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Research Article

Chebyshev Distance Entropy Combined with TODIM Method in Lq-ROFS Multiattribute Group Decision Making

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Linguistic q-rung orthopair fuzzy numbers (Lq-ROFNs) are composed of a set of q-rung orthopair fuzzy numbers (q-ROFNs) with membership and nonmembership degrees of linguistic variables, which can be regarded as an extension of linguistic intuitionistic fuzzy numbers (LIFNs) and linguistic Pythagorean fuzzy numbers (LPFNs). Compared with LIFNs and LPFNs, Lq-ROFNs have a wider range of applicability because the value of q can have a wider range, thus allowing to handle more multiattribute group decision making (MAGDM) problems. Based on Lq-ROFS, this paper first proposes the Chebyshev distance metric, then develops the Chebyshev distance entropy for deriving objective solution of decision makers' (DMs') weight vector and attribute weight vector by combining the metric and entropy measure. The TODIM method has been widely valued by the society and has achieved good results in many MAGDM problems. Subsequently, the TODIM decision method in Lq-ROFS is presented and combined with the Chebyshev distance entropy model as a new decision method (CDE-TODIM) to solve the MAGDM problems. This method process is objective and direct and takes into account the decision maker's preferences, and the decision results are more persuasive. Finally, the effectiveness and rationality of this MAGDM method are illustrated by a case as well as comparisons.

1. Introduction

In today's human social activities, decision-making is becoming increasingly vital, so how to seek an optimal solution is the key to the problem [1–4]. When choosing the optimal solution, the choice is analyzed by collecting the evaluation information of a group of DMs, in which the decision making behaviors involving multiple criteria become multicriteria group decision-making (MCGDM) [5–8] and those involving multiple attributes become MAGDM, and the problem studied in this paper focuses on MAGDM [9–11]. In the beginning, to express their subjective decisions, DMs often describe them by real numbers [12]. The size of real numbers represents the intensity of subjective will, but this means that the evaluation information is not so crisp. Therefore, to solve the situation mentioned above in the MAGDM problem, Zadeh [13] first proposed the concept of the fuzzy set (FS), which only considers the membership function. Once this method was proposed, it aroused widespread discussion in society [14, 15]. However, considering the membership function alone means that only membership degree (MD) information is included in the evaluation information so it will cause the loss of nonmembership degree (NMD) information, and it is not easy to guarantee the accuracy of the final decision result. Aiming at the fuzzy set (FS) proposed by Zadeh, Atanassov made further improvements to its deficiencies. Based on this, Atanassov [16] proposed the intuitionistic fuzzy set (IFS) in 1986; that is, the degree of membership μ and the degree of nonmembership ν are included in the evaluation information at the same time. The IFS requires two degrees cannot exceed one, that is, $\mu + \nu \leq 1$.

However, as the MAGDM problems are becoming more and more complex, this constraint is becoming too strong for decision-makers (DMs). For example, under the evaluation criteria, a DM's preference for a certain alternative is recorded as 0.8 and NMD is recorded as 0.5, obviously, $0.8 + 0.5 > 1$; so it cannot meet the conditions of IFS proposed by Atanassov. Thus, based on the predecessors, Yager and Abbasov [17] introduced Pythagorean fuzzy sets (PFSs), and the corresponding constraint conditions are relaxed compared with IFS and only need to meet $\mu^2 + \nu^2 \leq 1$. IFS and PFS have solved many MAGDM problems, respectively, have been recognized in the society and have also made some achievements in the academic field. In recent years, Yager [18] has further expanded fuzzy sets and proposed the concept of q-rung orthopair fuzzy sets (q-ROFSs); that is, MD and NMD only need to satisfy the following formula: $\mu^q + \nu^q \leq 1, q > 1$. In fact, it is easy to see that q-ROFSs are much looser than PFSs, also can be better applied to more uncertain MAGDM problems than IFSs and PFSs. And, people found that if $q = 1$ in q-ROFS, q-ROFS becomes IFS, if $q = 2$, it becomes PFS. From here, it can be understood that q-ROFS is a kind of promotion of IFS or PFS [19–22].

For different research objects, such as information aggregation and decision problems under uncertainty, Ullah [23] propose picture fuzzy set (PFS'), so as to study the MAGDM problem in this set, and Mahmood et al. [24] make further extensions based on PFS' by proposing spherical fuzzy set (SFS) and T spherical fuzzy set (T-SFS) concept, which makes the decision process more explicit.

In the methods like IFSs, PFSs, and q-ROFSs, MD and NMD given by DMs are expressed in the form of real numbers, but for DMs themselves, this group tends to use language to describe and express vague evaluation information. This is actually more convenient [25]. For example, when people select outstanding students to issue scholarships, the linguistic evaluation system is utilized to evaluate students' ability such as "very good," "ordinary," and "poor". In the early days, Zadeh [26] established the linguistic term set (LTS) and proposed some linguistic calculation models to model the evaluation information, thereby improving the accuracy [27–29]. However, these linguistic calculation models only contain MD, without NMD. Based on this, Zhang [30] combined the established IFS and LTS to create a new concept named the linguistic intuitionistic fuzzy set (LIFS), which elements are called the linguistic intuitionistic fuzzy numbers (LIFNs). Each element in LIFSs, named LIFN, is composed of a pair of linguistic terms, including MD and NMD. And, it needs to meet the condition: the sum of the subscripts of MD and NMD must less than the cardinal number of LTS. The development of linguistic fuzzy sets is similar to the fuzzy sets. Garg [31] proposed a novel linguistic fuzzy set called linguistic Pythagorean fuzzy set (LPFS), and the elements in the set are called LPFNs. The analogy LIFN describes MD and NMD through a pair of linguistic terms, and LPFN is also the same. But the difference is that LPFN only needs to satisfy that the sum of MD and NMD subscripts less than the square of the cardinality number of the LTS. For easily understanding, we assumed that continuous LTS (CLTS) as $S = \{s_\alpha \mid \alpha \in (0, \sigma)\}$. Thus, for LIFN, its related conditions are $\mu \in (0, \sigma), \nu \in (0, \sigma), \mu + \nu \in (0, \sigma)$. And, as comparison, condition that LPFNs needs to meet $\mu^2 + \nu^2 \in (0, \sigma)$. From here, people have concluded that all LIFNs will certainly meet the conditions of LPFNs, but the converse is not necessarily true. The conclusion comes naturally, that LPFNs cover a larger range than LIFNs.

In order to further expand the coverage of linguistic evaluation information for decision makers (DMs), Liu and Liu [32] proposed the linguistic q-rung orthopair fuzzy numbers (Lq-ROFNs). And condition is $\mu^q + \nu^q \in (0, \sigma)$, $q \geq 1$. Over time, more and more MAGDM problems have been built on Lq-ROFS, and thus more and more authors have proposed decision methods for Lq-ROFNs [33–49]. If the evaluation of DM is inscribed by selecting real numbers directly from the number field, it is not precise enough and does not aggregate all the information provided by DM, while Lq-ROFNs can precisely describe the linguistic information of DM, so this is the main reason why we choose to solve the MAGDM problem in Lq-ROFS in this paper.

It has been shown in the previous section that people collect evaluation information by using Lq-ROFNs, the information will be more accurate and the decision results obtained will be more convincing and valid. The TODIM method has not been used in Lq-ROFS yet, therefore, the decision making method proposed in this paper is mainly implemented in Lq-ROFS. The following section will illustrate the superiority of choosing the TODIM method.

