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Research Article

A Multistage Stochastic Programming Model with Multiple Objectives for the Optimal Issuance of Corporate Bonds

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Large corporations usually cover their capital and operating expenses by issuing bonds with fixed rates and different maturities. This paper proposes a multistage stochastic programming (MSP) model with multiple objectives to optimize bond issuance by satisfying the three common objectives of corporate managers, as follows: (i) Minimizing expected discounted cost under cash liquidity and financial leverage risk constraints. (ii) Minimizing financial leverage risk under expected discounted cost and cash liquidity risk constraints. (iii) Minimizing cash liquidity risk under expected discounted cost and financial leverage risk constraints. We measure liquidity risk as conditional payment-at-risk (*CPaR*), according to the corporation's financial characteristics. Financial leverage risk is captured by conditional financial leverage-at-risk (*CFLaR*), which we design based on conditional value-at-risk (*CVaR*). Through empirical analysis of a company in China, we explore the efficient frontier curves for the three above objectives and provide the corresponding issuance compositions of an optimal bond portfolio. Our MSP model offers guidance for corporations on achieving a trade-off between cost and risk when issuing corporate bonds.

1. Introduction

The dynamic issuance of bonds provides an optimal supply of funds for corporations' capital and operating expenses under uncertainty. Compared with bank loans, bonds issued in tranches are a more flexible source of funding, which facilitates bond repayment (Sierpińska and Bąk, [1]). The composition of the bond portfolio should be considered when issuing bonds in tranches. Numerous studies have investigated the structure of corporate bond portfolios. Aydin [2] showed empirically that corporations with more growth opportunities tend to have more short-term bonds in their portfolios. Kailan, Richard, and Yilmaz [3] found that corporate asset maturity and liquidity were significantly positively associated with bond maturity and that corporate equity structure had some influence on bond portfolios. Körner [4] discussed the determinants of the bond maturity structure of Czech national corporations and found that the issuance of long-term bonds increased with corporation size, leverage, and asset maturity. Orman and Köksal [5] argued that rationalizing debt maturity can help corporations in developing countries grow rapidly and that macroeconomic environmental factors affect bond maturity. Besides illustrating the importance of bond portfolio structure to corporate management, these studies indicated that bond management should consider the volatility in cash savings and other financial uncertainty indicators that affect a corporation's financial structure and cash liquidity expenditure.

This paper proposes a multistage stochastic programming (MSP) model with multiple objectives to deal with the uncertainty of bond issuance in tranches. The stochastic programming model, the most effective solution to asset management problems, is now a dynamic multistage stochastic programming (MSP) model that combines multiple stages of asset and bond simulation and forecasting. In recent years, the MSP model has been widely used in various fields. It offers a very flexible solution to bond portfolio and liquidity management problems. Bradley and Crane [6] were the first to propose a multistage model of bond portfolio management. Subsequently, Carino et al. [7] applied the MSP theory to the problem of asset liability management in the insurance industry. Many studies have since used the MSP model to solve asset management problems (see Ziemba and Mulvey, Dupacova et al., Hilli, Koivu, Pennanen, and Ranne, Topaloglou N et al, and so on) [8–13]. Regarding sovereign bond management, MSP specializes in optimizing sovereign bond issuance and finding a trade-off between minimum expected cost and minimum risk (Balibek and Köksalan; Consiglio and Staino; Date et al.,) [14–16]. From the perspective of corporate cases, Davi, Álvaro, and Veiga [17] proposed an MSP model that described the dynamic decision-making process behind the issuance of corporate bonds and explored the average risk trade-off between expected bond servicing costs and the expected value of corporate bankruptcy. However, these studies have focused on using MSP to explore the trade-off between minimum cost and minimum risk. Little effort has been made to explore an MSP model with multiple objectives, such as minimizing costs or risk under the constraints of various objectives.

In the development process of scholars for solving the multiobjective bond portfolio optimization problems, many kinds of algorithms have been explored. The solution method of the multiobjective bond portfolio optimization function often lies in the optimization problem of converting many incomparable objectives into a single objective. Since the end of the twentieth century, some scholars have focused on improving algorithms with the aim of improving the iterative solution methods for various multiobjective bond portfolio functions to promote the diversity and validity of solutions (Nakayama, Sharma et al., Nakayama et al., Pai et al., Lam et al., Wang et al.,) [18–23]. It is worth noting that the widely studied nondominated sorting genetic algorithm II (NSGA-II) can find the corresponding optimal solution in the true Pareto optimal front in most problems, and the convergence and calculation speed are constantly improving (Altiparmak et al., Kalyanmoy Deb, Ali Hojjati et al.) [24–26]. For reference in this paper, García et al. combine factors such as return, risk, and liquidity to measure portfolio performance and use L-R type fuzzy numbers to determine the future return and liquidity of each asset. They considered cardinality constraints and upper and lower bound constraints, and improved the algorithm NSGA-II for solving optimization problems that constrained portfolio expected return, semivariance, and expected liquidity (García et al.,) [27]. The improved algorithm of NSGA-II provides an idea for this paper to obtain the optimal bond issuance portfolio of the MSP model with multiple objectives and provides a probabilistic optimization method idea. We can automatically obtain and guide the optimized search space according to the set threshold and make the algorithm adaptively adjust the search direction.

Considering the multistage multiobjective bond portfolio optimization is the main direction of the current multiobjective bond portfolio optimization evolutionary algorithm. In multiclass combinatorial optimization problems, decision-makers perform multiobjective (Pareto) optimization in multiple stages can explore various reasonable and effective optimal solutions in complex situations, and strive to develop algorithms that minimize the computational burden (Balıbek, Banihashemi, Radulescu et al., Zhou et al., Kim et al.,) [28–32]. Ma et al. considered multiple objectives and complex constraint problems at the same time and explored that most of the current multiobjective optimization problems in complex constraint regions are inefficient and have insufficient convergence. They proposed a multistage evolutionary algorithm with constraints stacked on top of each other, which can be processed in different stages of the multiobjective optimization evolutionary algorithm. This algorithm can search for the optimal solution for the next stage adding new constraints under the optimal solution obtained in the current stage. They also propose a strategy for prioritizing constrained processing based on the impact on the unconstrained Pareto front, which can speed up the convergence of the algorithm (Ma et al) [33]. This paper also seeks the optimal solution for MSP model with multiple objectives under complex constraints, we consider the optimal solution at each stage when formulating the bond issuance strategy, and designing the program code for this article by taking into account the ideas of the above solution algorithm.

In this study, we propose an MSP model with multiple objectives that consider the expected discounted cost, cash liquidity risk, and financial leverage risk. We design a conditional financial leverage-at-risk (*CFLaR*) construct to measure corporate financial leverage risk in extreme cases, inspired by the conditional value-at-risk *CVaR* risk measurement approach. We also develop a corporate cash liquidity risk measurement construct, conditional payment-at-risk (*CPaR*), that improves on the existing liquidity risk measurement method (Balibek and Köksalan) [14]. Most importantly, we derive efficient frontier curves to explore the following three objectives: (i) minimizing the expected discounted cost under different liquidity and financial leverage risk constraints; (ii) minimizing financial leverage risk under different cost and liquidity risk constraints; and (iii) minimizing liquidity risk under different cost and financial leverage risk constraints.

The remainder of this paper is organized as follows. In Section 2, we define the corporate bond management problem and describe its main features. In Section 3, we describe our MSP model in detail. In Section 4, we provide three objective functions that capture corporate managers' preferences for minimizing costs and risks when issuing bonds. In Section 5, we empirically test our model and examine the effective frontiers of our three objectives. Finally, we summarize our results and suggest future research directions in Section 6.

2. Characteristics of the Corporate Bond Management Strategy Formulation Problem

An effective management strategy for issuing corporate bonds needs to meet the expected financing requirements over a specific period and consider the trade-off between costs and risk (Bradley S P and Crane D B) [6]. Because the repayment cost of corporate bonds is generally higher than that of bank loans, bond repayment may impose a greater financial burden on a corporation and increase the risk of corporate liquidity in the future.

When managing the issuance of corporate bonds, decision-makers mainly consider the repayment cost of issuing new bonds. Here, "the repayment cost" refers to a bond's principal and the interest at different repayment points in the future. The sum of the discounted value of the principal and interest of a bond is defined as the bond's expected repayment cost (O'Connell and Zeldes) [34]. As multistage repayment costs are distributed over several years, it is necessary to discount the principals and interests of bonds issued on each year as a first step when corporate managers consider formulating a bond issuance management strategy.

A multiobjective corporate bond management strategy is characterized by decision-making under uncertainty. Therefore, the corporation needs to consider the evolution of the coupon rates of newly issued bonds in the future, which is a random process (Kim, Ramaswamy, and Sundaresan) [35]. Corporations usually issue corporate bonds at fair prices, and the coupon rates of new bonds are affected by the forward yields of bonds with the same credit rating and the same maturity in the market. The coupon rates of corporate bonds issued in the future are dynamic; therefore, managers cannot predict the size of interest payments with certainty (Koltsaklis and Dagoumas) [36]. Moreover, corporations cannot be sure about the future circumstances of relevant macroeconomic variables. A degree of uncertainty is associated with the evolution of macroeconomic variables such as interest and deposit rates that drive the cost of borrowing (Were and Wambua) [37]. The outcomes of the decision on bond issuance made are subject to the realization of these variables. Hence, managers must analyze the risks caused by these uncertainties.

To a large extent, the risks faced by a corporation when issuing bonds arise from the uncertain payments of new bonds with different maturity terms. These risks mainly include financial leverage risk and liquidity risk. Financial leverage risk, which causes a corporation's losses to increase exponentially, can be measured by the degree of financial leverage, which is mainly determined by bond interest payments. Liquidity risk is based on cash expenses during a cash liquidity planning horizon, and it increases with the proportion of bonds that mature within the cash liquidity planning horizon (Diamond D W) [38]. Controlling the issuance amount of corporate bonds with different maturities at distinct time points can affect both the financial leverage risk and liquidity risk simultaneously. In most cases, there should be a trade-off between cost and liquidity risk. Reducing liquidity risk may require the issuance of long-term bonds with high costs and the maintenance of an excess cash reserve, both of which incur additional costs for the corporation. Given the cost of bonds and the two kinds of risk driven by the payments of the principals and interests of newly issued bonds, corporations must consider the composition of their bond issuance portfolios, which can be adjusted to manage costs and risks simultaneously (Gilchrist et al) [39].

When formulating corporate bond issuance strategies, decision-makers need to consider constraints on bond issuance. Funds raised in the decision-making stage should be sufficient to repay existing bond on the planning horizon. A corporation needs to maintain a sound cash account to meet its operating cash needs (Jindřichovská I and Körner P) [40]. However, the issuance of bonds by a corporation is subject to restrictions imposed by national laws and policies. In particular, the scale of newly issued bonds cannot be too large (Hurst J. W) [41].

The management strategy behind corporate bond issuance involves multiple interrelated decision-making processes, not one-off decisions. The decisions made should work well together to allow the corporation to issue a portfolio of bonds with different maturities to facilitate adaptation to future changes in macroeconomic conditions (Weill P and Ross J. W) [42]. On the basis of observed and predicted macroeconomic trends, the strategy may need to be adjusted in the future. Corporate bond managers need to consider the impact of future adjustment decisions based on the current market environment as well as the corporation's current financial situation and existing corporate outstanding bonds (Donaldson G) [43].

3. Multistage Stochastic Programming Model with Multiple Objectives

3.1. Model Framework. Based on a scenario tree of macroeconomic factors that affect the discounted costs of bonds and the risks faced by corporations, we propose an MSP model with multiple objectives. As a linear equation, our model specifies a sequence of bond issuance decisions at discrete time points during a multistage stochastic planning horizon. We assume that corporations with strong financing needs set a bond issuance strategy at the initial stage of the multistage stochastic planning horizon, which considers the number of bonds to be issued each year and revise this strategy annually. We start with an existing fixed corporate outstanding bond portfolio and a set of anticipated scenarios regarding the future states of relevant macroeconomic variables, such as interest and deposit rates.

Following research on multistage stochastic planning models (Topaloglou et al [11]; Consiglio and Staino [15]; Shapiro et al [44]), we divide the planning horizon of our model into multiple stages, $t\,(t = 1, 2, 3, \ldots, T)$, where T is the final stage. Considering interest payments and discount rates, the total planning period in our MSP model is T years, and a planning interval is 1 year. The formulation of a bond issuance strategy at the beginning of each year is regarded as a decision node, the scenarios between decision nodes combine to form a sequence of joint realizations for a multistage stochastic planning horizon. These sequences of scenarios are linked at each decision node, and the scenario paths cover the entire planning horizon.

Generally, a scenario tree simulates the evolutionary paths of random variables across different stages of the planning horizon by discretizing their joint probability distributions (Boender G) [45]. Uncertainty is the most prominent feature of the scenario tree, and its importance

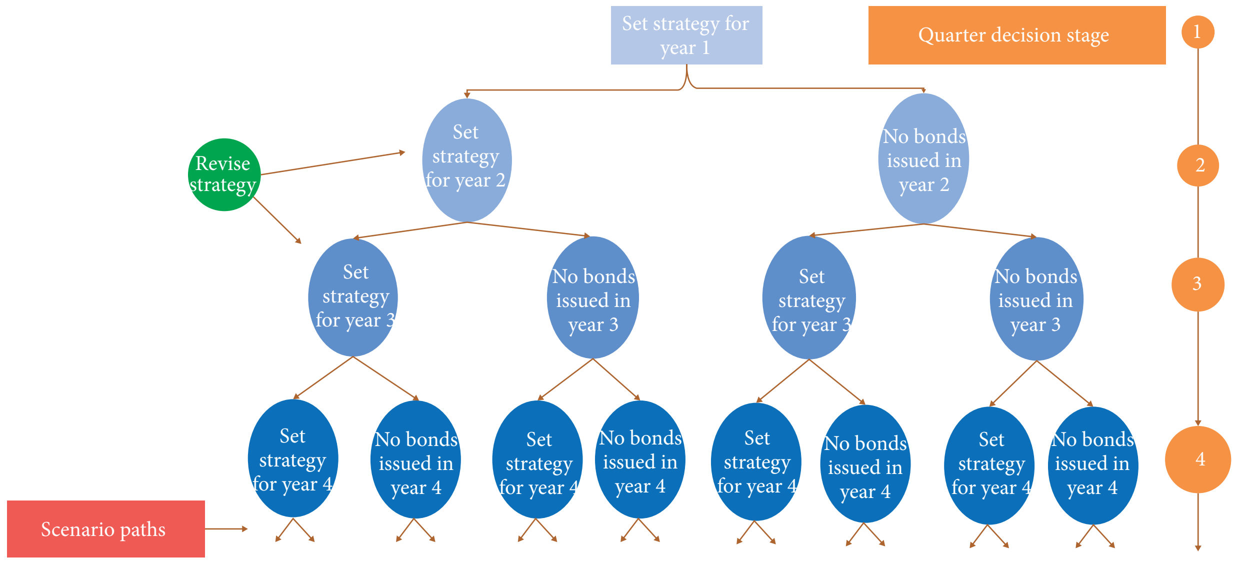


Figure 1: scenario tree of corporate debt issuance strategies.

increases with the number of decision stages. Issuance planning in the first year is affected by the corporation's outstanding bond portfolio, and at the end of the first year, the corporation has a new outstanding bond portfolio and makes a new set of decisions incorporating this new portfolio structure; thus, the updated cash-flow scheme is contingent on the scenario's realization in the first year and the tree branches in the current scenario. Decisions other than the first-stage decision are divided into two categories: issuing a fixed amount of bonds and not issuing bonds. Figure 1 shows a schematic diagram of the paths of the macroeconomic variables in the scenario tree.

