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Research Article

Antisynchronization and Generalized Pinning Control of Multiweighted Coupled Complex-Valued Delayed Memristive Neural Networks

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In this article, antisynchronization problem of multiweighted coupled complex-valued delayed memristive neural networks (MWCCVDMNNs) with and without coupling delays are investigated. First, via devising a suitable controller and constructing an appropriate Lyapunov functional, a criterion for ensuring antisynchronization of MWCCVDMNNs is derived. Second, we research the generalized pinning antisynchronization of MWCCVDMNNs by creating a generalized pinning controller to guarantee that the considered networks can accomplish antisynchronization. Similarly, several sufficient conditions guaranteeing the antisynchronization and generalized pinning antisynchronization of MWCCVDMNNs with coupling delays are also presented. Third, two numerical examples are provided to verify the correctness of the obtained antisynchronization results.

1. Introduction

In recent decades, the applications of neural networks (NNs) in automatic control [1], pattern recognition [2], associative memory [3], and many other fields have attracted much attention. As a particular case of complex networks, coupled NNs (CNNs) are generated from a large number of single NN through complex interactions between different NNs. Enormous practical problems such as target recognition, edge detection, and image segmentation can be described and studied with the help of CNNs. Therefore, increasing interest has been dedicated to exploring the dynamic behaviors of CNNs, such as the passivity and synchronization of CNNs [4–8]. The authors in [4] obtained the global exponential synchronization and passivity conditions of CNNs under impulse control. Wang et al. [5] derived some adequate conditions to ensure the finite-time passivity of directed and undirected CNNs.

Chua [9] first presented the concept of memristor in 1971. Numerous studies have shown that memristor shows the characteristic of pinched hysteresis, that is to say, there is a lag between the application and removal of a field and its succeeding effect, just like neurons in the human brain have [10, 11]. Due to this feature, the past dynamic history of memristor can be remembered, with a substantial similarity to the function of synapses. For this reason, many scholars have been trying to use memristors to simulate neurons to achieve brain-like computation, which leads to the development of memristive neural networks (MNNs) [12, 13]. An increasing number of scholars have recently investigated the dynamical behaviors of coupled MNNs (CMNNs) [14–16]. In [14], a distributed impulsive control strategy is employed to explore the multisynchronization problem of CMNNs. As discussed in [15], several synchronization conditions in finite-time and fixed-time for CMNNs were derived. Wu and Zeng [16] investigated a class of memristive recurrent NNs (MRNNs) and realized the exponential antisynchronization of the drive-response-based coupled MRNNs (CMRNNs).

As a generalization of the real-valued NNs (RVNNs), the state, activation function, connection weight matrix, and external input of the complex-valued NNs (CVNNs) are all defined in the range of complex numbers. The CVNNs can solve many practical problems which cannot be solved by RVNNs. Thus, it is urgent to explore the dynamic properties of CVNNs, and some achievements have been made in recent years [17–19]. As discussed by Wei et al. [17], the antisynchronization and synchronization problems of complex-valued inertial NNs were studied, and some corresponding adequate conditions for synchronization and antisynchronization of complex-valued inertial NNs were established. The authors in [18] researched the multistability and robustness of CVNNs with delays and input perturbation. Moreover, Wei et al. [19] analyzed the matter of antisynchronization for CVNNs with time-varying delays and leakage delay and proposed several corresponding antisynchronization criteria. Furthermore, a large number of researchers have discussed the synchronization and passivity of complex-valued MNNs (CVMNNs) [20, 21]. In [20], the model of delayed CVMNNs is established for the first time, and some sufficient criteria for the passivity of the given CVMNNs are derived. Zhang et al. [21] discussed the global asymptotical synchronization of fractional-order CVMNNs with both parameter uncertainties and multiple time delays. To the best of the authors' knowledge, there is no research result on the coupled CVMNNs (CCVMNNs) except that some conditions for ensuring passivity and synchronization of CCVMNNs were established in [22].

In the works as mentioned above, the network models about CCVMNNs are coupled by single-weight. As a matter of fact, in the real world, it is more intuitive and convenient to describe some large-scale networks with multiweighted complex dynamical networks (MWCDNs), such as social networks, bus line networks, and communication networks. Hence, the multiweighted coupled neural networks (MWCNNs) as a special type of MWCDNs have received extensive attention [23, 24]. Wang et al. [23] obtained several novel criteria for ensuring finite-time passivity and synchronization of MWCNNs without and with coupling delays by proposing several novel concepts about passivity in finite-time and designing appropriate controllers. The authors derived some passivity and synchronization of MWCNNs by means of designing proportional-integral-derivative controllers [24]. As is known to all, stability is an essential theoretical problem in studying NNs. The introduction of time delays may lead to oscillations or instability or even chaos in the neural network system. Thus, multiweighted coupled complex-valued delayed MNNs (MWCCVDMNNs) is more worthy of being studied. Moreover, it is also important to discuss the MWCCVDMNNs with coupling delays. In [25, 26], the authors studied the multiweighted coupled delayed MNNs (MWCDMNNs) for event-triggered passivity and passivity-based synchronization, but the networks considered in these publications are all real-valued.

It is generally known that antisynchronization of chaotic oscillators is a fascinating phenomenon. Antisynchronization is basically the same type of synchronization that Pecora and Carroll [27] first studied. The only discrepancy is that antisynchronization allows mutually symmetrical chaotic attractors to coexist. More precisely, antisynchronization refers to a phenomenon that the state vectors of the slave systems have the opposite signs but the same amplitude as those of the master system. Hence, the sum of two signals is anticipated to converge to 0 when antisynchronization occurs. So far, a wide range of methods have been proposed for the antisynchronization of chaotic systems, such as adaptive control, direct linear coupling, and nonlinear control. In [28], Ren et al. investigated the antisynchronization of chaotic systems and deduced some sufficient conditions for the given chaotic system to realize antisynchronization.

In many practical situations, antisynchronization has gained widespread use in many areas including image processing, communication systems, and laser. Hence, antisynchronization, like synchronization, also plays a critical role in the study of the dynamic behaviors of CNNs [29, 30]. In [29], some finite-time antisynchronization conditions were derived for the MWCNNs. By integrating the definition of lag synchronization into the definition of decay synchronization, the concept of general decay lag antisynchronization was proposed and obtained several criteria for guaranteeing that multiweighted delayed CNNs with reaction-diffusion terms achieve decay lag antisynchronization [30].

Different from other control methods, pinning control is a very effective control scheme in which only a small fraction of nodes are chosen to be controlled. So far, some scholars have investigated pinning synchronization control problem of the coupled delayed NNs (CDNNs) [31] and CMNNs [32]. In [31], several criteria were proposed to ensure the cluster synchronization of nonlinear CDNNs in both finite-time and fixed-time aspects based on the pinning control strategy. In a previous study [32], Yue et al. addressed the passivity and synchronization of CMNNs by making use of two effective pinning control strategies. However, the investigation on pinning antisynchronization of MWCNNs is very rare [33]. Hou et al. [33] analyzed the pinning antisynchronization of the MWCNNs. To the best of the authors' knowledge, antisynchronization and pinning antisynchronization problems of MWCCVDMNNs have not been researched yet which motivates the study in this paper. Different from the classical pinning control method, what we study in this paper is the networks with discontinuous neurons; in this case, we develop the generalized antisynchronization pinning scheme for this kind of network.

From what has been discussed above, this paper studies two classes of NNs: MWCCVDMNNs and MWCCVDMNNs with coupling delays. The main novelties of our work can be outlined as follows. (1) The network model of CCVDMNNs with multiweights is presented for the first time. (2) By virtue of employing appropriate Lyapunov functional and several inequality techniques, several adequate conditions are proposed to ensure the antisynchronization of MWCCVDMNNs with and without coupling delays. (3) Considering that MWCCVDMNNs are discontinuous neural networks, a novel pinning controller for this type of network is designed to realize the generalized pinning antisynchronization.

2. Preliminaries

2.1. Notations. Let $\mathbb{R}^N$ and $\mathbb{C}^N$ be the N-dimensional real and complex vector space, respectively. The smallest and largest eigenvalue of the corresponding matrix are denoted by $\lambda_m(\cdot)$ and $\lambda_M(\cdot)$, respectively. $\otimes$ represents the Kronecker product. For any complex number $a = a^R + ia^I$, in which $i = \sqrt{-1}$ is the imaginary unit, $a^I, a^R \in \mathbb{R}$ are the imaginary part and real part of a, respectively. For any vector $a(t) \in \mathbb{C}^N$, $\|a(t)\| = \sqrt{a^H(t)a(t)}$, where $a^H(t)$ represents the conjugate transposition of $a(t)$. If the real and imaginary parts of $a(t) \in \mathbb{C}^N$ are denoted by $a^R(t)$, $a^I(t) \in \mathbb{R}^N$, then one has $\|a(t)\| = \sqrt{(a^R(t))^T a^R(t) + (a^I(t))^T a^I(t)}$. Obviously, the vector norm in our paper is Euclidean norm, i.e., L_2 norm. Note that the vector $a(t)$ is a complex vector, and the norm of $a(t)$ is the square of $a^H(t)a(t) = (a^R(t))^T a^R(t) + (a^I(t))^T a^I(t)$.

2.2. Lemmas

Lemma 1 (see [34]). *For any real vectors α_1, $\alpha_2 \in \mathbb{R}^n$ and matrix $0 < \Xi \in \mathbb{R}^{n\times n}$,*

$$2\alpha_1^T\alpha_2 \le \alpha_1^T \Xi \alpha_1 + \alpha_2^T \Xi^{-1}\alpha_2. \tag{1}$$

Lemma 2 (see [35]). *Let A, B, C, and D be matrices with compatible dimensions and $a \in \mathbb{R}$. Then,*

$$\begin{aligned}
&(1)\,(aA)\otimes B = A\otimes (aB);\\
&(2)\,(A\otimes B)^T = A^T \otimes B^T;\\
&(3)\,(A\otimes B)(C\otimes D) = (AC)\otimes (B\ D);\\
&(4)\,(A+B)\otimes C = A\otimes C + B\otimes C.
\end{aligned} \tag{2}$$

3. Antisynchronization and Generalized Pinning Antisynchronization of MWCCVDMNNs

3.1. Antisynchronization of MWCCVDMNNs. In this paper, a single model of CVDMNN is presented as follows:

$$\dot{a}_s(t) = -d_s a_s(t) + \sum_{h=1}^{n} p_{sh}(a_s(t))\, y_h(a_h(t-u_h(t))), \quad s = 1,2,\ldots,n, \tag{3}$$

where the complex-valued $a_s(t)$ indicates the state variable associated with the s-th neuron; $d_s > 0$ represents the self-feedback coefficient; $p_{sh}(a_s(t))$ stands for the weight of the synaptic connection of complex-valued memristor; the complex-valued nonlinear function $y_h(\cdot)$ denotes the activation function of the h-th neuron; $u_h(t)$ is the time-varying delay with $0 \le u_h(t) \le u_h \le u = \max_{h=1,2,\ldots,n}\{u_h\}$ and $\dot{u}_h(t) \le \varrho_h < 1$.