Of course, in addition to the TODIM method, there is no shortage of methods to solve the MAGDM problem, such as VIKOR [50, 51], TOPSIS [52], ELECTRE [53, 54], and DEMATEL [55] (some of them have not yet been extended to Lq-ROFS, which will also be considered for future work). These methods have their suitable applications in different fields and different directions. However, it is a pity that all the above-given methods have a common shortcoming. That is, these methods do not consider the orientation of the DMs' subjective to strengthen the decision-making results. For the time being, it is called the bounded rationality of the DMs. Gomes and Lima [56] considered the bounded rationality of DMs and proposed the TODIM method. This is the first time that the bounded rationality of DMs has been considered in the MAGDM problem, which is an unprecedented breakthrough [57]. TODIM method constructs the dominance matrix of pairwise comparison of schemes according to the different preferences of DMs for benefits and costs and then aggregates the individual dominance to form the dominance of each scheme. The ranking of schemes can be obtained by the size of comparative dominance. At present, this method has been widely valued by the society, and has achieved good results in many MAGDM problems. For example, Zhao et al. [58] applied the TODIM method in scientific and technological assessment in risk management, He et al. [59] proposed a shadowed set-based TODIM method, and applied it to large-scale group decision making, Wu et al. [60] developed a linguistic distribution behavioral MCGDM model integrating extended generalized TODIM method and quantum decision theory. Decision making using the TODIM method involves solving for different attribute weights. If the weights are unknown, the subjective assignment of weights tends to create a preference for a particular scheme, thus making it impossible to objectively evaluate the decision scheme, so an objective method of solving for the weights is considered. In the paper, Liu and Liu [32] propose the Hamming distance and combine it with the entropy method to solve the attribute weights. The superiority of Chebyshev distance will be discussed and the distance will be extended to distance entropy to solve for expert weights and attribute weights.

In this paper, we propose a new distance measure based on the Hamming distance in Lq-ROFS, which is the Chebyshev distance. It has similar properties to the Hamming distance. However, it is easier to define and considers fewer factors than the Hamming distance. For example, it considers only one between membership, nonmembership, and uncertainty, and it is more able to bring out the subjective tendency of DMs, which is objective mixed with subjective. And, to some extent, it is more effective than the metrics established by considering only subjective or objective. Furthermore, in most papers proposing methods for solving the MAGDM problem in Lq-ROFS, the authors only explain the steps of the method but do not indicate how to solve for the DMs' weights and attribute weights. For example, Lin et al. [61] propose a new decision operator when the DM and attribute weights are known. Therefore, in this paper, we propose the Chebyshev distance entropy model for solving the DMs' weights and attribute weights by using the definition of Chebyshev distance measure and entropy, which can explain the accuracy of the decision results and the effectiveness of the method more objectively.

The benefits and weaknesses of existing methods are listed in Table 1.

From the comparison in Table 1, it can be concluded that the innovation of this paper mainly focuses on extending the TODIM method to Lq-ROFS, defining the concept of Chebyshev distance entropy, proposing an objective method for solving expert weights and attribute weights, and finally combining the two to solve the MAGDM problem. And, we can also know that the traditional model for solving the MCDM problem has its own shortcomings, MABAC, RAFSI, and other methods are applicable to different environments, while for DMs' linguistic evaluation to collect information, Lq-ROFS is more accurate.

Therefore, the advantages of the CDE-TODIM method are as follows:

The first point is that the Chebyshev distance only considers the largest one of MD, NMD, and uncertainty, which can better express the preference of DM compared to the Hamming distance which directly takes the average of the three.

The second point is to expand the Chebyshev distance to Chebyshev distance entropy, so as to solve the weight of DM and the attribute weight. The overall process is objective and direct, and the results are more convincing than those obtained by direct or subjective assignment.

The third point is the extension of the TODIM method to Lq-ROFS, which is an unprecedented work that retains the advantageous nature of the TODIM method itself, i.e., considering the bounded rationality of DM and uses Lq-ROFNs for decision making, which makes the linguistic expressions of DMs transformed into real numbers and thus solves the MAGDM problem objectively.

For ease of reading, Table 2 will explain the abbreviations in the paper.

The rest of this paper is arranged as follows. In Section 2, some preliminary knowledge is introduced, including the concepts of q-ROFS, LTS, Lq-ROFS, and LqROFWA; in Section 3, how to use the Chebyshev distance entropy measure model to solve for DMs' and attribute weight vector is proposed; in Section 4, we combine the Chebyshev distance entropy model with the TODIM method for solving the MAGDM problem in Lq-ROFS; in Section 5, a case is given to illustrate the effectiveness of the proposed method. Then, the paper solves the case using different methods and the results obtained are compared and analyzed. At the end of this paper, the practicability of CDE-TODIM method will be explained.

2. Preliminaries

The following concepts will be briefly introduced: q-ROFS, LTS, Lq-ROFS, and LqROFWA.

2.1. Q-Rung Orthopair Fuzzy Set (Q-ROFS)

Definition 1 (Liu and Wang [20]). Assuming that the given q-ROFS T is in finite universe $K = \{k_1, k_2, \ldots, k_n\}$, it can be expressed as

$$T = \{\langle k, \mu_T(k), \nu_T(k)\rangle \mid k \in K\}, \quad (1)$$

where $\mu_T: q \longrightarrow [0,1], \nu_T: q \longrightarrow [0,1]$, respectively, represent the membership degree and the nonmembership degree of element $k \in K$ belonging to the q-ROFS T. The corresponding constraints are as follows: for each $k \in K$, it holds the following results:

$$\mu_T(k)^q + \nu_T(k)^q \in [0,1], q \geq 1. \quad (2)$$

Due to $\mu_T(k)^q + \nu_T(k)^q + \pi_T(k)^q = 1$, the indeterminacy degree of q-ROFS can be expressed as $\pi_T(k) = \sqrt[q]{1 - \mu_T(k)^q - \nu_T(k)^q}$, $k \in K$. Thereby, q-ROFN corresponds to $\langle \mu_t(k), \nu_t(k)\rangle$, and it can be recorded as $t = \langle \mu_t, \nu_t\rangle$.

2.2. Linguistic Term Set (LTS)

Definition 2 (Herrera et al. [72]). LTS is a finite-ordered discrete with odd cardinality set, which is recorded as $A = \{\alpha_0, \alpha_1, \ldots, \alpha_l\}$. The elements in A, like $\alpha_0, \alpha_1, \ldots, \alpha_l$, can be defined according to different semantic environments, but in any case, the following conditions need to be met:

(1) Ordered: if $i < j$, it means $\alpha_i < \alpha_j$, and α_i is worse than α_j
(2) Negative operator: neg $(\alpha_j) = \alpha_i, i = l - j$
(3) Min operator: min $(\alpha_i, \alpha_j) = \alpha_k$, if $i < j, k = i$, else $k = j$

Table 1: Benefits and weaknesses of existing methods.

Methods	Environment	Benefits	Weaknesses	Targeted issues
MABAC [62, 63]	Triangular fuzzy number	Condition stability	The linguistic terms are not accurate enough when converted to real numbers	MCDM
RAFSI [64]	Interval number	Transfer data into interval, eliminates the rank reversal problem	Less of objective and subjective criteria weighting techniques	MADM
MAIRCA [65, 66]	Interval rough number	High stability	Lack of model of determining the weight coefficients of criteria	MCDM
MARCOS [67]	Real number	Flexibility, less affected by increases in attributes or criteria	The scale of five degrees is not accurate enough to describe the degree	MCDM
LMAW [68]	Triangular fuzzy number	Conducted practice	The quality of this method cannot be determined in uncertain field	MCDM
TOPSIS [69]	Lq-ROFS	The linguistic scale function is proposed in the method for decision making	Have no weight calculation model	MAGDM
LqROFWA	Lq-ROFS	Easy to use and understand	Do not consider the partition of input values	MAGDM
LqROFIWPGHM	Lq-ROFS	Consider the interactions between the MD and NMD	Using hamming distance	MAGDM
LqROFWG	Lq-ROFS	Easy to use and understand	Do not consider the inter-relationships between input values	MAGDM
Fuzzy measures and choquet integrals [70]	Rough	Simplify the process of MCGDM and remove the redundant data for the sort result	Have no model to solve for criteria weight	MCDM
WAGE [71]	Grey	Consider multiple attributes simultaneously	The AHP method for solving the weights is too subjective	MCDM

Table 2: Abbreviations explained.

