| + \left\|\sqrt{\Omega_i}\,C_iB_i\widetilde{C}_i\right\| \\ + \left\|\sqrt{\Omega_i}\,C_iM_i\right\| \cdot \left\|F_j'\widetilde{C}_i\right\| + \left\|\sqrt{\Omega_i}\,M_i\right\| \cdot \left\|F_j''B_i\widetilde{C}_i\right\| + \left\|\sqrt{\Omega_i}\,M_i\right\| \cdot \left\|F_j''M_i\right\| \cdot \left\|F_j'\widetilde{C}_i\right\| \end{pmatrix}\beta \\ + \left\|\sqrt{\Omega_i}\,C_iB_i\widetilde{D}_i\right\| + \left\|\sqrt{\Omega_i}\,C_iM_i\right\| \cdot \left\|F_j'\widetilde{D}_i\right\| + \left\|\sqrt{\Omega_i}\,M_i\right\| \cdot \left\|F_j''B_i\widetilde{D}_i\right\| + \left\|\sqrt{\Omega_i}\,M_i\right\| \cdot \left\|F_j''M_i\right\| \cdot \left\|F_j'\widetilde{D}_i\right\| \end{matrix}\right] \\ + \left\|\sqrt{\Omega_i}\,C_iB_i\widetilde{D}_i\sqrt{\Omega_i}^{-1}\right\| + \left\|\sqrt{\Omega_i}\,C_iM_i\right\| \cdot \left\|F_j'\widetilde{D}_i\sqrt{\Omega_i}^{-1}\right\| \\ + \left\|\sqrt{\Omega_i}\,M_i\right\| \cdot \left\|F_j''B_i\widetilde{D}_i\sqrt{\Omega_i}^{-1}\right\| + \left\|\sqrt{\Omega_i}\,M_i\right\| \cdot \left\|F_j''M_i\right\| \cdot \left\|F_j'\widetilde{D}_i\sqrt{\Omega_i}^{-1}\right\| \cdot \mathfrak{R}(t) \end{matrix}\right\} \\ &\leq (1+\mathfrak{R}(t))\left(\Lambda_a + \Lambda_b\mathfrak{R}(t)\right) \leq (1+\mathfrak{R}(t))\left(\overline{\Lambda} + \widehat{\Lambda}\mathfrak{R}(t)\right), \end{aligned} \tag{57}$$

where

$$\begin{aligned}\Lambda_a &= \max_{i\in\mathscr{L}} \frac{1}{\lambda_{\min}\sqrt{\Omega_i}} \\ &\left[\begin{matrix}\begin{pmatrix} \left\|\sqrt{\Omega_i}\,C_iA_i\right\| + \left\|\sqrt{\Omega_i}\,C_iM_i\right\| \cdot \|F\|_i + \left\|\sqrt{\Omega_i}\,C_iM_i\right\| \cdot \left\|F_j''A_i\right\| \\ + \left\|\sqrt{\Omega_i}\,M_i\right\| \cdot \left\|F_j''M_i\right\| \cdot \left\|F_i\right\| + \left\|\sqrt{\Omega_i}\,C_iB_i\widetilde{C}_i\right\| + \left\|\sqrt{\Omega_i}\,C_iM_i\right\| \cdot \left\|F_i'\widetilde{C}_i\right\| \\ + \left\|\sqrt{\Omega_i}\,M_i\right\| \cdot \left\|F_j''B_i\widetilde{C}_i\right\| + \left\|\sqrt{\Omega_i}\,M_i\right\| \cdot \left\|F_j''M_i\right\| \cdot \left\|F_i'\widetilde{C}_i\right\| \end{pmatrix}\beta \\ + \left\|\sqrt{\Omega_i}\,C_iB_i\widetilde{D}_i\right\| + \left\|\sqrt{\Omega_i}\,C_iM_i\right\| \cdot \left\|F_i'\widetilde{D}_i\right\| + \left\|\sqrt{\Omega_i}\,M_i\right\| \cdot \left\|F_j''B_i\widetilde{D}_i\right\| + \left\|\sqrt{\Omega_i}\,M_i\right\| \cdot \left\|F_j''M_i\right\| \cdot \left\|F_i'\widetilde{D}_i\right\| \end{matrix}\right], \\ \Lambda_b &= \max_{i\in\mathscr{L}} = \left\|\sqrt{\Omega_i}\,C_iB_i\widetilde{D}_i\sqrt{\Omega_i}^{-1}\right\| + \left\|\sqrt{\Omega_i}\,C_iM_i\right\| \cdot \left\|F_i'\widetilde{D}_i\sqrt{\Omega_i}^{-1}\right\| \\ &+ \left\|\sqrt{\Omega_i}\,M_i\right\| \cdot \left\|F_j''B_i\widetilde{D}_i\sqrt{\Omega_i}^{-1}\right\| + \left\|\sqrt{\Omega_i}\,M_i\right\| \cdot \left\|F_i''M_i\right\| \cdot \left\|F_i'\widetilde{D}_i\sqrt{\Omega_i}^{-1}\right\|. \end{aligned} \tag{58}$$

