Hindawi
Discrete Dynamics in Nature and Society
Volume 2023, Article ID 2013300, 20 pages
https://doi.org/10.1155/2023/2013300

Research Article

Stability Analysis of Cohen–Grossberg Type BAM Neural Network with Piecewise Constant Argument

Wenqing Zheng, Tao Xie, and Wenxiang Fang

School of Mathematics and Statistics, Hubei Normal University, Huangshi 435002, Hubei, China

Correspondence should be addressed to Tao Xie; xt0216@hbnu.edu.cn

Received 20 April 2023; Revised 27 May 2023; Accepted 29 May 2023; Published 15 June 2023

Academic Editor: Luca Pancioni

This paper introduces the stability problems of Cohen–Grossberg type BAM neural network (BAMCGNN) with piecewise constant argument (PCA). By employing the homeomorphism theory, sufficient conditions for the existence and uniqueness of the equilibrium point are obtained; using inequality technique and Lyapunov method, sufficient stability criteria for BAMCGNN with PCA are presented. Finally, a numerical case shows the significance of the results of this paper.

1. Introduction

Neural networks (NNs) are complex network systems which are formed by a vast number of simple neurons widely connected to each other. NNs are typical nonlinear dynamic systems, and research studies on NNs first began in the 1940s; McCulloch and Pitts proposed the McCulloch–Pitts model in 1943, which led to the growth of NN research. With the development and improvement of NNs, the results on NNs have been widely presented [1–6] and have a wide range of applications in many fields, such as sensing information processing, automatic control, information analysis, aerospace, and military fields [7–9]. So far, numerous researches have proposed different NN models, such as Hopfield network network (HNN), cellular neural network (CNN), and Cohen–Grossberg neural network (CGNN). CGNN was proposed by Cohen and Grossberg in 1983 [10]. CGNN is a very broad model that includes multiple ecosystems and NNs, such as the Volterra–Lotka system, the Gilpin–Ayala competitive ecosystem, the Eigen–Schusterxit system, and HNN [10–12]. CGNN has its own unique advantages; it is not only closely connected with the biological network but can also solve the nonlinear and uncertain problems in practical applications. At present, CGNN models have been widely used in parallel processing, associative memory, optimization calculation, etc. [10, 13, 14], and the research studies of CGNN have aroused widespread interest. Many scholars have conducted in-depth research studies on the stability of CGNN and have achieved some excellent results [15–23]. For instance, in [16], Arik and Orman studied global asymptotic stability (GAS) and global exponential stability (GES) of the equilibrium point for CGNNs with time delays. Based on the LMI optimization approach and Halanay inequality technique, Cao and Li gave the global stability criteria for delayed CGNNs in [19]. In [20], Zhu and Cao investigated the robust exponential stability problem for a class of Markovian jump stochastic CGNNs with mixed time delays and unknown parameters.

The previously proposed systems are all single-layer associative memory neural networks; Kosko established a neural network with bidirectional associative memory in 1987, known as BAM neural network (BAMNN) [24–26]. The BAMNN is different from the previously proposed systems in that the BAMNN model popularizes the common single-layer associative memory neural network to realize the mutual transmission of information between two-layer neurons [27]. Over the years, research studies on the BAMNN have yielded many results. Such as, the authors of [28] studied GES of delayed BAMNNs. The authors dealt with the uniform stability in mean square of stochastic fractional-order memristor fuzzy BAMNNs with delay and leakage terms in [29]. The authors of [30] investigated the stability problem of impulsive stochastic BAMNNs with both Markovian jump parameters and mixed time delays. For more results, see [28–37].

In [38], a new kind of BAM model with Cohen–Grossberg dynamics was proposed; the author studied the Cohen–Grossberg BAM model with time delays, sufficient conditions for the existence and uniqueness of the equilibrium point are obtained, and GAS of the model is proved. In recent years, since applications of NNs depend heavily on the dynamic behaviors, researchers have paid increasing attention to the stability of Cohen–Grossberg type BAM neural network (BAMCGNN), and many findings have been reported in [39–43]. Many factors can affect the stability of BAMCGNN, such as time delays [39–41] and impulses [42, 43]. As we all know, the stability of the system is not only related to the delayed state but also to the advanced state. Many physical models involve piecewise constant argument (PCA), for example, Geneva wheel, Froude pendulum, workpiece-cutter system, amped loading system, undamped systems, and vibration systems. Systems with PCA include advanced systems and delayed systems and have the characteristics of differential systems and difference systems, which can alternately change the types of advance and retard with the evolution of process; this system have important applications in cybernetics and biomedical problems [44–46]. The theory of differential equations for piecewise constant argument (EQPCA) was proposed by Cooke and Wiener in [47]; EQPCA is a hybrid system of continuous and discrete dynamical systems and can be applied to mathematics, engineering, biology, and other fields. PCA has a profound impact on NNs; many scholars have analyzed the impacts of PCA on the system [45, 48–58]. For example, Akhmet studied and improved the theory of EQPCA and obtained many useful results [48–53]. In [51–53], Akhmet et al. explored the stability of CNN, RNN, and HNN with PCA, respectively. From previous results, it can be seen that the authors of [55] studied the GES of CGNN with PCA and impulses; the authors of [56] investigated the global robust exponential stability of interval fuzzy CGNN with PCA. The authors of [39, 41, 42] considered a class of BAMCGNN with time delays, and few articles have examined the stability of BAMCGNN with PCA. To fill this gap, we will consider stability problem of BAMCGNN with PCA. First, by using the homeomorphic mapping theorem, we obtain sufficient conditions for the existence and uniqueness of the equilibrium point, and then we derive the desired stability criteria by constructing a Lyapunov function. Briefly, the following is a list of our works and contributions:

(1) This paper gives sufficient conditions to ensure the existence and uniqueness of the equilibrium point and proves the solution is unique.

(2) In this paper, the homeomorphic mapping theorem is applied to the BAMCGNN model to obtain sufficient conditions to guarantee the uniqueness of the equilibrium point. Compared with contraction mapping principle and Brouwer fixed points theorem, it is practical and convenient to apply the homomorphic mapping theorem in BAMCGNN.

(3) The GES criteria of BAMCGNN with PCA are developed by using inequality method and constructing an appropriate Lyapunov functional. Lyapunov theorem is a classical method to solve the stability problems of system, and by using the inequality method, it will be more convenient to get the desired results.

Here, the structure of the article is as follows. In Section 2, we demonstrate the existence and uniqueness of the equilibrium point; we ensure that the solution of the system is exist and unique, and then, we establish the criteria for ensuring the GES of BAMCGNN with PCA. In Section 3, the article's findings are supported by a numerical case.

Notations: let $N = \{1, 2, \ldots\}$ and $R^+ = [0, +\infty)$, denote R^n be the n-dimensional Euclidean space. The Euclidean norm of a vector $\zeta \in R^n$ is defined by $\|\zeta\| = \sum_{i=1}^{n} |\zeta_i|$. Fix two real-valued sequences $\{\theta_k\}$, $\{\xi_k\}$ and $k \in N$, such that $\theta_k < \theta_{k+1}$, $\theta_k \le \xi_k \le \theta_{k+1}$ for all $k \in N$, and $\theta_k \longrightarrow \infty$ as $k \longrightarrow \infty$. Denote $A = (a_{ij})_{n\times n}$ is a real matrix. If $A > 0\,(A < 0)$, it implies that A is symmetric and positive (negative) definite. The A^T represents the transpose of a matrix A and A^{-1} means the inverse of a matrix A. The Euclidean vector norm of A is expressed as $\|A\|_2$, and $\|A\|_2 = (\lambda(A^T A))^{1/2}$, where $\lambda(A)$ means the maximum eigenvalue of matrix A.

2. Main Results

Consider the following BAMCGNN with PCA:

$$\begin{aligned}
\frac{\mathrm{d}\varsigma_i(t)}{\mathrm{d}t} &= a_i(\varsigma_i(t))\left[-b_i(\varsigma_i(t)) + \sum_{j=1}^{m} k_{ij} f_j\big(\vartheta_j(\eta(t))\big) + I_i\right],\\
\frac{\mathrm{d}\vartheta_j(t)}{\mathrm{d}t} &= c_j\big(\vartheta_j(t)\big)\left[-d_j\big(\vartheta_j(t)\big) + \sum_{i=1}^{n} w_{ji} g_i(\varsigma_i(\eta(t))) + J_j\right].
\end{aligned} \tag{1}$$

for $t > 0$, $i = 1, \ldots, n, j = 1, \ldots, m$, where $\eta(t) = \xi_k$, if $t \in [\theta_k, \theta_{k+1}]$; $\varsigma(t) = (\varsigma_1(t), \ldots, \varsigma_n(t))^T$, $\vartheta(t) = (\vartheta_1(t), \ldots, \vartheta_m(t))^T$, $\varsigma_i(t)$ and $\vartheta_j(t)$ are the states of the i th neuron from the neuron field F_ς and the j th neuron from the neuron field F_ϑ at time t; the functions $a_i(\cdot)$ and $c_j(\cdot)$ mean amplification functions; denote $f_j(\cdot)$, $g_i(\cdot)$ be the activation functions of the j th neuron from F_ϑ and the i th neuron from F_ς; $b_i(\cdot)$, and $d_j(\cdot)$ are appropriately behaved functions such that the solutions of BAMCGNN (1) remain bounded; k_{ij} and w_{ji} represent the connection strengths; I_i and J_j are constants representing the external inputs.

The model (1) is a hybrid system. Fix $k \in N$, and on the interval $[\theta_k, \theta_{k+1}]$, $\eta(t) = \xi_k$. If $\theta_k \le t < \xi_k$ holds for the argument t, that is, $t < \eta(t)$, then system (1) is a network with advanced argument. Similarly, if $\xi_k \le t < \theta_{k+1}$, then $t > \eta(t)$, system (1) is a network with delayed argument.

Rewrite system (1) in the following form:

$$\begin{aligned}\dot{\varsigma}(t) &= \mathscr{A}(\varsigma(t))[-\mathscr{B}(\varsigma(t)) + \mathscr{K}\Gamma(\vartheta(\eta(t))) + \mathscr{I}],\\ \dot{\vartheta}(t) &= \mathscr{C}(\vartheta(t))[-\mathscr{D}(\vartheta(t)) + \mathscr{W}\Theta(\varsigma(\eta(t))) + \mathscr{J}],\end{aligned} \tag{2}$$

where

$$\begin{aligned}\mathscr{A}(\varsigma(t)) &= \operatorname{diag}(a_1(\varsigma_1(t)),\ldots,a_n(\varsigma_n(t))),\\ \mathscr{B}(\varsigma(t)) &= (b_1(\varsigma_1(t)),\ldots,b_n(\varsigma_n(t)))^T,\\ \mathscr{C}(\vartheta(t)) &= \operatorname{diag}(c_1(\vartheta_1(t)),\ldots,c_m(\vartheta_m(t))),\\ \mathscr{D}(\vartheta(t)) &= d_1(\vartheta_1(t)),\ldots,d_m(\vartheta_m(t))^T,\\ \Gamma(\vartheta(\eta(t))) &= f_1(\vartheta_1(\eta(t))),\ldots,f_m(\vartheta_m(\eta(t)))^T,\\ \Theta(\varsigma(\eta(t))) &= g_1(\varsigma_1(\eta(t))),\ldots,g_n(\varsigma_n(\eta(t)))^T,\\ \mathscr{K} &= (k_{ij})_{n\times m},\\ \mathscr{W} &= (w_{ji})_{m\times n},\\ \mathscr{I} &= (I_1,\ldots,I_n)^T,\\ \mathscr{J} &= (J_1,\ldots,J_m)^T.\end{aligned} \tag{3}$$

After that, we provide some mathematical explanations and assumptions:

Assumption 1. $a_i(\cdot)$, $c_j(\cdot)$ are continuously bounded, and then there are positive constants $\overline{a}$, $\overline{c}$, $\underline{a}$, $\underline{c}$, such that

$$\underline{a} < a_i(\varsigma) \leq \overline{a}, \underline{c} < c_j(\vartheta) \leq \overline{c}; \tag{4}$$

besides, the functions $a_i(\cdot)$, $c_j(\cdot)$ satisfy the Lipschitz condition:

$$\left|a_i(\varsigma) - a_i(\varsigma^*)\right| \leq e_i\left|\varsigma - \varsigma^*\right|, \left|c_j(\vartheta) - c_j(\vartheta^*)\right| \leq l_j\left|\vartheta - \vartheta^*\right|, \tag{5}$$

for $\forall \varsigma, \varsigma^* \in R^n$, $\vartheta, \vartheta^* \in R^m$, where e_i and l_j are known constants.

Assumption 2. $b_i(\cdot)$ and $d_j(\cdot)$ are monotonically increasing functions, that is, there are the following four positive matrices:

$$\begin{aligned}\underline{\mathscr{B}} &= \operatorname{diag}(\underline{b}_1,\ldots,\underline{b}_n), \overline{\mathscr{B}} = \operatorname{diag}(\overline{b}_1,\ldots,\overline{b}_n),\\ \underline{\mathscr{D}} &= \operatorname{diag}(\underline{d}_1,\ldots,\underline{d}_m), \overline{\mathscr{D}} = \operatorname{diag}(\overline{d}_1,\ldots,\overline{d}_m),\end{aligned} \tag{6}$$

such that

$$\begin{aligned}\underline{b}_i &\leq \frac{b_i(\varsigma_i^*) - b_i(\varsigma_i)}{\varsigma_i^* - \varsigma_i} \leq \overline{b}_i,\\ \underline{d}_j &\frac{d_j(\vartheta_j^*) - d_j(\vartheta_j)}{\vartheta_j^* - \vartheta_j} \leq \overline{d}_j,\end{aligned} \tag{7}$$

for $\varsigma_i^* \neq \varsigma_i$, $\vartheta_j^* \neq \vartheta_j$.