Abbreviations	Original words
MAGDM	Multiattribute group decision making
MCGDM	Multicriteria group decision-making
FS	Fuzzy set
IFS	Intuitionistic fuzzy set
PFS	Pythagorean fuzzy set
PFS’	Picture fuzzy set
SFS	Spherical fuzzy set
T-SFS	T spherical fuzzy set
q-ROFNs	Linguistic term set
LTS	Continuous linguistic term set
CLTS	Q-rung orthopair fuzzy numbers
LIFNs	Linguistic intuitionistic fuzzy numbers
LPFNs	Linguistic Pythagorean fuzzy numbers
Lq-ROFNs	Linguistic q-rung orthopair fuzzy numbers
Lq-ROFVs	Linguistic q-rung orthopair fuzzy vectors
Lq-ROFMs	Linguistic q-rung orthopair fuzzy matrices
Lq-ROFR	Linguistic q-rung orthopair fuzzy result
LqROFWA	Lq-ROFNs weighted average operator
LqROFIWPGHM	Linguistic q-rung orthopair fuzzy interactional weighted partitioned geometric Heronian mean
LqROFWG	Linguistic q-rung orthopair fuzzy weighted geometric
DM	Decision maker
MD	Membership degree
NMD	Nonmembership degree
CDE-TODIM	Chebyshev distance entropy model with the TODIM method
MABAC	Multiattributive border approximation area comparison
WAGE	Weighting with AHP, grey numbers, and entropy

(4) Max operator: max $(\alpha_i, \alpha_j) = \alpha_k$, if $i < j$, $k = j$, else $k = i$

Due to the discrete language set cannot well include all experts decision information, Xu [73] extends it to a continuous set, which is recorded as CLTS $A = \{\alpha_\tau \mid \tau \in [0, l]\}$ (l is a positive integer), and its conditions are as same as LTS.

2.3. Linguistic Q-Rung Orthopair Fuzzy Set (Lq-ROFS)

Definition 3 (Liu and Liu [32]). A Lq-ROFN in K is recorded as

$$\tilde{\eta} = \left\{ \left(k, \alpha_i(k), \alpha_j(k) \right) \mid k \in K \right\}, \quad (3)$$

where $\alpha_i(k), \alpha_j(k) \in A_{[0,l]}$ stand for linguistic MD and linguistic NMD, respectively. The condition $i^q + j^q \in [0, l^q]\ (q \geq 1)$ holds for any $k \in K$. For this q-ROFS, q-ROFN is $\tilde{\eta} = (\alpha_i, \alpha_j)$, and the linguistic indeterminacy degree of $\tilde{\eta}$ is $\pi(k) = \alpha_{(l^q - i^q - j^q)^{1/q}}$.

The complete Lq-ROFNs on the basis of $A_{[0,l]}$ just recorded as $\Lambda_{[0,l]}$.

Definition 4 (Liu and Liu [74]). Suppose $\tilde{\eta}_1 = (\alpha_{i_1}, \alpha_{j_1})$, $\tilde{\eta}_2 = (\alpha_{i_2}, \alpha_{j_2}) \in \Lambda_{[0,l]}$ are two Lq-ROFNs, $q \geq 1$, p is a constant, and $p > 0$.

$$\tilde{\eta}_1 \oplus \tilde{\eta}_2 = \left(\alpha_{\left(i_1^q + i_2^q - \frac{i_1^q i_2^q}{l^q} \right)^{1/q}}, \alpha_{\left(\frac{j_1 j_2}{l} \right)} \right), \quad (4)$$

$$\tilde{\eta}_1 \otimes \tilde{\eta}_2 = \left(\alpha_{\left(\frac{i_1 i_2}{l} \right)}, \alpha_{\left(j_1^q + j_2^q - \frac{j_1^q j_2^q}{l^q} \right)^{1/q}} \right), \quad (5)$$

$$p\tilde{\eta}_1 = \left(\alpha_{l\left(1 - \left(1 - i_1^q / l^q\right)^p\right)^{1/q}}, \alpha_{l\left(i_1 / l\right)^p} \right), \quad (6)$$

$$\tilde{\eta}_1^p = \left(\alpha_{l\left(i_1 / l\right)}, \alpha_{l\left(1 - \left(1 - j_1^q / l^q\right)^p\right)^{1/q}} \right). \quad (7)$$

2.4. LqROFWA (Lq-ROFNs Weighted Average Operator)

Definition 5 (Lin et al. [61]). Let $\tilde{\eta}_k = (\alpha_{i_k}, \alpha_{j_k}), k = 1, 2, \ldots, n$ be n Lq-ROFNs, the LqROFWA has the following equation:

$$\text{LqROFWA}\left(\tilde{\eta}_1, \tilde{\eta}_2, \cdots, \tilde{\eta}_n\right) = \omega_1 \tilde{\eta}_1 \oplus \omega_2 \tilde{\eta}_2 \oplus \cdots \oplus \omega_n \tilde{\eta}_n, \quad (8)$$

where ω_i denotes the weight of $\tilde{\eta}_i$ that satisfies $0 \leq \omega_i \leq 1$ and $\sum_{i=1}^{n} \omega_i = 1$.

3. Chebyshev Distance Entropy Measure Model

In this section, the definition of Chebyshev distance between Lq-ROFNs, Lq-ROFV (Linguistic Q-rung Orthopair Fuzzy Vectors), and Lq-ROFM (Linguistic Q-rung Orthopair Fuzzy Matrices) will be introduced, and then, the methods to solve for DMs' weights and attribute weights by Chebyshev distance entropy model will be presented.

3.1. Chebyshev Distance

Definition 6. The Chebyshev distance between any two Lq-ROFNs $\tilde{\eta}_1 = (\alpha_{i_1}, \alpha_{j_1})$, $\tilde{\eta}_2 = (\alpha_{i_2}, \alpha_{j_2})$ can be defined as follows:

$$d_C(\tilde{\eta}_1, \tilde{\eta}_2) = \frac{1}{l^q} \max\left(\left|i_1^q - i_2^q\right|, \left|j_1^q - j_2^q\right|, \left|\left(l^q - i_1^q - j_1^q\right) - \left(l^q - i_2^q - j_2^q\right)\right|\right). \tag{9}$$

Theorem 1. *The Chebyshev distance between any two Lq-ROFNs $\tilde{\eta}_1 = (\alpha_{i_1}, \alpha_{j_1})$, $\tilde{\eta}_2 = (\alpha_{i_2}, \alpha_{j_2})$ satisfies the following properties:*

(1) (Symmetry) $d_C(\tilde{\eta}_1, \tilde{\eta}_2) = d_C(\tilde{\eta}_2, \tilde{\eta}_1)$

(2) (Non-negativity) $d_C(\tilde{\eta}_1, \tilde{\eta}_2) \geq 0$

(3) (Triangle rule) $d_C(\tilde{\eta}_1, \tilde{\eta}_3) \leq d_C(\tilde{\eta}_1, \tilde{\eta}_2) + d_C(\tilde{\eta}_2, \tilde{\eta}_3)$

Proof. (Symmetry) $d_C(\tilde{\eta}_1, \tilde{\eta}_2) = d_C(\tilde{\eta}_2, \tilde{\eta}_1)$:

$$d_C(\tilde{\eta}_1, \tilde{\eta}_2) = \frac{1}{l^q} \max\left(\left|i_1^q - i_2^q\right|, \left|j_1^q - j_2^q\right|, \left|\left(l^q - i_1^q - j_1^q\right) - \left(l^q - i_2^q - j_2^q\right)\right|\right), \tag{10}$$

$$= \frac{1}{l^q} \max\left(\left|i_2^q - i_1^q\right|, \left|j_2^q - j_1^q\right|, \left|\left(l^q - i_2^q - j_2^q\right) - \left(l^q - i_1^q - j_1^q\right)\right|\right) = d_C(\tilde{\eta}_2, \tilde{\eta}_1). \tag{11}$$

(Non-negativity).