By introducing

$$\dot{\nu}(t) = (1 + \nu(t))\,(\overline{\Lambda} + \widehat{\Lambda}\nu(t)), \tag{59}$$

inequality $\mathfrak{R}(t) \leq \nu(t)$ holds apparently for the initial value $\mathfrak{R}(s_k) = \nu(s_k) = 0$ from comparison Lemma 4.

As t adds from s_k to s_{k+1}, $\mathfrak{R}(t)$ varies and increases from 0 to $\sqrt{\epsilon}$. By integrating both sides of (59) and supposing (2) is triggered at $t = T_1 + s_k$, the inequality $\nu(T_1 + s_k) \geq \sqrt{\epsilon}$ can be derived. Subsequently, the lower bound of $s_{k+1} - s_k$ can be calculated to be $1/\overline{\Lambda} - \widehat{\Lambda} \ln (1 + ((\overline{\Lambda} - \widehat{\Lambda})\sqrt{\epsilon}/\overline{\Lambda} + \widehat{\Lambda}\sqrt{\epsilon}))$, which yields expressions (51).

Step 2. When the system switches $n(\in \mathbb{N}^+)$ between the successive triggering moments s_k and s_{k+1}, one could suppose that $s_k < t_{q+1} < \cdots < t_{q+n} \leq s_{k+1}$. Considering the subinterval $[s_k, t_{q+1})$, system (50) has the identical form as (53); hence, the following inequality is deduced for $t \in [s_k, t_{q+1})$:

$$\dot{\nu}(t) \leq (1 + \mathfrak{R}(t))\,(\overline{\Lambda} + \widehat{\Lambda}\mathfrak{R}(t)). \tag{60}$$

For the subinterval $[t_{q+1}, t_{q+2})$, uncertain system (50) could be expressed by

$$\begin{aligned}\dot{x}(t) &= \left(A_j + \Delta A_j\right)x(t) + \left(B_j + \Delta B_j\right)\widetilde{D}_i\left(y(t) + e_{s_k}(t)\right)\\ &\quad + (B_j + \Delta B_j)\widetilde{C}_i\widetilde{x}(t),\ t \in [t_{q+1}, t_{q+2}),\end{aligned} \tag{61}$$

with $\sigma(t_{q+1}) = j \in \mathscr{L}$. Parallelling to the deduction of Step 1, one derives

