Hindawi
Discrete Dynamics in Nature and Society
Volume 2022, Article ID 8901180, 10 pages
https://doi.org/10.1155/2022/8901180

Research Article

Modeling the Linkages between Bitcoin, Gold, Dollar, Crude Oil, and Stock Markets: A GARCH-EVT-Copula Approach

Feng Jin,[1] **Jingwei Li**,[2] **and Guangchen Li**[3]

[1]*School of International Trade and Economics, University of International Business and Economics, Beijing 100029, China*
[2]*School of Insurance and Economics, University of International Business and Economics, Beijing 100029, China*
[3]*School of Economics, Beijing Wuzi University, Beijing 101149, China*

Correspondence should be addressed to Feng Jin; 201800152003@uibe.edu.cn

Received 7 June 2022; Accepted 27 July 2022; Published 27 August 2022

Academic Editor: Lele Qin

This paper aims to analyze and compare the ability of bitcoin, gold, and dollar to diversify the risk of traditional market such as crude oil and stock markets. Specifically, we model the linkages between bitcoin, gold, dollar, crude oil, and stock markets using the GARCH-EVT-copula approach. The results show that the gold market is in the central position among these markets, which is consistent with the status of gold as a major safe asset. Before the outbreak of COVID-19, bitcoin and the dollar also had the ability to diversify risks, although less effective than gold. However, during the COVID-19 period, gold loses its dominant position and gold, bitcoin, and dollar can no longer act as a hedge. We measure the value at risk (VaR) and expected shortfall (ES) of simulated portfolios constructed based on these five markets and use several backtesting methods to check the validity of the risk measures. The backtesting results show that our model can provide accurate risk measures before and within the COVID-19 period, which may help investors and risk managers construct the optimal portfolios.

1. Introduction

Bitcoin has attracted great attention around the world since its introduction in 2008. Proposed by Nakamoto [1], bitcoin is a peer-to-peer electric cash system that allows online payments to be sent directly from one party to another without going through a financial institution. The essence of bitcoin indicates that it may serve as a medium of exchange, which is one of the core functions of money. However, there have been many debates on whether bitcoin can be seen as a currency, speculative asset, or just a bubble [2–5]. Some studies find that the return of bitcoin is much more volatile than traditional financial markets and fiat currency [6, 7]. In early 2013, the unit price of bitcoin was only less than 100 dollars. However, it has multiplied several hundred times and reached more than 67,000 dollars at the end of 2021. Therefore, many researchers believe that bitcoin behaves more like a speculative asset. The bitcoin market has been increasingly attractive to international investors.

Recently, a growing literature has focused on the relationship between bitcoin and conventional markets, which has important implications for investors and policymakers [8–11]. Bitcoin is found to be connected with commodity markets [12–14], foreign exchange markets [15, 16], and stock markets [17–19]. However, the relationship between bitcoin and conventional markets is not as strong as that between the conventional markets, making it possible to use bitcoin as a hedge or safe haven for investors during market turmoil [20, 21].

Historically, gold has long been seen as a store of value and a trustful safe asset. The US dollar, the most popular currency in the world, can also maintain a relatively stable value and hedge risks. Dyhrberg [22] compares bitcoin with gold and the dollar and finds bitcoin has several similarities, which indicate that bitcoin has some hedging capabilities and can be useful in risk management. Bouri et al. [20] use the dynamic conditional correlation model to examine whether bitcoin can act as a hedge and safe haven for major world stock indices, bonds, oil, gold, the general commodity index, and the US dollar index. They find that bitcoin has hedge and safe haven properties against Asia Pacific stocks. Shahzad et al. [23] compare gold and bitcoin and find that bitcoin can also be used as a safe haven and hedging instrument for G7 stock markets. However, whether bitcoin can act as “digital gold” and become a safe haven are still under discussion. Smales [7] investigates the asset characteristics of Bitcoin over the periods 2011–2018 and believes that bitcoin does not have the potential to be a safe haven. Conlon and McGee [24] also cast doubt on the ability of bitcoin to provide shelter from turbulence in traditional markets. They find that bitcoin fails to be a safe haven for S&P 500 during the COVID-19 bear market. There are still some gaps in existing studies, which leave space for our research. On one hand, opinions have not reached a consensus on whether bitcoin can be used to diversify portfolio risk. More empirical evidence is needed to analyze the characteristics of bitcoin. On the other hand, many studies only focus on the linear relationship between bitcoin and other markets, which may lead to inaccurate results. In this paper, we try to fill these gaps.

In this paper, we study the linkages between bitcoin, gold, dollar, and two conventional markets—the crude oil and stock market. Bouri et al. [20] have investigated similar markets using the DCC-GARCH model. However, GARCH type models can only depict the linear relationship and need to assume asset distribution beforehand [25]. This paper employs the GARCH-EVT-copula approach to characterize the intermarket dependency structure. Specifically, first we use the ARMA-EGARCH-t to model the volatility of each market. Then, the extreme value theory (EVT) is used to model the tail risk of each market. Finally, we employ the R-vine copula to depict the dependence structure of the five markets and analyze the market relationships. This model allows us to provide accurate risk measures of the portfolio of these assets, which can help investors and risk managers control the portfolio risk.

This paper employs the GARCH-EVT-copula method to study the links between bitcoin, gold, dollar, crude oil, and stock markets. We focus on describing the dependence structure between markets. We find that gold is the central market during the whole sample period, which is consistent with the status of gold as a major safe haven asset. The outbreak of COVID-19, however, changed the dependence structure between these markets. Before COVID-19, bitcoin can act as a hedge, as it is negatively correlated with the crude oil and stock markets. When COVID-19 began to spread worldwide, gold is no longer in the central position, and gold, bitcoin, and dollar can no longer be seen as hedges but diversifiers. We also show that our model can provide accurate risk measures before and within the COVID-19 period, which facilitates the risk management for international investors and risk managers.

Our paper contributes to the existing literature in three parts. First, we study the relationship between bitcoin, gold, dollar, crude oil, and stock markets simultaneously. As far as we know, there have not been enough studies investigating the relationship between the five markets. Moreover, the existing literature usually only compares bitcoin with gold, to analyze whether bitcoin can diversify investment risks. However, few studies consider the dollar and analyze the interaction between these markets. This paper studies the relationship between five markets, which is a more comprehensive analysis framework. Second, the extreme correlations of the five markets are considered. When analyzing the bitcoin market, it is necessary to take the tail behavior of markets into consideration [26]. Based on the extreme value theory, we depict the extreme risk between markets by modeling the tail distribution of market returns. It can allow us to analyze the relationship between these markets more comprehensively and study whether bitcoin can diversify risks in terms of traditional market fluctuations. Third, the copula method is used to analyze the relationship between markets. The GARCH type models, most commonly used in existing literature, can only depict the linear relationship between markets [27]. In this paper, the R-vine copula is presented to overcome the limitation.

