

Research Article

Connectivity Index in Vague Graphs with Application in Construction

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The vague graph (VG), which has recently gained a place in the family of fuzzy graph (FG), has shown good capabilities in the face of problems that cannot be expressed by fuzzy graphs and interval-valued fuzzy graphs. Connectivity index (CI) in graphs is a fundamental issue in fuzzy graph theory that has wide applications in the real world. The previous definitions' limitations in the connectivity of fuzzy graphs directed us to offer new classifications in vague graph. Hence, in this paper, we investigate connectivity index, average connectivity index, and Randic index in vague graphs with several examples. Also, one of the motives of this research is to introduce some special types of vertices such as vague connectivity enhancing vertex, vague connectivity reducing vertex, and vague connectivity neutral vertex with their properties. Finally, an application of connectivity index in the selected town for building hospital is presented.

1. Introduction

The FG concept serves as one of the most dominant and extensively employed tools for multiple real-world problem representations, modeling, and analysis. To specify the objects and the relations between them, the graph vertices or nodes and edges or arcs are applied, respectively. Graphs have long been used to describe objects and the relationship between them. Many of the issues and phenomena around us are associated with complexities and ambiguities that make it difficult to express certainty. These difficulties were alleviated by the introduction of fuzzy sets by Zadeh [1]. This concept established well-grounded allocation membership degree to elements of a set. Actually, fuzzy set theory is one of the best and most powerful tools for modeling problems in examining the relationship between uncertainties in the real world. Rosenfeld [2] proposed the idea of FG in 1975. Kauffman [3] represented FGs based on Zadeh's fuzzy relation [4, 5]. Bhutani and Rosenfeld [6] introduced the concept of strong edges. Bhattacharya [7] presented some observations on FGs, and some operations on FGs were described by Mordeson and Peng [8]. The existence of a

single degree for a true membership could not resolve the ambiguity on uncertain issues, so the need for a degree of membership was felt. Afterward, to overcome the existing ambiguities, Gau and Buehrer [9] gave false membership degrees and defined a vague set as the sum of degrees not greater than 1. Ramakrishna in [10] proposed the vague graph concept. New VG concept was introduced and analyzed by Borzooei et al. [11–13]. The vague graph, which has recently gained a place in the family of FG, has shown good capabilities in the face of problems that cannot be expressed by FGs. A vague graph is referred to as a generalized structure of an FG that conveys more exactness, adaptability, and compatibility to a system when coordinated with systems running on FGs. Furthermore, a VG is able to concentrate on determining the uncertainly coupled with the inconsistent and indeterminate information of any real-world problem, where FGs may not lead to adequate results. Connectivity index is one of the most important topics that has many applications in dynamic detection of competition condition in parallel programming, finding phylogenetic trees based on protein domain information, and social networks. The connectivity index problem can also be used

to model many real-world situations in the fields of circuit design, telecommunications, network flow, and so on. Binu et al. [14] introduced connectivity index of a fuzzy graph. Mathew and Sunitha [15] defined several types of arc in fuzzy graph. Naeem et al. [16] investigated connectivity indices of intuitionistic fuzzy graphs (IFGs). Poulik and Ghorai [17] studied indices of graphs under bipolar fuzzy environment. Sebastian et al. [18] presented connectivity parameters in generalized fuzzy graphs. Several concepts and results in VGs were proposed and investigated by Akram et al. [19, 20]. Ghorai et al. [21] studied regular product vague graphs and product vague line graphs. References [22, 23] introduced competition-FGs and some remarks on bipolar-FGs. References [24–29] investigated several concepts on FGs and VGs. Kou et al. [30] studied g-eccentric node and vague detour g-boundary nodes in VGs. Rashmanlou et al. [31–33] described categorical properties and paired domination in IFGs. Kosari et al. [34] introduced the restrained K-rainbow reinforcement number of graphs.

In this paper, we investigated connectivity index, average connectivity index, and Randic index in VGs with several examples. Likewise, we introduced some special types of nodes such as vague connectivity enhancing node, vague connectivity reducing node, and vague connectivity neutral node with their properties. Finally, an application of connectivity index in construction is presented.

2. Preliminaries

A FG $G = (V, \nu, \eta)$ is a nonempty set V together with a pair of functions $\nu: V \rightarrow [0, 1]$ and $\eta: V \times V \rightarrow [0, 1]$ such that $\eta(ab) \leq \min\{\nu(a), \nu(b)\}$, for all $a, b \in V$. Here, η is a symmetric fuzzy relation on $V \times V$.

Definition 1 (see [9]). A VS M is a pair (t_M, f_M) on set V where t_M and f_M are taken as real valued functions which can be defined on $V \rightarrow [0, 1]$, so that $t_M(a) + f_M(a) \leq 1$, for all $a \in V$.

Definition 2 (see [10]). $G = (M, N)$ is called a VG on a crisp graph G^* , in which $M = (t_M, f_M)$ is a VS on V and $N = (t_N, f_N)$ is a VS on $E \subseteq V \times V$ so that $t_N(ab) \leq \min(t_M(a), t_M(b))$ and $f_N(ab) \geq \max(f_M(a), f_M(b))$, $\forall ab \in E$.

Definition 3 (see [11]). Let $G = (M, N)$ be a VG.

- (i) A VG $G' = (M', N')$ is said to be a PVSG of G if $t_{M'}(a) \leq t_M(a)$, $f_{M'}(a) \geq f_M(a)$, $\forall a \in V$ and $t_{N'}(ab) \leq t_N(ab)$, $f_{N'}(ab) \geq f_N(ab)$, for each edge $ab \in G$.

- (ii) A VG $G' = (M', N')$ is said to be a VSG of G if $t_M(a) = t_{M'}(a)$, $f_M(a) = f_{M'}(a)$, $\forall a \in V$ and $t_{N'}(ab) = t_N(ab)$, $f_{N'}(ab) = f_N(ab)$, for each edge $ab \in G'$.

Definition 4 (see [11]). Let $G = (M, N)$ be a VG.

- (i) A path $p: a = a_0, a_1, \dots, a_{k-1}, a_k = b$ in G is a sequence of distinct nodes where $t_N(a_{i-1}a_i) > 0$, $f_N(a_{i-1}a_i) > 0$, $i = 1, 2, \dots, k$. The length of P is k .
- (ii) If $p: a = a_0, a_1, \dots, a_{k-1}, a_k = b$ is a path between a and b of length k , then $(t_N(ab))^k = \sup \{t_N(aa_1) \wedge t_N(a_1a_2) \wedge \dots \wedge t_N(a_{k-1}b)\}$ and $(f_N(ab))^k = \inf \{f_N(aa_1) \vee f_N(a_1a_2) \vee \dots \vee f_N(a_{k-1}b)\}$.

$\text{CONN}_G(a, b) = (\text{CONN}_G^t(a, b), \text{CONN}_G^f(a, b)) = ((t_N(ab))^\infty, (f_N(ab))^\infty)$ is called the strength of connectedness between any two nodes a and b in G where $\text{CONN}_G^t(a, b) = \sup_{k \in \mathbb{N}} \{(t_N(ab))^k\}$ and $\text{CONN}_G^f(ab) = \inf_{k \in \mathbb{N}} \{(f_N(ab))^k\}$. If $t_N(ab) > 0$ and $f_N(ab) > 0$, $\forall (a, b)$ in G , then the VG G is called a CVG. If $t_N(ab) = \min \{t_M(a), t_M(b)\}$ and $f_N(ab) = \max \{f_M(a), f_M(b)\}$, $\forall a, b \in V$, then G is named a complete VG.

Definition 5 (see [11]). For a VG G , if $t_N(ab) \geq (t_N(ab))^\infty$ and $f_N(ab) \leq (f_N(ab))^\infty$, then the arc (ab) is named a strong arc of G .

Definition 6 (see [11]). Let $G = (M, N)$ be a CVG.