Assumption 3. For activation functions f_j and g_i, let $\mathscr{F} = \operatorname{diag}(F_1,\ldots,F_m)$ and $\mathscr{G} = \operatorname{diag}(G_1,\ldots,G_n)$ be two positive matrices, then the following formulae hold:

$$0 \leq \frac{f_j(\vartheta_j^*) - f_j(\vartheta_j)}{\vartheta_j^* - \vartheta_j} \leq F_j, 0 \leq \frac{g_i(\varsigma_i^*) - g_i(\varsigma_i)}{\varsigma_i^* - \varsigma_i} \leq G_i, \tag{8}$$

for all ς_i^*, $\varsigma_i \in R^n$, $\varsigma_i^* \neq \varsigma_i$; ϑ_j^*, $\vartheta_j \in R^m$, and $\vartheta_j^* \neq \vartheta_j$.

Assumption 4. There is a positive number θ such that $\theta_{k+1} - \theta_k \leq \theta$, $k \in N$.

Assumption 5. $R < 1$, $R = \max\{\theta(\varepsilon_1 + \tau_2), \theta(\varepsilon_2 + \tau_1)\}$.

Assumption 6. $(\varepsilon e^{(\varepsilon+\tau e^{\varepsilon\theta}/1-\nu)\theta} + \tau e^{\varepsilon\theta})\theta < 1$.

Assumption 7. $\tau\theta + (1 + \tau\theta)m_1\theta e^{m_1\theta} < 1$.

Here are the notations we adopt:

$$\begin{aligned}\varepsilon_1 &= \max_{1\leq i\leq n}\{e_iP + \overline{a}\overline{b}_i\},\\ \varepsilon_2 &= \max_{1\leq j\leq m}\{l_jQ + \overline{c}\overline{d}_j\},\\ \tau_1 &= \max_{1\leq i\leq n}\overline{a}\sum_{j=1}^{m}\left|k_{ij}\right|F_j,\\ \tau_2 &= \max_{1\leq j\leq m}\overline{c}\sum_{i=1}^{n}\left|w_{ji}\right|G_i,\\ \varepsilon &= \max(\varepsilon_1,\varepsilon_2),\\ \tau &= \max(\tau_1,\tau_2),\\ m_1 &= \max\{\overline{a}\overline{b}_i, \overline{c}\overline{d}_j\},\\ m_2 &= \min\{\underline{b}_i, \underline{d}_j\},\\ m_3 &= \max\left\{\sum_{j=1}^{m}k_{ij}F_j, \sum_{i=1}^{n}w_{ji}G_i\right\},\\ m_4 &= \min\left\{\frac{1}{\overline{a}}, \frac{1}{\overline{c}}\right\},\\ m_5 &= \max\left\{\frac{1}{\underline{a}}, \frac{1}{\underline{c}}\right\}.\end{aligned} \tag{9}$$

Definition 8. The equilibrium point χ^* of (1) is globally exponentially stable and $\chi^* = (\varsigma^*, \vartheta^*)$, if for any intial value $s_0 \in R^+$ and $\chi_0 \in R^{n+m}$, there are constants $\nu > 0$ and $\kappa > 0$ such that

$$\left\|\chi(s) - \chi^*\right\| \leq \nu\left\|\chi_0 - \chi^*\right\|e^{-\kappa(s-s_0)}, \tag{10}$$

for $s \geq s_0$, where $\chi(s)$ is the solution of the model (1).

Before discussing, we first introduce the following lemmas:

Lemma 9 (see [59]). *Let $\varsigma(t)$, z_i: $R \longrightarrow [0,\infty]$, $i = 1,2$ be continuous functions and α be a nonnegative real constant. Suppose that for all $t \geq t_0$, the inequality*

$$\varsigma(t) \leq \alpha + \int_{t_0}^{t} [z_1(s)\varsigma(s) + z_2(s)\varsigma(\eta(s))]\mathrm{d}s, \tag{11}$$

holds. Assume

$$\begin{aligned} v_i &= \int_{t_i}^{\xi_i}\left[z_2(s)\exp\left\{\int_s^{\xi_i} z_1(k)\mathrm{d}k\right\}\right]\mathrm{d}s \leq v: \\ &= \sup_{i\in N} v_i < 1. \end{aligned} \tag{12}$$

Then for $t \geq t_0$,

$$\begin{aligned} \varsigma(t) &\leq \alpha\exp\left\{\int_{t_0}^{t} z_1(s)\mathrm{d}s + \frac{1}{1-v}\int_{t_0}^{t}\left[z_2(s)\times\exp\left\{\int_{t_{i(s)}}^{\eta(s)} z_1(k)\mathrm{d}k\right\}\right]\mathrm{d}s\right\}, \\ \varsigma(\eta(t)) &\leq \frac{\alpha}{1-v}\exp\left\{\int_{t}^{\eta(t)} z_1(s)\mathrm{d}s + \frac{1}{1-v}\int_{t_0}^{t_{i(s)}}\left[z_2(s)\times\exp\left\{\int_{t_{i(s)}}^{\eta(s)} z_1(k)\mathrm{d}k\right\}\right]\mathrm{d}s\right\}, \\ \varsigma(\xi_i) &\leq \frac{1}{1-v}\varsigma(t_i)\exp\left\{\int_{t_{i(s)}}^{\xi_i} z_1(s)\mathrm{d}s\right\}. \end{aligned} \tag{13}$$

Lemma 10 (see [60]). *For every real matrices A, B, P of appropriate dimensions and a positive scalar ε_0, where $P > 0$. Then, the inequality holds:*

$$A^T B + B^T A \leq \varepsilon_0 A^T P A + \frac{1}{\varepsilon_0} B^T P^{-1} B. \tag{14}$$

Especially, if A and B are vectors, the above formula can be converted into:

$$A^T B \leq \frac{\left(A^T A + B^T B\right)}{2}. \tag{15}$$

After that, we obtain sufficient criteria for the equilibrium point to be exist and unique.

Theorem 11. *Let assumptions (A1)–(A3) hold, if there have positive matrices $S_l = \mathrm{diag}(s_1^{(l)}, s_2^{(l)}, \ldots, s_n^{(l)})$, $l = 1,2$, $Q_1 > 0, Q_2 > 0$, and factorizations of $\mathscr{K} = K_1K_2$, $\mathscr{W} = W_1W_2$ such that*

$$\begin{aligned} \Omega_1 &= 2S_1\underline{\mathscr{B}}\mathscr{G}^{-1} - S_1K_1Q_1^{-1}K_1^TS_1 - W_2^TQ_2W_2 > 0, \\ \Omega_2 &= 2S_2\underline{\mathscr{D}}\mathscr{F}^{-1} - S_2W_1Q_2^{-1}W_1^TS_2 - K_2^TQ_1K_2 > 0, \end{aligned} \tag{16}$$

where $Q_1, Q_2, K_1, K_2, W_1, W_2$ are constant matrices with appropriate dimensions, then the equilibrium point $\chi^* = (\varsigma^*, \vartheta^*)^T$ of (1) is unique.

Proof. Let χ^* be an equilibrium point of (1) and $\chi^* = (\varsigma^*, \vartheta^*)^T$. Then, χ^* satisfies the following equation:

$$\begin{cases} \mathscr{A}(\varsigma^*)[-\mathscr{B}(\varsigma^*) + \mathscr{K}\Gamma(\vartheta^*) + \mathscr{I}] = 0, \\ \mathscr{C}(\vartheta^*)[-\mathscr{D}(\vartheta^*) + \mathscr{W}\Theta(\varsigma^*) + \mathscr{J}] = 0, \end{cases} \tag{17}$$

then we have

$$\begin{cases} -\mathscr{B}(\varsigma^*) + \mathscr{K}\Gamma(\vartheta^*) + \mathscr{I} = 0, \\ -\mathscr{D}(\vartheta^*) + \mathscr{W}\Theta(\varsigma^*) + \mathscr{J} = 0. \end{cases} \tag{18}$$

Let

$$\begin{aligned} \Phi(\rho) &= (\Phi_1(\rho), \Phi_2(\rho))^T \\ &= (-\mathscr{B}(\rho) + \mathscr{K}\Gamma(\rho) + \mathscr{I}, -\mathscr{D}(\rho) + \mathscr{W}\Theta(\rho) + \mathscr{J})^T \\ &= 0, \end{aligned} \tag{19}$$

where

$$\begin{aligned} \Gamma(\rho) &= (f_1(\vartheta_1), \ldots, f_m(\vartheta_m))^T, \\ \Theta(\rho) &= (g_1(\varsigma_1), \ldots, g_n(\varsigma_n))^T, \\ \rho &= (\varsigma, \vartheta), \\ \varsigma &= (\varsigma_1, \ldots, \varsigma_n)^T, \\ \vartheta &= (\vartheta_1, \ldots, \vartheta_m)^T. \end{aligned} \tag{20}$$

This can be seen from formula (20) that the solution of equation (19) is the equilibrium point of BAMCGNN (1). Thus, if $\Phi(\rho)$ is homeomorphism of R^{n+m}, then (1) has a unique solution. From the reference [61], we can gain the following conclusion: if $\Phi(\rho) \neq \Phi(\delta)$, $\forall \rho \neq \delta$, $\rho, \delta \in R^{n+m}$, and $\|\Phi(\rho)\| \longrightarrow \infty$ as $\|\rho\| \longrightarrow \infty$, then $\Phi(\rho)$ is homeomorphism of R^{n+m}.

Assume $P(\rho) = (\Gamma(\rho), \Theta(\rho))^T$, and ρ, δ be two different vectors, i.e., $\rho \neq \delta$. Based on the assumptions of the activation functions, $\rho \neq \delta$ represents the following two cases:

(1) $\rho \neq \delta$, $P(\rho) \neq P(\delta)$
(2) $\rho \neq \delta$, $P(\rho) = P(\delta)$

Then, we have

$$\begin{aligned} \Phi_1(\rho) - \Phi_1(\delta) &= -\mathscr{B}\left(\varsigma_\rho\right) + \mathscr{B}(\varsigma_\delta) + \mathscr{K}(\Gamma(\rho) - \Gamma(\delta)), \\ \Phi_2(\rho) - \Phi_2(\delta) &= -\mathscr{D}\left(\vartheta_\rho\right) + \mathscr{D}(\vartheta_\delta) + \mathscr{W}(\Theta(\rho) - \Theta(\delta)), \end{aligned} \tag{21}$$

where

$$\begin{aligned} \Phi_1(\rho) &= \left(\phi_1^1(\rho), \phi_2^1(\rho), \ldots, \phi_n^1(\rho)\right)^T, \\ \Phi_2(\rho) &= \left(\phi_1^2(\rho), \phi_2^2(\rho), \ldots, \phi_m^2(\rho)\right)^T, \\ \Phi_1(\delta) &= \left(\phi_1^1(\delta), \phi_2^1(\delta), \ldots, \phi_n^1(\delta)\right)^T, \\ \Phi_2(\delta) &= \left(\phi_1^2(\delta), \phi_2^2(\delta), \ldots, \phi_m^2(\delta)\right)^T, \\ \rho &= \left(\varsigma_\rho, \vartheta_\rho\right), \\ \delta &= \left(\varsigma_\delta, \vartheta_\delta\right), \\ \varsigma_\rho &= \left(\varsigma_{\rho 1}, \varsigma_{\rho 2}, \ldots, \varsigma_{\rho n}\right), \\ \varsigma_\delta &= \left(\varsigma_{\delta 1}, \varsigma_{\delta 2}, \ldots, \varsigma_{\delta n}\right) \in R^n, \\ \vartheta_\rho &= \left(\vartheta_{\rho 1}, \vartheta_{\rho 2}, \ldots, \vartheta_{\rho m}\right), \\ \vartheta_\delta &= \left(\vartheta_{\delta 1}, \vartheta_{\delta 2}, \ldots, \vartheta_{\delta m}\right) \in R^m. \end{aligned} \tag{22}$$

First, we consider the case (i): $\rho \neq \delta$ and $P(\rho) \neq P(\delta)$, that is, $\Gamma(\rho) - \Gamma(\delta) \neq 0$, $\Theta(\rho) - \Theta(\delta) \neq 0$.

