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Research Article

Applications of AG-Groupoids in Decision-Making via Linear Diophantine Fuzzy Sets

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In this paper, we investigated the notion of a linear Diophantine fuzzy set (LDFS) by using the concept of a score function to build the LDF-score left (right) ideals and LDF-score (0,2)-ideals in an AG-groupoid. We used these newly developed LDF-score ideals to characterize an AG-groupoid. We then use the proposed structure in multiattribute decision-making by considering bridge design selection and artificial intelligence-based chatbot selection.

1. Introduction

The commutative law in ternary operations is given by abc = cba. By adding brackets on the left of this equation, that is, $(ab)c = (cb)a$, Kazim and Naseeruddin proposed a novel algebraic structure called an LA-semigroup (left almost semigroup) [1]. The left invertive law identifies this identity. An AG-groupoid (Abel–Grassmann's groupoid) was coined by Protic and Stevanovic to describe the same structure [2]. This nonassociative (noncommutative) algebraic structure falls between a groupoid and a commutative semigroup [3]. In [1], it was shown that an AG-groupoid S is medial; that is, $(ab)(cd) = (ac)(bd)$ holds for all $a, b, c, d \in S$. A left identity may or may not exist in an AG-groupoid. The left identity of an AG-groupoid allows the inverses of elements. If an AG-groupoid has a left identity, then it is unique [3]. An AG-groupoid with a left identity is called an AG-group if it has inverses [4]. The paramedial law $(ab)(cd) = (dc)(ba)$ holds for all $a, b, c, d \in S$ in an AG-groupoid S with a left identity. We can get $a(bc) = b(ac)$ for all $a, b, c \in S$ by applying the medial law with the left identity. For the interest of the reader, some recent applications of AG-groupoids in decision-making can be found, for example, in [5, 6].

In traditional set theory, an element is either in or out of the set. Fuzzy set theory, on the other hand, allows for the gradual determination of the membership of elements in a set, which is represented using a membership function having a value in the real unit interval [0, 1]. To deal with real-world uncertain and ambiguous problems, strategies commonly used in classical mathematics are not always useful. In 1965, Zadeh [7] proposed the concept of a fuzzy set (FS) as an extension of the classical notion of sets. In many cases, however, because the membership function is a single-valued function, it cannot be used to represent both support and objection evidence. The intuitionistic fuzzy set (IFS), which is a generalization of Zadeh's fuzzy set, was introduced by Atanassov [8]. IFS has both a membership and a nonmembership function, allowing it to better express the fuzzy character of data than Zadeh's fuzzy set, which only has a membership function. In some real-life scenarios, however, the sum of membership and nonmembership degrees acquired by alternatives satisfying a decision-maker (DM) characteristic may be larger than 1, while their sum of squares is less than or equal to 1. Therefore, Yager [9] introduced the idea of the Pythagorean fuzzy set (PFS) with membership and nonmembership degrees that fulfill the condition that the total squares of their membership and nonmembership degrees are less than or equal to 1. By Atanassov [10], PFS is also known as IFS of type 2. Many scholars have researched another model known as a q-rung orthopair fuzzy set (q-ROFS) to expand the space of IFS and PFS [11–13].

In real life, variations in the cycle (periodicity) of the data happen simultaneously as vagueness and uncertainty in the data. The existing concepts and approaches available for the fuzzy information were not capable of dealing with membership and nonmembership functions taken from any part of the domain; instead, they impose strict conditions on them, resulting in some information loss during the process. To overthrow it, the concept of linear Diophantine fuzzy sets (LDFSs) was given in [14] to express uncertainty in decision-making. LDFS is more versatile and dependable than current ideas such as intuitionistic fuzzy sets (IFSs), Pythagorean fuzzy sets (PFSs), and q-rung orthopair fuzzy sets (q-ROFSs) because it includes reference or control factors with membership and nonmembership functions. Almagrabi et al. suggested a new generalization of the Pythagorean fuzzy set, q-rung orthopair fuzzy set, and linear Diophantine fuzzy set, named q-linear Diophantine fuzzy set (q-LDFS), and analyzed its key properties [15].

The main aim or motivation of this paper is to develop an algorithm for multiattribute decision-making by considering LDF-score (0,2)-ideals of AG-groupoids. An ideal of an AG-groupoid decomposes it, which makes it easy to study the characteristics of the AG-groupoid. Besides the (0,2)-ideals, some other ideals include interior ideals, bi-ideals, etc. To construct an LDF-score (0,2)-ideal, the concept of LDFSs is considered, and some characterization problems in terms of this ideal are constructed. Then, an algorithm to rank alternatives of a decision-making problem via the LDF-score (0,2)-ideal is developed. In the end, some practical applications of LDFNs are also thoroughly discussed.

There are a total of six sections in this paper. In Section 1, a brief introduction of AG-groupoids is given along with the historical literature review of fuzzy set theory. Section 2 provides the basic definitions to develop an understanding of the forthcoming sections. Section 3 deals with some novel results regarding the structural properties of LDF-score (0,2)-ideals and also provides the algorithm for multiattribute decision-making with the help of the LDF-score (0,2)-ideals. In Section 4, some real-life applications of the proposed algorithm are given. In Section 5, a discussion and comparison of various spaces of fuzzy sets are given in detail, and in Section 6, a comprehensive conclusion comprising the summary, limitations, and future work is given.

2. Preliminaries

In this section, we discuss the score and accuracy functions for the comparative analysis of linear Diophantine fuzzy numbers (LDFNs). Note that the concept of LDFS is similar to that of the well-known linear Diophantine equation $\alpha s + \beta t = \gamma$ from number theory.

Definition 1 (see [14]). Let S be a nonempty reference set. A linear Diophantine fuzzy set (LDFS) is an object of the form:

$$A = \{(s, \langle f_A(s), g_A(s)\rangle, \langle \alpha_A, \beta_A\rangle) : s \in S\}, \quad (1)$$

where $f_A(s)$ and $g_A(s)$ are real-valued membership and nonmembership functions, respectively, such that

$$f_A(s), g_A(s), \alpha_A, \beta_A \in [0,1] \text{ and } \alpha_A + \beta_A \leq 1, \quad (2)$$

satisfying

$$0 \leq f_A(s)\alpha_A + g_A(s)\beta_A \leq 1. \quad (3)$$

For convenience, let $A = (\langle f_A, g_A\rangle, \langle \alpha_A, \beta_A\rangle)$ be a linear Diophantine fuzzy number (LDFN), where α_A and β_A are reference parameters. These reference parameters can contribute to the categorization of a particular system. By altering the physical meaning of these parameters, we can categorize the system.

In order to rank the LDFNs, we now propose the idea of the score function as follows.

Definition 2. Let S be an AG-groupoid and A be the set of all LDFNs. The score function on A can be defined by the mapping ξ: $A \longrightarrow [-1, 1]$ and given by

$$\xi(A) = |f_A - g_A| - |\alpha_A - \beta_A|, \quad (4)$$

where $A = (\langle f_A, g_A\rangle, \langle \alpha_A, \beta_A\rangle)$, ξ is the score function, and $\xi(A)$ is the score of A.

In particular, if $\xi(A) = 1$, then the LDFN $A = (\langle f_A, g_A\rangle, \langle \alpha_A, \beta_A\rangle)$ takes the largest value $f_A = 1$, $g_A = 0$, and $\alpha_A = \beta_A$ or $g_A = 1$, $f_A = 0$, and $\alpha_A = \beta_A$. On the other hand, if the score function attains the minimum value, i.e., if $\xi(A) = -1$, then the LDFN $A = (\langle f_A, g_A\rangle, \langle \alpha_A, \beta_A\rangle)$ takes the smallest value $\alpha_A = 1, \beta_A = 0$ and $f_A = g_A$ or $\beta_A = 1, \alpha_A = 0$ and $f_A = g_A$.