- (i) G is called a VT if G has a VSSG $G' = (A', B')$ which is also a tree and for all arcs (ab) not in G , $(t_{N'}(ab))^\infty < (t_N(ab))^\infty$ and $(f_{N'}(ab))^\infty > (f_N(ab))^\infty$.
- (ii) An edge (ab) is said to be a VB if $(t_N(ab))^\infty > (t_{N'}(ab))^\infty$ and $(f_N(ab))^\infty < (f_{N'}(ab))^\infty$.
- (iii) G is called a vague cycle if G^* is a cycle and there does not exist unique edge (ab) in G for which $t_N(ab) = \min \{t_N(a_1b_1) \mid (a_1b_1) \in E\}$ and $f_N(ab) = \max \{f_N(a_1b_1) \mid (a_1b_1) \in E\}$.

All the basic notations are shown in Table 1.

3. Connectivity Index of a Vague Graph

Definition 7. The CI of a VG G denoted by $\text{CI}_{\text{VG}}(G)$ is defined as

$$\text{CI}_{\text{VG}}(G) = (\text{CI}_{\text{VG}}^t(G), \text{CI}_{\text{VG}}^f(G)) = \left(\sum_{a,b \in V} t_M(a)t_M(b) \cdot \text{CONN}_G^t(a, b), \sum_{a,b \in V} f_M(a)f_M(b) \cdot \text{CONN}_G^f(a, b) \right), \quad (1)$$

TABLE 1: Some basic notations.

Notation	Meaning
FG	Fuzzy graph
VG	Vague graph
CVG	Connected vague graph
VS	Vague set
MV	Membership value
PVSG	Partial vague subgraph
VSG	Vague subgraph
VT	Vague tree
ACI	Average connectivity index
VCRN	Vague connectivity reducing node
VCEN	Vague connectivity enhancing node
VCNN	Vague connectivity neutral node
CEVG	Connectivity enhancing vague graph
CRVG	Connectivity reducing vague graph
NVG	Neutral vague graph
OND	Open neighborhood degree
RI	Randic index
CI	Connectivity index

where $CI_{VG}^t(G)$ and $CI_{VG}^f(G)$, respectively, denote the true-CI and false-CI of G .

Example 1. Consider the CVG G of the graph G^* shown in Figure 1, where $V = \{a, b, c, d, e\}$ and $E = \{(a, b), (b, c), (c,$

$d), (d, e), (a, e), (b, d)\}$. Here, we suppose that the true-MV and false-MV of every node are 0.2 and 0.5, respectively. Here,

$$\begin{aligned}
 & \text{CONN}_G^t(a, b) = 0.1, \quad \text{CONN}_G^t(a, c) = 0.1, \quad \text{CONN}_G^t(a, d) = 0.1, \\
 & \text{CONN}_G^t(a, e) = 0.2, \quad \text{CONN}_G^t(b, c) = 0.2, \quad \text{CONN}_G^t(b, d) = 0.2, \\
 & \text{CONN}_G^t(b, e) = 0.1, \quad \text{CONN}_G^t(c, d) = 0.2, \quad \text{CONN}_G^t(c, e) = 0.1, \\
 & \text{CONN}_G^t(d, e) = 0.1. \\
 & \text{CONN}_G^f(a, b) = 0.6, \quad \text{CONN}_G^f(a, c) = 0.6, \quad \text{CONN}_G^f(a, d) = 0.6, \\
 & \text{CONN}_G^f(a, e) = 0.6, \quad \text{CONN}_G^f(b, c) = 0.5, \quad \text{CONN}_G^f(b, d) = 0.5, \\
 & \text{CONN}_G^f(b, e) = 0.6, \quad \text{CONN}_G^f(c, d) = 0.5, \quad \text{CONN}_G^f(c, e) = 0.6, \\
 & \text{CONN}_G^f(d, e) = 0.6.
 \end{aligned} \tag{2}$$

$$CI_{VG}^t(G) = 0.004 + 0.004 + 0.004 + 0.008 + 0.008 + 0.008 + 0.004 + 0.008 + 0.004 + 0.004 = 0.056,$$

$$CI_{VG}^f(G) = 0.15 + 0.15 + 0.15 + 0.15 + 0.125 + 0.125 + 0.15 + 0.125 + 0.15 + 0.15 = 1.425.$$

Hence, $CI_{VG}(G) = (CI_{VG}^t(G), CI_{VG}^f(G)) = (0.056, 1.425)$

Theorem 1. For a PVSG $G' = (M', N')$ of a VG $G = (M, N)$, $CI_{VG}^t(G') \leq CI_{VG}^t(G)$ and $CI_{VG}^f(G') \geq CI_{VG}^f(G)$.

Proof. Let $G' = (M', N')$ be a PVSG of $G = (M, N)$. Hence, $t_{M'}(a) \leq t_M(a)$, $f_{M'}(a) \geq f_M(a)$, $\forall a \in G'$ and $t_{N'}(ab) \leq t_N(ab)$, $f_{N'}(ab) \geq f_N(ab)$, $\forall ab \in G'$. Since $\text{CONN}_{G'}^t(a, b)$ and $\text{CONN}_G^t(a, b)$ lie on one or many edges of G' and $\text{CONN}_G^t(a, b)$ and $\text{CONN}_G^f(a, b)$ lie on one or

many edges of G and $t_{N'}(ab) \leq t_N(ab)$, $f_{N'}(ab) \geq f_N(ab)$, $\forall ab \in G'$, and hence we have

$$\begin{aligned}
 \text{CONN}_{G'}^t(a, b) & \leq \text{CONN}_G^t(a, b), \text{CONN}_{G'}^f(a, b) \\
 & \geq \text{CONN}_G^f(a, b), \forall (a, b) \in G'.
 \end{aligned} \tag{3}$$

Since

$$\begin{aligned}
 f_{M'}(a)f_{M'}(b) & \geq 0, \\
 f_M(a)f_M(b) & \geq 0.
 \end{aligned} \tag{4}$$

So,

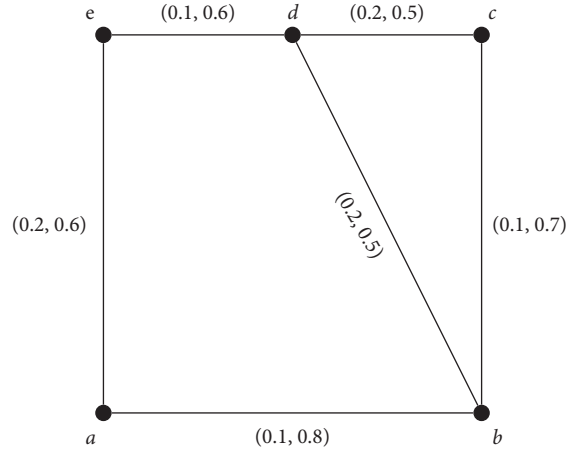


FIGURE 1: Connected VG G.

$$\begin{aligned} t_{M'}(a)t_{M'}(b)\text{CONN}_{G'}^t(a,b) &\leq t_M(a)t_M(b)\text{CONN}_G^t(a,b), \\ f_{M'}(a)f_{M'}(b)\text{CONN}_{G'}^f(a,b) &\geq f_M(a)f_M(b)\text{CONN}_G^f(a,b). \end{aligned} \quad (5)$$

Therefore,

$$\begin{aligned} &\sum_{a,b \in V'} t_{M'}(a)t_{M'}(b)\text{CONN}_{G'}^t(a,b) \\ &\leq \sum_{a,b \in V} t_M(a)t_M(b)\text{CONN}_G^t(a,b) \end{aligned} \quad (6)$$

$$\begin{aligned} &\sum_{a,b \in V'} f_{M'}(a)f_{M'}(b)\text{CONN}_{G'}^f(a,b) \\ &\geq \sum_{a,b \in V} f_M(a)f_M(b)\text{CONN}_G^f(a,b). \end{aligned} \quad (7)$$

Therefore, $\text{CI}_{\text{VG}}^t(G') \leq \text{CI}_{\text{VG}}^t(G)$ and $\text{CI}_{\text{VG}}^f(G') \leq \text{CI}_{\text{VG}}^f(G)$. \square

Remark 1. For a VSG $G' = (M', N')$ of a VG G , $\text{CI}_{\text{VG}}^t(G') \leq \text{CI}_{\text{VG}}^t(G)$ and $\text{CI}_{\text{VG}}^f(G') \leq \text{CI}_{\text{VG}}^f(G)$.