For $S_1 = \operatorname{diag}(s_1^{(1)}, s_2^{(1)}, \ldots, s_n^{(1)})$ and Q_1, multiplying both sides of the first equation in (21) by $2(\Theta(\rho) - \Theta(\delta))^T S_1$, we have

$$2(\Theta(\rho) - \Theta(\delta))^T S_1 (\Phi_1(\rho) - \Phi_1(\delta)) = -2(\Theta(\rho) - \Theta(\delta))^T S_1 \left(\mathscr{B}(\varsigma_\rho) - \mathscr{B}(\varsigma_\delta)\right) + 2(\Theta(\rho) - \Theta(\delta))^T S_1 \mathscr{K}(\Gamma(\rho) - \Gamma(\delta)). \tag{23}$$

From assumption (A3), we have

$$G_i^{-1}\left(g_i(\varsigma_{\rho i}) - g_i(\varsigma_{\delta i})\right)^2 \leq \left(\varsigma_{\rho i} - \varsigma_{\delta i}\right)\left(g_i(\varsigma_{\rho i}) - g_i(\varsigma_{\delta i})\right), \tag{24}$$

then

$$(\Theta(\rho) - \Theta(\delta))^T S_1 \left(\mathscr{B}(\varsigma_\rho) - \mathscr{B}(\varsigma_\delta)\right) \geq (\Theta(\rho) - \Theta(\delta))^T S_1 \underline{\mathscr{B}} \mathscr{G}^{-1} (\Theta(\rho) - \Theta(\delta)). \tag{25}$$

By using the Cholesky factorization, Q_1 can be written as $Q_1 = U_1 U_1^T$, and rewriting $\mathscr{K} = (K_1 U_1^{-1})(U_1 K_2)$, we obtain

$$\begin{aligned} 2(\Theta(\rho) - \Theta(\delta))^T S_1 (\Phi_1(\rho) - \Phi_1(\delta)) \leq & -2(\Theta(\rho) - \Theta(\delta))^T S_1 \underline{\mathscr{B}} \mathscr{G}^{-1} (\Theta(\rho) - \Theta(\delta)) \\ & + 2\left[(\Theta(\rho) - \Theta(\delta))^T S_1 K_1 U_1^{-1}\right][U_1 K_2 (\Gamma(\rho) - \Gamma(\delta))]. \end{aligned} \tag{26}$$

By Lemma 10, we get

$$\begin{aligned} 2(\Theta(\rho) - \Theta(\delta))^T S_1 (\Phi_1(\rho) - \Phi_1(\delta)) \leq & -2(\Theta(\rho) - \Theta(\delta))^T S_1 \underline{\mathscr{B}} \mathscr{G}^{-1} (\Theta(\rho) - \Theta(\delta)) \\ & + (\Theta(\rho) - \Theta(\delta))^T S_1 K_1 U_1^{-1} \left(U_1^{-1}\right)^T K_1^T S_1^T (\Theta(\rho) - \Theta(\delta)) \\ & + (\Gamma(\rho) - \Gamma(\delta))^T K_2^T U_1^T U_1 K_2 (\Gamma(\rho) - \Gamma(\delta)) \\ = & -2(\Theta(\rho) - \Theta(\delta))^T S_1 \underline{\mathscr{B}} \mathscr{G}^{-1} (\Theta(\rho) - \Theta(\delta)) \\ & + (\Theta(\rho) - \Theta(\delta))^T S_1 K_1 Q_1^{-1} K_1^T S_1 (\Theta(\rho) - \Theta(\delta)) \\ & + (\Gamma(\rho) - \Gamma(\delta))^T K_2^T Q_1 K_2 (\Gamma(\rho) - \Gamma(\delta)). \end{aligned} \tag{27}$$

Similarly,

$$
\begin{aligned}
2(\Gamma(\rho)-\Gamma(\delta))^T S_2(\Phi_2(\rho)-\Phi_2(\delta)) \leq & -2(\Gamma(\rho)-\Gamma(\delta))^T S_2 \underline{\mathscr{D}} \mathscr{F}^{-1}(\Gamma(\rho)-\Gamma(\delta)) \\
& +(\Gamma(\rho)-\Gamma(\delta))^T S_2 W_1 Q_2^{-1} W_1^T S_2(\Gamma(\rho)-\Gamma(\delta)) \\
& +(\Theta(\rho)-\Theta(\delta))^T W_2^T Q_2 W_2(\Theta(\rho)-\Theta(\delta)),
\end{aligned} \tag{28}
$$

which implies that

$$
\begin{aligned}
2(P(\rho)-P(\delta))^T \operatorname{diag}(S_2, S_1)(\Phi(\rho)-\Phi(\delta)) \leq & -(\Gamma(\rho)-\Gamma(\delta))^T \Omega_2(\Gamma(\rho)-\Gamma(\delta)) \\
& -(\Theta(\rho)-\Theta(\delta))^T \Omega_1(\Theta(\rho)-\Theta(\delta)) \\
& < 0.
\end{aligned} \tag{29}
$$

Since $\operatorname{diag}(S_2, S_1)$ is a positive diagonal matrix, it implies $\Phi(\rho) \neq \Phi(\delta)$. Hence, when $\rho \neq \delta$ and $P(\rho) \neq P(\delta)$, we have $\Phi(\rho) - \Phi(\delta) \neq 0$.

Now, we consider case (ii): $\rho \neq \delta$, $P(\rho) = P(\delta)$. In this case, we have

$$
\Phi(\rho)-\Phi(\delta) = -\begin{pmatrix} \mathscr{B}(\varsigma_\rho)-\mathscr{B}(\varsigma_\delta) & 0 \\ 0 & \mathscr{D}(\vartheta_\rho)-\mathscr{D}(\vartheta_\delta) \end{pmatrix} \leq -\begin{pmatrix} \underline{\mathscr{B}} & 0 \\ 0 & \underline{\mathscr{D}} \end{pmatrix}(\rho-\delta), \tag{30}
$$

it means that $\Phi(\rho) - \Phi(\delta) \neq 0$ for any $\rho \neq \delta$.

Next, we shall prove that $\|\Phi(\rho)\| \longrightarrow \infty$ as $\|\rho\| \longrightarrow \infty$. Let $\delta = 0$, from (29), we have

$$
\begin{aligned}
& 2(P(\rho)-P(0))^T \operatorname{diag}(S_2, S_1)(\Phi(\rho)-\Phi(0)) \\
& \quad \leq -(\Gamma(\rho)-\Gamma(0))^T \Omega_2(\Gamma(\rho)-\Gamma(0))-(\Theta(\rho)-\Theta(0))^T \Omega_1(\Theta(\rho)-\Theta(0)) \\
& \quad \leq -\lambda_{\min}\left[(\Gamma(\rho)-\Gamma(0))^T(\Gamma(\rho)-\Gamma(0))+(\Theta(\rho)-\Theta(0))^T(\Theta(\rho)-\Theta(0))\right] \\
& \quad = -\lambda_{\min}(P(\rho)-P(0))^T(P(\rho)-P(0)),
\end{aligned} \tag{31}
$$

where $\lambda_{\min}$ is the minimum eigenvalue of Ω_1 and Ω_2, and $\Omega_1 > 0$, $\Omega_2 > 0$.

From (31), yields

$$\begin{aligned}\lambda_{\min}\|P(\rho)-P(0)\|_2^2 &\leq \left|\sum_{i=1}^{n} 2s_i^{(2)}\left(f_i\left(\vartheta_{\rho i}\right)-f_i(0)\right)\left(\phi_i^1\left(\vartheta_{\rho i}\right)-\phi_i^{(1)}(0)\right)\right| \\ &\quad + \left|\sum_{i=1}^{m} 2s_i^{(1)}\left(g_i\left(\varsigma_{\rho i}\right)-g_i(0)\right)\left(\phi_i^2\left(\varsigma_{\rho i}\right)-\phi_i^{(2)}(0)\right)\right| \\ &\leq 2s\sum_{i=1}^{n}\left|f_i\left(\vartheta_{\rho i}\right)-f_i(0)\right|\left|\phi_i^1\left(\vartheta_{\rho i}\right)-\phi_i^{(1)}(0)\right| \\ &\quad + 2s\sum_{i=1}^{m}\left|g_i\left(\varsigma_{\rho i}\right)-g_i(0)\right|\left|\phi_i^2\left(\varsigma_{\rho i}\right)-\phi_i^{(2)}(0)\right| \\ &\leq 2s\|P(\rho)-P(0)\|_\infty\left(\sum_{i=1}^{n}\left|\phi_i^1\left(\vartheta_{\rho i}\right)-\phi_i^{(1)}(0)\right|+\sum_{i=1}^{m}\left|\phi_i^2\left(\varsigma_{\rho i}\right)-\phi_i^{(2)}(0)\right|\right) \\ &= 2s\|\Phi(\rho)-\Phi(0)\|_1\|P(\rho)-P(0)\|_\infty,\end{aligned} \tag{32}$$

where $s=\max(s_1^{(1)},\ldots,s_n^{(1)},s_1^{(2)},\ldots,s_m^{(2)})$.

From $\|P(\rho)-P(0)\|_\infty \leq \|P(\rho)-P(0)\|_2$, there have

$$\lambda_{\min}\left\|P(\rho)-P(0)\right\|_\infty \leq 2s\left\|\Phi(\rho)-\Phi(0)\right\|_1. \tag{33}$$

Since $\|P(\rho)-P(0)\|_\infty \geq \|P(\rho)\|_\infty-\|P(0)\|_\infty$ and $\|\Phi(\rho)\|_1+\|\Phi(0)\|_1 \geq \|\Phi(\rho)-\Phi(0)\|_1$, then

$$\lambda_{\min}\left\|P(\rho)\right\|_\infty-\lambda_{\min}\left\|P(0)\right\|_\infty \leq 2s\left\|\Phi(\rho)\right\|_1+2s\left\|\Phi(0)\right\|_1, \tag{34}$$

that is,

$$\|\Phi(\rho)\|_1 \geq \frac{\|P(\rho)\|_\infty-\lambda_{\min}\left\|P(0)\right\|_\infty-2s\left\|\Phi(0)\right\|_1}{2s}, \tag{35}$$

hence, we can draw the following conclusion: $\|\Phi(\rho)\| \longrightarrow \infty$ as $\|P(\rho)\| \longrightarrow \infty$, it is equivalent to $\|\Phi(\rho)\| \longrightarrow \infty$ as $\|\rho\| \longrightarrow \infty$. Thereby, we have demonstrated that $\Phi(\rho)$ is homeomorphism of R^{n+m}, that is, representing BAMCGNN (1) has a unique equilibrium point. □ □

Next, we prove the existence and uniqueness of the solution of (1).

Theorem 12. *Suppose assumptions (A1)–(A6) hold. For any $(t_0,\chi^0)\in R^+\times R^{n\times m}$, there is a unique solution $\chi(t)=(\varsigma(t),\vartheta(t))^T$ of (1), such that $\chi(t_0)=\chi^0$.*

Proof. Existence.

Fix $k\in N$, generally, let $\theta_k\leq\xi_k<t_0<\theta_{k+1}$. For $\forall t\in[\theta_k,\theta_{k+1}]$, $(\varsigma^0,\vartheta^0)\in R^{n\times m}$, we have the following equivalent integral equations:

$$\begin{aligned}\varsigma_i(t) &= \varsigma_i^0+\int_{t_0}^{t} a_i(\varsigma_i(s))\left[-b_i(\varsigma_i(s))+\sum_{j=1}^{m}k_{ij}f_j\left(\vartheta_j(\xi_k)\right)+I_i\right]\mathrm{d}s, \\ \vartheta_j(t) &= \vartheta_j^0+\int_{t_0}^{t} c_j\left(\vartheta_j(s)\right)\left[-d_j\left(\vartheta_j(s)\right)+\sum_{i=1}^{n}w_{ji}g_i(\varsigma_i(\xi_k))+J_j\right]\mathrm{d}s.\end{aligned} \tag{36}$$

Define $\|\chi(t)\|_0 = \max_{[\theta_k,\theta_{k+1}]}\|\chi(t)\|$, construct the following sequences $\{\varsigma_i^r(t)\}, \{\vartheta_j^r(t)\}$, $(\varsigma_i^0, \vartheta_j^0) = \chi(t_0) = \chi^0, i = 1, \ldots, n, j = 1, \ldots, m$ such that

$$\begin{aligned}\varsigma_i^{r+1}(t) &= \varsigma_i^0 + \int_{t_0}^t a_i(\varsigma_i^r(s))\left[-b_i(\varsigma_i^r(s)) + \sum_{j=1}^m k_{ij} f_j\left(\vartheta_j^r(\xi_k)\right)\right] + I_i \mathrm{d}s,\\ \vartheta_j^{r+1}(t) &= \vartheta_j^0 + \int_{t_0}^t c_j\left(\vartheta_j^r(s)\right)\left[-d_j\left(\vartheta_j^r(s)\right) + \sum_{i=1}^n w_{ji} g_i(\varsigma_i^r(\xi_k)) + J_j\right]\mathrm{d}s,\end{aligned} \tag{37}$$

then

$$\begin{aligned}\left|\varsigma_i^{r+1}(t) - \varsigma_i^r(t)\right| &= \left|\int_{t_0}^t a_i(\varsigma_i^r(s))\left[-b_i(\varsigma_i^r(s)) + \sum_{j=1}^m k_{ij} f_j\left(\vartheta_j^r(\xi_k)\right) + I_i\right]\mathrm{d}s - \int_{t_0}^t a_i\left(\varsigma_i^{r-1}(s)\right)\left[-b_i\left(\varsigma_i^{r-1}(s)\right) + \sum_{j=1}^m k_{ij} f_j\left(\vartheta_j^{r-1}(\xi_k)\right) + I_i\right]\mathrm{d}s\right|\\ &= \left|\int_{t_0}^t \left\{a_i(\varsigma_i^r(s))\left[-b_i(\varsigma_i^r(s)) + \sum_{j=1}^m k_{ij} f_j\left(\vartheta_j^r(\xi_k)\right) + I_i\right] - a_i\left(\varsigma_i^{r-1}(s)\right)\left[-b_i(\varsigma_i^r(s)) + \sum_{j=1}^m k_{ij} f_j\left(\vartheta_j^r(\xi_k)\right) + I_i\right]\right.\right.\\ &\quad \left.\left. + a_i\left(\varsigma_i^{r-1}(s)\right)\left[-b_i(\varsigma_i^r(s)) + \sum_{j=1}^m k_{ij} f_j\left(\vartheta_j^r(\xi_k)\right) + I_i\right] - a_i\left(\varsigma_i^{r-1}(s)\right)\left[-b_i\left(\varsigma_i^{r-1}(s)\right) + \sum_{j=1}^m k_{ij} f_j\left(\vartheta_j^{r-1}(\xi_k)\right) + I_i\right]\right\}\mathrm{d}s\right|\\ &\le \left|\int_{t_0}^t \left(a_i(\varsigma_i^r(s)) - a_i\left(\varsigma_i^{r-1}(s)\right)\right)\left[-b_i(\varsigma_i^r(s)) + \sum_{j=1}^m k_{ij} f_j\left(\vartheta_j^r(\xi_k)\right) + I_i\right]\mathrm{d}s\right|\\ &\quad + \left|\int_{t_0}^t a_i\left(\varsigma_i^{r-1}(s)\right)\left[-\left(b_i(\varsigma_i^r(s)) - b_i\left(\varsigma_i^{r-1}(s)\right)\right) + \sum_{j=1}^m k_{ij}\left(f_j\left(\vartheta_j^r(\xi_k)\right) - f_j\left(\vartheta_j^{r-1}(\xi_k)\right)\right)\right]\mathrm{d}s\right|.\end{aligned} \tag{38}$$

