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*Research Article*

# Bipolar-Valued Fuzzy Social Network and Centrality Measures

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The concept of bipolar-valued fuzzy relationships and their role in modeling bipolar-valued fuzzy social networks (BVFSN) are discussed. The goal of this study is to represent the bipolarity and fuzziness that are always present in the relationship between actors. Ranking the most central actors to achieve maximum spreading ability has been a difficult and critical topic thus far. Many centrality measures have been proposed to determine the significance of nodes in the central person detection process. In this article, we proposed two new centrality measures, BF-degree centrality and BF-closeness centrality, based on the natural characteristics of a bipolar-valued fuzzy social network. The proposed centrality measures highlight the fuzziness and bipolarity of the relationship. We consider the fuzzy stability index for direct connections here, as well as the strengths and weaknesses of the relationship. With the help of the effective and total effective strength of the path, we find the favorable path in this centrality measure. Moreover, to investigate the validity and reliability of these new centrality measures, we gathered Google Scholar data and built a G-S Research network. Experimental results show that the model and centrality measures can be used to objectively rank the most central node in the network.

## 1. Introduction

The network structure is usually drawn in the form of a graph with actors and their relationships. Graphs appear naturally here because they are useful for showing how the objects are linked together both biologically and socially. Actors and relationships in social networking sites are portrayed by vertices and edges in a graph, respectively. The concept of social network analysis (SNA) originated in the western world, usually dated to 1930s and 1940s anthropological and cognitive science research projects [1]. Social network analysis is a series of rules and techniques used to analyze the framework and characteristics of social relationships. Due to its recent growth and its often-technical character especially attractive to mathematically minded people, it is widely regarded as the most recent innovation and among a large number of specialized technical methods used by sociologists. Many social network models [2–7] have been proposed in recent years, including community detection, online social media networks, clustering in social networks, social networks including fuzziness, relational networks based on intuitionistic fuzzy graph, and so on. The analysis of the relationship among actors in social platforms aids in the integration of the correlation among individuals and the social architecture of the system. Benevenunto et al. [8] examined the use of online social network (OSN) facilities from the perspective of a social network aggregator, providing an accurate picture of how users react when they link to OSN web pages. Chen et al. [9] presented a feature set that combines the feature of traditional heuristics and social networking. Nair and Sarasamma [10] developed data mining techniques and a new binary operation called consolidation operation for the fuzzy social network. Samanta and Pal [11] introduced a new concept of registration for a new user in the fuzzy social network so that the chances of fake users may be reduced. Kundu and Pal [4] develop a unified framework based on the fuzzy granular theory to demonstrate the social network model. Chu and Wang [12] presented a fuzzy clustering-based method for detecting social media communities, as well as three separate conventional wisdom reaching techniques to maintain the computational complexity.

In general, the relationship among actors is regarded as a binary association in social network analysis, with "0" indicating no relationship and "1" indicating there are relationships [13]. We frequently feel that we cannot express our relationship with two values 1 (yes) and 0 (no), even though there may be various levels of relationship in social environments, for example, we enjoy talking with some friends and with some friends we try to avoid talking. The important thing to remember here is that in both cases, the relationship is friendship, but its dimension is different; additionally, in every relationship, bipolarity exists as a strength and weakness of the relationship. Akram [14, 15] introduced the notion of bipolar-valued fuzzy relation and bipolar-valued fuzzy graph (BVFG) in 2011 after applying Zhang's [16] bipolar-valued fuzzy sets (BVFS) to graph theory. BVFSN's motivation is to represent social actors and relationships with bipolar membership grades, which are very useful for measuring the extent and bipolarity of relationships. Many research scholars have generalized the concepts, definitions, algorithms, as well as characteristics of BVFG's and developed new notions and results based on Zhang and Akram's bipolarity theory in recent years.

The detection of influential nodes (node centrality measure) is among the most critical topics in investigating the nature of diffusion. Finding the appropriate actor is the best way to prevent and control the spread of negative types by protecting the most adequate actors [17]. For example, in social network marketing, most of the ads are distributed in few amount of time and with minimum effort by utilizing the influential actors. According to popular belief, the person with the most connections is the most important. However, a person with just a large number of connections does not necessarily spread information faster because the dissemination of information is dependent not only on the number of connections but also depends on the strength and weakness of the link. Our main assumption is that each relationship has some positive and negative attributes that affect the network's spreading system. In the past few years, various types of centrality measures have been introduced to end this, but all of these measures ignore the presence of bipolarity and fuzziness in the relationship. Bavelas [18] developed the centrality measure for integrated networks and suggested its use in the analysis of communication networks. Shimble [19] introduced stress centrality as a method for measuring the amount of communication based on the shortest path. Kartz [20] introduced kartz centrality to quantify a node's corresponding level of influence inside a system. Freeman [21] first created a computational formula related to the centrality that counts the number of other vertices directly joining the target vertex. As for the degree of relative power and centrality, Bonachich [22] introduced a generalized concept of degree centrality. Barrat et al. [23], Newman [24], and Opshl and Panzarasa [25] generalized and extended the degree of the nodes by taking the sum of the weights into account instead of the ties number.

Many centrality measures, like degree centrality, closeness centrality, eigenvector centrality, sub-graph centrality, K-shell centrality, neighborhood centrality, P-mean centrality, H-index centrality, and others, are available in the literature. Singh [26] summarizes a few of the centrality measures that are widely used for mining social network data and discusses multiple research directions related to such measures. The main thing in social networks is the relationships between actors. We have mostly seen that these relationships are not fixed and that there is no technique to measure the strengths of social relationships with one another. For example, if the links between two actors are given by "friendship," the corresponding graph is not a classical graph. The entire system can be expressed as a fuzzy graph, which gives rise to the concept of social network with fuzziness and has piqued the interest of several researchers in fuzzified social relational networks. Nair and Sarasamma [10] are the first researchers to discuss social network fuzzy parameters. They measured social actors and their relationships using the fuzzy techniques in the relational network. Qian and Zeng [27] discussed the structural properties of centrality measures based on fuzzy hypergraphs and defined some centrality measures such as relative degree centrality, relative closeness centrality, and relative betweenness centrality. The primary goal of this research is to develop an effective and efficient bipolar framework for modeling social networks that include the degree of bipolarity and fuzziness of actors and their relationships, namely bipolar-valued fuzzy social network (BVFSN). In this article, we extend the concept of centrality measure to BVFSN, proposing a few centrality measurements such as BF-degree centrality and BF-closeness centrality. To demonstrate the significance of these centrality measures, we discuss an example of locating the most popular researcher in the G-S research network.

The rest of this article is designed as follows: Section 2 goes over previous relevant studies. Section 3 introduces a bipolar-valued fuzzy social network model as well as some definitions such as the fuzzy stability index, fuzzy effective strength, and fuzzy total strength. Section 4 discusses the proposed centrality measures, BF-degree centrality and BF-closeness centrality. Section 5 presents the mathematical and graphical analysis based on two different centrality measures applied to the G-S Research Network. Section 6 wraps up the article and provides a discussion.

## 2. Preliminaries

The purpose of this section is to refresh your memory on the fundamental concepts that appear frequently in this article. In this section, we will go over the fundamental concept of bipolarity with fuzziness, some basic centrality measures as well as bipolar-valued fuzzy sets and graph-theoretical concepts.

Throughout the article, indicate $I^P = [0,1]$, $I^N = [-1,0]$, and unless stated separately, $X$ always denote the universe of discourse.