Example 2. Let G' be a CVSG of the VG G as in Example 2 (see Figure 2). Here,

$$\begin{aligned} \text{CI}_{\text{VG}}^t(G') &= 0.004 + 0.004 + 0.008 + 0.008 + 0.004 + 0.004 = 0.032, \\ \text{CI}_{\text{VG}}^f(G') &= 0.15 + 0.15 + 0.15 + 0.125 + 0.15 + 0.15 = 0.875. \end{aligned} \quad (8)$$

So, $\text{CI}_{\text{VG}}(G') = (0.032, 0.875)$. Therefore, Example 2 shows that $\text{CI}_{\text{VG}}^t(G') \leq \text{CI}_{\text{VG}}^t(G)$ and $\text{CI}_{\text{VG}}^f(G') \leq \text{CI}_{\text{VG}}^f(G)$.

Theorem 2. If $G' = (M', N')$ is a VSG of a CVG $G = (M, N)$, where $V' = V - \{s\}$, $s \in V$, then $\text{CI}_{\text{VG}}^t(G') < \text{CI}_{\text{VG}}^t(G)$ and $\text{CI}_{\text{VG}}^f(G') < \text{CI}_{\text{VG}}^f(G)$.

Proof. Let G have n nodes, i.e., $|V| = n$ and $s \in V$. Then, $V' = V - \{s\}$ i.e., $|V'| = n - 1$. Suppose $V = \{a_1, a_2, \dots, a_n = s\}$. Hence, $V' = \{a_1, a_2, \dots, a_{n-1}\}$. Since G' is VSG of G , G' must be a PVS of G . So,

$$\begin{aligned} \text{CI}_{\text{VG}}^t(G) &= \text{CI}_{\text{VG}}^t(G') + \sum_{i=1}^{n-1} t_M(a_n)t_M(a_i)\text{CONN}_G^t(a_n, a_i), \\ \text{CI}_{\text{VG}}^f(G) &= \text{CI}_{\text{VG}}^f(G') + \sum_{i=1}^{n-1} f_M(a_n)f_M(a_i)\text{CONN}_G^f(a_n, a_i). \end{aligned} \quad (9)$$

Thus, $\text{CI}_{\text{VG}}^t(G') < \text{CI}_{\text{VG}}^t(G)$ and $\text{CI}_{\text{VG}}^f(G') < \text{CI}_{\text{VG}}^f(G)$. \square

Theorem 3. If G is a complete VG and $V = \{a_1, a_2, \dots, a_n\}$ so that $t_M(a_i) = p_n$ and $f_M(a_i) = q_i$, $i = 1, 2, \dots, n$, where

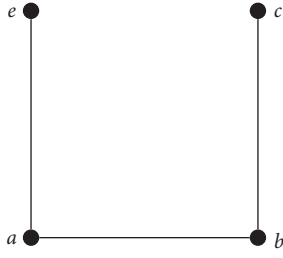


FIGURE 2: Connected VSG G' .

$p_1 \leq p_2 \leq \dots \leq p_n$ and $q_1 \geq q_2 \geq \dots \geq q_n$, then $CI_{VG}^t(G) = \sum_{i=1}^{n-1} p_i^2 \sum_{j=i+1}^n p_j$ and $CI_{VG}^f(G) = \sum_{i=1}^{n-1} q_i^2 \sum_{j=i+1}^n q_j$.

Proof. Let $a_1 \in V$, $t_M(a_1) = p_1$, and $f_M(a_1) = q_1$. Since G is complete, $CONN_G^t(a_i, a_j) = t_M(a_i, a_j)$ and $CONN_G^f(a_i, a_j) = f_M(a_i, a_j)$. Then, $CONN_G^t(a_1, a_j) = t_M(a_1) = p_1$, $CONN_G^f(a_1, a_j) = f_M(a_1) = q_1$, $j > 1$. Since $p_1 \leq p_2 \leq \dots \leq p_n$ and $q_1 \geq q_2 \geq \dots \geq q_n$, then $t_M(a_1) \leq t_M(a_2) \leq \dots \leq t_M(a_n)$ and $f_M(a_1) \geq f_M(a_2) \geq \dots \geq f_M(a_n)$. Therefore,

$$\begin{aligned} t_M(a_1)t_M(a_j)CONN_G^t(a_1, a_j) &= p_1 p_j p_1 = p_1^2 p_j, \\ f_M(a_1)f_M(a_j)CONN_G^f(a_1, a_j) &= q_1 q_j q_1 = q_1^2 q_j. \end{aligned} \tag{10}$$

It shows that

$$\begin{aligned} \sum_{j=2}^n t_M(a_1)t_M(a_j)CONN_G^t(a_1, a_j) &= \sum_{j=2}^n p_1^2 p_j, \\ \sum_{j=2}^n f_M(a_1)f_M(a_j)CONN_G^f(a_1, a_j) &= \sum_{j=2}^n q_1^2 q_j. \end{aligned} \tag{11}$$

Therefore,

$$\begin{aligned} \sum_{i=1}^{n-1} \sum_{j=2}^n t_M(a_i)t_M(a_j)CONN_G^t(a_i, a_j) &= \sum_{i=1}^{n-1} \sum_{j=2}^n p_i^2 p_j, \\ \sum_{i=1}^{n-1} \sum_{j=2}^n f_M(a_i)f_M(a_j)CONN_G^f(a_i, a_j) &= \sum_{i=1}^{n-1} \sum_{j=2}^n q_i^2 q_j. \end{aligned} \tag{12}$$

Thus, $CI_{VG}^t(G) = \sum_{i=1}^{n-1} \sum_{j=i+1}^n p_j$ and $CI_{VG}^f(G) = \sum_{i=1}^{n-1} \sum_{j=i+1}^n q_j$. \square

Example 3. Consider the complete VG G shown in Figure 3 where $V = \{a, b, c\}$. Here, $t_M(v_i) = p_i$ and $f_M(v_i) = q_i$, for $i = 1, 2, 3$.

$t_M(a) = p_1 = 0.1$, $f_M(a) = q_1 = 0.4$, $t_M(b) = p_2 = 0.2$, $f_M(b) = q_2 = 0.3$, $t_M(c) = p_3 = 0.3$, $f_M(c) = q_3 = 0.2$.

Hence, $p_1 \leq p_2 \leq p_3$ and $q_1 \geq q_2 \geq q_3$.

$$\begin{aligned} CI_{VG}^t(G) &= \sum_{i=1}^2 p_i^2 \sum_{j=i+1}^3 p_j \\ &= p_1^2 p_2 + p_1^2 p_3 + p_2^2 p_3 \\ &= (0.1)^2 (0.2) + (0.1)^2 (0.3) + (0.2)^2 (0.3) = 0.017, \end{aligned} \tag{13}$$

$$\begin{aligned} CI_{VG}^f(G) &= \sum_{i=1}^2 q_i^2 \sum_{j=i+1}^3 q_j \\ &= q_1^2 q_2 + q_1^2 q_3 + q_2^2 q_3 \\ &= 0.048 + 0.032 + 0.018 = 0.098. \end{aligned} \tag{14}$$

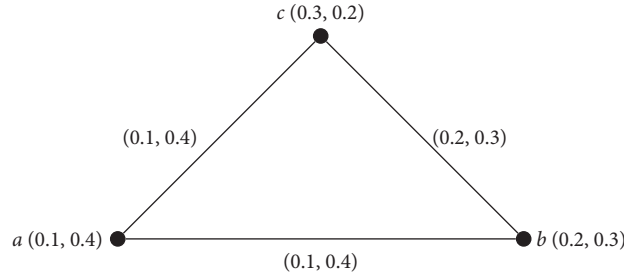
Thus, $CI_{VG}(G) = (CI_{VG}^t(G), CI_{VG}^f(G)) = (0.017, 0.098)$.

