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Research Article

Switching Delay Effects on Input-to-State Stability of Switched Systems

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In this paper, input-to-state stability (ISS) is investigated for a class of nonlinear switched systems with time-varying switching delay, in which both ISS and non-ISS subsystems are considered simultaneously. By means of the Lyapunov function method, we show that ISS can be ensured for switched systems with time-varying switching delay if the activation time of ISS subsystems is sufficiently large and switching delays satisfy certain conditions. Moreover, inspired by (Zhang et al. 2020), a time-dependent multiple Lyapunov function is considered for linear switched systems with switching delay to obtain less conservative results, where the conservativeness can be reduced by explicitly providing the lower and upper bounds of switching intervals. Finally, simulations including an example of coordination of multiagent systems are offered to verify the effectiveness of the proposed results.

1. Introduction

Switched systems, as a kind of special hybrid systems, have gained increasing research attention since switched systems can be efficiently used to model various real-world systems displaying switching features [1–5]. Typically, a switched system comprises a family of subsystems and a switching signal governing the switching among subsystems. Stability of switched systems can be classified into two categories: stability of switched systems with stable subsystems and stability of switched systems containing unstable subsystems [6–10]. Clearly, the stability of switched systems with unstable subsystems renders more practical significance than that of the systems one with only stable subsystems [6, 10].

Usually, the performance of a real control system is always affected by uncertainties such as unmodeled dynamics, parameter perturbations, exogenous disturbances, and measurement errors. This arouses the investigation of the input-to-state stability (ISS), since ISS can well characterize the effects of external inputs on a control system [11, 12]. Therefore, in the past decade, various extensions of ISS have been made for different types of dynamical systems, such as discrete-time systems, impulsive systems, and switched systems (see [11] and the references therein). Among them, the study of the ISS of switched systems constitutes an important component. In the past decade, there have been some well-studied results about the ISS of switched systems. In [13], some sufficient conditions were derived to ensure that the whole switched nonlinear system is ISS when each mode is ISS. In [9], input/output-to-state stability (IOSS) of switched nonlinear systems was studied, in which IOSS and non-IOSS systems were considered simultaneously. Moreover, very recently, in [14], a general class of switching signals was considered to ensure that switched systems with both ISS and non-ISS subsystems are stable, where it allows the number of switches to grow faster than an affine function of the length of a time interval.

However, in the results mentioned above, it is implicitly assumed that switching among system modes is synchronous, which is unpractical. In practice, it usually takes some time to detect the switching signal [15], i.e., a switching delay is unavoidable.

Actually, when detecting a switching signal, due to sensor/actuator failures and to changing the mode of controllers, the switching law cannot be detected instantly. It usually takes a period of time to detect the switching signal

[16], i.e., a time delay widely exists in switching signals. Thus, it is important to investigate the ISS of switched systems with switching delays. It is noted that some researchers have focused on asynchronously switched control for switched systems [7, 15, 17, 18]. Nevertheless, in [7, 15, 17, 18], exogenous disturbances have been overlooked, which is unrealistic since exogenous disturbances cannot be averted in practical systems. In addition, in [15, 17, 18], only a constant time delay was considered, and it is assumed that the switching delay only exists in the switching controller. To the best of the authors' knowledge, there are few results on ISS of switched systems with time-varying switching delays, primarily due to the difficulties in characterizing the effects of switching delays, exogenous disturbances, ISS subsystems, and non-ISS subsystems on the performance of switched systems.

On the other hand, the multiple Lyapunov function method (MLFM) is a very popular method for the stability analysis of switched systems [15, 17, 19]. However, as was shown in [20], the results obtained by using the MLFM may be conservative since the upper bound of the dwell time was overlooked. The results in [20] indicated that the upper bound of the dwell time is useful for reducing conservativeness, and the time-dependent multiple Lyapunov function is a better method since the conservativeness can be reduced by explicitly providing the information on both the lower and upper bounds of the dwell time.

Based on the above discussions, in this paper, we aim to carry out the ISS of switched systems with time-varying switching delay, in which both ISS and non-ISS subsystems are considered simultaneously. A sufficient condition is obtained to ensure that the switched system with switching delay is ISS. Then, a time-dependent multiple Lyapunov function is considered to obtain less conservative results for linear switched systems with switching delay. The main contributions of this paper can be listed as follows: (1) Switching delay is considered in this paper, where it will be shown that the existence of the switching delay may destroy or enhance the ISS due to the fact that the ISS-subsystem/non-ISS subsystems can be replaced by non-ISS subsystem/ISS subsystems. (2) In addition to the switching delay, both ISS subsystems, non- ISS subsystems, and disturbances are taken into account in a unified model, which also brings difficulties to our theoretical analysis.

1.1. Notations. The notations of this paper are shown in the following Table 1.

Moreover, κ represent the class of continuous strictly increasing function ϕ: $[0, \infty) \longrightarrow [0, \infty)$ with $\phi(0) = 0$. κ_∞ is the subset of κ functions that are unbounded. A function β: $[0, \infty) \times [0, \infty) \longrightarrow [0, \infty)$ is said to belong to the class of $\kappa\mathscr{L}$, if β (., t) is of class κ for each fixed $t > 0$ and β (s, t) decreases to zero as $t \longrightarrow \infty$ for each fixed $s \geq 0$.

Table 1: Notations.

$n \times n$ real matrices	$\mathbb{R}^{n\times n}$
The natural number	$\mathbb{N}$
The absolute value	$\lvert\bullet\rvert$
Transpose of $x \in \mathbb{R}^n$	x^T

2. Model Formulation and Preliminaries

In this section, we present the model formulation of this paper. Moreover, some useful definitions and lemmas are given.

Consider a class of switched nonlinear systems with a time-varying switching delay given by

$$\dot{x}(t) = f_{\sigma(t-\tau(t))}(x(t), u(t)), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ with $x(t_0) = x_0$ is the state vector and the switching signal σ: $[0, \infty) \longrightarrow Q = \{1, 2, \ldots, N\}$ is a piecewise constant function, in which N is the number of subsystems of the switched system. t_0 is the initial time and $t_k (k \in \mathbb{N})$ denotes the switching sequence. Without loss of generality, it is assumed that there are no switching effects at the initial time. $\tau(t)$ is a continuous function representing the time-varying switching delay that satisfies $0 \leq \tau(t) \leq \tau . f_i$. $i \in Q$ is the nonlinear function satisfying the Lipschitz condition, and u(t) denotes the disturbance input.

Definition 1 (see [11]). The switched system (1) is said to be input-to-state stable (ISS) if there exist class κ_∞ functions α and γ and a class $\kappa\mathscr{L}$ function β such that for the input $u(t)$ and initial state x_0, the following inequality holds:

$$\alpha(\|x(t)\|) \leq \beta\left(\|x_0\|, t\right) + \sup_{0\leq s\leq t}(\|u(s)\|). \tag{2}$$

The following dwell time constraint is made on the switching signal $\sigma(t)$.

Assumption 2. There are two positive constants $0 < \mu_1 < \mu_2$ such that the following condition holds true.

$$\mu_1 \leq t_k - t_{k-1} \leq \mu_2, \quad k \in N. \tag{3}$$

Remark 3. The above constraint is called as the ranged dwell time constraint [20, 21]. In the presence of non-ISS subsystems, in order to retain stability, the activation time of ISS subsystems should not be too small, while the activation time of non-ISS subsystems should not be too large [9, 19]. The concept of dwell time has been introduced early in [2] and has been proven to be efficient for the analysis of switched systems. It is due to the fact that the lower and upper bounds of switching intervals guarantee that the activation time of ISS subsystems is not too small and the activation time of non-ISS subsystems is not too large simultaneously.