As shown in Figure 1, corporations must consider whether to issue bonds at the beginning of each year on the basis of their current financial conditions and the macroeconomic environment after the first year. All discrete joint results are mapped onto the nodes of the scenario tree, i.e., the scenario tree describes a dynamic setting in which a decision is made regarding the future at a given stage. Once the decision is implemented, the information related to the issuance of new corporate bonds is revealed in the following period, and the associated process is repeated.

3.2. Definitions. To prepare a complete formal statement of the model, we first define the parameters and index sets, stochastic variables, auxiliary variables, and decision variables used in the model.

3.2.1. Parameters and Index Sets

(i) T Length of the multi-stage stochastic planning horizon.

(ii) t Decision time. $t = 0, 1, \ldots, T$

(iii) S Set of macro-scenario when corporation issuing new bonds.

(iv) S_{total} Total number of macro-scenario when corporation issuing new bonds.

s Scenario index.

(v) H Cash liquidity planning horizon of the corporation.

(vi) p^s Probability of the state associated with a scenario. s

(vii) J Set of variable fixed-rate bonds (with different issuance maturities issued by a corporation).

(viii) j Serial numbers of fixed-rate bonds issued by a corporation with different issuance maturities.

(ix) M_j Maturity of new bond j issued by the corporation.

(x) M_{old} Remaining maturity of the outstanding bonds that existed at the initial decision time.

(xi) NP_t^s Corporate average net profit for the last three years before time t under scenario s.

(xii) PS_t Corporate available cash surplus at time t.

(xiii) β^s Discount rate under scenario s.

(xiv) λ Ratio of the accumulated bond balance to the corporation's net assets.

(xv) CB_0 Cash balance of the corporation at the initial moment.

(xvi) $f_s(t)$ Amount of cash flow expected by the corporation at time t under scenario s.

The dependent inputs of the scenario are given below:

3.2.2. Stochastic Variables

(i) $r^s_{M_j,t}$ Coupon interest rate of bond j issued by the corporation at time t under scenario s.

(ii) $r^s_{w,t}$ One-year treasury bond yield at time t under scenario s.

(iii) L_t^s Liquidity payments amount of the corporation at time t under scenario s.

The decision variables are defined for each node of the scenario tree:

3.2.3. Decision Variable. X_{t,M_j}^s Amounts of types j of bonds issued by corporations at time t under scenario s.

The auxiliary variables are defined for each node of the scenario tree:

3.2.4. Auxiliary Variables

(i) DEBT_t^s Total number of bonds of the corporation at time t under scenario s.

(ii) ASSET_t^s Corporate net assets at time t under scenario s.

(iii) $\text{cost\_}i_t^s$ Amount of bond interests that the corporation has to pay at time t under scenario s.

(vi) D_t^s Sum of the principal discounted of all new bonds issued by the corporation at time t to time 0 under scenario s.

(v) D_H^s Sum of the principal discounted to the initial stage of all old and new bonds mature within H under scenario s.

(vi) C_t^s Corporate cash amount at time t under scenario s.

(vii) $FB_{t,j}^{s,i}$ Amount of j bonds issued at time t and the remaining financing balance after the interest paid in year $t+i$ under scenario s.

(viii) $I_{t,i}^{s,j}$ Annual interests paid by the corporate bonds at time t is discounted to the amount at time 0 under scenario s.

(ix) I_t^s The sum of the discounted net interests of new bonds issued at time t to time 0 under the scenario. s

(x) I_H^s Sum of the interests of all outstanding bonds and the net interests of newly issued bonds during the period discounted to the initial stage under scenario s.

(x) I_{old} Outstanding bond interests that existed at the initial decision time.

(xi) $NI_{t,t+1}^s$ Non-bond cash inflows of the corporation from t to t+1, under scenario s.

(xii) $X_{t,t+1,j}^s$ Amount of types of bonds issued by corporations from t to t+1 under scenario s.

(xiii) $L_{t,t+1}^s$ Liquidity payments amount of the corporation from t to t+1, under scenario s.

(xiv) L_H^s Sum of the liquidity payment amount of the corporation within H under scenario s.

(xv) TC^s Total cost of issuing new bonds under scenario s.

(xvi) OLD_t^s Total amount of outstanding bonds issued by the corporation at time t under scenario s.

(xvii) CB_t^s Corporation's available cash balance at time t under scenario s.

(xviii) FLR Variables used in the definition of conditional financial leverage risk equal to VaR under the optimal solution.

(xix) PR Variables used in the definition of conditional payment-at-risk (similar to FLR).

3.3. Constraints. Our corporate bond issuance strategy management model is subject to three types of constraints:

(i) Balance constraints on the payments of principals and net interests of newly issued bonds by the corporation.

(ii) Balance constraints on the state of corporate cash flow.

(iii) Government constraints on the number of bonds issued by corporations.

3.3.1. Balance Constraint Equation for the Payments of Principals and Net Interest of Newly Issued Bonds. The balance constraint equation for the payments of the principals and net interests of newly issued bonds represents the corporate repayment cost and lays the foundation for the balance constraint equation for the corporate cash flow state.

(i) Total discounted principal value at the initial decision node of bonds issued at time t under scenario s:

$$D_t^s = \sum_{j\in J} \frac{X_{t,M_j}^s}{\left(1+\beta^s\right)^{Tj+t}}. \quad (1)$$

Based on the bond maturity, we can obtain the discounted value at the initial decision node of the principals of the bonds issued at time t. This equation sums up all the discounted principal values of each bond issued at time t for a specific scenario s. It is worth noting that in the calculation process, the WACC of the corporation under scenario s is used as the discount rate β^s.

(ii) Under scenario s, the total discounted interest at the initial decision node of bonds issued at time t:

$$I_{t,i}^{s,j} = \sum_{j\in J} \sum_{i=1}^{T_j} \frac{X_{t,M_j}^s \cdot r_{M_j,t}^s}{\left(1+\beta^s\right)^{i+t}}. \quad (2)$$

The discounted value at the initial decision node of the interests of bonds issued at time t can be calculated by the coupon rates and maturity of the bonds. Equation (2) sums the discounted interest value of all bonds issued at time t under a specific scenario s.

(iii) Under scenario s, the total discounted net interest at the initial decision node of bonds issued at time t:

$$\left.\begin{aligned} &FB_{t,j}^{s,i} = X_{t,M_j}^{s} - i \cdot I_{t,i}^{s,j}, \\ &DF_t^{s,i} = \frac{r_{w,t}^{s}}{\left(1+\beta^{s}\right)^{i+t}}, \\ &R_t^{s} = \sum_{j\in J}\sum_{i=1}^{T_j}\left(\left(FB_{t,j}^{s,i} + r_{w,t}^{s}\cdot\left(FB_{t,j}^{s,i-1}\right)\right)\cdot DF_t^{s,i}\right)(i\geq 1) \\ &R_t^{s}\geq 0 \end{aligned}\right\}, \tag{3}$$

where $FB_{t,j}^{s,i}$ represents the balance of financing after paying interests in year $t+i$ under scenario s, and $DF_t^{s,i}$ is a deposit discount factor. Formula (3) represents the cash deposits due to issuing bonds, it depends on $FB_{t,j}^{s,i}$ and $r_{w,t}^{s}$ as well as the discount factor $DF_t^{s,i}$.

$$I_t^{s} = \sum_{j\in J} I_{t,i}^{s,j} - R_t^{s}. \tag{4}$$

Equation (4) is the net interest cash flow equation for a corporation issuing new bonds under scenario s, which consists of the discounted interest of newly issued bonds and the cash deposits due to issuing bonds before repaying the principals of the bonds.

3.3.2. Balance Constraint Equation for Corporate Cash Flow. The balance constraint equation for corporate cash flow takes into account corporate cash liquidity, and the cash flow can be calculated on the basis of the balance state of the cash account each year.

Cash flow balance equation for a single corporation:

$$C_{t+1} = \sum_{j\in J} X_{t,t+1,M_j}^{s} + C_t^{s} + NI_{t,t+1}^{s} - L_{t,t+1}^{s} \quad \forall t, s. \tag{5}$$

The cash flow balance (5) shows that under scenario s, the cash balance of the corporation at t+1 equals the sum of the cash balance account at time t and the difference between cash inflow and cash outflow from t to t+1; here, the cash inflow includes corporate debt financing cash $\sum_{j\in J} X_{t,t+1,j}^{s}$ and non-bond cash inflow $NI_{t,t+1}^{s}$. $L_{t,t+1}^{s}$ represents cash liquidity payments from t to t+1.

3.3.3. Government Constraints on the Number of Bonds Issued by Corporations. In China, the number of corporate bonds that a corporation can be issued is subject to the following governmental restrictions. The average distributable profit of the corporation in the last 3 years must be sufficient to pay the interest on corporate bonds for 1 year, and the cumulative bond balance must not exceed 40% of net corporate assets (excluding minority shareholders' equity).

Below are mathematical descriptions of the constraint equations of our model:

(i) Principle of non-negativity (constraints on the number of corporate bonds):

$$X_{t,M_j}^{s} \geq 0. \tag{6}$$

(ii) Principle of market discipline:

$$\sum_{j\in J} X_{t,M_j}^{s} \cdot r_{M_j,t}^{s} \leq NP_t^{s}, \tag{7}$$

where NP_t^{s} represents the average distributable profit (net profit) for the 3 years before time t under the scenario s. Formula (7) shows that the average net profit for the 3 years before time t is enough to pay the interest on corporate bonds for 1 year.

(iii) Constraints on the amount of accumulated outstanding bonds: The government controls the leverage ratio to ensure that the corporation has an appropriate cash flow to deal with unexpected crises.

$$\begin{aligned} &\mathrm{DEBT}_t^{s} = \sum_{j\in J} X_{t,M_j}^{s} + \mathrm{OLD}_t^{s}, \\ &\mathrm{ASSET}_t^{s} = C_t^{s} + NI_{t,t+1}^{s} + \frac{1}{S}\cdot\left(\sum_{K=1}^{T}\frac{f_k^{s}}{\left(1+\beta^{s}\right)^{k}}\right), \\ &\lambda = \frac{\mathrm{DEBT}_t^{s}}{\mathrm{ASSET}_t^{s}}, \end{aligned} \tag{8}$$

λ is the corporate bond leverage ratio such that $0\leq\lambda\leq\overline{\lambda}$, where $\overline{\lambda}$ is the maximum debt ratio specified by the government, meaning that the accumulated outstanding bonds do not exceed $\overline{\lambda}$ of corporate net assets (excluding minority shareholders' equity).

3.4. Basic Concepts. Our model has three basic components: expected discounted cost, financial leverage risk, and cash liquidity risk.

3.4.1. Expected Discounted Cost. The expected discounted cost is the sum of the discounted costs of all bonds with different maturities issued at each decision time during the planning horizon [46]. For simplicity, we assume that the discount factor is fixed across the decision period and use the same discount factor to calculate the discounted costs of newly issued bonds with different maturities. For convenience, we regard all newly issued bonds of different maturities at the same decision time as one bond portfolio. The expected discounted cost of bonds issued within the planning horizon is the sum of the discounted costs of issuing bonds in the bond portfolio at each decision time.

We denote TC_t^{s} as the discounted cost at the initial decision time when issuing a new bond portfolio at time t under the scenario s,

$$\begin{aligned} &TC_t^{s} = D_t^{s}(x) + I_t^{s}(x), \\ &TC^{s} = \sum_{t=1}^{T} TC_t^{s}, t = 1, 2, \ldots, T, \end{aligned} \tag{9}$$

where TC^s represents the total discounted cost at the initial decision time of newly issued bonds within the decision horizon. Formula (9) shows that the discounted cost to be repaid by the corporation is the sum of the principals and interests in the bond portfolio et al. l decision-making points under the scenario s.

We focus on the expected discounted cost of bond portfolios at the initial decision time in each scenario within the decision horizon,

$$\mathrm{COST} = \sum_{s\in S} p^s \cdot TC^s, \tag{10}$$

where p^s is the occurrence probability under the scenario s.

3.4.2. Financial Leverage Risk in-a Worst-Case Scenario. Traditionally, corporate financial leverage risk is measured by the degree of financial leverage (*DFL*) (Xu, Gunarathna, Balasubramaniam et al.) [47–49]. *DFL* is a leverage ratio that measures the sensitivity of a corporation's earnings per share to fluctuations in its operating income as a result of changes in its capital structure. It is determined by the ratio of earnings before interest and tax (*EBIT*) to post-interest profit. The *DFL* of a corporation at the time t under scenario s is

$$\begin{aligned} DFL_t^s &= \frac{EBIT_t^s}{(EBIT_t^s - \mathrm{Interest}_t^s)}, \\ \mathrm{Interest}_{t+1}^s &= \sum_{j\in J}\left(X_{t,M_j}^s \cdot r_{M_j,t}^s\right) + \sum_{j\in J}\left(X\_B_t^s \cdot r\_B_t^s\right), \end{aligned} \tag{11}$$

where $EBIT_t^s$ is earnings before interest and taxes at the time t, $\mathrm{Interest}_{t+1}^s(x)$ is the amount the corporation needs to pay at time t+1, $\sum_{j\in J}(X\_B_t^s \cdot r\_B_t^s)$ is the interest paid on outstanding bonds at time t+1 except for those issued at time t, and $\sum_{j\in J}(X_{t,M_j}^s \cdot r_t^{M_j})$ is the number of interest payments at time t+1 of the bond portfolio issued at time t.