*2.1. Bipolarity and Fuzziness.* Fuzziness and bipolarity are natural aspects of human psychology. On the one hand, the fuzzy set theory has been developed to deal with imprecision in a systematic manner that occurs when the boundaries of a class of objects are not clearly defined. Bipolarity, on the other hand, presumes the presence of the two reference poles, P/N, such as good/bad, true/false, and effect/side effect. Han and Shi [28] classified bipolarity into three categories.

(1) Type I: in this type of bipolarity, the second tuple in ordered pair is completely regarded as the complement of the first tuple, i.e., $N = -P$ (bad = not good), implying that the study of one tuple is sufficient to know all the facts. In fuzzy case, although it is not explicitly stated, the concept of the fuzzy set is closely related to the concept of this type of bipolarity $\beta^N(x) = -(\beta^P(x)) = 1 - \beta^P(x)$.

(2) Type II: these types of bipolarity share a similar behavior in both poles, it is also naturally thought that the consistency condition must be satisfied, i.e., no conflict exists between the two poles. The negative pole, on the other hand, may not be the positive pole's complement, i.e., $N \neq -P$ (bad $\neq$ not good). Throughout this situation, an element can neither fulfill nor contradict a concept.

(3) According to the intuitionistic fuzzy set (IFS), if $\beta^P(x)$ is viewed as the positive polarity's membership function and $\beta^N(x)$ is viewed as the negative polarity's membership function, then $\beta^P(x) + \beta^N(x) \leq 1$ make sure the consistency condition. IFS beautifully includes the concept of this type of bipolarity in terms of semantics.

(4) Type III: both poles even now share a similar behavior in type II bipolarity, whereas bipolarity of judgments is sometimes associated with using two distinct valuation ranges, in other words, both poles represents the degree of truthiness and the degree of falsity, respectively, that are supposed to occur simultaneously and independently of each other. In this regard, overall freedom of positive and negative conditions is supposed, and controversy is assumed to persist, implying that the IFS-assumed situation of stability should be left in this scenario.

It is clear that none of the fuzzy set theories mentioned above can assert this type of conflicting bipolarity. In this context, BVFS, a new fuzzy set theory that combines fuzziness with inconsistent bipolarity, was introduced.

*2.2. Bipolar-Valued Fuzzy Set.* A bipolar-valued fuzzy set (BVFS) in $X$ is defined by the mapping $\beta = (\beta^P, \beta^N)$: $X \longrightarrow I^P \times I^N$, $x \longrightarrow (\beta^P(x), \beta^N(x))$, $\forall x \in X$, where $\beta^P$: $X \longrightarrow I^P$, $x \longrightarrow \beta^P(x) \in I^P$ and $\beta^N$: $X \longrightarrow I^N$, $x \longrightarrow \beta^N(x) \in I^N$ are membership functions. The positive and negative membership degree $(\beta^P(x), \beta^N(x))$, respectively, define the satisfaction degree of an element $x \in X$ as the corresponding property of BVFS $\beta$ and some implicit counter property of BVFS $\beta$. Note that, the inspiration of BVFS is nothing but simply join the bipolarity and fuzziness, BVFS concentrates on expressing the bipolarity and extending the membership function from $I^P \longrightarrow I^P \times I^N$. It is obvious that $I^P \times I^N$ are mathematically equivalent, but as we mentioned, mathematically equivalent is one thing and semantics is another thing. In the semantic aspects of the real world, $I^P \times I^N$ is quite appropriate for human behavior to express bipolarity than $I^P \times I^P$ [28].

*2.3. Bipolar-Valued Fuzzy Relation.* Bipolar-valued fuzzy relations are a couple of fuzzy relations, particularly regarding membership and non-membership functions that reflect the benefits and drawbacks of the available data. Some authors have also called bipolar-valued fuzzy relations "bifuzzy relations" [28].

Let $U$ be a reference set. A mapping $\alpha = (\alpha^P, \alpha^N)$: $U \times U \longrightarrow [0,1] \times [-1,0]$ is said to be bipolar-valued fuzzy relation on $U$ such that $\alpha^P(x,y) \in [0,1]$ and $\alpha^N(x,y) \in [-1,0]$. If $\alpha = (\alpha^P, \alpha^N)$ and $\beta = (\beta^P, \beta^N)$ are bipolar-valued fuzzy sets on $U$ and if $\alpha$ is bipolar-valued fuzzy relation on $U$, then $\alpha$ is said to be bipolar-valued fuzzy relation of $\beta$ if $\alpha^P(x,y) \leq \min\{\beta^P(x), \beta^P(y)\}$ and $\alpha^N(x,y) \geq \max\{\beta^N, \beta^y\}$ $\forall x, y \in U$.

*2.4. Bipolar-Valued Fuzzy Graph.* Let $\alpha = (\alpha^P, \alpha^N)$ and $\beta = (\beta^P, \beta^N)$ are the BVFS's of $U$ and $\widetilde{U^2}$, respectively, for which $\beta^P(xy) \leq \min\{\alpha^P(x), \alpha^P(y)\} \forall xy \in \in \widetilde{U^2}$ and $\beta^N(xy) \geq \max\{\alpha^N(x), \alpha^N(y)\} \forall xy \in \widetilde{U^2}$ and $\beta^P(xy) = \beta^N(xy) = 0 \, \forall xy \in (U^2 - E)$, then $G = (U, \alpha, \beta)$ is said to be bipolar-valued fuzzy graph (BVFG) over the graph $G^* = (U, E)$ [14, 29], an example of bipolar-valued fuzzy graph is shown in Figure 1.

*Definition 1* (see [14]). A strong bipolar-valued fuzzy graph $\widetilde{G}$ over the graph $G^*$ is an ordered triple $\widetilde{G} = (U, \alpha, \beta)$, where $\alpha = (\alpha^P, \alpha^N)$: $U \longrightarrow [0,1] \times [-1,0]$ and $\beta = (\beta^P, \beta^N)$: $\widetilde{U^2} \longrightarrow [0,1] \times [-1,0]$ are the bipolar-valued fuzzy sets in $U$ and $\widetilde{U^2}$, respectively, satisfies the following conditions

$$\begin{aligned} \beta^P(x_1x_2) &= \min\{\alpha^P(x_1), \alpha^P(x_2)\} \quad \forall x_1x_2 \in U^2, \\ \beta^N(x_1x_2) &= \max\{\alpha^N(x_1), \alpha^N(x_2)\} \quad \forall x_1x_2 \in U^2. \end{aligned} \tag{1}$$

*Definition 2* (see [5]). Let $\{u_1, u_2 \ldots u_m\}$ be the collection of vertices in BVFG $(U, \alpha, \beta)$. A path $\rho = u_1u_2 \cdots u_m$ is a series of distinguishable nodes that satisfy one of the following conditions for all $u_i$, $u_j$ in $\{u_1, u_2 \cdots u_m\}$.

(1) $\beta^P(u_iu_j) \geq 0$ and $\beta^N(u_iu_j) = 0$ for some $i$, $j$.
(2) $\beta^P(u_iu_j) = 0$ and $\beta^N(u_iu_j) \leq 0$ for some $i$, $j$.
(3) $\beta^P(u_iu_j) \geq 0$ and $\beta^N(u_iu_j) \leq 0$ for some $i$, $j$.