Since inputs I_i are bounded, $\chi^0 \in R^{n+m}$, and the functions $b_i(\cdot)$, $f_j(\cdot)$ are continuous on closed interval $[\theta_k, \theta_{k+1}]$, then there are large enough positive constants P_i, such that

$$-b_i(\varsigma_i^r(s)) + \sum_{j=1}^m k_{ij} f_j\left(\vartheta_j^r(\xi_k)\right) + I_i < P_i, \tag{39}$$

for all $n \in N$. Denote $P = \max_{1\le i\le n}\{P_i\}$, we obtain

$$\begin{aligned}&\left|\int_{t_0}^t \left(a_i(\varsigma_i^r(s)) - a_i\left(\varsigma_i^{r-1}(s)\right)\right)\left[-b_i(\varsigma_i^r(s)) + \sum_{j=1}^m k_{ij}\times f_j\left(\vartheta_j^r(\xi_k)\right) + I_i\right]\mathrm{d}s\right|\\ &\le \int_{t_0}^t e_i P_i\left|\varsigma_i^r(s) - \varsigma_i^{r-1}(s)\right|,\\ &\left|\int_{t_0}^t a_i\left(\varsigma_i^{r-1}(s)\right)\left[-\left(b_i(\varsigma_i^r(s)) - b_i\left(\varsigma_i^{r-1}(s)\right)\right) + \sum_{j=1}^m k_{ij}\times\left(f_j\left(\vartheta_j^r(\xi_k)\right) - f_j\left(\vartheta_j^{r-1}(\xi_k)\right)\right)\right]\mathrm{d}s\right|\\ &\le \int_{t_0}^t \overline{a}\left[\overline{b}_i\left|\varsigma_i^r(s) - \varsigma_i^{r-1}(s)\right| + \sum_{j=1}^m \left|k_{ij}\right|F_j\left|\vartheta_j^r(\xi_k) - \vartheta_j^{r-1}(\xi_k)\right|\right]\mathrm{d}s,\end{aligned} \tag{40}$$

that is

$$\left|\varsigma_i^{r+1}(t) - \varsigma_i^r(t)\right| \le \int_{t_0}^t \left(e_i P_i + \overline{a}\overline{b}_i\right)\left|\varsigma_i^r(s) - \varsigma_i^{r-1}(s)\right|\mathrm{d}s + \overline{a}\theta\sum_{i=1}^n \left|k_{ij}\right|F_j\left|\vartheta_j^r(\xi_k) - \vartheta_j^{r-1}(\xi_k)\right|, \tag{41}$$

then

$$\|\varsigma^{r+1}(t)-\varsigma^{r}(t)\|=\sum_{i=1}^{n}\|\varsigma_i^{r+1}(t)-\varsigma_i^{r}(t)\|\le\varepsilon_1\int_{t_0}^{t}\|\varsigma^{r}(s)-\varsigma^{r-1}(s)\|\mathrm{d}s+\tau_1\theta\|\vartheta^{r}(\xi_k)-\vartheta^{r-1}(\xi_k)\|. \tag{42}$$

Similarly,

$$\begin{aligned}\left|\vartheta_j^{r+1}(t)-\vartheta_j^{r}(t)\right|&=\left|\int_{t_0}^{t}c_j\left(\vartheta_j^{r}(s)\right)\left[-d_j\left(\vartheta_j^{r}(s)\right)+\sum_{i=1}^{n}w_{ji}g_i\left(\varsigma_i^{r}(\xi_k)\right)+J_j\right]\mathrm{d}s-\int_{t_0}^{t}c_j\left(\vartheta_j^{r-1}(s)\right)\left[-d_j\left(\vartheta_j^{r-1}(s)\right)+\sum_{i=1}^{n}w_{ji}g_i\left(\varsigma_i^{r-1}(\xi_k)\right)+J_j\right]\mathrm{d}s\right|\\&\le\left|\int_{t_0}^{t}\left(c_j\left(\vartheta_j^{r}(s)\right)-c_j\left(\vartheta_j^{r-1}(s)\right)\right)\left[-d_j\left(\vartheta_j^{r}(s)\right)+\sum_{i=1}^{n}w_{ji}g_i\left(\varsigma_i^{r}(\xi_k)\right)+J_j+\right]\mathrm{d}s\right|\\&+\left|\int_{t_0}^{t}c_j\left(\vartheta_j^{r-1}(s)\left[-\left(d_j\left(\vartheta_j^{r}(s)\right)-d_j\left(\vartheta_j^{r-1}(s)\right)\right)+\sum_{i=1}^{n}w_{ji}\left(g_i\left(\varsigma_i^{r}(\xi_k)\right)-g_i\left(\varsigma_i^{r-1}(\xi_k)\right)\right)\right]\mathrm{d}s\right|.\end{aligned} \tag{43}$$

Since inputs J_j are bounded, $\chi^0\in R^{n+m}$, and the functions $d_j(\cdot)$, $g_i(\cdot)$ are continuous on the closed interval $[\theta_k,\theta_{k+1}]$, there also have large enough positive constants Q_j, such that

$$-d_j\left(\vartheta_j^{r}(s)\right)+\sum_{i=1}^{n}w_{ji}g_i\left(\varsigma_i^{r}(\xi_k)\right)+J_j<Q_j, \tag{44}$$

for all $n\in N$. Denote $Q=\max_{1\le j\le m}Q_j$.

$$\left|\vartheta_j^{r+1}(t)-\vartheta_j^{r}(t)\right|\le\int_{t_0}^{t}\left(l_jQ_j+\bar{c}\bar{d}_j\right)\left|\vartheta_j^{r}(s)-\vartheta_j^{r-1}(s)\right|\mathrm{d}s+\bar{c}\theta\sum_{i=1}^{n}\left|w_{ji}\right|G_i\left|\varsigma_i^{r}(\xi_k)-\varsigma_i^{r-1}(\xi_k)\right|, \tag{45}$$

then

$$\|\vartheta^{r+1}(t)-\vartheta^{r}(t)\|=\sum_{j=1}^{m}\|\vartheta_j^{r+1}(t)-\vartheta_j^{r}(t)\|\le\varepsilon_2\int_{t_0}^{t}\|\vartheta^{r}(s)-\vartheta^{r-1}(s)\|\mathrm{d}s+\tau_2\theta\|\varsigma^{r}(\xi_k)-\varsigma^{r-1}(\xi_k)\|. \tag{46}$$

From (42) and (46), we have

$$\begin{aligned}&\|\varsigma^{r+1}(t)-\varsigma^{r}(t)\|+\|\vartheta^{r+1}(t)-\vartheta^{r}(t)\|\\&\quad\le\varepsilon_1\int_{t_0}^{t}\|\varsigma^{r}(s)-\varsigma^{r-1}(s)\|\mathrm{d}s+\tau_1\theta\|\vartheta^{r}(\xi_k)-\vartheta^{r-1}(\xi_k)\|\\&\qquad+\varepsilon_2\int_{t_0}^{t}\|\vartheta^{r}(s)-\vartheta^{r-1}(s)\|\mathrm{d}s+\tau_2\theta\|\varsigma^{r}(\xi_k)-\varsigma^{r-1}(\xi_k)\|,\end{aligned} \tag{47}$$

then

$$\begin{aligned}&\|\varsigma^{r+1}(t)-\varsigma^{r}(t)\|_0+\|\vartheta^{r+1}(t)-\vartheta^{r}(t)\|_0\\&\quad\le\theta(\varepsilon_1+\tau_2)\|\varsigma^{r}(s)-\varsigma^{r-1}(s)\|_0\\&\qquad+\theta(\varepsilon_2+\tau_1)\|\vartheta^{r}(s)-\vartheta^{r-1}(s)\|_0.\end{aligned} \tag{48}$$

Let $R=\max_{1\le i\le n,1\le j\le m}\{\theta(\varepsilon_1+\tau_2),\theta(\varepsilon_2+\tau_1)\}$, then

$$\begin{aligned}&\|\varsigma^{r+1}(t)-\varsigma^{r}(t)\|_0+\|\vartheta^{r+1}(t)-\vartheta^{r}(t)\|_0\\&\quad\le R\left(\|\varsigma^{r}(s)-\varsigma^{r-1}(s)\|_0+\|\vartheta^{r}(s)-\vartheta^{r-1}(s)\|_0\right)\\&\quad\le\ldots\\&\quad\le R^{r}\left(\|\varsigma^{1}(s)-\varsigma^{0}(s)\|_0+\|\vartheta^{1}(s)-\vartheta^{0}(s)\|_0\right)\\&\quad\le R^{r+1}\left(\|\varsigma^{0}\|_0+\|\vartheta^{0}\|_0\right),\end{aligned} \tag{49}$$

according to (A5), the sequences $\{\varsigma_i^r(t)\}$, $\{\vartheta_j^r(t)\}$ are convergent, and their limits satisfy the integral equation on $[\theta_k, \theta_{k+1}]$.

Uniqueness.

For each $t \in [\theta_k, \theta_{k+1}]$, $\varsigma^1 = (\varsigma_1^1, \varsigma_2^1, \ldots, \varsigma_n^1)^T$, $\varsigma^2 = (\varsigma_1^2, \varsigma_2^2, \ldots, \varsigma_n^2)^T$, $\vartheta^1 = (\vartheta_1^1, \vartheta_2^1, \ldots, \vartheta_m^1)^T$, $\vartheta^2 = (\vartheta_1^2, \vartheta_2^2, \ldots, \vartheta_m^2)^T$, and $\varsigma^1 \neq \varsigma^2, \vartheta^1 \neq \vartheta^2$. Let $\chi^1(t) = (\varsigma^1(t), \vartheta^1(t)), \chi^2(t) = (\varsigma^2(t), \vartheta^2(t))$ be two solutions of (1), and $\varsigma^1(t) = \varsigma(t; t_0, \varsigma^1) = (\varsigma_1^1(t), \ldots, \varsigma_n^1(t))^T$, $\varsigma^2(t) = \varsigma(t; t_0, \varsigma^2) = (\varsigma_1^2(t), \ldots, \varsigma_n^2(t))^T$, $\vartheta^1(t) = \vartheta(t; t_0, \vartheta^1) = (\vartheta_1^1(t), \ldots, \vartheta_m^1(t))^T$, $\vartheta^2(t) = \vartheta(t; t_0, \vartheta^2) = (\vartheta_1^2(t), \ldots, \vartheta_m^2(t))^T$, then $\chi^1(t) \neq \chi^2(t)$, we have

$$\begin{aligned}
\varsigma_i^1(t) &= \varsigma_i^1 + \int_{t_0}^{t} a_i\left(\varsigma_i^1(s)\right)\left[-b_i\left(\varsigma_i^1(s)\right) + \sum_{j=1}^{m} k_{ij} f_j\left(\vartheta_j^1(\xi_k)\right) + I_i\right] \mathrm{d}s, \\
\varsigma_i^2(t) &= \varsigma_i^2 + \int_{t_0}^{t} a_i\left(\varsigma_i^1(s)\right)\left[-b_i\left(\varsigma_i^2(s)\right) + \sum_{j=1}^{m} k_{ij} f_j\left(\vartheta_j^2(\xi_k)\right) + I_i\right] \mathrm{d}s, \\
\vartheta^1(t) &= \vartheta_j^1 + \int_{t_0}^{t} a_j\left(\vartheta_j^1(s)\right)\left[-b_i\left(\vartheta_i^1(s)\right) + \sum_{j=1}^{m} k_{ij} f_i\left(\varsigma_i^1(\xi_k)\right) + I_j\right] \mathrm{d}s, \\
\vartheta^2(t) &= \vartheta_j^2 + \int_{t_0}^{t} a_j\left(\vartheta_j^1(s)\right)\left[-b_i\left(\vartheta_i^2(s)\right) + \sum_{j=1}^{m} k_{ij} f_i\left(\varsigma_i^2(\xi_k)\right) + I_j\right] \mathrm{d}s,
\end{aligned} \tag{50}$$