Let

$$\begin{aligned}
a_s(t) &= a_s^R(t) + ia_s^I(t),\ p_{sh}(a_s(t))\\
&= p_{sh}^R\big(a_s^R(t)\big) + ip_{sh}^I\big(a_s^I(t)\big),\ y_h(a_h(t-u_h(t)))\\
&= y_h^R\big(a_h^R(t-u_h(t))\big) + iy_h^I a_h^I(t-u_h(t))\big),
\end{aligned} \tag{4}$$

where $a_s^I(t), p_{sh}^I(a_s^I(t)), y_h^I(a_h^I(t-u_h(t)))$ are the imaginary parts of $a_s(t), p_{sh}(a_s(t)), y_h(a_h(t-u_h(t)))$ and $a_s^R(t)$, $p_{sh}^R(a_s^R(t)), y_h^R(a_h^R(t-u_h(t)))$ are the real parts of $a_s(t), p_{sh}(a_s(t)), y_h(a_h(t-u_h(t)))$.

According to the characteristics of memristor's current and voltage, one has

$$\begin{aligned}
p_{sh}^R\big(a_s^R(t)\big) &= \begin{cases} \widehat{p}_{sh}^R, & \big|a_s^R(t)\big| \le \Lambda_s,\\ \check{p}_{sh}^R, & \big|a_s^R(t)\big| > \Lambda_s,\end{cases}\\
p_{sh}^I\big(a_s^I(t)\big) &= \begin{cases} \widehat{p}_{sh}^I, & \big|a_s^I(t)\big| \le \Lambda_s,\\ \check{p}_{sh}^I, & \big|a_s^I(t)\big| > \Lambda_s,\end{cases}
\end{aligned} \tag{5}$$

where $s,h \in \{1,2,\ldots,n\}$; $\widehat{p}_{sh}^R, \check{p}_{sh}^R, \widehat{p}_{sh}^I, \check{p}_{sh}^I$ are constants; $\Gamma_s > 0$ represents the threshold level. Let $\widetilde{p}_{sh}^R = \max\{|\widehat{p}_{sh}^R|, |\check{p}_{sh}^R|\}$, $\widetilde{p}_{sh}^I = \max\{|\widehat{p}_{sh}^I|, |\check{p}_{sh}^I|\}$, $\overline{p}_{sh}^R = |\widehat{p}_{sh}^R - \check{p}_{sh}^R|$, $\overline{p}_{sh}^I = |\widehat{p}_{sh}^I - \check{p}_{sh}^I|$, $\overline{P}^R = (\overline{p}_{sh}^R)_{n\times n}$, $\overline{P}^I = (\overline{p}_{sh}^I)_{n\times n}$, $\widetilde{P}^R = \mathrm{diag}(\sum_{h=1}^n (\widetilde{p}_{1h}^R)^2, \sum_{h=1}^n (\widetilde{p}_{2h}^R)^2, \ldots, \sum_{h=1}^n (\widetilde{p}_{nh}^R)^2)$, and $\widetilde{P}^I = \mathrm{diag}(\sum_{h=1}^n (\widetilde{p}_{1h}^I)^2, \sum_{h=1}^n (\widetilde{p}_{2h}^I)^2, \ldots, \sum_{h=1}^n (\widetilde{p}_{nh}^I)^2)$.

In the following, we consider the MWCCVDMNNs made up of N CVDMNNs as (3)

$$\begin{aligned}
\dot{A}_\delta(t) &= -DA_\delta(t) + P(A_\delta(t))\,y\big(\overline{A_\delta(t)}\big) + c_\delta(t)\\
&\quad + \sum_{\epsilon=1}^{\eta}\sum_{\kappa=1}^{N} l_\epsilon K_{\delta\kappa}^{\epsilon} H_\epsilon A_\kappa(t),\ \delta = 1,2,\ldots,N,
\end{aligned} \tag{6}$$

in which the complex-valued vector $A_\delta(t) = (A_{\delta 1}(t), A_{\delta 2}(t), \ldots, A_{\delta n}(t))^T \in \mathbb{C}^n$ denotes the state vector for the δ-th node; $0 < D = \mathrm{diag}(d_1, d_2, \ldots, d_n) \in \mathbb{R}^{n\times n}$, $\overline{A_\delta(t)} = (A_{\delta 1}(t-u_1(t)),\ A_{\delta 2}(t-u_2(t)), \ldots, A_{\delta n}(t-u_n(t)))^T \in \mathbb{C}^n$, $y(\overline{A_\delta(t)}) = (y_1(A_{\delta 1}(t-u_1(t))), y_2(A_{\delta 2}(t-u_2(t))), \cdots, y_n(A_{\delta n}(t-u_n(t))))^T \in \mathbb{C}^n$, and $H_\epsilon \in \mathbb{R}^{n\times n}$ $(\epsilon = 1,2,\ldots,\eta)$ is an internally coupled matrix; $P(A_\delta(t)) = (p_{sh}(A_{\delta s}(t)))_{n\times n} \in \mathbb{C}^{n\times n}$; $c_\delta(t) = c_\delta^R(t) + ic_\delta^I(t) = (c_{\delta 1}(t), c_{\delta 2}(t), \ldots, c_{\delta n}(t))^T \in \mathbb{C}^n$ represents the controller; $\mathbb{R} \ni l_\epsilon > 0$ $(\epsilon = 1,2,\ldots,\eta)$ symbols coupling strength for the ϵ-th coupling form; $K^\epsilon = (K_{\delta\kappa}^\epsilon)_{N\times N} \in \mathbb{R}^{N\times N}$ $(\epsilon = 1,2,\ldots,\eta)$ describes the outer coupling matrix, where $\mathbb{R} \ni K_{\delta\kappa}^\epsilon > 0$, if there is a connection from node ι to node $\kappa(\delta \ne \kappa)$; otherwise, $\mathbb{R} \ni K_{\delta\kappa}^\epsilon = 0$.

$$K_{\delta\delta}^\epsilon = -\sum_{\substack{k=1\\ k\ne\delta}}^{N} K_{\delta\kappa}^\epsilon,\ \delta = 1,2,\ldots,N. \tag{7}$$

Remark 1. CVNNs, as an extension of RVNNs, can solve some problems which cannot be dealt by RVNNs because their states, activation functions, and connection weights are all complex-valued. For instance, the detection of symmetric problem and the XOR problem cannot be modelled by a single real-valued neuron, but they can be achieved by a single complex-valued neuron with the orthogonal decision boundaries [36]. Moreover, it is more suitable to describe various physical phenomena (e.g., the phase progression and retardation, the superposition of fields, and the wave propagation) through complex numbers in reality, all of which can be drawn as the applications of CVNNs. Naturally, CVNNs have found widespread practical applications in physical systems dealing with quantum waves, ultrasonic, and electromagnetic waves. Therefore, it is very meaningful to consider the characteristic of complex-value when studying CMNNs.

Then, separating network (6) into the following imaginary and real parts:

$$\begin{aligned}\dot{A}_\delta^R(t) &= -DA_\delta^R(t) + P^R\left(A_\delta^R(t)\right)y^R\left(\overline{A_\iota^R(t)}\right) + \sum_{\epsilon=1}^{\eta}\sum_{\kappa=1}^{N} l_\epsilon K_{\delta\kappa}^\epsilon H_\epsilon A_\kappa^R(t)\\ &\quad -P^I\left(A_\delta^I(t)\right)y^I\left(\overline{A_\delta^I(t)}\right) + c_\delta^R(t),\\ \dot{A}_\delta^I(t) &= -DA_\delta^I(t) + P^R\left(A_\delta^R(t)\right)y^I\left(\overline{A_\delta^I(t)}\right) + \sum_{i=1}^{\eta}\sum_{\kappa=1}^{N} l_\epsilon K_{\iota\kappa}^\epsilon H_\epsilon A_\kappa^I(t)\\ &\quad +P^I\left(A_\delta^I(t)\right)y^R\left(\overline{A_\delta^R(t)}\right) + c_\delta^I(t),\end{aligned} \tag{8}$$

where $y^R(\overline{A_\delta^R(t)}) = (y_1^R(A_{\delta 1}^R(t-u_1(t))),\ y_2^R(A_{\delta 2}^R(t-u_2(t))),\ \ldots, y_n^R(A_{\delta n}^R(t-u_n(t))))^T$, $y^I(\overline{A_\delta^I(t)}) = (y_1^I(A_{\delta 1}^I(t-u_1(t))), y_2^I(A_{\delta 2}^I(t-u_2(t))),\ldots, y_n^I(A_{\delta n}^I(t-u_n(t))))^T$, $A_\delta^R(t) = (A_{\delta 1}^R(t), A_{\delta 2}^R(t),\ldots,A_{\delta n}^R(t))^T$, $P^R(\cdot) = (p_{sh}^R(\cdot))_{n\times n}$, $A_\delta^I(t) = (A_{\delta 1}^I(t), A_{\delta 2}^I(t),\ldots,A_{\delta n}^I(t))^T$, $c_\delta^R(t) = (c_{\delta 1}^R(t),c_{\delta 2}^R(t),\ldots,c_{\delta n}^R(t))^T$, and $P^I(\cdot) = (p_{sh}^I(\cdot))_{n\times n}$, $c_\delta^I(t) = (c_{\delta 1}^I(t),c_{\delta 2}^I(t),\ldots,c_{\delta n}^I(t))^T$.

Assumption 1. For all $\beta_1,\beta_2 \in \mathbb{R}$, there exist real numbers $Y_\delta^R, Y_\delta^I, j_\delta^R, j_\delta^R > 0$ such that

$$\begin{aligned}&\left|y_\delta^R(\cdot)\right| \le Y_\delta^R, \left|y_\delta^I(\cdot)\right| \le Y_\delta^I,\\ &\left|y_\delta^R(\beta_1) + y_\delta^R(\beta_2)\right| \le j_\delta^R|\beta_1 + \beta_2|,\\ &\left|y_\delta^I(\beta_1) + y_\delta^I(\beta_2)\right| \le j_\delta^I|\beta_1 + \beta_2|.\end{aligned} \tag{9}$$

For the network (6), assume that $A_*(t) = (A_{*1}(t), A_{*2}(t),\ldots,A_{*n}(t))^T \in \mathbb{C}^n \in \mathbb{C}^n$ is an arbitrary solution, then

$$\dot{A}_*(t) = -DA_*(t) + P(A_*(t))y(A_*(t)). \tag{10}$$

Here, $A_*(t) = A_*^R(t) + iA_*^I(t)$. Then, (10) can be expressed by separating it into the following two real systems:

$$\begin{aligned}\dot{A}_*^R(t) &= -DA_*^R(t) + P^R\left(A_*^R(t)\right)y^R\left(A_*^R(t)\right)\\ &\quad - P^I\left(A_*^I(t)\right)y^I\left(A_*^I(t)\right),\\ \dot{A}_*^I(t) &= -DA_*^I(t) + P^R\left(A_*^R(t)\right)y^I\left(A_*^I(t)\right)\\ &\quad + P^I\left(A_*^I(t)\right)y^R\left(A_*^R(t)\right).\end{aligned} \tag{11}$$

Let $z_\delta(t) = A_\delta(t) + A_*(t)$, then

$$\begin{aligned}\dot{z}_\delta(t) &= -Dz_\delta(t) + P(A_\delta(t))y\left(\overline{A_\delta(t)}\right)\\ &\quad + c_\delta(t) + P(A_*(t))y(A_*(t))\\ &\quad + \sum_{\epsilon=1}^{\eta}\sum_{\kappa=1}^{N} l_\epsilon K_{\delta\kappa}^\epsilon H_\epsilon z_\kappa(t),\end{aligned} \tag{12}$$

where $z_\delta(t) = (z_{\delta 1}(t), z_{\delta 2}(t),\ldots,z_{\delta n}(t))^T$, $\delta = 1,2,\ldots,N$.