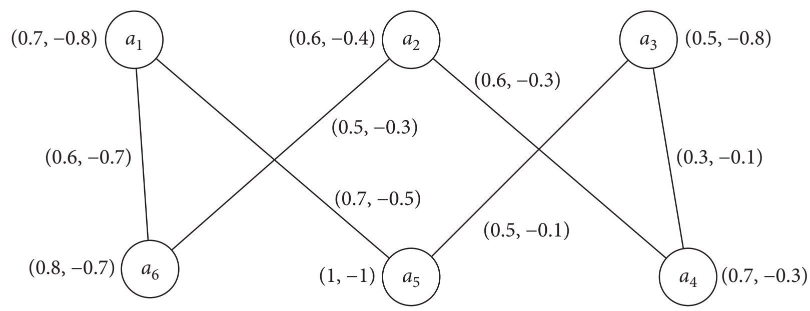


Figure 1: Example of the bipolar-valued fuzzy graph over graph G.

*Definition 3.* Let $G = (U, \alpha, \beta)$ be a bipolar-valued fuzzy graph (BVFG) over the graph $G^* = (U, E)$. Bipolar-valued fuzzy strength of a path $\rho = u_1 u_2 \cdots u_m$ is defined as $\xi(\rho) = [\xi^P(\rho), \xi^N(\rho)]$, where $\xi^P(\rho) = \min\{\beta^P(u_{ij})\}$, $\xi^N(\rho) = \max\{\beta^N(u_{ij})\}$, $(i, j = 1, 2, \ldots n)$.

*Definition 4.* Bipolar-valued fuzzy connecting strength between two vertices $u_i$ and $u_j$ is given by $CONN(u_i, u_j) = (CONN^P(u_i, u_j), CONN^N(u_i, u_j))$, where $CONN^P(u_i, u_j) = \max_{\rho \in \gamma(u_i, u_j)} \{S^P(\rho)\}$ and $CONN^N(u_i, u_j) = \min_{\rho \in \gamma(u_i, u_j)} \{S^N(\rho)\}$. Here $\gamma(u_i, u_j)$ indicates the set of all possible paths between the vertices $u_i$ and $u_j$.

*2.5. Some Basic Centrality Measures.* Degree centrality: a node's degree centrality represents how many edges are directly linked to the target node. Formally, degree centrality is represented by $C_d(i) = d_i$, where $d_i$ is the degree of node $i$ [30]. The degree centrality, however, demonstrates the importance of vertices up to a certain level, but we cannot say that vertices of the same degree perform the same function in the network. Because the scope of this centrality measure is limited and does not take into account the large-scale relationships in the network, this measure does not have a high level of accuracy.

*2.5.1. Closeness Centrality.* Sabidussi [31] introduced closeness centrality as the inverse of the sum of the geodetic distances of all nodes in the network from each node. The closeness centrality of a node $i$ is represented by $C_c(i) = 1/\sum_{j \in N} d(i, j)$, where $N$ is the collection of nodes in a network. This method is preferable to degree centrality because it considers both direct and indirect connections between vertices. If a network has disconnected components, this measure cannot be used because the finite distance between nodes in unconnected parts of the network cannot be calculated.

*2.5.2. Betweenness Centrality.* The goal of betweenness centrality is to determine the significance of a node based on the information flow in the graph. It is determined by the number of times a node appears in the shortest paths between all pairs of nodes in the graph. The betweenness centrality [32] of a vertex $i$ in the network is given by $C_b(i) = \sum_{y \neq z \in N} \sigma_{yz}(i)/\sigma_{yz}$, where $\sigma_{yz}(i)$ is the number of shortest paths between pair of nodes $y$ and $z$ containing $i$, and $\sigma_{yz}$ is the number of all shortest paths between pair of nodes $y$ and $z$. This is a global centrality measure but very complex in a large network.

## 3. BVFSN Model and Bipolar-Valued Fuzzy Centrality

A social network comprises actors (such as individuals or organizations) and other social instructions between actors. The purpose of social network analysis (SNA) is to understand the nature, structure, consequences, and conditions of the relationship between actors. These social network relations are typically characterized by vague notions such as "good," "average," "strong," "very strong," and "extreme." The fuzzy set comes normally here to solve this problem. However, a wide variety of network analyses, especially social network analysis is based on bipolar or two-sided relationships, usually reflected as the strength and weakness of the relationship. This leads us to model a social network in terms of fuzziness with bipolarity. This section introduces two centrality measures, BF-degree centrality and BF-closeness centrality, which will be useful in determining the central person in a bipolar-valued fuzzy social network.

*3.1. The Model.* Take a look at a social networking site, here we assume that $V$ is the collection of all actors and $E$ is the collection of ties between actors. The bipolar-valued fuzzy graph over the graph $G = (V, E)$ is defined as a couple $(\alpha, \beta)$, where $\alpha = (\alpha^P, \alpha^N)$ and $\beta = (\beta^P, \beta^N)$ are the bipolar-valued fuzzy sets in $V$ and $\widetilde{V^2}$ satisfies the condition $\beta^P(v_1 v_2) \leq \min\{\alpha^P(v_1), \alpha^P(v_2)\}$, $\forall xy \in \widetilde{V^2}$, $\beta^N(v_1 v_2) \geq \max\{\alpha^N(v_1), \alpha^N(v_2)\} \forall v_1 v_2 \in \widetilde{V^2}$, and $\beta^P(v_1 v_2) = \beta^N(v_1 v_2) = 0 \, \forall xy \in \widetilde{V^2} - E$. Thus bipolar-valued fuzzy social network (BVFSN) is represented by a triplet: $\mathbb{B} = (V, \alpha, \beta)$, where

(1) $V$ is the finite set of actors in the social network.

(2) $\alpha$: $V \longrightarrow [0, 1] \times [-1, 0]$ is a bipolar-valued fuzzy set, representing the bipolar satisfaction degree of actors in social networks.

(3) $\beta$: $V \times V \longrightarrow [0, 1] \times [-1, 0]$ is the bipolar-valued fuzzy relation, representing the bipolar satisfaction degree of ties between social actors.

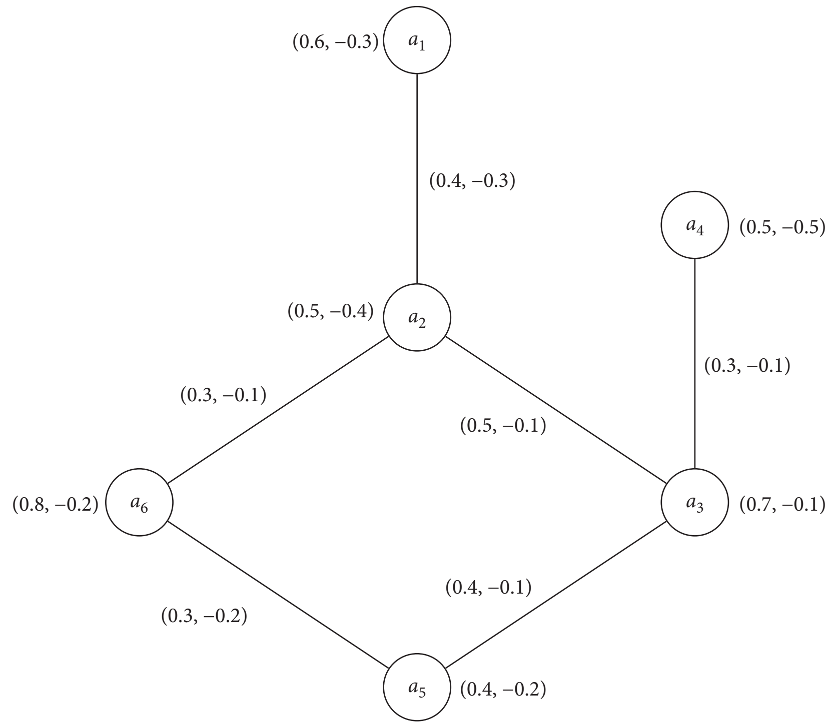


Figure 2: Bipolar-valued fuzzy social network (BVFSN).