then

$$\begin{aligned}
\left|\varsigma_i^1(t) - \varsigma_i^2(t)\right| &\leq \left|\varsigma_i^1 - \varsigma_i^2\right| + \left|\int_{t_0}^{t} \left\{a_i\left(\varsigma_i^1(s)\right)\left[-b_i\left(\varsigma_i^1(s)\right) + \sum_{j=1}^{m} k_{ij} f_j\left(\vartheta_j^1(\xi_k)\right) + I_i\right] - a_i\left(\varsigma_i^2(s)\right)\left[-b_i\left(\varsigma_i^2(s)\right) + \sum_{j=1}^{m} k_{ij} f_j\left(\vartheta_j^2(\xi_k)\right) + I_i\right]\right\} \mathrm{d}s\right| \\
&= \left|\varsigma_i^1 - \varsigma_i^2\right| + \left|\int_{t_0}^{t} \left\{\left\{a_i\left(\varsigma_i^1(s)\right)\left[-b_i\left(\varsigma_i^1(s)\right) + \sum_{j=1}^{m} k_{ij} f_j\left(\vartheta_j^1(\xi_k)\right) + I_i\right] - a_i\left(\varsigma_i^2(s)\right)\left[-b_i\left(\varsigma_i^2(s)\right) + \sum_{j=1}^{m} k_{ij} f_j\left(\vartheta_j^2(\xi_k)\right) + I_i\right]\right.\right. \\
&\quad \left.\left. + a_i\left(\varsigma_i^2(s)\right)\left[-b_i\left(\varsigma_i^1(s)\right) + \sum_{j=1}^{m} k_{ij} f_j\left(\vartheta_j^1(\xi_k)\right) + I_i\right] - a_i\left(\varsigma_i^2(s)\right)\left[-b_i\left(\varsigma_i^1(s)\right) + \sum_{j=1}^{m} k_{ij} f_j\left(\vartheta_j^1(\xi_k)\right) + I_i\right] \mathrm{d}s \right\}\right| \\
&= \left|\varsigma_i^1 - \varsigma_i^2\right| + \left|\int_{t_0}^{t} \left\{\left(a_i\left(\varsigma_i^1(s)\right) - a_i\left(\varsigma_i^2(s)\right)\right)\left[-b_i\left(\varsigma_i^1(s)\right) + \sum_{j=1}^{m} k_{ij} f_j\left(\vartheta_j^1(\xi_k)\right) + I_i\right]\right.\right. \\
&\quad \left.\left. + a_i\left(\varsigma^2(s)\right)\left[-\left(b_i\left(\varsigma_i^1(s)\right) - b_i\left(\varsigma_i^2(s)\right)\right) + \sum_{j=1}^{m} k_{ij}\left(f_j\left(\vartheta_j^1(\xi_k)\right) - f_j\left(\vartheta_j^2(\xi_k)\right)\right)\right]\right\} \mathrm{d}s\right| \\
&\leq \left|\varsigma_i^1 - \varsigma_i^2\right| + \int_{t_0}^{t} \left(e_i P_i \left|\varsigma_i^1(s) - \varsigma_i^2(s)\right| + \overline{a}\overline{b}_i \left|\varsigma_i^1(s) - \varsigma_i^2(s)\right| + \overline{a} \sum_{j=1}^{m} \left|k_{ij}\right| F_j \left|\vartheta_j^1(\xi_k) - \vartheta_j^2(\xi_k)\right|\right) \\
&\leq \left|\varsigma_i^1 - \varsigma_i^2\right| + \int_{t_0}^{t} \left[\left(e_i P_i + \overline{a}\overline{b}_i\right)\left|\varsigma_i^1(s) - \varsigma_i^2(s)\right| \mathrm{d}s + \overline{a} \sum_{j=1}^{m} \left|k_{ij}\right| F_j \left|\vartheta_j^1(\xi_k) - \vartheta_j^2(\xi_k)\right|\right] \mathrm{d}s,
\end{aligned} \tag{51}$$

similarly,

$$\left|\vartheta_j^1(t) - \vartheta_j^2(t)\right| \leq \left|\vartheta_j^1 - \vartheta_j^2\right| + \int_{t_0}^{t} \left[\left(l_i Q_j + \overline{c}\overline{d}_i\right)\left|\vartheta_j^1(s) - \vartheta_j^2(s)\right| + \overline{c} \sum_{i=1}^{n} \left|w_{ji}\right| G_i \left|\varsigma_i^1(\xi_k) - \varsigma_i^2(\xi_k)\right|\right] \mathrm{d}s, \tag{52}$$

we obtain

$$\begin{aligned}&\|\varsigma^1(t)-\varsigma^2(t)\|\le\|\varsigma^1-\varsigma^2\|+\int_{t_0}^{t}\left(\varepsilon_1\|\varsigma^1(s)-\varsigma^2(s)\|+\tau_1\|\vartheta^1(\xi_k)-\vartheta^2(\xi_k)\|\right)\mathrm{d}s,\\&\|\vartheta^1(t)-\vartheta^2(t)\|\le\|\vartheta^1-\vartheta^2\|+\int_{t_0}^{t}\left(\varepsilon_2\|\vartheta^1(s)-\vartheta^2(s)\|\mathrm{d}s+\tau_2\|\varsigma^1(\xi_k)-\varsigma^2(\xi_k)\|\right)\mathrm{d}s.\end{aligned}\tag{53}$$

Then,

$$\begin{aligned}&\|\varsigma^1(t)-\varsigma^2(t)\|+\|\vartheta^1(t)-\vartheta^2(t)\|\\&\le\|\varsigma^1-\varsigma^2\|+\|\vartheta^1-\vartheta^2\|+\int_{t_0}^{t}\left[\varepsilon\left(\|\varsigma^1(s)-\varsigma^2(s)\|+\|\vartheta^1(s)-\vartheta^2(s)\|\right)+\tau\left(\|\varsigma^1(\xi_k)-\varsigma^2(\xi_k)\|+\|\vartheta^1(\xi_k)-\vartheta^2(\xi_k)\|\right)\right]\mathrm{d}s.\end{aligned}\tag{54}$$

Using the Lemma 9, we have

$$\|\varsigma^1(t)-\varsigma^2(t)\|+\|\vartheta^1(t)-\vartheta^2(t)\|\le\left(\|\varsigma^1-\varsigma^2\|+\|\vartheta^1-\vartheta^2\|\right)e^{\left(\varepsilon+\tau e^{\varepsilon\theta}/1-\nu\right)\theta},\tag{55}$$

where $\int_{t_i}^{\xi_i}(\tau e^{\int_s^{\xi_i}\varepsilon \mathrm{d}k})\mathrm{d}s\le\nu<1$.

In particular,

$$\|\varsigma^1(\xi_k)-\varsigma^2(\xi_k)\|+\|\vartheta^1(\xi_k)-\vartheta^2(\xi_k)\|\le\left(\|\varsigma^1-\varsigma^2\|+\|\vartheta^1-\vartheta^2\|\right)e^{\varepsilon\theta}.\tag{56}$$

By contrary, assume there exists $t\in[\theta_k,\theta_{k+1}]$ such that $\chi^1(t)=\chi^2(t)$, then

$$\begin{aligned}&\|\varsigma^1-\varsigma^2\|\le\int_{t_0}^{t}\left(\varepsilon_1\|\varsigma^1(s)-\varsigma^2(s)\|+\tau_1\|\vartheta^1(\xi_k)-\vartheta^2(\xi_k)\|\right)\mathrm{d}s,\\&\|\vartheta^1-\vartheta^2\|\le\int_{t_0}^{t}\left(\varepsilon_2\|\vartheta^1(s)-\vartheta^2(s)\|+\tau_2\|\varsigma^1(\xi_k)-\varsigma^2(\xi_k)\|\right)\mathrm{d}s.\end{aligned}\tag{57}$$

So

$$\|\varsigma^1-\varsigma^2\|+\|\vartheta^1-\vartheta^2\|+\int_{t_0}^{t}\left[\varepsilon\left(\|\varsigma^1(s)-\varsigma^2(s)\|+\|\vartheta^1(s)-\vartheta^2(s)\|\right)+\tau\left(\|\varsigma^1(\xi_k)-\varsigma^2(\xi_k)\|+\|\vartheta^1(\xi_k)-\vartheta^2(\xi_k)\|\right)\right]\mathrm{d}s.\tag{58}$$

From (56) and (58), we obtain

$$\|\varsigma^1-\varsigma^2\|+\|\vartheta^1-\vartheta^2\|\le\varepsilon\theta\left(\|\varsigma^1(s)-\varsigma^2(s)\|+\|\vartheta^1(s)-\vartheta^2(s)\|\right)+\theta\tau e^{\varepsilon\theta}\times\left(\|\varsigma^1-\varsigma^2\|+\|\vartheta^1-\vartheta^2\|\right),\tag{59}$$

that is

$$\|\varsigma^1-\varsigma^2\|+\|\vartheta^1-\vartheta^2\|\le\left(\varepsilon e^{\left(\varepsilon+\tau e^{\varepsilon\theta}/1-\nu\right)\theta}+\tau e^{\varepsilon\theta}\right)\theta\left(\|\varsigma^1-\varsigma^2\|+\|\vartheta^1-\vartheta^2\|\right).\tag{60}$$

Hence, we can see that (A6) and inequality (60) are contradictory. For all $t \in [\theta_k, \theta_{k+1}]$, the uniqueness of the solution is proved. □

Remark 13. In Theorem 12, we use a global Gronwall-type lemma to obtain a weaker condition to guarantee the existence and uniqueness of the solution. Since $\int_{t_i}^{\xi_i} [z_2(s) \exp\left\{\int_t^{\xi_i} z_1(k)\mathrm{d}k\right\}]\mathrm{d}s < 1$ is weaker than $\int_{t_i}^{\xi_i} [z_1(s) + z_2(s)]\mathrm{d}s < 1$, Lemma 9 in this paper generalizes the lemmas in [62–65], and the conditions of Theorem 12 are wider than the conditions in [52, 53].

To establish the criteria for GES of the equilibrium point, we translate the equilibrium point to the origin. Let $\zeta(t) = \varsigma(t) - \varsigma^*$, $\varrho(t) = \vartheta(t) - \vartheta^*$, (1) can be converted into:

$$\begin{aligned}
\frac{\mathrm{d}\zeta_i}{\mathrm{d}t} &= \alpha_i(\zeta_i(t))\left[-\beta_i(\zeta_i(t)) + \sum_{j=1}^{m} k_{ij}\varphi_j\left(\varrho_j(\eta(t))\right)\right],\\
\frac{\mathrm{d}\varrho_j}{\mathrm{d}t} &= \mu_j\left(\varrho_j(t)\right)\left[-\nu_j\left(\varrho_j(t)\right) + \sum_{i=1}^{n} w_{ji}\psi_i(\zeta_i(\eta(t)))\right].
\end{aligned} \tag{61}$$

where

$$\begin{aligned}
\alpha_i(\zeta_i(t)) &= a_i(\zeta_i(t) + \varsigma_i^*),\\
\beta_i(\zeta_i(t)) &= b_i(\zeta_i(t) + \varsigma_i^*) - b_i(\varsigma_i^*),\\
\varphi_j\left(\varrho_j(\eta(t))\right) &= f_j\left(\varrho_j(\eta(t)) + \vartheta_j^*\right) - f_j\left(\vartheta_j^*\right),\\
\mu_j\left(\varrho_j(t)\right) &= c_j\left(\varrho_j(t) + \vartheta^*\right),\\
\nu_j\left(\varrho_j(t)\right) &= d_j\left(\varrho_j(t) + \vartheta_j^*\right) - d_j\left(\vartheta_j^*\right),\\
\psi_i(\zeta_i(\eta(t))) &= g_i(\zeta_i(\eta(t)) + \varsigma_i^*) - g_i(\varsigma_i^*).
\end{aligned} \tag{62}$$

Assumption (A1)–(A3) imply that

$$\begin{aligned}
&\underline{a} < \alpha_i(\zeta) \le \overline{a},\\
&\underline{c} < \mu_j(\varrho) \le \overline{c},\\
&\left|\alpha_i(\zeta) - \alpha_i(\varrho)\right| \le e_i|\zeta - \varrho|,\\
&\left|\mu_j(\zeta) - \mu_j(\varrho)\right| \le l_j|\zeta - \varrho|,\\
&\underline{b}_i \le \frac{\beta_i(\varsigma_i^*) - \beta_i(\varsigma_i)}{\varsigma_i^* - \varsigma_i} \le \overline{b}_i,\\
&\underline{d}_j \le \frac{\nu_j\left(\vartheta_j^*\right) - \nu_j\left(\vartheta_j\right)}{\vartheta_j^* - \vartheta_j} \le \overline{d}_j,\\
&0 \le \frac{\varphi_j\left(\vartheta_j^*\right) - \varphi_j\left(\vartheta_j\right)}{\vartheta_j^* - \vartheta_j} \le F_j,\\
&0 \le \frac{\psi_i(\varsigma_i^*) - \psi_i(\varsigma_i)}{\varsigma_i^* - \varsigma_i} \le G_i.
\end{aligned} \tag{63}$$

Obviously, (61) has the same stability for the zero solution as (1)'s equilibrium point. Thereby, we will explore the stability of the zero solution of (24).

A significant auxiliary content of this article is the following lemma.

Lemma 14. *Assume (A1)–(A7) be satisfied, $z(t)$ be a solution of (60), $z(t) = (\zeta(t), \varrho(t))^T$. Then, for any $t \in R^+$, the following inequality*

$$\|\zeta(\eta(t))\| + \|\varrho(\eta(t))\| \le \lambda(\|\zeta(t)\| + \|\varrho(t)\|), \tag{64}$$

holds, where $\lambda = (1 - \tau\theta - (1 + \tau\theta)m_1\theta e^{m_1\theta})^{-1}$.