Now we define average connectivity index (ACI) of a VG.

Definition 8. The ACI of a VG $G = (M, N)$ denoted by $ACI_{VG}(G)$ is defined as

$$\begin{aligned} ACI_{VG}(G) &= (ACI_{VG}^t(G), ACI_{VG}^f(G)) \\ &= \frac{1}{\binom{n}{2}} \left[\sum_{a,b \in V} t_M(a)t_M(b)CONN_G^t(a, b), \sum_{a,b \in V} f_M(a)f_M(b)CONN_G^f(a, b) \right]. \end{aligned} \tag{15}$$

Example 4. From Example 2, for VG G , the ACI of G is

FIGURE 3: A complete VG G .

$$\begin{aligned} \text{ACI}_{\text{VG}}(G) &= (\text{ACI}_{\text{VG}}^t(G), \text{ACI}_{\text{VG}}^f(G)) = \left(\frac{0.056}{\binom{5}{2}}, \frac{1.425}{\binom{5}{2}} \right) \\ &= \left(\frac{0.056}{10}, \frac{1.425}{10} \right) = (0.0056, 0.1425). \end{aligned}$$

(16)

- (i) VCRN if $\text{AC}_{\text{VG}}^t(G - \{a\}) < \text{AC}_{\text{VG}}^t(G)$ and $\text{ACI}_{\text{VG}}^f(G - \{a\}) > \text{ACI}_{\text{VG}}^f(G)$.
- (ii) VCEN if $\text{ACI}_{\text{VG}}^t(G - \{a\}) > \text{ACI}_{\text{VG}}^t(G)$ and $\text{ACI}_{\text{VG}}^f(G - \{a\}) < \text{ACI}_{\text{VG}}^f(G)$.
- (iii) VCNN if $\text{ACI}_{\text{VG}}^t(G - \{a\}) = \text{ACI}_{\text{VG}}^t(G)$ and $\text{ACI}_{\text{VG}}^f(G - \{a\}) = \text{ACI}_{\text{VG}}^f(G)$.

Example 5. Consider the VG G shown in Figure 4. We have

Definition 9. Let $G = (M, N)$ be a CVG. A node a is called a

$$\begin{aligned} \text{CONN}_G^t(a, b) &= 0.1, \text{CONN}_G^t(a, c) = 0.1, \text{CONN}_G^t(a, d) = 0.1, \\ \text{CONN}_G^t(b, c) &= 0.2, \text{CONN}_G^t(b, d) = 0.2, \text{CONN}_G^t(c, d) = 0.2, \\ \text{CONN}_G^f(a, b) &= 0.4, \text{CONN}_G^f(a, c) = 0.5, \text{CONN}_G^f(a, d) = 0.4, \\ \text{CONN}_G^f(b, c) &= 0.5, \text{CONN}_G^f(b, d) = 0.4, \text{CONN}_G^f(c, d) = 0.5. \end{aligned} \tag{17}$$

Hence,

$$\text{CI}_{\text{VG}}^t(G) = 0.004 + 0.004 + 0.004 + 0.008 + 0.008 + 0.008 = 0.036,$$

$$\text{CI}_{\text{VG}}^f(G) = 0.036 + 0.045 + 0.045 + 0.036 + 0.045 + 0.036 + 0.045 = 0.243.,$$

$$\text{ACI}_{\text{VG}}^t(G) = \frac{0.036}{6} = 0.006, \quad \text{ACI}_{\text{VG}}^f(G) = 0.040,$$

$$\text{ACI}_{\text{VG}}^t(G - \{a\}) = 0.008, \quad \text{ACI}_{\text{VG}}^f(G - \{a\}) = 0.039,$$

$$\text{ACI}_{\text{VG}}^t(G - \{b\}) = 0.005, \quad \text{ACI}_{\text{VG}}^f(G - \{b\}) = 0.042,$$

$$\text{ACI}_{\text{VG}}^t(G - \{c\}) = 0.005, \quad \text{ACI}_{\text{VG}}^f(G - \{c\}) = 0.036,$$

$$\text{ACI}_{\text{VG}}^t(G - \{d\}) = 0.005, \quad \text{ACI}_{\text{VG}}^f(G - \{d\}) = 0.042.$$

(18)

Hence, a is a VCEN of G and b and d are VCRN of G .

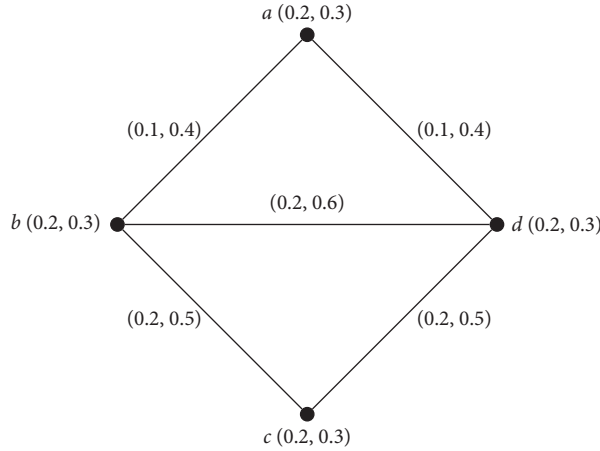


FIGURE 4: A connected VG G .

Theorem 4. Let $G = (M, N)$ be a complete VG with $|V| = n \geq 3$, and $p_1 = CI_{VG}^t(G)/CI_{VG}^t(G - \{a\})$, $p_2 = CI_{VG}^f(G)/CI_{VG}^f(G - \{a\})$, $a \in V$. The node a is a

- (i) VCEN if and only if $p_1 \leq n/n - 2$ and $p_2 > n/n - 2$.
- (ii) VCRN if and only if $p_1 > n/n - 2$ and $p_2 < n/n - 2$.
- (iii) VCNN if and only if $p_1 = n/n - 2$ and $p_2 = n/n - 2$.

Proof. (i) Assume that a is a VCEN. Then, Definition 9 shows that

$$\begin{aligned} ACI_{VG}^t(G) &< ACI_{VG}^t(G - \{a\}), \\ ACI_{VG}^f(G) &> ACI_{VG}^f(G - \{a\}). \end{aligned} \tag{19}$$

So, $CI_{VG}^t(G)/(n/2) < CI_{VG}^t(G - \{a\})/(n - 1/2)$ and $CI_{VG}^f(G)/(n/2) < CI_{VG}^f(G - \{a\})/(n - 1/2)$. Hence, $CI_{VG}^t(G)/CI_{VG}^t(G - \{a\}) < (n/2)/(n - 1/2)$ and $CI_{VG}^f(G)/CI_{VG}^f(G - \{a\}) > (n/2)/(n - 1/2)$. Thus, $p_1 < n/n - 2$ and $p_2 > n/n - 2$.

Conversely, let $p_1 < n/n - 2$ and $p_2 > n/n - 2$. So, $CI_{VG}^t(G)/CI_{VG}^t(G - \{a\}) < n/n - 2$ and $CI_{VG}^f(G)/CI_{VG}^f(G - \{a\}) > n/n - 2$. Hence, $CI_{VG}^t(G)/(n/2) < CI_{VG}^t(G - \{a\})/(n - 1/2)$ and $CI_{VG}^f(G)/(n/2) > CI_{VG}^f(G - \{a\})/(n - 1/2)$. Thus, $ACI_{VG}^t(G) < ACI_{VG}^t(G - \{a\})$ and $ACI_{VG}^f(G) > ACI_{VG}^f(G - \{a\})$. Therefore, a is a VCEN of G . In the same way, (ii) and (iii) can be proved. \square

Theorem 5. Let $G = (M, N)$ be a complete VG with $|V| = n \geq 3$. Suppose $L^t = \sum_{a \in V - \{b\}} t_M(a)t_M(b)CONN_G^t(a, b)$, $L^f = \sum_{a \in V - \{b\}} f_M(a)f_M(b)CONN_G^f(a, b)$, and b is an end node of G . Then,

- (i) $L^t < 2/n - 2$ and $L^f > 2/2 - n$ iff b is a VCEN.
- (ii) $L^t > 2/n - 2$ and $L^f < 2/2 - n$ iff b is a VCRN.
- (iii) $L^t = 2/n - 2$ and $L^f = 2/2 - n$ iff b is a VCNN.