The degree of financial leverage cannot reflect the real level of financial leverage risk caused by some extreme events. These extreme events often emerge as fat tails, and tail risk can be measured by conditional value-at-risk (*CVaR*) (see Pflug, 2000; Uryasev and Rockafellar, Mansini et al; Agarwal and Naik, Liang and Park) [50–54]. Moreover, known as "expected shortfall," *CVaR* is derived by taking the weighted average of "extreme" losses in the tail of the distribution of possible returns. Building on *CVaR*, we formulate the construct of conditional financial leverage risk (*CFLaR*) to measure the financial leverage risk faced by a corporation in a worst-case scenario. Essentially, *CFLaR* extends *CVaR* by considering the size of the financial leverage ratio beyond a given *DFL* faced by the corporation at the end of year *t*. To compute the value of *CFLaR*, we define an auxiliary variable *FLR* that represents a certain financial leverage risk for a given level α, and we denote *CFLaR* as

$$CFLaR = FLR(\alpha) + \frac{1}{(1-\alpha)} \cdot \frac{1}{S} \cdot \sum_{s=1}^{S} \left(DFL_t^s - FLR(\alpha)\right)^+, \tag{12}$$

where $(DFL_t^s - FLR(\alpha))^+ = \max\{DFL_t^s(x) - FLR(\alpha), 0\}$. Thus, *CFLaR* is the extended risk measure of *DFL* that

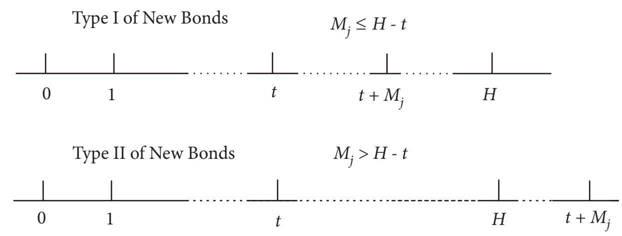


FIGURE 2: The new bonds are issued at time t within the cash liquidity planning horizon (H).

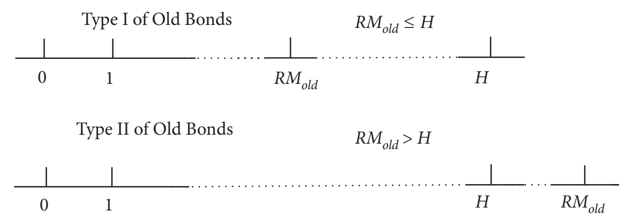


FIGURE 3: The old outstanding corporate bonds were issued before the initial decision time 0 but still existed at the initial decision time 0 within the cash liquidity planning horizon (H).

quantifies the average *DFL* values that exceed a given *FLR* (α) value, and it accounts for the expected possible *DFL* level in a worst-case scenario.

3.4.3. Corporate Short-Term Cash Liquidity Risk in Worst-Case Scenario. Cash liquidity risk is associated with a corporation's actual debt repayments and total cash flow. In practice, corporations tend to focus on short-term liquidity risk. Here, short-term liquidity is quantified as corporate liquidity income and expenses within a certain period (Owolabi and Obida) [55]. To accurately measure the short-term liquidity risk of a corporation over some time, we consider a corporation's short-term liquidity risk during a cash liquidity planning period H and assume that H is longer than the multistage stochastic planning horizon.

We identify two types of bond payments during H. The first is the payment of newly issued bonds at a time t within H. The second is the payment of outstanding corporate bonds that were issued before the initial decision time 0 but exist at the initial decision time 0 within H. For convenience, we denote these two types of bond payments as "new bonds" and "old bonds," respectively and divide "new bonds" and "old bonds" both into a further two types. New bonds issued at time t that mature within H are the first type of new bonds. New bonds issued at time t that mature beyond H are the second type of new bonds. The first type of old bonds is outstanding bonds issued before the initial decision time 0 that still exist at the initial decision time 0 and mature within H, while the second type of old bonds is outstanding bonds issued before the initial decision time 0 that still exist at the initial decision time 0 and mature beyond H. The classification of "new bonds" can be seen in Figure 2, and the classification of "old bonds" is shown in Figure 3.

Therefore, we denote $D_H^s(x)$ as the discounted bond principal payment at the initial decision time 0 during H as follows:

$$\left.\begin{aligned} &D_H^s(x) = \sum_{M_j \le H-t} \frac{X_{t,M_j}^s}{(1+\beta^s)^{M_j}} \cdot 1_{A_1}(M_j) + \sum_{RM_{\text{old}} \le H} \frac{DEBT_{\text{old}}}{(1+\beta^s)^{RM_{\text{old}}}} \cdot 1_{A_2}(M_{\text{old}}), \forall s \in S \\ &A_1 = \{M_j\colon M_j \le H-t\} \\ &A_2 = \{M_{\text{old}}\colon M_{\text{old}} \le H\} \\ &1_A(\omega) = \begin{cases} 1 & \omega \in A \\ 0 & \omega \notin A \end{cases} \end{aligned}\right\}, \tag{13}$$

where

(i) DEBT_{old} denotes the principals of outstanding bonds that exist at the initial decision time.

(ii) RM_{old} is the remaining maturity of outstanding bonds at the initial decision time.

(iii) $\sum_{M_j \le H-t} X_{t,M_j}^s/(1+\beta^s)^{M_j} \cdot 1_{\{M_j \le H-t\}}$ is the discounted principals of new corporate bonds issued at time t within H.

(iv) $\sum_{RM_{\text{old}} \le H} DEBT_{\text{old}}/(1+\beta^s)^{RM_{\text{old}}}$ is the discounted principals of outstanding bonds issued before the initial decision time 0 that exists at the initial decision time 0 within H.

$D_H^s(x)$ is the sum of $\sum_{M_j \le H-t} X_{t,M_j}^s/(1+\beta^s)^{M_j}$ and $\sum_{RM_{\text{old}} \le H} DEBT_{\text{old}}/(1+\beta^s)^{RM_{\text{old}}}$ during H.

We denote $I_H^s(x)$ as discounted bond interest payment at the initial decision time 0 during H:

$$\left.\begin{aligned} &I_H^s(x) = \sum_{M_j \le H-t} \sum_{i=1}^{M_j} \frac{X_{t,M_j}^s \cdot r_{M_j,t}^s}{(1+\beta^s)^{i+t}} \cdot 1_{A_1}(M_j) + \sum_{M_j > H-t} \sum_{i=1}^{H-t} \frac{X_{t,M_j}^s \cdot r_{M_j,t}^s}{(1+\beta^s)^{i+t}} \cdot 1_{A_3}(M_j) + \\ &\sum_{RM_{\text{old}} \le H} \sum_{i=1}^{RM_{\text{old}}} \frac{I_{\text{old}}}{(1+\beta^s)^i} \cdot 1_{A_2}(RM_{\text{old}}) + \sum_{RM_{\text{old}} > H} \sum_{i=1}^{H} \frac{I_{\text{old}}}{(1+\beta^s)^i} \cdot 1_{A_4}(RM_{\text{old}}) \\ &\forall s \in S \\ &A_3 = \{M_j\colon M_j > H-t\} \\ &A_4 = \{M_{\text{old}}\colon M_{\text{old}} > H\} \\ &1_A(\omega) = \begin{cases} 1 & \omega \in A \\ 0 & \omega \notin A \end{cases} \end{aligned}\right\}, \tag{14}$$

where I_{old} is the outstanding bond interest that exists at the initial decision time 0. For convenience, we define the first type of new bonds as those issued at time t that matures within H. We define the second type of new bonds as those issued at time t that matures beyond H. We define the first type of old bonds as those outstanding at the initial decision time that matures within H; and the second type of old bonds is defined as those outstanding at the initial decision time that matures beyond H. In the above formulation (20):

(i) $\sum_{M_j \le H-t} \sum_{i=1}^{M_j} X_{t,M_j}^s \cdot r_{M_j,t}^s / (1+\beta^s)^{i+t} \cdot 1_{A_1}(M_j)$ is the discounted interest payments at the initial decision time of the first type of new bonds during H.

(ii) $\sum_{M_j > H-t} \sum_{i=1}^{H-t} X_{t,M_j}^s \cdot r_{M_j,t}^s / (1+\beta^s)^{i+t} \cdot 1_{A_3}(M_j)$ is the discounted interest payments at the initial decision time of the second type of new bonds during H.

(iii) $\sum_{RM_{\text{old}} \le H} \sum_{i=1}^{RM_{\text{old}}} I_{\text{old}} / (1+\beta^s)^i \cdot 1_{A_2}(RM_{\text{old}})$ is the discounted interest payments at the initial decision time 0 of the first type of outstanding bonds during H.

(iv) $\sum_{RM_{\text{old}} > H} \sum_{i=1}^{H} I_{\text{old}} / (1+\beta^s)^i \cdot 1_{A_4}(RM_{\text{old}})$ is the discounted interest payments at the initial decision time 0 of the second type of outstanding bonds during H.

We use conditional payment-at-risk (*CPaR*) to measure the highest possible payment level in a worst-case scenario:

$$CPaR(\alpha) = PR(\alpha) + \frac{1}{(1-\alpha)} \cdot \frac{1}{S} \cdot \sum_{s=1}^{S} \left(D_H^s(x) + I_H^s(x) + L_H^s - PR(\alpha) \right)^+, \tag{15}$$

where *PR* is the value of cash liquidity payments for a given level α, and $(D_H^s(x) + I_H^s(x) + L_H^s - PR(\alpha))^+ = \max(D_H^s(x) + I_H^s(x) + L_H^s - PR(\alpha), 0)$. That is, *CPaR* is the average liquidity payment value paid by a corporation when the corporate liquidity payment exceeds a given *PR* value during H. The above-mentioned *CPaR* formula considers the expected possible payment level of a corporation during H in a worst-case scenario. Corporations can compare the value of *CPaR* with the level of funds that they generate under extreme market conditions to manage liquidity risks.

4. Objective Functions

In the real world, bond issuance managers mainly focus on bond repayment cost (*COST*), financial leverage risk in worst-case scenarios (*CFLaR*), and short-term cash liquidity risk in worst-case scenarios (*CpaR*). To meet the specific requirements of their corporations, managers usually prioritize the following three common cases about objective functions.

Case I. Minimizing *COST* under the constraints of different *CFLaR* and *CPaR*.

Under the constraints of different *CFLaR* and *CPaR*, we minimize *COST* in the debt planning horizon as follows:

$$\min_{x_{t,m_j}^s, s\in S} \text{COST},$$
$$\text{s.t.} \begin{cases} \underline{b} \le CFLaR \le \overline{b}, \\ \underline{c} \le CPaR \le \overline{c}, \\ \sum_{j\in J} x_{t,m_j}^s = X_t, \\ \underline{b} \ge 0, \\ \underline{c} \ge 0, \end{cases} \tag{16}$$

where $\underline{b}$ and $\overline{b}$ are the lower and upper thresholds of *CFLaR*, respectively, $\underline{c}$ and $\overline{c}$ are the lower and upper thresholds of CPaR, respectively, and $X_t \ge 0$, X_t is the financing size at decision time t.

Case II. Minimizing *CFLaR* under the constraints of different COST and *CPaR*.

Similarly, we minimize the financial leverage risk *CFLaR* under the constraints of different COST and *CPaR* in the debt planning horizon as follows:

$$\min_{x_{t,m_j}^s, s\in S} CFLaR(\alpha),$$
$$\text{s.t.} \begin{cases} \underline{a} \le \text{COST} \le \overline{a}, \\ \underline{c} \le CPaR \le \overline{c}, \\ \sum_{j\in J} x_{t,m_j}^s = X, \\ \underline{a} \ge 0, \\ \underline{c} \ge 0, \end{cases} \tag{17}$$

where $\underline{a}$ and $\overline{a}$ are the lower and upper thresholds of COST, respectively.

Case III. Minimizing *CPaR* under the constraints of different *COST* and *CFLaR*.

We minimize the corporate cash liquidity risk (*CPaR*) under the constraints of different *COST* and *CFLaR* in the debt planning horizon as follows:

$$\min_{x_{t,m_j}^s, s\in S} CPaR,$$
$$\text{s.t.} \begin{cases} \underline{a} \le CPaR \le \overline{a}, \\ \underline{b} \le CFLaR \le \overline{b}, \\ \sum_{j\in J} x_{t,m_j}^s = X, \\ \underline{a} \ge 0, \\ \underline{b} \ge 0. \end{cases} \tag{18}$$

5. Empirical Analysis

We empirically test the model proposed in Section 4 by applying it to the Chinese company Zhejiang Oriental Holding Group Co., Ltd. (ZOH), whose credit rating is AAA. For convenience, we assume that ZOH company makes an annual issuance plan at the beginning of each year and that the plan can be revised at the beginning of the following year on the basis of the company's finances. ZOH has three outstanding bonds, comprising one medium-term bond and two short-term bonds (see Table 1).

Based on the information on these outstanding bonds, we make the following assumptions:

(i) We assume that the company issues 1 billion bonds each year in three stages (3 years).

(ii) Our model presents a selection of four types of bonds, 3-year, 5-year, 10-year, and 15-year maturity

Table 1: Basic Information of outstanding bonds issued by ZOH company.

	Bond I (21 Dongfang01)	Bond II (20 Dongfang01)	Bond III (20 Dongfang02)
Category of coupon rate	Fixed rate	Fixed rate	Progressive interest rate
Coupon rate (year)	3.9% (2022–2024)	3.63% (2021–2023)	3.4% (2021, 2022, 2023) 3.4% +Basis point (2024, 2025)
Remaining maturity	2.781 Years	1.523 Years	3.929 Years
Issuance size	1 billion yuan	1 billion yuan	0.5 billion yuan

bonds, based on the actual portfolios of bonds issued by ZOH.

(iii) Because the length of our multistage stochastic planning horizon is 3 years, we set the cash liquidity planning horizon H as 6 years to examine corporate cash payments in the current three-stage stochastic planning horizon and the following three-stage stochastic planning horizon.