Then, the decomposition form of (12) can be expressed as

$$\begin{aligned}\dot{z}_\delta^R(t) &= -Dz_\delta^R(t) + P^R\left(A_\delta^R(t)\right)G^R\left(\overline{z_\delta^R(t)}\right)\\ &\quad - P^I\left(A_\delta^I(t)\right)G^I\left(\overline{z_\delta^I(t)}\right)\\ &\quad -\left(P^R\left(A_\delta^R(t)\right) - P^R\left(A_*^R(t)\right)\right)y^R\left(A_*^R(t)\right) + c_\delta^R(t)\\ &\quad +\left(P^I\left(A_\delta^I(t)\right) - P^I\left(A_*^I(t)\right)\right)y^I\left(A_*^I(t)\right)\\ &\quad + \sum_{\epsilon=1}^{\eta}\sum_{\kappa=1}^{N} l_\epsilon K_{\iota\kappa}^\epsilon H_\epsilon z_\kappa^R(t),\\ \dot{z}_\delta^I(t) &= -Dz_\delta^I(t) + P^R\left(A_\delta^R(t)\right)G^I\left(\overline{z_\delta^I(t)}\right)\\ &\quad + P^I\left(A_\delta^I(t)\right)G^R\left(\overline{z_\delta^R(t)}\right)\\ &\quad -\left(P^R\left(A_\delta^R(t)\right) - P^R\left(A_*^R(t)\right)\right)y^I\left(A_*^I(t)\right) + c_\delta^I(t)\\ &\quad -\left(P^I\left(A_\delta^I(t)\right) - P^I\left(A_*^I(t)\right)\right)y^R\left(A_*^R(t)\right)\\ &\quad + \sum_{\epsilon=1}^{\eta}\sum_{\kappa=1}^{N} l_\epsilon K_{\epsilon\kappa}^\epsilon H_\epsilon z_\kappa^I(t),\end{aligned} \tag{13}$$

where $z_\delta^R(t) = (z_{\delta 1}^R(t),z_{\delta 2}^R(t),\ldots,z_{\delta n}^R(t))^T$, $z_\delta^I(t) = (z_{\delta 1}^I(t), z_{\delta 2}^I(t),\ldots,z_{\delta n}^I(t))^T$, $\overline{z_\delta^R(t)} = (z_{\delta 1}^R(t-u_1(t)),z_{\delta 2}^R(t-u_2(t)),\ldots,z_{\delta n}^R(t-u_n(t)))^T$, $\overline{z_\delta^I(t)} = (z_{\delta 1}^I(t-u_1(t)),z_{\delta 2}^I(t-u_2(t)),\ldots, z_{\delta n}^I(t-u_n(t)))^T$, $G^R(\overline{z_\delta^R(t)}) = y^R(\overline{A_\delta^R(t)}) + y^R(A_*^R(t))$, and $G^I(\overline{z_\delta^I(t)}) = y^I(\overline{A_\delta^I(t)}) + y^I(A_*^I(t))$.

Definition 1. The network (6) is said to be antisynchronized if

$$\lim_{t\longrightarrow +\infty}\left\|A_\delta(t) + A_*(t)\right\| = 0, \text{for all } \delta = 1,2,\ldots,N. \tag{14}$$

In this paper, the authors construct the following controller for the MWCCVDMNNs (6):

$$\begin{aligned} c_\delta^R(t) &= -\Theta_\delta^R z_\delta^R(t) - \operatorname{sign}\left(z_\delta^R(t)\right)\left(\overline{P}^R\overline{Y}^R + \overline{P}^I\overline{Y}^I\right), \\ c_\delta^I(t) &= -\Theta_\delta^I z_\delta^I(t) - \operatorname{sign}\left(z_\delta^I(t)\right)\left(\overline{P}^R\overline{Y}^I + \overline{P}^I\overline{Y}^R\right), \end{aligned} \tag{15}$$

where $\delta = 1, 2, \ldots, N$; $\mathbb{R} \ni v_\delta^R > 0$ and $\mathbb{R} \ni v_\delta^I > 0$; $\overline{Y}^R = (Y_1^R, Y_2^R, \ldots, Y_n^R)^T$; $\Theta_\delta^R = \operatorname{diag}(v_{\delta 1}^R, v_{\delta 2}^R, \ldots, v_{\delta n}^R) \in \mathbb{R}^{n\times n}$ and $\Theta_\delta^I = \operatorname{diag}(v_{\delta 1}^I, v_{\delta 2}^I, \ldots, v_{\delta n}^I) \in \mathbb{R}^{n\times n}$ are the positive definite control gain matrices; sign $(z_\delta^R(t))$ = diag (sign $(z_{\delta 1}^R(t))$, sign $(z_{\delta 2}^R(t)), \ldots,$ sign $(z_{\delta n}^R(t)))$; $\overline{Y}^I = (Y_1^I, Y_2^I, \ldots, Y_n^I)^T$; and sign $(z_\delta^I(t))$ = diag (sign $(z_{\delta 1}^I(t))$, sign $(z_{\delta 2}^I(t)), \ldots,$ sign $(z_{\delta n}^I(t)))$.

For convenience, some symbols are denoted as follows:

$$\begin{aligned} z^R(t) &= \left(\left(z_1^R(t)\right)^T, \left(z_2^R(t)\right)^T, \ldots, \left(z_N^R(t)\right)^T\right)^T, \\ z^I(t) &= \left(\left(z_1^I(t)\right)^T, \left(z_2^I(t)\right)^T, \ldots, \left(z_N^I(t)\right)^T\right)^T, \\ \overline{z^R(t)} &= \left(\left(\overline{z_1^R(t)}\right)^T, \left(\overline{z_2^R(t)}\right)^T, \ldots, \left(\overline{z_N^R(t)}\right)^T\right)^T, \\ \overline{z^I(t)} &= \left(\left(\overline{z_1^I(t)}\right)^T, \left(\overline{z_2^I(t)}\right)^T, \ldots, \left(\overline{z_N^I(t)}\right)^T\right)^T, \\ J^R &= \operatorname{diag}\left(\left(j_1^R\right)^2, \left(j_2^R\right)^2, \ldots, \left(j_n^R\right)^2\right), \\ J^I &= \operatorname{diag}\left(\left(j_1^I\right)^2, \left(j_2^I\right)^2, \ldots, \left(j_n^I\right)^2\right), \\ \Lambda &= \operatorname{diag}\left(\frac{1}{1-\varrho_1}, \frac{1}{1-\varrho_2}, \ldots, \frac{1}{1-\varrho_n}\right). \end{aligned} \tag{16}$$

Theorem 1. *The antisynchronization of network (6) can be realized under the controller (15) if the following conditions hold:*

$$\Phi_1^R < 0 \text{and} \Phi_1^I < 0, \tag{17}$$

where $\Phi_1^R = I_N \otimes (-2D + \widetilde{P}^R + \widetilde{P}^I + 2J^R\Lambda) - 2\Theta^R + \sum_{\epsilon=1}^{\eta} l_\epsilon [K^\epsilon \otimes H_\epsilon + (K^\epsilon)^T \otimes H_\epsilon^T]$, $\Phi_1^I = I_N \otimes (-2D + \widetilde{P}^R + \widetilde{P}^I + 2J^I\Lambda) - 2\Theta^I + \sum_{\epsilon=1}^{\eta} l_\epsilon [K^\epsilon \otimes H_\epsilon + (K^\epsilon)^T \otimes H_\epsilon^T]$; $\Theta^R = diag(\Theta_1^R, \Theta_2^R, \ldots, \Theta_N^R) \in \mathbb{R}^{nN\times nN}$, *and* $\Theta^I = diag(\Theta_1^I, \Theta_2^I, \ldots, \Theta_N^I) \in \mathbb{R}^{nN\times nN}$.

Proof. Consider the Lyapunov function as

$$\begin{aligned} V_1(t) = &\sum_{\delta=1}^{N}\left(z_\delta^R(t)\right)^T z_\delta^R(t) + 2\sum_{\delta=1}^{N}\sum_{h=1}^{n}\int_{t-u_h(t)}^{t}\frac{\left(j_h^R z_{\delta h}^R(\varepsilon)\right)^2}{1-\varrho_h}d\varepsilon \\ &+ 2\sum_{\delta=1}^{N}\sum_{h=1}^{n}\int_{t-u_h(t)}^{t}\frac{\left(j_h^I z_{\delta h}^I(\varepsilon)\right)^2}{1-\varrho_h}d\varepsilon + \sum_{\delta=1}^{N}\left(z_\delta^I(t)\right)^T z_\delta^I(t). \end{aligned} \tag{18}$$

Then, one can yield

$$\begin{aligned} \dot{V}_1(t) \leq\; & 2\sum_{\delta=1}^{N}\left(z_\delta^R(t)\right)^T\Bigg\{-Dz_\delta^R(t) + P^R\left(A_\delta^R(t)\right)G^R\left(\overline{z_\delta^R(t)}\right) - P^I\left(A_\delta^I(t)\right)G^I\left(\overline{z_\delta^I(t)}\right) + \sum_{\epsilon=1}^{\eta}\sum_{\kappa=1}^{N} l_\epsilon K_{\iota\kappa}^\epsilon H_\epsilon z_\kappa^R(t) \\ & -\Theta_\delta^R z_\delta^R(t) - \operatorname{sign}\left(z_\delta^R(t)\right)\left(\overline{P}^R\overline{Y}^R\right. \\ & \left.+\overline{P}^I\overline{Y}^I\right) - \left[P^R\left(A_\delta^R(t)\right) - P^R\left(A_*^R(t)\right)\right]y^R\left(A_*^R(t)\right) + \left[P^I\left(A_\delta^I(t)\right) - P^I\left(A_*^I(t)\right)\right]y^I\left(A_*^I(t)\right)\Bigg\} + 2\sum_{\delta=1}^{N}\left(z_\delta^I(t)\right)^T\Big\{-Dz_\delta^I(t) \\ & + P^R\left(A_\delta^R(t)\right)G^I\left(\overline{z_\delta^I(t)}\right) + P^I\left(A_\delta^I(t)\right)G^R\left(\overline{z_\delta^R(t)}\right) + \sum_{\epsilon=1}^{\eta}\sum_{\kappa=1}^{N} l_\epsilon K_{\iota\kappa}^\epsilon H_\epsilon z_\kappa^I(t) - \Theta_\delta^I z_\delta^I(t) - \operatorname{sign}\left(z_\delta^I(t)\right)\left(\overline{P}^R\overline{Y}^I + \overline{P}^I\overline{Y}^R\right) - \left[P^R\left(A_\delta^R(t)\right)\right. \\ & \left.- P^R\left(A_*^R(t)\right)\right]y^I\left(A_*^I(t)\right) - \left[P^I\left(A_\delta^I(t)\right) - P^I\left(A_*^I(t)\right)\right]y^R\left(A_*^R(t)\right)\Big\} + 2\left(z^R(t)\right)^T\left(I_N\otimes\left(J^R\Lambda\right)\right)z^R(t) \\ & - 2\left(\overline{z^R(t)}\right)^T\left(I_N\otimes J^R\right)\overline{z^R(t)} + 2\left(z^I(t)\right)^T\left(I_N\otimes\left(J^I\Lambda\right)\right)z^I(t) - 2\left(\overline{z^I(t)}\right)^T\left(I_N\otimes J^I\right)\overline{z^I(t)}. \end{aligned} \tag{19}$$