*3.2. Bipolar-Valued Fuzzy Adjacency Relations.* Several prescripts already have been presented to address the issue of inadequate knowledge about the sharpness of the correlation between pairs of elements. As an example, a discrete measure may be adopted and each entry $a_{ij}$ is assigned a value to represent the sharpness of the relationship between $a_i$ and $a_j$. This method, depends on the valued adjacency relations is commonly preferred to solve the problems of un-valued relationships. Here, we would like to attempt an alternative outlook through bipolar-valued fuzzy set theory to obtain bipolar-valued fuzzy adjacency relationship between actors. A bipolar-valued fuzzy relation $R$ on a non-empty set $X$ is given by a mapping $R = (R^P, R^N)$: $X \times X \longrightarrow [0,1] \times [-1,0]$ and we write in short $r_{ij} = R(a_i, a_j)$. By means of the matrix $M = [r_{ij}]_{n\times n}$, we can easily represent the bipolar-valued fuzzy relation in which each entry $r_{ij}$ is given by the bipolar satisfaction degree of the relation between $a_i$ and $a_j$. That is, the value of each entry $r_{ij}$ is a response to the question: "what are the degree of strengths and weakness in the relationship between $a_i$ and $a_j$?" Therefore, in the context of BVFSN

$$R(a_i, a_j) = \begin{cases} 0, & \text{if } a_i \text{ is not related to } a_j, \\ (0,1], & \text{degree of strength in the relationship between } a_i \text{ and } a_j, \\ [-1,0), & \text{degree of weakness in the relationship between } a_i \text{ and } a_j. \end{cases} \quad (2)$$

To be more specific, suppose $X$ is a universe of discourse, $R$ is a bipolar-valued fuzzy relation on $X$, and thus a bipolar-valued fuzzy relation is frequently expressed by the matrix:

$$\mathbf{M_R} = \begin{bmatrix} (r^P_{11}, r^N_{11}) & (r^P_{12}, r^N_{12}) & \cdots & (r^P_{1n}, r^N_{1n}) \\ (r^P_{21}, r^N_{21}) & (r^P_{22}, r^N_{22}) & \cdots & (r^P_{2n}, r^N_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ (r^P_{n1}, r^N_{n1}) & (r^P_{n2}, r^N_{n2}) & \cdots & (r^P_{nn}, r^N_{nn}) \end{bmatrix}, \quad (3)$$

where $(r^P_{ij}, r^N_{ij}) \in [0,1] \times [-1,0]$ represents the degree of strength and weakness in the relationship between $a_i$ and $a_j$.

*Definition 5.* Let us $\mathbb{B} = (V, \alpha, \beta)$ be a bipolar-valued fuzzy social network. The stability of actors is important in any network; for example, if $a_i$ is any actor in BVFSN, then we define the fuzzy stability index of $a_i$ is given by

$$S(a_i) = \begin{cases} \dfrac{d}{\eta}, & \text{if } d < \eta, \\ 1, & \text{if } d \geq \eta, \end{cases} \quad (4)$$

where $d$ is the number of direct connections of $a_i$ and $\eta$ is the stable connection number that is constant in the network.

*Definition 6.* The connectivity index of a BVFSN $\mathbb{B} = (V, \alpha, \beta)$ over the bipolar-valued fuzzy graph G denoted by $CI(\mathbb{B}) = (CI^P(\mathbb{B}), CI^N(\mathbb{B}))$, where $CI^P(\mathbb{B}) = \sum_{a,b\in V} \alpha^P(a)\alpha^P(b) CONN^P(a,b)$ and $CI^N(\mathbb{B}) = \sum_{a,b\in V} \alpha^N(a)\alpha^N(b) CONN^N(a,b)$.

*Definition 7.* Suppose $a_{ih_1}, a_{h_1h_2}, a_{h_2h_3} \ldots a_{hj}$ is any path between the vertices $a_i$ and $a_j$ then bipolar-valued fuzzy effective strength of the path is given by

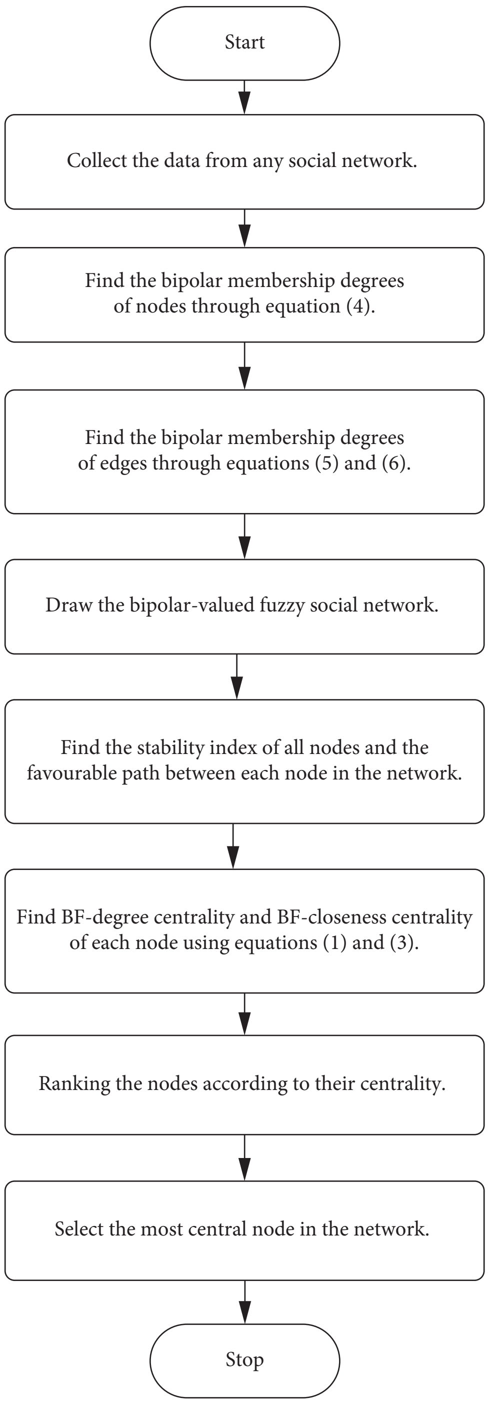


Figure 3: Flowchart to determine the most central actor in BVFSN.

$$\tau\left(a_i, a_j\right) = \frac{\min\left\{\left(a_{ih_1}^{P} + a_{ih_1}^{N}\right), \ldots, \left(a_{hj}^{P} + a_{hj}^{N}\right)\right\}}{\text{Number of edges between } a_i \text{ and } a_j}. \quad (5)$$

*Definition 8.* Suppose $a_{ih_1}, a_{h_1h_2}, a_{h_2h_3} \ldots a_{hj}$ is any path between the vertices $a_i$ and $a_j$ then the fuzzy total strength of the path is given by

$$\tau * \left(a_i, a_j\right) = \frac{a_{ih_1}^{P} + a_{ih_1}^{N} + \cdots + a_{hj}^{P} + a_{hj}^{N}}{\text{Number of edges between } a_i \text{ and } a_j}. \quad (6)$$

*3.3. Bipolar-Valued Fuzzy Centrality Measures.* At the moment, information about the entire world can be obtained in a fraction of a second, and social networks play a significant part in this process. Identifying the influential broadcasters to maximize broadcast potential has been a difficult and important topic so far. Most traditional influence measures evaluate the significance of adjacent nodes in a graph based on their functional and spatial characteristics and are unconcerned with the merits, strengths, and weaknesses of the relationship. As a result, we proposed two basic centrality measures based on bipolar-valued fuzzy relations: BF-degree centrality and BF-closeness centrality.