Due to

$$l^q > 0, \left|i_1^q - i_2^q\right| \geq 0, \left|j_1^q - j_2^q\right| \geq 0\Big|, \left(l^q - i_1^q - j_1^q\right) - \left(l^q - i_2^q - j_2^q\right)\Big| \geq 0. \tag{12}$$

So, $d_C(\tilde{\eta}_1, \tilde{\eta}_2) \geq 0$.

And $d_C(\tilde{\eta}_1, \tilde{\eta}_1) = 0$. Reasons are as follows:

$$\left|i_1^q - i_1^q\right| = 0, \left|j_1^q - j_1^q\right| = 0, \left|\left(l^q - i_1^q - j_1^q\right) - \left(l^q - i_1^q - j_1^q\right)\right| = 0. \tag{13}$$

(Triangle rule) $d_C(\tilde{\eta}_1, \tilde{\eta}_3) \leq d_C(\tilde{\eta}_1, \tilde{\eta}_2) + d_C(\tilde{\eta}_2, \tilde{\eta}_3)$:

$$\left|i_1^q - i_3^q\right| = \left|i_1^q - i_2^q + i_2^q - i_3^q\right| \leq \left|i_1^q - i_2^q\right| + \left|i_2^q - i_3^q\right|, \tag{14}$$

$$\left|j_1^q - j_3^q\right| = \left|j_1^q - j_2^q + j_2^q - j_3^q\right| \leq \left|j_1^q - j_2^q\right| + \left|j_2^q - j_3^q\right|, \tag{15}$$

$$\begin{aligned} \left|\left(l^q - i_1^q - j_1^q\right) - \left(l^q - i_3^q - j_3^q\right)\right| &= \left|\left(l^q - i_1^q - j_1^q\right) - \left(l^q - i_2^q - j_2^q\right) + \left(l^q - i_2^q - j_2^q\right) - \left(l^q - i_3^q - j_3^q\right)\right| \\ &\leq \left|\left(l^q - i_1^q - j_1^q\right) - \left(l^q - i_2^q - j_2^q\right)\right| + \left|\left(l^q - i_2^q - j_2^q\right) - \left(l^q - i_3^q - j_3^q\right)\right|. \end{aligned} \tag{16}$$

Thus, $d_C(\tilde{\eta}_1, \tilde{\eta}_3) = (1/l^q) \max(|i_1^q - i_3^q|, |j_1^q - j_3^q|, |(l^q - i_1^q - j_1^q) - (l^q - i_3^q - j_3^q)|)$.

$$\leq \frac{1}{l^q}\max\left|i_1^q - i_2^q\right| + \left|i_2^q - i_3^q\right|, \left|j_1^q - j_2^q\right| + \left|j_2^q - j_3^q\right|, \tag{17}$$

$$\left|\left(l^q - i_1^q - j_1^q\right) - \left(l^q - i_2^q - j_2^q\right)\right| + \left|\left(l^q - i_2^q - j_2^q\right) - \left(l^q - i_3^q - j_3^q\right)\right|\Big), \tag{18}$$

$$\leq \frac{1}{l^q}\max\Big(\left|i_1^q - i_2^q\right|, \left|j_1^q - j_2^q\right|, \left|\left(l^q - i_1^q - j_1^q\right) - \left(l^q - i_2^q - j_2^q\right)\right|\Big) + \frac{1}{l^q}\max\Big(\left|i_2^q - i_3^q\right|, \left|j_2^q - j_3^q\right|, \left|\left(l^q - i_2^q - j_2^q\right) - \left(l^q - i_3^q - j_3^q\right)\right|\Big), \tag{19}$$

$$= d_C\left(\tilde{\eta}_1, \tilde{\eta}_2\right) + d_C\left(\tilde{\eta}_2, \tilde{\eta}_3\right). \tag{20}$$

Comparing to the Hamming distance presented in paper [74],

$$d_{Hd}\left(\tilde{\eta}_1, \tilde{\eta}_2\right) = \left(1/2l^q\right)\cdot\left(\left|i_1^q - i_2^q\right| + \left|j_1^q - j_2^q\right| + \left|\left(l^q - i_1^q - j_1^q\right) - \left(l^q - i_2^q - j_2^q\right)\right|\right). \tag{21}$$

The Chebyshev distance also operates with the subscripts of Lq-ROFN and converts Lq-ROFN to real number in a similar way, which means that they have similar properties. However, compared to Hamming distance, which averages the sum of subscript differences, Chebyshev distance considers only the one with the largest difference among membership, nonmembership, and uncertainty, and is more able to take into account the subjective tendencies of decision makers. □

Definition 7. The Chebyshev distance between any two Lq-ROFVs

$$\overrightarrow{\psi_1} = \left(\tilde{\eta}_{11}, \tilde{\eta}_{12}, \cdots, \tilde{\eta}_{1,n-1}, \tilde{\eta}_{1,n}\right)\ \overrightarrow{\psi_2} = \left(\tilde{\eta}_{21}, \tilde{\eta}_{22}, \cdots, \tilde{\eta}_{2,n-1}, \tilde{\eta}_{2n}\right), \tag{22}$$

And can be defined as follows:

$$d_C\left(\overrightarrow{\psi_1}, \overrightarrow{\psi_2}\right) = d_C\left(\frac{1}{n}\left(\tilde{\eta}_{11}\oplus\tilde{\eta}_{12}\oplus\cdots\oplus\tilde{\eta}_{1,n-1}\oplus\tilde{\eta}_{1n}\right), \frac{1}{n}\left(\tilde{\eta}_{21}\oplus\tilde{\eta}_{22}\oplus\cdots\oplus\tilde{\eta}_{2,n-1}\oplus\tilde{\eta}_{2n}\right)\right). \tag{23}$$

Theorem 2. *$d_C\left(\overrightarrow{\psi_1}, \overrightarrow{\psi_2}\right)$ still satisfies symmetry, nonnegativity, and triangle rule.*

Definition 8. The Chebyshev distance between any two Lq-ROFMs:

$$\mathrm{M}_{n\times n}^1 = \begin{pmatrix} \tilde{\eta}_{11}^1 & \cdots & \tilde{\eta}_{1n}^1 \\ \cdots & \cdots & \cdots \\ \tilde{\eta}_{n1}^1 & \cdots & \tilde{\eta}_{nn}^1 \end{pmatrix} \mathrm{M}_{n\times n}^2 = \begin{pmatrix} \tilde{\eta}_{11}^2 & \cdots & \tilde{\eta}_{1n}^2 \\ \cdots & \cdots & \cdots \\ \tilde{\eta}_{n1}^2 & \cdots & \tilde{\eta}_{nn}^2 \end{pmatrix}. \tag{24}$$