Proof. (i) Let $L^t < 2/2 - n$ and $L^f > 2/2 - n$. Now,

$$\begin{aligned} CI_{VG}^t(G) &= CI_{VG}^t(G - \{b\}) + \sum_{a \in V - \{b\}} t_M(a)t_M(b)CONN_G^t(a, b), \\ CI_{VG}^f(G) &= CI_{VG}^f(G - \{b\}) + \sum_{a \in V - \{b\}} f_M(a)f_M(b)CONN_G^f(a, b). \end{aligned} \tag{20}$$

So, $CI_{VG}^t(G) = CI_{VG}^t(G - \{b\}) + L^t$ and $CI_{VG}^f(G) = CI_{VG}^f(G - \{b\}) + L^f$. Hence, $CI_{VG}^t(G)/(n/2) = CI_{VG}^t(G - \{b\})/(n/2) + L^t/(n/2)$ and $CI_{VG}^f(G)/(n/2) = CI_{VG}^f(G - \{b\})/(n/2) + L^f/(n/2)$. Thus, $ACI_{VG}^t(G) < CI_{VG}^t(G - \{b\})/(n - 1/2) \cdot (n - 2)/n + 2/n - 2/(n/2)$ and $ACI_{VG}^f(G) > CI_{VG}^f(G - \{b\})/(n - 1/2) \cdot (n - 2)/n + 2/2 - n/(n/2)$. Therefore,

$$\begin{aligned} ACI_{VG}^t(G) &< ACI_{VG}^t(G - \{b\}) - \frac{2}{n} \left[ACI_{VG}^t(G - \{b\}) - \frac{2}{(n - 1)(n - 2)} \right], \\ ACI_{VG}^f(G) &> ACI_{VG}^f(G - \{b\}) - \frac{2}{n} \left[ACI_{VG}^f(G - \{b\}) + \frac{2}{(n - 1)(n - 2)} \right]. \end{aligned} \tag{21}$$

So, $ACI_{VG}^t(G) < ACI_{VG}^t(G - \{b\})$ and $ACI_{VG}^f(G) > ACI_{VG}^f(G - \{b\})$.

In the same way, (ii) and (iii) can be proved. \square

Definition 10. Let $G = (M, N)$ be a CVG.

- (i) G is named CEVG, if it has at least one VCEN.
- (ii) G is named CRVG, if it has no VCRN.
- (iii) G is named NVG, if all nodes of G are VCNN.

Example 6. For the VG G in Example 5, the node a is a VCEN of G , so G is a CEVG, but G is neither a CRVG nor a NVG.

Theorem 6. For each pair of numbers (L_1, L_2) , where L_1 is a positive true real number and L_2 is a positive false real number, there always exist VG $G = (M, N)$ so that $CI_{VG}^t(G) = L_1$ and $CI_{VG}^f(G) = L_2$, $|V| = n$.

Proof. Suppose $t_M(a) = 1, \forall a \in V$, and $f_M(a) = 0, \forall a \in V$. Now, we consider a path in G so that $t_N(ab) = L_1/(n/2)$ and $f_N(ab) = L_2/(n/2)$ for all edges $(a, b) \in G$. Hence, we get

$$CI_{VG}^t(G) = \sum_{a,b \in V} t_M(a)t_M(b)CONN_G^t(a,b) = \binom{n}{2} \frac{L_1}{\binom{n}{2}} = L_1,$$

$$CI_{VG}^f(G) = \sum_{a,b \in V} f_M(a)f_M(b)CONN_G^f(a,b) = \binom{n}{2} \frac{L_2}{\binom{n}{2}} = L_2. \quad (22)$$

Definition 11. The OND or degree of a node c in a VG G is described as $\deg(c) = (\deg^t(c), \deg^f(c))$, where $\deg^t(c) = \sum_{c \neq d} t_N(cd)$ and $\deg^f(c) = \sum_{c \neq d} f_N(cd)$. If $\deg(c) = (t_1, t_2), \forall c \in V$, then G is named (t_1, t_2) -regular.

Now, we introduce the RI of a VG with examples. \square

Definition 12. The RI of a VG $G = (M, N)$ is shown by $RI_{VG}(G)$ and described as

$$RI_{VG}(G) = (RI_{VG}^t(G), RI_{VG}^f(G))$$

$$= \left(\sum_{\substack{i \neq j \\ a_i, a_j \in E}} (t_M(a_i)t_M(a_j)\deg^t(a_i)\deg^t(a_j))^{-1/2}, \sum_{\substack{i \neq j \\ a_i, a_j \in E}} (f_M(a_i)f_M(a_j)\deg^f(a_i)\deg^f(a_j))^{-1/2} \right). \quad (23)$$

Example 7. Consider the VG G shown in Figure 5. Here, $\deg(a) = (0.3, 1.3)$, $\deg(b) = (0.6, 1.8)$, $\deg(c) = (0.4, 1.5)$, $\deg(d) = (0.4, 1.7)$, and $\deg(e) = (0.3, 1.1)$.

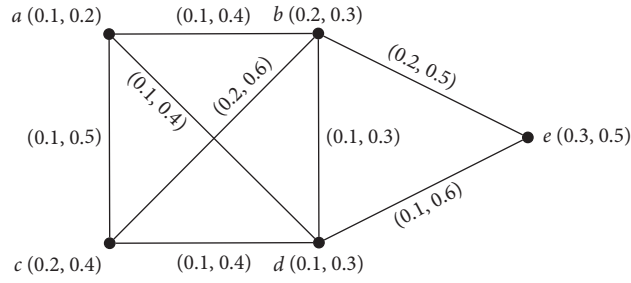


FIGURE 5: VG G.

$$\sum_{\substack{i \neq j \\ v_i, v_j \in E}} (t_M(v_i)t_M(v_j)\text{deg}^t(v_i)\text{deg}^t(v_j))^{-1/2}$$

$$= 16.667 + 20.449 + 28.901 + 10.214$$

$$+ 17.699 + 14.450 + 9.624 + 16.667$$

$$= 134.671,$$

$$\sum_{\substack{i \neq j \\ v_i, v_j \in E}} (f_M(v_i)f_M(v_j)\text{deg}^f(v_i)\text{deg}^f(v_j))^{-1/2}$$

$$= 2.669 + 2.532 + 2.746 + 1.756$$

$$+ 1.807 + 1.905 + 1.835 + 1.889$$

$$= 17.139.$$

Therefore, $RI_{VG}(G) = (RI_{VG}^t(G), RI_{VG}^f(G)) = (134.671, 17.139)$.

Theorem 7. Let $G = (M, N)$ be a connected VG and $G' = (M', N')$ so that $V' = V - \{c_n\}$, $c_n \in V$ with $|V| = n$. Then, $RI_{VG}^t(G) \geq RI_{VG}^t(G')$ and $RI_{VG}^f(G) \geq RI_{VG}^f(G')$.

Proof. G has n nodes, i.e., $V = \{c_1, c_2, \dots, c_n\}$. Then, $V' = \{c_1, c_2, \dots, c_{n-1}\}$. So, V' should be a subset of V , so G' is a VSG of G . Hence, $t_M(c_i) = t_{M'}(c_i)$, $f_M(c_i) = f_{M'}(c_i)$, $t_N(c_i c_j) = t_{N'}(c_i c_j)$, and $f_N(c_i c_j) = f_{N'}(c_i c_j)$, $\forall c_i \in V'$ and $\forall c_i c_j \in E'$.