Let A and B be two LDFSs on a domain U;then, for $s \in U$, $A(s)$ and $B(s)$ are LDFNs. Let $A(s)$ and $B(s)$ be given as follows:

$$\begin{aligned} A(s) &= (\langle 0.4, 0.6\rangle, \langle 0.7, 0.1\rangle),\\ B(s) &= (\langle 0.8, 0.4\rangle, \langle 0.7, 0.2\rangle), \end{aligned} \quad (5)$$

and then by Definition 2, $\xi(A)(s) = -0.4$ and $\xi(B)(s) = -0.1$, implying that $\xi(B)(s) \geq \xi(A)(s)$.

If we now define another LDFN, $C(s)$ is as follows:

$$C(s) = (\langle 0.9, 0.7\rangle, \langle 0.8, 0.2\rangle), \quad (6)$$

and then again by Definition 2, $\xi(C)(s) = -0.4$. From here, we see $\xi(A)(s) = \xi(C)(s)$. To distinguish score-equivalent LDFNs, we give the following definition.

Definition 3. Let S be an AG-groupoid and A be the set of all LDFNs. The accuracy function on A can be defined by the mapping ϱ: $A \longrightarrow [0, 1]$ and given by

$$\varrho(A) = \frac{|f_A + \alpha_A - (g_A + \beta_A)|}{2}, \quad (7)$$

where $A = (\langle f_A, g_A \rangle, \langle \alpha_A, \beta_A \rangle)$, ϱ is the accuracy function, and $\varrho(A)$ is the accuracy degree of A.

Considering the same LDFN $A(s)$ and $C(s)$ given above, by Definition 3, we see that $\varrho(A)(s) = 0.2$ and $\varrho(C)(s) = 0.4$, implying that, although $\xi(A)(s) = \xi(C)(s)$, we have $\varrho(C)(s) \geq \varrho(A)(s)$.

The relationship between the score function and the accuracy function has been established to be similar to the relationship between the mean and variance in statistics [16]. In statistics, an efficient estimator is described as a measure of the variance of an estimate's sampling distribution; the lower the variance, the better the estimator's performance. On this basis, it is reasonable and appropriate to say that the higher an LDFN's accuracy degree, the better the LDFN. In [17, 18], the techniques were developed for comparing and rating two IFNs and IVIFNs, respectively, based on the score and accuracy functions, which were motivated by the aforementioned study. We can now compare and rate two LDFNs, in the same way, using the score and accuracy functions, as shown below.

Definition 4. Let $A_k = (\langle f_{A_k}, g_{A_k} \rangle, \langle \alpha_{A_k}, \beta_{A_k} \rangle)$; $(k = 1, 2)$, and then, the comparison of A_1 and A_2 is given as follows:

(i) If $\xi(A_1) < \xi(A_2)$, then $A_1 < A_2$

(ii) If $\xi(A_1) = \xi(A_2)$, then

If $\varrho(A_1) < \varrho(A_2)$, then $A_1 < A_2$

If $\varrho(A_1) = \varrho(A_2)$, then $A_1 = A_2$.

Remark 1. Let A be a set of all LDFNs and $\xi\colon A \longrightarrow [-1, 1]$ be a score function, and then, we have the following:

(i) $\xi(A)$ increases with respect to the membership functions f_A and α_A

(ii) $\xi(A)$ decreases with respect to the nonmembership functions g_A and β_A.

3. LDF-Score (0,2)-Ideals of an AG-Groupoid

In this section, we introduced the concepts of linear Diophantine fuzzy score left (right) ideals and linear Diophantine fuzzy score (LDF-score) $(0, 2)$-ideals in an AG-groupoid using the notion of a score function. We intended to answer a question about the connection between LDF-idempotent subsets of an AG-groupoid S and its LDF-score (0,2)-ideals, particularly when an LDF-idempotent subset of S is an LDF-score (0,2)-ideal in terms of an LDF-score right ideal and an LDF-score left ideal of S. Some characterization problems are also provided in terms of LDF-score (0,2) -ideals. Finally, we give a method to rank different alternatives on the basis of given reference parameters.

3.1. Characterization Problems. Note that the results of this section can be followed simply for the case of fuzzy sets, which will be an extension of the results obtained in [19, 20].

If S is an AG-groupoid with the product $\cdot\colon S \times S \longrightarrow S$, then $ab \cdot c = (ab)c$ and $a \cdot bc = a(bc)$, andboth will denote the product $(a \cdot b) \cdot c$ and $a \cdot (b \cdot c)$. Similarly, $ab \cdot cd = (ab)(cd)$ will denote the product $(a \cdot b) \cdot (c \cdot d)$.

Definition 5 (see [21]). An AG-groupoid S is called a regular AG-groupoid if for each $a \in S$, there is an $s \in S$, with $a = \mathrm{as} \cdot a$.

Definition 6 (see [21]). An AG-groupoid S is called a strongly regular AG-groupoid if for each $a \in S$, there exists $s \in S$, such that $a = \mathrm{as} \cdot a$ and $\mathrm{as} = \mathrm{sa}$.

Definition 7 (see [22]). A completely inverse AG-groupoid S is an AG-groupoid satisfying the identity $\mathrm{as} = \mathrm{sa}$, where s is an inverse of a; that is, $a = \mathrm{as} \cdot a$ and $s = \mathrm{sa} \cdot s$ for all $a \in S$.

Let S be an AG-groupoid. Then, $V(a) = \{s \in S / a = as \cdot a, s = \mathrm{sa} \cdot s, \mathrm{as} = \mathrm{sa}\}$ represents a collection of complete inverses of $a \in S$.

An AG**-groupoid is an AG-groupoid S if $a \cdot bc = b \cdot ca$ for all $a, b, c \in S$. The paramedial law is also satisfied by an AG**-groupoid. It is worth noting that an AG**-groupoid is the generalization of an AG-groupoid with the left identity since an AG-groupoid with the left identity is an AG**-groupoid, but not the other way around.

Lemma 1. *An* AG***-groupoid S is strongly regular if and only if $V(a) \neq \varnothing$ for all $a \in S$.*

Proof. Necessity. Let $a \in S$. Then, there exists some $s \in S$ such that $a = \mathrm{as} \cdot a$ and $\mathrm{as} = \mathrm{sa}$. Now, $= (\mathrm{as} \cdot a)s \cdot s = (\mathrm{sa} \cdot \mathrm{as})s = (\mathrm{as} \cdot \mathrm{as})s = (s \cdot \mathrm{as})(\mathrm{as}) = (\mathrm{as})(\mathrm{as} \cdot s) = ((\mathrm{as} \cdot a)s)(\mathrm{as} \cdot s) = (\mathrm{sa} \cdot \mathrm{as})(\mathrm{as} \cdot s) = (\mathrm{as} \cdot \mathrm{as})(\mathrm{as} \cdot s) = ((\mathrm{as} \cdot s)a)(\mathrm{as} \cdot s)$. It is easy to prove that $a(\mathrm{as} \bullet s) \bullet a = a$ and $(\mathrm{as} \cdot s)a = a(\mathrm{as} \cdot s)$, which implies that $V(a) \neq \varnothing$ for all $a \in S$ sufficiency. It is obvious. □

Corollary 1. *An AG-groupoid S with the left identity is strongly regular if and only if $V(a) \neq \varnothing$ for all $a \in S$.*

The proof of the following two lemmas is the same as in [23].