And can be defined as follows:

$$d_C\left(\mathrm{M}_{n\times n}^1, \mathrm{M}_{n\times n}^2\right) = d_C\left(\overrightarrow{\psi}^1, \overrightarrow{\psi}^2\right), \tag{25}$$

where $\overrightarrow{\psi}^1 = \begin{pmatrix} (1/n) & \left(\tilde{\eta}_{11}^1\oplus\cdots\oplus\tilde{\eta}_{1n}^1\right) \\ (1/n) & (\cdots\oplus\cdots\oplus\cdots) \\ (1/n) & \left(\tilde{\eta}_{n1}^1\oplus\cdots\oplus\tilde{\eta}_{nn}^1\right) \end{pmatrix}, \overrightarrow{\psi}^2 = \begin{pmatrix} (1/n) & \left(\tilde{\eta}_{11}^2\oplus\cdots\oplus\tilde{\eta}_{1n}^2\right) \\ (1/n) & (\cdots\oplus\cdots\oplus\cdots) \\ (1/n) & \left(\tilde{\eta}_{n1}^2\oplus\cdots\oplus\tilde{\eta}_{nn}^2\right) \end{pmatrix}.$

Theorem 3. *$d_c\left(M_{n\times n}^1, M_{n\times n}^2\right)$ also satisfies symmetry, nonnegativity, and triangle rule.*

3.2. Chebyshev Distance Entropy for DMs' Weights. Entropy measures the uncertainty of information based on probability theory, which reveals the law that the more dispersed the distribution of information, the greater the uncertainty. After being inspired by the definition of the entropy power method based on the linguistic intuition fuzzy number in one paper (2017), the definition of Chebyshev distance entropy will be subsequently proposed in the following steps.

Before presenting the method, first assume that $DM_t, t = 1, 2, .., p$ evaluate each scheme $S_j, j = 1, \ldots, m$ based on every attribute $C_i, i = 1, \ldots, n$, and each appraised value can be recorded as Lq-ROFN variable $k = \left(\alpha_{ij}(k), \beta_{ij}(k)\right)$. Here, we also assumed that the DMs' weight vector is $\left(\lambda_1, \lambda_2, \ldots, \lambda_p\right)$ and the attribute weight vector is $\left(\omega_1, \omega_2, \ldots, \omega_n\right)$.

In this way, we can obtain the Lq-ROFS decision matrix $K^t = [k_{ij}^t]_{n\times m}, t = 1, 2, ..., p$, with $k_{ij}^t = (\alpha_{ij}^t(k), \beta_{ij}^t(k))$, where $i = 1,\ldots,n$, and $j = 1,\ldots,m$. Then, the specific steps of Chebyshev distance entropy to solve for *DMs*' weight vector are as follows:

Step 1: calculate the Chebyshev distance between the decision matrices given by each DM, then the DMs' distance matrix can be obtained such that

$$d_C(M_K) = \begin{pmatrix} d_C(K^1, K^1) & \cdots & d_C(K^1, K^p) \\ \cdots & \cdots & \cdots \\ d_C(K^p, K^1) & \cdots & d_C(K^p, K^p) \end{pmatrix}. \tag{26}$$

Step 2: calculate the deviation $D_t, t = 1, 2, \ldots, p$ of the $DM_t, t = 1, 2, \ldots, p$ from each of the other DMs:

$$D_t = \sum_{l=1}^{p} d_C(K^t, K^l) t = 1, 2, \cdots, p. \tag{27}$$

Step 3: calculate the entropy value $E_t\ (t = 1, 2, \ldots, p)$ of each DM_t to represent the DM's information:

$$E_t = -\frac{1}{\ln p} \cdot D_t \ln D_t, t = 1, 2, \cdots p \text{ with } 0 \ln 0 \equiv 0. \tag{28}$$

Step 4: calculate the variance $G_t, t = 1, 2, \ldots, p$ of each DM_t:

$$G_t = 1 - E_t, \quad t = 1, 2, \cdots, p. \tag{29}$$

Step 5: calculate the weights corresponding to each DM_t:

$$\lambda_t = \frac{G_t}{\sum_{t=1}^{p} G_t}, t = 1, 2, \cdots, p. \tag{30}$$

3.3. Chebyshev Distance Entropy for Attribute Weights. The following section will describe how to solve for attribute weights using the Chebyshev distance entropy model:

Step 1: calculate the deviation $D_j\ (j = 1, 2, \ldots, n)$ of the scheme $S_i\ (i = 1, 2, \ldots, m)$ from each of the other schemes under attribute $C_j\ (j = 1, 2, \ldots, n)$:

$$D_j = \sum_{l=1}^{m} d_C(k_{xj}, k_{yj}), x, y = 1, \cdots, m, j = 1, \cdots, n. \tag{31}$$

Here, $d_C(k_{xj}, k_{yj})$ indicates the Chebyshev distance between Lq-ROFNs.

Step 2: calculate the entropy value $E_j\ (j = 1, 2, \ldots, n)$ of each attribute C_j to represent the decision information:

$$E_j = \frac{-1}{\text{lnm}} \cdot \sum_{t=1}^{m} D_j \ln D_j j = 1, 2, \cdots n \text{ with } 0 \ln 0 \equiv 0. \tag{32}$$

Step 3: calculate the variance $G_j\ (j = 1, 2, \ldots, n)$ of attribute C_j:

$$G_j = 1 - E_j, j = 1, 2, \cdots, n. \tag{33}$$

Step 4: calculate the weights corresponding to each attribute C_j:

$$\omega_j = \frac{G_j}{\sum_{j=1}^{n} G_j}, j = 1, 2, \cdots, n. \tag{34}$$

As a way to obtain attribute weights objectively, the weights solved are more representative and can make the decision results more accurate and effective, so it will be a powerful tool for solving MAGDM problems.

The following will introduce how distance entropy will be combined with the TODIM method.

4. Lq-ROFS CDE-TODIM

In this section, based on the intuitionistic linguistic TODIM method, we extend this method to Lq-ROFS. The followings are the specific steps of this method.

We know that attributes can be divided into two types according to their properties, namely, benefit-type attributes and cost-type attributes. Therefore, the first step should be normalization, which is the following:

Step 1: the decision matrix $K_t = [k_{ij}^t]_{n\times m}, t = 1, ..., p$ given by each DM is normalized to give the matrix $Y_t = [y_{ij}^t]_{n\times m}$.

When the elements $k_{ij}^t = (\alpha_{ij}^t(k), \beta_{ij}^t(k))$ in the decision matrix K are normalized, there are following conclusions:

$$\begin{cases} y_{ij}^t = (\alpha_{ij}^t(k), \beta_{ij}^t(k)) & C_j \text{ belongs to benefit} - \text{type}, \\ y_{ij}^t = (\beta_{ij}^t(k), \alpha_{ij}^t(k)) & C_j \text{ belongs to cost} - \text{type}. \end{cases} \tag{35}$$

Step 2: assuming that the DMs' weights are derived by the Chebyshev distance entropy model, or are given in the case. Then, the DMs' decision matrices can be assembled by LqROFWA operator; in this way, they will turn into a new decision matrix $R = [r_{ij}]_{n\times m}$ containing all the wishes of each DM.

Step 3: assuming that attribute weights are derived by the Chebyshev distance entropy measure model or given in the case, which are noted as $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$, and each $\omega_i \geq 0, i = 1, \ldots, n$, $\sum_{i=1}^{n} \omega_i = 1$. Then, calculate the relative weight $\omega_{aj} = (\omega_j / \omega_a)$ of each attribute relative to the highest weight ω_a.