$$\begin{aligned}\dot{\mathfrak{R}}(t) &\leq \frac{1 + \mathfrak{R}(t)}{\left\|\sqrt{\Omega_i}\,y(t)\right\|}\begin{pmatrix} \left\|\sqrt{\Omega_i}\left(C_j + \Delta C_j\right)\left(A_j + \Delta A_j\right)\right\| \cdot \|x(t)\| \\ + \left\|\sqrt{\Omega_i}\left(C_j + \Delta C_j\right)\left(B_j + \Delta B_j\right)\widetilde{C}_i\right\| \cdot \|\widetilde{x}(t)\| \\ + \left\|\sqrt{\Omega_i}\left(C_j + \Delta C_j\right)\left(B_j + \Delta B_j\right)\widetilde{D}_i\right\| \cdot \|y(t)\| \\ + \left\|\sqrt{\Omega_i}\left(C_j + \Delta C_j\right)\left(B_j + \Delta B_j\right)\widetilde{D}_i e_{s_k}(t)\right\| \end{pmatrix}\\ &\leq (1 + \mathfrak{R}(t))\left(\Lambda_c + \Lambda_d\mathfrak{R}(t)\right) \leq (1 + \mathfrak{R}(t))\,(\overline{\Lambda} + \widehat{\Lambda}\mathfrak{R}(t)),\end{aligned} \tag{62}$$

where

$$\begin{aligned}\Lambda_c &= \max_{i \neq j \in \mathscr{L}} \frac{1}{\lambda_{\min}\sqrt{\Omega_i}}\\ &\left[\begin{matrix}\begin{pmatrix} \left\|\sqrt{\Omega_i}\,C_jA_j\right\| + \left\|\sqrt{\Omega_i}\,C_jM_j\cdot\right\|\left\|F_j\right\| + \left\|\sqrt{\Omega_i}\,M_j\right\| \cdot \left\|F_j''A_j\right\| \\ + \left\|\sqrt{\Omega_i}\,M_j\right\| \cdot \left\|F_j''M_j\right\| \cdot \left\|F_j\right\| + \left\|\sqrt{\Omega_i}\,C_jB_j\widetilde{C}_i\right\| + \left\|\sqrt{\Omega_i}\,C_jM_j\right\| \cdot \left\|F_j'\widetilde{C}_i\right\| \\ + \left\|\sqrt{\Omega_i}\,M_j\right\| \cdot \left\|F_j''B_j\widetilde{C}_i\right\| + \left\|\sqrt{\Omega_i}\,M_j\right\| \cdot \left\|F_j''M_j\right\| \cdot \left\|F_j'\widetilde{C}_i\right\| \end{pmatrix}\beta \\ + \left\|\sqrt{\Omega_i}\,C_jB_j\widetilde{D}_i\right\| + \left\|\sqrt{\Omega_i}\,C_jM_j\right\| \cdot \left\|F_j'\widetilde{D}_i\right\| \\ + \left\|\sqrt{\Omega_i}\,M_j\right\| \cdot \left\|F_j''B_j\widetilde{D}_i\right\| + \left\|\sqrt{\Omega_i}\,M_j\right\| \cdot \left\|F_j''M_j\right\| \cdot \left\|F_j'\widetilde{D}_i\right\| \end{matrix}\right],\\ \Lambda_d &= \max_{i \neq j \in \mathscr{L}} \left\|\sqrt{\Omega_i}\,C_jB_j\widetilde{D}_i\sqrt{\Omega_i}^{-1}\right\| + \left\|\sqrt{\Omega_i}\,C_jM_j\right\| \cdot \left\|F_j'\widetilde{D}_i\sqrt{\Omega_i}^{-1}\right\|\\ &\quad + \left\|\sqrt{\Omega_i}\,M_j\right\| \cdot \left\|F_j''B_j\widetilde{D}_i\sqrt{\Omega_i}^{-1}\right\| + \left\|\sqrt{\Omega_i}\,M_j\right\| \cdot \left\|F_j''M_j\right\| \cdot \left\|F_j'\widetilde{D}_i\sqrt{\Omega_i}^{-1}\right\|.\end{aligned} \tag{63}$$