Now, $\text{deg}^t(c_i) = \sum_{c_j c_i \in E} c_j t_N(c_i c_j)$ and $\text{deg}^f(c_i) = \sum_{c_j c_i \in E} c_j f_N(c_i c_j)$, i.e., $\text{deg}^t(c_i)$ and $\text{deg}^f(c_i)$ are sum of the true-MVs and false-MVs of the edges incident in c_i in G , respectively. Then, $\text{deg}^t(c_i)\text{deg}^f(c_j)$ is a positive real number. So, $\text{deg}^t(c_i) = \sum_{c_j c_i \in E'} c_j t_N(c_i c_j)$ and $\text{deg}^f(c_i) = \sum_{c_j c_i \in E'} c_j f_N(c_i c_j)$, i.e., $\text{deg}^t(c_i)$ and $\text{deg}^f(c_i)$ are sum of the T-MVs and false-MVs of the edges incident in c_i in G' ,

respectively. Then, $\text{deg}^t(c_i)\text{deg}^f(c_j)$ is a positive real number. Thus,

$$(t_M(c_i)t_M(c_j)\text{deg}^t(c_i)\text{deg}^t(c_j))^{-1/2} \geq 0,$$

$$(t_{M'}(c_i)t_{M'}(c_j)\text{deg}^t(c_i)\text{deg}^t(c_j))^{-1/2} \geq 0,$$

$$(f_M(c_i)f_M(c_j)\text{deg}^f(c_i)\text{deg}^f(c_j))^{-1/2} \geq 0,$$

$$(f_{M'}(c_i)f_{M'}(c_j)\text{deg}^f(c_i)\text{deg}^f(c_j))^{-1/2} \geq 0.$$

So,

$$\sum_{\substack{1 \leq i \neq j \leq n \\ c_i, c_j \in E}} (t_M(c_i)t_M(c_j)\text{deg}_M^t(c_i)\text{deg}_M^t(c_j))^{-1/2}$$

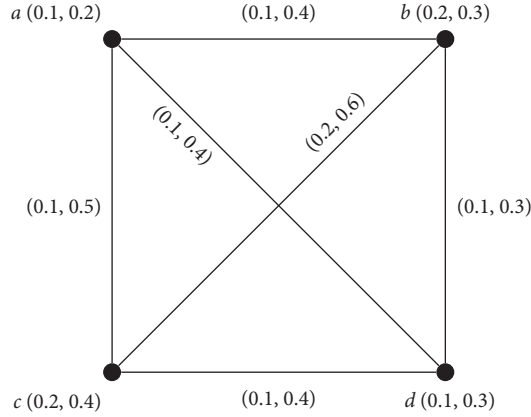
$$\geq \sum_{\substack{1 \leq i \neq j \leq n-1 \\ c_i, c_j \in E'}} (t_{M'}(c_i)t_{M'}(c_j)\text{deg}^t(c_i)\text{deg}^t(c_j))^{-1/2}$$

$$\sum_{\substack{1 \leq i \neq j \leq n \\ c_i, c_j \in E}} (f_M(c_i)f_M(c_j)\text{deg}_M^f(c_i)\text{deg}_M^f(c_j))^{-1/2}$$

$$\geq \sum_{\substack{1 \leq i \neq j \leq n-1 \\ c_i, c_j \in E'}} (f_{M'}(c_i)f_{M'}(c_j)\text{deg}^f(c_i)\text{deg}^f(c_j))^{-1/2}.$$

Therefore, $RI_{VG}^t(G) \geq RI_{VG}^t(G')$ and $RI_{VG}^f(G) \geq RI_{VG}^f(G')$. \square

Example 8. Consider VG $G' = (M', N')$ of Figure 6 and VG G of Figure 5. Clearly, $t_M(v_i) \geq t_{M'}(v_i)$, $f_M(v_i) \leq f_{M'}(v_i)$, $\forall v_i \in V'$, and $t_N(v_i v_j) \geq t_{N'}(v_i v_j)$, $f_N(v_i v_j) \leq f_{N'}(v_i v_j)$, $\forall v_i, v_j \in E'$. Hence, G' is a VSG of the VG G .

FIGURE 6: Vague subgraph G' of the vague graph G of Figure 5.

$$\begin{aligned}
 RI_{VG}^t(G') &= 20.449 + 20.449 + 33.333 + 12.5 + 20.449 + 20.449 \\
 &= 127.629, \\
 RI_{VG}^f(G') &= 3.140 + 2.532 + 3.414 + 2.067 + 2.247 + 2.788 \\
 &= 16.188.
 \end{aligned} \tag{28}$$

Therefore, $RI_{VG}^t(G) \geq RI_{VG}^t(G')$ and $RI_{VG}^f(G) \geq RI_{VG}^f(G')$.

Remark 2. Let $G = (M, N)$ be a connected VG and $G' = (M', N')$ so that $E' = E - \{c_i c_j\}$, $c_i c_j \in E$. Then, $RI_{VG}^t(G) \geq RI_{VG}^t(G')$ and $RI_{VG}^f(G) \geq RI_{VG}^f(G')$.

Proof. Assume G has n nodes, i.e., $|V| = n$ and $c_i \in V$. Let $V = \{c_1, c_2, \dots, c_n\}$. So, $V' = \{c_1, c_2, \dots, c_{i-1}, c_{i+1}, \dots, c_n\}$. Then, V' must be a subset of V , and so G' is a VGS of G . So, $RI_{VG}^t(G) \geq RI_{VG}^t(G')$ and $RI_{VG}^f(G) \geq RI_{VG}^f(G')$. \square

Theorem 8. Let G be a complete VG so that M is a constant function. Then, $RI_{VG}(G) = (n/2\nu_1^2, n/2\nu_2^2)$, where $n = |V|$ and $(\nu_1, \nu_2) = (t_M(c_i), f_M(c_i))$, $c_i \in V$.

Proof. Since M is constant and $t_M(c_i) = \nu_1$, $f_M(c_i) = \nu_2$, $c_i \in V$, then $t_M(c_i) = \nu_1$, $f_M(c_i) = \nu_2$, $\forall c_i \in V$. Since G is a complete VG, $t_N(c_i c_j) = \min\{t_M(c_i), t_M(c_j)\} = \nu_1$ and $f_N(c_i c_j) = \max\{f_M(c_i), f_M(c_j)\} = \nu_2$, $\forall c_i c_j \in E$. Again, G is complete and $|V| = n$, and thus there are $n(n-1)/2$ nodes and $n(n-1)/2$ edges and each node in G is neighbor to $(n-1)$ nodes. Therefore,

$$\begin{aligned}
 \deg^t(c_i) &= \sum_{\substack{c_j \\ c_i c_j \in E}} t_N(c_i c_j) = \nu_1 \nu_1 \dots \nu_1 = (n-1)\nu_1, \\
 \deg^f(c_i) &= \sum_{\substack{c_j \\ c_i c_j \in E}} f_N(c_i c_j) = \nu_2 \nu_2 \dots \nu_2 = (n-1)\nu_2.
 \end{aligned} \tag{29}$$

Hence,

$$\begin{aligned}
 RI_{VG}^t(G) &= \sum_{\substack{i \neq j \\ c_i c_j \in E}} (t_M(c_i) t_M(c_j) \deg^t(c_i) \deg^t(c_j))^{-1/2} \\
 &= \frac{n(n-1)}{2} (\nu_1 \nu_1 (n-1) \nu_1 (n-1) \nu_1)^{-1/2} = \frac{n}{2\nu_1^2}, \\
 RI_{VG}^f(G) &= \sum_{\substack{i \neq j \\ c_i c_j \in E}} (f_M(c_i) f_M(c_j) \deg^f(c_i) \deg^f(c_j))^{-1/2} \\
 &= \frac{n(n-1)}{2} (\nu_2 \nu_2 (n-1) \nu_2 (n-1) \nu_2)^{-1/2} = \frac{n}{2\nu_2^2}.
 \end{aligned} \tag{30}$$

Therefore, $RI_{VG}(G) = (RI_{VG}^t(G), RI_{VG}^f(G)) = (n/2\nu_1^2, n/2\nu_2^2)$. \square

Example 9. Let G be a VG as shown in Figure 7. Here, $V = \{a, b, c, d, e\}$, $n = |V| = 5$, and $t_M(v_i) = 0.3 = \nu_1$, $f_M(v_i) = 0.5 = \nu_2$, $1 \leq i \leq 5$. Hence, M is a constant function.

$$E = \{ab, ac, ad, ae, bc, bd, be, cd, ce, de\}, \tag{31}$$

and $t_N(v_i v_j) = 0.3$, $f_N(v_i v_j) = 0.5$, for all edges $v_i v_j \in E$. Now, $t_N(v_i v_j) = 0.3 = \min\{t_M(v_i), t_M(v_j)\}$ and $f_N(v_i v_j) = 0.5 = \max\{f_M(v_i), f_M(v_j)\}$.