Lemma 2. *Let S be an AG-groupoid. For $\varnothing \neq A, B \subseteq S$, the following holds.*

(i) $\mathbb{C}_A \cap \mathbb{C}_B = \mathbb{C}_{A \cap B}$

(ii) $\mathbb{C}_A \circ \mathbb{C}_B = \mathbb{C}_{AB}$.

Lemma 3. *If ξ is any score function of an AG-groupoid S, then ξ is an LDF-score right* (left) *ideal of S if and only if $\xi \circ S \subseteq \xi \circ (S \circ \xi \subseteq \xi)$.*

Definition 8 (see [24]). A non-empty subset A of an AG-groupoid S is called a $(0, 2)$ -ideal of S if $SA^2 \subseteq A$.

Definition 9. Let τ be a score function of an AG-groupoid S and $s, y, z \in S$. Then, τ is called an LDF-score (0,2)-ideal of S if

$$\tau(s \cdot yz) \geq \tau(y) \wedge \tau(z). \quad (8)$$

The proof of the following two lemmas is the same as in [25].

Lemma 4. *If τ is any score function of an AG-groupoid S, then τ is an LDF-score (0,2)-ideal of S if and only if $S \circ \tau^2 \subseteq \tau$.*

Lemma 5. *Let S be an AG-groupoid and $\varnothing \neq O \subseteq S$. Then, O is a $(0,2)$-ideal of S if and only if $\mathbb{C}_O$ is an* LDF-score *$(0,2)$-ideal of S.*

Theorem 1. *Let τ be an* LDF-*idempotent subset of an AG-groupoid S with the left identity. Then, the following conditions are equivalent:*

(i) $\tau = \xi \circ \sigma$, where ξ is an LDF-*score right ideal and σ is an* LDF-*score left ideal of S*

(ii) τ is an LDF-*score $(0,2)$-ideal of S.*

Proof. $(i) \Longrightarrow (ii)$: We can obtain the following by using Lemma 3:

$$\begin{aligned} S \circ \tau^2 &= (S \circ S)\circ(\tau \circ \tau) = (S \circ \tau)\circ(S \circ \tau) = (S \circ (\xi \circ \sigma))\circ(S \circ (\xi \circ \sigma)) \\ &= (\xi \circ (S \circ \sigma)) \circ ((S \circ S) \circ (\xi \circ \sigma)) \subseteq (\xi \circ S)\circ((\sigma \circ \xi) \circ (S \circ S)) \\ &\subseteq \xi \circ ((S \circ \xi) \circ \sigma) \subseteq \xi\circ(S \circ \sigma) \subseteq \xi\circ\sigma = \tau. \end{aligned} \quad (9)$$

As a result of Lemma 4, τ is an LDF-score $(0,2)$-ideal of S.

$(ii) \Longrightarrow (i)$: Setting $\sigma = S \circ \tau$ and $\xi = S \circ \tau^2$ and then using Lemma 4, we obtain

$$\begin{aligned} \xi \circ \sigma &= \left(S \circ \tau^2\right)\circ(S \circ \tau) = (\tau \circ S)\circ\left(\tau^2 \circ S\right) = (\tau \circ \tau)\circ((\tau \circ S) \circ S) \\ &= (S \circ (\tau \circ S)) \circ \tau \subseteq S\circ\tau^2 \subseteq \tau = \tau\circ\tau = \left(\tau \circ \tau^2\right)\circ(\tau \circ \tau) \\ &\subseteq \left(S \circ \tau^2\right)^{\circ}(S \circ \tau) = \xi\circ\sigma. \end{aligned} \quad (10)$$

This is what we set out to show. □

Remark 2. Assume that S is an AG-groupoid with the left identity and $a \in S$. The smallest $(0,2)$-ideal of S containing a^2 is $O_{a^2} = Sa^2$.

Theorem 2. *Assume that S is an AG-group. Then, the following conditions are equivalent:*

(i) $V(a) \neq \varnothing$

(ii) $O_{a^2} = O_{a^2}^2$, where O_{a^2} is the smallest $(0,2)$-ideal of S containing a^2

(iii) $O_1 \cap O_2 = O_2^2 O_1^2$, where both O_1 and O_2 are any $(0,2)$-ideals of S

(iv) $\xi \cap \sigma = \sigma^{2\circ}\xi^2$, where both ξ and σ are any LDF-*score (0,2)-ideals of S.*

Proof. $(i) \Longrightarrow (iv)$: Let ξ and σ be both LDF-score $(0,2)$-ideals of S with the left identity e such that $V(a) \neq \varnothing$. Now, for $a \in S$, there exists some $s \in S$ such that

$$\begin{aligned} a &= \text{as} \cdot a = (\text{as} \cdot a)s \cdot (\text{as})(\text{sa} \cdot a) = (\text{sa} \cdot \text{as})\left(\text{as} \cdot a^2 s\right) \\ &= a(\text{sa} \cdot s) \cdot a^2(\text{as} \cdot s) = (\text{sa} \cdot a)(\text{sa} \cdot s) \cdot \left(a^2 \cdot s^2 a\right) \\ &= (\text{aa} \cdot \text{es})(\text{sa} \cdot s) \cdot \left(\text{as}^2 \cdot a^2\right) \\ &= (\text{se} \cdot \text{aa})\left(s^2(\text{as} \cdot a)\right) \cdot \left(a^2 s^2\right)(\text{sa} \cdot a) \\ &= (\text{se} \cdot \text{aa})\left(\text{as} \cdot s^2 a\right) \cdot (\text{aa} \cdot \text{ss})(\text{sa} \cdot a) \\ &= (\text{se} \cdot \text{aa})\left(s^2(\text{sa} \cdot a)\right) \cdot (\text{ss} \cdot \text{aa})(\text{aa} \cdot \text{es}) \\ &= (\text{se} \cdot \text{aa})\left(s^2 \cdot a^2 s\right) \cdot \left(s^2 \cdot \text{aa}\right)(se \cdot \text{aa}) \\ &= (se \cdot \text{aa})\left(ss^2 \cdot \text{aa}\right) \cdot \left(s^2 \cdot \text{aa}\right)(se \cdot \text{aa}) = st \cdot uv, \\ \left(\sigma^2\circ\xi^2\right)(a) &= \left(\left(\sigma^{\circ}\xi\right)^{\circ}\left(\sigma^{\circ}\xi\right)\right)(a) = \bigvee_{a=st\cdot uv}\left\{\left(\sigma^{\circ}\xi\right)(st) \wedge \left(\sigma^{\circ}\xi\right)(uv)\right\} \\ &\geq \sigma(a) \wedge \xi(a). \end{aligned} \quad (11)$$

Thus, by using Lemma 4, we get $\xi \cap \sigma = \sigma^2\circ\xi^2$.

