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*Research Article*

# Bifurcation Analysis and Single Traveling Wave Solutions of the Variable-Coefficient Davey–Stewartson System

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This paper mainly studies the bifurcation and single traveling wave solutions of the variable-coefficient Davey–Stewartson system. By employing the traveling wave transformation, the variable-coefficient Davey–Stewartson system is reduced to two-dimensional nonlinear ordinary differential equations. On the one hand, we use the bifurcation theory of planar dynamical systems to draw the phase diagram of the variable-coefficient Davey–Stewartson system. On the other hand, we use the polynomial complete discriminant method to obtain the exact traveling wave solution of the variable-coefficient Davey–Stewartson system.

## 1. Introduction

Partial differential equations (PDEs) play a major role in the fields of plasma, quantum mechanics, and engineering [1]. In the study of PDEs, the most important thing is to analyze the dynamic behavior and find the exact traveling wave solution. In recent years, the study of exact traveling wave solutions of nonlinear PDEs with the variable coefficients has always been the focus of mathematicians and physicists, and many experts and scholars [2, 3] have proposed many methods to find PDEs with the variable coefficients, such as variable-coefficient extended mapping method [4], Hirota's bilinear method [5], Lax integrability [6], and dynamical system approach [7, 8].

One of the most important PDEs is the variable-coefficient Davey–Stewartson system. In this paper, we consider the variable-coefficient Davey–Stewartson system [9, 10]:

$$\begin{cases} i\dfrac{\partial u}{\partial t}+a_1(t)\dfrac{\partial^2 u}{\partial x^2}+a_2(t)\dfrac{\partial^2 u}{\partial y^2}-b_1(t)|u|^2u+b_2(t)uv=0, \\ s_1\dfrac{\partial^2 v}{\partial x^2}-\dfrac{\partial^2 v}{\partial y^2}=s_2\dfrac{\partial^2 |u|^2}{\partial x^2}, \end{cases} \tag{1}$$

where $u=u(t,x,y)$ is the complex wave envelope, $v=v(t,x,y)$ is the real forcing function, $a_1(t)$ and $a_2(t)$ are real functions with respect to time $t$, which stand for group velocity dispersion terms, $b_1(t)$ and $b_2(t)$ represent the quadratic nonlinearity and cubic nonlinear coefficient term, respectively, and $s_1$ and $s_2$ are constants. When the coefficients in equation (1) are constant, equation (1) is called the Davey–Stewartson system [11–15]. It is a very important nonlinear Schrödinger equation, which is usually used to describe the nonlinear wave packet of finite depth.

In [10], Wei and her collaborators investigated equation (1) by the Lie group method and obtained the periodic solutions and elliptic function solutions. In [9], El-Shiekh and Gaballah obtained the dark soliton solutions and bright soliton solutions of equation (1) by the modified sine-Gordon equation method. Although some exact solutions of equation (1) have been obtained in references [9, 10], the analysis of the dynamic behavior and the classification of traveling wave solutions of this kind of equation have not been reported. Therefore, in this paper, we will further study the above two problems.

The structure of this paper is as follows. In Section 2, we use the bifurcation theory of planar dynamical systems to draw the phase diagram of the variable-coefficient Davey–Stewartson system. In Section 3, we construct the classification of all single traveling wave solutions of the variable-coefficient Davey–Stewartson system by the complete discrimination system. In Section 4, we give a summary.

## 2. Bifurcation Analysis of System (1)

Consider the traveling wave transformation as follows:

$$\begin{aligned} u(t,x,y) &= U(\xi)e^{i\eta},\\ v(t,x,y) &= V(\xi),\\ \xi &= \mu_1 x + \mu_2 y - \lambda(t),\\ \eta &= k_1 x + k_2 y - \theta(t), \end{aligned} \tag{2}$$

where $\mu_1$, $\mu_2$, $k_1$, and $k_2$ are constants and $\lambda(t)$ and $\theta(t)$ are functions defined on $t$.