The remainder of this paper is organized as follows. Section 2 introduces the methodology. Section 3 deals with the data and presents the empirical results. Section 4 concludes the paper.

2. Methodology

2.1. The Framework of GARCH-EVT-Copula. This paper proposes the GARCH-EVT-copula method to study the links between bitcoin, gold, dollar, crude oil, and stock markets. Specifically, we first calculate the return of each market, and the ARMA-EGARCH-t model is used to model the volatility. Then, the extreme value theory is employed to model the tail risk by using the standardized residual of the ARMA-EGARCH-t model. The ARMA-GARCH-t and EVT are used to model the marginal distribution of each market. Next, we use the R-vine copula model to describe the dependence structure of these markets. Finally, we calculate and backtest the value at risk (VaR) and expected shortfall (ES) of simulated portfolios to test the performance of the model. The steps are presented in Figure 1.

2.2. ARMA-EGARCH-T Model. Abundant studies show that the time series of financial markets exhibit leptokurtosis, fat tails, and volatility clustering. The GARCH type models are commonly used to model these features. Besides, the returns of financial assets usually respond differently to positive and negative information, which is called the asymmetric effect. In this paper, we employ ARMA (1, 1)-EGARCH (1, 1)-t model to characterize each market:

$$\mathrm{r}_t = a_0 + a_1 r_{t-1} + b_1 \varepsilon_{t-1} + \varepsilon_t. \tag{1}$$

In the conditional mean model, r_t is the asset return on day t, a_0 is the constant term, a_1 is the coefficient of the lagged return, ε_t is the residual, and b_1 is the coefficient of the lagged residual. We assume e_t follows the t-distribution with degree of freedom v:

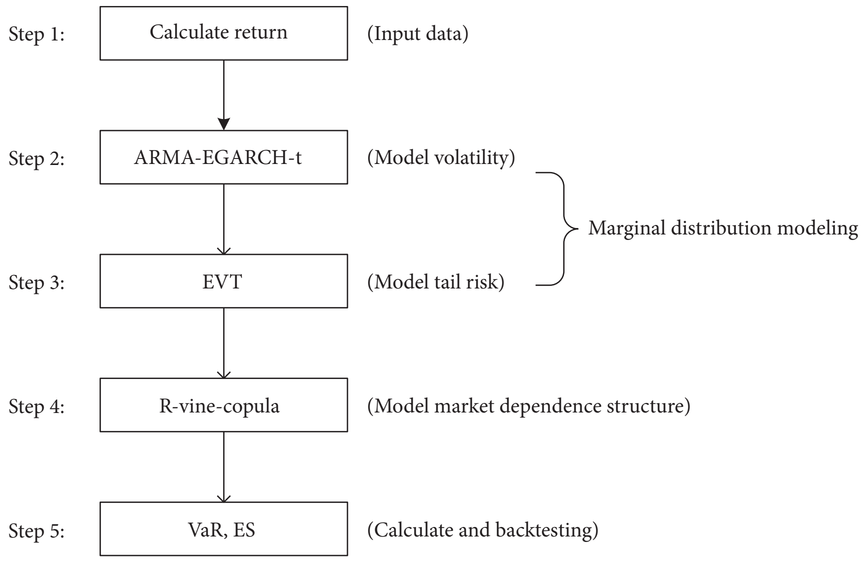


FIGURE 1: The steps of GARCH-EVT-copula approach.

$$\varepsilon_t = \sigma_t e_t,\ e_t \sim t(v),$$
$$\ln\sigma_t^2 = \omega + \alpha\frac{|\varepsilon_{t-1}|}{\sigma_{t-1}} + +\gamma\frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta\ln\sigma_{t-1}^2. \tag{2}$$

In the conditional variance equation, σ_t^2 is the conditional variance on day t, and γ is the parameter measuring the asymmetric effect. If γ is significantly different from 0, the asymmetric effect exists in the market.

After modeling the market return, we calculate the standardized residual, which is the conditional return filtered by volatility estimates using the ARMA-EGARCH-t model. The standardized residual of each market will be used in the following EVT analysis.

2.3. Extreme Value Theory. Some big events may cause the market prices to rise or fall simultaneously, which may affect the dependence structure between markets. Therefore, it is very necessary to measure the extreme correlation before investigating the dependence structure among markets [25]. Since the tail risks of financial assets are usually associated with huge gains or losses, we measure the extreme risks of these markets by using the extreme value theory (EVT) to model these returns' tail distributions.

We first set the upper and lower tail thresholds, which will divide the data into three parts. For the data between the upper and lower tail thresholds, we use the Gaussian kernel density estimation to obtain the cumulative distribution function (CDF). The upper and lower parts are modeled using the peaks over threshold (POT) method. The conditional tail distribution function $F_u(y)$ can be written as

$$F_u(y) = p(z - u \leq y \mid z > u),\ 0 \leq y \leq z_F - u. \tag{3}$$

Here, z is the conditional return filtered by the ARMA-EGARCH-t model, u is the preset threshold, y represents the extreme statistic, and $z_F \leq \infty$ represents the right endpoint of the distribution. $F_u(y)$ can be rewritten as

$$F_u(y) = \frac{F(u+y) - F(u)}{1 - F(u)} = \frac{F(z) - F(u)}{1 - F(u)}. \tag{4}$$

Balkema and Haan (1974) and Pickands (1975)show that the distribution beyond the threshold can be approximately modeled as the generalized Pareto distribution (GPD) for a sufficiently large threshold.

$$F_u(y) \approx G_{\xi,\beta}(y) = \begin{cases} 1 - \left(1 + \dfrac{\xi}{\beta}y\right)^{1/\xi}, & \xi \neq 0 \\ 1 - e^{-y/\beta}, & \xi = 0 \end{cases}, \tag{5}$$

where β is the scale parameter and ξ is the shape parameter. When $\xi \geq 0$, we have $z \geq u$; when $\xi < 0$, we have $u \leq z \leq u - \beta/\xi$, and for any $z > u$, $y = z - u$. By combining equations (5) and (6), the $F(z)$ can be expressed as

$$F(z) = (1 - F(u))G_{\xi,\beta}(z - u) + F(u). \tag{6}$$

We use the historical simulation method to estimate $F(u)$, that is, $F(u) = n - N_u/n$, where n and N_u represent sample size and observation that exceeds the tail threshold, respectively. We introduce $F(u)$ into equations (7) and obtain the tail estimation $\widehat{F}(z)$.

$$\widehat{F}(z) = \begin{cases} 1 - \dfrac{N_u}{n}\left(1 - \dfrac{\xi}{\beta}(z - u)\right)^{1/\xi}, & \xi \neq 0 \\ 1 - \dfrac{N_u}{n}e^{-z-u/\beta}, & \xi = 0 \end{cases}. \tag{7}$$

The copula function requires uniform margins. Therefore, we use the probability density transformation and make the series uniformly distributed on [0, 1].