*3.4. BF-Degree Centrality.* Shaw was the first to introduce degree centrality as a critical vertex index, which was later defined by Niemann. Freeman was the first to propose centrality as a mathematical model based on the number of ties connected to a vertex. In fact, a node with a higher degree is considered more important. To some extent, this measure would demonstrate the significance of nodes, but it does not consider the strength and weakness present in the relationship, also we observe that vertices with the same degree do not necessarily play equal importance in the network.

Bipolar-valued fuzzy degree centrality emphasizes the degree of the node as well as the sum of the strengths and weaknesses of the relationship. Bipolar-valued fuzzy degree centrality of a vertex $a_i$ is given by

$$BF_d\left(a_i\right) = S + \sum_{j\in N} \left|\left(a_{ij}^{P}\right)^{q_1}\right| - \left|\left(a_{ij}^{N}\right)^{q_2}\right|, \quad (7)$$

where $S$ is the fuzzy stability index as we defined in Definition 5 and $q_1$, $q_2$ are positive real numbers that are used to ensure the consistency of positive and negative satisfaction degrees. If $q_1$ and $q_2$ are the same numbers, the ranking of influential actors remains unchanged. We can choose different values of $q_1$ and $q_2$ based on network properties to maintain uniformity. For example, if we have a terrorism network, negativity is much higher than positivity; similarly, if we have a social welfare network, positivity is much higher than negativity. These numbers are only used to adjust the values of positive and negative satisfaction degrees. By default, these can be interpreted as unity.

*3.5. BF-Closeness Centrality.* Bavelas proposed the closeness centrality measure, which Sabidusi defined as the inverse of the sum of the geodesic distances from each vertex in the network to every other vertex. It is a centrality measure based on distance, and distance is measured using the shortest paths. Closeness centrality refers to the ease and convenience of connecting the focused node to the other nodes. Using Dijkstra's algorithm for shortest paths, Newman generalized closeness centrality to weighted networks.

Under this segment, we introduce the idea of bipolar-valued fuzzy closeness centrality, which recognizes not just the shortest distance of a path, but as well as its effective and total strength, as the inverse of the sum of the number of

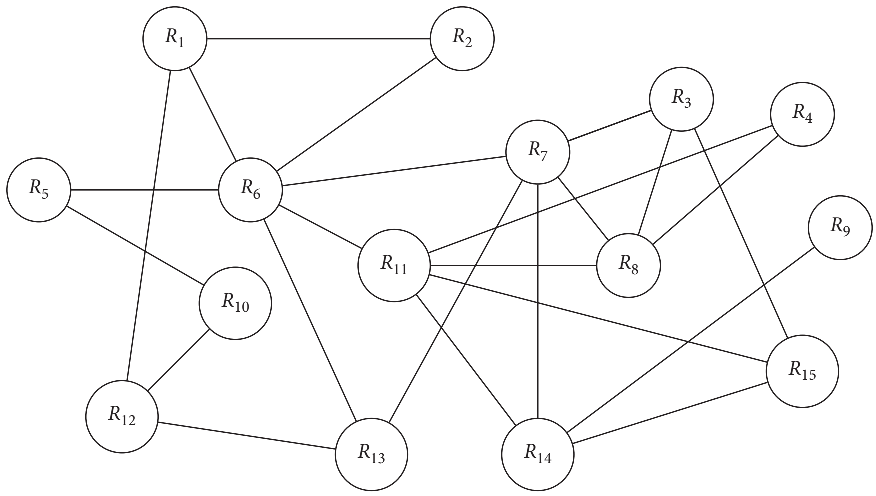


Figure 4: Bipolar-valued fuzzy G-S Research Team.

edges in the favorable path between two vertices and the sum of the relationship's strength and weaknesses.

If there are two or more paths between vertices then we take the favorable path. A path is said to be favorable if the effective strength of the path is maximum. The effective strength of the path $a_{ih_1}, a_{h_1h_2}, a_{h_2h_3} \ldots a_{hj}$ is given by

$$\tau\left(a_{ih_1}, a_{hj}\right) = \frac{\min\left\{\left(a_{ih_1}^P + a_{ih_1}^N\right), \ldots, \left(a_{hj}^P + a_{hj}^N\right)\right\}}{\text{No. of edges between } a_{ih_1} \text{ and } a_{hj}}. \quad (8)$$

If the effective strength of two or more paths is equal, we can calculate the total strength of the path, which is given by

$$\tau^*\left(a_{ih_1}, a_{hj}\right) = \frac{a_{ih_1}^P + a_{ih_1}^N + \cdots + a_{hj}^P + a_{hj}^N}{\text{No. of edges between } a_{ih_1} \text{ and } a_{hj}}. \quad (9)$$

If $a_{ih_1}, a_{h_1h_2}, \ldots, a_{hj}$ is the favorable path between $a_i$ and $a_j$, then the bipolar fuzzy distance is denoted by $\delta(a_i, a_j)$ and defined as

$$\delta\left(a_i, a_j\right) = \lambda \sum \left|1 - a_{ih}^P\right| + \lambda \sum \left|a_{ih}^N\right|, \quad (10)$$

where $\lambda$ is the number of the edges between the nodes $a_i$ and $a_j$ in the favorable path. If positive membership degree of edges in the path between $a_i$ and $a_j$ are maximum then the bipolar fuzzy distance is minimum and if negative membership degree of edges in the path between $a_i$ and $a_j$ are maximum then the bipolar fuzzy distance is maximum.

Bipolar-valued fuzzy closeness centrality of a vertex $a_i$ is given by

$$\mathrm{BF}_c\left(a_i\right) = \frac{n-1}{\sum_{a_j \in N} \delta\left(a_i, a_j\right)}, \quad (11)$$

where $\delta(a_i, a_j)$ is the bipolar-valued fuzzy distance and $n$ is the number of vertices in the network.

*3.6. Example.* Suppose $G = (V, E)$ be the graph, where $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and $E = \{v_1v_2, v_2v_3, v_3v_4, v_3v_5, v_5v_6, v_6v_2\}$. Then bipolar-valued fuzzy social network and bipolar-valued membership degrees of actors and ties are represented in Figure 2 and Table 1.

### *3.7. Bipolar Fuzzy Centrality Measures*

*3.7.1. BF-Degree Centrality.* Bipolar fuzzy degree centrality of a node $a_i$ is given by

$$\mathrm{BF}_d\left(a_i\right) = S + \sum_{j \in N} \left|\left(a_{ij}^P\right)^{q_1}\right| - \left|\left(a_{ij}^N\right)^{q_2}\right|, \quad (12)$$

where $S$ is the fuzzy stability index, here we may take stable connection number $d =$ maximum degree of a node in the network that is 3 and $q_1$, $q_2$ are positive real numbers. For simplicity, let us take $q_1 = q_2 = 1$.

$$\begin{aligned}
\mathrm{BF}_d\left(a_1\right) &= \frac{1}{3} + (0.4 - 0.3) = 0.43, \\
\mathrm{BF}_d\left(a_2\right) &= 1 + [(0.4 - 0.3) + (0.3 - 0.1) + (0.5 - 0.1)] = 1.70, \\
\mathrm{BF}_d\left(a_3\right) &= 1 + [(0.5 - 0.1) + (0.3 - 0.1) + (0.4 - 0.1)] = 1.90, \\
\mathrm{BF}_d\left(a_4\right) &= \frac{1}{3} + [(0.3 - 0.1)] = 0.53, \\
\mathrm{BF}_d\left(a_5\right) &= \frac{2}{3} + [(0.4 - 0.1) + (0.3 - 0.2)] = 1.07, \\
\mathrm{BF}_d\left(a_6\right) &= \frac{2}{3} + [(0.3 - 0.1) + (0.3 - 0.2)] = 0.96.
\end{aligned} \quad (13)$$

Figure 2 shows that nodes $a_1$ and $a_4$ have the same number of directly connected edges, and the positive membership degree of node $a_1's$ edge is higher than the positive membership degree of node $a_4's$ edge, but the bipolar fuzzy degree centrality of node $a_4$ is higher than the bipolar fuzzy degree centrality (BF-degree centrality) of node $a_1$. This is because bipolar fuzzy degree centrality takes into account not only the direct connection but also the strength and weaknesses of the relationship. Ranking of the nodes according to BF-degree centrality is given as $a_3 > a_2 > a_5 > a_6 > a_4 > a_1$.