Step 4: calculate the overall dominance $\Phi(S_x, S_y)$ of scheme S_x over S_y.

$$\Phi(S_x, S_y) = \sum_{j=1}^{n} \Phi_j(S_x, S_y)\ x, y = 1, 2, \cdots, m, \tag{36}$$

where

$$\Phi_j\left(S_x,S_y\right)=\begin{cases}\sqrt{\dfrac{\omega_{aj}d\left(r_{xj},r_{yj}\right)}{\sum_{j=1}^{n}\omega_{aj}}} & d\left(r_{xj},r_{yj}\right)\geq 0\\ -\dfrac{1}{\theta}\sqrt{\dfrac{d\left(r_{xj},r_{yj}\right)\left(\sum_{j=1}^{n}\omega_{aj}\right)}{\omega_{aj}}} & d\left(r_{xj},r_{yj}\right)<0\end{cases}\quad j=1,2,\cdots,n, \tag{37}$$

where $d(r_{xj}, r_{yj})$ represents the Chebyshev distance between two Lq-ROFNs, and $d(r_{xj}, r_{yj}) \geq 0$ represents profit or 0, while $d(r_{xj}, r_{yj}) \leq 0$ indicates costs. θ represents the attenuation factor of the losses. When $\theta > 1$, the DMs are risk averters, the higher value of θ, the higher the degree of avoidance; when $0 < \theta < 1$, DMs are risk appetite. From paper [75], it suggests DM give the value of θ between [1, 2.5]. And $\theta = 1$ or $\theta = 2.5$ are common values.

Then, prospect value function $\delta(S_x)$ of scheme S_x is shown as follows:

$$\delta(S_x)=\sum_{y=1}^{m}\Phi\left(S_x,S_y\right)x=1,2,\cdots,m. \tag{38}$$

Step 5: normalize $\delta(S_x)$:

$$\widetilde{\delta}(S_x)=\frac{\delta(S_x)-\min\{\delta(S_x)\}}{\max_x\{\delta(S_x)\}-\min_x\{\delta(S_x)\}},x=1,2,\cdots,m. \tag{39}$$

Step 6: the scheme is sorted according to the numerical size of $\widetilde{\delta}(S_x)$. The larger $\widetilde{\delta}(S_x)$, the better scheme S_x is.

In Figure 1, we can see the specific method flow, which is presented in a more macroway.

5. Case Study

In this session, a case of decision making utilizing the CDE-TODIM method under Lq-ROFS will be given, and the results will be profiled and illustrated and then compared with other decision making methods on this basis. As a result, the validity of the CDE-TODIM method will be analyzed from the above-given solution conclusions.

5.1. Case Study Based on the CDE-TODIM Model. The epidemic caused by COVID-19 is a great concern for humans worldwide at this time, and while hoping for a speedy improvement in the epidemic, business opportunities have been identified in home isolation. This case will apply the CDE-TODIM method to decide the best investment option for investors and to illustrate the final results. An investor currently has a sum of money on hand and while isolating his home, he has identified several business opportunities and intends to invest in one of these options, which are as follows: P_1 is a pharmaceutical company, P_2 is a mask manufacturing plant, P_3 is an online medical platform, and P_4 is an online educational platform. In order to find the optimal investment solution, the investor asks three decision makers $DM_x\,(x = 1, 2, 3)$ to make decisions and takes into account all the wishes of these three DMs. The DMs considered the following four main evaluation criteria in selecting the schemes: profitability (C_1), market prospects (C_2), risk (C_3), and cost (C_4). The first two attributes are benefit-type and last two are cost-type; therefore, these can be divided into two classes $W_1 = \{C_1, C_2\}$ and $W_2 = \{C_3, C_4\}$. On the basis of the CLTS $S = \{s_\tau \mid s \in [0, 8]\}$, the DMs give the decision result (Lq-ROFR) for each scheme P_i under each attribute C_i.

$S = \{s_0 =$ extremely poor, $s_1 =$ very poor, $s_3 =$ poor, $s_4 =$ slightly poor, $s_5 =$ general, $s_6 =$ slightly good, $s_7 =$ good, $s_8 =$ very good, and $s_9 =$ extremely good$\}$

The decision making steps using the CDE-TODIM method are as follows:

Step 1. The Lq-ROFR given by the DMs is in Tables 3–5.

Step 2. The results of normalizing the Lq-ROFR in Tables 6–8.

Step 3. The Chebyshev distance entropy model is used to find the DMs' weights as Table 9.

Step 4. LqROFWA method of DM's decision matrices for aggregation ($q = 3$).

Step 5. Applying Chebyshev distance entropy measure method to solve for attribute weights. And calculated attribute weights are shown in Table 10.

Step 6. Calculate the relative weight $\omega_{ac_i} = \omega_{c_i}/\omega_a$ of each attribute relative to the highest weight ω_a.

Step 7. Calculate the total dominance $\Phi(P_x, P_y)$ of scheme P_x over scheme P_y (As a result of the conservative decision analysis, first take $\theta = 2.5$).

Step 8. Calculate $\delta(P_x)$.

Step 9. Normalize $\delta(P_x)$, then we can obtain $\widetilde{\delta}(P_x)$.

Step 10. Sort schemes: $P_4 \succ P_2 \succ P_1 \succ P_3$. Therefore, the optimal scheme is P_4.

Step 11. End.

Tables 9–15 show the intermediate results obtained from the model calculations, which more visually reflect the feasibility of the method.

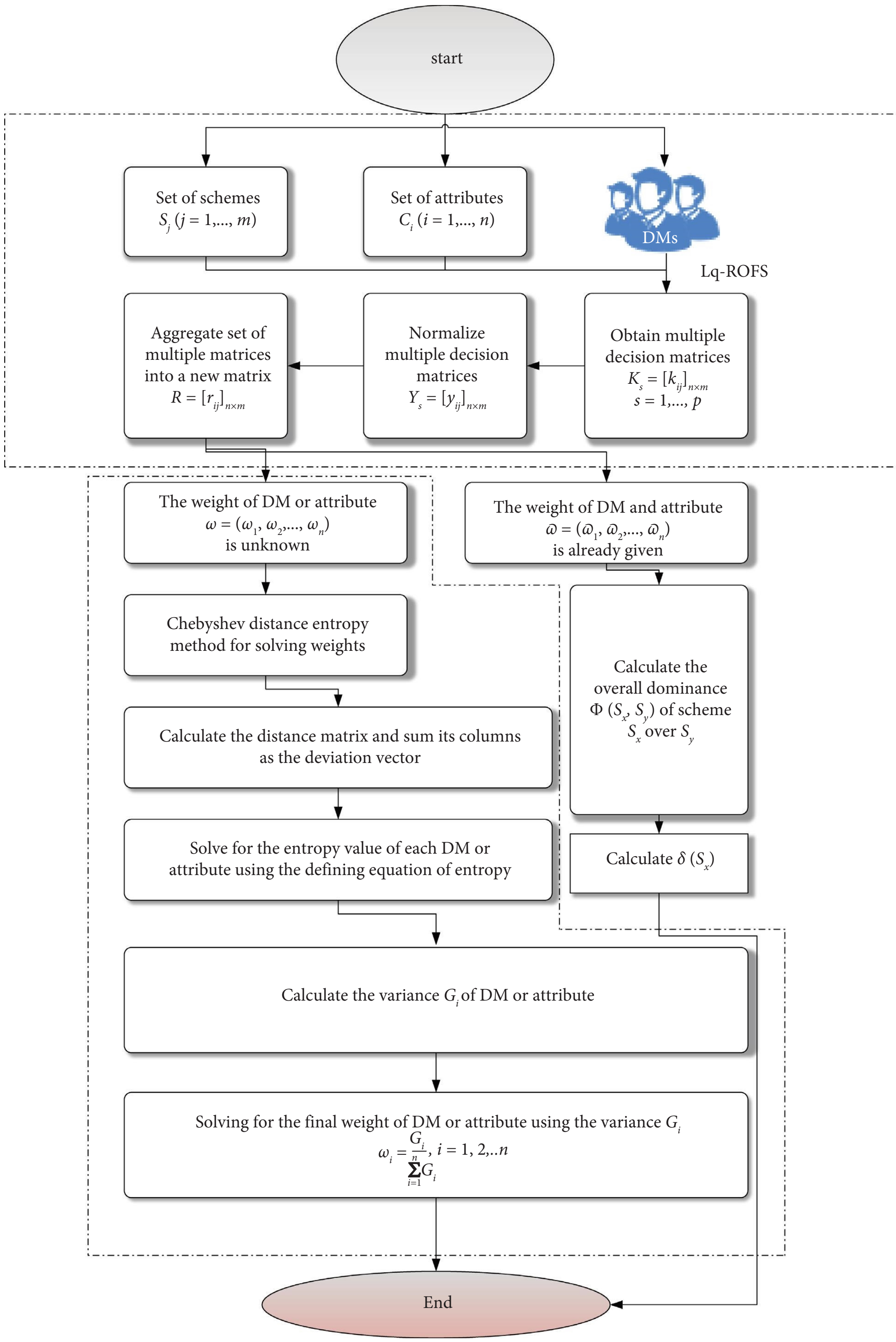

Figure 1: Specific process for multiattribute group decision making.