Based on the analysis above, inequality (60) is guaranteed for all subintervals $[s_k, t_{q+1})t, n[qt_{q+1}, t_{q+2}h), x\ldots 7, C[; t_{q+n}, s_{k+1})$. Defining $\varpi(t) = \dot{\nu}(t) = (1 + \nu(t))\,(\overline{\Lambda} + \widehat{\Lambda}\nu(t))$, we could derive from $\dot{\mathfrak{R}}(t) \leq \varpi(t)$ and $\mathfrak{R}(s_k) = \nu(s_k) = 0$ that $\mathfrak{R}(t_{q+1}) \leq \nu(t_{q+1})$. By repetitively applying Lemma 4, we deduce $\mathfrak{R}(t_{q+2}) \leq \nu(t_{q+2}), \ldots, \mathfrak{R}(t_{q+n}) \leq \nu(t_{q+n})$. Then, for $t \in [t_{q+n}, s_{k+1})$,

$$
\begin{aligned}
\mathfrak{R}(t) \leq \nu(t) &= \int_{t_{q+n}}^{t} \varpi(\iota)\mathrm{d}\iota + \nu(t_{q+n}) \\
&= \int_{t_{q+n}}^{t} \varpi(\iota)d\iota + \int_{t_{q+n-1}}^{t_{q+n}} \varpi(\iota)\mathrm{d}\iota + \cdots + \int_{s_k}^{t_q} \varpi(\iota)\mathrm{d}\iota + \nu(s_k) \\
&= \int_{s_k}^{t} \varpi(\iota)\mathrm{d}\iota.
\end{aligned} \tag{64}
$$

Due to Step 1, the lower bound of $s_{k+1} - s_k$ can be deduced to be $1/\overline{\Lambda} - \widehat{\Lambda}\ln(1 + (\overline{\Lambda} - \widehat{\Lambda})\sqrt{\epsilon}/\overline{\Lambda} + \widehat{\Lambda}\sqrt{\epsilon})$. Consequently, the presence of the lower boundary on adjacent event intervals is ultimately demonstrated. Based on (51), we could reason out the following relationship:

$$
\begin{aligned}
\widehat{\Lambda} &= \left\| \max_{i,j\in\mathscr{L}} \sqrt{\Omega_i} C_j B_j \widetilde{D}_i \sqrt{\Omega_i}^{-1} \right\| + \left\| \sqrt{\Omega_i} C_j M_j \right\| \cdot \left\| F_j' \widetilde{D}_i \sqrt{\Omega_i}^{-1} \right\| \\
&\quad + \left\| \sqrt{\Omega_i} M_j \right\| \cdot \left\| F_j'' B_j \widetilde{D}_i \sqrt{\Omega_i}^{-1} \right\| + \left\| \sqrt{\Omega_i} M_j \right\| \left\| F_j'' M_j \right\| \cdot \left\| F_j' \widetilde{D}_i \sqrt{\Omega_i}^{-1} \right\| \\
&\leq \max_{i,j\in\mathscr{L}} \left\| \sqrt{\Omega_i} C_j B_j \widetilde{D}_i \right\| \cdot \left\| \sqrt{\Omega_i}^{-1} \right\| + \left\| \sqrt{\Omega_i} C_j M_j \right\| \cdot \left\| F_j' \widetilde{D}_i \right\| \cdot \left\| \sqrt{\Omega_i}^{-1} \right\| \\
&\quad + \left\| \sqrt{\Omega_i} M_j \right\| \cdot \left\| F_j'' B_j \widetilde{D}_i \right\| \cdot \left\| \sqrt{\Omega_i}^{-1} \right\| + \left\| \sqrt{\Omega_i} M_j \right\| \cdot \left\| F_j'' M_j \right\| \cdot \left\| F_j' \widetilde{D}_i \right\| \cdot \left\| \sqrt{\Omega_i}^{-1} \right\| \\
&\leq \max_{i,j\in\mathscr{L}} \frac{1}{\lambda_{\min}\sqrt{\Omega_i}} \begin{pmatrix} \left\| \sqrt{\Omega_i} C_j B_j \widetilde{D}_i \right\| + \left\| \sqrt{\Omega_i} C_j M_j \right\| \cdot \left\| F_j' \widetilde{D}_i \right\| \\ + \left\| \sqrt{\Omega_i} M_j \right\| \cdot \left\| F_j'' B_j \widetilde{D}_i \right\| + \left\| \sqrt{\Omega_i} M_j \right\| \cdot \left\| F_j'' M_j \right\| \cdot \left\| F_j'' \widetilde{D}_i \right\| \end{pmatrix} < \overline{\Lambda},
\end{aligned} \tag{65}
$$

which symbolizes that the lower boundary T_1 is positive invariably; therefore, the Zeno behavior could be eliminated.