From Assumption 1, we can deduce

$$\begin{aligned}&2\sum_{\delta=1}^{N}\left(z_\delta^R(t)\right)^T P^R\left(A_\delta^R(t)\right)G^R\left(\overline{z_\delta^R(t)}\right)\\&\leq 2\sum_{\delta=1}^{N}\sum_{s=1}^{n}\sum_{h=1}^{n}\left|z_{\delta s}^R(t)\right|\widetilde{p}_{sh}^R\left|y_h^R\left(\overline{A_{\delta h}^R(t)}\right)+y_h^R\left(A_{*h}^R(t)\right)\right|\\&\leq\sum_{\delta=1}^{N}\sum_{s=1}^{n}\sum_{h=1}^{n}\left(z_{\delta s}^R(t)\right)^2\left(\widetilde{p}_{sh}^R\right)^2+\sum_{\delta=1}^{N}\sum_{h=1}^{n}\left(j_h^R\right)^2\left(\overline{z_{\delta h}^R(t)}\right)^2\\&=\left(z^R(t)\right)^T\left(I_N\otimes\widetilde{P}^R\right)z^R(t)+\left(\overline{z^R(t)}\right)^T\left(I_N\otimes J^R\right)\overline{z^R(t)}\end{aligned}\tag{20}$$

and

$$\begin{aligned}&-2\sum_{\delta=1}^{N}\left(z_\delta^R(t)\right)^T P^I\left(A_\delta^I(t)\right)G^I\left(\overline{z_\delta^I(t)}\right)\\&\quad\leq\left(z^R(t)\right)^T\left(I_N\otimes\widetilde{P}^I\right)z^R(t)+\left(\overline{z^I(t)}\right)^T\left(I_N\otimes J^I\right)\overline{z^I(t)},\end{aligned}\tag{21}$$

$$\begin{aligned}&2\sum_{\delta=1}^{N}\left(z_\delta^I(t)\right)^T P^R\left(A_\delta^R(t)\right)G^I\left(\overline{z_\delta^I(t)}\right)\\&\quad\leq\left(z^I(t)\right)^T\left(I_N\otimes\widetilde{P}^R\right)z^I(t)+\left(\overline{z^I(t)}\right)^T\left(I_N\otimes J^I\right)\overline{z^I(t)},\end{aligned}\tag{22}$$

$$\begin{aligned}&2\sum_{\delta=1}^{N}\left(z_\delta^I(t)\right)^T P^I\left(A_\delta^I(t)\right)G^R\left(\overline{z_\delta^R(t)}\right)\\&\quad\leq\left(z^I(t)\right)^T\left(I_N\otimes\widetilde{P}^I\right)z^I(t)+\left(\overline{z^R(t)}\right)^T\left(I_N\otimes J^R\right)\overline{z^R(t)}.\end{aligned}\tag{23}$$

In addition,

$$\begin{aligned}&2\sum_{\epsilon=1}^{\eta}\sum_{\delta=1}^{N}\sum_{\kappa=1}^{N}l_\epsilon K_{\delta\kappa}^\epsilon\left(z_\delta^R(t)\right)^T H_\epsilon z_\kappa^R(t)\\&\quad=\left(z^R(t)\right)^T\left[\sum_{\epsilon=1}^{\eta}l_\epsilon\left(K^\epsilon\otimes H_\epsilon+\left(K^\epsilon\right)^T\otimes H_\epsilon^T\right)\right]z^R(t).\end{aligned}\tag{24}$$

Similarly,

$$\begin{aligned}&2\sum_{\epsilon=1}^{\eta}\sum_{\delta=1}^{N}\sum_{\kappa=1}^{N}l_\epsilon K_{\iota\kappa}^\epsilon\left(z_\delta^I(t)\right)^T H_\delta z_\kappa^I(t)\\&\quad=\left(z^I(t)\right)^T\left[\sum_{\epsilon=1}^{\eta}l_\epsilon\left(K^\epsilon\otimes H_\epsilon+\left(K^\epsilon\right)^T\otimes H_\epsilon^T\right)\right]z^I(t).\end{aligned}\tag{25}$$

Furthermore,

$$\begin{aligned}&2\sum_{\delta=1}^{N}\left(z_\delta^R(t)\right)^T\left[P^R\left(A_\delta^R(t)\right)-P^R\left(A_*^R(t)\right)\right]y^R\left(A_*^R(t)\right)\\&=2\sum_{\delta=1}^{N}\sum_{h=1}^{n}\sum_{s=1}^{n}\left|z_{\delta s}^R(t)\right|\left[p_{sh}^R\left(A_{\delta s}^R(t)\right)-p_{sh}^R\left(A_{*s}^R(t)\right)\right]y_h^R\left(A_{*h}^R(t)\right)\\&\leq 2\sum_{\delta=1}^{N}\sum_{h=1}^{n}\sum_{s=1}^{n}\left|z_{\delta s}^R(t)\right|\left|\widehat{p}_{sh}^R-\breve{p}_{sh}^R\right|Y_h^R\\&=2\sum_{\delta=1}^{N}\left|\left(z_\delta^R(t)\right)^T\right|\overline{P}^R\overline{Y}^R.\end{aligned}\tag{26}$$

Likewise,

$$\begin{aligned}&2\sum_{\delta=1}^{N}\left(z_\delta^R(t)\right)^T\left[P^I\left(A_\delta^I(t)\right)-P^I\left(A_*^I(t)\right)\right]y^I\left(A_*^I(t)\right)\\&\quad\leq 2\sum_{\delta=1}^{N}\left|\left(z_\delta^R(t)\right)^T\right|\overline{P}^I\overline{Y}^I,\end{aligned}\tag{27}$$

$$\begin{aligned}&2\sum_{\delta=1}^{N}\left(z_\delta^I(t)\right)^T\left[P^R\left(A_\delta^R(t)\right)-P^R\left(A_*^R(t)\right)\right]y^I\left(A_*^I(t)\right)\\&\quad\leq 2\sum_{\delta=1}^{N}\left|\left(z_\delta^I(t)\right)^T\right|\overline{P}^R\overline{Y}^I,\end{aligned}\tag{28}$$

$$\begin{aligned}&2\sum_{\delta=1}^{N}\left(z_\delta^I(t)\right)^T\left[P^I\left(A_\delta^I(t)\right)-P^I\left(A_*^I(t)\right)\right]y^R\left(A_*^R(t)\right)\\&\quad\leq 2\sum_{\delta=1}^{N}\left|\left(z_\delta^I(t)\right)^T\right|\overline{P}^I\overline{Y}^R.\end{aligned}\tag{29}$$

It then follows from (19)–(29) that

$$\begin{aligned}\dot{V}_1(t)\leq&\left(z^R(t)\right)^T\left\{I_N\otimes\left(-2D+\widetilde{P}^R+\widetilde{P}^I+2J^R\Lambda\right)-2\Theta^R+\sum_{\epsilon=1}^{\eta}l_\epsilon\left[K^\epsilon\otimes H_\epsilon+\left(K^\epsilon\right)^T\otimes H_\epsilon^T\right]\right\}z^R(t)\\&+\left(z^I(t)\right)^T\left\{I_N\otimes\left(-2D+\widetilde{P}^R+\widetilde{P}^I+2J^I\Lambda\right)-2\Theta^I+\sum_{\epsilon=1}^{\eta}l_\epsilon\left[K^\epsilon\otimes H_\epsilon+\left(K^\epsilon\right)^T\otimes H_\epsilon^T\right]\right\}z^I(t)\\\leq&\,\alpha_1\|z(t)\|^2,\end{aligned}\tag{30}$$

where $\alpha_1=\max\{\lambda_M(\Phi_1^R),\lambda_M(\Phi_1^I)\}<0$.

From (30) and the construction of $V_1(t)$, one can obtain that $V_1(t)$ is a bounded function that does not increase. Therefore, $V_1(t)$ converges to a nonnegative constant when $t\longrightarrow+\infty$. In addition, according to (30), we can get

$$\|z(t)\|^2\leq\frac{\dot{V}_1(t)}{\alpha_1}.\tag{31}$$

From (31), one can infer that $\lim_{t\longrightarrow+\infty}\int_0^t\|z(\varepsilon)\|^2d\varepsilon$ exists and is a nonnegative real number. Furthermore,

$$\begin{aligned}0 &\le \lim_{t\longrightarrow+\infty}\sum_{\delta=1}^{N}\sum_{h=1}^{n}\int_{t-u_h(t)}^{t}\frac{2\left(j_h^R z_{\delta h}^R(\varepsilon)\right)^2}{1-\varrho_h}d\varepsilon\\ &\le \lim_{t\longrightarrow+\infty}\int_{t-u}^{t}\left(z^R(\varepsilon)\right)^T\left[I_N\otimes\left(2J^R\Lambda\right)\right]z^R(\varepsilon)d\varepsilon\\ &\le \lambda_M\left(I_N\otimes\left(2J^R\Lambda\right)\right)\lim_{t\longrightarrow+\infty}\int_{t-u}^{t}\|z(\varepsilon)\|^2 d\varepsilon\\ &= 0.\end{aligned}\tag{32}$$

Similarly,

$$0\le \lim_{t\longrightarrow+\infty}\sum_{\delta=1}^{N}\sum_{h=1}^{n}\int_{t-u_h(t)}^{t}\frac{2\left(j_h^I z_{\delta h}^I(\varepsilon)\right)^2}{1-\varrho_h}d\varepsilon = 0.\tag{33}$$