2.4. R-Vine Copula Model. A copula is a multidimensional joint distribution function that can capture the dependence structure of various assets. In general, the dependence structures between different assets are different. Using the same copula function to depict the dependence structure between each pair of markets is not a good choice. To solve this problem, Bedford and Cooke [30] propose the R-vine (regular vine) copula model, which can select the most suitable copula functions from various pair-copula families.

According to Aas et al. [31]; the joint density function of R-vine can be decomposed as

$$f(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} f(x_i) \prod_{k=n-1}^{1} \prod_{j=n}^{k+1} C_{m_{k,k}, m_{j,k} | m_{j+1,k}, \ldots, m_{n,k}} \left(F_{m_{k,k} | m_{j+1}, k, \ldots, m_{n,k}}, F_{m_{j,k} | m_{j+1,k}, \ldots, m_{n,k}} \right), \tag{8}$$

where $f(x_i)$ is the marginal density and $C_{m_{k,k}, m_{j,k} | m_{j+1,k}, \ldots, m_{n,k}}(.,.)$ represents the pair-copula density. F is the conditional distribution function, which can be expressed as

$$F_{p|q} = \frac{\partial C_{pq_j|q_{-j}}\left(F(p \mid q_{-j}), F(q_j \mid q_{-j})\right)}{\partial F(q_j \mid q_{-j})}. \tag{9}$$

Here, q_j represents one arbitrarily chosen component of vector q and q_{-j} represents the vector that excludes this component. For more information on the R-vine copula method, we can refer to Aas et al. [31].

2.5. VaR and ES Calculation and Backtesting. In this section, we test the performance of the GARCH-EVT-copula model on risk management. We build several portfolios using the five assets and calculate the value at risk (VaR) and expected shortfall (ES) of the simulated portfolios.

We first calculate the simulated return of each asset. Using the dependence structure of the R-vine copula model, we perform the Monte Carlo simulation to simulate the marginal series of each asset, as suggested by Janekova et al. [32, 33]. In our analysis, we generate a 5-dimension array with 5000 random observations. Note that each simulated series is uniformly distributed on [0, 1]. Then, we use the EVT fitting inversely and transform the simulated marginal series into standard residuals $z_{i,t+1}$. And, we use the ARMA (1,1)-EGARCH (1, 1)-t model in Section 2.2 to forecast the conditional volatility $\sigma_{i,t+1}$ and conditional average $\mu_{i,t+1}$ of each asset. Finally, the simulated return $X_{i,t+1}$ of each asset can be calculated by

$$X_{i,t+1} = \mu_{i,t+1} + z_{i,t+1}\sigma_{i,t+1}, i = 1, 2, \ldots, 5. \tag{10}$$

We then construct several portfolios and calculate the value at risk (VaR):

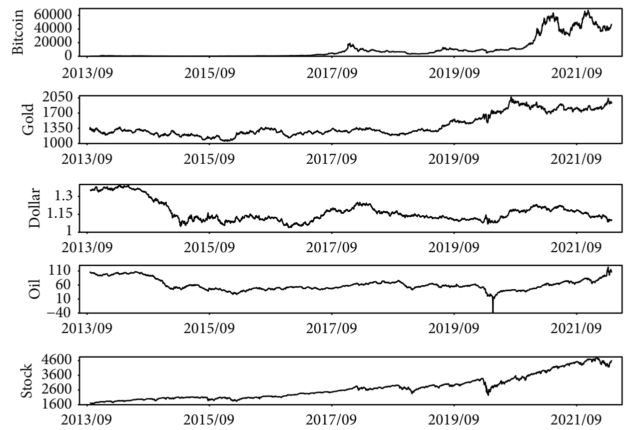


Figure 2: The price trends of five markets.

$$VaR_\alpha(R) = \min\{c\colon P(R \le c) \ge \alpha\}. \tag{11}$$

Here, $R = \sum_{i=1}^{5} w_i X_i$. and w_i is the portfolio weight of asset i. The VaR represents the minimum loss no more than the given value c with probability α. In this paper, we use two methods to backtest the VaR, the unconditional coverage test proposed by Kupiec (1995) and the conditional coverage test suggested by Christoffersen and Pelletier (2004).

To better measure the risk of the portfolio, the ES is also introduced:

$$\mathrm{ES}_\alpha(R) = E\left[R \ge VaR_\alpha(R)\right]. \tag{12}$$

Following Rockafellar and Uryasev [36] and Acerbi and Tasche [37], the ES can be calculated using the following equation:

$$\mathrm{ES}_\alpha(R) = \frac{1}{1-\alpha} \int_\alpha^1 VaR_\beta(R) d\beta. \tag{13}$$

To check the validity of ES, we use the bootstrap method suggested by McNeil and Frey [38]. For the tests for VaR and ES, the null hypotheses are the same, i.e., the model can provide accurate risk measurement. If the p value is larger than the given significant level such as 5%, we cannot reject the null hypothesis.

3. Data and Empirical Results

In this study, we use the daily closing price of bitcoin, gold, crude oil, dollar, and stock market from September 17, 2013, to March 28, 2022. Specifically, the bitcoin price data is retrieved from the coinmarketcap website (https://www.coinmarketcap.com). We use the per ounce of gold futures prices on the New York Mercantile Exchange to represent the gold market, the price of WTI crude oil futures to represent the crude oil market, the dollar-euro rate to represent the dollar market, and the S&P 500 Index to represent the stock market. The crude oil data is obtained from the EIA website, and the gold price, dollar, and S&P 500 data are obtained from the Wind database. For simplicity, we call them bitcoin market, gold market, dollar market, oil market, and stock market, respectively. The