The systems in the real world have more or less nonlinear characteristics, so the nonlinear switched systems have a wide range of practicability. However, due to the complexity of nonlinear systems, there are still many problems to be solved in the analysis and control of nonlinear switched systems. The system considered in this paper is a linear switched system, but the analysis framework and analytical method can be extended to the nonlinear switched system.

Time delay is a common phenomenon in the establishment of a mathematical model of control systems. It is one of the key factors leading to the performance degradation of the systems. Therefore, the analysis and design of time-delay switched systems are an important research field in control theory. How to extend the results of this paper to time-delayed switched systems is not only a more general problem but also a more complex problem.

4. Simulation Results

To verify the availability of the derived results, in this section, uncertain switched linear system (1) that incorporates two subsystems is considered, whose parameters are as follows:

$$
\begin{aligned}
A_1 &= \begin{bmatrix} -4.5 & 0 \\ 0 & -4.4 \end{bmatrix}, \\
A_2 &= \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}, \\
B_1 &= \begin{bmatrix} -4.5 & -2 \\ 1.5 & 5 \end{bmatrix}, \\
B_2 &= \begin{bmatrix} -5 & -2 \\ 1 & 5 \end{bmatrix}, \\
C_1 &= \begin{bmatrix} -1.5 & 1.5 \end{bmatrix}, \\
C_2 &= \begin{bmatrix} -1 & 1 \end{bmatrix}, \\
M_1 &= M_2 = 0.05, \\
F_1 &= F_2 = \begin{bmatrix} -0.3 & 0 \\ 0 & 0.1 \end{bmatrix}, \\
F_1 &= F_2 = \begin{bmatrix} -0.3 & 0 \\ 0 & 0.1 \end{bmatrix}, \\
F_1' &= F_2' = \begin{bmatrix} -1 & 0.01 \\ 0 & -0.1 \end{bmatrix}, \\
F_1'' &= F_2'' = \begin{bmatrix} -1 & 1 \end{bmatrix}.
\end{aligned} \tag{66}
$$

The resultant uncertain closed-loop system form can be described by

$$\widetilde{A}_1 = \begin{bmatrix} -20 & -0.1 \\ 0.4 & -4 \end{bmatrix},$$
$$\widetilde{A}_2 = \begin{bmatrix} -15 & -0.01 \\ 0.3 & -4 \end{bmatrix},$$
$$\mathcal{T}_{11} = \begin{bmatrix} 0.001 \\ 0.0025 \end{bmatrix},$$
$$\mathcal{T}_{21} = \begin{bmatrix} 0.0015 \\ 0.002 \end{bmatrix},$$
$$\mathcal{T}_{12} = \begin{bmatrix} -0.1 & -0.4998 \\ 0 & -0.002 \end{bmatrix},$$
$$\mathcal{T}_{22} = \begin{bmatrix} -0.2 & -0.0496 \\ 0 & -0.004 \end{bmatrix}, \quad (67)$$
$$\mathcal{T}_{13} = \begin{bmatrix} 0.003 \\ 0.002 \end{bmatrix},$$
$$\mathcal{T}_{23} = \begin{bmatrix} 0.004 \\ -0.002 \end{bmatrix},$$
$$\mathcal{T}_{14} = \begin{bmatrix} -0.00016 \\ -0.001 \end{bmatrix},$$
$$\mathcal{T}_{24} = \begin{bmatrix} -0.00019 \\ 0.001 \end{bmatrix}.$$