(iv) We calculate *CPaR* and *CFLaR* at the 95th percentile of their respective distributions (i.e., $\alpha = 5\%$) .

(v) For simplicity, we set the issuance strategy in each year as time-variant, i.e., the weight of each maturity bond is unfixed within the planning horizon.

(vi) The interest payments of outstanding bond II (20 Dongfang01), which matures in the second year, are not paid; therefore, the interest payments of outstanding bond III (20 Dongfang02) will increase in the third year. For convenience, we assume that the total interest payments for outstanding bonds in each year are stable across the three stages.

Next, we describe our method of generating scenario trees to simulate the paths of relevant macroeconomic variables.

5.1. Generation of Scenario Trees. We generate our scenario trees by filtered historical simulation (FHS), which combines a relatively sophisticated model-based treatment of volatility (generalized autoregressive conditional heteroskedasticity) with a nonparametric specification of the probability distribution of asset returns (Anderson, Høyland Wallace, and Tao et al.) [56–58]. Compared with other simulation methods, such as traditional historical simulation and Monte Carlo simulation, FHS has two advantages (Roy) [59]. First, it can capture the conditional heteroskedasticity in the data and be unrestrictive about the shape of the distribution of the risk factors. Second, the method does not involve the estimation of the correlation matrix of risk factors.

The scenario trees generated for the three associated stochastic variables, comprising the coupon rates of newly issued fixed-rate corporate bonds, *EBIT*, and 1-year treasury bond yield, demonstrate the potential evolution of the path of each variable into the future. The three scenario trees help us to measure COST, *CFLaR*, and *CPaR* and examine their relationships with different constraints. For instance, we can use the FHS technique (Pritsker) [60] to simulate a scenario tree of coupon rates based on historical data on forward yields. Specifically, we assume that the conditional mean and variance-covariance matrix of the market forward yields of different bond maturities depend on the history of the market forward yields. r_t is the coupon rate at time t and h_{r_t} is the historical data on the market forward yields of bonds before time t. We define θ as the parameters of the market forward yields generation process; $\mu(h_{r_t}, \theta)$ represents the mean of the yields at time t, conditional on the history of market forwarding yields and θ; $\sum(h_{r_t}, \theta)$ is the variance-covariance matrix of r_t, conditional on h_{r_t} and θ. The coupon rate r_t is driven by the following equation:

$$r_t = \mu\left(h_{r_t}, \theta\right) + \sum\left(h_{r_t}, \theta\right) \cdot \epsilon_t, \tag{19}$$

where θ denotes the parameters of the conditional mean and volatility model and ϵ_t is independent and identically distributed through time with a mean of 0 and variance of I; the θ parameters can be estimated by quasimaximum likelihood under appropriate regularity conditions. Because h_{r_t} is observable, ϵ_t can be identified, θ parameters can be estimated, and the FHS method can be implemented in more general cases. Based on historical data on the market forward yields of bonds with maturities of 3 years, 5 years, 10 years, and 15 years for the target corporation, which has an AAA credit rating, from October 2016 to April 2021, using (17), the scenario trees were generated using MATLAB 2018. It is worth noting that FHS can predict the future trend of corporate bond market forward yields with different maturities based on its historical data. In this paper, the market forward yield of corporate bonds with different maturities for each of the next three years can be simulated according to the historical data on corporate bond's market forward yields with different maturities, for example, the market forward yield of 3-year corporate bonds for each of the next three years can be simulated according to the historical data on 3-year corporate bond's market forward yields, and the forward market yield of corporate bonds can be used as the coupon rate for newly issued bonds. The bond issuance decision at each stage will be affected by the simulated changes in the market forward yield curve of different maturities, that is, the decision-makers in the next stage must take into account the changes in the coupon rate of bond portfolios with different maturities at the current stage and the previous stage when formulating the bond issuance strategy, this is achieved in the program code we designed.

5.2. Robustness Analysis of Scenario Tree Generation. Using FHS, historical data on corporate bond market forward yields (3-years, 5-years, 10-years, and 15-year) and 1 year treasury bond yields from October 2016 to April 2021, we generate three independent and identically distributed scenario trees within a 3 year planning period, and we test the robustness of the three given models in, and (10) and

Table 2: Robustness results.

Scenario tree	Case 1: Minimum of *COST* (billion yuan)		Minimum of *CPaR* (billion yuan)		Minimum of *CFLaR*	
	Average	St. Dev.	Average	St. Dev.	Average	St. Dev.
$10 \times 10 \times 10$	2.8821	0.0003	3.7450	0.0035	1.1367	0.000007
$20 \times 10 \times 10$	2.8720	0.0070	3.7473	0.0002	1.1380	0.000014
$30 \times 10 \times 10$	2.8808	0.0069	3.7453	0.0307	1.1375	0.000003
$50 \times 10 \times 10$	2.8739	0.0073	3.7392	0.0338	1.1378	0.000001
$80 \times 10 \times 10$	2.8720	0.0070	3.7369	0.0181	1.1367	0.0000001

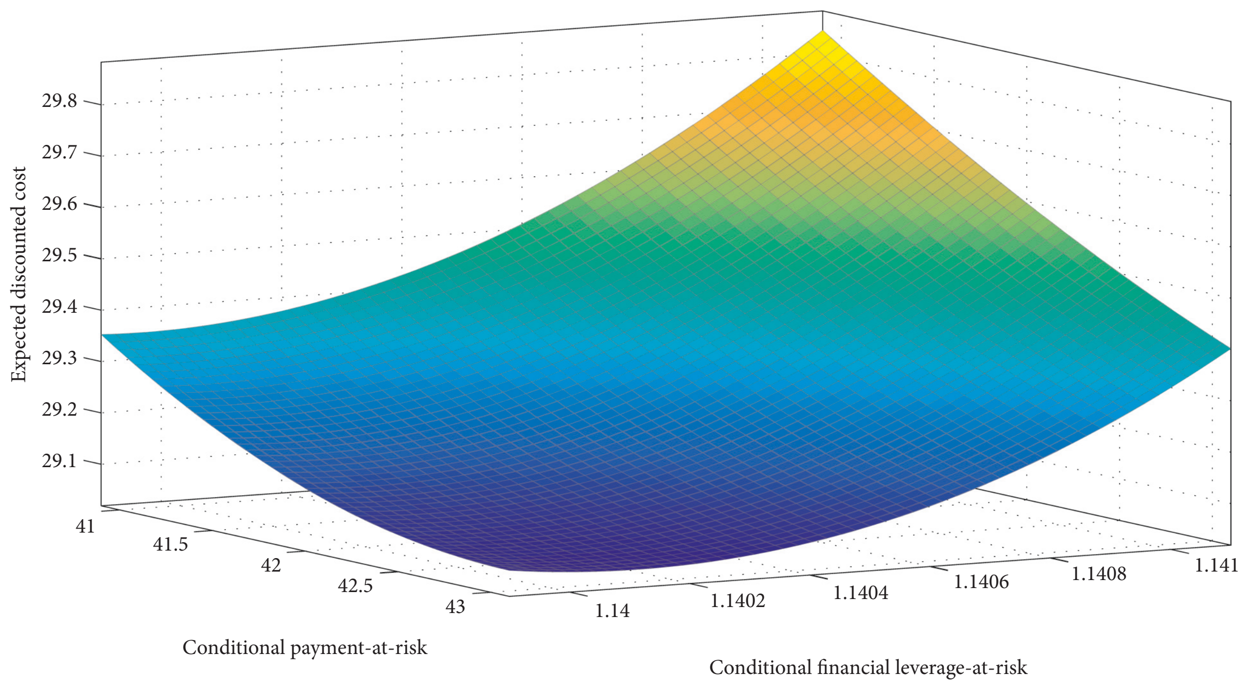


Figure 4: Efficient frontier of a minimum of **COST** with the different **CPaR** and **CFLaR** constraints.

(12), (13). All of the scenario trees are created within the same planning period (3 years); only the number of branches differs. For example, the notation $10 \times 10 \times 10$ corresponds to a three-stage tree with 10 branches from each node in each stage, with 1,000 branches in the final stage. Table 2 depicts the average and standard deviation of the minimum *COST*, *CPaR*, and *CFLaR*, respectively, within the 3-year planning period. The average minimum value of *COST* represents the lowest possible level of expected discounted cost, while the average minimum values of *CFLaR* and *CPaR* indicate the minimum level of corporate financial leverage risk and the minimum level of corporate cash liquidity risk, respectively, in worst-case scenarios.

From Table 2, we find that the average minimum COST for ZOH company is approximately 2.8 billion yuan for 3 years , while the average minimum CPaR is around 3.7 billion yuan. The corresponding variance of minimum *COST*, *CFLaR*, and *CPaR* is very small, which shows that the scenario tree generation process is stable.

5.3. Empirical Results. We describe the empirical results for cases *I-III* (introduced in Section 4) in Section 5.3.1, Section 5.3.2, and Section 5.3.3, respectively.

5.3.1. Case 1: Minimizing COST under the Constraints of Different **CFLaR** *and* **CPaR**. For convenience, we examine the efficient frontier of minimum *COST* under the constraints of different $CPaR \in [41, 42]$ and $CFLaR \in [1.139, 1.141]$ and show our results in Figure 4.

As shown in Figure 4, the minimum COST increases with *CFLaR* and decreases as *CPaR* increases. *CFLaR* is generated by the interest payments of outstanding bonds and newly issued bonds; however, under the assumption that the total interest payments of outstanding bonds in each year are stable across the three stages, *CFLaR* is mainly determined by the interest payments of newly issued bonds. Furthermore, because the interest of long-term bonds is higher than that of short-term bonds, increasing the proportion of long-term bonds will cause *CFLaR* and minimum COST to increase. *CPaR*, which represents the average cash payment level in a worst-case scenario, and is mainly determined by the total payments of principals and interests associated with bonds that mature within H. Thus, increasing the proportion of long-term bonds will lead to a decrease in *CPaR* and an increase in minimum COST.

To explore the evolution of the efficient frontier more precisely, we examine different efficient frontier curves for minimum *COST* with two categories of constraints: (i)

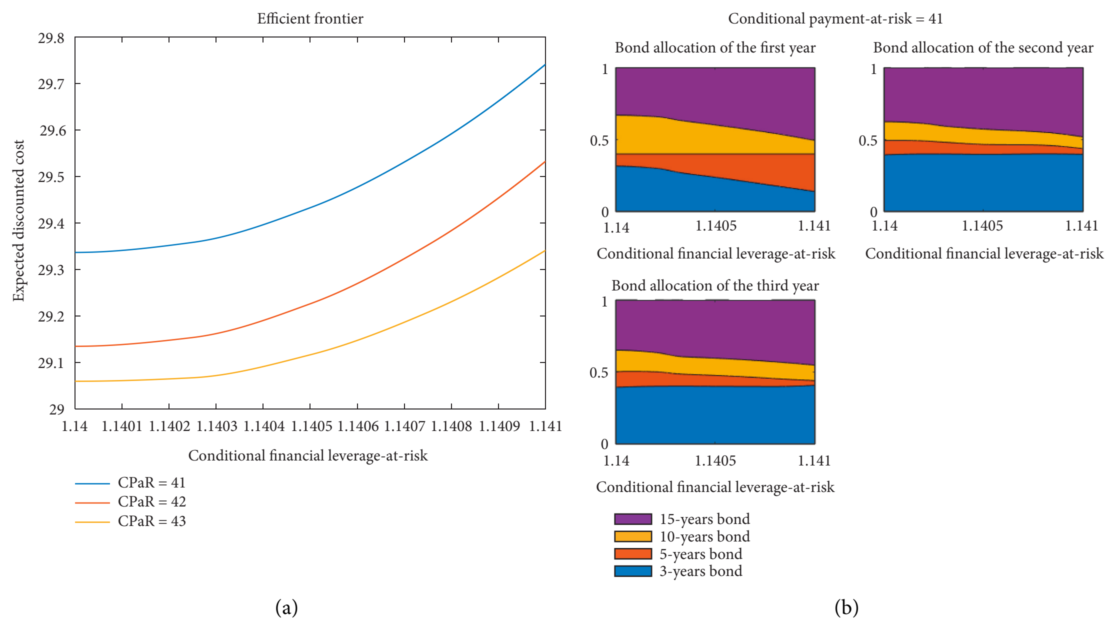


FIGURE 5: The efficient frontier curves of minimal **COST** under different **CFLaR** and three fixed **CPaR** = 40, 41, 42 (a), the optimal bond composition with constraints of **CPaR** = 41 and different **CFLaR** (b).

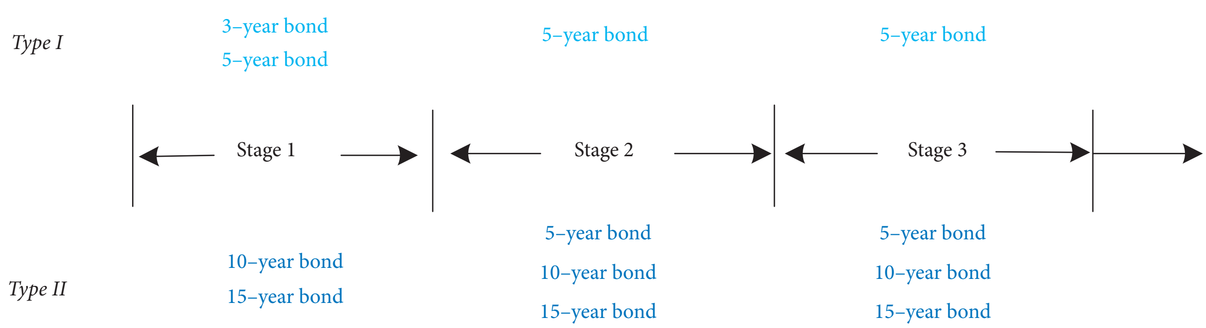


FIGURE 6: The bonds mature within the liquidity planning horizon **H** = 6 years (*Type I*) and the bonds mature after **H** = 6 years (*Type II*).

different *CFLaR* and fixed *CPaR*; and (ii) different *CPaR* and fixed *CFLaR*.

(i) The efficient frontier curves of the minimum of COST with constraints of different *CFLaR* and three fixed *CPaR*.