Let QS and Qu denote respectively the sets of indices of ISS and non-ISS systems. Obviously, $Q_s \cup Q_u = Q$, $Q_s \cap Q_u = \varnothing$. For $t \geq s \geq t_0$, $T_s^\tau[s,t]$ and $T_s^\tau[s,t]$ represent the total activation time of ISS and non-ISS subsystems under $\sigma(t-\tau(t))$during the time interval [s, t), respectively. Denote by $t_{\tau,i}$ the ith switching instant of $\sigma(t-\tau(t))$, i.e., $\sigma(-\tau(t)) = \overline{r}$, $\overline{r} \in Q$, $t \in [t_{\tau,i}, t_{\tau,i+1})$. Let

$$\begin{aligned}\Delta_j^S[s,t] &:= \left|[s,t) \cap \left(\bigcup_{i=0,\sigma(t_{\tau,i}-\tau(t_{\tau,i}))=j \in Q_S}^{\infty} \left[t_{\tau,i}, t_{\tau,,i+1}\right)\right)\right|,\\ \Delta_l^U[s,t] &:= \left|[s,t) \cap \left(\bigcup_{i=0,\sigma(t_{\tau,i}-\tau(t_{\tau,i}))=l \in Q_U}^{\infty} \left[t_{\tau,i}, t_{\tau,,i+1}\right)\right)\right|.\end{aligned} \tag{4}$$

i.e., Δ_j^s [s, t) and Δ_l^u [s, t) denote, respectively, the total activation time of the jth ISS and lth non-ISS modes in the interval [s, t). Denote by Ns (s, t) and Nu (s, t), respectively, the number of ISS and non-ISS modes in the interval [s, t) under $\sigma(t)$. N_s^τ (s, t) and N_u^τ (s, t), represent the number of ISS and non-ISS modes in the interval [s, t) under $\sigma(t-\tau(t))$, respectively.

Assumption 4. There exists a positive constant $a_k \in [0, 1]$ such that the number of stable modes under $\sigma(t)$ in the interval [0, t), $t \in [t_k, t_{k+1})$ satisfies

$$Ns\ (0,t) = ka_k. \tag{5}$$

It can be seen that $a_k = 1$ means that there are no non-ISS modes in the interval [0, t), while $a_k = 0$ means that there are no ISS modes in the interval [0, t). Due to the existence of the time-varying switching delay, when $t \in [t_k, t_{k+1})$, $\sigma(t) \in Q_S$ (Q_U), there exist the following two cases (Please refer to Figure 1 for more details):

(1) $t \in [t_{\tau,i}, t_{\tau,i+1})$, $\sigma(t-\tau(t)) \in Q_S$ (Q_U), i.e., in the presence of switching delay, ISS (non-ISS) modes are still activated in $[t_k, t_{k+1})$ under $\sigma(t-\tau(t))$.

(2) $t \in [t_{\tau,i}, t_{\tau,i+1})$, $\sigma(t-\tau(t)) \in Q_U$ (Q_S), i.e., in the presence of switching delay, unstable (stable) modes take the place of stable (unstable) modes and are activated in $[t_k, t_{k+1})$ under $\sigma(t-\tau(t))$.

The following assumption is given to characterize the lower and upper bounds of switching intervals and the total activation time of ISS and non-ISS modes under $\sigma(t-\tau(t))$.

Assumption 5. There exist positive constants b, $c \in [0, 1]$, $d \in (0, 1)$ and a positive constant T_S, such that the following conditions hold:

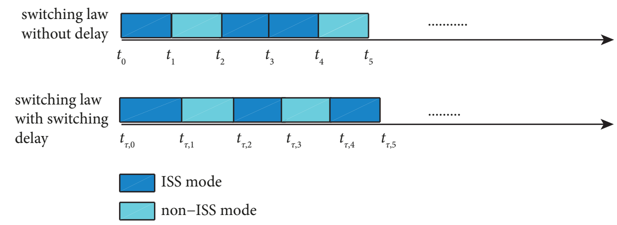

Figure 1: A switching law with and without switching delay.

$$\begin{aligned}N_s^\tau(t_0,t) &\geq a_k k(1-b) + (1-\mathrm{a_k})\,\mathrm{kc},\\ N_u^\tau(t_0,t) &\leq (1-\mathrm{a_k})k(1-c) + a_k \mathrm{kb},\\ T_u^\tau[s,t) &\leq T_-(S) + d(t-s),\end{aligned} \tag{6}$$

where $t \in [t_k, t_{k+1})$, $t \geq s \geq t_0$.

Remark 6. In Assumption 5, b can be regarded as the upper bound of the ratio of the number of ISS subsystems, that is replaced by non-ISS subsystems with Ns (0, t). Moreover, c can be regarded as the lower bound of the ratio of the number of non-ISS subsystems, that is replaced by ISS subsystems with Nu (0, t). In fact, $b = 1$ means that all the ISS subsystems will be replaced by non-ISS subsystems, while $c = 1$ means that all the non-ISS subsystems will be replaced by ISS subsystems. In addition, as $T_u^\tau[s,t) \leq T_s + d(t-s)$ is assumed to hold for any interval [s, t), the selection T_S is based on the selection of a and the upper bound of unstable subsystems.

3. Main Results

3.1. Input-to-State Stability of Switched Systems with Time-Varying Switching Delay. In this section, we will present the condition to ensure ISS of the switched system (1) with time-varying switching delay, where an important lemma is first proved.

Lemma 7. *Suppose that Assumption 2 holds. If there are a positive constant $\varrho > 1$ and κ_∞ functions αi ($i = 1, 2$) and γ, continuous differential functions V_i: $\mathbb{R}^n \longrightarrow [0, +\infty)$ and constants λ_i, $i \in Q$, such that*

$$\begin{aligned}&\alpha 1\,(\|x(t)\|) \leq V_{\sigma(t)}(x(t)) \leq \alpha 2\,(\|x(t)\|),\\ &\dot{V}_{\sigma(t)}(x(t)) \leq -\lambda\sigma(t-\tau(t))\,V\sigma(t)(x(t)) + \gamma\|u\|[0,t],\\ &V_i\,(x(t)) \leq \varrho V_j\,(x(t)), \quad i, j \in Q.\end{aligned} \tag{7}$$

Then for $t \in [t_k, t_{k+1})$,

$$\begin{aligned}V_{\sigma(t)}(x(t)) \leq{}& \varrho^k e^{-\lambda_1 T_S^\tau[t_0-t)-\lambda_2 T_U^\tau[t_0-t)} V_{\sigma(t_0)}(x(t_0)) + \left[\sum_{i=0}^{k-2} \varrho^{k-i} e^{\Gamma(t_k,t)+\sum_{j=i+1}^{k-1}\Gamma(t_j,t_{j+1})} \times \Xi(t_i,t_{i+1})\right.\\ &\left. + \varrho e^{\Gamma(tk,t)}\Xi(t_{k-1},t_k) + \Xi(t_k,t)\right]\gamma\|u\|_{[0,t]},\end{aligned} \tag{8}$$

where

$$\Gamma\left(t_k,t\right) = \sum_{i=0}^{n_k-1} -\lambda_{\sigma\left(t^{i}_{\tau,k}-\tau\left(t^{i}_{\tau,k}\right)\right)}\left(t^{i+1}_{\tau,k}-t^{i}_{\tau,k}\right) - \lambda_{\sigma\left(t^{n_k}_{\tau,k}-\tau\left(t^{n_k}_{\tau,k}\right)\right)}\left(t-t^{n_k}_{\tau,k}\right)$$