*3.7.2. BF-Closeness Centrality.* If $a_{ih_1}, a_{h_1h_2}, \ldots, a_{hj}$ is the favorable path between $a_i$ and $a_j$ then

$$\delta(a_i, a_j) = \lambda \sum \left|1 - a_{ih}^P\right| + \lambda \sum \left|a_{ih}^N\right|, \tag{14}$$

where $\lambda$ is the number of the edges between $a_i$ and $a_j$ in the favorable path. Bipolar-valued fuzzy closeness centrality of a vertex $a_i$ is given by

$$\mathrm{BF}_c(a_i) = \frac{n-1}{\sum_{a_j \in N} \delta(a_i, a_i)}, \tag{15}$$

where $\delta(a_i, a_j)$ is the bipolar-valued fuzzy distance and $n$ is the number of vertices in the network.

$$\begin{aligned}
&\delta(a_1, a_2) = 1 \times 0.6 + 1 \times 0.3, \delta(a_1, a_3) = 2 \times (0.6 + 0.5) + 2 \times (0.3 + 0.1),\\
&\delta(a_1, a_4) = 3 \times (0.6 + 0.5 + 0.7) + 3 \times (0.3 + 0.1 + 0.1),\\
&\delta(a_1, a_5) = 3 \times (0.6 + 0.5 + 0.6) + 3 \times (0.3 + 0.1 + 0.1), \delta(a_1, a_6) = 2 \times (0.6 + 0.7) + 2 \times (0.3 + 0.1),\\
&\mathrm{BF}_c(a_1) = \frac{5}{0.90 + 3.00 + 6.90 + 6.60 + 3.40},\\
&\quad = \frac{5}{20.8} = 0.240.
\end{aligned} \tag{16}$$

Similarly,

$$\begin{aligned}
\mathrm{BF}_c(a_2) &= \frac{5}{0.90 + 0.60 + 2.80 + 2.60 + 0.80}\\
&= \frac{5}{7.70} = 0.649,\\
\mathrm{BF}_c(a_3) &= \frac{5}{3.00 + 0.60 + 0.80 + 0.70 + 2.80}\\
&= \frac{5}{7.90} = 0.633,\\
\mathrm{BF}_c(a_4) &= \frac{5}{6.90 + 2.80 + 0.80 + 3.00 + 6.60}\\
&= \frac{5}{20.1} = 0.248,\\
\mathrm{BF}_c(a_5) &= \frac{5}{6.60 + 2.60 + 0.70 + 3.00 + 0.90}\\
&= \frac{5}{13.80} = 0.362,\\
\mathrm{BF}_c(a_6) &= \frac{5}{3.40 + 0.80 + 2.80 + 6.60 + 0.90}\\
&= \frac{5}{14.50} = 0.345.
\end{aligned} \tag{17}$$

Closeness centrality refers to how close a node is to other nodes in a network using the shortest distance, but only the shortest distance cannot be used to measure two nodes' closeness. When the distance between two nodes is small, but their relationship is poor, they are not close. By using a favorable path, BF-closeness centrality considers the shortest distance as well as the path's strength and weakness. According to BF-closeness centrality ranking of the node in Figure 2 is given as $a_2 > a_3 > a_5 > a_6 > a_4 > a_1$.

## 4. Application and Analysis of BVFSN

Inside this portion, we demonstrate how the presented Bipolar-valued fuzzy social network framework and centrality measures can provide a solution to a well-known social network analysis problem, namely, detecting the most central node. We present an application related to the G-S Research network here and explain the process of finding the most central actor in Figure 3.

We select 15 authors from Google Scholar, as shown in Tables 2 and 3, which have coauthor relationship with one another. The two poles in this bipolar-valued fuzzy social network represent each author's publications number and citations number from 1998 to 2016. $\mathbb{B} = (V_{15}, \alpha, \beta)$ is a BVFSN of G-S Research Team, where $V_{15} = \{R_1, R_2, \ldots, R_{15}\}$ is a set of 15 authors, $\alpha = (\alpha^P, \alpha^N)$: $V_{15} \longrightarrow [0,1] \times [-1,0]$ and $\beta = (\beta^P, \beta^N)$: $V_{15} \times V_{15} \longrightarrow [0,1] \times [-1,0]$ are bipolar-valued fuzzy sets and bipolar-valued fuzzy relations on $V_{15}$, respectively.

*4.1. Description of Bipolar Membership Grades.* In Figure 4 each node is treated as an author, the bipolar membership grade of the nodes, representing the degree of publications and the degree of citations. Here, we determine that the maximum number of article publications and citations per article within the period 1998 to 2016 are 500 and 30, respectively.

The positive and negative membership grades of the nodes shown in Table 4 are determined by

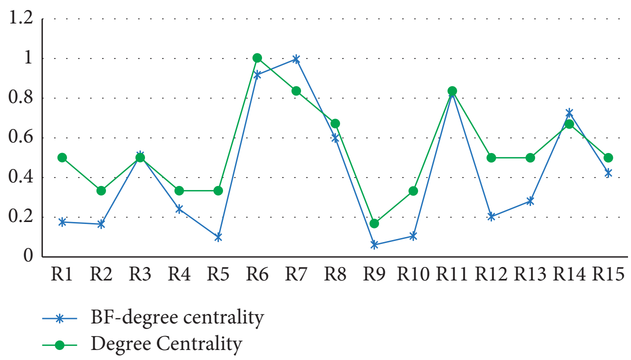


FIGURE 5: Scattered chart marked with lines depending on BF-degree and degree centrality measures (normalized) on G-S Research network.

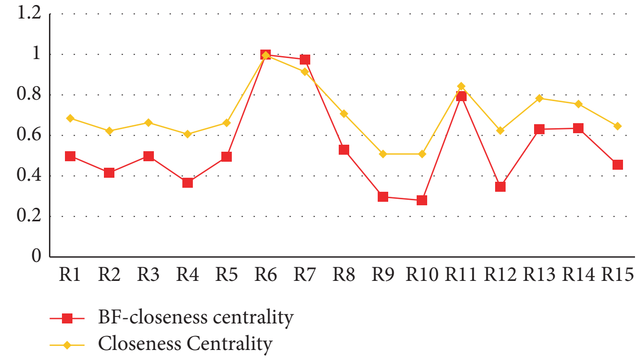


FIGURE 6: Scattered chart marked with lines depending on BF-closeness and closeness centrality measures (normalized) on G-S Research network.

$$\alpha^{+}(R_i) = \begin{cases} \dfrac{k_i}{500}, & \text{if } k_i < 500, \\ 1, & \text{if } k_i \geq 500, \end{cases} \quad \text{and } \alpha^{-}(R_i) = \begin{cases} -\left(1 - \dfrac{h_i}{30}\right), & \text{if } h_i < 30, \\ 0, & \text{if } h_i \geq 30, \end{cases} \tag{18}$$

where $k_i$ and $h_i$ represent the number of publications and citations of an author $R_i$ as shown in Table 2.