Proof. Fix $k \in N$,

$$\begin{aligned} \zeta(t) &= \zeta(\xi_k) + \int_{\xi_k}^{t} \alpha_i(\zeta_i(s))\left[-\beta_i(\zeta_i(s)) + \sum_{j=1}^{m} k_{ij}\varphi_j(\varrho_j(\xi_k))\right]\mathrm{d}s, \\ \varrho(t) &= \varrho(\xi_k) + \int_{\xi_k}^{t} \mu_j(\varrho_j(s))\left[-\nu_j(\varrho_j(s)) + \sum_{i=1}^{n} w_{ji}\psi_i(\zeta_i(\xi_k))\right]\mathrm{d}s, \end{aligned} \tag{65}$$

for any $t \in [\theta_k, \theta_{k+1}]$, then

$$\begin{aligned} \|\zeta(t)\| &\le \|\zeta(\xi_k)\| + \sum_{i=1}^{n}\left|\int_{\xi_k}^{t} \alpha_i(\zeta_i(s))\left[-\beta_i(\zeta_i(s)) + \sum_{j=1}^{m} k_{ij}\varphi_j(\varrho_j(\xi_k))\right]\mathrm{d}s\right| \\ &\le \|\zeta(\xi_k)\| + \sum_{i=1}^{n}\int_{\xi_k}^{t} \bar{a}\bar{b}_i|\zeta_i(s)|\mathrm{d}s + \sum_{i=1}^{n}\sum_{j=1}^{m}\int_{\xi_k}^{t} \bar{a}|k_{ij}| \times F_j|\varrho_j(\xi_k)|\mathrm{d}s \le \|\zeta(\xi_k)\| + \bar{a}\bar{b}_i\int_{\xi_k}^{t}\|\zeta(s)\|\mathrm{d}s + \sum_{j=1}^{m}\theta\bar{a}|k_{ij}|F_j\|\varrho(\xi_k)\|, \\ \|\varrho(t)\| &\le \|\varrho(\xi_k)\| + \sum_{j=1}^{m}\left|\int_{\xi_k}^{t} \mu_j(\varrho_j(s))\left[-\nu_j(\varrho_j(s)) + \sum_{i=1}^{n}|w_{ji}|\psi_i(\zeta_i(\xi_k))\right]\mathrm{d}s\right| \\ &\le \|\varrho(\xi_k)\| + \sum_{i=1}^{n}\int_{\xi_k}^{t} \bar{c}\bar{d}_j|\varrho_j(s)|\mathrm{d}s + \sum_{i=1}^{n}\sum_{j=1}^{m}\int_{\xi_k}^{t} \bar{c}|w_{ji}| \times G_i|\zeta_i(\xi_k)|\mathrm{d}s \le \|\varrho(\xi_k)\| + \bar{c}\bar{d}_j\int_{\xi_k}^{t}\|\varrho(s)\|\mathrm{d}s + \sum_{j=1}^{m}\theta\bar{c}|w_{ji}|G_i\|\zeta(\xi_k)\|, \end{aligned} \tag{66}$$

thus

$$\begin{aligned} \|\zeta(t)\| + \|\varrho(t)\| \le{}& \|\zeta(\xi_k)\| + \|\varrho(\xi_k)\| \\ &+ \bar{a}\bar{b}_i\int_{\xi_k}^{t}\|\zeta(s)\|\mathrm{d}s + \sum_{j=1}^{m}\theta\bar{a}|k_{ij}|F_j\|\varrho(\xi_k)\| \\ &+ \bar{c}\bar{d}_j\int_{\xi_k}^{t}\|\varrho(s)\|\mathrm{d}s + \sum_{i=1}^{n}\theta\bar{c}|w_{ji}|G_i\|\zeta(\xi_k)\| \\ \le{}& \|\zeta(\xi_k)\| + \|\varrho(\xi_k)\| + m_1\int_{\xi_k}^{t}(\|\zeta(s)\| + \|\varrho(s)\|)\mathrm{d}s \\ &+ \theta\tau(\|\zeta(\xi_k)\| + \|\varrho(\xi_k)\|) \\ ={}& (1 + \theta\tau)(\|\zeta(\xi_k)\| + \|\varrho(\xi_k)\|) \\ &+ m_1\int_{\xi_k}^{t}(\|\zeta(s)\| + \|\varrho(s)\|)\mathrm{d}s. \end{aligned} \tag{67}$$

From the Gronwall–Bellman Lemma, we obtain

$$\|\zeta(t)\| + \|\varrho(t)\| \le (1 + \theta\tau)e^{m_1\theta}(\|\zeta(\xi_k)\| + \|\varrho(\xi_k)\|). \tag{68}$$

Similarly,

$$\begin{aligned}\left\|\zeta(\xi_k)\right\|+\left\|\varrho(\xi_k)\right\| \le{}& \|\zeta(t)\|+\|\varrho(t)\| \\ &+\overline{a}\overline{b}_i\int_{\xi_k}^{t}\|\zeta(s)\|\mathrm{d}s+\sum_{j=1}^{m}\theta\overline{a}\left|k_{ij}\right|F_j\left\|\varrho(\xi_k)\right\| \\ &+\overline{c}\overline{d}_j\int_{\xi_k}^{t}\|\varrho(s)\|\mathrm{d}s+\sum_{i=1}^{n}\theta\overline{c}\left|w_{ji}\right|G_i\left\|\zeta(\xi_k)\right\| \\ \le{}& \|\zeta(t)\|+\|\varrho(t)\|+m_1\int_{\xi_k}^{t}(\|\zeta(s)\|+\|\varrho(s)\|)\mathrm{d}s \\ &+\theta\tau\left(\left\|\zeta(\xi_k)\right\|+\left\|\varrho(\xi_k)\right\|\right),\end{aligned} \tag{69}$$

together (68) and (69), we have

$$\begin{aligned}\left\|\zeta(\xi_k)\right\|+\left\|\varrho(\xi_k)\right\|\le{}&\|\zeta(t)\|+\|\varrho(t)\| \\ &+\theta\tau\left(\left\|\zeta(\xi_k)\right\|+\left\|\varrho(\xi_k)\right\|\right) \\ &+m_1\theta(1+\tau\theta)e^{m_1\theta}\left(\left\|\zeta(\xi_k)\right\|+\left\|\varrho(\xi_k)\right\|\right).\end{aligned} \tag{70}$$

Thus, from assumption (A7), for $t\in[\theta_k,\theta_{k+1}]$, we obtain

$$\left\|\zeta(\xi_k)\right\|+\left\|\varrho(\xi_k)\right\|\le\lambda(\|\zeta(t)\|+\|\varrho(t)\|). \tag{71}$$

where $\lambda=(1-\tau\theta-(1+\tau\theta)m_1\theta e^{m_1\theta})^{-1}$.

So (64) holds for any $t\in R^+$. □

Theorem 15. *Suppose assumptions (A1)–(A3) hold. Then, the system (61) is globally exponentially stable, if the following conditions hold:*

$$\begin{aligned}&m_3\lambda<m_2,\\ &\beta\le\min\{\underline{a}(m_2-m_3\lambda),\underline{c}(m_2-m_3\lambda)\},\end{aligned} \tag{72}$$

where $m_2=\min\{\underline{b}_i,\underline{d}_j\}$, $m_3=\max\{\sum_{j=1}^{m}k_{ij}F_j,\sum_{i=1}^{n}w_{ji}G_i\}$, *β is a positive number.*

Proof. Construct the following Lyapunov functional:

$$\begin{aligned}V(t)={}&\sum_{i=1}^{n}\operatorname{sgn}(\zeta_i(t))\int_0^{\varrho_j(t)}\frac{1}{\alpha_i(s)}\mathrm{d}s \\ &+\sum_{j=1}^{m}\operatorname{sgn}\left(\varrho_j(t)\right)\times\int_0^{\varrho_j(t)}\frac{1}{\mu_j(s)}\mathrm{d}s,\end{aligned} \tag{73}$$

then

$$\frac{1}{\overline{a}}\|\zeta(t)\|+\frac{1}{\overline{c}}\|\varrho(t)\|\le V(t)\le\frac{1}{\underline{a}}\|\zeta(t)\|+\frac{1}{\underline{c}}\|\varrho(t)\|. \tag{74}$$

When $t\ne\theta_k$, the derivative of $V(t)$ satisfies

$$\begin{aligned}\dot V(t)={}&\sum_{i=1}^{n}\operatorname{sgn}\zeta_i(t)\left[-\beta_i(\zeta_i(t))+\sum_{j=1}^{m}k_{ij}\varphi_j\left(\varrho_j(\eta(t))\right)\right] \\ &+\sum_{j=1}^{m}\operatorname{sgn}\varrho_j(t)\left[-\mu_j\left(\varrho_j(t)\right)+\sum_{i=1}^{n}w_{ji}\psi_i(\zeta_i(\eta(t)))\right] \\ \le{}&-\sum_{i=1}^{n}\underline{b}_i\left|\zeta_i(t)\right|+\sum_{i=1}^{n}\sum_{j=1}^{m}k_{ij}F_j\left|\varrho_j(\eta(t))\right| \\ &-\sum_{j=1}^{m}\underline{d}_j\left|\varrho_j(t)\right|+\sum_{j=1}^{m}\sum_{i=1}^{n}w_{ji}G_i\left|\zeta_i(\eta(t))\right| \\ \le{}&-\min\left\{\underline{b}_i,\underline{d}_j\right\}(\|\zeta(t)\|+\|\varrho(t)\|) \\ &+\max\left\{\sum_{j=1}^{m}k_{ij}F_j,\sum_{i=1}^{n}w_{ji}G_i\right\}(\|\zeta(\eta(t))\|+\|\varrho(\eta(t))\|) \\ ={}&-m_2(\|\zeta(t)\|+\|\varrho(t)\|)+m_3(\|\zeta(\eta(t))\|+\|\varrho(\eta(t))\|).\end{aligned} \tag{75}$$

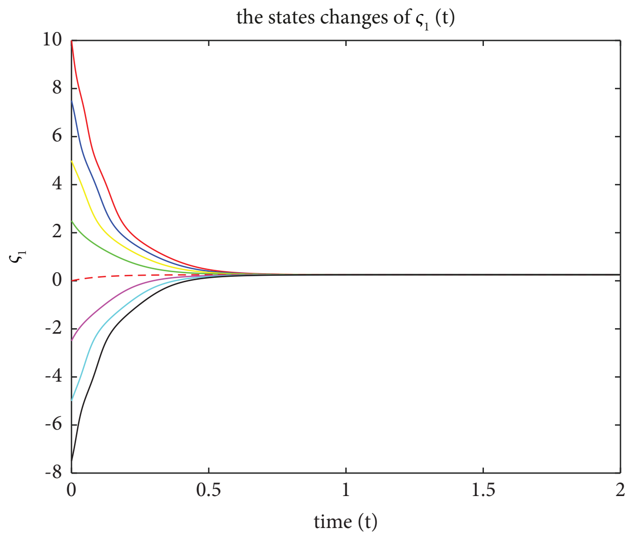

Figure 1: The states of $\varsigma_1(t)$ of BAMCGNN with PCA.

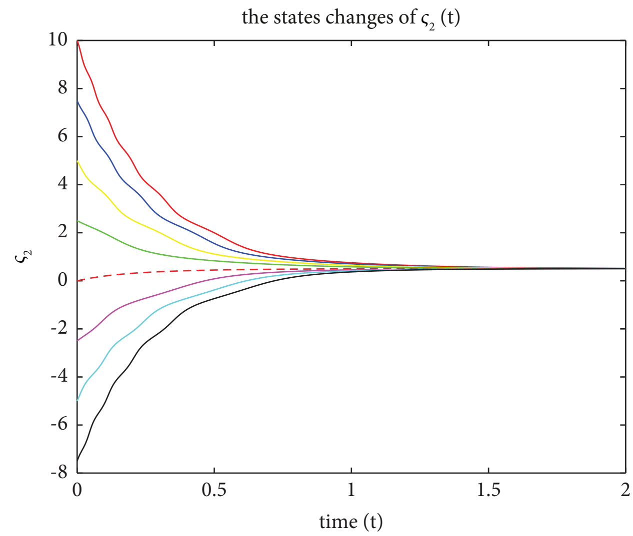

Figure 2: The states of $\varsigma_2(t)$ of BAMCGNN with PCA.

From Lemma 14, we obtain

$$\dot{V}(t) \leq -m_2(\|\zeta(t) + \|\varrho(t)\|) + m_3\lambda(\|\zeta(t)\| + \|\varrho(t)\|) = -(m_2 - m_3\lambda)(\|\zeta(t)\| + \|\varrho(t)\|). \quad (76)$$

Denote σ for convenience as following:

$$\sigma = m_2 - m_3\lambda > 0. \quad (77)$$

Then, for $t \neq \theta_k$ and exist a positive number $\beta \leq \min\{\underline{a}(m_2 - m_3\lambda), \underline{c}(m_2 - m_3\lambda)\}$, we obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(e^{\beta t}V(t)\right) = \beta e^{\beta t}V(t) + e^{\beta t}\dot{V}(t) \leq \beta e^{\beta t}\left(\frac{1}{\underline{a}}\|\zeta(t)\| + \frac{1}{\underline{c}}\|\varrho(t)\|\right) - e^{\beta t}\sigma(\|\zeta(t)\| + \|\varrho(t)\|) \leq 0. \quad (78)$$

Since the continuity of the solution $(\zeta(t), \varrho(t))^T$ and the function V, we have

$$e^{\beta t}V(t) \leq e^{\beta t_0}V(t_0), \quad (79)$$

then

$$e^{\beta t}\left(\frac{1}{\overline{a}}\|\zeta(t)\| + \frac{1}{\overline{c}}\|\varrho(t)\|\right) \leq e^{\beta t_0}\left(\frac{1}{\underline{a}}\|\zeta(t_0)\| + \frac{1}{\underline{c}}\|\varrho(t_0)\|\right), \quad (80)$$

let $m_4 = \min\{1/\overline{a}, 1/\overline{c}\}$, $m_5 = \max\{1/\overline{a}, 1/\overline{c}\}$, so

$$\|\zeta(t)\| + \|\varrho(t)\| \leq \frac{m_5}{m_4}e^{-(\beta(t-t_0))}\left(\|\zeta(t_0)\| + \|\varrho(t_0)\|\right). \quad (81)$$

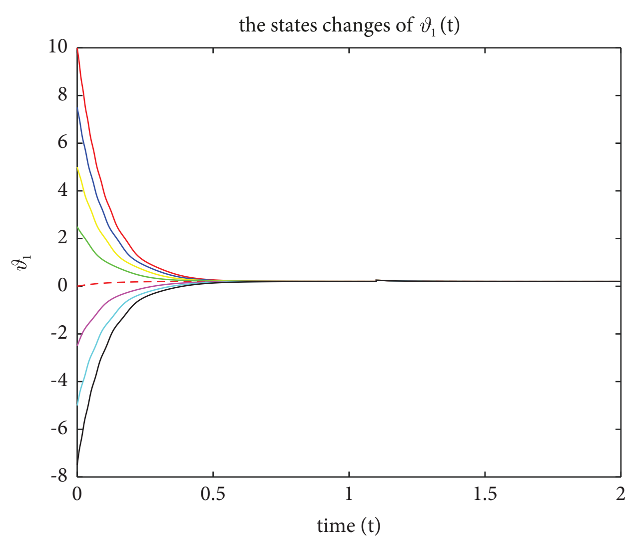

Figure 3: The states of $\vartheta_1(t)$ of BAMCGNN with PCA.