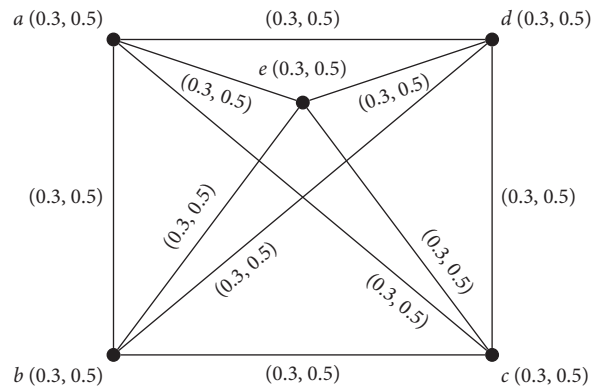


FIGURE 7: A VG G .

$v_j) = 0.5 = \max\{f_M(v_i), f_M(v_j)\}$. So, G is a complete VG. Using Theorem 8,

$$\begin{aligned}
 RI_{VG}^t(G) &= \frac{n}{2v_1^2} = \frac{5}{2(0.3)^2} = 27.77, \\
 RI_{VG}^f(G) &= \frac{n}{2v_2^2} = \frac{5}{2(0.5)^2} = 10.
 \end{aligned}
 \tag{32}$$

4. Connectivity Index in the Selected Town for Building Hospital

Today, the treatment and medical services issue sits among the most important issues for every country. Governments are always trying to provide the best possible services and medical facilities for patients. In the past, unfortunately, as a result of the lack of medical services and the lack of hospitals and clinics, patients were not transferred to these medical centers for the required time and so they lost their lives. This crisis was because of the governments' constraints for revenue and financial problems. So, governments decided to build hospitals in the cities so that patients could seek treatment without stress as soon as possible. But determining the right place to build a hospital is very important

because firstly, it must be built in a place where there is no traffic and crowds nearby, and secondly, patients living in neighboring cities can easily access it. Therefore, in this paper, by using the concept of connectivity index, we try to determine the most appropriate places to establish a hospital. For this purpose, we consider six cities in China (Guangdong state), namely, Shaoguan, Qingyuan, Heyuan, Zhaoqing, Foshan, and Guangzhou, and the attributing symbols for each city are given in the graph as A, B, C, D, E , and F , respectively. Location of cities is shown in Figure 8. Also, distance between cities is shown in Table 2.

In this vague graph, the nodes representing cities and edges also represent the quality of roads as well as the amount of traffic of cars during most hours of the day. Weight of nodes and edges is shown in Tables 3 and 4.

The vertex $B(0.6, 0.3)$ shows that the city of Qingyuan has 60% of the necessary facilities and equipment to build a hospital, of which the necessary manpower is also part of this equipment. Clearly, it does not have 30% of the equipments for construction. The AB edge shows that the Shaoguan-Qingyuan route has 50% quality and the necessary road and transportation standards (road quality and traffic signs), and the amount of car traffic on this route is equal to 50%. For the vague graph G shown in Figure 9, we have

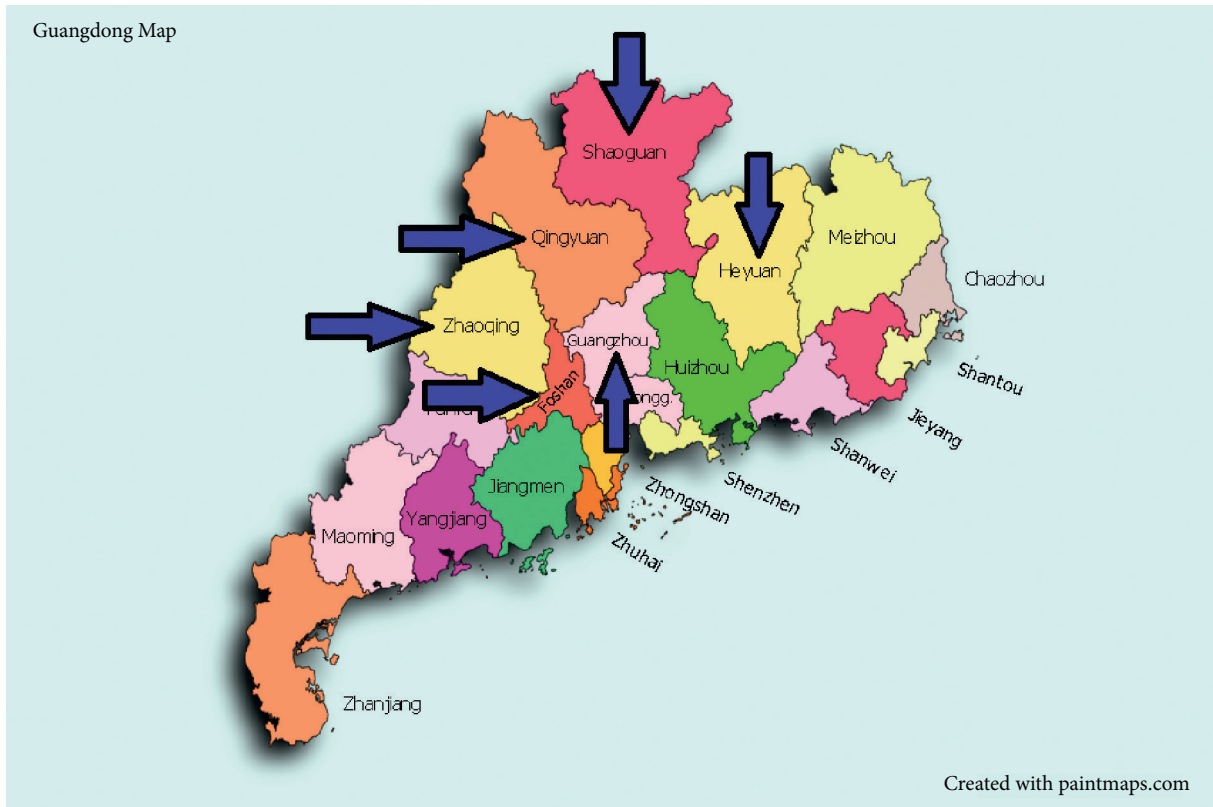


FIGURE 8: Location of cities.

TABLE 2: Distance between cities.

Cities	Shaoguan-Foshan	Foshan-Zhaoqing
Distance	202 km	69 km
Cities	Shaoguan-Zhaoqing	Shaoguan-Qingyuan
Distance	225 km	134 km
Cities	Qingyuan-Heyuan	Heyuan-Zhaoqing
Distance	168 km	293 km
Cities	Zhaoqing-Guangzhou	
Distance	81 km	

TABLE 3: Weight of nodes in G .

G	A	B	C	D	E	F
(t_M, f_M)	(0.5, 0.3)	(0.6, 0.2)	(0.5, 0.4)	(0.3, 0.5)	(0.6, 0.3)	(0.7, 0.2)

TABLE 4: Weight of edges in G .