Figure 5(a) depicts the corresponding efficient frontier curves for minimum COST under the constraints of different *CFLaR* and three fixed *CPaR* = 41, 42, 43. As shown in Figure 5(a), each efficient frontier of minimum COST is an increasing function of *CFLaR* under every fixed *CPaR*. An increase in *CFLaR* represents an increase in the proportion of long-term bonds, which leads to an increase in minimum COST, and vice versa. To further examine the composition of optimal bond portfolios, we consider the example *CPaR* = 41 and give the corresponding issuance compositions in Figure 5(b).

Corresponding to a minimum COST with the constraints of different *CFLaR* and fixed *CPaR* = 41, Figure 5(b) shows the optimal bond portfolio composition for each year of three-stage bond issuance with a 3 year planning horizon. We find all four types of bonds in the composition of bonds issued at each issuance stage.

To further explore the total payments of bonds issued within the three-stage bond issuance planning horizon when $H =$ 6 years, we divide bonds issued within the three-stage bond issuance planning horizon into the following two types (see Figure 6):

(i) *Type I*: Bonds that mature within the liquidity planning horizon when $H = 6$ years, including 3 year and 5 year bonds issued in the first year, 3 year bonds issued in the second year, and 3 year bonds issued in the third year.

(ii) *Type II*: Bonds that mature after the liquidity planning horizon when $H = 6$ years, including 10 year and 15 year bonds issued in the first year, 5 year, 10 year, and 15 year bonds issued in the second year, and 5 year, 10 year, and 15 year bonds issued in the third year.

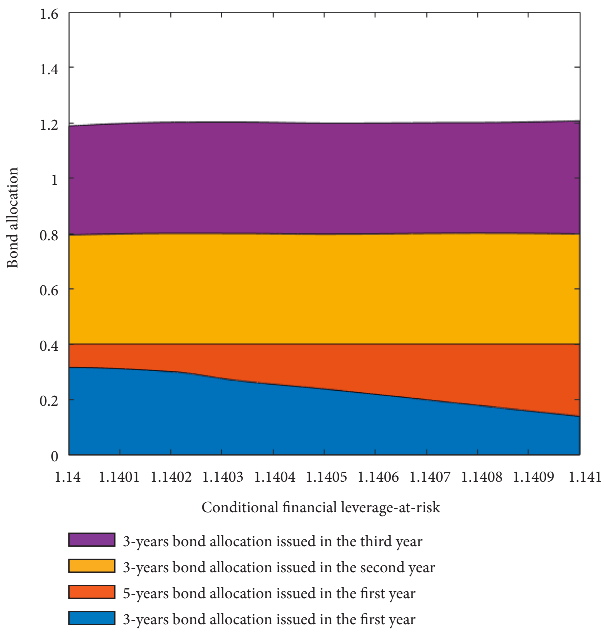


FIGURE 7: The total proportion of bonds that mature within the liquidity planning horizon **H** = 6 years under the fixed **CPaR** = 41.

Based on Figure 5(b), we derive the total proportion of *Type I* bonds as shown in Figure 7, including 3 year and 5 year bonds issued in the first stage, 3 year bonds issued in the second stage, and 3-year bonds issued in the third stage. Figure 7 shows that the total proportion of *Type I* bonds in the three stages is relatively stable. This can be explained as follows. Because *CPaR* is determined by the total payments of principal and interest of *Type I* bonds when the cash liquidity planning horizon $H = 6$ years, a fixed *CPaR* should be driven by the fact that the total payments of principal and interest associated with *Type 1* bonds in the three stages are fixed.

Regarding $CPaR = 41$, because 3 year and 5 year bonds represent nearly half of the total issued bond portfolio in Figure 5(b), assuming that the same amount of new bonds (1 billion yuan) is issued in each stage, the total issuance amount of 3 year and 5 year bonds is close to the total issuance amount of 10 year and 15 year bonds. Because a bond's interest payments are a very small proportion of its principal, the total interest payments of bonds can be ignored. Thus, for a fixed $CPaR = 41$, the total payment of principals of *Type I* bonds when the liquidity planning horizon $H = 6$ years should be stable, which is consistent with Figure 7.

(ii) The efficient frontiers of the minimum of COST with constraints of different *CPaR* and three fixed *CFLaR*.

Figure 8(a) shows the corresponding efficient frontier curves for minimizing COST with different *CPaR* and three fixed $CFLaR = 1.14, 1.1405, 1.141$. As shown in Figure 8(a), each efficient frontier of minimum COST is a decreasing function of *CPaR* under a fixed *CFLaR*. As mentioned above, *CPaR* is mainly determined by the total payments of principals associated with bonds that mature within *H*. An increase in *CPaR* means that the proportion of short-term bonds increases, which in turn decreases the minimum COST, and vice versa. To further explore the composition of optimal bond portfolios, we consider $CFLaR = 1.141$ and provide the corresponding composition in Figure 8(b).

Figure 8(b) shows the optimal bond portfolio composition corresponding to a minimum COST with the constraints of different CPaR and fixed $CFLaR = 1.141$ for each year of a three-stage bond issuance planning horizon. It shows that the portfolio includes all four types of bonds in each issuance year.

To explain the evolution of the slopes of the efficient frontier curves in Figure 8(a), we examine the total discounted payments of new bonds issued in three stages, which include all four bond types, and give the total proportion of each kind of newly issued bonds in Figure 9. As *CPaR* shifts from 41 to 43, the total proportions of 3 year and 5 year bonds in the three stages gradually increase, while the total proportions of 10 year and 15 year bonds in the three stages gradually decrease. Because COST is mainly determined by the discounted principal and interest payments of the newly issued bonds, increasing the proportion of long-term bonds leads to an increase in COST. The total proportion curve for each new bond becomes smooth as *CPaR* shifts from 41 to 43 in Figure 9. This shows that the incremental magnitude of the total proportions of 3-year and 5-year bonds gradually decreases, while the magnitude of the decrease in the total proportions of 10 year and 15-year bonds gradually decreases when the cash liquidity planning horizon $H = 6$ years. Consequently, the rate of decrease of minimum COST gradually falls, as shown in Figure 8(a).

5.3.2. *Case 2: Minimizing* **CFLaR** *under the Constraints of Different* **COST** *and* **CPaR**.

Similar to Case 1, we examine the efficient frontier of minimum *CFLaR* with the constraints of different $CPaR \in [41, 43]$ and different $COST \in [29, 30]$, as shown in Figure 10.

As shown in Figure 10, the minimum *CFLaR* increases as COST increases and decreases as *CPaR* increases. COST is determined by the discounted principals and interests of newly issued bonds during the three stages. The discounted principals and interests of short-term bonds are both smaller than those of long-term bonds when the issued amounts of short- and long-term bonds are the same; therefore, an increase in the proportion of long-term bonds leads to an increase in COST. As in Case 1, minimum *CFLaR* increases with the proportion of long-term bonds, while an increase in the proportion of long-term bonds causes *CPaR* to decrease.

Again, to explore the evolution of the efficient frontier more precisely, we consider two types of constraints: (i) different COST and fixed *CPaR*; and (ii) different *CPaR* and fixed COST.

(i) The efficient frontier curves of the minimum of *CFLaR* with constraints of different COST and three fixed *CPaR*.

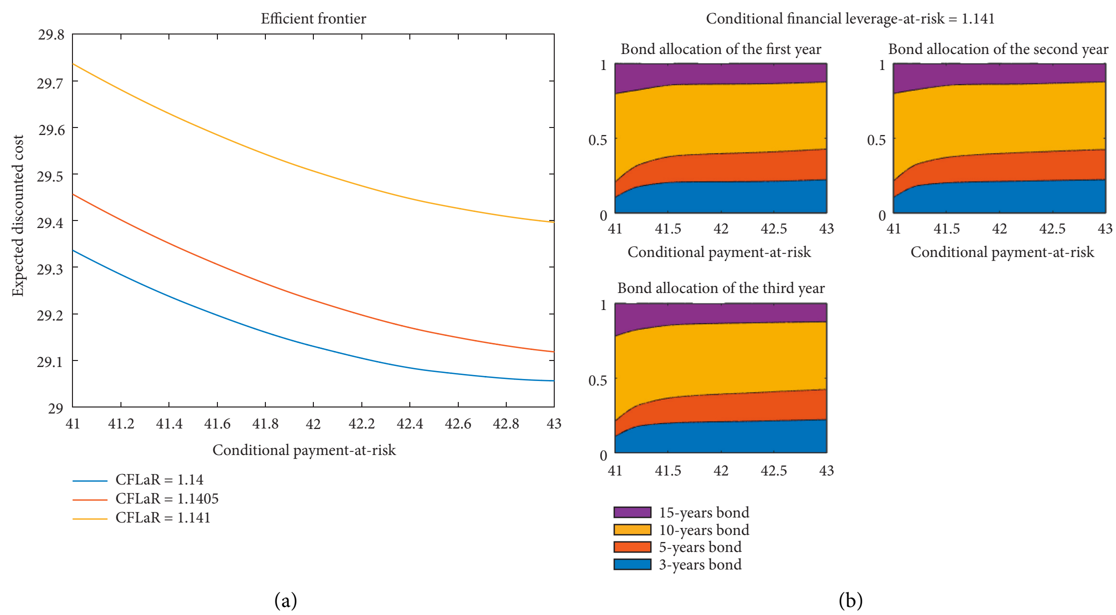


Figure 8: The efficient frontier curves to minimize the **COST** with the different **CPaR** and three fixed **CFLaR** $= 1.14, 1.1405, 1.141$ (a), the optimal bond composition with constraints of **CFLaR** $= 1.141$ and different **CPaR** (b).

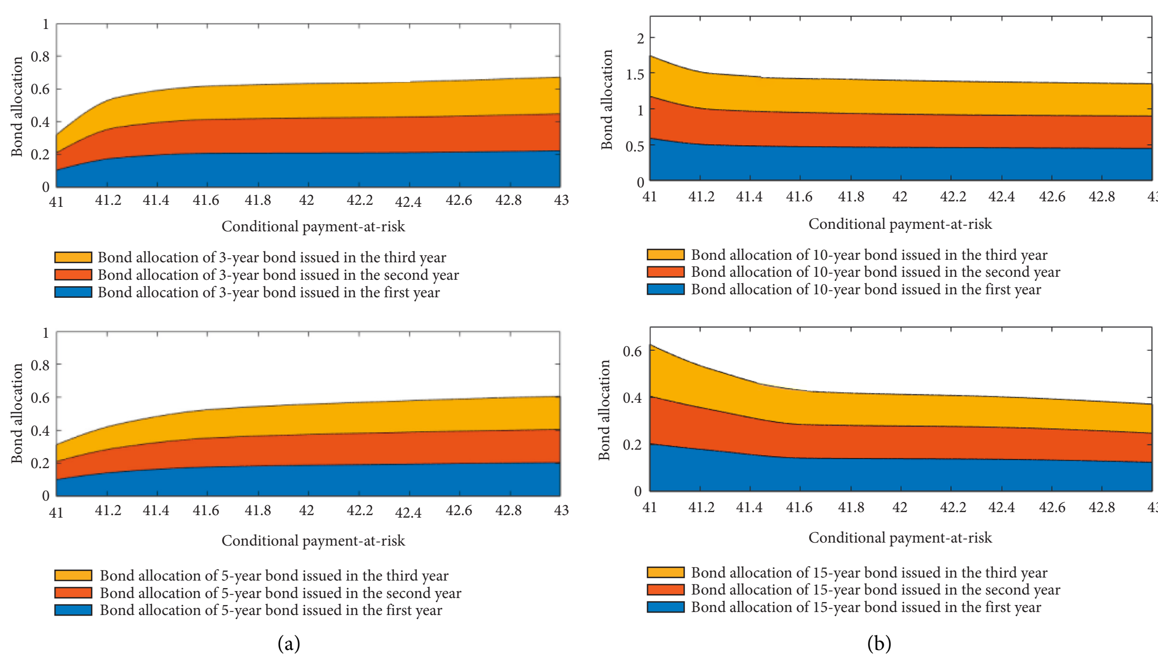


Figure 9: The total proportion of each kind of new issue which includes issued in the three stages as **CPaR shift** from 41 to 43.

Figure 11(a) shows the efficient frontier curves for minimum *CFLaR* under different COST and three fixed $CPaR = 41, 42, 43$. Each efficient frontier of minimum *CFLaR* is an increasing function of COST under a fixed *CPaR*. Regarding the bond portfolio composition, the proportion of long-term bonds increases as COST gradually increases, and an increase in the proportion of long-term bonds leads to an increase in minimum *CFLaR*, and vice versa. We also provide the optimal bond issuance compositions for $CPaR = 41$ in Figure 11(b).

Figure 11(b) displays the optimal bond portfolio compositions for each year of the three-stage bond issuance period within a 3-year planning horizon under fixed $CPaR = 42$, which include all four types of bonds at each issuance stage.

We further analyze the total payments of bonds issued within the three-stage bond issuance planning horizon when $H = 6$ years, showing the total proportion of *Type I* bonds in Figure 12. Although the proportion of each of the *Type I* bonds in the three stages is time-variant as COST shifts from 29 to 30,

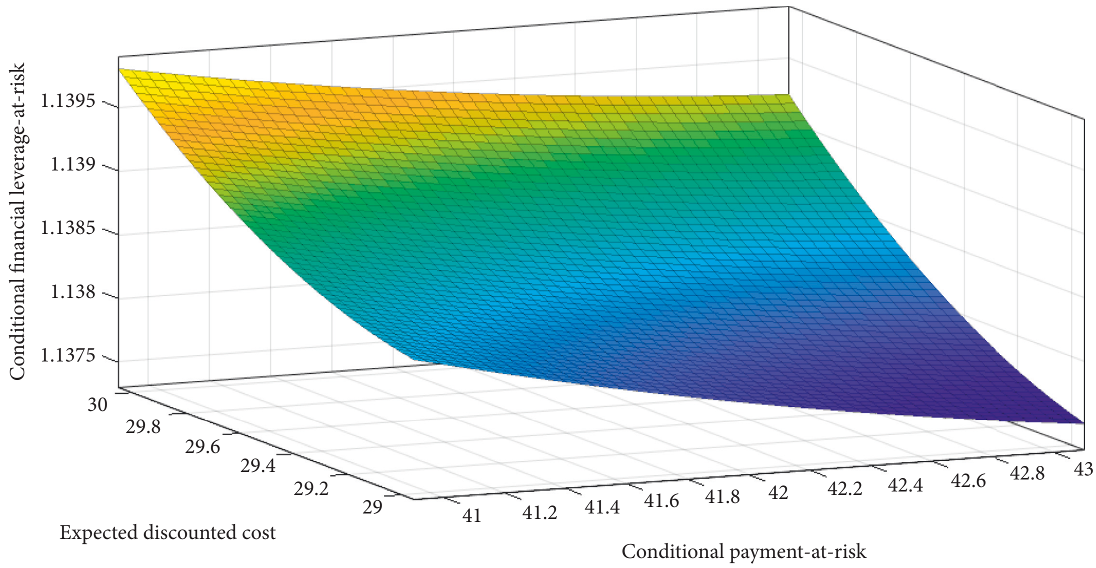


FIGURE 10: Efficient frontier of a minimum of **CFLaR** with different **CPaR** and **COST** constraints.