Hence, the GES of the system (61) is proved, it implies that BAMCGNN (1) is globally exponentially stable. □

Remark 16. The main tools for studying the stability of the system generally include Lyapunov's first method and Lyapunov's second method. With the introduction of dynamic system research and the birth of modern control theory by state-space analysis method, Lyapunov's second method has attracted the attention of people in the field of control and has gradually become the most important method for studying stability. In Theorem 15, we apply Lyapunov's second method to establish stability criteria. By constructing a suitable Lyapunov function with symbolic function and using inequality techniques, sufficient conditions for the GES of BAMCGNN are derived.

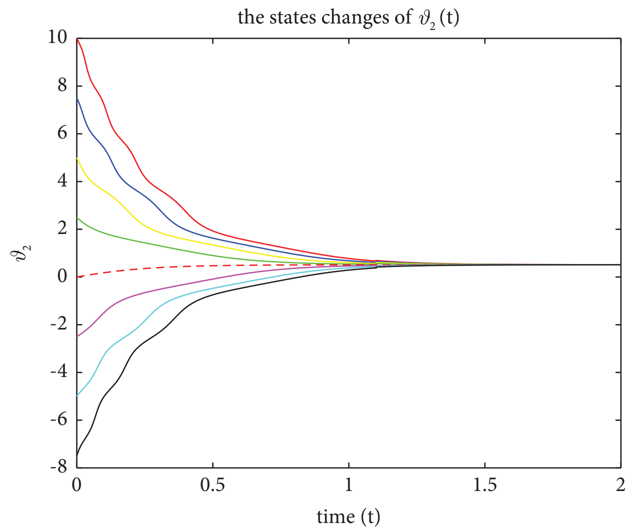

Figure 4: The states of $\vartheta_2(t)$ of BAMCGNN with PCA.

3. Conclusion

This paper introduces a class of BAMCGNN with PCA. Firstly, by using the homeomorphic mapping theorem, we obtain sufficient conditions for the existence and uniqueness of the solution. Then, by constructing a Lyapunov function, stability criteria for GES of BAMCGNN with PCA are derived. In this process, we estimate the norm of the piecewise constant state and reveal the relationship between the piecewise constant state and the current state and then discuss the stability of BAMCGNN with PCA. In the future, we can consider the robustness of the GES of BAMCGNN with PCA or the robustness of the GES of CGNN disturbed by various factors and further investigate the dynamic behaviors of CGNN.

4. Numerical Examples

Example 17. Consider a two dimensional BAMCGNN (1), the associated parameters of BAMCGNN are follows:

$$K = \begin{pmatrix} 0.01 & 0.02 \\ 0.02 & 0.06 \end{pmatrix}, \quad W = \begin{pmatrix} 0.01 & 0.03 \\ 0.01 & 0.05 \end{pmatrix}. \tag{82}$$

Take the values:

$P = Q = 2$, $\underline{a} = \underline{c} = 1$, $\overline{a} = \overline{c} = 2$, $e_1 = e_2 = l_1 = l_2 = 0.5$, $\overline{b}_1 = \overline{b}_2 = \overline{d}_1 = \overline{d}_2 = 1$, $\underline{b}_1 = \underline{b}_2 = \underline{d}_1 = \underline{d}_2 = 0.5$, $F_1 = F_2 = G_1 = G_2 = 1$, and

$$\begin{aligned} a_1(\varsigma_1(t)) &= 2 + 0.5\cos(2\varsigma_1(t)), \\ a_2(\varsigma_2(t)) &= 2 + 0.6\cos(4\varsigma_2(t)), \\ b_1(\varsigma_1(t)) &= 2\varsigma_1(t), \underline{b}_2(\underline{\varsigma}_2(t)) \\ &= 2\varsigma_2(t), \\ c_1(\vartheta_1(t)) &= 2 + 0.4\sin(5\vartheta_1(t)), \\ c_2(\vartheta_2(t)) &= 2 + 0.8\sin(3\vartheta_2(t)), \\ d_1(\vartheta_1(t)) &= 5\vartheta_1(t), \\ d_2(\vartheta_2(t)) &= 2\vartheta_2(t), \\ I_i &= J_j = 1, \\ i &= 1, 2, j = 1, 2. \end{aligned} \tag{83}$$

The following activation functions are playing in BAMCGNN:

$$\begin{aligned} f(\vartheta(t)) &= \tanh(\vartheta(t)), \\ g(\varsigma(t)) &= \tanh(\varsigma(t)). \end{aligned} \tag{84}$$

That is,

$$\begin{aligned} \frac{d\varsigma_1(t)}{dt} &= (2 + 0.5\cos(2\varsigma_1(t)))\left[-2\varsigma_1(t) + \sum_{j=1}^{2} k_{ij}\tanh(\vartheta_j(\xi_k)) + 1\right], \\ \frac{d\varsigma_2(t)}{dt} &= (2 + 0.6\cos(4\varsigma_2(t)))\left[-2\varsigma_2(t) + \sum_{j=1}^{2} k_{ij}\tanh(\vartheta_j(\xi_k)) + 1\right], \\ \frac{d\vartheta_1(t)}{dt} &= (2 + 0.4\sin(5\vartheta_1(t)))\left[-5\vartheta_1(t) + \sum_{i=1}^{2} w_{ji}\tanh(\varsigma_i(\xi_k)) + 1\right], \\ \frac{d\vartheta_2(t)}{dt} &= (2 + 0.8\sin(3\vartheta_2(t)))\left[-2\vartheta_2(t) + \sum_{i=1}^{2} w_{ji}\tanh(\varsigma_i(\xi_k)) + 1\right], \end{aligned} \tag{85}$$

where $\xi_k = 2k + 1/18$ when $t \in [\theta_k, \theta_{k+1}]$, and $\theta_k = k/9$, $k \in N$.

By calculations, we obtain

$$\begin{aligned} \varepsilon &= 3, \\ \tau &= 0.22, \\ m_1 &= 2, \\ m_2 &= 0.5, \\ m_3 &= 0.3, \\ m_4 &= 0.5, \\ m_5 &= 1, \\ R &\approx 0.3577 < 1, \\ \left(\varepsilon e^{\left(\varepsilon + \tau e^{\varepsilon\theta}/1-v\right)\theta} + \tau e^{\varepsilon\theta}\right)\theta &\approx 0.5161 < 1, \\ \tau\theta + (1+\tau\theta) m_1 \theta e^{m_1\theta} &\approx 0.7033 < 1, \\ m_2 &= 0.5 > m_3\lambda \approx 0.3707. \end{aligned} \tag{86}$$

Obviously, (A5)–(A7) are satisfied, by Theorems 11 and 12, the equilibrium point and the solution of the above model exist uniquely. According to Theorem 15, it shows that the BAMCGNN with PCA is global exponentially stable. Simulation results of the stable state trajectory can be shown in the following figures.

Remark 18. The above-given calculations confirm that the conditions in the obtained results are valid for this example, and Figures 1–4 show the stable state trajectories of BAMCGNN. Since the GES of the model in this paper cannot be demonstrated by the previous conditions, and if $F_\varsigma = F_\vartheta$, we take $a_i(\varsigma_i(t)) = c_j(\vartheta_j(t)) = 1$, $b_i(\varsigma_i(t)) = d_j(\vartheta_j(t)) = h_i\varsigma_i$, and $f_j = g_i = \tanh$, then the BAMCGNN can be reduced to HNN, that is, CGNN includes HNN and CNN as its special cases, so the results of this paper are more general. This shows the advantages of our results.

Appendix

We write the following MATLAB program with reference to literature [57], which is divided into three.m files, i.e., BAMCGNNFUNC.m, BAMCGNNLOOP.m, and BAMCGNNALL.m, where BAMCGNNLOOP.m invoked BAMCGNNFUNC.m and looped 30 times, and BAMCGNNALL.m invoked BAMCGNNLOOP.m. Finally, we can obtain the simulation result. The details are as follows:

BAMCGNNFUNC.m

```
function dxdt = BAMCGNNs_func (t, x, p1, p2, p3, p4)
dxdt (1, 1) = (2 + cos (2*x(1)))*(−4*x(1) + 0.01*tanh (p1) + 0.02*tanh (p2) + 1); dxdt (2, 1) = (2 + cos (4*x(2)))*(−2*x(2) + 0.02*tanh (p1) + 0.06*tanh (p2) + 1); dxdt (3, 1) = (2 + sin (5*x(3)))*(−5*x(3) + 0.01*tanh (p3) + 0.03*tanh (p4) + 1); dxdt (4, 1) = (2 + sin (3*x(4)))*(−2*x(4) + 0.01*tanh (p3) + 0.05*tanh (p4) + 1);
```

BAMCGNNLOOP.m

```
function [total_tt, total_dxdt] = BAMCGNNs_loop (x01, x02, x03, x04)
p1 = x01;
p2 = x02;
p3 = x03;
p4 = x04;
for j = 1 : 1 : 30
for i = 1 : 1 : 500
[tt1, dxdt1] = ode45(@BAMCGNNs_func, [0/9, 1/9], [p1, p2, p3, p4], [], p1, p2, p3, p4);
end
a = dxdt1 (:, 1); p1 = a (end);
b = dxdt1 (:, 2); p2 = b (end);
c = dxdt1 (:, 3); p3 = c (end);
d = dxdt1 (:, 4); p4 = d (end);
total_tt = [total_tt; tt1];
total_dxdt = [total_dxdt; dxdt1];
end
```

BAMCGNNALL.m

```
x10 = [10; 7.5; 5;2.5; 0; −2.5; −5; −7.5; −10];
x20 = [10; 7.5; 5;2.5; 0; −2.5; −5; −7.5; −10];
x30 = [10; 7.5; 5;2.5; 0; −2.5; −5; −7.5; −10];
x40 = [10; 7.5; 5;2.5; 0; −2.5; −5; −7.5; −10];
[tk1, dxdtk1] = BAMCGNNs_loop(x10(1), x20(1), x30(1), x40(1));
[tk2, dxdtk2] = BAMCGNNs_loop(x10(2), x20(2), x30(2), x40(2));
[tk3, dxdtk3] = BAMCGNNs_loop(x10(3), x20(3), x30(3), x40(3));
[tk4, dxdtk4] = BAMCGNNs_loop(x10(4), x20(4), x30(4), x40(4));
[tk5, dxdtk5] = BAMCGNNs_loop(x10(5), x20(5), x30(5), x40(5));
[tk6, dxdtk6] = BAMCGNNs_loop(x10(6), x20(6), x30(6), x40(6));
[tk7, dxdtk7] = BAMCGNNs_loop(x10(7), x20(7), x30(7), x40(7));
[tk8, dxdtk8] = BAMCGNNs_loop(x10(8), x20(8), x30(8), x40(8));
plot (tk1, dxdtk1 (:, 1), “r,” tk2, dxdtk2 (:, 1), “b,” tk3, dxdtk3 (:, 1), “y,” tk4, dxdtk4 (:, 1), “g,” tk5, dxdtk5 (:, 1), “r−−,” tk6, dxdtk6 (:, 1), “m,” tk7, dxdtk7 (:, 1), ‘c”, tk8, dxdtk8 (:, 1), “k”);
xlabel (“timet”);
ylabel (“x_1”);
figure;
plot (tk1, dxdtk1 (:, 2), “r,” tk2, dxdtk2 (:, 2), “b,” tk3, dxdtk3 (:, 2), “y,” tk4, dxdtk4 (:, 2), “g,” tk5, dxdtk5 (:, 2), “r−−,” tk6, dxdtk6 (:, 2), “m,” tk7, dxdtk7 (:, 2), “c,” tk8, dxdtk8 (:, 2), “k”);
xlabel (‘timet”);
ylabel (“x_2”);
```

figure;

plot (tk1, d$x$dtk1 (:, 3), "r," tk2, d$x$dtk2 (:, 3), "b," tk3, d$x$dtk3 (:, 3), "y," tk4, d$x$dtk4 (:, 3), "g," tk5, d$x$dtk5 (:, 3), "r−−," tk6, d$x$dtk6 (:, 3), "m," tk7, d$x$dtk7 (:, 3), "c," tk8, d$x$dtk8 (:, 3), "k");

xlabel ("timet");

ylabel ("y_1");

figure;

plot (tk1, dxdtk1 (:, 4), "r," tk2, dxdtk2 (:, 4), "b," tk3, dxdtk3 (:, 4), "y," tk4, dxdtk4 (:, 4), "g," tk5, dxdtk5 (:, 4), "r−−," tk6, dxdtk6 (:, 4), "m," tk7, dxdtk7 (:, 4), "c," tk8, dxdtk8 (:, 4), "k");

xlabel ("timet");

ylabel ("y_2");

figure;

Data Availability

No data were generated or analyzed during the current study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

The study was supported by NSFC 12101213 of China.

References

[1] S. Haykin, *Neural Networks*, Prentice-Hall, Englewood Cliffs, NJ, USA, 1994.

[2] J. D. Cao and J. Wang, "Global asymptotic and robust stability of recurrent neural networks with time delays," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 52, no. 2, pp. 417–426, 2005.

[3] S. Arik, "Global asymptotic stability of a class of dynamical neural networks," *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, vol. 47, no. 4, pp. 568–571, 2000.