Each edge in this network represents a coauthorship relationship between two nodes. Following the collection of data from Google Scholar, we determined that the maximum number of publications and citations per article as a co-author is 25 and 30, respectively.

The positive and negative membership grades of edges shown in Table 5 are determined by

$$\beta^{+}(R_{ij}) = \begin{cases} \dfrac{p_{ij}}{25} \times \min\{\alpha^{+}(R_i), \alpha^{+}(R_j)\}, & \text{if } p_{ij} < 25, \\ \min\{\alpha^{+}(R_i), \alpha^{+}(R_j)\}, & \text{if } p_{ij} \geq 25, \end{cases}$$

And (19)

$$\beta^{-}(R_{ij}) = \begin{cases} \left(1 - \dfrac{g_{ij}}{30}\right) \times \max\{\alpha^{-}(R_i), \alpha^{-}(R_j)\}, & \text{if } g_{ij} < 30, \\ \max\{\alpha^{-}(R_i), \alpha^{-}(R_j)\}, & \text{if } g_{ij} \geq 30. \end{cases}$$

As shown in Table 3, $p_{ij}$ and $g_{ij}$ represent the number of publications and citations for which $R_i$ and $R_j$ are coauthors.

We calculate the BF-degree centrality and BF-closeness centrality of each node in the network after modeling the problem in BVFSN. First, we fix the stable connection number $\eta = 6$ for G-S Research network and use Definition 5 to calculate the stability index of all nodes. Find the favorable path between any two nodes in the network and calculate the bipolar fuzzy distance between them. Finally, using (7) and (11), we may calculate BF-degree centrality and BF-closeness centrality for each node. We also compute degree centrality and closeness centrality in the crisp sense at the same time for comparative analysis. The four centrality measurements of the G-S Research Team are listed in Table 6. The ranking of actors are also highlighted. In Figures 5 and 6, we also present detailed information about these 15 researchers based on the four centrality measures, with the $x$-axis representing the researchers and the $y$-axis corresponding to their scores. As we will see, $R_7$ has the highest BF-degree centrality score. It means that $R_7$ is the central researcher who has better interpersonal relationships with others, i.e., $R_7$ is liked by other researchers with good influence. While $R_9$ has the lowest BF-degree centrality score. It means that $R_9$ is the researcher whose strength of relationship with other researchers is weak. $R_6$ receives the highest BF-closeness centrality score. This means that $R_6$ is the closest researcher, having the shortest bipolar fuzzy distance with the majority of the researchers. $R_{10}$, on the other hand, receives the lowest score based on BF-closeness centrality, which reflects $R_{10}'s$ low connectivity from the people present in the network.

*4.2. Comparative Analysis.* If there is a relationship between nodes in a crisp graphical model, the score value is 1, otherwise, it is 0. However, in real-life relationships, we must

Table 1: Bipolar-valued fuzzy membership degrees.

| V | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ |
|---|---|---|---|---|---|---|
| degree | (0.6, −0.3) | (0.5, −0.4) | (0.7, −0.1) | (0.5, −0.5) | (0.4, −0.2) | (0.8, −0.2) |
| E | $a_1a_2$ | $a_2a_3$ | $a_3a_4$ | $a_2a_6$ | $a_6a_5$ | $a_5a_3$ |
| degree | (0.4, −0.3) | (0.5, −0.1) | (0.3, −0.1) | (0.3, −0.1) | (0.3, −0.2) | (0.4, −0.1) |

Table 2: Authors publications and citations since 1998 to 2016.

| Author | Publication | Citation |
|---|---|---|
| $R_1$ | 184 | 19407 |
| $R_2$ | 97 | 2514 |
| $R_3$ | 240 | 7578 |
| $R_4$ | 219 | 4594 |
| $R_5$ | 104 | 1720 |
| $R_6$ | 396 | 8833 |
| $R_7$ | 444 | 8761 |
| $R_8$ | 334 | 3816 |
| $R_9$ | 118 | 1291 |
| $R_{10}$ | 123 | 1748 |
| $R_{11}$ | 592 | 8839 |
| $R_{12}$ | 354 | 3092 |
| $R_{13}$ | 310 | 4822 |
| $R_{14}$ | 414 | 5104 |
| $R_{15}$ | 193 | 4590 |

Table 3: Coauthors publications and citations since 1998 to 2016.

| Coauthor | Publication | Citation | Coauthor | Publication | Citation |
|---|---|---|---|---|---|
| $R_1, R_2$ | 6 | 137 | $R_6, R_{11}$ | 19 | 353 |
| $R_1, R_6$ | 11 | 256 | $R_6, R_{13}$ | 13 | 340 |
| $R_1, R_{12}$ | 14 | 211 | $R_7, R_8$ | 32 | 348 |
| $R_2, R_6$ | 21 | 490 | $R_7, R_{13}$ | 8 | 279 |
| $R_3, R_7$ | 25 | 533 | $R_7, R_{14}$ | 28 | 437 |
| $R_3, R_8$ | 17 | 441 | $R_8, R_{11}$ | 15 | 234 |
| $R_3, R_{15}$ | 14 | 387 | $R_9, R_{14}$ | 13 | 315 |
| $R_4, R_8$ | 13 | 321 | $R_{10}, R_{12}$ | 12 | 297 |
| $R_4, R_{11}$ | 18 | 341 | $R_{11}, R_{14}$ | 21 | 366 |
| $R_5, R_6$ | 9 | 193 | $R_{11}, R_{15}$ | 18 | 410 |
| $R_5, R_{10}$ | 16 | 297 | $R_{12}, R_{13}$ | 14 | 279 |
| $R_6, R_7$ | 23 | 446 | $R_{14}, R_{15}$ | 26 | 419 |

recognize that some positive and negative properties exist, which we cannot judge with only two degrees 0 and 1. We present the BVFSN model, which has nodes and edges with bipolar membership degrees. We extend the membership degrees \{0, 1}\ in this network to the interval $[0, 1] \times [-1, 0]$, which allows us to model the problem with fuzziness and bipolarity. The main difference between crisp centrality measures and our centrality measures is that it only takes into account the number of direct connections and shortest distances, but nodes with the same number of direct connections do not necessarily have the same strength, and it also does not take into account the bipolarity of the relationship, which is always present in real life. The strengths, connectivity, and stability of paths are also important factors that we take into account in our proposed method. The differences between our proposed centrality measures and other centrality measures are shown in Table 6.

In case of degree centrality the ranking of influential authors from Figure 5 is given by $R_6 > R_7 = R_{11} > R_{14} = R_8 > R_1 = R_3 = R_{12} = R_{13} = R_{15} > R_2 > R_4 = R_5 = R_{10} > R_9$, the main issue is that it only depends on the number of connections, so most of the authors have the same rank. While a ranking of influential authors based on BF-degree centrality is provided here. $R_7 > R_6 > R_{11} > R_{14} > R_8 > R_3 > R_{15} > R_{13} > R_4 > R_{12} > R_1 > R_2 > R_{10} > R_5 > R_9$, which is more accurate than degree centrality. According to degree centrality $R_6$ is the most influential author, but according to BF-degree centrality $R_7$ is the most influential author. $R_6$ has a direct connection with six authors, and $R_7$ has a direct connection with five authors, however the author $R_7$ connections strength is much better than the author $R_6$ connections strength.