Substituting (2) into (1) and separating the real and imaginary parts, we obtain

$$\begin{aligned} \left(a_1(t)\mu_1^2 + a_2(t)\mu_2^2\right)U(\xi) + \left(\theta'(t) - a_1(t)k_1^2 - a_2(t)k_2^2\right)U - b_1(t)U^3 + b_2(t)UV &= 0,\\ \left(\lambda'(t) - 2k_1\mu_1 a_1(t) - 2k_2\mu_2 a_2(t)\right)U' &= 0,\\ \left(s_1\mu_1^2 - \mu_2^2\right)V'' &= \mu_1^2 s_2\left(U^2(\xi)\right)''. \end{aligned} \tag{3}$$

In order to eliminate the terms $U'(\xi)$, we set $\lambda(t) = 2\int(\mu_1 k_1 a_1(t) + \mu_2 k_2 a_2(t))\mathrm{d}t$, $\theta(t) = \int(k_1^2 a_1(t) + k_2^2 a_2(t))\mathrm{d}t + \theta_0$, $b_1(t) = b_1(\mu_1^2 a_1(t) + \mu_2^2 a_2(t))$, and $b_2(t) = b_2(\mu_1^2 a_1(t) + \mu_2^2 a_2(t))$, where $b_1$, $b_2$, and $\theta_0$ are arbitrary constants. Then, equation (3) can be reduced to

$$\begin{aligned} U''(\xi) - b_1 U^3(\xi) + b_2 U(\xi)V(\xi) &= 0,\\ \left(s_1\mu_1^2 - \mu_2^2\right)V'' &= \mu_1^2 s_2\left(U^2(\xi)\right)''. \end{aligned} \tag{4}$$

Integrating the second equation of (4) twice with respect to $\xi$, we have

$$V(\xi) = \frac{s_2\mu_1^2}{s_1\mu_1^2 - \mu_2^2}U^2(\xi) + c_1\xi + c_0, \tag{5}$$

where $c_0$ and $c_1$ are integration constants. Substituting equation (5) into the first equation of (4), we obtain

$$U''(\xi) + \left(\frac{s_2\mu_1^2 b_2}{s_1\mu_1^2 - \mu_2^2} - b_1\right)U^3(\xi) + b_2 c_0 U(\xi) = 0. \tag{6}$$

Next, by assuming $(\mathrm{d}U/\mathrm{d}\xi) = \psi$, we can rewrite equation (6) as the following planar dynamical system:

$$\begin{cases} \dfrac{\mathrm{d}U}{\mathrm{d}\xi} = \psi,\\[2ex] \dfrac{\mathrm{d}\psi}{\mathrm{d}\xi} = -\left(\dfrac{s_2\mu_1^2 b_2}{s_1\mu_1^2 - \mu_2^2} - b_1\right)U^3(\xi) - b_2 c_0 U(\xi), \end{cases} \tag{7}$$

with the Hamiltonian system

$$H(U,\psi) = \frac{1}{2}\psi^2 + \frac{1}{4}\left(\frac{s_2\mu_1^2 b_2}{s_1\mu_1^2 - \mu_2^2} - b_1\right)U^4(\xi) + \frac{b_2 c_0}{2}U^2(\xi) = h. \tag{8}$$

Here, we assume that $G(U) = -((s_2\mu_1^2 b_2/(s_1\mu_1^2 - \mu_2^2)) - b_1)U^3(\xi) - b_2 c_0 U(\xi)$. When $((s_2\mu_1^2 b_2/(s_1\mu_1^2 - \mu_2^2)) - b_1)b_2 c_0 < 0$, we easily obtain three zeros of $G(U)$ including $U_0 = 0$, $U_1 = \sqrt{-(b_2 c_0(s_1\mu_1^2 - \mu_2^2)/(s_2\mu_1^2 b_2 - b_1((s_1\mu_1^2 - \mu_2^2)))}$, and $U_2 = -\sqrt{-(b_2 c_0(s_1\mu_1^2 - \mu_2^2)/(s_2\mu_1^2 b_2 - b_1(s_1\mu_1^2 - \mu_2^2)))}$. When $((s_2\mu_1^2 b_2/s_1\mu_1^2 - \mu_2^2) - b_1)b_2 c_0 > 0$, we obtain $U_3 = 0$. Then, we suppose that $M_i(U_i, 0)$ are the equilibrium points of equation (8), and we obtain that $M_i(U_i, 0)$ is the saddle point when $G'(U_i) > 0$; $M_i(U_i, 0)$ is the degraded saddle point when $G'(U_i) = 0$; $M_i(U_i, 0)$ is the center point when $G'(U_i) < 0$. With the help of Maple software, we draw the phase portraits of (8) as shown in Figure 1.