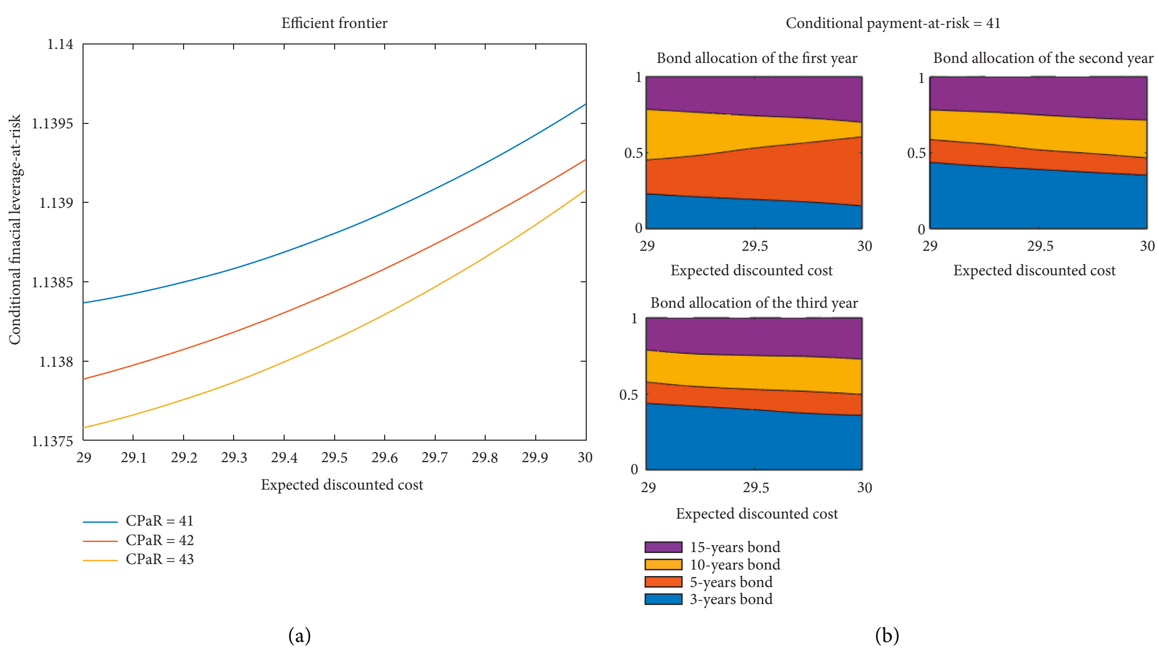


FIGURE 11: The efficient frontier curves of the minimum of **CFLaR** under different **COST** and three fixed **CPaR** = 41, 42, 43 (a), the bond composition with **CPaR** = 42 and different **COST** (b).

the total proportion of *Type I* bonds in the three stages is relatively stable. A fixed *CPaR* should be driven by the total payments of principals associated with *Type I* bonds in the three stages. When *CPaR* is fixed in each stage, the total proportion of *Type I* bond principals should be relatively stable for a liquidity planning horizon $H = 6$ years, regardless of whether the proportion of each *Type I* bond is stable. This is consistent with Figure 11(a).

(ii) The efficient frontiers of the minimum of *CFLaR* with constraints of different *CPaR* and three fixed COST.

Figure 13(a) shows the efficient frontier curves for minimum *CFLaR* with different *CPaR* and three fixed COST = 29, 29.5, 30. As shown in Figure 13(a), each efficient frontier of minimum *CFLaR* is a decreasing function with respect to *CPaR* under a fixed COST. As in Case 1, *CPaR*, which is mainly decided by the total proportion of *Type I* bonds in the three stages, increases with the proportion of short-term bonds, and an increase in the proportion of short-term bonds leads to a decrease in minimum *CFLaR*, and vice versa. We examine the composition of the optimal bond portfolio again for COST = 30, as shown in Figure 13(b).

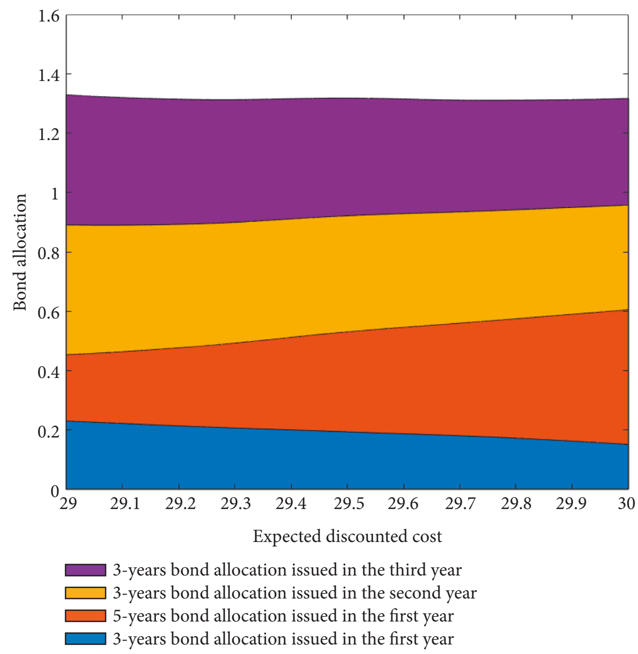
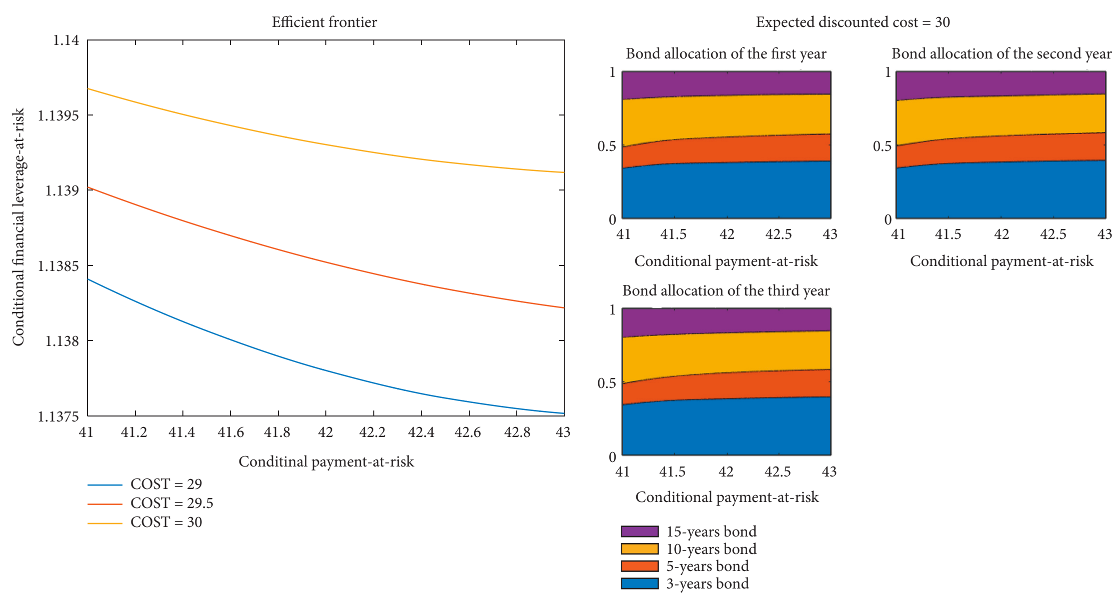


Figure 12: The total proportion of bonds that mature within the liquidity planning horizon $\mathbf{H} = 6$ years under the fixed $\mathbf{CPaR} = 4\ 2$.

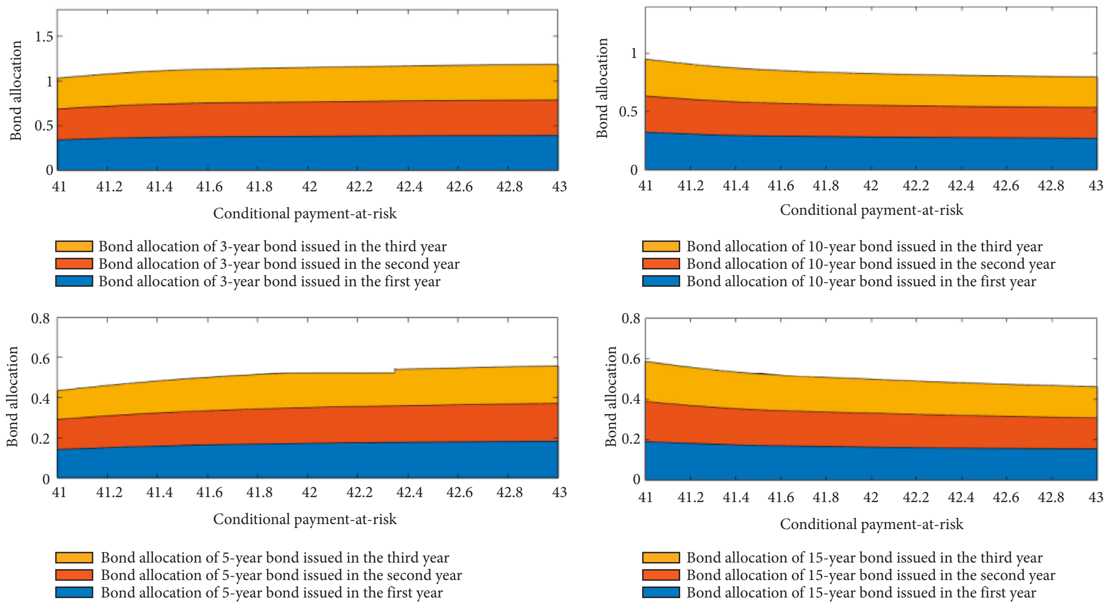


(a) (b)

Figure 13: The efficient frontier curves of the minimum of **CFLaR** with the different **CPaR** and three fixed **COST** = 29, 29.5, 30 (a), the bond composition with **COST** = 30 and different **CPaR** (b).

Corresponding to a minimum *CFLaR* under fixed COST = 30, the optimal bond portfolio compositions for each year of the three-stage bond issuance planning horizon are shown in Figure 13(b). The bond portfolio includes all four types of bonds in each issuance year, and the proportions of 3-year and 5-year bond in each year in the three stages both increase as *CPaR* shifts from 41 to 43.

To further explore the evolution of the total payments of newly issued bonds when $H = 6$ years, we depict the total proportion of each kind of newly issued bond across the three stages in Figure 14. As shown in Figure 14, the total proportions of 3 years and 5 years bonds increase and the total proportions of 10 year and 15 year bonds gradually decrease across the three stages as *CPaR* shifts from 41 to 43. Additionally, Figure 14 shows that the total proportion curve for each newly issued bond becomes smooth as *CPaR* shifts from 41 to 43, i.e., the magnitude of the increase of the total proportions of 3 year and 5 year bonds gradually decreases and the magnitude of the decrease of the total proportion of 10 year and 15 year bonds gradually decreases. Thus, the rate



(a) (b)

Figure 14: The total proportion of each kind of new issue that includes issued in the three stages as **CPaR shift** from 41 to 43.

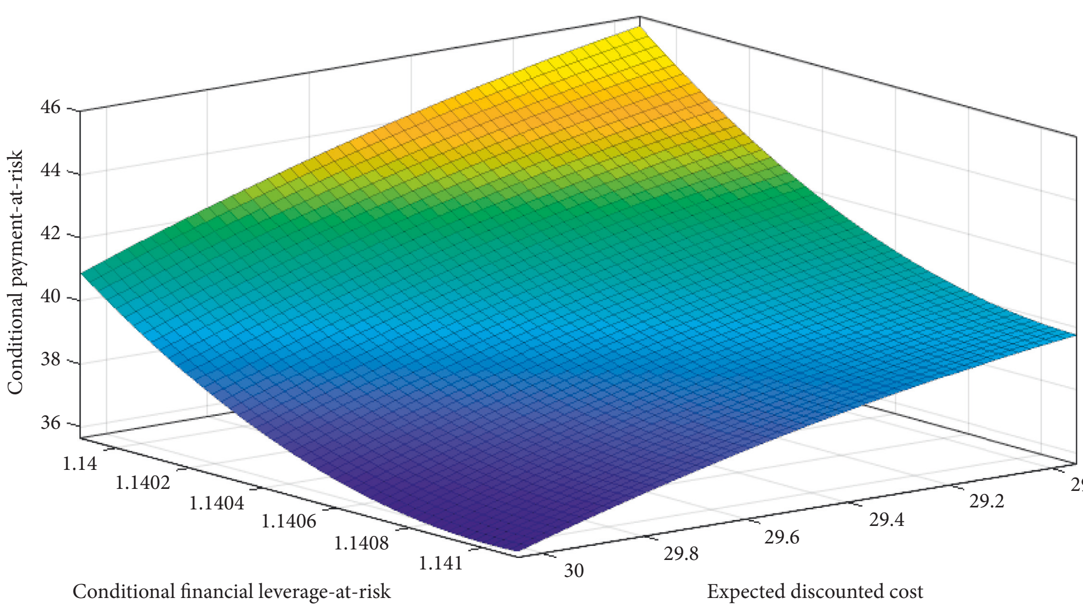


Figure 15: Efficient frontier of minimum of **CPaR** with different **CFLaR** and **COST** constraints.