$(iv) \Longrightarrow (iii)$: Let O_1 and O_2 be any $(0,2)$-ideals of S. Then, by Lemma 5, $\mathbb{C}_{O_1}$ and $\mathbb{C}_{O_2}$ are the LDF-score $(0,2)$-ideals of S. Let $s \in O_1 \cap O_2$. Then, by using Lemma 2, we have

$$\begin{aligned} 1 &= \mathbb{C}_{O_1 \cap O_2}(s) = \left(\mathbb{C}_{O_1} \cap \mathbb{C}_{O_2}\right)(s) \\ &= \left(\mathbb{C}_{O_2^2}{}^{\circ}\mathbb{C}_{O_1^2}\right)(s) = \mathbb{C}_{O_2^2 O_1^2}(s), \end{aligned} \quad (12)$$

which implies that $a \in O_2^2 O_1^2$, and therefore, $O_1 \cap O_2 \subseteq O_2^2 O_1^2$. It is easy to see that $O_2^2 O_1^2 \subseteq O_1 \cap O_2$, and therefore, $O_1 \cap O_2 = O_2^2 O_1^2$.

$(iii) \Longrightarrow (ii)$: It is obvious.

$(ii) \Longrightarrow (i)$: Since Sa^2 is the smallest $(0,2)$-ideal of S containing a^2, therefore $a^2 \in Sa^2 = Sa^2 \cdot Sa^2 = a^2 S \cdot a^2 S = a^2(a^2 S \cdot S) = a^2(SS \cdot \text{aa}) = a^2 \cdot aS$, which implies that $a^2 = \text{aa} \cdot \text{as} = (\text{as} \cdot a)a$ for some $s \in S$. Thus, $a^2 = (\text{as} \cdot a)a \Longrightarrow (\text{aa})a' = ((\text{as} \cdot a)a)a' \Longrightarrow (a'a)a =$

$(a'a)(\text{as}\cdot a)\Longrightarrow a=\text{as}\cdot a$. Similarly, we can get $\text{as}=sa$. Hence, by Lemma 1, $V(a)\neq\varnothing$. □

Definition 10. An AG-groupoid S is called left zero if $sy=s$ for all $s,y\in S$.

Lemma 6. *Let S be a left zero AG-groupoid with the left identity and τ be a score function of S. Then, for any idempotent elements e and f of S, $\tau(e)=\tau(f)$.*

Proof. It is simple. □

Lemma 7. *The following conditions are equivalent for an AG-groupoid S with the left identity:*

(i) The set of all idempotent elements of S forms a left zero AG-subgroupoid of S

(ii) For every LDF-score (0, 2)-ideal τ of S, $\tau(e)=\tau(f)$ for all idempotent elements e and f of S.

Proof. $(i)\Longrightarrow(ii)$: It can be followed from Lemma 6.

$(ii)\Longrightarrow(i)$: Since S contains a left identity, it is obvious that $E_S\neq\varnothing$. If we consider $f\in E_S$, we can see that $Sf^2=Sf$ is the (0,2)-ideal of S. Using Lemma 5, the characteristic function $\mathbb{C}_{Sf}$ is an LDF-score (0,2)-ideal of S. As a result of the assumption, we get $\mathbb{C}_{Sf}(e)=\mathbb{C}_{Sf}(f)=1$, and hence, $e\in Sf$. Thus, for some $s\in S$, we have $e=s\cdot ff=f\cdot sf=fe=ef$. □

Lemma 8. *Every LDF-score (0,2)-ideal of an AG-group S is a constant function.*

Proof. If $a\in S$, then

$$\begin{aligned}\tau(a)&=\tau(\text{ea})=\tau(\text{ee}\cdot a)=\tau(\text{ae}\cdot\text{ee})\geq\tau(e)\wedge\tau(e)=\tau(\text{ee})=\tau\left(\text{aa}^{-1}\cdot\text{aa}^{-1}\right)\\&=\tau\left(\text{aa}\cdot a^{-1}a^{-1}\right)=\tau\left(a^{-1}a^{-1}\cdot\text{aa}\right)\geq\tau(a)\wedge\tau(a)=\tau(a).\end{aligned}\tag{13}$$

This indicates that $\tau(e)=\tau(a)$. As a conclusion, τ is a constant function. □

Theorem 3. *Let S be an AG-groupoid with the left identity such that $V(a)\neq\varnothing$. Then, the following conditions are equivalent:*

(i) S is an AG-groupoid

(ii) For every LDF-score (0,2)-ideal τ of S, $\tau(e)=\tau(f)$ for all idempotent elements e and f of S

(iii) The set of all idempotent elements of S forms a left zero AG-subgroupoid of S.

Proof. $(i)\Longrightarrow(ii)$: It can be simply followed from Lemma 8.

$(ii)\Longrightarrow(i)$: Since the characteristic function C_{Sf} is an LDF-score (0, 2)-ideal of S and $f\in Sf$, so $C_{Sf}(e)=C_{Sf}(f)=1$, and hence, $e\in Sf$, implying that $e=sf$ for some $s\in S$. Similarly, for some $y\in S$, one may obtain $f=\text{ye}$. Consequently, $e=sf=s\cdot ff=f\cdot sf=fe=ef=e\cdot\text{ye}=y\cdot\text{ee}=\text{ye}=f$. As $E_S=\{e\}$ and $V(a)\neq\varnothing$, we obtain $(\text{as})^2=\text{as}\cdot\text{as}=\text{sa}\cdot\text{as}=(\text{as}\cdot a)s=\text{as}$. Thus, $e=\text{as}=\text{sa}$. Hence, $ea=\text{as}\cdot a=a$. This shows that S is an AG-groupoid.

$(ii)\Longleftrightarrow(iii)$: It can be followed from Lemma 7. □

3.2. MADM through LDF-Score (0,2)-Ideals of AG-Groupoids. Life is all about making decisions. A lot of people avoid taking responsibility when faced with problems or making important decisions. The impact of decision-making comes in the way it helps you decide among various alternatives. Decision-making is a conceptual process that assists you in visualizing the implications of your choices. It enables you to determine the optimal strategy for achieving your goals and objectives, eventually determining your outcome. According to contemporary decision-making theory, the multiattribute decision-making (MADM) approach is important for addressing the significant problems in our everyday life.

It is supposed that a decision-maker (DM) must review and evaluate a set of alternatives with various characteristics. MADM seeks to identify or rate the most preferable alternatives in order to improve decision-making. Certain traditional methods, such as the consensus-based TOPSIS-sort-B method [26] and techniques for decision-making with multigranular unbalanced linguistic information [27], have been attempted to solve MADM problems.

Attribute values are required decision-making data in MADM situations. The attribute values represent the options' features, benefits, and abilities. Because of the complexity of the real world and humans' limited information and perceptual abilities, the values of attributes are unknown. As a result, decision-makers are unable to express their preferences or evaluations directly. Therefore, a mechanized mathematical algorithm is required to solve such a problem.

We now devise a MADM technique to see which alternative is a good choice for further analysis on the basis of the given reference parameters.

Let us assume that a decision maker is trying to make a decision for a collection of n alternatives. Then, the steps of MADM via the LDF-score (0,2)-ideal are broken down as follows:

Step 1: take the collection of alternatives and label them as

$$C=\{c_i: i=1,2,3\cdots,n\}.\tag{14}$$

Step 2: construct an AG-groupoid on collection C under a combination rule "∘"

Step 3: define the reference parameters α and β

Step 4: take the attributes with reference parameters α and β and define an LDFS on them

Step 5: define LDFNs on C

Step 6: create an LDF-score $(0,2)$-ideal of $(C,^\circ)$

Step 7: rank all the LDFNs by using the score function (use the accuracy function if scores are equal)

4. Applications of LDFNs and LDF-Score (0,2)-Ideals

We can utilize mathematical modeling for ranking different alternatives, but an LDFN approach is preferable to others due to the expanded space and unrestricted choice of the parameters α and β that are chosen based on the preferences of the decision-makers.