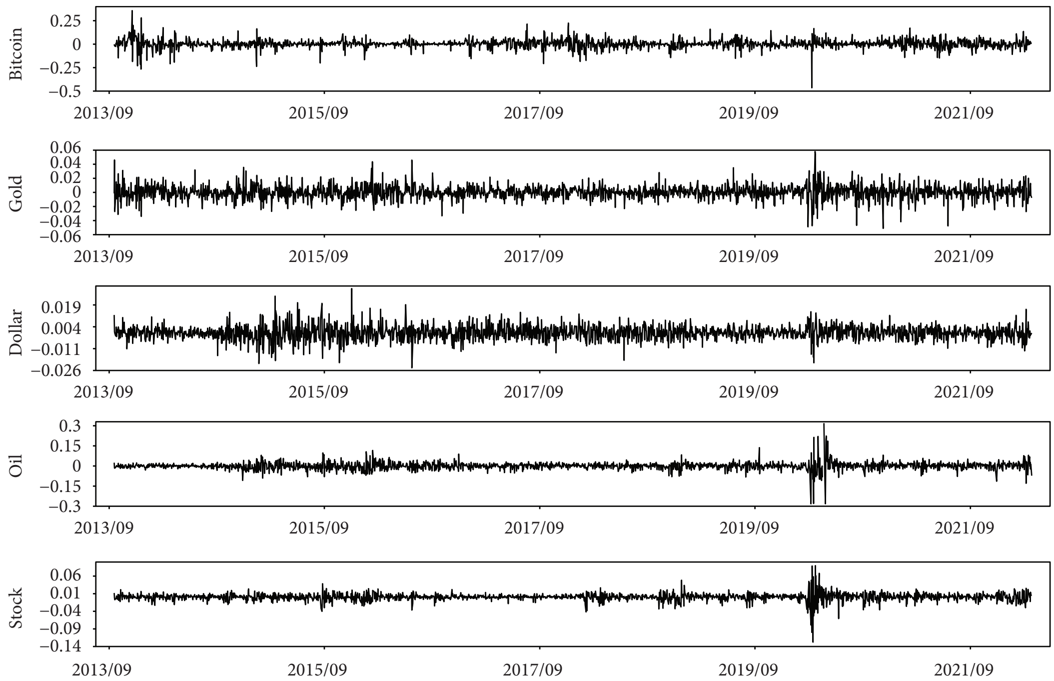


Figure 3: The daily returns of five markets.

Table 1: Descriptive statistics of the returns.

	Bitcoin	Gold	Dollar	Oil	Stock
Observations	2134	2134	2134	2134	2134
Mean	0.0023	0.0002	−0.0001	0.0003	0.0005
S.D.	0.0457	0.0095	0.0048	0.0295	0.0109
Min	−0.4647	−0.0511	−0.0242	−0.2822	−0.1277
Max	0.3575	0.0581	0.0302	0.3196	0.0897
Skewness	−0.5200	−0.0299	0.0585	0.1661	−0.9862
Kurtosis	14.2187	7.0228	5.5921	28.0320	23.8222
J-B statistics	11000^{***} (0.0000)	1439^{***} (0.0000)	599^{***} (0.0000)	56000^{***} (0.0000)	39000^{***} (0.0000)
ARCH	62.023^{***} (0.0000)	40.645^{***} (0.0000)	39.411^{***} (0.0000)	273.683^{***} (0.0000)	352.842^{***} (0.0000)
ADF	-12.099^{***} (0.0000)	-9.083^{***} (0.0000)	-8.055^{***} (0.0000)	-7.665^{***} (0.0000)	-11.516^{***} (0.0000)

Note. p values are in the parentheses. *** indicates the significant level at 1%.

Table 2: The results of ARMA (1,1)-EGARCH (1, 1)-t model.

	Bitcoin	Gold	Dollar	Oil	Stock
α_0	0.0017^{***}	0.0002^{*}	−0.0001	−0.0000	0.0006^{***}
α_1	-0.8894^{***}	-0.3706^{***}	0.8455^{***}	-0.3197^{***}	-0.2424^{***}
b_1	0.8802^{***}	0.3053^{***}	-0.8662^{***}	0.2814^{***}	0.1731^{***}
ω	-0.1567^{***}	-0.1344^{***}	-0.0777^{***}	-0.0807^{***}	-0.3775^{***}
α	0.4196^{***}	0.0902^{***}	0.1044^{***}	0.1291^{***}	0.2407^{***}
β	0.9726^{***}	0.9856^{***}	0.9928^{***}	0.9898^{***}	0.9619^{***}
γ	0.0645^{**}	0.0285^{**}	0.0138	-0.0882^{***}	-0.1839^{***}
LL	3530.112	5949.549	7047.405	4370.925	6184.461

Note. LL is the log-likelihood value of the estimation; *, **, and *** indicate the significant level at 10%, 5%, and 1%, respectively.

logarithmic return is calculated for each market using $r_t = \ln(P_t/P_{t-1})$, where r_t is the daily return and P_t is the closing price at day t. The prices and returns of the five markets are shown in Figures 2 and 3, respectively. We can observe the volatility clustering of each market. Moreover, all the markets showed significant price changes in early 2020, when COVID-19 began to spread around the world. This change may also influence the market relationship, which needs further analysis.

The descriptive statistics of the returns are shown in Table 1. The standard deviations indicate that bitcoin market is the most volatile market. All returns show the obvious

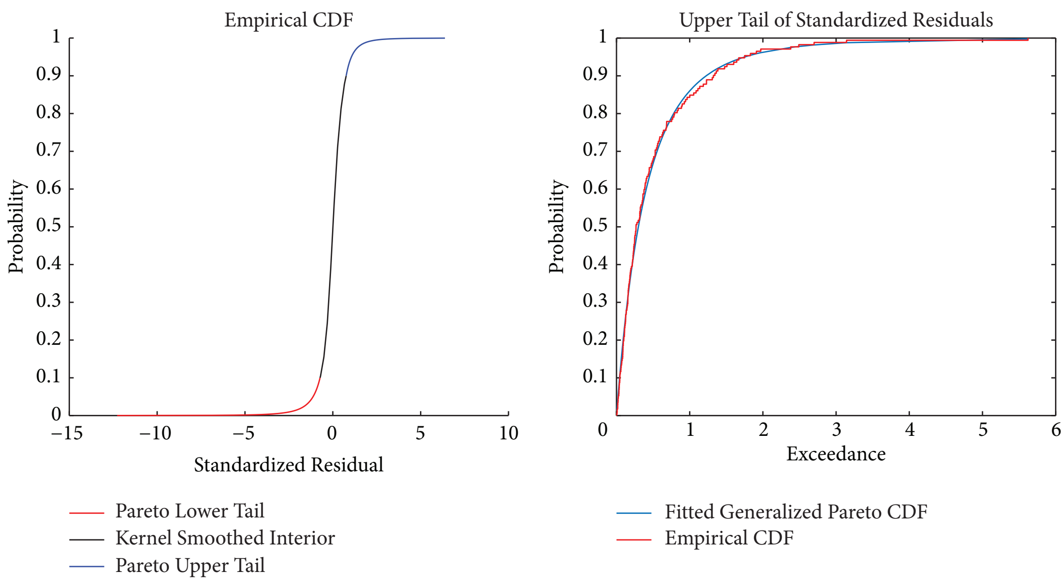
Table 3: BDS test results of standard residuals.

	Bitcoin	Gold	Dollar	Oil	Stock
BDS	1.2275 (0.2196)	−0.4898 (0.6243)	0.4229 (0.6724)	1.2273 (0.2197)	1.2972 (0.1946)

Note. p values are in the parentheses.