$$\Xi\left(t_k,t\right) = \frac{1-e^{-\lambda_{\sigma\left(t^{n_k}_{\tau,k}-\tau\left(t^{n_k}_{\tau,k}\right)\right)}\left(t-t^{n_k}_{\tau,k}\right)}}{\lambda_{\sigma\left(t^{n_k}_{\tau,k}-\tau\left(t^{n_k}_{\tau,k}\right)\right)}} + e^{-\lambda_{\sigma\left(t^{n_k}_{\tau,k}-\tau\left(t^{n_k}_{\tau,k}\right)\right)}\left(t-t^{n_k}_{\tau,k}\right)} \times \frac{1-e^{-\lambda_{\sigma\left(t^{n_k-1}_{\tau,k}-\tau\left(t^{n_k-1}_{\tau,k}\right)\right)}\left(t^{n_k}_{\tau,k}-t^{n_k-1}_{\tau,k}\right)}}{\lambda_{\sigma\left(t^{n_k-1}_{\tau,k}-\tau\left(t^{n_k-1}_{\tau,k}\right)\right)}} + \sum_{i=0}^{n_k-2} e^{-\sum_{j=i+1}^{n_k-1}\lambda_{\sigma\left(t^{j}_{\tau,k}-\tau\left(t^{j}_{\tau,k}\right)\right)}\left(t^{j+1}_{\tau,k}-t^{j}_{\tau,k}\right)} \times \frac{1-e^{-\lambda_{\sigma\left(t^{i}_{\tau,k}-\tau\left(t^{i}_{\tau,k}\right)\right)}\left(t^{i+1}_{\tau,k}-t^{i}_{\tau,k}\right)}}{\lambda_{\sigma\left(t^{i}_{\tau,k}-\tau\left(t^{i}_{\tau,k}\right)\right)}},$$

$$\Gamma\left(t_p,t_{p+1}\right) = \sum_{i=0}^{n_p-1} -\lambda_{\sigma\left(t^{i}_{\tau,p}-\tau\left(t^{i}_{\tau,p}\right)\right)}\left(t^{i+1}_{\tau,p}-t^{i}_{\tau,p}\right) - \lambda_{\sigma\left(t^{n_p}_{\tau,p}-\tau\left(t^{n_p}_{\tau,p}\right)\right)}\left(t_{p+1}-t^{n_p}_{\tau,p}\right)$$

$$\Xi\left(t_p,t_{p+1}\right) = \sum_{i=0}^{n_p-2} e^{-\sum_{j=i+1}^{n_p-1}\lambda_{\sigma\left(t^{j}_{\tau,p}-\tau\left(t^{j}_{\tau,p}\right)\right)}\left(t^{j+1}_{\tau,p}-t^{j}_{\tau,p}\right)} \times \frac{\left(1-e^{-\lambda_{\sigma\left(t^{i}_{\tau,p}-\tau\left(t^{i}_{\tau,p}\right)\right)}\left(t^{i+1}_{\tau,p}-t^{i}_{\tau,p}\right)}\right)}{\lambda_{\sigma\left(t^{i}_{\tau,p}-\tau\left(t^{i}_{\tau,p}\right)\right)}} + \frac{\left(1-e^{-\lambda_{\sigma\left(t^{n_p}_{\tau,p}-\tau\left(t^{n_p}_{\tau,p}\right)\right)}\left(t_{p+1}-t^{n_p}_{\tau,p}\right)}\right)}{\lambda_{\sigma\left(t^{n_p}_{\tau,p}-\tau\left(t^{n_p}_{\tau,p}\right)\right)}} + e^{-\lambda_{\sigma\left(t^{n_p}_{\tau,p}-\tau\left(t^{n_p}_{\tau,p}\right)\right)}\left(t_{p+1}-t^{n_p}_{\tau,p}\right)} \times \frac{\left(1-e^{-\lambda_{\sigma\left(t^{n_p-1}_{\tau,p}-\tau\left(t^{n_p-1}_{\tau,p}\right)\right)}\left(t^{n_p}_{\tau,p}-t^{n_p-1}_{\tau,p}\right)}\right)}{\lambda_{\sigma\left(t^{n_p-1}_{\tau,p}-\tau\left(t^{n_p-1}_{\tau,p}\right)\right)}}, p = 0, 1, 2, \ldots, k-1. \tag{9}$$

in which $t_k = t^0_{\tau,k} \le t^1_{\tau,k} < t^2_{\tau,k} < \ldots < t^{n_k}_{\tau,k} \le t$, $t_p = t^0_{\tau,p} \le t^1_{\tau,p} < t^2_{\tau,p} < \ldots < t^{n_p}_{\tau,p} \le t_{p+1}$, where $t^i_{\tau,k}$, $t^j_{\tau,p}$, $1 \le i \le n_k$, $1 \le j \le n_p$ are switching instants under $\sigma(t-\tau(t))$.

Proof. In view of (5), we have for $t \in [t_k, t_{k+1})$:

$$
\begin{aligned}
V_{\sigma(t)}(x(t)) &\le e^{-\lambda_{\sigma\left(t^{n_k}_{\tau,k}-\tau\left(t^{n_k}_{\tau,k}\right)\right)}\left(t-t^{n_k}_{\tau,k}\right)} V_{\sigma(t_k)}\left(x\left(t^{n_k}_{\tau,k}\right)\right) + \gamma\left(\|u\|_{[0,t]}\right)\int_{t^{n_k}_{\tau,k}}^{t} e^{-\lambda_{\sigma\left(t^{n_k}_{\tau,k}-\tau\left(t^{n_k}_{\tau,k}\right)\right)}(t-s)} ds \\
&\le e^{-\lambda_{\sigma\left(t^{n_k}_{\tau,k}-\tau\left(t^{n_k}_{\tau,k}\right)\right)}\left(t-t^{n_k}_{\tau,k}\right)} \times e^{\sum_{i=0}^{n_k-1} -\lambda_{\sigma\left(t^{i}_{\tau,k}-\tau\left(t^{i}_{\tau,k}\right)\right)}\left(t^{i+1}_{\tau,k}-t^{i}_{\tau,k}\right)} V_{\sigma(t_k)}(x(t_k)) \\
&\quad + e^{-\lambda_{\sigma\left(t^{n_k}_{\tau,k}-\tau\left(t^{n_k}_{\tau,k}\right)\right)}\left(t-t^{n_k}_{\tau,k}\right)} \sum_{i=0}^{n_k-2} e^{-\sum_{j=i+1}^{n_k-1}\lambda_{\sigma\left(t^{j}_{\tau,k}-\tau\left(t^{j}_{\tau,k}\right)\right)}\left(t^{j+1}_{\tau,k}-t^{j}_{\tau,k}\right)} \\
&\quad \times \gamma\left(\|u\|_{[0,t]}\right)\int_{t^{i}_{\tau,k}}^{t^{i+1}_{\tau,k}} e^{-\lambda_{\sigma\left(t^{j}_{\tau,k}-\tau\left(t^{j}_{\tau,k}\right)\right)}(t-s)} ds + e^{-\lambda_{\sigma\left(t^{n_k}_{\tau,k}-\tau\left(t^{n_k}_{\tau,k}\right)\right)}\left(t-t^{n_k}_{\tau,k}\right)} \gamma\left(\|u\|_{[0,t]}\right) \\
&\quad \times \int_{t^{n_k-1}_{\tau,k}}^{t^{n_k}_{\tau,k}} e^{-\lambda_{\sigma\left(t^{n_k-1}_{\tau,k}-\tau\left(t^{n_k-1}_{\tau,k}\right)\right)}(t-s)} ds + \gamma\left(\|u\|_{[0,t]}\right)\int_{t^{n_k}_{\tau,k}}^{t} e^{-\lambda_{\sigma\left(t^{n_k}_{\tau,k}-\tau\left(t^{n_k}_{\tau,k}\right)\right)}(t-s)} ds \\
&\le e^{\Gamma(t_k,t)} V_{\sigma(t_k)}(x(t_k)) + \Xi(t_k,t)_{\gamma}\left(\|u\|_{[0,t]}\right).
\end{aligned}
\tag{10}
$$