For instance, let us consider an example of an automated traffic signal at an intersection. Traffic lights are controlled by counting the number of cars waiting to cross on any side, and a fuzzy function bridges the gap between the number of cars and traffic light timing. Let us consider the scenario where the counting of cars can no longer happen due to camera malfunction. The automated system has the historic data on traffic at any specific time of the day and can keep using it to automate traffic lights. The problem is that the system is not being actively monitored and that there is always a doubt about the accuracy of implementing historic data to real-time problems. If this system is working with simple intuitionistic fuzzy sets of the type $\langle f_A, g_A\rangle$, there is no way to introduce uncertainty for the values of f_A and g_A. LDFS can solve this problem by introducing the reference parameters α and β.

4.1. Selection of Bridge Designs. In civil engineering, bridge construction is among the most demanding task a civil engineer is required to perform. Since ancient times till now, bridges have been used to cross rivers, valleys, and roadways, allowing people to travel between different parts of the country. Because each structure has distinct needs to meet, such as span clearance, traffic flow, geometry, and the peculiarities of the construction site, a wide range of bridges can be built. When it comes to creating a road network, a civil engineer's choice of bridge design is critical.

Assume that a construction company wants to construct bridges for a highway project. It wants to select the best construction design with lots of features and having less completion time. Let $\mathscr{C} = \{b_1, b_2, b_3\}$ be the set of some bridge designs elaborated as follows:

(i) $b_1 \longrightarrow$ Arch de sign: a design with support beams in the form of curved arches at each end

(ii) $b_2 \longrightarrow$ Truss de sign: a design whose superstructure or load-bearing portion is made up of connected triangle-shaped sections called trusses

(iii) $b_3 \longrightarrow$ Suspension de sign: a design that has its deck suspended on vertical suspenders from below suspension cables.

Table 1: Composition of $\mathscr{C}$ under the rule $\circledcirc$.

$\circledcirc$	b_2	b_1	b_3
b_2	b_2	b_2	b_2
b_1	b_2	b_2	b_3
b_3	b_2	b_1	b_2

Table 2: Reference parameters α and β for $(\mathscr{C},\circledcirc)$.

α	Adapts to environmental conditions
β	Environment effects the bridge

Table 3: LDFNs A_is along with scores, accuracies and rankings on $(\mathscr{C},\circledcirc)$.

b_i	A_i	$f_A(b_i)$	$g_A(b_i)$	$\alpha_A(b_i)$	$\beta_A(b_i)$	Score	Accuracy	Rank
b_1	A_2	0.9	0.4	0.3	0.2	0.4	0.35	2nd
b_2	A_1	1.0	0.2	0.5	0.4	0.7	0.45	1st
b_3	A_3	0.2	0.4	0.6	0.4	0.0	0.0	3rd

To take a decision that will rank the available alternatives according to the attribute "environmental condition," we will utilize an LDF-score (0,2) -ideal of an AG-groupoid. The rankings of the LDFNs associated with each design in the collection $\mathscr{C} = \{b_1, b_2, b_3\}$ will decide the rankings.

Let us consider the collection of bridge designs $\mathscr{C} = \{b_1, b_2, b_3\}$ under the combination rule $\circledcirc$ given in Table 1.

It is easy to see that $(\mathscr{C},\circledcirc)$ is an AG-groupoid.

Let us define the parameters α and β as in Table 2.

Let us define the set of LDFS $\mathscr{F}_1$ on $\mathscr{C}$ as follows:

$$\mathscr{F}_1 = \{A_i = (\langle f_A(b_i), g_A(b_i)\rangle \langle \alpha_A(b_i), \beta_A(b_i)\rangle); i = 1,2,3\}, \tag{15}$$

where the LDFNs A_i are defined in Table 3.

Table 3 shows that $\xi(\mathscr{F}_1)$ is an LDF-score (0,2)-ideal of $(\mathscr{C},\circledcirc)$.

All of the alternatives are sorted according to their respective scores. If two scores are equal, the accuracy function can be used to sort the alternatives. Figure 1 represents the visualization of the score, accuracy, and ranking comparison for $\mathscr{F}_1$.

The preferences of the alternatives based on an LDF-score $(0,2)$-ideal on $(\mathscr{C},\circledcirc)$ can be seen in Table 4.

Let us consider the same collection of bridge designs $\mathscr{C} = \{b_1, b_2, b_3\}$ under the combination rule $\ominus$ given in Table 5.

One can easily verify that $(\mathscr{C},\ominus)$ forms an AG-groupoid.

Defining an LDFS, we obtain

$$\mathscr{F}_2 = \{B_i = (\langle f_B(b_i), g_B(b_i)\rangle \langle \alpha_B(b_i), \beta_B(b_i)\rangle); i = 1,2,3\}, \tag{16}$$

on $\mathscr{C}$, and the corresponding values are listed in Table 6.

We can see that $\xi(\mathscr{F}_2)$ is an LDF-score $(0,2)$-ideal of $(\mathscr{C},\ominus)$. Figure 2 represents the visualization of score, accuracy, and ranking comparison for $\mathscr{F}_2$.

The preferences of the alternatives based on an LDF-score $(0,2)$-ideal on $(\mathscr{C},\ominus)$ can be seen in Table 7.

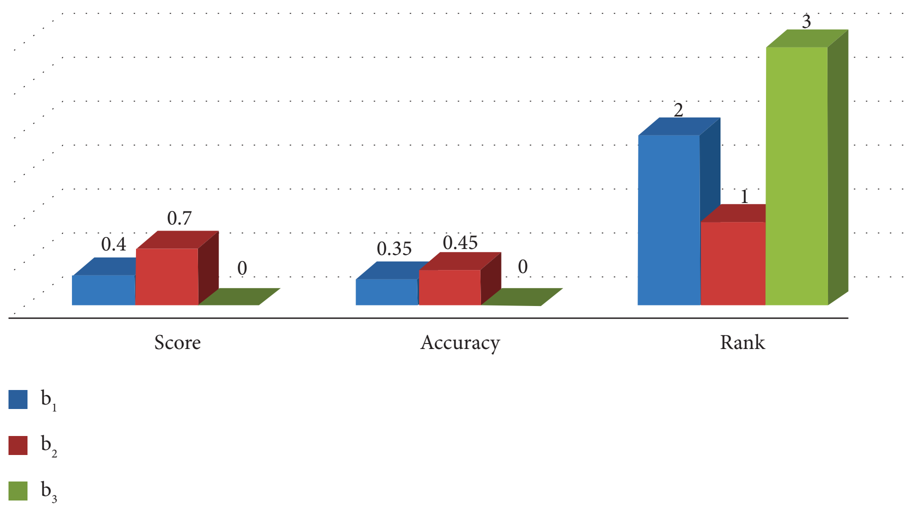

FIGURE 1: Score, Accuracy & Ranking Comparison for $\mathscr{F}_1$ on $(\mathscr{C}, \circledcirc)$.

TABLE 4: Rankings of bridge designs under $(\mathscr{C}, \circledcirc)$.

Rankings	1st	2nd	3rd
Bridge design	b_2	b_1	b_3

TABLE 5: Composition of $\mathscr{C}$ under the rule $\ominus$.