Table 4: Results of tail thresholds and EVT fitting.

	Lower tail			Upper tail		
	μ_L	ξ_L	β_L	μ_U	ξ_U	β_U
Bitcoin	−−0.7004	0.2951	0.5180	0.7258	0.2463	0.3953
Gold	−1.0964	−0.0310	0.6878	1.1099	0.0970	0.6028
Dollar	−1.2238	0.1736	0.4190	1.2080	−0.0305	0.6330
Oil	−1.2179	0.0721	0.6637	1.1191	0.0243	0.4836
Stock	−0.2661	0.1416	0.6547	1.1181	−0.0031	0.4107



(a) (b)

Figure 4: Distribution curve of residual and CDF fitting analysis of the bitcoin market.

feature of leptokurtosis, which is also confirmed by the J-B test. The ARCH test demonstrates that the returns have noticeable ARCH effects, which denotes that the GARCH type model is necessary to model the marginal distribution of each market. The ADF test reveals that all the returns are stationary.

Table 2 shows the coefficients of the ARMA (1, 1)-EGARCH (1, 1)-*t* model in each market. We can find that most of the coefficients are significant at 1% level, denoting that the model can fit the return series of five markets well. The leverage effect exists in all the markets except the dollar market. The coefficients γ are positive for the bitcoin and gold market, denoting that the positive news would cause larger volatilities. For oil and stock markets, the coefficients γ are negative, indicating that the negative news has a larger effect on the market volatilities. We then extract the standard residual sequence from the ARMA (1, 1)-EGARCH (1, 1)-*t* model. Before using the EVT method, the standard residual sequence needs to be independent identically distributed.

We use the BDS test proposed by Broock et al. [39], which is designed to test whether a random sequence is i.i.d (identical independent distributed). Table 3 shows that we cannot reject the null hypothesis that the sequence is i.i.d. Therefore, the standard residual series of each market can be used in the EVT analysis.

An appropriate threshold needs to be determined before employing the EVT method. A very small threshold will lead to too much tail data, which will not meet the preconditions of the EVT method. However, a very large threshold will result in too little tail data and affect the model's performance. According to DuMouchel [40], we select 10% as the threshold and obtain five corresponding upper (μ_U) and lower tail (μ_L) threshold values of market returns. The EVT fitting parameters are reported in Table 4. We find that the upper tail parameters (ξ_U) of bitcoin, and gold and oil markets are larger than 0, indicating that these markets present fat tails. Taking the bitcoin market as an example, we draw the

Table 5: K–S test results of marginal distribution.

	Bitcoin	Gold	Dollar	Oil	Stock
K–S	0.0222 (0.3409)	0.0176 (0.6332)	0.0124 (0.9436)	0.0146 (0.8383)	0.0154 (0.7865)

Note. p values are in the parentheses.

Table 6: Estimation results of R-vine copula.

<table>
<tr><th></th><th></th><th colspan="4">The whole period</th><th colspan="4">Without COVID-19</th><th colspan="4">During COVID-19</th></tr>
<tr><td></td><td>Edge</td><td>2,1</td><td>2,3</td><td>5,2</td><td>5,4</td><td>2,1</td><td>2,3</td><td>5,2</td><td>5,4</td><td>2,1</td><td>2,3</td><td>5,1</td><td>5,4</td></tr>
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<tr><td>Tree1</td><td>Kendall’s τ</td><td>0.05</td><td>0.24</td><td>−0.06</td><td>0.18</td><td>0.02</td><td>0.24</td><td>-0.09</td><td>0.17</td><td>0.15</td><td>0.21</td><td>0.18</td><td>0.20</td></tr>
<tr><td></td><td>Up</td><td>0.07</td><td>—</td><td>0.00</td><td>—</td><td>0.05</td><td>—</td><td>0.00</td><td>—</td><td>0.06</td><td>0.01</td><td>—</td><td>0.11</td></tr>
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<tr><td></td><td>Edge</td><td>3,1|5,2</td><td>4,3|5,2</td><td></td><td></td><td>5,3|1,2</td><td>4,1|5,2</td><td></td><td></td><td>5,3|1,2</td><td>4,2|5,1</td><td></td><td></td></tr>
<tr><td></td><td>Copula</td><td>Joe</td><td>Student t</td><td></td><td></td><td>Student t</td><td>Gaussian</td><td></td><td></td><td>Gumbel</td><td>Student t</td><td></td><td></td></tr>
<tr><td>Tree3</td><td>Kendall’s τ</td><td>0.01</td><td>0.01</td><td></td><td></td><td>-0.00</td><td>-0.01</td><td></td><td></td><td>0.08</td><td>-0.01</td><td></td><td></td></tr>
<tr><td></td><td>Up</td><td>0.03</td><td>0.00</td><td></td><td></td><td>0.02</td><td>—</td><td></td><td></td><td>0.11</td><td>0.00</td><td></td><td></td></tr>
<tr><td></td><td>Low</td><td>—</td><td>0.00</td><td></td><td></td><td>0.02</td><td>—</td><td></td><td></td><td>—</td><td>0.00</td><td></td><td></td></tr>
<tr><td></td><td>Edge</td><td>4,1| 3,5,2</td><td></td><td></td><td></td><td>4,3| 5,1,2</td><td></td><td></td><td></td><td>4,3| 5,1,2</td><td></td><td></td><td></td></tr>
<tr><td></td><td>Copula</td><td>Gaussian</td><td></td><td></td><td></td><td>Clayton</td><td></td><td></td><td></td><td>Student t</td><td></td><td></td><td></td></tr>
<tr><td>Tree4</td><td>Kendall’s τ</td><td>0.00</td><td></td><td></td><td></td><td>0.03</td><td></td><td></td><td></td><td>0.01</td><td></td><td></td><td></td></tr>
<tr><td></td><td>Up</td><td>—</td><td></td><td></td><td></td><td>—</td><td></td><td></td><td></td><td>0.02</td><td></td><td></td><td></td></tr>
<tr><td></td><td>Low</td><td>—</td><td></td><td></td><td></td><td>0.00</td><td></td><td></td><td></td><td>0.02</td><td></td><td></td><td></td></tr>
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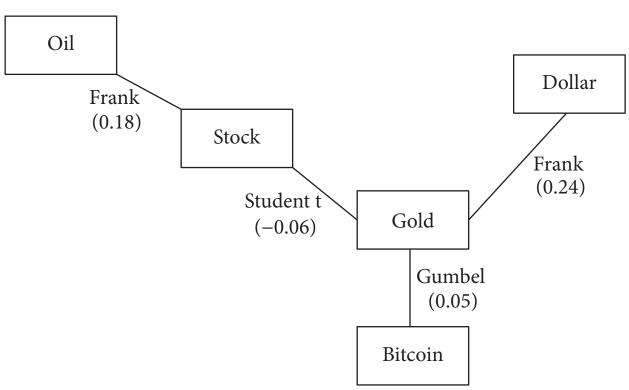

Figure 5: The estimated five-dimensional R-vine tree of the whole sample.