## 3. Traveling Wave Solutions of System (1)

The complete discriminant system method was first introduced by Lu and his collaborators in 1996 [16]. In recent years, many experts and scholars [17–22] have applied this method to construct the exact traveling wave solutions of partial differential equations. In this section, we intend to use this method to analyze the exact traveling wave solution of the variable-coefficient Davey–Stewartson system.

In fact, in Section 2, we have simplified equation (1) to nonlinear equation (6). Next, multiply both ends of equation (6) by $U'$ and integrate once, and we obtain

$$(U')^2 = d_4 U^4(\xi) + d_2 U^2(\xi) + d_0, \tag{9}$$

where $d_4 = -1/2((s_2\mu_1^2 b_2/s_1\mu_1^2 - \mu_2^2) - b_1)$, $d_2 = -b_2 c_0$, and $d_0$ is the integration constant.

Consider the following transformations:

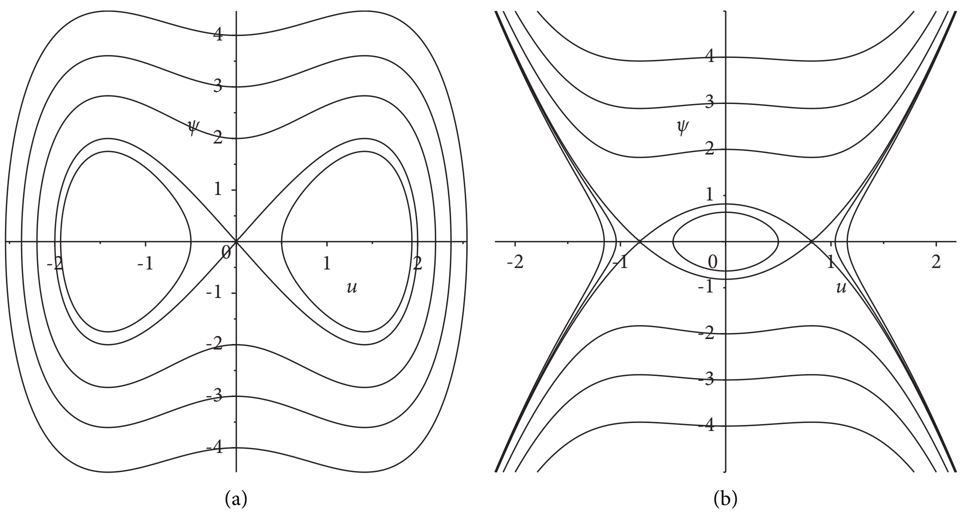


FIGURE 1: Phase portraits of system (7). (a) $(s_2\mu_1^2 b_2/s_1\mu_1^2 - \mu_2^2) - b_1 > 0,\ (b_2c_0/2) < 0$. (b) $(s_2\mu_1^2 b_2/s_1\mu_1^2 - \mu_2^2) - b_1 < 0,\ (b_2c_0/2) > 0$.

$$\begin{cases} U = \pm\sqrt{(4d_4)^{-(1/3)}w}, \\ p = 4d_2(4d_4)^{-(2/3)}, \\ q = 4d_0(4d_4)^{-(1/3)}, \\ \xi_1 = (4d_4)^{(1/3)}\xi. \end{cases} \tag{10}$$

Then, equation (9) can be rewritten as

$$\left(w_{\xi_1}\right)^2 = w\left(w^2 + pw + q\right). \tag{11}$$

Integrating equation (11) once, we obtain

$$\pm(\xi_1 - \xi_0) = \int \frac{\mathrm{d}w}{\sqrt{w\left(w^2 + pw + q\right)}}, \tag{12}$$

where $\xi_0$ is an integration constant. Setting $F(w) = w^2 + pw + q$, its complete discrimination system is

$$\Delta = p^2 - 4q. \tag{13}$$

According to the root of equation (13), the traveling wave solution of equation (1) has four cases.