$\ominus$	b_2	b_1	b_3
b_2	b_1	b_3	b_2
b_1	b_2	b_1	b_3
b_3	b_3	b_2	b_2

TABLE 6: LDFNs B_is along with scores, accuracies and rankings on $(\mathscr{C}, \ominus)$.

b_i	B_i	$f_B(b_i)$	$g_B(b_i)$	$\alpha_B(b_i)$	$\beta_B(b_i)$	Score	Accuracy	Rank
b_1	B_2	0.4	0.6	0.6	0.2	−0.2	0.1	2nd
b_2	B_1	0.1	0.4	0.7	0.3	−0.1	. . .	1st
b_3	B_3	0.6	0.8	0.5	0.1	−0.2	0.1	2nd

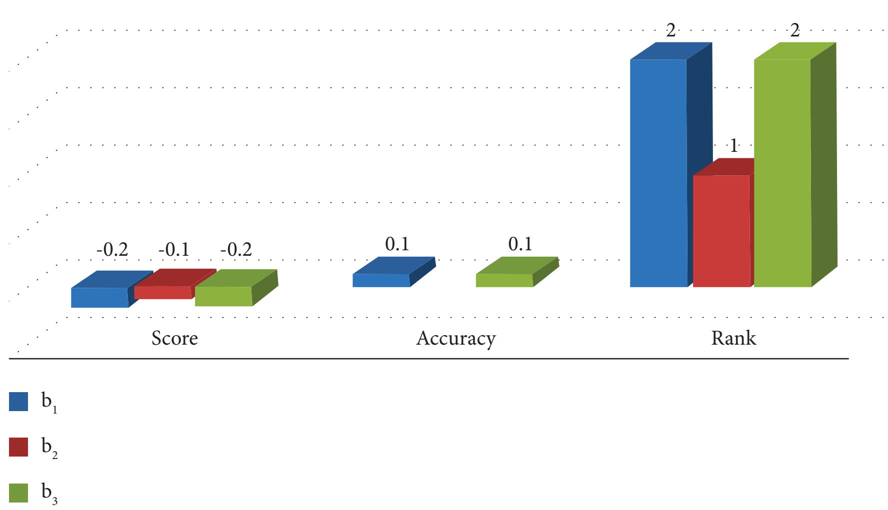

FIGURE 2: Score, Accuracy & Ranking Comparison for $\mathscr{F}_2$ on $(\mathscr{C}, \ominus)$.

Table 7: Rankings of bridge designs under $(\mathscr{C}, \ominus)$.

Rankings	1st	2nd	2nd
Bridge design	b_2	b_1	b_3

4.2. Selection of AI-Based Chatbots. Artificial intelligence (AI) chatbots use machine learning to interact with humans. Weizenbaum, an MIT scientist, created the first AI chatbot in the 1960s [28]. Chatbot technology has advanced substantially in recent years. It interacts with people on a personal and emotional level. Artificial intelligence-powered chatbots are revolutionizing customer care experience. They understand the context and meaning of words. They can use questions to elicit purpose and assist in the resolution of customer issues. The programs in chatbots analyze human speech and respond appropriately using modern natural language processing (NLP) algorithms.

Assume that an IT company is looking for the best AI chatbot for customer engagement and customer service. As a consequence, the company's top executives agreed to do a feasibility study on several AI bots.

The selection has to be made from the collection $\mathscr{B} = \{d_1, d_2, d_3, d_4, d_5\}$ of five chatbots elaborated as follows:

(i) $d_1 \longrightarrow$ Mitsuku: It is the world's most human-like talking bot. The chatbot has received the Loebner Prize for the most human-like conversation on many occasions.

(ii) $d_2 \longrightarrow$ Tidio: It distinguishes from its competitors by providing a solution that allows one to construct his own chatbots without requiring any coding skills.

(iii) $d_3 \longrightarrow$ ProProfs: It improves your company's customer service, marketing, and sales activities all at the same time.

(iv) $d_4 \longrightarrow$ Botsify: It is easy to use and does not require any coding knowledge. Advanced chatbots can be built for education, customer support, sales, and the HR department.

(v) $d_5 \longrightarrow$ MobileMonkey: It has advanced automation and integration assists to connect with the audience in real-time. Anyone with no coding experience can use the program to construct effective chatbots.

To take a decision that will rank the available alternatives according to the attribute "customer rating," we will utilize an LDF-score (0, 2)-ideal of an AG-groupoid. The rankings of the LDFNs associated with each bot in the collection $\mathscr{B}$ will decide the rankings.

Let us consider the collection of bots $\mathscr{B} = \{d_1, d_2, d_3, d_4, d_5\}$ under the combination rule $\boxplus$ given in Table 8.

One can easily verify that $(\mathscr{B}, \boxplus)$ forms an AG-groupoid.

Let us define the parameters α and β as shown in Table 9.

Now, we consider the following LDFS $\mathscr{F}_1$ on $\mathscr{B}$:

$$\mathscr{F}_1 = \{A_i = (\langle f_A(d_i), g_A(d_i)\rangle, \langle \alpha_A(d_i), \beta_A(d_i)\rangle); i = 1, 2, 3, 4, 5\}, \tag{17}$$

where A_i are LDFNs as given in Table 10.

Table 8: Composition of $\mathscr{B}$ under the rule $\boxplus$.

$\triangleright$	d_3	d_1	d_4	d_2	d_5
d_3	d_3	d_3	d_3	d_3	d_3
d_1	d_3	d_5	d_5	d_4	d_5
d_4	d_3	d_5	d_5	d_1	d_5
d_2	d_3	d_1	d_4	d_2	d_5
d_5	d_3	d_5	d_5	d_5	d_5

Table 9: Reference parameters α and β for chatbot selection.

α	Good customer rating
β	Bad customer rating

Table 10: LDFNs A_is along with scores, accuracies and rankings on $(\mathscr{B}, \boxplus)$.

d_i	A_i	$f_A(d_i)$	$g_A(d_i)$	$\alpha_A(d_i)$	$\beta_A(d_i)$	Score	Accuracy	Rank
d_1	A_2	0.5	0.5	0.6	0.6	0.0	...	4th
d_2	A_4	0.3	0.7	0.9	0.1	−0.4	...	5th
d_3	A_1	1	0	0.5	0.5	1.0	...	1st
d_4	A_3	0.7	0.1	0.5	0.1	0.2	0.55	3rd
d_5	A_5	0.9	0.1	0.8	0.2	0.2	0.7	2nd

It is obvious from the table that $\xi(\mathscr{F}_1)$ is an LDF-score (0, 2)-ideal of $(\mathscr{B}, \boxplus)$.

All of the alternatives are sorted according to their respective scores. If two scores are equal, the accuracy function can be used to sort the alternatives. Figure 3 represents the visualization of the score, accuracy, and ranking comparison for $\mathscr{F}_1$.

The preferences of the alternatives based on an LDF-score (0, 2)-ideal on $(\mathscr{B}, \boxplus)$ can be seen in Table 11.

Now, we consider the collection of bots $\mathscr{B} = \{d_3, d_1, d_4, d_2, d_5\}$ under the combination rule $\boxminus$ given in Table 12.

It is easy to verify that $(\mathscr{B}, \boxminus)$ is an AG-groupoid.

Now, we define the collection of LDFS on $\mathscr{B}$ as follows:

$$\mathscr{F}_2 = \{B_i = (\langle f_B(d_i), g_B(d_i)\rangle, \langle \alpha_B(d_i), \beta_B(d_i)\rangle); i = 1, 2, 3, 4, 5\}, \tag{18}$$

where B_i are LDFN and are provided in Table 13.

It is obvious from Table 13 that $\xi(\mathscr{F}_2)$ is an LDF-score (0, 2)-ideal of $(\mathscr{B}, \boxminus)$. Figure 4 represents the visualization of score, accuracy, and ranking comparison for $\mathscr{F}_2$.