By repeating similar steps as in (10), we obtain for $t \in [t_{k-1}, t_k)$:

$$
\begin{aligned}
V_{\sigma(t)}(x(t)) &\le \varrho e^{\Gamma(t_k,t)+\Gamma(t_{k-1},t_k)} V_{\sigma(t_{k-1})}(x(t_{k-1})) \\
&\quad + \left\{\Xi(t_k,t) + \varrho e^{\Gamma(t_k,t)}\Xi(t_{k-1},t_k)\right\} \times \gamma\left(\|u\|_{[0,t]}\right).
\end{aligned}
\tag{11}
$$

Then by induction method, we obtain (8) and thus the proof is completed. □

Theorem 8. *Suppose that the following conditions and Lemma 7 hold true*

$$
\begin{aligned}
\eta = &-\lambda_1\left[a_k(1-b)+(1-a_k)c\right]\mu^\tau_1 \\
&-\lambda_2\left[a_k b+(1-a_k)(1-c)\right]k\mu^\tau_2 \ln\varrho < 0,
\end{aligned}
\tag{12}
$$

$$
\varepsilon = \frac{\ln\varrho}{\mu_1} - \lambda_1 + (\lambda_1-\lambda_2)d < 0,
\tag{13}
$$

where are the lower and upper bounds of the switching interval under $\sigma(t-\tau(t))$, respectively, i.e., $\mu^\tau_1 \le t_{\tau,k+1} - t_{\tau,k} \le \mu^\tau_2$. Then, the switched system in (1) with time-varying switching delay is ISS.

Proof. From Definition 1, in order to prove ISS of (1), we need to prove the following two propositions:

(1) $\varrho^k e^{-\lambda_1 T^\tau_u[t_0-t)-\lambda_2 T^\tau_u[t_0-t)}$ is bounded by a $\kappa\mathscr{L}$ function;

(2) the upper bound $\sum_{i=0}^{k-2}\varrho^{k-i}e^{\Gamma(t_k,t)+\sum_{j=i+1}^{k-1}\Gamma(t_j,t_{j+1})}\Xi$ $(t_i, t_{i+1}) + \varrho e^{\Gamma(t_k,t)}\Xi(t_{k-1},t_k) + \Xi$ (t_k, t) exists.

(1) In view of Assumption 5, the following inequality can be derived:

$$
\varrho^k e^{-\lambda_1 T^\tau_u[t_0-t)-\lambda_2 T^\tau_u[t_0-t)} \le e^{k\ln\varrho} e^{-\lambda_1\left[a_k(1-b)+(1-a_k)c\right]k\mu^\tau_1} \times e^{-\lambda_2\left[(1-a_k)(1-c)+a_k b\right]k\mu^\tau_2} \le e^{\eta k}.
\tag{14}
$$

It can be checked from $\mu_1 \le k \le \mu_2$ that $(k+1)\mu_2 \ge t - t_0$. Then, we have $e^{\eta k} \le e^{-\eta} e^{\eta t - t_0/\mu_2}$, which means that $e^{-\lambda_1 T^\tau_u[t_0-t)-\lambda_2 T^\tau_u[t_0-t)}$ is bounded by a $\kappa\mathscr{L}$ function $e^{-\eta}e^{\eta t - t_0/\mu_2}$.

(2) There exist two cases:

(i) $\lambda_{\sigma(t^i_{\tau,k}-\tau(t^i_{\tau,k}))} > 0$, then and

$$
\frac{1-e^{-\lambda_{\sigma\left(t^{i}_{\tau,k}-\tau\left(t^{i}_{\tau,k}\right)\right)}\left(t^{i+1}_{\tau,k}-t^{i}_{\tau,k}\right)}}{\lambda_{\sigma\left(t^{i}_{\tau,k}-\tau\left(t^{i}_{\tau,k}\right)\right)}} < \frac{1}{\lambda_1}.
\tag{15}
$$

(ii) $\lambda_{\sigma(t^i_{\tau,k}-\tau(t^i_{\tau,k}))} < 0$, then

$$\frac{1-e^{-\lambda_{\sigma\left(t^i_{\tau,k}-\tau\left(t^i_{\tau,k}\right)\right)}\left(t^{i+1}_{\tau,k}-t^i_{\tau,k}\right)}}{\lambda_{\sigma\left(t^i_{\tau,k}-\tau\left(t^i_{\tau,k}\right)\right)}}<\frac{e^{-\lambda_{\sigma\left(t^i_{\tau,k}-\tau\left(t^i_{\tau,k}\right)\right)}\left(t^{i+1}_{\tau,k}-t^i_{\tau,k}\right)}}{|\lambda_2|}. \tag{16}$$

For $t\in[t_k,\ t_{k+1})$, the definition of $\Xi(t_k,t)$ and Assumption 5 yields

$$\begin{aligned}\Xi(t_k,t)&\le\frac{1}{\lambda_1}\sum_{1\le i\le n_k\sigma\left(t^i_{\tau,k}-\tau\left(t^i_{\tau,k}\right)\right)=p\epsilon Q_S} e^{-\lambda_1\sum_{p\epsilon Q_S}\Delta^S_p\left[t^i_{\tau,k},t\right)}\times e^{-\lambda_2\sum_{q\epsilon Q_U}\Delta^U_q\left[t^i_{\tau,k},t\right)}\\
&\quad+\sum_{0\le i\le n\sigma\left(t^i_{\tau,k}-\tau\left(t^i_{\tau,k}\right)\right)=q\epsilon Q_{uk}} e^{-\lambda_1\sum_{p\epsilon Q_S}\Delta^S_p\left[t^i_{\tau,k},t\right)}\times\frac{1}{|\lambda_2|}e^{-\lambda_2\sum_{q\epsilon Q_U}\Delta^U_q\left[t^i_{\tau,k},t\right)}\\
&\le\frac{1}{\lambda_1}\sum_{1\le i\le n_k}e^{-\lambda_1\left(t-t^i_{\tau,k}\right)}e^{(\lambda_1-\lambda_2)T^\tau_u\left[t^i_{\tau,k},t\right)}\\
&\quad+\frac{1}{|\lambda_2|}\sum_{0\le i\le n_k}e^{-\lambda_1\left(t-t^i_{\tau,k}\right)}e^{(\lambda_1-\lambda_2)T^\tau_u\left[t^i_{\tau,k},t\right)}\\
&\le e^{(\lambda_1-\lambda_2)T_S}\left(\frac{1}{\lambda_1}+\frac{1}{|\lambda_2|}\right)\left(\sum_{0\le i\le n_k}e^{\varepsilon\left(t-t^i_{\tau,k}\right)}+\sum_{0\le i\le n_k}e^{\varepsilon\left(t-t^i_{\tau,k}\right)}\right).\end{aligned} \tag{17}$$