Table 3: The decision matrix given by DM_1.

	C_1	C_2	C_3	C_4
P_1	(s_1, s_6)	(s_4, s_3)	(s_4, s_3)	(s_4, s_3)
P_2	(s_5, s_2)	(s_6, s_1)	(s_1, s_5)	(s_2, s_5)
P_3	(s_5, s_2)	(s_2, s_3)	(s_2, s_5)	(s_3, s_4)
P_4	(s_1, s_5)	(s_3, s_4)	(s_4, s_1)	(s_1, s_5)

Table 4: The decision matrix given by DM_2.

	C_1	C_2	C_3	C_4
P_1	(s_3, s_2)	(s_1, s_4)	(s_2, s_5)	(s_6, s_1)
P_2	(s_4, s_3)	(s_5, s_1)	(s_3, s_1)	(s_2, s_3)
P_3	(s_4, s_2)	(s_3, s_4)	(s_1, s_3)	(s_3, s_4)
P_4	(s_2, s_2)	(s_5, s_1)	(s_4, s_2)	(s_3, s_2)

Table 5: The decision matrix given by DM_3.

	C_1	C_2	C_3	C_4
P_1	(s_1, s_4)	(s_5, s_1)	(s_4, s_2)	(s_4, s_2)
P_2	(s_7, s_2)	(s_3, s_1)	(s_4, s_3)	(s_3, s_4)
P_3	(s_1, s_6)	(s_2, s_2)	(s_3, s_2)	(s_5, s_2)
P_4	(s_6, s_1)	(s_2, s_5)	(s_2, s_5)	(s_1, s_3)

Table 6: The normalized decision matrix given by DM_1.

	C_1	C_2	C_3	C_4
P_1	(s_1, s_6)	(s_4, s_3)	(s_3, s_4)	(s_3, s_4)
P_2	(s_5, s_2)	(s_6, s_1)	(s_5, s_1)	(s_5, s_2)
P_3	(s_5, s_2)	(s_2, s_3)	(s_5, s_2)	(s_4, s_3)
P_4	(s_1, s_5)	(s_3, s_4)	(s_1, s_4)	(s_5, s_1)

Table 7: The normalized decision matrix given by DM_2.

	C_1	C_2	C_3	C_4
P_1	(s_3, s_2)	(s_1, s_4)	(s_5, s_7)	(s_1, s_6)
P_2	(s_4, s_3)	(s_5, s_1)	(s_1, s_3)	(s_3, s_2)
P_3	(s_4, s_2)	(s_3, s_4)	(s_3, s_1)	(s_4, s_3)
P_4	(s_2, s_2)	(s_5, s_1)	(s_2, s_4)	(s_2, s_3)

Table 8: The normalized decision matrix given by DM_3.

	C_1	C_2	C_3	C_4
P_1	(s_1, s_4)	(s_5, s_1)	(s_2, s_4)	(s_2, s_4)
P_2	(s_7, s_2)	(s_3, s_1)	(s_3, s_4)	(s_4, s_3)
P_3	(s_1, s_6)	(s_2, s_2)	(s_2, s_3)	(s_2, s_5)
P_4	(s_6, s_1)	(s_2, s_5)	(s_5, s_2)	(s_3, s_1)

Table 9: The value of DMs' weights.

λ_1	λ_2	λ_3
0.3340	0.3318	0.3341

Table 10: Attribute weights.

ω_{C_1}	ω_{C_2}	ω_{C_3}	ω_{C_4}
0.2345	0.1657	0.3732	0.2266

5.1.1. Sensitivity Analysis. The scores of different schemes when q is taken from 1 to 20 are displayed in the figure.

When θ is taken from 1 to 10 ($q = 3$), the figure of the change in scheme scores.

It can be summarized from Figure 2 that when θ is fixed at 2.5, which means that DMs are risk averse, then for q from 1 to 20 (different Lq-ROFS environments), it can be found that the optimal solution is P_4 when $q \leq 4$, followed by P_2, followed by P_1, and the worst is P_3, but when $5 \leq q \leq 15$, the optimal solution becomes P_2, and when $q = 16$, 17, 18, the optimal solution becomes P_4 again, and when $q > 18$, the optimal solution is P_2. The above mentioned discussion shows that the optimal solution fluctuates between P_2 and P_4 and is influenced by the value of q.

From Figure 3, when q is taken as 3 and θ takes different values, it can be seen that when $1 \leq \theta \leq 2.5$, the scheme ranking is $P_4 \succ P_2 \succ P_1 \succ P_3$, when $\theta > 2.5$, the scheme ranking is $P_2 \succ P_4 \succ P_1 \succ P_3$, which indicates that the value of θ also has an effect on the scheme ranking, and the optimal scheme also fluctuates between P_2 and P_4.

Therefore, it can be seen from the figure that the scheme ordering changes significantly when the q value changes, while the scheme does not change much when the θ value changes, which also indicates that the choice of which Lq-ROFS has a greater impact on the scheme.

5.2. Comparison of Effectiveness of Multiple Approaches. To illustrate the effectiveness and feasibility of the CDE-TODIM method, the results were compared with the use of other methods to solve this case, and the comparison is presented in Table 16 to obtain an evaluation of the CDE-TODIM method.

The results of other methods for solving this example show that the difference between the decision results obtained by the CDE-TODIM method and other methods is not significant, which shows that the CDE-TODIM method works.

For this case, the optimal solution of decision making using the CDE-TODIM method is better in investment in mask manufacturing plant and online education platform when the value of q is less than 4, and the education platform solution is better when it is greater than 4. The optimal solution found by the rest of the methods is investment in mask manufacturing plant, which indicates that the decision results of different solutions tend to be consistent and the results are more credible. According to the data, more people in real life choose to invest in the mask manufacturing plant and a small number of people choose to invest in the education platform. This means that the best way to measure the decision results is to find similar cases from real life and compare them with the results, and if the deviation is large, further careful consideration is needed, and if the deviation is small, the optimal result of the decision can be chosen.

6. CDE-TODIM Model Effectiveness Evaluation

After a comparative analysis with different methods, other advantages of the CDE-TODIM method, in addition to the previously proposed advantages, are as follows:

Table 11: The matrix formed by the set of three matrices.