The preferences of the alternatives based on an LDF-score (0, 2)-ideal on $(\mathscr{B}, \boxminus)$ can be observed in Table 14.

Again, we consider the collection of bots $\mathscr{B} = \{d_3, d_1, d_4, d_2, d_5\}$ under the combination rule $\boxtimes$ given in Table 15.

Clearly, $(\mathscr{B}, \boxtimes)$ forms an AG-groupoid.

Now, we define the collection of LDFS on $\mathscr{B}$ as follows:

$$\mathscr{F}_3 = \{C_i = (\langle f_C(d_i), g_C(d_i)\rangle, \langle \alpha_C(d_i), \beta_C(d_i)\rangle); i = 1, 2, 3, 4, 5\}, \tag{19}$$

where C_i are LDFN and are provided in Table 16.

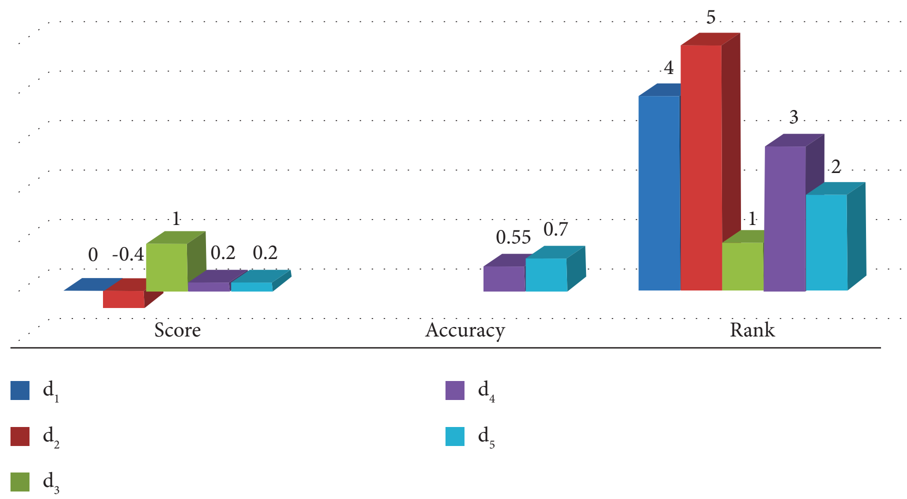

FIGURE 3: Score, Accuracy & Ranking Comparison for $\mathscr{F}_1$ on $(\mathscr{B},\boxplus)$.

TABLE 11: Rankings of chatbots under $(\mathscr{B},\boxplus)$.

Rankings	1st	2nd	3rd	4th	5th
Chatbots	d_3	d_5	d_4	d_1	d_2

TABLE 12: Composition of $\mathscr{B}$ under the rule $\boxminus$.

$\boxminus$	d_3	d_1	d_4	d_2	d_5
d_3	d_1	d_3	d_3	d_3	d_3
d_1	d_3	d_1	d_1	d_1	d_1
d_4	d_3	d_1	d_4	d_2	d_5
d_2	d_3	d_1	d_5	d_4	d_2
d_5	d_3	d_1	d_2	d_5	d_4

TABLE 13: LDFNs B_is along with scores, accuracies and rankings on $(\mathscr{B},\boxminus)$.

d_i	B_i	$f_B(d_i)$	$g_B(d_i)$	$\alpha_B(d_i)$	$\beta_B(d_i)$	Score	Accuracy	Rank
d_1	B_2	0.6	0.3	0.3	0.0	0.0	0.3	1st
d_2	B_4	0.4	0.6	0.6	0.2	−0.2	0.1	2nd
d_3	B_1	0.5	0.2	0.4	0.1	0.0	0.3	1st
d_4	B_3	0.3	0.5	0.5	0.1	−0.2	0.1	2nd
d_5	B_5	0.5	0.7	0.7	0.3	−0.2	0.1	2nd

Again, it can be checked from the table that $\xi(\mathscr{F}_3)$ is an LDF-score (0, 2)-ideal of $(\mathscr{B},\boxtimes)$. Figure 5 represents the visualization of the score, accuracy, and ranking comparison for $\mathscr{F}_3$.

The preferences of the alternatives based on an LDF-score (0, 2)-ideal on $(\mathscr{B},\boxtimes)$ are given in Table 17.

The parameters α and β in the above examples are chosen based on the preferences of decision-makers, whereas attribute functions are determined based on actual facts. The primary advantage of reference parameters is that we can choose the attribute functions we want without being constrained by the $0 \leq f_A + g_A \leq 1$ condition. The evaluation is parameterized by these parameters, which expand the space of our mathematical model. On the same reference set $\mathscr{Y}$, we can define various LDFSs for distinct sets of parameters. Simply expressed, it is clear to see that the input data values chosen start outside of the IFS and PFS spaces.

5. Discussion and Comparison

Fuzzy information techniques include fuzzy sets, intuitionistic fuzzy sets, Pythagorean fuzzy sets, and q-rung orthopair fuzzy sets. There are, however, significant restrictions, such as FS's inability to perform nonmembership functions. IFS overcame this problem; however, it imposed strong restrictions on membership and nonmembership functions, restricting the potential space. To address this issue, PFS and q-ROFS boosted potential space even further, yet the vast majority of it remained unused. LDFS makes use of the entire region, allowing users to freely select membership and nonmembership functions from any point in space. In various MADM scenarios, we encounter various types of criteria and input data depending on the circumstance. The space comparison is given in Figure 6.

The optimum choice of bridge designs carried out through LDF-score (0, 2) -ideals of two different AG-groupoids is given in Table 18.

The optimum choice of chatbots carried out through LDF-score (0, 2)-ideals of three different AG-groupoids is given in Table 19.

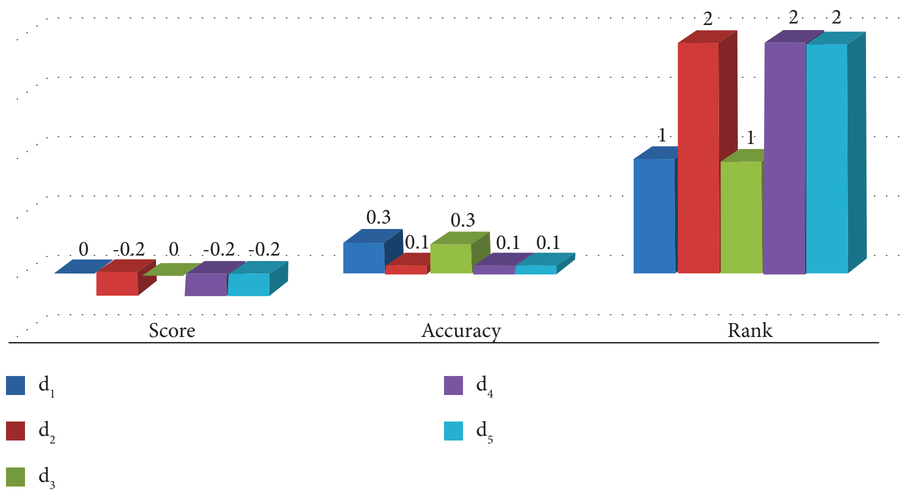

Figure 4: Score, Accuracy & Ranking Comparison for $\mathscr{F}_2$ on $(\mathscr{B}, \boxminus)$.

Table 14: Rankings of chatbots under $(\mathscr{B}, \boxminus)$.