From (32) and (33), we know that $\lim_{t\longrightarrow+\infty}\sum_{\delta=1}^{N}[(z_\delta^R(t))^T z_\delta^R(t)+(z_\delta^I(t))^T(z_\delta^I(t))]$ exists and is a nonnegative real number. In the next, we will prove $\lim_{t\longrightarrow+\infty}\sum_{\delta=1}^{N}[(z_\delta^R(t))^T z_\delta^R(t)+(z_\delta^I(t))^T z_\delta^I(t)]=0$. If this is not true, we have

$$\lim_{t\longrightarrow+\infty}\sum_{\delta=1}^{N}\left[\left(z_\delta^R(t)\right)^T z_\delta^R(t)+\left(z_\delta^I(t)\right)^T z_\delta^I(t)\right]=\theta>0.\tag{34}$$

In other words, there exists a positive real constant $\widehat{\varpi}$ such that

$$\sum_{\delta=1}^{N}\left[\left(z_\delta^R(t)\right)^T z_\delta^R(t)+\left(z_\delta^I(t)\right)^T z_\delta^I(t)\right]>\frac{\theta}{2}\text{ for } t\ge\widehat{\varpi}.\tag{35}$$

Hence,

$$\|z(t)\|^2>\frac{\theta}{2},\text{ when } t\ge\widehat{\varpi}.\tag{36}$$

From (30) and (36), we can obtain

$$\dot{V}_1(t)<\frac{\alpha_1\theta}{2}, t\ge\widehat{\varpi}.\tag{37}$$

Then, according to (37), one gets

$$\begin{aligned}-V_1(\widehat{\varpi})&\le V_1(+\infty)-V_1(\widehat{\varpi})\\ &=\int_{\widehat{\varpi}}^{+\infty}\dot{V}_1(t)dt<\int_{\widehat{\varpi}}^{+\infty}\frac{\alpha_1\theta}{2}dt=-\infty,\end{aligned}\tag{38}$$

which leads to contradiction. Therefore,

$$\lim_{t\longrightarrow+\infty}\sum_{\delta=1}^{N}\left[\left(z_\delta^R(t)\right)^T z_\delta^R(t)+\left(z_\delta^I(t)\right)^T z_\delta^I(t)\right]=0.\tag{39}$$

Then, we have

$$\lim_{t\longrightarrow+\infty}\|z(t)\|=0,\tag{40}$$

which means that the network (6) achieves antisynchronization. □

3.2. Generalized Pinning Antisynchronization of MWCCVDMNNs. In the following, we generalize the traditional pinning control method. One part of the controller we designed controls all the nodes, and the other part controls the first m nodes of network (6). Then, the MWCCVDMNNs with corresponding pinning controller is described by

$$\begin{aligned}\dot{A}_\delta(t)&=-DA_\delta(t)+P\left(A_\delta(t)\right)y\left(\overline{A_\delta(t)}\right)\\ &\quad+v_\delta(t)+\sum_{\epsilon=1}^{\eta}\sum_{\kappa=1}^{N}l_\epsilon K_{\iota\kappa}^{\epsilon}H_\epsilon A_\kappa(t),\ \delta=1,2,\ldots,N,\end{aligned}\tag{41}$$

where $v_\delta(t)=v_\delta^R(t)+iv_\delta^I(t)\in\mathbb{C}^n$ is the generalized pinning adaptive controller and $0<D\in\mathbb{R}^{n\times n}$, $A_\delta(t)\in\mathbb{C}^n$, $P(A_\delta(t))\in\mathbb{C}^{n\times n}$, $y(\overline{A_\delta(t)})\in\mathbb{C}^n$, $\mathbb{R}\ni l_\epsilon>0$, $K^\epsilon\in\mathbb{R}^{N\times N}$, and $H_\epsilon\in\mathbb{R}^{n\times n}$ have the same meanings as in Section 3.1. Without loss of generality, we select the first m nodes and pin them with the following generalized pinning adaptive controller designed as

$$v_\delta(t)=c_\delta(t)+\widehat{c}_\delta(t),\ \delta=1,2,\ldots,N,\tag{42}$$

where $c_\delta(t)=c_\delta^R(t)+ic_\delta^I(t)$, $\widehat{c}_\delta(t)=\widehat{c}_\delta^R(t)+i\widehat{c}_\delta^I(t)$, in which $c_\delta^R(t)$ and $c_\delta^I(t)$ are given as in (15) for $\delta=1,2,\ldots,N$, and

$$\begin{aligned}\widehat{c}_\delta^R(t)&=\begin{cases}-\sum_{\epsilon=1}^{\eta}l_\epsilon\left(\rho_\delta^\epsilon(t)\right)^R H_\epsilon z_\delta^R(t), & \delta=1,2,\ldots,m,\\ 0, & \delta=m+1,m+2,\ldots,N,\end{cases}\\ \widehat{c}_\delta^I(t)&=\begin{cases}-\sum_{\epsilon=1}^{\eta}l_\epsilon\left(\rho_\delta^\epsilon(t)\right)^I H_\epsilon z_\delta^I(t), & \delta=1,2,\ldots,m,\\ 0, & \delta=m+1,m+2,\ldots,N,\end{cases}\end{aligned}\tag{43}$$

with

$$\begin{aligned}\left(\dot{\rho}_\delta^\epsilon(t)\right)^R&=\zeta_\delta^\epsilon\left(z_\delta^R(t)\right)^T\frac{H_\epsilon+H_\epsilon^T}{2}z_\delta^R(t),\\ \left(\dot{\rho}_\delta^\epsilon(t)\right)^I&=\zeta_\delta^\epsilon\left(z_\delta^I(t)\right)^T\frac{H_\epsilon+H_\epsilon^T}{2}z_\delta^I(t),\end{aligned}\tag{44}$$

when $\delta=1,2,\ldots,m$, where $\mathbb{R}\ni l_\epsilon>0$, $z_\delta^R(t)=A_\delta^R(t)+A_*^R(t)$, $z_\delta^I(t)=A_\delta^I(t)+A_*^I(t)$, $z_\delta^R(t)=(z_{\delta1}^R(t),z_{\delta2}^R(t),\ldots,z_{\delta n}^R(t))^T$, $z_\delta^I(t)=(z_{\delta1}^I(t),z_{\delta2}^I(t),\ldots,z_{\delta n}^I(t))^T$, $H_\epsilon\in\mathbb{R}^{n\times n}$, $H_\epsilon+H_\epsilon^T>0$, $1\le m<N$, and $\mathbb{R}\ni\zeta_\delta^\epsilon>0$; $\mathbb{R}\ni(\rho_\delta^\epsilon(0))^R>0$; and $\mathbb{R}\ni(\rho_\delta^\epsilon(0))^I>0$ for $\delta=1,2,\ldots,m$.

Similar to Section 3.1, the error vector $z_\delta(t)=A_\delta(t)+A_*(t)$ can be governed by

$$\begin{aligned}\dot{z}_\delta(t)&=-Dz_\delta(t)+P\left(A_\delta(t)\right)f\left(\overline{A_\delta(t)}\right)+v_\delta(t)\\ &\quad+P\left(A_*(t)\right)y\left(A_*(t)\right)\\ &\quad+\sum_{\epsilon=1}^{\eta}\sum_{\kappa=1}^{N}l_\epsilon K_{\delta\kappa}^{\epsilon}H_\epsilon z_\kappa(t),\ \delta=1,2,\ldots,N.\end{aligned}\tag{45}$$

In order to get the desired result, we separate networks (45) into the following equivalent imaginary and real parts:

$$\begin{aligned}
\dot{z}_\delta^R(t) &= -Dz_\delta^R(t) + P^R\left(A_\delta^R(t)\right)G^R\left(\overline{z_\delta^R(t)}\right) \\
&\quad - P^I\left(A_\delta^I(t)\right)G^I\left(\overline{z_\delta^I(t)}\right) \\
&\quad - \left[P^R\left(A_\delta^R(t)\right) - P^R\left(A_*^R(t)\right)\right]y^R\left(A_*^R(t)\right) \\
&\quad + \sum_{\epsilon=1}^{\eta}\sum_{\kappa=1}^{N} l_\epsilon K_{\delta\kappa}^\epsilon H_\epsilon z_\kappa^R(t) \\
&\quad + \left[P^I\left(A_\delta^I(t)\right) - P^I\left(A_*^I(t)\right)\right]y^I\left(A_*^I(t)\right) + v_\delta^R(t), \\
\dot{z}_\delta^I(t) &= -Dz_\delta^I(t) + P^R\left(A_\delta^R(t)\right)G^I\left(\overline{z_\delta^I(t)}\right) \\
&\quad + P^I\left(A_\delta^I(t)\right)G^R\left(\overline{z_\delta^R(t)}\right) \\
&\quad - \left[P^R\left(A_\delta^R(t)\right) - P^R\left(A_*^R(t)\right)\right]y^I\left(A_*^I(t)\right) \\
&\quad + \sum_{\epsilon=1}^{\eta}\sum_{\kappa=1}^{N} l_\epsilon K_{\delta\kappa}^\epsilon H_\epsilon z_\kappa^I(t) \\
&\quad - \left[P^I\left(A_\delta^I(t)\right) - P^I\left(A_*^I(t)\right)\right]y^R\left(A_*^R(t)\right) + v_\delta^I(t),
\end{aligned} \tag{46}$$

where

$$\begin{aligned}
v_\delta^R(t) &= -\Theta_\delta^R z_\delta^R(t) - \operatorname{sign}\left(z_\delta^R(t)\right)\left(\overline{P}^R\overline{Y}^R + \overline{P}^I\overline{Y}^I\right) \\
&\quad - \sum_{\epsilon=1}^{\eta} l_\epsilon\left(\rho_\delta^\epsilon(t)\right)^R H_\epsilon z_\delta^R(t), \\
v_\delta^I(t) &= -\Theta_\delta^I z_\delta^I(t) - \operatorname{sign}\left(z_\delta^I(t)\right)\left(\overline{P}^R\overline{Y}^I + \overline{P}^I\overline{Y}^R\right) \\
&\quad - \sum_{\epsilon=1}^{\eta} l_\epsilon\left(\rho_\delta^\epsilon(t)\right)^I H_\epsilon z_\delta^I(t),
\end{aligned} \tag{47}$$

in which $(\dot{\rho}_\delta^\varepsilon(t))^R = \zeta_\delta^\varepsilon(z_\delta^R(t))^T(H_\varepsilon + H_\varepsilon^T)/2z_\delta^R(t)$, $(\dot{\rho}_\delta^\varepsilon(t))^I = \zeta_\delta^\varepsilon(z_\delta^I(t))^T(H_\varepsilon + H_\varepsilon^T)/2z_\delta^I(t)$ for $\delta = 1, 2, \ldots, m$, and $(\rho_\delta^\epsilon(t))^R = (\rho_\delta^\epsilon(t))^I \equiv 0$ for $\delta = m+1, m+2, \ldots, N$.