From Assumption 2, we can obtain that $k-i\le t-t_i/\mu_1\ (i<k)$. Then for $t\in[t_k,\ t_{k+1})$

$$\begin{aligned}&\sum_{i=0}^{k-2}\varrho^{k-i}e^{\Gamma(t_k,t)+\sum_{j=i+1}^{k-1}\Gamma(t_j,t_{j+1})}\Xi(t_i,t_{i+1})\\
&\le\sum_{i=0}^{k-1}\left\{\frac{e^{\ln\varrho\,(t-t_i)/\mu_1}}{\lambda_1}\left[\sum_{\substack{0\le q\le n_i\\ \sigma\left(t^q_{\tau,i}-\tau\left(t^q_{\tau,i}\right)\right)=l\epsilon Q_s}}e^{-\lambda_1\sum_{l\epsilon Q_s}\Delta^s_l\left(t^q_{\tau,i},t\right)}\times e^{-\lambda_2\sum_{m\epsilon Q_u}\Delta^u_m\left(t^q_{\tau,i},t\right)}+\frac{1}{|\lambda_2|}\sum_{\substack{0\le q\le n_i\\ \sigma\left(t^q_{\tau,i}-\tau\left(t^q_{\tau,i}\right)\right)=m\epsilon Q_u}}e^{-\lambda_1\sum_{l\epsilon Q_s}\Delta^s_l\left(t^q_{\tau,i},t\right)}\times e^{-\lambda_2\sum_{m\epsilon Q_u}\Delta^u_m\left(t^q_{\tau,i},t\right)}\right.\right\}\\
&\le\sum_{i=0}^{k-1}\left\{e^{(\lambda_1-\lambda_2)T_s}\left[\sum_{\substack{0\le q\le n_i\\ \sigma\left(t^q_{\tau,i}-\tau\left(t^q_{\tau,i}\right)\right)=l\epsilon Q_s}}\frac{1}{\lambda_1}e^{\ln\varrho\left(t^q_{\tau,i}-t_i\right)/\mu_1}\times e^{\varepsilon\left(t-t^q_{\tau,i}\right)}+\sum_{\substack{0\le q\le n_i\\ \sigma\left(t^q_{\tau,i}-\tau\left(t^q_{\tau,i}\right)\right)=m\epsilon Q_u}}e^{\ln\varrho\left(t^q_{\tau,i}-t_i\right)/\mu_1}\frac{1}{|\lambda_2|}\right]\times e^{\varepsilon\left(t-t^q_{\tau,i}\right)}\right\}.\end{aligned} \tag{18}$$

Then $\sum_{i=0}^{k-2}\varrho^{k-i}e^{G(t_k,t)+\sum_{j=i+1}^{k-1}G(t_j,t_{j+1})}H(t_i,t_{i+1})+H(t_k,t)+\varrho e^{G(t_k,t)}H(t_{k-1},t_k)$ is bounded due to the fact that Q is finite, thus completing the proof. □

Remark 9. It can be obtained from (10) that the stability criteria heavily depend on the values of a_k, b, c, u_1^τ and u_2^τ. In (11), for a fixed a_k, if b is relatively large, then a large c is required such that (11) can be satisfied. This means that if the number of ISS-subsystems that replaced by non-ISS subsystems is large, then larger number of non-ISS subsystems replaced by ISS subsystems is needed to make that the total activation time of ISS subsystems is large enough to ensure stability.

Remark 10. Recently, the stability problem of switched systems with switching delay has been investigated by some existing well-studied results [5, 6, 17, 22–24]. To compare with existing results, here we take the following special kind of nonlinear systems as an example.

$$\dot{x}(t)=A_{\sigma(t-\tau(t))}+B_{\sigma(t-\tau(t))}f_1(x(t))+C_{\sigma(t-\tau(t))}u(t). \tag{19}$$

In [5, 6, 17, 22–24], the asynchronously switching only exists in the input $u(t)$ and the switching delay is shown in the form of $\tau(t)=\tau$. In fact, the switching delay may exist in the whole switching signal rather than in the switching controller. For instance, in application layer multicast (ALM) networks, the switching delay exists in the whole switching signal due to the complexity of ALM networks [25]. Clearly, the time-varying switching delay is more general than the constant delay and it will make the results more challenging. Actually, the switching instants under the constant switching delay are $t_k+\tau, k=1,2,\ldots,$ *i.e.*, the switching instants are fixed and the delayed switching interval is also the same as the case without switching delay. However, from (19), we know that the switching instants considered here are time-varying and therefore the switching interval is time-varying as well. In addition to this main difference, the switching system considered in this paper is also more general than those in [5, 6, 17, 22–24]. Moreover, the ISS of nonlinear switching systems is considered here, which is also more general than the exponential stability of nonlinear switching systems.

Remark 11. If there is no switching delay in (1), then condition (10) and (11) becomes

$$\frac{\ln\varrho}{\mu_1}-\lambda_1+(\lambda_1-\lambda_2)d<0. \tag{20}$$

In the past few years, there have been some existing results on the stability analysis of switched systems with both stable and unstable subsystems [8, 19, 26], where it was shown that the stability of switched systems can be achieved if the total activation time of stable modes is large enough such that the negative effect of unstable modes can be prevailed over by stable modes. This point is consistent with the expression in (20). In addition, a necessary condition for (20) is that-$\lambda_1+(\lambda_1-\lambda_2)d<0$, which further means that $d<\lambda_1/\lambda_1-\lambda_2$. This is also consistent with Theorem 2 in [9], which manifests the advantage of this paper.

3.2. The Linear Switched System Case. In this section, we will extend the results derived in Section A into the linear case. A time-dependent multiple Lyapunov function is considered to get the main results. The advantage of the time-dependent multiple Lyapunov function is that the time-dependent multiple Lyapunov function can explicitly provide the information involving the bounds on switching interval.

Consider the following linear switched systems with time-varying switching delay:

$$\dot{x}(t)=A_{\sigma(t-\tau(t))}x(t). \tag{21}$$

In the following, we will provide a dwell-time dependent stability criterion for the linear switched system in (21).

Theorem 12. *Suppose that Assumption 2 holds. Assume that there exist positive definite matrices $P_{i,1},P_{i,2},i\in Q$, a positive constant $\varrho>1$, and constants λ_1, λ_2 such that (10) and the following conditions hold:*

$$\begin{aligned}&A_{\sigma(t-\tau(t))}^TP_{\sigma(t),l}+P_{\sigma(t),l}A_{\sigma(t-\tau(t))}+\lambda_{\sigma(t-\tau(t))}P_{\sigma(t),l}+\frac{P_{\sigma(t),1}-P_{\sigma(t),2}}{\mu_m}\beta(t)<0,\\&P_{i,2}<\varrho P_{j,1},\quad i,j\in Q,\end{aligned} \tag{22}$$

where $l,m=1,2$. λ_1 and λ_2 are the same as the ones in Theorem 8. Then the linear switched system in (21) with time-varying switching delay is exponentially stable.