In view of closeness centrality, from Figure 6 ranking of the authors is given as $R_6 > R_7 > R_{11} > R_{13} > R_{14} > R_8 > R_1 > R_3 = R_5 > R_{15} > R_{12} > R_2 > R_4 > R_9 = R_{10}$. According to

Table 4: Bipolar-valued fuzzy membership degrees of authors.

| V | $R_1$ | $R_2$ | $R_3$ | $R_4$ | $R_5$ | $R_6$ |
|---|---|---|---|---|---|---|
| degree | (0.37, −0.65) | (0.20, −0.14) | (0.48, −0.05) | (0.44, −0.30) | (0.21, −0.45) | (0.79, −0.25) |
| V | $R_7$ | $R_8$ | $R_9$ | $R_{10}$ | $R_{11}$ | $R_{12}$ |
| degree | (0.89, −0.34) | (0.67, −0.62) | (0.24, −0.63) | (0.25, −0.53) | (1.00, −0.50) | (0.72, −0.62) |
| V | $R_{13}$ | $R_{14}$ | $R_{15}$ | | | |
| degree | (0.62, −0.48) | (0.71, −0.68) | (0.39, −0.21) | | | |

Table 5: Bipolar-valued fuzzy membership degrees of coauthor relations.

| E | $R_{1,2}$ | $R_{1,6}$ | $R_{1,12}$ | $R_{2,6}$ | $R_{3,7}$ |
|---|---|---|---|---|---|
| degree | (0.048, −0.033) | (0.162, −0.057) | (0.207, −0.310) | (0.168, −0.030) | (0.480, −0.014) |
| E | $R_{3,8}$ | $R_{3,15}$ | $R_{4,8}$ | $R_{4,11}$ | $R_{5,6}$ |
| degree | (0.326, −0.007) | (0.218, −0.004) | (0.228, −0.054) | (0.316, −0.111) | (0.075, −0.072) |
| E | $R_{5,10}$ | $R_{6,7}$ | $R_{6,11}$ | $R_{6,13}$ | $R_{7,8}$ |
| degree | (0.134, −0.175) | (0.726, −0.087) | (0.600, −0.097) | (0.322, −0.032) | (0.670, −0.217) |
| E | $R_{7,13}$ | $R_{7,14}$ | $R_{8,11}$ | $R_{9,14}$ | $R_{10,12}$ |
| degree | (0.198, −0.340) | (0.830, −0.163) | (0.402, −0.255) | (0.124, −0.112) | (0.120, −0.095) |
| E | $R_{11,14}$ | $R_{11,15}$ | $R_{12,13}$ | $R_{14,15}$ | |
| degree | (0.697, −0.210) | (0.281, −0.052) | (0.347, −0.163) | (0.390, −0.098) | |

Table 6: BF-degree centrality, BF-closeness centrality, degree centrality, and closeness centrality with ranking of authors.

| Author | BF-degree centrality with rank $\text{BF}_d\ (R_i)$ | BF-closeness centrality with rank $\text{BF}_c\ (R_i)$ | degree centrality with rank $d\ (R_i)$ | closeness centrality with rank $C\ (R_i)$ |
|---|---|---|---|---|
| $R_1$ | 0.517 **(11)** | 0.231 **(7)** | 0.214 **(4)** | 0.437 **(7)** |
| $R_2$ | 0.486 **(12)** | 0.192 **(11)** | 0.142 **(5)** | 0.400 **(10)** |
| $R_3$ | 1.499 **(6)** | 0.230 **(8)** | 0.214 **(4)** | 0.424 **(8)** |
| $R_4$ | 0.712 **(9)** | 0.169 **(12)** | 0.142 **(5)** | 0.389 **(11)** |
| $R_5$ | 0.295 **(14)** | 0.227 **(9)** | 0.142 **(5)** | 0.424 **(8)** |
| $R_6$ | 2.678 **(2)** | 0.459 **(1)** | 0.428 **(1)** | 0.636 **(1)** |
| $R_7$ | 2.916 **(1)** | 0.450 **(2)** | 0.357 **(2)** | 0.583 **(2)** |
| $R_8$ | 1.759 **(5)** | 0.246 **(6)** | 0.285 **(3)** | 0.451 **(6)** |
| $R_9$ | 0.179 **(15)** | 0.136 **(14)** | 0.071 **(6)** | 0.325 **(12)** |
| $R_{10}$ | 0.317 **(13)** | 0.129 **(15)** | 0.142 **(5)** | 0.325 **(12)** |
| $R_{11}$ | 2.404 **(3)** | 0.365 **(3)** | 0.357 **(2)** | 0.538 **(3)** |
| $R_{12}$ | 0.606 **(10)** | 0.159 **(13)** | 0.214 **(4)** | 0.400 **(10)** |
| $R_{13}$ | 0.832 **(8)** | 0.291 **(5)** | 0.214 **(4)** | 0.500 **(4)** |
| $R_{14}$ | 2.124 **(4)** | 0.292 **(4)** | 0.285 **(3)** | 0.482 **(5)** |
| $R_{15}$ | 1.235 **(7)** | 0.210 **(10)** | 0.214 **(4)** | 0.412 **(9)** |

BF-closeness centrality, the authors are ranked as follows: $R_6 > R_7 > R_{11} > R_{14} > R_{13} > R_8 > R_1 > R_3 > R_5 > R_{15} > R_2 > R_4 > R_{12} > R_9 > R_{10}$. We can easily see that the ranking of authors has changed significantly using both centrality measures. The main reason for the disparities in ranking is the capability of centrality measures; closeness centrality only considers the shortest distance between two nodes, whereas BF-closeness centrality considers the shortest distances as well as the effective and total strength of the path between two nodes. For example, there are two shortest paths between the nodes $R_6$ and $R_8$ such as $R_6 \longrightarrow R_7 \longrightarrow R_8$ and $R_6 \longrightarrow R_{11} \longrightarrow R_8$. According to closeness centrality, we may consider any of the path both have same importance, but in the BF-closeness centrality, effective strength of the path $R_6 \longrightarrow R_7 \longrightarrow R_8$ is maximum and it is most favorable path between these two nodes.

## 5. Conclusion

We named the bipolar-valued fuzzy social network (BVFSN) after a novel modeling technique based on the bipolar-valued fuzzy set theory that we used to describe the network with fuzziness and bipolarity. The determination of the most central node in a complex network is a critical and unresolved problem. As a result, new measures for identifying central nodes with greater accuracy are appealing. In this article, we extend the centrality theory to BVFSN and investigate some of its fundamental properties. First, in BVFSN, we define the concepts of the fuzzy stability index, effective strength, total strength, and connectivity strength. We can find the favorable path between any two authors in the BVFSN based on their effectiveness and total strength. Then, for the BVFSN, we propose two new centrality measures: BF-degree centrality and BF-closeness centrality. To determine the most central actor in BVFSN, BF-degree centrality measures combine the degree of the node, bipolarity of the relations, and the fuzzy stability index of the network. The second one is the BF-closeness centrality measure, which combines bipolar fuzzy distance and effective path strength to help us choose the closest node in the network. We used the G-S Research network to assess the performance of our centrality measures. The mathematical results demonstrate that our method can clearly choose the most central node to evaluate the coauthor relationship. We discovered that our centrality measure could better rank actors based on their popularity in the network. This study adds to the centrality theory and lays the theoretical groundwork for future research on BVFSN.

## Data Availability

There is no data associated with this study.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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