*Case 1.* Assume that $\Delta = 0$. Since $w > 0$, we can obtain

$$\pm(\xi_1 - \xi_0) = \int \frac{\mathrm{d}w}{\sqrt{w}(w + (p/2))}. \tag{14}$$

If $p < 0$, the explicit solution of equation (1) is

$$\begin{aligned} u_1(t,x,y) &= \pm\sqrt{-\frac{1}{2}d_2d_4^{-1}\tanh^2\left[\frac{1}{2}\left(-2d_2(4d_4)^{-(2/3)}\right)^{(1/2)}\left((4d_4)^{(1/3)}\xi - \xi_0\right)\right]}e^{i(k_1x+k_2y-\theta(t))}, \\ u_2(t,x,y) &= \pm\sqrt{-\frac{1}{2}d_2d_4^{-1}\coth^2\left[\frac{1}{2}\left(-2d_2(4d_4)^{-(2/3)}\right)^{(1/2)}\left((4d_4)^{(1/3)}\xi - \xi_0\right)\right]}e^{i(k_1x+k_2y-\theta(t))}. \end{aligned} \tag{15}$$

If $p > 0$, the explicit solution of equation (1) is

$$u_3(t,x,y) = \sqrt{\frac{1}{2}d_2d_4^{-1}\tan^2\left[\frac{1}{2}\left(2d_2(4d_4)^{-(2/3)}\right)^{(1/2)}\left((4d_4)^{(1/3)}\xi - \xi_0\right)\right]}e^{i(k_1x+k_2y-\theta(t))}. \tag{16}$$

If $p = 0$, the explicit solution of equation (1) is

$$u_4(t,x,y) = 2\sqrt{\frac{(4d_4)^{-(1/3)}}{\left((4d_4)^{(1/3)}\xi - \xi_0\right)^2}}e^{i(k_1x+k_2y-\theta(t))}. \tag{17}$$

*Case 2.* Assume that $\Delta > 0$ and $q = 0$. Since $w > -p$, we can obtain

$$\pm(\xi_1 - \xi_0) = \int \frac{\mathrm{d}w}{w\sqrt{w + p}}. \tag{18}$$

According to Case 1, the solution of equation (18) is as follows.

If $p>0$, the explicit solution of equation (1) is

$$\begin{aligned} u_5(t,x,y) &= \pm\sqrt{\frac{1}{2}d_2d_4^{-1}\tanh^2\left[\frac{1}{2}\left(2d_2(4d_4)^{-(1/3)}\right)^{(1/2)}\left((4d_4)^{(1/3)}\xi-\xi_0\right)\right]-d_2d_4^{-1}}e^{i(k_1x+k_2y-\theta(t))},\\ u_6(t,x,y) &= \pm\sqrt{\frac{1}{2}d_2d_4^{-1}\coth^2\left[\frac{1}{2}\left(2d_2(4d_4)^{-(1/3)}\right)^{(1/2)}\left((4d_4)^{(1/3)}\xi-\xi_0\right)\right]-d_2d_4^{-1}}e^{i(k_1x+k_2y-\theta(t))}. \end{aligned} \tag{19}$$

If $p<0$, the explicit solution of equation (1) is

$$u_7(t,x,y)=\sqrt{-\frac{1}{2}d_2d_4^{-1}\tan^2\left[\frac{1}{2}\left(2d_2(4d_4)^{-(1/3)}\right)^{(1/2)}\left((4d_4)^{(1/3)}\xi-\xi_0\right)\right]-d_2d_4^{-1}}e^{i(k_1x+k_2y-\theta(t))}. \tag{20}$$

*Case 3.* Assume that $\Delta>0$, $q\neq 0$, and $\alpha<\beta<\gamma$, where one of $\alpha$, $\beta$, and $\gamma$ is zero, and the rest of them are the two roots of $F(w)$. Since $\alpha<w<\beta$, make the transformation $w=\alpha+(\beta-\alpha)\sin^2\varphi$. It can be obtained from equation (13) that

$$\pm(\xi_1-\xi_0)=\frac{2}{\sqrt{\gamma-\alpha}}\int\frac{d\varphi}{\sqrt{1-m^2\sin^2\varphi}}, \tag{21}$$

where $m^2=((\beta-\alpha)/(\gamma-\alpha))$.