Proof. Define by

$$\beta(t)=\frac{t_{k+1}-t}{t_{k+1}-t_k},\quad t\in[t_k,t_{k+1}). \tag{23}$$

Obviously, we can conclude that $\beta(t_k)=1$. Note that $t_k^-\in[t_{k-1},t_k)$, $\beta(t_k^-)=t_k-t_k^-/t_k-t_{k-1}=0$ due to the continuity of time domain. Denote by $P_{\sigma(t)}(t)=(1-\beta(t))P_{\sigma(t),l}+\beta(t)P_{\sigma(t),2}$. Then, construct the following Lyapunov function.

$$V_{\sigma(t)}(x(t))=x^T(t)P_{\sigma(t)}(t)x(t). \tag{24}$$

Then the derivative of $V_{\sigma(t)}(x(t))$ along system (21) is as follows,

$$\begin{aligned}\dot{V}_{\sigma(t)}(x(t)) &= x^T(t)P_{\sigma(t)}(t)A_{\sigma(t-\tau(t))}x(t) + x^T(t)A^T_{\sigma(t-\tau(t))}P_{\sigma(t)}(t)x(t) \\ &\quad + x^T(t)\dot{P}_{\sigma(t)}(t)x(t) \\ &= x^T(t)P_{\sigma(t)}(t)A_{\sigma(t-\tau(t))}x(t) + x^T(t)A^T_{\sigma(t-\tau(t))}P_{\sigma(t)}(t)x(t) \\ &\quad + x^T(t)\frac{P_{\sigma(t),1} - P_{\sigma(t),2}}{t_{k+1} - t_k}x(t).\end{aligned} \tag{25}$$

Note that $\mu_1 \le t_{k+1} - t_k \le \mu_2$. Then

$$\frac{1}{\mu_2} \le \frac{1}{t_{k+1} - t_k} \le \frac{1}{\mu_1}. \tag{26}$$

Therefore,

$$\begin{aligned}\frac{P_{\sigma(t),1} - P_{\sigma(t),2}}{t_{k+1} - t_k} &= (1 - \nu(t))\frac{P_{\sigma(t),1} - P_{\sigma(t),2}}{\mu_1} \\ &\quad - \nu(t)\frac{P_{\sigma(t),1} - P_{\sigma(t),2}}{\mu_2},\end{aligned} \tag{27}$$

where $\nu(t)$: $[0,\infty) \longrightarrow [0,1]$. From (25)–(27), we get

$$\begin{aligned}\dot{V}_{\sigma(t)}(x(t)) \le x^T(t)\Big\{&1 - \alpha(t)\left[A^T_{\sigma(t-\tau(t))}P_{\sigma(t),1}(t) + P_{\sigma(t),1}A_{\sigma(t-\tau(t))}\right] + \alpha(t)\left[A^T_{\sigma(t-\tau(t))}P_{\sigma(t),2}(t) + P_{\sigma(t),2}A_{\sigma(t-\tau(t))}\right] \\ &+ \varrho(t)\frac{P_{\sigma(t),1} - P_{\sigma(t),2}}{\mu_2} + (1 - \varrho(t))\frac{P_{\sigma(t),1} - P_{\sigma(t),2}}{\mu_1}\Big\}.\end{aligned} \tag{28}$$

From (22) and the well-known Schur complement lemma, it is easy to see that

$$\dot{V}_{\sigma(t)}(x(t)) \le -\lambda_{\sigma(t-\tau(t))}V_{\sigma(t)}(x(t)). \tag{29}$$

Then by repeating the similar steps in Theorem 8 and by considering $u(t) = 0$, we can conclude that the linear switched system in (21) with time-varying switching delay is exponentially stable. This completes the proof.

In Theorem 12, if the switching delay is overlooked, then the following dwell-time dependent stability criterion for linear switched system can be obtained directly, which has been investigated in [20]. □

Corollary 13. *Assume that there exist positive definite matrices $P_{i,1}$, $P_{i,2}$, $i \in Q$, a positive constant $\varrho > 1$, and constants λ_1, λ_2 such that (11) and the following conditions hold:*

$$\begin{aligned}&A_i^T P_{i,l} + P_{i,l}A_i + \lambda_i P_{i,l} + \frac{P_{i,1} - P_{i,2}}{\mu_m} < 0, \\ &\epsilon = \frac{\ln \varrho}{\mu_1} - \lambda_1 + (\lambda_1 - \lambda_2)d < 0, \\ &P_{i,2} < \varrho P_{j,1}, \quad i, j \in Q,\end{aligned} \tag{30}$$

where $l, m = 1, 2.\lambda_1 = \min_{j\in Q_S}\{\lambda_j\}.\lambda_1 = \max_{q\in Q_u}\{\lambda_q\}$. Then the linear switched system in (21) without time-varying switching delay is exponentially stable.

Remark 14. The time-dependent multiple Lyapunov function was proposed in [20] and it was shown that it can explicitly provide more information on upper and lower bounds of switching interval, so as to obtain less

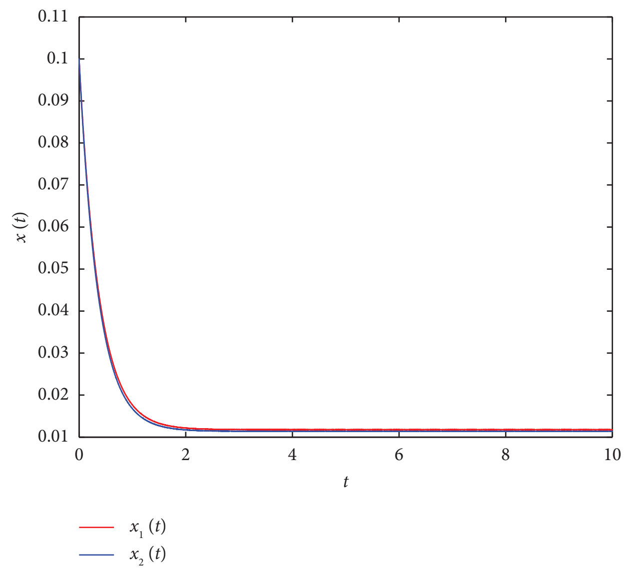

FIGURE 2: States of the subsystem 1 in (32).

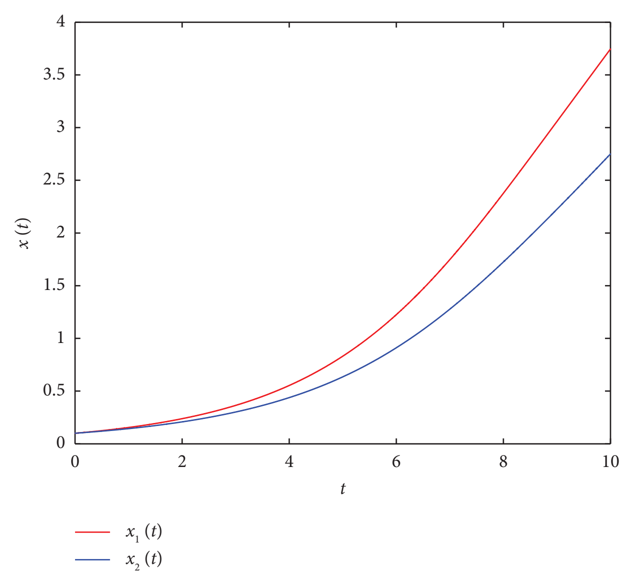

FIGURE 3: States of the subsystem 2 in (32).

conservative results. Compared with [20], the novelties of this paper can be summarized as follows:

(1) Model difference: The switched system with switching delay is considered in this paper. It can be seen from Figure 1 that due to the existence of switching delay, some number of ISS subsystems may be replaced by non-ISS subsystems, and some number of non-ISS subsystems may be replaced by ISS subsystems. It is known to all that, if too many ISS subsystems are replaced by non-ISS subsystems, then the ISS will be destroyed and if there are sufficiently many non-ISS subsystems are replaced by ISS subsystems, then the ISS can be enhanced. Therefore, it is very important to characterize how many ISS subsystems will be replaced by non-ISS subsystems under the switching signal $\sigma(t-\tau(t))$. In this paper, an assumption (Assumption 5) is proposed to characterize the ratio of the number of ISS subsystems that is replaced by non-ISS subsystems.