Rankings	1st	1st	2nd	2nd	2nd
Chatbots	d_3	d_1	d_4	d_2	d_5

Table 15: Composition of $\mathscr{B}$ under the rule $\boxtimes$.

$\boxtimes$	d_3	d_1	d_4	d_2	d_5
d_3	d_3	d_3	d_3	d_3	d_3
d_1	d_3	d_1	d_1	d_1	d_1
d_4	d_3	d_1	d_2	d_5	d_4
d_2	d_3	d_1	d_4	d_2	d_5
d_5	d_3	d_1	d_5	d_4	d_2

Table 16: LDFNs $C_i s$ along with scores, accuracies, and rankings.

d_i	C_i	$f_C(d_i)$	$g_C(d_i)$	$\alpha_C(d_i)$	$\beta_C(d_i)$	Score	Accuracy	Rank
d_1	C_2	0.8	0.3	0.2	0.1	0.4	...	2nd
d_2	C_4	0.3	0.5	0.4	0.3	0.1	0.05	3rd
d_3	C_1	0.9	0.1	0.4	0.3	0.7	...	1st
d_4	C_3	0.1	0.3	0.5	0.4	0.1	0.05	3rd
d_5	C_5	0.5	0.7	0.3	0.2	0.1	0.05	3rd

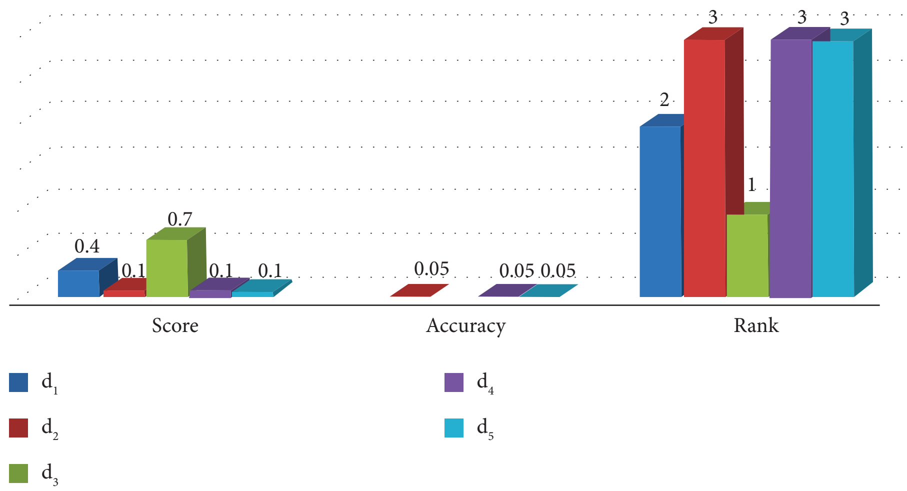

FIGURE 5: Score, accuracy, and ranking comparison for $\mathscr{F}_3$.

TABLE 17: Rankings of chatbots under $(\mathscr{B}, \boxtimes)$.

Rankings	1st	2nd	3rd	3rd	3rd
Chatbots	d_3	d_1	d_4	d_2	d_5

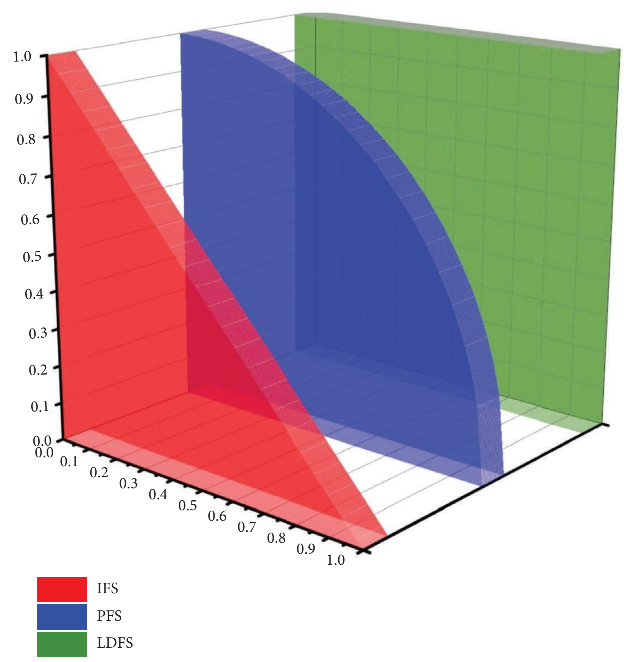

FIGURE 6: Space comparison of IFS, PFS, and LDFS.

TABLE 18: Ranking comparison with the optimal choice of bridge designs.

AG-groupoids	Ranking Comparison	Choice
$(\mathscr{C}, \circledcirc)$	$b_2 > b_1 > b_3$	b_2
$(\mathscr{C}, \ominus)$	$b_2 > b_1 = b_3$	b_2

It is evident that, for both AG-groupoids $(\mathscr{C}, \circledcirc)$ and $(\mathscr{C}, \ominus)$, the "truss design bridge" b_2 is ranked as no. 1. For the selection of AI-based chatbot problems, the chatbot d_3, which is "ProProfs," is the most optimum option to select via all AG-groupoids $(\mathscr{B}, \boxplus)$, $(\mathscr{B}, \boxminus)$, and $(\mathscr{B}, \boxtimes)$.

TABLE 19: Ranking comparison with the optimal choice of chatbots.

AG-groupoids	Ranking Comparison	Choice
$(\mathscr{B}, \boxplus)$	$d_3 > d_5 > d_4 > d_1 > d_2$	d_3
$(\mathscr{B}, \boxminus)$	$d_3 = d_1 > d_4 = d_2 = d_5$	d_3
$(\mathscr{B}, \boxtimes)$	$d_3 > d_1 > d_4 = d_2 = d_5$	d_3

6. Conclusion

A crucial and essential area of research for multiattribute decision-making (MADM) is how to encode these perplexing pieces of information. IFSs, PFSs, and q-ROFSs are all great approaches to dealing with ambiguous data. Although LDFSs are more generic, by integrating reference/control parameters, they excel at easing the restrictive limits of IFS, PFS, and q-ROFS. The tactics used for this assignment are mostly determined by the type of problem being examined. Our everyday lives are erratic, imprecise, and blurry. The present structures are based on the assumption that decision-makers take into account defined limitations while evaluating various alternatives and characteristics. However, given other circumstances, this type of scenario prevents decision-makers from allocating membership grades and nonmembership grades. To address these constraints, the LDFS technique utilizes two reference or control parameters in place of membership grades and nonmembership grades.

In this research, we used LDF data along with the AG-groupoid to tackle real-world multiattribute decision-making problems. We reviewed the advantages of LDFS over other strategies and compared the ranks of other alternatives by changing the AG-groupoids over the exact same problem.

As the applications considered in this paper are from very diverse fields of science, i.e., one from engineering and the other from information technology. It is therefore observed that the propound method is a very useful tool for decision-making in a wide variety of real-life scenarios. Another advantage of this method is the freedom of choice for LDFNs in the space of LDFSs because an LDFS does not impose restrictions on membership and nonmembership functions; therefore, the proposed method could also be applied to other types of fuzzy sets such as intuitionistic fuzzy sets, picture fuzzy sets, and q-rung orthopair fuzzy sets. Since the LDFS could be generalized to interval-valued LDFS by considering membership and nonmembership functions as intervals instead of numbers, hence the proposed method could also be generalized to interval-valued LDF-score (0,2)-ideals.

Data Availability

The data used to support the findings of this study are included in the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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