Obviously, the controller gains can be calculated as
Setting $\gamma = \delta = 0.1$, $\eta = 1.3$, $T = 4$, $\epsilon = 0.5$, $\varsigma = 1$,

$$\mathcal{K}_1 = \begin{bmatrix} -20 & -0.1 & 0.02 \\ 0.4 & -4 & 0.05 \\ 0.2 & 0.5 & -0.0032 \\ 0 & 0.02 & -0.02 \end{bmatrix},$$
$$\mathcal{K}_2 = \begin{bmatrix} -15 & -0.01 & 0.03 \\ 0.3 & -4 & 0.04 \\ 0.2 & 0.05 & -0.0038 \\ 0 & 0.04 & 0.02 \end{bmatrix}. \quad (68)$$

by employing the aforementioned event-triggered strategy and controller gains, the state trajectory of the resultant uncertain closed-loop system can be clearly portrayed in Figures 3-4. Since the system remains stable and converges to its original point, it follows that Theorem 2 is effective. Moreover, Figure 5 exhibits the dynamics of the corresponding controller. Some release instants and release intervals, which are yielded by the above-mentioned adaptive event generator, are illustrated in Figure 6. Meanwhile, viewed from the curve trend of Figure 5 and the triggered dynamics of Figure 6, the event-triggered dynamic output-feedback controller adopted by us can monitor the uncertain switched system in real time and reduce unnecessary data sampling, thus achieving the effect of resource-saving.

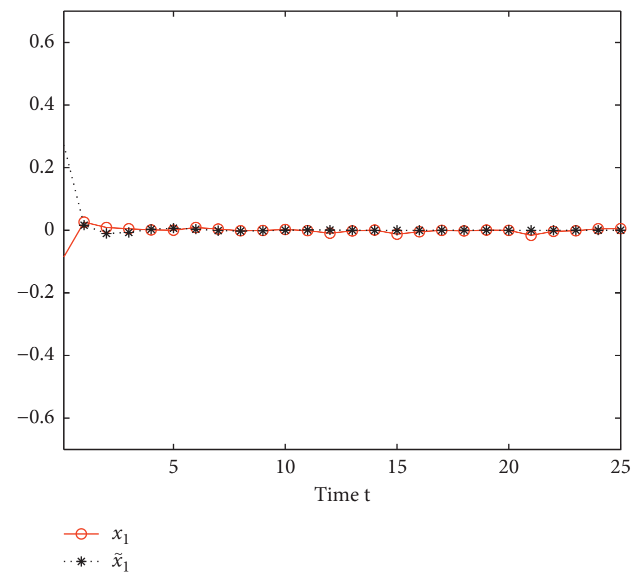

Figure 3: Responses of $x_1(t)$ and $\tilde{x}_1(t)$.

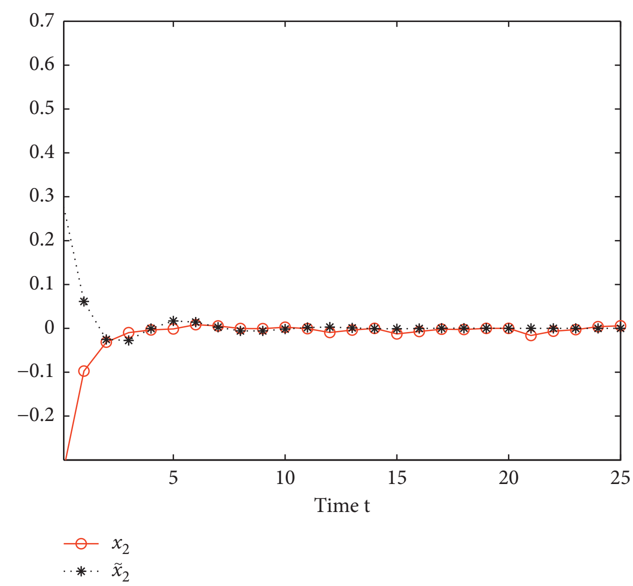

Figure 4: Responses of $x_2(t)$ and $\tilde{x}_2(t)$.