(2) Objective differences: The main aim of [20] is to investigate the quasi-consensus problem of nonlinear multiagent systems with both cooperation and competition interactions. The main purpose is to derive some sufficient conditions such that quasi-consensus is achieved. However, in this paper, our main aim is to investigate the ISS problem of switched systems with switching delay, where the system is subjected to disturbance input. The

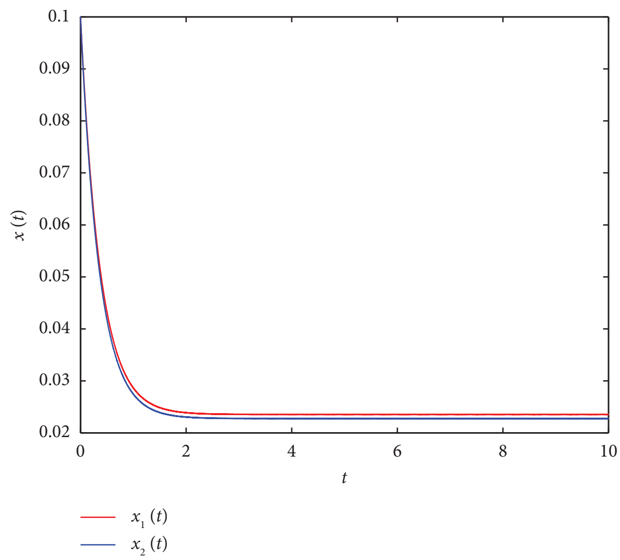

Figure 4: ISS of the switched system in (32) with time-varying switching delay.

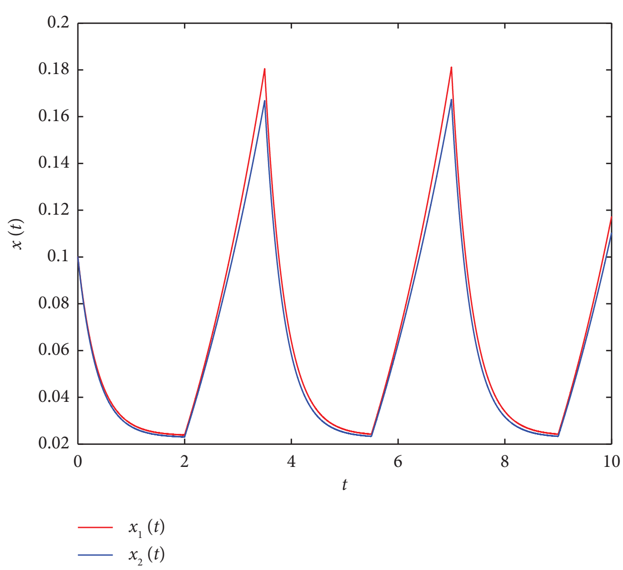

Figure 5: ISS of the switched system in (32) without time-varying switching delay.

existence of the disturbance makes the system cannot converge to zero, which also brings difficulties in our theoretical analysis.

4. Numerical Example

In this section, two examples will be given to illustrate the effectiveness of our theoretical results. The first example is given to verify the theoretical results of the paper. The second example is presented to show that our results can be applied to consensus of multi-agent systems with co-operation and competition interactions.

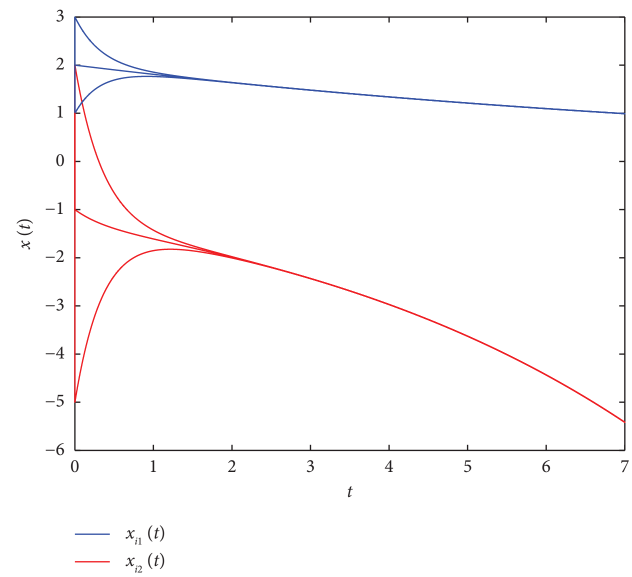

Figure 6: States of the multi-agent system in (33) with time-varying switching delay.

Example 1. In this example, we consider the following nonlinear switched system in (1) with switching delay.

$$\dot{x}(t) = f_{\sigma(t-\tau(t))}(x(t), u(t)), \quad (31)$$

where $f_{\sigma(t-\tau(t))}(x(t), u(t)) = A_{\sigma(t-\tau(t))}x(t) + g_{\sigma(t-\tau(t))}(x(t)) + B_{\sigma(t-\tau(t))}u(t)$. For simplicity, we assume that there are two modes and the parameters are chosen as follows:

$$\begin{aligned} A_1 &= \mathrm{diag}\{-3, -3\}, \\ A_2 &= \mathrm{diag}\{0.2, 0.2\}, \\ g_1(x(t)) &= \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 01 \end{bmatrix} \begin{bmatrix} \sin\ (x_1(t)) \\ \sin\ (x_2(t)) \end{bmatrix}, \\ B &= [0.1\, 0.1]^T, \\ g_2(x(t)) &= 0.8 * \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.1 \end{bmatrix} \begin{bmatrix} \sin\ (x_1(t)) \\ \sin\ (x_2(t)) \end{bmatrix}. \end{aligned} \quad (32)$$

The state trajectories of subsystems 1 and 2 are shown in Figures 2 and 3, respectively. From which, it can be seen that the subsystem 1 is ISS stable, while the subsystem 2 is non-ISS stable. The Lyapunov function is chosen as: $V_1(x(t)) = x_1^2 + x_1x_2 + x_2^2$, $V_2(x(t)) = x^T(t)x(t)$. Then, we can obtain $\mu_1 = 1.5, \lambda_1 = 2.1879$ and $\lambda_2 = -1.0587$. Let $T_S = 0.75$ and $d = 0.6$. Assuming that $a_k = 0.4$, $b = 0.2$ and $c = 0.5$. The switching signal is chosen as follows: $1 \longrightarrow 2 \longrightarrow 1 \longrightarrow \ldots$, where the activation time of ISS and non-ISS subsystems are 2 s and 1.5 s, respectively. Choosing $\tau(t) = 0.5|\sin(t)|$, then $\mu_{\tau 1} = 0.9597$ and $\mu_{\tau 2} = 2.422$. It is easy to see that $\sigma(t)$ satisfies Assumption 5. Letting $u(t) = \sin(t)|$, the state of the switched system in (31) with and without switching delay are plotted in Figures 4 and 5, respectively. It can be seen from Figures 4 and 5 that although both the ISS of the switching systems in (31) with and without switching delay can be guaranteed, the system with switching delay shows better convergence.