Theorem 2. *If there are matrices* $0 < (\widehat{\rho}^\epsilon)^R = diag((\widehat{\rho}_1^\epsilon)^R, (\widehat{\rho}_2^\epsilon)^R, \ldots, (\widehat{\rho}_m^\epsilon)^R, 0, \ldots, 0) \in \mathbb{R}^{N\times N}$ *and* $0 < (\widehat{\rho}^\epsilon)^I = diag((\widehat{\rho}_1^\epsilon)^I, (\widehat{\rho}_2^\epsilon)^I, \ldots, (\widehat{\rho}_m^\epsilon)^I, 0, \ldots, 0) \in \mathbb{R}^{N\times N}$, $\epsilon = 1, 2, \ldots, u$, *such that*

$$\Phi_2^R < 0 \text{and} \Phi_2^I < 0, \tag{48}$$

where $\Phi_2^R = I_N \otimes (-2D + \widetilde{P}^R + \widetilde{P}^I + 2J^R\Lambda) - 2\Theta^R + \sum_{\epsilon=1}^{\eta} l_\epsilon[(K^\epsilon - (\widehat{\rho}^\epsilon)^R)\otimes H_\epsilon + ((K^\epsilon)^T - (\widehat{\rho}^\epsilon)^R)\otimes H_\epsilon^T]$, $\Phi_2^I = I_N \otimes (-2D + \widetilde{P}^R + \widetilde{P}^I + 2J^I\Lambda) - 2\Theta^I + \sum_{\varepsilon=1}^{\eta} l_\varepsilon[(K^\varepsilon - (\widehat{\rho}^\varepsilon)^I)\otimes H_\varepsilon + (K^\varepsilon)^T - (\widehat{\rho}^\varepsilon)^I \otimes H_\varepsilon^T]$; $\Theta^R = diag(\Theta_1^R, \Theta_2^R, \ldots, \Theta_N^R) \in \mathbb{R}^{nN\times nN}$, $\Theta^I = diag(\Theta_1^I, \Theta_2^I, \ldots, \Theta_N^I) \in \mathbb{R}^{nN\times nN}$; $(\widehat{\rho}_\delta^\epsilon)^R > 0$ *and* $(\widehat{\rho}_\delta^\epsilon)^I > 0$ *for* $\delta = 1, 2, \ldots, m$, *then the network (41) is pinning adaptive antisynchronized.*

Proof. Take the Lyapunov functional for network (45) as follows:

$$\begin{aligned}
V_2(t) &= \sum_{\epsilon=1}^{\eta}\sum_{\delta=1}^{N}\frac{l_\epsilon}{\zeta_\delta^\epsilon}\left((\rho_\delta^\epsilon(t))^R - (\widehat{\rho}_\delta^\epsilon)^R\right)^2 \\
&\quad + \sum_{\epsilon=1}^{\eta}\sum_{\delta=1}^{N}\frac{l_\epsilon}{\zeta_\delta^\epsilon}\left((\rho_\delta^\epsilon(t))^I - (\widehat{\rho}_\delta^\epsilon)^I\right)^2 + \sum_{\delta=1}^{N}\left(z_\delta^R(t)\right)^T z_\delta^R(t) \\
&\quad + \sum_{\delta=1}^{N}\left(z_\delta^I(t)\right)^T z_\delta^I(t) \\
&\quad + 2\sum_{\delta=1}^{N}\sum_{h=1}^{\eta}\int_{t-u_h(t)}^{t}\frac{\left(j_h^R z_{\delta h}^R(\varepsilon)\right)^2}{1-\varrho_h}d\varepsilon \\
&\quad + 2\sum_{\delta=1}^{N}\sum_{h=1}^{\eta}\int_{t-u_h(t)}^{t}\frac{\left(j_h^I z_{\delta h}^I(\varepsilon)\right)^2}{1-\varrho_h}d\varepsilon.
\end{aligned} \tag{49}$$

Then, we have

$$\begin{aligned}
\dot{V}_2(t) &\le 2\sum_{\delta=1}^{N}\left(z_\delta^R(t)\right)^T\Big\{-Dz_\delta^R(t) + P^R\left(A_\delta^R(t)\right)G^R\left(\overline{z_\delta^R(t)}\right) - P^I\left(A_\delta^I(t)\right)G^I\left(\overline{z_\delta^I(t)}\right) - \left[P^R\left(A_\delta^R(t)\right) - P^R\left(A_*^R(t)\right)\right]y^R\left(A_*^R(t)\right) \\
&\quad + \sum_{\epsilon=1}^{\eta}\sum_{\kappa=1}^{N} l_\epsilon K_{\delta\kappa}^\epsilon H_\epsilon z_\kappa^R(t) - \Theta_\delta^R z_\delta^R(t) - \operatorname{sign}\left(z_\delta^R(t)\right)\left(\overline{P}^R\overline{Y}^R + \overline{P}^I\overline{Y}^I\right) - \sum_{\epsilon=1}^{\eta} l_\epsilon\left(\rho_\delta^\epsilon(t)\right)^R H_\epsilon z_\delta^R(t) + \left[P^I\left(A_\delta^I(t)\right)\right. \\
&\quad \left. - P^I\left(A_*^I(t)\right)\right]y^I\left(A_*^I(t)\right)\Big\} + 2\sum_{\delta=1}^{N}\left(z_\delta^I(t)\right)^T\Big\{-Dz_\delta^I(t) + P^R\left(A_\delta^R(t)\right)G^I\left(\overline{z_\delta^I(t)}\right) + P^I\left(A_\delta^I(t)\right)G^R\left(\overline{z_\delta^R(t)}\right) - \left[P^R\left(A_\delta^R(t)\right)\right. \\
&\quad \left. - P^R\left(A_*^R(t)\right)\right]y^I\left(A_*^I(t)\right) + \sum_{\epsilon=1}^{\eta}\sum_{\kappa=1}^{N} l_\epsilon K_{\delta\kappa}^\epsilon H_\epsilon z_\kappa^I(t) - \Theta_\delta^I z_\delta^I(t) - \operatorname{sign}\left(z_\delta^I(t)\right)\left(\overline{P}^R\overline{Y}^I + \overline{P}^I\overline{Y}^R\right) - \sum_{\epsilon=1}^{\eta} l_\epsilon\left(\rho_\delta^\epsilon(t)\right)^I H_\epsilon z_\delta^I(t) \\
&\quad - \left[P^I\left(A_\delta^I(t)\right) - P^I\left(A_*^I(t)\right)\right]y^R\left(A_*^R(t)\right)\Big\} + \sum_{\epsilon=1}^{\eta}\sum_{\delta=1}^{m} l_\epsilon\left[\left(\rho_\delta^\epsilon(t)\right)^R - (\widehat{\rho}_\delta^\epsilon)^R\right]\left(z_\delta^R(t)\right)^T\left(H_\epsilon + H_\epsilon^T\right)z_\delta^R(t) + \sum_{\epsilon=1}^{\eta}\sum_{\delta=1}^{m} l_\epsilon\left[\left(\rho_\delta^\epsilon(t)\right)^I\right. \\
&\quad \left. - (\widehat{\rho}_\delta^\epsilon)^I\right]\left(z_\delta^I(t)\right)^T\left(H_\epsilon + H_\epsilon^T\right)z_\delta^I(t) + 2\left(z^R(t)\right)^T\left(I_N \otimes \left(J^R\Lambda\right)\right)z^R(t) - 2\left(\overline{z^R(t)}\right)^T\left(I_N \otimes J^R\right)\overline{z^R(t)} \\
&\quad + 2\left(z^I(t)\right)^T\left(I_N \otimes \left(J^I\Lambda\right)\right)z^I(t) - 2\left(\overline{z^I(t)}\right)^T\left(I_N \otimes J^I\right)\overline{z^I(t)}.
\end{aligned} \tag{50}$$

Moreover, we can derive that

$$
\begin{aligned}
&-2\sum_{\delta=1}^{N}\sum_{\epsilon=1}^{\eta} l_\epsilon (\rho_\delta^\epsilon(t))^R \left(z_\delta^R(t)\right)^T H_\epsilon z_\delta^R(t) + \sum_{\epsilon=1}^{\eta}\sum_{\delta=1}^{m} l_\epsilon \left[(\rho_\delta^\epsilon(t))^R - (\widehat{\rho}_\delta^\epsilon)^R\right]\left(z_\delta^R(t)\right)^T \left(H_\epsilon + H_\epsilon^T\right) z_\delta^R(t)\\
&= -2\sum_{\delta=1}^{N}\sum_{\epsilon=1}^{\eta} l_\epsilon (\rho_\delta^\epsilon(t))^R \left(z_\delta^R(t)\right)^T H_\epsilon z_\delta^R(t) + \sum_{\epsilon=1}^{\eta}\sum_{\delta=1}^{m} l_\epsilon (\rho_\delta^\epsilon(t))^R \left(z_\delta^R(t)\right)^T \left(H_\epsilon + H_\epsilon^T\right) z_\delta^R(t) - \sum_{\epsilon=1}^{\eta}\sum_{\delta=1}^{m} l_\epsilon (\widehat{\rho}_\delta^\epsilon)^R \left(z_\delta^R(t)\right)^T \left(H_\epsilon + H_\epsilon^T\right) z_\delta^R(t)\\
&= -\sum_{\epsilon=1}^{\eta}\sum_{\delta=1}^{m} l_\epsilon (\widehat{\rho}_\delta^\epsilon)^R \left(z_\delta^R(t)\right)^T \left(H_\epsilon + H_\epsilon^T\right) z_\delta^R(t)\\
&= -\left(z^R(t)\right)^T \left[\sum_{\epsilon=1}^{\eta} l_\epsilon (\widehat{\rho}^\epsilon)^R \otimes \left(H_\epsilon + H_\epsilon^T\right)\right] z^R(t).
\end{aligned}
\tag{51}
$$

Similarly,

$$
\begin{aligned}
&-2\sum_{\delta=1}^{N}\sum_{\epsilon=1}^{\eta} l_\epsilon (\rho_\delta^\epsilon(t))^I \left(z_\delta^I(t)\right)^T H_\epsilon z_\delta^I(t) + \sum_{\epsilon=1}^{\eta}\sum_{\delta=1}^{m} l_\epsilon \left[(\rho_\delta^\epsilon(t))^I - (\widehat{\rho}_\delta^\epsilon)^I\right]\left(z_\delta^I(t)\right)^T \left(H_\epsilon + H_\epsilon^T\right) z_\delta^I(t)\\
&= -\sum_{\epsilon=1}^{\eta}\sum_{\delta=1}^{m} l_\epsilon (\widehat{\rho}_\delta^\epsilon)^I \left(z_\delta^I(t)\right)^T \left(H_\epsilon + H_\epsilon^T\right) z_\delta^I(t)\\
&= -\left(z^I(t)\right)^T \left[\sum_{\epsilon=1}^{\eta} l_\epsilon (\widehat{\rho}^\epsilon)^I \otimes \left(H_\epsilon + H_\epsilon^T\right)\right] z^I(t).
\end{aligned}
\tag{52}
$$

Thus, it is easy to get that by (20)–(29) and (50)–(52)

$$
\begin{aligned}
\dot{V}_2(t) \le{}& \left(z^R(t)\right)^T \left\{ I_N \otimes \left(-2\,D + \widetilde{P}^R + \widetilde{P}^I + 2J^R\Lambda\right) - 2\Theta^R + \sum_{\epsilon=1}^{\eta} l_\epsilon \left[\left(K^\epsilon - (\widehat{\rho}^\epsilon)^R\right)\otimes H_\epsilon + \left((K^\epsilon)^T - (\widehat{\rho}^\epsilon)^R\right)\otimes H_\epsilon^T\right]\right\} z^R(t)\\
&+\left(z^I(t)\right)^T \left\{ I_N \otimes \left(-2\,D + \widetilde{P}^R + \widetilde{P}^I + 2J^I\Lambda\right) - 2\Theta^I + \sum_{\epsilon=1}^{\eta} l_\epsilon \left[\left(K^\epsilon - (\widehat{\rho}^\epsilon)^I\right)\otimes H_\epsilon + \left((K^\epsilon)^T - (\widehat{\rho}^\epsilon)^I\right)\otimes H_\epsilon^T\right]\right\} z^I(t)\\
\le{}& \alpha_2 \|z(t)\|^2,
\end{aligned}
\tag{53}
$$

where $\alpha_2 = \max\{\lambda_M(\Phi_2^R), \lambda_M(\Phi_2^I)\} < 0$.