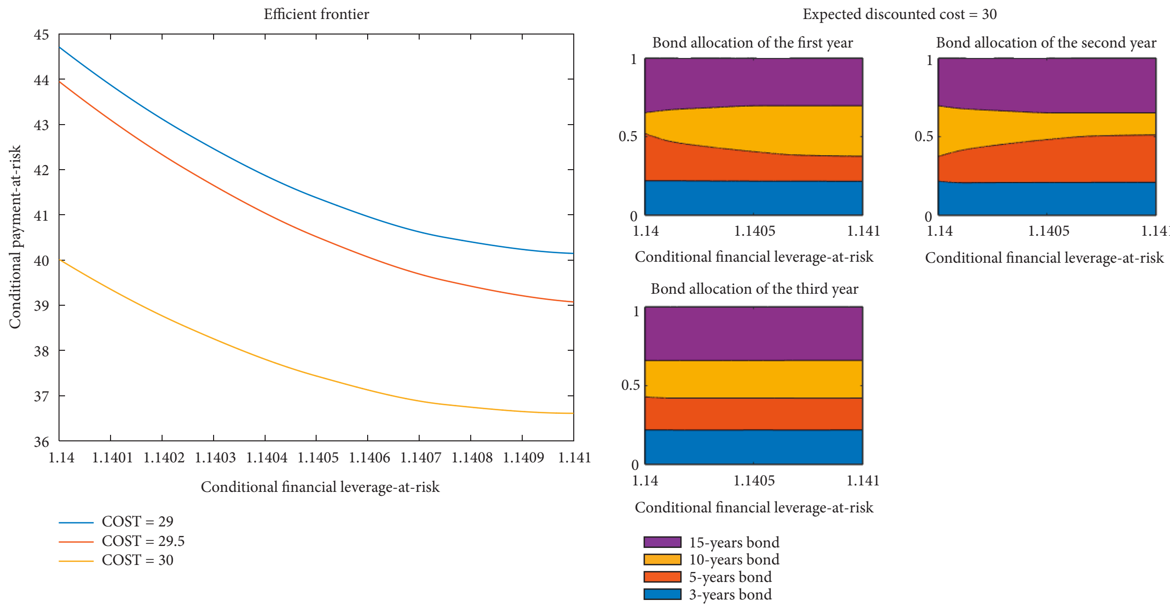
of the decrease of minimum *CFLaR* gradually increases, as shown in Figure 13(a).

5.3.3. Case 3: Minimizing **CPaR** *under the Constraints of Different* **COST** *and* **CFLaR**. We provide the efficient frontier for minimum *CPaR* with the constraints of different $CFLaR \in [1.14, 1.141]$ and different $\text{COST} \in [29, 30]$ in Figure 15.

As shown in Figure 15, the minimum *CPaR* decreases as *CFLaR* increases and also decreases as COST increases. From Case I and Case II, we know that *CFLaR* is mainly determined by the interest payments of newly issued bonds and that COST is mainly determined by the total discounted payments of the principals of these newly issued bonds. Thus, an increase in the proportion of long-term bonds issued leads to an increase in COST and *CFLaR*; conversely, it leads to a decrease in minimum *CPaR*.

To explore the evolution of the efficient frontier more precisely, similar to our analyses for Cases 1 and *II*, we also consider the following two constraints.

(i) The efficient frontier curves of the minimal *CPaR* with the constraints of different *CFLaR* and three fixed COST.



(a) (b)

Figure 16: The efficient frontier curves of the minimum of **CPaR** under different **CFLaR** and three fixed **COST** = 29, 29.5, 30 (a), the bond composition with **COST** = 30 and different **CFLaR** (b).

The efficient frontier curves for minimum *CPaR* with the constraints of different *CFLaR* and three fixed COST. Figure 16(a) shows the corresponding efficient frontier curves for minimum *CPaR* under different *CFLaR* and three fixed COST = 29, 29.5, 30. As shown in Figure 16(a), each efficient frontier of minimum *CPaR* is a decreasing function of *CFLaR* for each fixed COST. An increase in *CFLaR* coincides with an increase in the proportion of long-term bonds, which leads to an increase in minimum *CPaR*, and vice versa. To further explore the optimal portfolio, we investigate an example with COST = 30 and provide the corresponding issuance of newly issued bonds within three stages in Figure 16(b).

Figure 16(b) shows the optimal bond allocation for each year of three-stage bond issuance with a 3-year planning horizon under the fixed COST = 30. To further examine the evolution of payment of newly issued bonds within $H = 6$ years, we use *Type I* bonds which method in Case 1, and give the total proportion of *Type I* bonds in Figure 17.

Because *CPaR* is determined by the total payments of *Type I* bond principals when $H = 6$ years and a bond's interest payments are a very small proportion of its principal, a decrease in *CPaR* requires the total payments of *Type* 1 bond principals during $H = 6$ years to decrease; that is, the total proportion of *Type I* bonds in the three stages must decrease. Figure 17 shows that the total proportion curve of the *Type I* bonds becomes smooth, which implies that as *CFLaR* shifts from 1.14 to 1.141, the magnitude of the decrease in the total proportion of *Type I* bonds gradually decreases. Thus, the magnitude of the

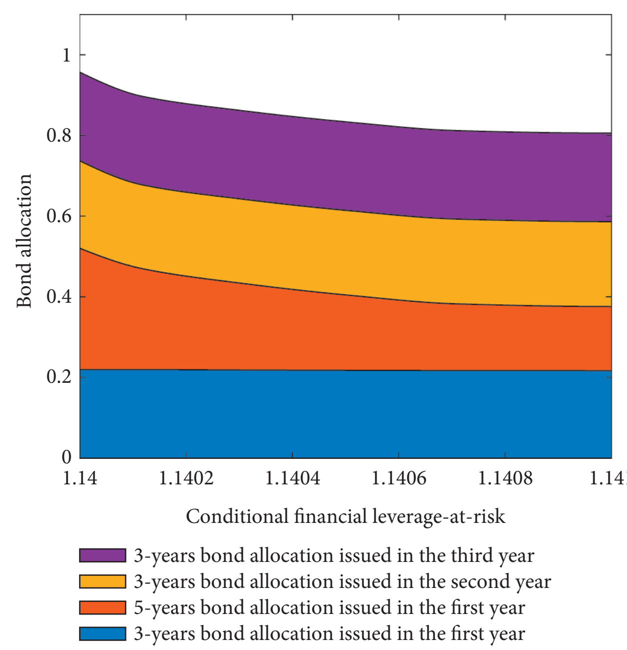
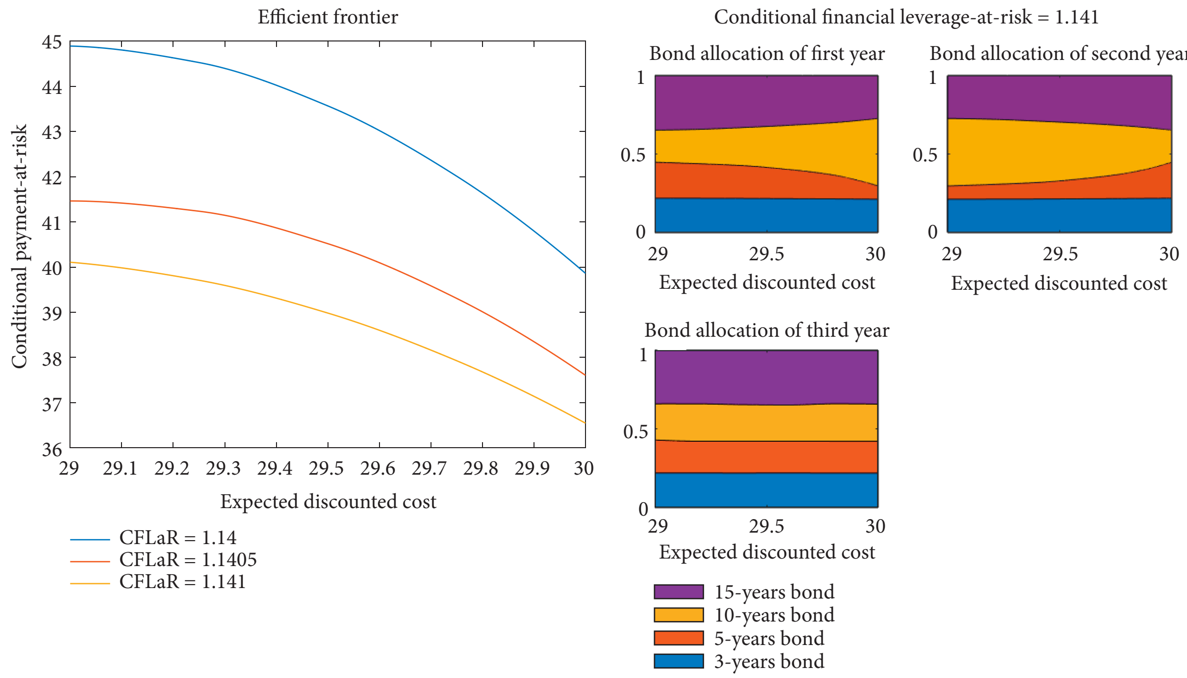


Figure 17: The total proportion of bonds that mature within the liquidity planning horizon **H** = 6 year under the fixed **COST** = 30.

reduction in *CPaR* gradually decreases; that is, the rate of decrease of minimum *CPaR* gradually falls, consistent with Figure 16(a).

(ii) The efficient frontier curves of the minimum of *CPaR* with the constraints of different COST and three fixed *CFLaR*.

The efficient frontier curves of minimum *CPaR* with the constraints of different COST and three fixed *CFLaR*.



(a) (b)

Figure 18: The efficient frontier curves of the minimum of **CPaR** under different **COST** and three fixed **CFLaR** = 1.14, 1.1405, 1.141 (a), the bond composition with **CFLaR** = 1.141 and different **COST** (b).

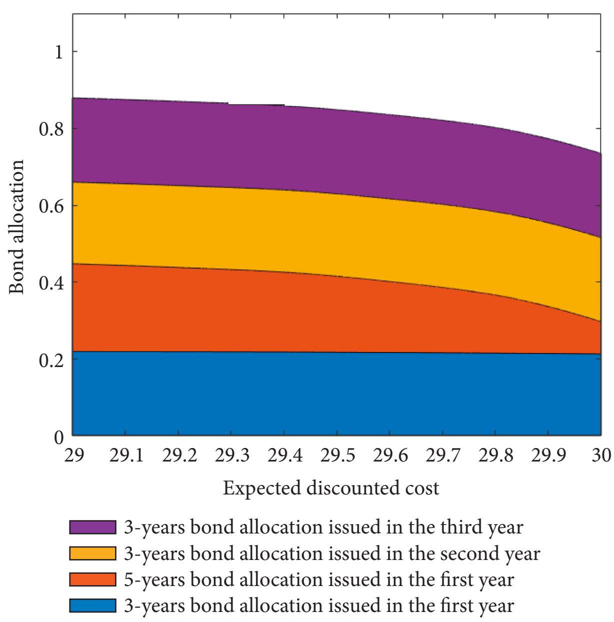


Figure 19: The total proportion of bonds that mature within the liquidity planning horizon **H** = 6 year under the fixed **CFLaR** = 1.141.

Figure 18(a) shows the corresponding efficient frontier curves for minimum *CPaR* under different COST and three fixed *CFLaR* = 1.14, 1.1405, 1.141. As shown in Figure 18(a), each efficient frontier of minimum *CPaR* is a decreasing function of COST under fixed *CFLaR* constraints. An increase in COST represents an increase in the proportion of long-term bonds, which leads to a decrease in minimum *CPaR*, with the order reversed. To further examine the optimal portfolio for minimum *CPaR*, we examine an example in which *CFLaR* = 1.141 and give the corresponding issuance allocation of newly issued bonds within the three stages in Figure 18(b).

Figure 18(b) shows the optimal bond allocation for each year of a three-stage bond issuance with a 3-year planning horizon under fixed *CFLaR* = 1.141. All four types of bonds are present in the bond allocation at each issuance stage, as shown in Figure 18(b).

We further explore the evolution of the minimum payments for the bonds issued within the three stages when $H = 6$ years, considering the proportion of *Type 1* bonds. As shown in Figure 19, the proportion of *Type I* bonds gradually decreases as COST shifts from 29 to 30, meaning that the minimum *CPaR* gradually decreases. Moreover, Figure 19 indicates that the total proportion curve of *Type I* bonds becomes steeper, which signifies that the magnitude of the decrease in the total proportion of *Type I* bonds gradually increases, and the magnitude of the decrease in *CPaR* gradually decreases when $H = 6$ years, i.e., the rate of decrease of the minimum *CPaR* gradually falls, which is consistent with Figure 18(a).

6. Conclusion

We propose an MSP model with multiple objectives to optimize bond issuance and design a *CFLaR* construct inspired by *CVaR* to measure financial leverage risk in worst-case scenarios. We further develop a corporate cash liquidity risk measurement construct, *CPaR*, that improves on the existing liquidity risk measurement method. To help corporations achieve a trade-off between the expected discounted bond repayment cost and the two types of risks mentioned above, we discuss the following three cases: (1) minimizing *COST* under the constraints of different *CFLaR* and *CPaR*, (2) minimizing *CFLaR* under the constraints of different COST and *CPaR*, and (3) minimizing *CPaR* under the constraints of different *COST* and *CFLaR*.

We empirically test our model by applying it to a real Chinese company, ZOH. We assume that the company issues a bond portfolio comprising 3 years, 5 years, 10 years, and 15 years bonds within a three-year bond issuance plan period. Scenario trees simulating the evolution of macroeconomic variables are generated by HFS, and a three-stage stochastic programming bond issuance model is established for $H = 6$ years. Specifically, we examine the efficient frontier of minimum COST with different *CFLaR* and *CPaR* constraints, and the empirical results show that the minimum COST increases as *CFLaR* increases and decreases as *CPaR* increases. We provide the efficient frontier of minimum *CFLaR* with the constraints of different *CPaR* and different COST, which shows that minimum *CFLaR* increases with COST and decreases as *CPaR* increases. Finally, we consider the efficient frontier of minimum *CPaR* with the constraints of different *CFLaR* and different COST. The results show that the minimum *CPaR* decreases as COST increases and decreases as *CFLaR* increases.

The main contributions of this paper are as follows:

(i) We propose an MSP model with multiple objectives that optimize the uncertain bond issuance by satisfying the three common objectives of corporate managers: (i) minimizing the expected discounted cost under cash liquidity and financial leverage risk constraints; (ii) minimizing financial leverage risk under expected discounted cost and cash liquidity risk constraints; (iii) and minimizing cash liquidity risk under expected discounted cost and financial leverage risk constraints.

(ii) We improve on the *CPaR* method of measuring corporate short-term liquidity risk in a worst-case scenario of corporate bond issuance. Furthermore, based on the degree of financial leverage, we design a *CFLaR* construct to measure the financial leverage risk faced by a corporation in a worst-case scenario.