[4] L. Chua and L. Yang, "Cellular neural networks: applications," *IEEE Transactions on Circuits and Systems*, vol. 35, no. 10, pp. 1273–1290, 1988.

[5] F. Kong, Q. Zhu, and T. Huang, "New fixed-time stability lemmas and applications to the discontinuous fuzzy inertial neural networks," *IEEE Transactions on Fuzzy Systems*, vol. 29, pp. 3711–3722, 2021.

[6] G. Narayanan, M. Syed Ali, H. Alsulami, B. Ahmad, and J. J. Trujillo, "A hybrid impulsive and sampled-data control for fractional-order delayed reaction-diffusion system of mrna and protein in regulatory mechanisms," *Communications in Nonlinear Science and Numerical Simulation*, vol. 111, Article ID 106374, 2022.

[7] T. Wilusz, "Neural networks a comprehensive foundation," *Neurocomputing*, vol. 8, no. 3, pp. 359-360, 1995.

[8] E. Domany, J. Hemmen, and K. Schulten, *Models of Neural Networks: Temporal Aspects of Coding and Information Processing in Biological Systems*, Springer, Heidelberg, Germany, 1994.

[9] W. E. Faller and S. J. Schreck, "Neural networks: applications and opportunities in aeronautics," *Progress in Aerospace Sciences*, vol. 32, no. 5, pp. 433–456, 1996.

[10] M. A. Cohen and S. Grossberg, "Absolute stability of global pattern formation and parallel memory storage by competitive neural networks," *IEEE Transactions on Systems, Man, and Cybernetics*, vol. 13, no. 5, pp. 815–826, 1983.

[11] K. Gopalsamy, "Global asymptotic stability in a periodic Lotka-Volterra system," *The Journal of the Australian Mathematical Society. Series B. Applied Mathematics*, vol. 27, no. 1, pp. 66–72, 1985.

[12] X. Liao, S. Yang, S. Cheng, and Y. Fu, "Stability of general neural networks with reaction-diffusion," *Science in China, Series A F*, vol. 44, no. 5, pp. 389–395, 2001.

[13] Y. Takahashi, "Solving optimization problems with variable-constraint by an extended Cohen-Grossberg model," *Theoretical Computer Science*, vol. 158, no. 1-2, pp. 279–341, 1996.

[14] P. Zheng, J. Zhang, and W. Tang, "Color image associative memory on a class of Cohen-Grossberg networks," *Pattern Recognition*, vol. 43, pp. 3255–3260, 2010.

[15] L. Wang and X. Zou, "Exponential stability of Cohen-Grossberg neural networks," *Neural Networks*, vol. 15, no. 3, pp. 415–422, 2002.

[16] S. Arik and Z. Orman, "Global stability analysis of cohen–grossberg neural networks with time varying delays," *Physics Letters A*, vol. 341, no. 5-6, pp. 410–421, 2005.

[17] L. Zhang and B. Shi, "Global exponential stability of Cohen-Grossberg neural networks with variable delays," *Applied Mathematics: A Journal of Chinese Universities*, vol. 24, no. 2, pp. 167–174, 2009.

[18] N. Ozcan, "New conditions for global stability of neutral-type delayed cohen-grossberg neural networks," *Neural Networks*, vol. 106, Article ID S0893608018301916, pp. 1–7, 2018.

[19] J. Cao and X. Li, "Stability in delayed cohen–grossberg neural networks: lmi optimization approach," *Physica D: Nonlinear Phenomena*, vol. 212, no. 1-2, pp. 54–65, 2005.

[20] Q. Zhu and J. Cao, "Robust exponential stability of markovian jump impulsive stochastic cohen-grossberg neural networks with mixed time delays," *IEEE Transactions on Neural Networks*, vol. 21, no. 8, pp. 1314–1325, 2010.

[21] Y. Li, X. Meng, and X. Zhang, "Almost automorphic solutions for fuzzy cohen-grossberg neural networks with mixed time delays," *Mathematical Problems in Engineering*, vol. 2015, no. 4, Article ID 812670, 14 pages, 2015.

[22] Y. Huang, S. Qiu, S. Ren, and Z. Zheng, "Fixed-time synchronization of coupled cohen-grossberg neural networks with and without parameter uncertainties," *Neurocomputing*, vol. 315, no. 13, pp. 157–168, 2018.

[23] J. Xiao, Z. Zeng, A. Wu, and S. Wen, "Fixed-time synchronization of delayed Cohen-Grossberg neural networks based on a novel sliding mode," *Neural Networks*, vol. 128, pp. 1–12, 2020.

[24] B. Kosko, "Adaptive bi-directional associative memories," *Applied Optics*, vol. 26, no. 23, pp. 4947–4960, 1987.

[25] B. Kosko, "Bi-directional associative memories," *IEEE Trans. Syst. Man Cybern*, vol. 18, no. 1, pp. 49–60, 1988.

[26] B. Kosko, *Neural Networks and Fuzzy Systems A Synamical System Approach to Machine Intelligence*, Prentice-Hall, Englewood Clifs, NJ, USA, 1992.

[27] M. Schuster and K. K. Paliwal, "Bidirectional recurrent neural networks," *IEEE Transactions on Signal Processing*, vol. 45, no. 11, pp. 2673–2681, 1997.

[28] J. Cao and M. Dong, "Exponential stability of delayed bidirectional associative memory networks," *Applied Mathematics and Computation*, vol. 135, pp. 105–112, 2003.

[29] M. S. Ali, G. Narayanan, V. Shekher, H. Alsulami, and T. Saeed, "Dynamic stability analysis of stochastic fractional-order memristor fuzzy BAM neural networks with delay and leakage terms," *Applied Mathematics and Computation*, vol. 369, 2020.

[30] Q. Zhu and J. Cao, "Stability analysis of Markovian jump stochastic BAM neural networks with impulse control and mixed time delays," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 23, no. 3, pp. 467–479, 2012.

[31] S. Arik, "Global asymptotic stability analysis of bidirectional associative memory neural networks with time delays," *IEEE Transactions on Neural Networks*, vol. 16, no. 3, pp. 580–586, 2005.

[32] F. Zhou and H. Yao, "Stability analysis for neutral-type inertial BAM neural networks with time-varying delays," *Nonlinear Dynamics*, vol. 92, no. 4, pp. 1583–1598, 2018.

[33] Q. Zhang, L. Yang, and D. Liao, "Global exponential stability of fuzzy BAM neural networks with distributed delays," *Arabian Journal for Science and Engineering*, vol. 38, no. 3, pp. 691–697, 2013.

[34] W. Zhao, H. S. Zhang, and S. Kong, "An analysis of global exponential stability of bidirectional associative memory neural networks with constant time delays," *Neurocomputing*, vol. 70, no. 7-9, pp. 1382–1389, 2007.

[35] B. Liu and L. Huang, "Global exponential stability of bam neural networks with recent-history distributed delays and impulses," *Neurocomputing*, vol. 69, no. 16-18, pp. 2090–2096, 2006.

[36] Y. Li and Y. Li, "Existence and exponential stability of almost periodic solution for neutral delay bam neural networks with time-varying delays in leakage terms," *Journal of the Franklin Institute*, vol. 350, no. 9, pp. 2808–2825, 2013.

[37] M. Syed Ali, G. Narayanan, S. Sevgen, V. Shekher, and S. Arik, "Global stability analysis of fractional-order fuzzy BAM neural networks with time delay and impulsive effects," *Communications in Nonlinear Science and Numerical Simulation*, vol. 78, Article ID 104853, 2019.

[38] C. Feng and R. Plamondon, "Stability analysis of bidirectional associative memory networks with time delays," *IEEE Transactions on Neural Networks*, vol. 14, no. 6, pp. 1560–1565, 2003.

[39] J. Cao and Q. Song, "Stability in Cohen–Grossberg-type bidirectional associative memory neural networks with time-varying delays," *Nonlinearity*, vol. 19, no. 7, pp. 1601–1617, 2006.

[40] H. Xiang and J. Cao, "Exponential stability of periodic solution to Cohen-Grossberg-type BAM networks with time-varying delays," *Neurocomputing*, vol. 72, no. 7–9, pp. 1702–1711, 2009.

[41] R. Sakthivel, A. Arunkumar, K. Mathiyalagan, and S. Marshal Anthoni, "Robust passivity analysis of fuzzy Cohen-Grossberg BAM neural networks with time-varying delays," *Applied Mathematics and Computation*, vol. 218, no. 7, pp. 3799–3809, 2011.

[42] K. Li and H. Zeng, "Stability in impulsive Cohen-Grossberg-type BAM neural networks with time-varying delays: a general analysis," *Mathematics and Computers in Simulation*, vol. 80, pp. 2329–2349, 2010.

[43] Y. Xia, Z. Huang, and M. Han, "Exponential p-stability of delayed Cohen–Grossberg-type BAM neural networks with impulses," *Chaos, Solitons & Fractals*, vol. 38, no. 3, pp. 806–818, 2008.

[44] G. A. Bocharov and F. A. Rihan, "Numerical modelling in biosciences using delay differential equations," *Journal of Computational and Applied Mathematics*, vol. 125, no. 1-2, pp. 183–199, 2000.

[45] S. M. Shah and J. Wiener, "Advanced differential equations with piecewise constant argument deviations," *International Journal of Mathematics and Mathematical Sciences*, vol. 6, no. 4, pp. 671–703, Article ID 958736, 1983.

[46] Y. Muroya, "Persistence, contractivity and global stability in logistic equations with piecewise constant delays," *Journal of Mathematical Analysis and Applications*, vol. 270, no. 2, pp. 602–635, 2002.

[47] K. L. Cooke and J. Wiener, "Retarded differential equations with piecewise constant delays," *Journal of Mathematical Analysis and Applications*, vol. 99, no. 1, pp. 265–297, 1984.

[48] M. U. Akhmet, "On the reduction principle for differential equations with piecewise constant argument of generalized type," *Journal of Mathematical Analysis and Applications*, vol. 336, no. 1, pp. 646–663, 2007.

[49] M. U. Akhmet, "Integral manifolds of differential equations with piecewise constant argument of generalized type," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 66, no. 2, pp. 367–383, 2007.

[50] M. Akhmet and D. Aruğaslan, "Lyapunov-razumikhin method for differential equations with piecewise constant argument," *Discrete & Continuous Dynamical Systems A*, vol. 25, no. 2, pp. 457–466, 2009.

[51] M. U. Akhmet, D. Aruğaslan, and E. YiLmaz, "Stability in cellular neural networks with a piecewise constant argument," *Journal of Computational and Applied Mathematics*, vol. 233, no. 9, pp. 2365–2373, 2010.

[52] M. U. Akhmet, D. Aruğaslan, and E. Yilmaz, "Stability analysis of recurrent neural networks with piecewise constant argument of generalized type," *Neural Networks*, vol. 23, no. 7, pp. 805–811, 2010.

[53] M. U. Akhmet and E. Yilmaz, "Impulsive hopfield-type neural network system with piecewise constant argument," *Nonlinear Analysis: Real World Applications*, vol. 11, no. 4, pp. 2584–2593, 2010.

[54] W. Si, T. Xie, and B. Li, "Exploration on robustness of exponentially global stability of recurrent neural networks with neutral terms and generalized piecewise constant arguments," *Discrete Dynamics in Nature and Society*, vol. 2021, Article ID 9941881, 5 pages, 2021.

[55] Q. Xi and Qiang, "Global exponential stability of cohen-grossberg neural networks with piecewise constant argument of generalized type and impulses," *Neural Computation*, vol. 28, no. 1, pp. 229–255, 2016.

[56] G. Bao, S. Wen, and Z. Zeng, "Robust stability analysis of interval fuzzy cohen-grossberg neural networks with piecewise constant argument of generalized type," *Neural Networks*, vol. 33, no. 3, pp. 32–41, 2012.

[57] J. E. Zhang and -E. Jin, "Robustness analysis of global exponential stability of nonlinear systems with deviating argument and stochastic disturbance," *IEEE Access*, vol. 5, pp. 13446–13454, 2017.

[58] A. R. Aftabizadeh, J. Wiener, and J. M. Xu, "Oscillatory and periodic solutions of delay differential equations with piecewise constant argument," *Proceedings of the American Mathematical Society*, vol. 99, no. 4, pp. 673–679, 1987.

[59] K.-S. Chiu and M. Pinto, "Periodic solutions of differential equations with a general piecewise constant argument and applications," *Electronic Journal of Qualitative Theory of Differential Equations*, vol. 46, pp. 1–19, 2010.

[60] M. Forti and A. Tesi, "New conditions for global stability of neural networks with application to linear and quadratic programming problems," *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, vol. 42, no. 7, pp. 354–366, 1995.

[61] L. Zhou and M. Zhou, "Stability analysis of a class of generalized neural networks with delays," *Physics Letters A*, vol. 337, no. 3, pp. 203–215, 2005.

[62] M. Pinto, "Asymptotic equivalence of nonlinear and quasilinear differential equations with piecewise constant arguments," *Mathematical and Computer Modelling*, vol. 49, no. 9-10, pp. 1750–1758, 2009.

[63] M. Pinto, "Cauchy and Green matrices type and stability in alternately advanced and delayed differential systems," *Journal of Difference Equations and Applications*, vol. 17, no. 2, pp. 235–254, 2011.

[64] K.-S. Chiu, M. Pinto, and J.-C. Jeng, "Existence and global convergence of periodic solutions in recurrent neural network models with a general piecewise alternately advanced and retarded argument," *Acta Applicandae Mathematica*, vol. 133, no. 1, pp. 133–152, 2014.

[65] K.-S. Chiu and M. Pinto, "Variation of parameters formula and Gronwall inequality for differential equations with a general piecewise constant argument," *Acta Mathematicae Applicatae Sinica, English Series*, vol. 27, no. 4, pp. 561–568, 2011.