	C_1	C_2	C_3	C_4
P_1	$(s_{2.1384}, s_{3.6391})$	$(s_{4.0483}, s_{2.2864})$	$(s_{3.8301}, s_{3.1781})$	$(s_{2.2986}, s_{4.5761})$
P_2	$(s_{5.8866}, s_{2.2880})$	$(s_{5.0713}, s_{1.0000})$	$(s_{3.7882}, s_{2.2882})$	$(s_{4.1982}, s_{2.2902})$
P_3	$(s_{4.0453}, s_{2.8871})$	$(s_{2.4321}, s_{2.8823})$	$(s_{3.8352}, s_{1.8196})$	$(s_{3.5835}, s_{3.5584})$
P_4	$(s_{4.4507}, s_{2.1546})$	$(s_{3.8301}, s_{2.7206})$	$(s_{3.6455}, s_{3.1730})$	$(s_{3.8361}, s_{1.4399})$

Table 12: Relative weight.

ω_{ac_1}	ω_{ac_2}	ω_{ac_3}	ω_{ac_4}
0.6282	0.4438	1	0.6072

Table 13: Final dominance matrix.

	P_1	P_2	P_3	P_4
P_1	0	−0.7994	−0.1432	−0.1445
P_2	0.2287	0	0.2974	−0.5619
P_3	−0.5647	−0.8461	0	−0.4628
P_4	−0.2309	0.0871	0.1132	0

Table 14: The value of $\delta(P_x)$.

$\delta(P_1)$	$\delta(P_2)$	$\delta(P_3)$	$\delta(P_4)$
−0.8006	−0.0358	−1.8735	−0.0306

Table 15: The value of $\tilde{\delta}(P_x)$.

$\tilde{\delta}(P_1)$	$\tilde{\delta}(P_2)$	$\tilde{\delta}(P_3)$	$\tilde{\delta}(P_4)$
0.5822	0.9972	0	1

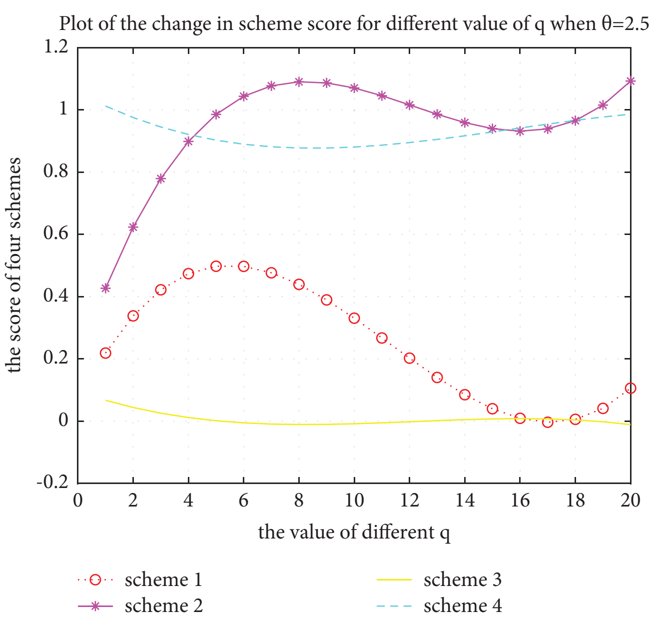

Figure 2: Scores of the schemes when q takes different values ($\theta = 2.5$).

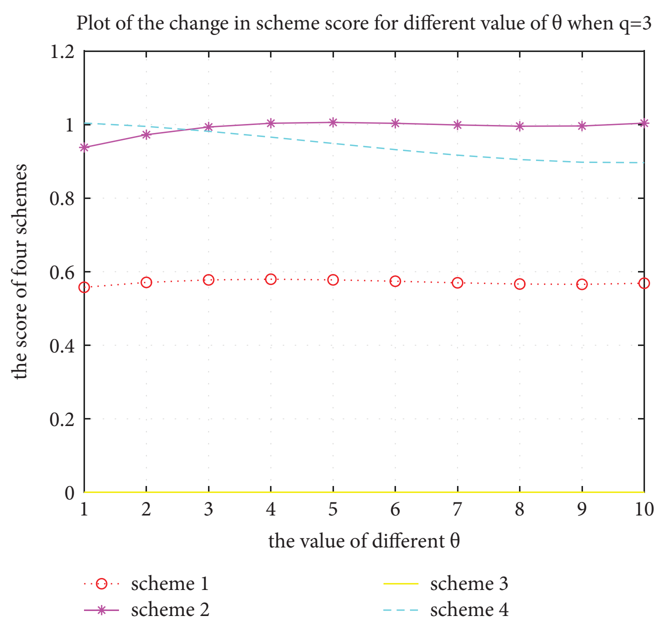

Figure 3: Scores of the schemes when θ takes different values $(q) = 3$).

(1) Considering the solutions of all DMs and using Chebyshev distance entropy method to solve the DMs' weights, and then assembling the decision

Table 16: Comparative analysis results.

Methods	Score values ($q = 3$)	Ranking results
LqROFIWPGHM operator	$\widetilde{\delta}(P_1) = s_{6.14360}$ $\widetilde{\delta}(P_2) = s_{6.85258}$ $\widetilde{\delta}(P_3) = s_{6.23779}$ $\widetilde{\delta}(P_4) = s_{6.42739}$	$P_2 \succ P_4 \succ P_3 \succ P_1$
LqROFWA operator	$\widetilde{\delta}(P_1) = s_{6.38924}$ $\widetilde{\delta}(P_2) = s_{6.80503}$ $\widetilde{\delta}(P_3) = s_{6.44116}$ $\widetilde{\delta}(P_4) = s_{6.58033}$	$P_2 \succ P_4 \succ P_3 \succ P_1$
LqROFWG operator	$\widetilde{\delta}(P_1) = s_{6.09989}$ $\widetilde{\delta}(P_2) = s_{6.48664}$ $\widetilde{\delta}(P_3) = s_{6.18986}$ $\widetilde{\delta}(P_4) = s_{6.25095}$	$P_2 \succ P_4 \succ P_3 \succ P_1$

matrix with the help of LqROFWA operator, which makes the results more objective and comprehensive.

(2) By applying the extended TODIM method under Lq-ROFS, not only the original advantages of the method are retained, but also the combination of language sets makes the evaluation system more realistic and accurate.

(3) Improved Hamming distance to Chebyshev distance for simplicity and ease of use

In general, the CDE-TODIM method is comprehensive and objective in its consideration of aspects that lead to better decision making results, and the method is simple and easy to follow with fewer steps.

7. Conclusions

In the context of the new crown epidemic and during home quarantine, the idea of how to find business opportunities, earn wealth, and subsidize family is on the minds of many people, but the exact aspect to start with is a difficult one. The method provided in this paper is to solve the realistic problem of difficult decisions and select the most suitable solution to achieve the goal.

In this paper, we have developed Chebyshev distance under Lq-ROFS and proposed Chebyshev distance entropy to objectively determine the DMs' weights and attribute weights based on the above distance and entropy. Then, we have combined the proposed weight method and the TODIM method to solve the MAGDM with Lq-ROFS problem whose decision matrix elements are Lq-ROFN. Moreover, a case study and a comparative analysis have been provided to verify the feasibility of the the CDE-TODIM MAGDM method.

The contribution of this paper is proposing the CDE-TODIM method for solving the investment option selection problem in the context of epidemic.

In the future, CDE-TODIM method will be extended to PFS' for research, and for Lq-ROFS will consider interval language, probabilistic language, etc. In addition to the investment case covered in this paper, extension applications in other fields such as health care assessment will also be considered.

Data Availability

The data used to support the study are included within the article.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

Consent

All authors agreed with the content of the manuscript and the accepted submission and agreed to be accountable for all aspects of the work.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

Authors' Contributions

Yangcan Wang conceptualized the study, curated the data, formally analysed the study, carried out the funding acquisition, proposed the methodology, and wrote the original draft; Xin Tang, Yutong Pan, and Mingqian Cai supervised the study; and Yangcan Wang, Xin Tang, Yutong Pan, and Mingqian Cai wrote the review and edited.

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