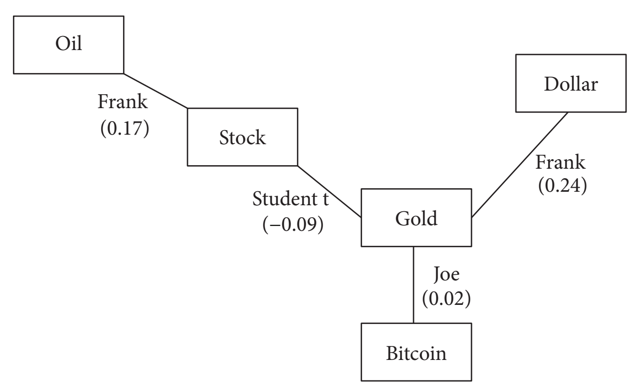

Figure 6: The estimated five-dimensional R-vine tree before COVID-19.

fitted cumulative distribution function curve in Figure 4, and we find it obtains an excellent fitting.

The K–S test (Kolmogorov–Smirnov test) is used to test whether the transformed marginal distribution of each series is uniformly distributed on [0, 1]. The results of Table 5 cannot reject the null hypothesis that the data follow the uniform distribution of [0, 1], indicating that the adjusted series satisfy the prerequisite conditions for the R-vine copula model.

Table 6 shows the R-vine structure between markets. An advantage of the R-vine copula is that it can make the choices of copula functions more extensive and flexible, which can depict the relationship between markets more accurately. The optimal copula functions are selected by the AIC criterion. In Table 6, numbers 1 to 5 represent bitcoin, gold, dollar, oil, and stock markets. We also present the structure of tree 1 of the R-vine copula in Figure 5. We annotate the most suitable copula function and Kendall’s τ on the lines linking two markets. The Kendall’s τ determines the direction and intensity of market dependence. As shown in Table 6 and Figure 5, during the whole sample period, gold is at the center of these markets, indicating its close interconnection with the rest of the markets, which is consistent with the current status of gold as a major global safe haven asset. We can also find a negative correlation between stock and gold markets. Moreover, the oil market is positively associated with the stock market. The relationships between bitcoin and gold markets, as well as gold and dollar markets,

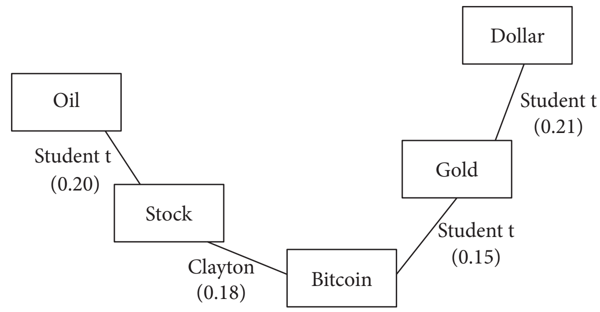

Figure 7: The estimated five-dimensional R-vine tree during COVID-19.

Table 7: Results of VaR and ES backtesting.

	Portfolio 1			Portfolio 2			Portfolio 3		
	VaR		ES	VaR		ES	VaR		ES
Test	LR_{uc}	LR_{cc}	McNeil	LR_{uc}	LR_{cc}	McNeil	LR_{uc}	LR_{cc}	McNeil
Panel A: the whole sample									
90%	0.8818	0.9876	0.4601	0.2761	0.5408	0.3311	0.8123	0.7888	0.1304
95%	0.8364	0.9673	0.3800	0.5471	0.3671	0.0919	0.7493	0.8044	0.1442
97.5%	0.8868	0.6991	0.2460	0.1656	0.2601	0.7326	0.3925	0.5375	0.1858
99%	0.2149	0.4068	0.9904	0.1038	0.2370	0.9998	0.0733	0.1767	0.6055
Panel B: without the COVID-19									
90%	0.1478	0.1099	0.3379	0.2319	0.4770	0.1812	0.5784	0.8485	0.2590
95%	0.2393	0.3926	0.3134	0.6267	0.5925	0.1166	0.8529	0.7138	0.1839
97.5%	0.5644	0.7834	0.4697	0.7458	0.7946	0.1570	0.8971	0.8881	0.1607
99%	0.5576	0.8060	0.8942	0.2414	0.4657	0.2653	0.2786	0.5227	0.4108
Panel C: within the COVID-19									
90%	0.3142	0.5646	0.9524	0.0313	0.0975	0.9961	0.6826	0.9185	0.4415
95%	0.7732	0.5354	0.7085	0.4092	0.2479	0.9929	0.1777	0.1134	0.7451
97.5%	0.5650	0.6567	0.9897	0.5650	0.6567	0.9999	0.1970	0.2912	0.9863
99%	0.8077	0.9373	0.9999	0.6967	0.9125	0.9999	0.8077	0.9373	0.9996

Note. The weights of bitcoin, gold, dollar, crude oil, and stock in portfolio 1 are 20%, 20%, 20%, 20%, and 20%. The weights in portfolio 2 are 10%, 10%, 10%, 10%, and 60%, respectively. The weights in portfolio 3 are 10%, 10%, 10%, 35%, and 35%, respectively.

are also positive. One possible explanation is that the price decline in the traditional capital market will drive investors to seek suitable safe haven markets, further leading to price increases in the markets such as gold, bitcoin, and dollar markets. Similarly, investors' demand for gold, bitcoin, and dollar may drop when the traditional assets are rolling, which leads to the decline of their prices [41].

Considering that the COVID-19 epidemic has brought great impacts worldwide and may bring changes to the market dependence structure, we investigate the subsamples without COVID-19 and during COVID-19. According to Azimli [42] and Umar et al. [43], the period without COVID-19 is defined as before January 1, 2020, and the period within COVID-19 covers the days from January 2, 2020, to the last day of the sample. Following Bouri et al. [20], a hedge is defined as an asset that is uncorrelated or negatively correlated with another asset on average, and a diversifier is defined as an asset that has a weak positive correlation with another asset on average. The Tree 1 structures of the period without COVID-19 and during COVID-19 are shown in Figures 6 and 7, respectively. We can find that, before the outbreak of COVID, the market structure of the five markets is the same as that in the whole sample period. The Kendall's τ of between each pair of markets also does not show a significant change. When the gold market acts as a conditional market, the relationship between the bitcoin market and the stock market is negative (−0.02), as shown in Tree 2, indicating that investors can incorporate bitcoin into their portfolios to reduce investment risks. Similarly, the dollar market can also diversify the risk of stock markets, though not as effective as gold and bitcoin markets. For the oil market, the Kendall's τ between bitcoin and oil markets is negative (−0.01) when choosing the gold and stock markets as conditional markets, as shown in Tree 3. Therefore, bitcoin is also a hedge for the oil market under this circumstance.