From (49) and (53), we can get $V_2(t)$ is nonincreasing, and any term of $V_2(t)$ is bounded. Therefore, $\lim_{t\longrightarrow+\infty} V_2(t) \ge 0$, $(\rho_\delta^\epsilon(t))^R$ and $(\rho_\delta^\epsilon(t))^I$ are bounded. According to (44), we can get $(\rho_\gamma^\epsilon(t))^R$ and $(\rho_\delta^\epsilon(t))^I$ are monotonically increasing; thus, $(\rho_\delta^\epsilon(t))^R$ and $(\rho_\delta^\epsilon(t))^I$ converge to a finite nonnegative value, which means both the limitation of $\sum_{\epsilon=1}^{\eta}\sum_{\delta=1}^{m} l_\epsilon/\zeta_\delta^\epsilon((\rho_\delta^\epsilon(t))^R - \widehat{\rho}_\delta^\epsilon)^2$ and $\sum_{\epsilon=1}^{\eta}\sum_{\delta=1}^{m} l_\epsilon/\zeta_\delta^\epsilon((\rho_\delta^\epsilon(t))^I - \widehat{\rho}_\delta^\epsilon)^2$ exist as well as $\lim_{t\longrightarrow+\infty}\sum_{\epsilon=1}^{\eta}\sum_{\delta=1}^{m} l_\epsilon/\zeta_\delta^\epsilon((\rho_\delta^\epsilon(t))^R - \widehat{\rho}_\delta^r)^2 \ge 0$ and $\lim_{t\longrightarrow+\infty}\sum_{\epsilon=1}^{\eta}\sum_{\delta=1}^{m} l_\epsilon/\zeta_\delta^r((\rho_\delta^\epsilon(t))^I - \widehat{\rho}_\delta^\epsilon)^2 \ge 0$. Thus, we have $\lim_{t\rightarrow+\infty}\Big\{\sum_{\delta=1}^{N}\sum_{h=1}^{n}[\int_{t-u_h(t)}^{t} (2(j_h^R x_{\delta h}^R(\varepsilon))^2/1-\varrho_h)\mathrm{d}\varepsilon + \int_{t-u_h(t)}^{t} 2(j_h^I z_{\delta h}^I(\varepsilon))^2/(1-\varrho_h)\mathrm{d}\varepsilon] + \sum_{\delta=1}^{N}[(z_\delta^R(t))^T z_\delta^R(t) + (z_\delta^I(t))^T z_\delta^I(t)]\Big\}$exist and is a nonnegative real number. Then, by virtue of the similar proof method as in Theorem 1, we can obtain that $\lim_{t\longrightarrow+\infty}\|z(t)\| = 0$. Therefore, the network (41) achieves antisynchronization under the generalized pinning adaptive controller (47). □

4. Antisynchronization and Pinning Antisynchronization of MWCCVDMNNs with Coupling Delays

4.1. Antisynchronization of MWCCVDMNNs with Coupling Delays. In this section, the MWCCVDMNNs with coupling delays is considered as follows:

$$
\begin{aligned}
\dot{A}_\delta(t) ={}& -DA_\delta(t) + P(A_\delta(t))y\left(\overline{A_\delta(t)}\right) + c_\delta(t)\\
&+ \sum_{\epsilon=1}^{\eta}\sum_{\kappa=1}^{N} l_\epsilon K_{\delta\kappa}^\epsilon H_\epsilon \widehat{A_\kappa(t)}, \delta = 1,2,\ldots,N,
\end{aligned}
\tag{54}
$$

where $D, A_\delta(t),\ P(A_\delta(t)), y(\overline{A_\delta(t)}),\ c_\delta(t), l_\epsilon, K^\epsilon, H_\epsilon$ have the similar meanings as in Section 3.1; $\widehat{A_\kappa(t)} = (A_{\kappa1}(t-w_1(t)), A_{\kappa2}(t-w_2(t)),\ldots,A_{\kappa n}(t-w_n(t)))^T \in \mathbb{R}^n$; $w_\epsilon(t)(\epsilon = 1,2,\ldots,\eta)$ denote the coupling delay with $0 \le w_\epsilon(t) \le w_\epsilon \le w = \max_{\epsilon=1,2,\ldots,n}\{w_\epsilon\}$ and $\dot{w}_\epsilon(t) \le \overline{\varrho}_\epsilon < 1$.

Let $z_\delta(t) = A_\delta(t) + A_*(t)$, then

$$\begin{aligned}\dot{z}_\delta(t) &= -Dz_\delta(t) + P(A_\delta(t))y\left(\overline{A_\delta(t)}\right) + c_\delta(t) \\ &\quad + P(A_*(t))y(Y_*(t)) \\ &\quad + \sum_{\epsilon=1}^{\eta}\sum_{\kappa=1}^{N} l_\epsilon K_{\delta\kappa}^{\epsilon} H_\epsilon \widehat{z_\kappa(t)}, \delta = 1,2,\ldots,N.\end{aligned} \tag{55}$$

Then, by separating the imaginary and real parts, system (55) can be shown as

$$\begin{aligned}\dot{z}_\delta^R(t) &= -Dz_\delta^R(t) + P^R A_\delta^R(t) G^R\left(\overline{z_\delta^R(t)}\right) - P^I\left(A_\delta^I(t)\right)G^I\left(\overline{z_\delta^I(t)}\right) - \left[P^R\left(A_\delta^R(t)\right) - P^R\left(A_*^R(t)\right)\right]y^R\left(A_*^R(t)\right) + \sum_{\epsilon=1}^{\eta}\sum_{\kappa=1}^{N} l_\epsilon K_{\delta\kappa}^{\epsilon} H_\epsilon \widehat{z_\kappa^R(t)} \\ &\quad + \left[P^I\left(A_\delta^I(t)\right) - P^I\left(A_*^I(t)\right)\right]y^I\left(A_*^I(t)\right) + c_\delta^R(t), \\ \dot{z}_\delta^I(t) &= -Dz_\delta^I + P^R\left(A_\delta^R(t)\right)G^I\left(\overline{z_\delta^I(t)}\right) + P^I\left(A_\delta^I(t)\right)G^R\left(\overline{z_\delta^R(t)}\right) - \left[P^R\left(A_\delta^R(t)\right) - P^R\left(A_*^R(t)\right)\right]y^I\left(A_*^I(t)\right) + \sum_{\epsilon=1}^{\eta}\sum_{\kappa=1}^{N} l_\epsilon K_{\delta\kappa}^{\epsilon} H_\epsilon \widehat{z_\kappa^I(t)} \\ &\quad - \left[P^I\left(A_\delta^I(t)\right) - P^I\left(A_*^I(t)\right)\right]y^R\left(A_*^R(t)\right) + c_\delta^I(t),\end{aligned} \tag{56}$$

where $z_\delta^R(t)$, $z_\delta^I(t)$, $\overline{z_\delta^R(t)}$, $\overline{z_\delta^I(t)}$, $G^R(\overline{z_\delta^R(t)})$, and $G^I(\overline{z_\delta^I(t)})$ have the same definitions as in Section 3.1; $\widehat{z_\kappa^R(t)} = (z_{\kappa 1}^R(t-w_1(t)), z_{\kappa 2}^R(t-w_2(t)), \ldots, z_{\kappa n}^R(t-w_n(t)))^T$ and $\widehat{z_\kappa^I(t)} = (z_{\kappa 1}^I(t-w_1(t)), z_{\kappa 2}^I(t-w_2(t)), \ldots, z_{\kappa n}^I(t-w_n(t)))^T$.

Theorem 3. *If there are some matrices* $0 < \Psi_\epsilon = diag(b_1^\epsilon, b_2^\epsilon, \ldots, b_{nN}^\epsilon) \in \mathbb{R}^{nN\times nN}$, $\epsilon = 1,2,\ldots,\eta$, *such that*

$$\Phi_3^R < 0 \text{ and } \Phi_3^I < 0, \tag{57}$$

where $\Phi_3^R = I_N \otimes (-2D + \widetilde{P}^R + \widetilde{P}^I + 2J^R\Lambda) - 2\Theta^R + \sum_{\varepsilon=1}^{\eta} l_\varepsilon[(K^\varepsilon \otimes H_\varepsilon)\Psi_\varepsilon^{-1}((K^\varepsilon)^T \otimes H_\varepsilon^T) + 1/(1-\overline{\varrho}_\varepsilon)\Psi_\varepsilon]$; $\Phi_3^I = I_N \otimes (-2D + \widetilde{P}^R + \widetilde{P}^I + 2J^R\Lambda) - 2\Theta^I + \sum_{\varepsilon=1}^{\eta} l_\varepsilon[(K^\varepsilon \otimes H_\varepsilon)\Psi_\varepsilon^{-1}((K^\varepsilon)^T \otimes H_\varepsilon^T) + 1/(1-\overline{\varrho}_\varepsilon)\Psi_\varepsilon]$; $\Theta^R = diag(\Theta_1^R, \Theta_2^R, \ldots, \Theta_N^R) \in \mathbb{R}^{nN\times nN}$; *and* $\Theta^I = diag(\Theta_1^I, \Theta_2^I, \ldots, \Theta_N^I) \in \mathbb{R}^{nN\times nN}$. *Then, the network (54) achieves antisynchronized under the controller (15).*