According to equation (21) and the definition of Jacobian elliptic function **sn**, the explicit solution of equation (1) is

$$u_8(t,x,y)=\sqrt{(4d_4)^{-(1/3)}\left[\alpha+(\beta-\alpha)\mathbf{sn}^2\left(\frac{\sqrt{\gamma-\alpha}}{2}\left((4d_4)^{(1/3)}\xi-\xi_0\right),m\right)\right]}e^{i(k_1x+k_2y-\theta(t))}. \tag{22}$$

Make the transformation $w=((-\beta\sin^2\varphi+\gamma)/\cos^2\varphi)$ and substitute it into equation (12). Similarly, the explicit solution of equation (1) can be obtained:

$$u_9(t,x,y)=\sqrt{(4d_4)^{-(1/3)}\frac{-\beta\mathbf{sn}^2\left((\sqrt{\gamma-\alpha}/2)\left((4d_4)^{(1/3)}\xi-\xi_0\right),m\right)+\gamma}{\mathbf{cn}^2\left((\sqrt{\gamma-\alpha}/2)\left((4d_4)^{(1/3)}\xi-\xi_0\right),m\right)}}e^{i(k_1x+k_2y-\theta(t))}. \tag{23}$$

*Case 4.* Assume that $\Delta<0$. Since $w>0$, we can make the following transformation:

$$w=\sqrt{q}\tan^2\frac{\varphi}{2}. \tag{24}$$

Substituting equation (24) into equation (13), we obtain

$$\pm(\xi_1-\xi_0)=q^{-(1/4)}\int\frac{d\varphi}{\sqrt{1-k^2\sin^2\varphi}}, \tag{25}$$

where $m^2=(1/2)(1-(p/2\sqrt{q}))$.

According to equation (25) and the definition of Jacobian elliptic function **cn**, we obtain

$$\mathbf{cn}\left((q)^{(1/4)}(\xi_1-\xi_0),m\right)=\cos\varphi. \tag{26}$$

From equation (24), we have

$$\cos\varphi=\frac{2\sqrt{q}}{w+\sqrt{q}}-1. \tag{27}$$

Comparing equation (26) with equation (27), we can obtain the explicit solution of equation (1):

$$u_{10}(t,x,y) = \sqrt{(4d_4)^{-(1/3)}\left[\frac{2\sqrt{4d_0(4d_4)^{-(1/3)}}}{1+\mathbf{cn}\left(\left(4d_0(4d_4)^{-(1/3)}\right)^{(1/4)}\left((4d_4)^{(1/3)}\xi-\xi_0\right),m\right)}-\sqrt{4d_0(4d_4)^{-(1/3)}}\right]}e^{i\left(k_1x+k_2y-\theta(t)\right)}. \quad (28)$$

*Remark 1.* In this paper, we obtained one of the solutions $(u(t,x,y))$ of equation (1). Using relation (5), we can obtain another solution $v(t,x,y)$ of equation (1).

## 4. Conclusion

In this paper, the bifurcation and single traveling wave solutions of the variable-coefficient Davey–Stewartson system have been investigated by employing the bifurcation theory of planar dynamical systems and the polynomial complete discriminant method. The phase portraits of the variable-coefficient Davey–Stewartson system are shown in Figure 1. Moreover, a series of new single traveling wave solutions is obtained. Compared with the published literature [9], this study not only obtains the hyperbolic function solution, trigonometric function solution, and rational function solution but also obtains the Jacobi function solution. We believe that the study of the variable-coefficient Davey–Stewartson system in the paper will help mathematicians and physicists.

## Data Availability

No data were used to support this study.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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