Example 2. In this example, we will show that our results can be applied to the consensus problem of multi-agent systems. Usually, the interactions among agents can be classified into two categories: one is called cooperation that is beneficial to consensus and the other is called competition that is harmful to consensus [20]. Assume that the dynamics of three linear agents are as follows:

$$\dot{x}(t) = \mathrm{A}x(t) + \mu_{i,\sigma(t-\tau(t))}, \quad i = 1, 2, 3, \tag{33}$$

where $A = \begin{bmatrix} -0.5 & 0 \\ 0 & 0.2 \end{bmatrix}$, $x_i \in R^2$ is the state the agent $i, u_i \in R^2$ is the control input. For simplicity, we also assume that there are two modes and the two kinds of protocols are designed as $\mu_{i,\sigma(t-\tau(t))} = \sum_{j=1}^{3}(x_i - x_j)$ if $\sigma(t-\tau(t)) \in Q_S$ and $\mu_{i,\sigma(t-\tau(t))} = \sum_{j=1}^{3}(x_j - x_i)$ if $\sigma(t-\tau(t)) \in Q_u$. Then the multi-agent system in (33) can be rewritten as the following compact form by using the Kronecker product technique:

$$\dot{x}(t) = (I_3 \otimes A)x(t) + \left(L_{\sigma(t-\tau(t))} \otimes I_n\right)x(t). \tag{34}$$

Choose $V_1(x(t)) = (x_1 - x_2)^2 + (x_2 - x_3)^2$ and $V_2(x(t)) = 1.2[(x_1 - x_2)^2 + (x_2 - x_3)^2]$, which leads to $\varrho = 1.4$. Then, we can get $\lambda_1 = 5.6$ and $\lambda_2 = -7.68$. For simplicity, we assume that there are two switching modes and the switching signals are the same as the ones in Example 1. According to Theorem 12, choosing $a_k = 0.5$, $b = 0.3$ and $c = 0.6$, we can get that the activation time of the cooperative interaction and competitive interaction be 2.5 s and 1 s, respectively. In addition, the switching delay is the same as the one in Example 1. Then the state trajectories of $x(t)$ are shown in Figure 6, which means that the consensus of multi-agent system in (33) with cooperative and competitive interaction can be reached.

5. Conclusion

In this paper, the ISS has been investigated for a class of nonlinear switched systems with time-varying switching delay, where ISS and non-ISS subsystems are considered simultaneously. Some sufficient conditions for ISS of switched systems with time-varying switching delay have been provided by using Lyapunv function based approach. The results are then extended into the linear switched systems with switching delay, and a time-dependent multiple Lyapunov function is constructed to obtain less conservative results mainly because that the useful information on lower and upper bounds of the switching interval can be provided by the time-dependent multiple Lyapunov function.

Data Availability

No underlying data was collected or produced in this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

[1] J. Hespanha, "Uniform stability of switched linear systems: extensions of lasalle's invariance principle," *IEEE Transactions on Automatic Control*, vol. 49, no. 4, pp. 470–482, 2004.

[2] A. Morse, "Supervisory control of families of linear set-point controllers Part I: exact matching," *IEEE Transactions on Automatic Control*, vol. 41, no. 10, pp. 1413–1431, 1996.

[3] R. Shorten, F. Wirth, O. Mason, K. Wulff, and C. King, "Stability criteria for switched and hybrid systems," *SIAM Review*, vol. 49, no. 4, pp. 545–592, 2007.

[4] Z. Wu, P. Shi, H. Su, and J. Chu, "Delay-dependent stability analysis for switched neural networks with time-varying delay," *IEEE Transactions on Systems, Man, and Cybernetics: B*, vol. 41, no. 6, pp. 1522–1530, 2011.

[5] L. Zhang and W. Xiang, "Mode-identifying time estimation and switching-delay tolerant control for switched systems: an elementary time unit approach," *Automatica*, vol. 64, pp. 174–181, 2016.

[6] H. Lin and P. J. Antsaklis, "Stability and stabilizability of switched linear systems: a survey of recent results," *IEEE Transactions on Automatic Control*, vol. 54, no. 2, pp. 308–322, 2009.

[7] H. Yang, B. Jiang, G. Tao, and D. Zhou, "Robust stability of switched nonlinear systems with switching uncertainties," *IEEE Transactions on Automatic Control*, vol. 61, no. 9, pp. 2531–2537, 2016.

[8] W. Zhang, Y. Tang, Q. Miao, and W. Du, "Exponential synchronization of coupled switched neural networks with mode-dependent impulsive effects," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 24, no. 8, pp. 1316–1326, 2013.

[9] M. A. Müller and D. Liberzon, "Input/output-to-state stability and statenorm estimators for switched nonlinear systems," *Automatica*, vol. 48, no. 9, pp. 2029–2039, 2012.

[10] H. Yang, B. Jiang, and V. Cocquempot, "A survey of results and perspectives on stabilization of switched nonlinear systems with unstable modes," *Nonlinear Analysis: Hybrid Systems*, vol. 13, pp. 45–60, 2014.

[11] X. Wu, Y. Tang, and W. Zhang, "Input-to-state stability of impulsive stochastic delayed systems under linear assumptions," *Automatica*, vol. 66, pp. 195–204, 2016.

[12] E. D. Sontag, "Smooth stabilization implies coprime factorization," *IEEE Transactions on Automatic Control*, vol. 34, no. 4, pp. 435–443, 1989.

[13] W. Xie, C. Wen, and Z. Li, "Input-to-state stabilization of switched nonlinear systems," *IEEE Transactions on Automatic Control*, vol. 46, no. 7, pp. 1111–1116, 2001.

[14] A. Kundu, D. Chatterjee, and D. Liberzon, "Generalized switching signals for input-to-state stability of switched systems," *Automatica*, vol. 64, pp. 270–277, 2016.

[15] L. Zhang and H. Gao, "Asynchronously switched control of switched linear systems with average dwell time," *Automatica*, vol. 46, no. 5, pp. 953–958, 2010.

[16] M. Wakaiki and Y. Yamamoto, “Stabilization of switched linear systems with quantized output and switching delays,” *IEEE Transactions on Automatic Control*, vol. 62, no. 6, pp. 2958–2964, 2017.

[17] X. Zhao, P. Shi, and L. Zhang, “Asynchronously switched control of a class of slowly switched linear systems,” *Systems and Control Letters*, vol. 61, no. 12, pp. 1151–1156, 2012.

[18] L. Hetel, J. Daafouz, and C. Iung, “Stability analysis for discrete time switched systems with temporary uncertain switching signal,” in *Proceedings of the 46th IEEE Conference Decision and Control*, pp. 5623–5628, New Orleans, LA, USA, August, 2007.

[19] G. Zhai, B. Hu, K. Yasuda, and A. N. Michel, “Stability analysis of switched systems with stable and unstable subsystems: an average dwell time approach,” *International Journal of Systems Science*, vol. 32, no. 8, pp. 1055–1061, 2001.

[20] W. Zhang, D. W. C. Ho, Y. Tang, and Y. Liu, “Quasi-consensus of heterogeneous-switched nonlinear multiagent systems,” *IEEE Transactions on Cybernetics*, vol. 50, no. 7, pp. 3136–3146, 2020.

[21] C. Briat, “Convex conditions for robust stability analysis and stabilization of linear aperiodic impulsive and sampled-data systems under dwell- time constraints,” *Automatica*, vol. 49, no. 11, pp. 3449–3457, 2013.

[22] Y. Wang, X. Sun, and B. Wu, “Lyapunov-Krasovskii functionals for switched nonlinear input delay systems under asynchronous switching,” *Automatica*, vol. 61, pp. 126–133, 2015.

[23] G. Xie and L. Wang, “Stabilization of switched linear systems with time- delay in detection of switching signal,” *Journal of Mathematical Analysis and Applications*, vol. 305, no. 1, pp. 277–290, 2005.

[24] J. Zhao and D. J. Hill, “On stability, L2-gain and H_1 control for switched systems,” *Automatica*, vol. 44, no. 5, pp. 1220–1232, 2008.

[25] T. Ako, H. Nishiyama, N. Ansari, and N. Kato, “A novel multichannel streaming scheme to reduce channel switching delay in application layer multicast,” *IEEE Systems Journal*, vol. 5, no. 4, pp. 545–554, 2011.

[26] W. Feng, J. Tian, and P. Zhao, “Stability analysis of switched stochastic systems,” *Automatica*, vol. 47, no. 1, pp. 148–157, 2011.