However, during the COVID-19 period, the gold market is no longer in the center position and the linkages between markets change a lot. During the COVID-19 period, the relationship between bitcoin market and stock market becomes positive (Kendall's τ is 0.18), indicating that the bitcoin market is no longer a hedge but a diversifier for stock market. It is similar to gold and dollar markets, which are positively related to the stock market. It indicates that COVID-19 has brought a great impact on many markets around the world, even on safe-haven markets. The gold, bitcoin, and dollar cannot diversify risks as they did before COVID-19. Nevertheless, as we will demonstrate below, the GARCH-EVT-copula model can provide accurate risk measures. The investors can control the portfolio risk, at least to some extent, by adjusting the weights of each asset.

Based on the dependence structure of the five markets, we construct three portfolios with different asset weights and conduct backtesting to test the risk measurement accuracy of the model. As shown in Table 7, we test the three portfolios under four upper tail quantiles, 90%, 95%, 97.5%, and 99%. We can find that almost all the P values are larger than 0.05, even in the period with COVID-19. The results denote that the GARCH-EVT-copula model of various portfolios has passed the VaR and ES backtesting (provide accurate risk measures). The model we constructed can measure the risks of the five market combinations very well.

The above empirical results show that, in general, gold is the most important and effective asset to diversify the risk of traditional markets, including the crude oil and stock markets in our analysis. Bitcoin, which is gaining popularity in recent years, also has the ability to diversify risk. Although not as effective as gold, bitcoin still behaves better than the US dollar in terms of risk diversification. The outbreak of COVID-19, however, changed the interdependence structure between these markets. Gold lost the central position and can no longer be a hedge. Bitcoin also becomes a diversifier rather than a hedge. Therefore, it may not be a good choice to invest in bitcoin after the market downturn caused by big events such as the COVID-19. Then, we construct different portfolios by changing the weight of each asset and test the performance of the GARCH-EVT-copula model on risk management. The results show that this model can obtain accurate risk measurement before and within COVID-19, making it a potential tool for portfolio construction and risk management.

4. Conclusions

This paper uses the GARCH-EVT-copula model to analyze the relationship between bitcoin, gold, dollar, crude oil, and stock markets. Our findings indicate that the gold market is central in these markets during the sample period, which is consistent with the status of gold as a major safe haven. We also find that, before the outbreak of COVID-19, bitcoin and dollar also had the ability to diversify risks, although not as effective as gold. However, when COVID-19 began to spread around the world, gold is no longer the center of these markets, and gold, bitcoin, and dollar can no longer be seen as a hedge. Nevertheless, we demonstrate that the model we use in this paper can provide accurate risk measures and help international investors or risk managers to control the risk of their portfolios.

Our results can provide some implications for investors and risk managers. In terms of the ability to diversify portfolio risk of crude oil and stock markets, we show that bitcoin is less effective than gold but better than the US dollar. However, bitcoin is also not a hedge during the market turmoil caused by external events such as COVID-19. Therefore, investors need to consider adding bitcoin to their portfolios carefully. In addition, the GARCH-EVT-copula method may help investors and risk managers analyze the relationship between multiple markets and control the risk of portfolios.

Unfortunately, like other approaches, the GARCH-EVT-copula approach used in this paper also has its limitations. A sufficiently long time series dataset is a prerequisite for modeling the relationship between multiple markets accurately, which makes this method unsuitable for short-term analysis such as one or two months. This method is also unable to detect the sudden structural changes in the market, as enough data after the structural changes are needed.

Data Availability

The data used in this study are derived from several sources. The bitcoin price data are downloaded from the coinmarketcap website at https://coinmarketcap.com/currencies/bitcoin/historical-data/. The WTI crude oil futures prices data are obtained from the EIA website at https://www.eia.gov/dnav/pet/hist/RCLC1D.htm. The rest data are obtained from the Wind database.

Conflicts of Interest

The authors declare no conflicts of interest.

References

[1] S. Nakamoto, “A peer-to-peer electronic cash system,” *Bitcoin.–URL*, vol. 4, 2008.
[2] E. T. Cheah and J. Fry, “Speculative bubbles in Bitcoin markets? An empirical investigation into the fundamental value of Bitcoin,” *Economics Letters*, vol. 130, pp. 32–36, 2015.
[3] D. G. Baur, K. Hong, and A. D. Lee, “Bitcoin: medium of exchange or speculative assets?” *Journal of International Financial Markets, Institutions and Money*, vol. 54, pp. 177–189, 2018.
[4] A. D. Lee, M. Li, and H. Zheng, “Bitcoin: speculative asset or innovative technology?” *Journal of International Financial Markets, Institutions and Money*, vol. 67, Article ID 101209, 2020.
[5] Y. Son and Y. S. Lee, “A smart contract weakness and security hole analyzer using virtual machine based dynamic monitor,” *Journal of Logistics, Informatics and Service Science*, vol. 9, no. 1, pp. 36–52, 2022.
[6] F. Almudhaf, “Pricing efficiency of Bitcoin trusts,” *Applied Economics Letters*, vol. 25, no. 7, pp. 504–508, 2018.
[7] L. A. Smales, “Bitcoin as a safe haven: is it even worth considering?” *Finance Research Letters*, vol. 30, pp. 385–393, 2019.
[8] S. Corbet, A. Meegan, C. Larkin, B. Lucey, and L. Yarovaya, “Exploring the dynamic relationships between cryptocurrencies and other financial assets,” *Economics Letters*, vol. 165, pp. 28–34, 2018.
[9] Y. Y. Gao and S. Y. Jin, “The impact mechanism between the incentive to hold financial assets, financial risk and innovation activities,” *Journal of Logistics, Informatics and Service Science*, vol. 8, no. 2, pp. 80–102, 2021.
[10] J. W. Kim, “Blockchain technology and its applications: case studies,” *Journal of System and Management Sciences*, vol. 10, no. 1, pp. 83–93, 2020.

[11] E. Bouri, M. Das, R. Gupta, and D. Roubaud, "Spillovers between Bitcoin and other assets during bear and bull markets," *Applied Economics*, vol. 50, no. 55, pp. 5935–5949, 2018.

[12] M. Y. Lin and C. L. An, "The relationship between Bitcoin and resource commodity futures: evidence from NARDL approach," *Resources Policy*, vol. 74, Article ID 102383, 2021.