$$\begin{aligned} h_{111} &= h_{121} = h_{131} = h_{141} = h_{151} = h_{161} = h_{211} = h_{221} \\ &= h_{231} = h_{241} = h_{251} = h_{261} = h_{112} = h_{122} = h_{132} \\ &= h_{142} = h_{152} = h_{162} = h_{212} = h_{222} = h_{232} = h_{242} \\ &= h_{252} = h_{262} = 20, \\ h_{271} &= h_{172} = h_{171} = h_{272} = 80, \\ h_{18} &= h_{28} = 30, \\ \Omega_1 &= 1, \\ \Omega_2 &= 0.5, \end{aligned} \quad (69)$$

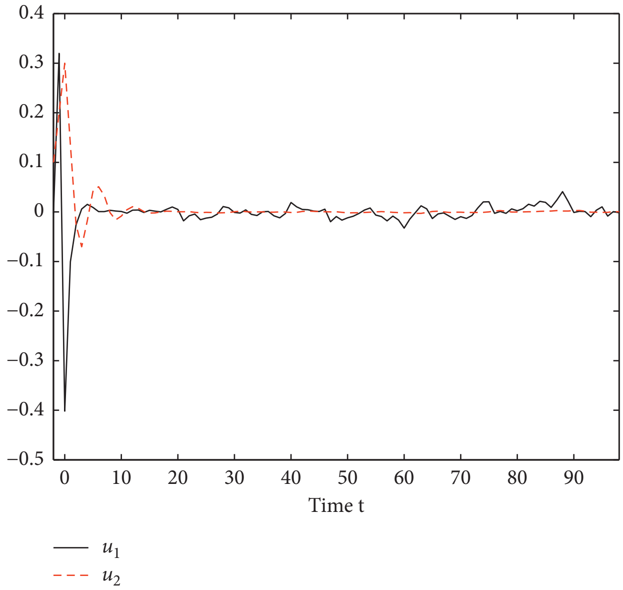

Figure 5: Responses of $u_1(t)$ and $u_2(t)$.

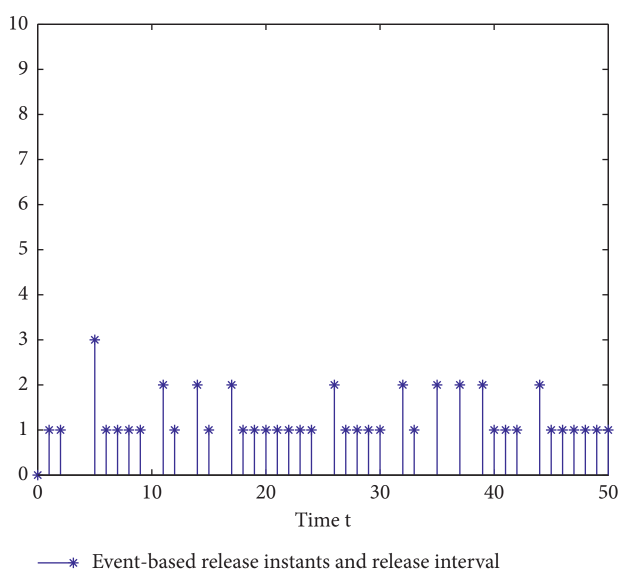

Figure 6: The dynamics of event-triggered mechanism.

5. Conclusion

In this article, we investigate innovatively the stabilization of an uncertain and frequently switched system that is equipped with an event-triggered dynamic output-feedback controller. By utilizing the average dwell-time strategy, the residence time of subsystems is arbitrarily small so that the uncertain system can be frequently switched. Moreover, we adopt dynamic output-feedback control for stabilization device, which replaces state-feedback control and is conducive to acquiring the whole information. Furthermore, a controller-pattern-related Lyapunov functional is constructed, and the pattern-related event-triggered scheme and the dynamic output-feedback controller are conjointly devised to guarantee the exponential stabilization of an uncertain closed-loop system. To reasonably deal with the uncertain parameters, the Lyapunov functional and controller gains are designed by the approach of a block matrix, as well as some linear matrix inequalities are exploited. Furthermore, the adopted event-triggered scheme can also avoid the Zeno phenomenon. Eventually, we provide a numerical simulation to confirm the availability of the derived results.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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