G	AE	AD	AB
(t_N, f_N)	(0.4, 0.5)	(0.2, 0.6)	(0.5, 0.5)
G	ED	DF	BC
(t_N, f_N)	(0.2, 0.6)	(0.3, 0.6)	(0.5, 0.4)
G	CD		
(t_N, f_N)	(0.2, 0.5)		

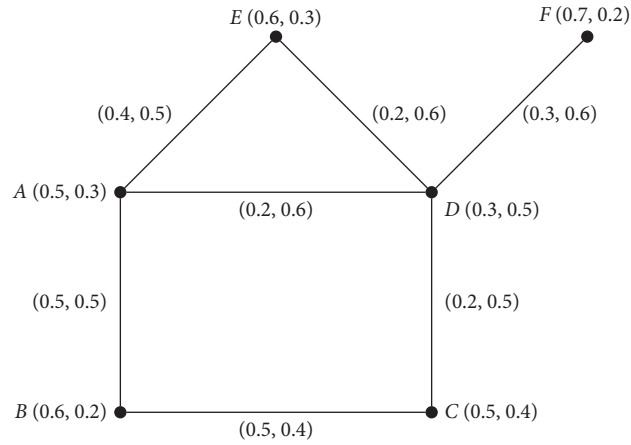


FIGURE 9: VG G.

$$\begin{aligned}
 \text{CONN}_G^t(A, E) &= 0.4, & \text{CONN}_G^t(A, D) &= 0.2, & \text{CONN}_G^t(A, F) &= 0.2, \\
 \text{CONN}_G^t(A, B) &= 0.5, & \text{CONN}_G^t(A, C) &= 0.5, & \text{CONN}_G^t(B, E) &= 0.4, \\
 \text{CONN}_G^t(B, D) &= 0.2, & \text{CONN}_G^t(B, C) &= 0.5, & \text{CONN}_G^t(B, F) &= 0.2, \\
 \text{CONN}_G^t(C, D) &= 0.2, & \text{CONN}_G^t(C, E) &= 0.4, & \text{CONN}_G^t(C, F) &= 0.2, \\
 \text{CONN}_G^t(D, E) &= 0.2, & \text{CONN}_G^t(D, F) &= 0.3, & \text{CONN}_G^t(E, F) &= 0.2, \\
 \text{CONN}_G^f(A, E) &= 0.5, & \text{CONN}_G^f(A, D) &= 0.5, & \text{CONN}_G^f(A, F) &= 0.6, \\
 \text{CONN}_G^f(A, B) &= 0.5, & \text{CONN}_G^f(A, C) &= 0.5, & \text{CONN}_G^f(B, E) &= 0.5, \\
 \text{CONN}_G^f(B, D) &= 0.5, & \text{CONN}_G^f(B, C) &= 0.4, & \text{CONN}_G^f(B, F) &= 0.6, \\
 \text{CONN}_G^f(C, D) &= 0.5, & \text{CONN}_G^f(C, E) &= 0.5, & \text{CONN}_G^f(C, F) &= 0.6, \\
 \text{CONN}_G^f(D, E) &= 0.5, & \text{CONN}_G^f(D, F) &= 0.6, & \text{CONN}_G^f(E, F) &= 0.6,
 \end{aligned}$$

$$\begin{aligned}
 \text{CI}_{\text{VG}}^t(G) &= 0.12 + 0.03 + 0.07 + 0.15 + 0.125 + 0.144 + 0.036 + 0.15 + 0.084 + 0.03 + 0.12 + 0.07 + 0.036 + 0.063 + 0.084 \\
 &= 1.312,
 \end{aligned}$$

$$\begin{aligned}
 \text{CI}_{\text{VG}}^f(G) &= 0.045 + 0.075 + 0.036 + 0.03 + 0.06 + 0.03 + 0.05 + 0.032 + 0.024 + 0.1 + 0.06 + 0.048 + 0.075 + 0.06 + 0.036 \\
 &= 0.761.
 \end{aligned}$$

$$\text{ACI}_{\text{VG}}^t(G) = \frac{1.312}{15} = 0.0874,$$

$$\text{ACI}_{\text{VG}}^f(G) = \frac{0.761}{15} = 0.0507.$$

(33)

In the same way, we have

$$\begin{aligned}
CI_{VG}^t(G - \{A\}) &= 1.817, & CI_{VG}^f(G - \{A\}) &= 0.515, \\
CI_{VG}^t(G - \{B\}) &= 0.748, & CI_{VG}^f(G - \{B\}) &= 0.595, \\
CI_{VG}^t(G - \{C\}) &= 0.817, & CI_{VG}^f(G - \{C\}) &= 0.461, \\
CI_{VG}^t(G - \{D\}) &= 1.117, & CI_{VG}^f(G - \{D\}) &= 0.401, \\
CI_{VG}^t(G - \{E\}) &= 0.808, & CI_{VG}^f(G - \{E\}) &= 0.515, \\
CI_{VG}^t(G - \{F\}) &= 0.941, & CI_{VG}^f(G - \{F\}) &= 0.557. \\
ACI_{VG}^t(G - \{A\}) &= 0.1817, & ACI_{VG}^t(G - \{B\}) &= 0.0748, \\
ACI_{VG}^t(G - \{C\}) &= 0.0817, & ACI_{VG}^t(G - \{D\}) &= 0.1117, \\
ACI_{VG}^t(G - \{E\}) &= 0.0808, & ACI_{VG}^t(G - \{F\}) &= 0.0941, \\
ACI_{VG}^f(G - \{A\}) &= 0.0515, & ACI_{VG}^f(G - \{B\}) &= 0.0595, \\
ACI_{VG}^f(G - \{C\}) &= 0.0461, & ACI_{VG}^f(G - \{D\}) &= 0.0401, \\
ACI_{VG}^f(G - \{E\}) &= 0.0515, & ACI_{VG}^f(G - \{F\}) &= 0.0557.
\end{aligned} \tag{34}$$

Hence,

$$\begin{aligned}
ACI_{VG}^t(G - \{B\}) &< ACI_{VG}^t(G) & ACI_{VG}^f(G - \{B\}) &> ACI_{VG}^f(G), \\
ACI_{VG}^t(G - \{D\}) &> ACI_{VG}^t(G) & ACI_{VG}^f(G - \{D\}) &< ACI_{VG}^f(G), \\
ACI_{VG}^t(G - \{E\}) &< ACI_{VG}^t(G) & ACI_{VG}^f(G - \{E\}) &> ACI_{VG}^f(G).
\end{aligned} \tag{35}$$

Clearly, the vertices B and E are VCEN and the vertex D is VCEN. If we remove the node D , then the true-CI is strictly increased and false-CI is strictly decreased. Also, by deleting vertex D , vertex F is automatically deleted (because it is completely disconnected).

Therefore, if the government wants to build a hospital in one of these six cities for the treatment of patients, it should avoid the cities of Zhaoqing and Guangzhou. Also, the cities of Qingyuan and Foshan are the best choices for building a hospital because firstly, they have the most facilities and human resources for construction, and secondly, the communication roads between other cities with these two cities are of better quality and have the least traffic and congestion compared to other routes.

5. Conclusion

The vague graphs can amplify flexibility and precision to model complex real-time problems better than a fuzzy graph. They have several applications in many decision-making processes among solution choice, weather forecasting, prognosis risks in business, and so on, and one of the most important features of VGs that has many applications in real problems is the concept of connectivity index. Connectivity index has many applications in psychology, medical sciences, social groups, and computer networks. Therefore, in this paper, we examined connectivity index, average connectivity index, and Randic index in VGs with

several examples. Likewise, some special types of nodes such as vague connectivity enhancing vertex, vague connectivity reducing vertex, and vague connectivity neutral vertex are presented. Finally, an application of connectivity index in construction has been introduced. In our future work, we will define the domination of the VGs in terms of strong edges and examine their properties. Also, we will study domination in terms of independent sets, and since many of the phenomena surrounding us are hybrid, we also discuss the domination concept on its fuzzy operations.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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