[13] Y. J. Song and J. K. Lee, "A blockchain and internet of things based architecture design for energy transaction," *Journal of System and Management Sciences*, vol. 10, no. 2, pp. 122–140, 2020.

[14] K. Syuhada, D. Suprijanto, and A. Hakim, "Comparing golds and Bitcoins safe-haven roles against energy commodities during the COVID-19 outbreak: a vine copula approach," *Finance Research Letters*, vol. 46, Article ID 102471, 2022.

[15] M. Ozyesil, "A research on i?nteraction between bitcoin and foreign exchange rates," *Pressacademia*, vol. 6, no. 1, pp. 55–62, 2019.

[16] R. B. Palazzi, G. D. S. R. Júnior, and M. C. Klotzle, "The dynamic relationship between Bitcoin and the foreign exchange market: a nonlinear approach to test causality between Bitcoin and currencies," *Finance Research Letters*, vol. 42, Article ID 101893, 2021.

[17] C. Dirican and I. Canoz, "The cointegration relationship between Bitcoin prices and major world stock indices: an analysis with ARDL model approach," *Pressacademia*, vol. 4, no. 4, pp. 377–392, 2017.

[18] A. Lahiani, A. jeribi, and N. B. Jlassi, "Nonlinear tail dependence in cryptocurrency-stock market returns: the role of Bitcoin futures," *Research in International Business and Finance*, vol. 56, Article ID 101351, 2021.

[19] K. Q. Nguyen, "The correlation between the stock market and Bitcoin during COVID-19 and other uncertainty periods," *Finance Research Letters*, vol. 46, Article ID 102284, 2022.

[20] E. Bouri, P. Molnár, G. Azzi, D. Roubaud, and L. I. Hagfors, "On the hedge and safe haven properties of Bitcoin: is it really more than a diversifier?" *Finance Research Letters*, vol. 20, pp. 192–198, 2017.

[21] A. Kliber, P. Marszałek, I. Musiałkowska, and K. Świerczyńska, "Bitcoin: safe haven, hedge or diversifier? Perception of bitcoin in the context of a countrys economic situation—a stochastic volatility approach," *Physica A: Statistical Mechanics and Its Applications*, vol. 524, pp. 246–257, 2019.

[22] A. H. Dyhrberg, "Bitcoin, gold and the dollar–A GARCH volatility analysis," *Finance Research Letters*, vol. 16, pp. 85–92, 2016.

[23] S. J. Shahzad, E. Bouri, D. Roubaud, and L. Kristoufek, "Safe haven, hedge and diversification for G7 stock markets: gold versus bitcoin," *Economic Modelling*, vol. 87, pp. 212–224, 2020.

[24] T. Conlon and R. McGee, "Safe haven or risky hazard? Bitcoin during the COVID-19 bear market," *Finance Research Letters*, vol. 35, Article ID 101607, 2020.

[25] W. Yu, K. Yang, Y. Wei, and L. Lei, "Measuring Value-at-Risk and Expected Shortfall of crude oil portfolio using extreme value theory and vine copula," *Physica A: Statistical Mechanics and Its Applications*, vol. 490, pp. 1423–1433, 2018.

[26] J. H. Kwon, "Tail behavior of Bitcoin, the dollar, gold and the stock market index," *Journal of International Financial Markets, Institutions and Money*, vol. 67, Article ID 101202, 2020.

[27] R. Chemkha, A. BenSaïda, and A. Ghorbel, "Connectedness between cryptocurrencies and foreign exchange markets: implication for risk management," *Journal of Multinational Financial Management*, vol. 59, 2021.

[28] A. A. Balkema and L. De Haan, "Residual life time at great age," *The Annals of Probability*, vol. 2, no. 5, pp. 792–804, 1974.

[29] J. Pickands III, "Statistical inference using extreme order statistics," *The Annals of Statistics*, pp. 119–131, 1975.

[30] T. Bedford and R. M. Cooke, "Vines: a new graphical model for dependent random variables," *Annals of Statistics*, vol. 30, no. 4, pp. 1031–1068, 2002.

[31] K. Aas, C. Czado, A. Frigessi, and H. Bakken, "Pair-copula constructions of multiple dependence," *Insurance: Mathematics and Economics*, vol. 44, no. 2, pp. 182–198, 2009.

[32] J. Janekova, J. Fabianova, and M. Fabian, "Assessment of economic efficiency and risk of the project using simulation," *International Journal of Simulation Modelling*, vol. 18, no. 2, pp. 242–253, 2019.

[33] J. Janekova, J. Fabianova, and J. Kadarova, "Selection of optimal investment variant based on Monte Carlo simulations," *International Journal of Simulation Modelling*, vol. 20, no. 2, pp. 279–290, 2021.

[34] P. Kupiec, "Techniques for verifying the accuracy of risk measurement models," *The Journal of Derivatives*, vol. 3, no. 2, pp. 173–184, 1995.

[35] P. Christoffersen and D. Pelletier, "Backtesting value-at-risk: a duration-based approach," *Journal of Financial Econometrics*, vol. 2, no. 1, pp. 84–108, 2004.

[36] R. T. Rockafellar and S. Uryasev, "Conditional value-at-risk for general loss distributions," *Journal of Banking & Finance*, vol. 26, no. 7, pp. 1443–1471, 2002.

[37] C. Acerbi and D. Tasche, "On the coherence of expected shortfall," *Journal of Banking & Finance*, vol. 26, no. 7, pp. 1487–1503, 2002.

[38] A. J. McNeil and R. Frey, "Estimation of tail-related risk measures for heteroscedastic financial time series: an extreme value approach," *Journal of Empirical Finance*, vol. 7, no. 3-4, pp. 271–300, 2000.

[39] W. A. Broock, J. A. Scheinkman, W. D. Dechert, and B. LeBaron, "A test for independence based on the correlation dimension," *Econometric Reviews*, vol. 15, no. 3, pp. 197–235, 1996.

[40] W. H. DuMouchel, "Estimating the stable index \$\alpha\$ in order to measure tail thickness: a critique," *Annals of Statistics*, vol. 11, no. 4, pp. 1019–1031, 1983.

[41] R. Selmi, W. Mensi, S. Hammoudeh, and J. Bouoiyour, "Is Bitcoin a hedge, a safe haven or a diversifier for oil price movements? A comparison with gold," *Energy Economics*, vol. 74, pp. 787–801, 2018.

[42] A. Azimli, "The impact of COVID-19 on the degree of dependence and structure of risk-return relationship: a quantile regression approach," *Finance Research Letters*, vol. 36, Article ID 101648, 2020.

[43] Z. Umar, F. Jareño, and A. Escribano, "Dynamic return and volatility connectedness for dominant agricultural commodity markets during the COVID-19 pandemic era," *Applied Economics*, vol. 54, no. 9, pp. 1030–1054, 2022.