(iii) Through the empirical analysis of ZOH company, we show that corporations can adjust the proportion of short-term bonds maturing within the cash liquidity horizon to reduce the minimum expected discounted cost and minimum financial leverage risk in worst-case scenarios while fixing the level of short-term cash liquidity risk. When the financial leverage risk is fixed, corporations can change the proportions of the two types of newly issued bonds (maturing during and after the cash liquidity horizon) to achieve a trade-off between minimum expected discounted costs and minimum short-term cash liquidity risk in worst-case scenarios. By controlling the future repayment cost of bonds, corporations can modify the proportions of these two bond types to manage the minimum financial leverage risk and minimum short-term cash liquidity risk simultaneously in a worst-case scenario.

Our MSP model with multiple objectives considers the expected discounted cost of newly issued bonds, cash liquidity risk, and financial leverage risk in worst-case scenarios, providing guidance for corporations on devising effective management strategies for the issuance of corporate bonds. In future research, we will assign different weights to cost, liquidity risk, and financial leverage risk to establish more comprehensive and objective equations and constraints, allowing us to more accurately determine the corresponding optimal issuance strategy.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Additional Points

Highlights. A multistage stochastic programming model with multiple objectives that optimizes the corporate bond issuance is proposed. In this paper, three common objectives of corporate managers are considered: (i) Minimizing expected discounted cost under cash liquidity and financial leverage risk constraints. (ii) Minimizing financial leverage risk under expected discounted cost and cash liquidity risk constraints. (iii) Minimizing cash liquidity risk under expected discounted cost and financial leverage risk constraints. Liquidity risk can be measured by conditional payment-at-risk (*CPaR*) model and financial leverage risk is captured by the conditional financial leverage-at-risk (*CFLaR*), which is based on conditional value-at-risk (*CVaR*). Corporations can adjust the proportions of the two types of newly issued bonds (maturing during and after the cash liquidity horizon) to reduce the minimum expected discounted cost, the minimum financial leverage risk in worst-case scenarios, and the minimum short-term cash liquidity risk in worst-case scenarios simultaneously.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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References

[1] M. Sierpińska and P. Bąk, "Financial structure of mining sector companies during an economic slowdown," *Archives of Mining Sciences*, vol. 57, no. 4, pp. 1089–1100, 2012.

[2] A. Aydin, "Fractures, faults, and hydrocarbon entrapment, migration and flow," *Marine and Petroleum Geology*, vol. 17, no. 7, pp. 797–814, 2000.

[3] K. Cai, R. Fairchild, and Y. Guney, "Debt maturity structure of Chinese companies," *Pacific-Basin Finance Journal*, vol. 16, no. 3, pp. 268–297, 2008.

[4] P. Körner, *Three Essays on Corporate Governance and Corporate finance*, VDM Verlag, Riga, Latvia, 2008.

[5] C. Orman and B. Köksal, "Debt maturity across firm types: evidence from a major developing economy," *Emerging Markets Review*, vol. 30, pp. 169–199, 2017.

[6] S. P. Bradley and D. B. Crane, "A dynamic model for bond portfolio management," *Management Science*, vol. 19, no. 2, pp. 139–151, 1972.

[7] D. R. Cariño, T. Kent, D. H. Myers et al., "The russell-yasuda kasai model: an asset/liability model for a Japanese insurance company using multistage stochastic programming," *Interfaces*, vol. 24, no. 1, pp. 29–49, 1994.

[8] W. T. Ziemba and J. H. Mulvey, *Worldwide Asset and Liability Modeling*, Vol. 10, Cambridge University Press, Cambridge, England UK, 1998.

[9] J. Dupačová, "Applications of stochastic programming: achievements and questions," *European Journal of Operational Research*, vol. 140, no. 2, pp. 281–290, 2002.

[10] P. Hilli, M. Koivu, T. Pennanen, and A. Ranne, "A stochastic programming model for asset liability management of a Finnish pension company," *Annals of Operations Research*, vol. 152, no. 1, pp. 115–139, 2007.

[11] N. Topaloglou, H. Vladimirou, and S. A. Zenios, "A dynamic stochastic programming model for international portfolio management," *European Journal of Operational Research*, vol. 185, no. 3, pp. 1501–1524, 2008.

[12] H. J. Yu and Z. M. Wang, "Construction of asset-liability management model of China's basic pension——based on the perspective of multi-stage stochastic programming," *Journal of Northwest University: Philosophy and Social Sciences Edition*, vol. 42, no. 3, p. 7, 2012.

[13] X. Jin, X. Huang, and Y. Feng, "Multi-period Stochastic Programming Model for Bank Asset and Liability Management," *Journal of Systems & Management*, vol. 13, 2007.

[14] E. Balibek and M. Köksalan, "A multi-objective multi-period stochastic programming model for public debt management," *European Journal of Operational Research*, vol. 205, no. 1, pp. 205–217, 2010.

[15] A. Consiglio and A. Staino, "A stochastic programming model for the optimal issuance of government bonds," *Annals of Operations Research*, vol. 193, no. 1, pp. 159–172, 2012.

[16] P. Date, A. Canepa, and M. Abdel-Jawad, "A mixed integer linear programming model for optimal sovereign debt issuance," *European Journal of Operational Research*, vol. 214, no. 3, pp. 749–758, 2011.

[17] D. M. Vallad, Á. Veiga, and G. Veiga, "A multistage linear stochastic programming model for optimal corporate debt management," *European Journal of Operational Research*, vol. 237, no. 1, pp. 303–311, 2014.

[18] H. Nakayama, "Aspiration level approach to interactive multi-objective programming and its applications," *Nonconvex Optimization and Its Applications*, Springer, Boston, MA, 1995pp. 147–174, Advances in Multicriteria Analysis.

[19] H. P. S. H. P. Sharma and D. K. Sharma, "A multi-objective decision-making approach for mutual fund portfolio," *Journal of Business & Economics Research*, vol. 4, no. 6, 2006.

[20] H. Nakayama, Y. Yun, and M. Yoon, "Interactive programming methods for multi-objective optimization," *Vector Optimization Sequential Approximate Multiobjective Optimization Using Computational Intelligence*, Springer, Berlin, Heidelberg, pp. 17–43, 2009.

[21] G. A. V. Pai and T. Michel, "Metaheuristic multi-objective optimization of constrained futures portfolios for effective risk management," *Swarm and Evolutionary Computation*, vol. 19, pp. 1–14, 2014.

[22] W. S. Lam, W. H. Lam, and S. H. Jaaman, "Portfolio optimization with a mean-absolute deviation-entropy multi-objective model," *Entropy*, vol. 23, no. 10, p. 1266, 2021.

[23] Z. Wang, X. Zhang, Z. Zhang, and D. Sheng, "Credit portfolio optimization: a multi-objective genetic algorithm approach," *Borsa Istanb Rev*, vol. 22, p. 01, 2021.

[24] F. Altiparmak, M. Gen, L. Lin, and T. Paksoy, "A genetic algorithm approach for multi-objective optimization of supply chain networks," *Computers & Industrial Engineering*, vol. 51, no. 1, pp. 196–215, 2006.

[25] K. Deb, "Multi-objective optimization," *Search Methodologies*, Springer, Boston, MA, pp. 403–449, 2014.

[26] A. Hojjati, M. Monadi, A. Faridhosseini, and M. Mohammadi, "Application and comparison of NSGA-II and MOPSO in multi-objective optimization of water resources systems," *Journal of Hydrology and Hydromechanics*, vol. 66, no. 3, pp. 323–329, 2018.

[27] F. García, J. González-Bueno, F. Guijarro, and J. Oliver, "A multiobjective credibilistic portfolio selection model. Empirical study in the Latin American integrated market," *Enterpreneurship and Sustainability Issues*, vol. 8, no. 2, pp. 1027–1046, 2020.

[28] E. Balıbek, *Multi-objective approaches to public debt management*, Middle East Technical University, Ankara, Turkey, Operational Research Department, 2008.

[29] S. Banihashemi, "Portfolio Optimization by Multi-Stage Stochastic model," Allameh Tabataba'i University, Tehran, Iran, 4th Seminar of Mathematics and Humanities, Financial Mathematics, 2016.

[30] M. Radulescu and C. Z. Radulescu, "A Multi-Objective Approach to Multi-Period: Portfolio Optimization with transaction costs," *Financial Decision Aid Using Multiple Criteria*, Springer, Cham, pp. 93–112, 2018.

[31] W. Zhou, W. Zhu, Y. Chen, and J. Chen, "Dynamic Changes and Multi-Dimensional Evolution of Portfolio optimization," *Economic Research-Ekonomska Istraživanja*, vol. 35, pp. 1–26, 2021.

[32] J. H. Kim, Y. Lee, W. C. Kim, and J. F. Frank, "Goal-based investing based on multi-stage robust portfolio optimization," *Annals of Operations Research*, vol. 313, pp. 1–18, 2022.

[33] H. Ma, H. Wei, Y. Tian, R. Cheng, and X. Zhang, "A multi-stage evolutionary algorithm for multi-objective optimization with complex constraints," *Information Sciences*, vol. 560, pp. 68–91, 2021.

[34] S. A. O'Connell and S. P. Zeldes, "Rational ponzi games," *International Economic Review*, vol. 29, no. 3, pp. 431–450, 1988.

[35] I. J. Kim, K. Ramaswamy, and S. Sundaresan, "Does default risk in coupons affect the valuation of corporate bonds?: a contingent claims model," *Financial Management*, vol. 22, no. 3, pp. 117–131, 1993.

[36] N. E. Koltsaklis and A. S. Dagoumas, "State-of-the-art generation expansion planning: a review," *Applied Energy*, vol. 230, pp. 563–589, 2018.

[37] M. Were and J. Wambua, "What factors drive interest rate spread of commercial banks? Empirical evidence from

Kenya," *Review of development Finance*, vol. 4, no. 2, pp. 73–82, 2014.

[38] D. W. Diamond, "Debt maturity structure and liquidity risk," *The Quarterly Journal of Economics*, vol. 106, no. 3, pp. 709–737, 1991.

[39] S. Gilchrist, V. Yankov, and E. Zakrajšek, "Credit market shocks and economic fluctuations: evidence from corporate bond and stock markets," *Journal of Monetary Economics*, vol. 56, no. 4, pp. 471–493, 2009.

[40] I. Jindřichovská and P. Körner, *Determinants of Corporate Financing Decisions: A Survey Evidence from Czech firms*Charles University Prague, Staré Město, Czechia, 2008.

[41] J. W. Hurst, *The Legitimacy of the Business Corporation in the Law of the United States, 1780-1970*, The Lawbook Exchange, Ltd, New Jersey, NJ, USA, 2004.

[42] P. Weill and J. W. Ross, *IT Savvy: What Top Executives Must Know to Go from Pain to gain*, Harvard Business Press, Massachusetts, MA, USA, 2009.

[43] G. Donaldson, *Corporate Debt Capacity: A Study of Corporate Debt Policy and the Determination of Corporate Debt capacity*, Beard Books, 2000.

[44] D. N. Shapiro, J. Chandler, and P. A. Mueller, "Using mechanical Turk to study clinical populations," *Clinical Psychological Science*, vol. 1, no. 2, pp. 213–220, 2013.

[45] G. C. E. Boender, "A hybrid simulation/optimisation scenario model for asset/liability management," *European Journal of Operational Research*, vol. 99, no. 1, pp. 126–135, 1997.

[46] Y. Rc, L. Li, J. Qi, and Q. Ji, "Optimal bond issuance with cost and liquidity constraints for Chinese local governments: a multi-period stochastic programming approach," *Empirical Economics*, Springer, Berlin, Germany, 2022.

[47] C. Xu, "On the use of financial leverage policy of enterprises' sustainable development ability," *Contemporary Finance & Economics*, no. 9, p. 7, 2006.

[48] V. Gunarathna, "Degree of Financial Leverage as a Determinant of Financial Risk: An Empirical Study in Sri Lanka," in *Proceedings of the 2nd International Conference on Management and Economics-Faculty of Management and Finance*, University of Ruhuna, Sri Lanka, 2013.

[49] V. Balasubramaniam, R. Sane, M. A. Sarah, and S. Karthik, *Do Indian financial firms have a robust Grievance Redress Framework in place?*, 2021.

[50] G. C. Pflug, "Some remarks on the value-at-risk and the conditional value-at-risk," *Probabilistic Constrained Optimization*, Springer, Boston, MA, USA, 2000.

[51] R. T. Rockafellar and S. Uryasev, "Optimization of conditional value-at-risk," *Journal of Risk*, vol. 2, no. 3, pp. 21–41, 2000.

[52] R. Mansini, W. Ogryczak, and M. G. Speranza, "LP solvable models for portfolio optimization: a classification and computational comparison," *IMA Journal of Management Mathematics*, vol. 14, no. 3, pp. 187–220, 2003.

[53] V. Agarwal and N. Y. Naik, "Risks and portfolio decisions involving hedge funds," *Review of Financial Studies*, vol. 17, no. 1, pp. 63–98, 2004.

[54] B. Liang and H. Park, "Predicting hedge fund failure: a comparison of risk measures," *Journal of Financial and Quantitative Analysis*, vol. 45, no. 1, pp. 199–222, 2010.

[55] S. A. Owolabi and S. S. Obida, "Liquidity management and corporate profitability: case study of selected manufacturing companies listed on the Nigerian stock exchange," *Business Management Dynamics*, vol. 2, no. 2, pp. 10–25, 2012.

[56] M. J. Anderson, "A new method for non-parametric multivariate analysis of variance," *Austral Ecology*, vol. 26, no. 1, pp. 32–46, 2001.

[57] K. Høyland and S. W. Wallace, "Generating scenario trees for multistage decision problems," *Management Science*, vol. 47, no. 2, pp. 295–307, 2001.

[58] J. Tao, J. Li, and Z. Wang, "Studying global sea level change with Jason-1 and GRACE satellite data," *Journal of Surveying and Mapping*, no. 2, p. 6, 2010.

[59] I. Roy, "Estimating value at risk (VaR) using filtered historical simulation in the indian capital market," *Reserve Bank of India Occasional Papers*, vol. 32, no. 2, pp. 81–98, 2011.

[60] M. Pritsker, "The Channels for Financial contagion," *International financial contagion*, Springer, Boston, MA, USA, 2001.