Proof. The Lyapunov function is chosen as

$$\begin{aligned}V_3(t) &= \sum_{\delta=1}^{N}\left(z_\delta^R(t)\right)^T z_\delta^R(t) + 2\sum_{\delta=1}^{N}\sum_{h=1}^{n}\int_{t-u_h(t)}^{t}\frac{\left(j_h^R z_{\delta h}^R(\varepsilon)\right)^2}{1-\varrho_h}d\varepsilon \\ &\quad + 2\sum_{\delta=1}^{N}\sum_{h=1}^{n}\int_{t-u_h(t)}^{t}\frac{\left(j_h^I z_{\delta h}^I(\varepsilon)\right)^2}{1-\varrho_h}d\varepsilon + \sum_{\delta=1}^{N}\left(z_\delta^I(t)\right)^T z_\delta^I(t) \\ &\quad + \sum_{\epsilon=1}^{\eta}\frac{l_\epsilon}{1-\overline{\varrho}_\epsilon}\int_{t-w_\epsilon(t)}^{t} z^R(\varepsilon)^T \Psi_\epsilon z^R(\varepsilon)d\varepsilon \\ &\quad + \sum_{\epsilon=1}^{\eta}\frac{l_\epsilon}{1-\overline{\varrho}_\epsilon}\int_{t-w_\epsilon(t)}^{t} z^I(\varepsilon)^T \Psi_\epsilon z^I(\varepsilon)d\varepsilon.\end{aligned} \tag{58}$$

Then, we have

$$\begin{aligned}\dot{V}_3(t) &\leq 2\sum_{\delta=1}^{N}\left(z_\delta^R(t)\right)^T\left\{\begin{array}{c}-Dz_\delta^R(t) + P^R\left(A_\delta^R(t)\right)G^R\left(\overline{z_\delta^R(t)}\right) - P^I\left(A_\delta^I(t)\right)G^I\left(\overline{z_\delta^I(t)}\right) + \sum_{\epsilon=1}^{\eta}\sum_{\kappa=1}^{N} l_\epsilon K_{\iota\kappa}^{\epsilon} H_\epsilon \widehat{z_\kappa^R(t)} - \Theta_\delta^R z_\delta^R(t) \\ -\text{sign}\left(z_\delta^R(t)\right)\left(\overline{P}^R\overline{Y}^R + \overline{P}^I\overline{Y}^I\right) - \left[P^R\left(A_\delta^R(t)\right) - P^R\left(A_*^R(t)\right)\right]y^R\left(A_*^R(t)\right) + \left[P^I\left(A_\delta^I(t)\right) - P^I\left(A_*^I(t)\right)\right]y^I\left(A_*^I(t)\right)\end{array}\right\} \\ &\quad + 2\sum_{\delta=1}^{N}\left(z_\delta^I(t)\right)^T\left\{\begin{array}{c}-Az_\delta^I(t) + P^R\left(A_\delta^R(t)\right)G^I\left(\overline{z_\delta^I(t)}\right) + P^I\left(A_\delta^I(t)\right)G^R\left(\overline{z_\delta^R(t)}\right) + \sum_{\epsilon=1}^{\eta}\sum_{\kappa=1}^{N} l_\epsilon K_{\delta\kappa}^{\epsilon} H_\epsilon \widehat{z_\kappa^I(t)} - \Theta_\delta^I z_\delta^I(t) \\ -\text{sign}\left(z_\delta^I(t)\right)\left(\overline{P}^R\overline{Y}^I + \overline{P}^I\overline{Y}^R\right) - \left[P^R\left(A_\delta^R(t)\right) - P^R\left(A_*^R(t)\right)\right]y^I\left(A_*^I(t)\right) - \left[P^I\left(A_\delta^I(t)\right) - P^I\left(A_*^I(t)\right)\right]y^R\left(A_*^R(t)\right)\end{array}\right\} \\ &\quad + 2\left(z^R(t)\right)^T\left(I_N \otimes \left(J^R\Lambda\right)\right)z^R(t) - 2\left(\overline{z^R(t)}\right)^T\left(I_N \otimes J^R\right)\overline{z^R(t)} + 2\left(z^I(t)\right)^T\left(I_N \otimes \left(J^I\Lambda\right)\right)z^I(t) - 2\left(\overline{z^I(t)}\right)^T\left(I_N \otimes J^I\right)\overline{z^I(t)} \\ &\quad + \sum_{\epsilon=1}^{\eta}\frac{l_\epsilon}{1-\overline{\varrho}_\epsilon}\left(z^R(t)\right)^T\Psi_\epsilon z^R(t) - \sum_{\epsilon=1}^{\eta} l_\epsilon\left(\widehat{z^R(t)}\right)^T\Psi_r\widehat{z^R(t)} + \sum_{\epsilon=1}^{\eta}\frac{l_\epsilon}{1-\overline{\varrho}_\epsilon}\left(z^I(t)\right)^T\Psi_\epsilon z^I(t) \\ &\quad - \sum_{\epsilon=1}^{\eta} l_\epsilon\left(\widehat{z^I(t)}\right)^T\Psi_\epsilon\widehat{z^I(t)}.\end{aligned} \tag{59}$$

Obviously,

$$\begin{aligned}
&2\sum_{\delta=1}^{N}\sum_{\epsilon=1}^{\eta}\sum_{\kappa=1}^{N} l_\epsilon K_{\iota\kappa}^{\epsilon}\left(z_\delta^R(t)\right)^T H_\epsilon \widehat{z_\kappa^R(t)}\\
&= 2\sum_{\epsilon=1}^{\eta} l_\epsilon \left(z^R(t)\right)^T \left(K^\epsilon \otimes H_\epsilon\right)\widehat{z^R(t)}\\
&\leq \sum_{\epsilon=1}^{\eta} l_\epsilon \left(z^R(t)\right)^T \left(K^\epsilon \otimes H_\epsilon\right)\Psi_\epsilon^{-1}\left(\left(K^\epsilon\right)^T \otimes H_\epsilon^T\right) z^R(t)\\
&\quad + \sum_{\epsilon=1}^{\eta} l_\epsilon \left(\widehat{z^R(t)}\right)^T \Psi_\epsilon \widehat{z^R(t)}.
\end{aligned} \tag{60}$$

Similarly,

$$\begin{aligned}
&2\sum_{\delta=1}^{N}\sum_{\epsilon=1}^{\eta}\sum_{\kappa=1}^{N} l_\epsilon K_{\iota\kappa}^{\epsilon}\left(z_\delta^I(t)\right)^T H_\epsilon \widehat{z_\kappa^I(t)}\\
&\leq \sum_{\epsilon=1}^{\eta} l_\epsilon \left(z^I(t)\right)^T \left(K^\epsilon \otimes H_\epsilon\right)\Psi_\epsilon^{-1}\left(\left(K^\epsilon\right)^T \otimes H_\epsilon^T\right) z^I(t)\\
&\quad + \sum_{\epsilon=1}^{\eta} l_\epsilon \left(\widehat{z^I(t)}\right)^T \Psi_\epsilon \widehat{z^I(t)}.
\end{aligned} \tag{61}$$

By (20)–(30), (26)–(29), and (60)-(61), one obtains

$$\begin{aligned}
\dot{V}_3(t) &\leq \left(z^R(t)\right)^T \left\{ I_N \otimes \left(-2D + \widetilde{P}^R + \widetilde{P}^I + 2J^R\Lambda\right) - 2\Theta^R + \sum_{\epsilon=1}^{\eta} l_\epsilon \left[\left(K^\epsilon \otimes H_\epsilon\right)\Psi_\epsilon^{-1}\left(\left(K^\epsilon\right)^T \otimes H_\epsilon^T\right) + \frac{1}{1-\varrho_\epsilon}\Psi_\epsilon\right]\right\} z^R(t)\\
&\quad + \left(z^I(t)\right)^T \left\{ I_N \otimes \left(-2D + \widetilde{P}^R + \widetilde{P}^I + 2J^I\Lambda\right) - 2\Theta^I + \sum_{\epsilon=1}^{\eta} l_\epsilon \left[\left(K^\epsilon \otimes H_\epsilon\right)\Psi_\epsilon^{-1}\left(\left(K^\epsilon\right)^T \otimes H_\epsilon^T\right) + \frac{1}{1-\varrho_\epsilon}\Psi_\epsilon\right]\right\} z^I(t)\\
&\leq \alpha_3 \|z(t)\|^2,
\end{aligned} \tag{62}$$

where $\alpha_3 = \max\{\lambda_M(\Phi_3^R), \lambda_M(\Phi_3^I)\} < 0$. Similar to the deduction of (31), we can obtain that $\lim_{t\longrightarrow+\infty}\int_0^t \|x(\varepsilon)\|^2 d\varepsilon$ exists and is a real nonnegative number. In addition,

$$\begin{aligned}
0 &\leq \sum_{\epsilon=1}^{\eta} \frac{l_\epsilon}{1-\varrho_\epsilon}\int_{t-w_\epsilon(t)}^{t} \left(z^R(\varepsilon)\right)^T \Psi_\epsilon z^R(\varepsilon) d\varepsilon\\
&\leq \sum_{\epsilon=1}^{\eta} \frac{l_\epsilon}{1-\varrho_\epsilon}\int_{t-w}^{t} \left(z^R(\varepsilon)\right)^T \Psi_\epsilon z^R(\varepsilon) d\varepsilon\\
&\leq \sum_{\epsilon=1}^{\eta} \frac{l_\epsilon}{1-\varrho_\epsilon}\lambda_M\left(\Psi_\epsilon\right)\int_{t-w}^{t} \|z(\varepsilon)\|^2 d\varepsilon\\
&= 0.
\end{aligned} \tag{63}$$

Similarly,

$$0 \leq \sum_{\epsilon=1}^{\eta} \frac{l_\epsilon}{1-\varrho_\epsilon}\int_{t-w_\epsilon(t)}^{t} \left(z^I(\varepsilon)\right)^T \Psi_\epsilon z^I(\varepsilon) d\varepsilon = 0. \tag{64}$$

From (32)-(33) and (63)-(64), one can obtain that $\lim_{t\longrightarrow+\infty}\sum_{\delta=1}^{N}[(z_\delta^R(t))^T z_\delta^R(t) + (z_\delta^I(t))^T z_\delta^I(t)]$ exists and is a nonnegative real number. Then, based on the method for proving Theorem 1, one can easily obtain $\lim_{t\longrightarrow+\infty}\|z(t)\| = 0$. Thus, under the controller (15), the network (54) is antisynchronized. □

4.2. Generalized Pinning Antisynchronization of MWCCVDMNNs with Coupling Delays. In this section, we add generalized pinning adaptive controller as in Section 3.2 to the MWCCVDMNNs with coupling delays, then the network (54) under the pinning adaptive controller can be expressed as follows:

$$\begin{aligned}
\dot{A}_\delta(t) &= -DA_\delta(t) + P\left(A_\delta(t)\right) y\left(\overline{A_\delta(t)}\right) + v_\delta(t)\\
&\quad + \sum_{\epsilon=1}^{\eta}\sum_{\kappa=1}^{N} l_\epsilon K_{\delta\kappa}^{\epsilon} H_\epsilon \widehat{A_\kappa(t)}, \delta = 1, 2, \ldots, N,
\end{aligned} \tag{65}$$

where $v_\delta(t) = v_\delta^R(t) + iv_\delta^I(t) \in \mathbb{C}^n$ is the generalized pinning adaptive controller as given in (42), $0 < D \in \mathbb{R}^{n\times n}$, $A_\delta(t) \in \mathbb{C}^n$, $P(A_\delta(t)) \in \mathbb{C}^{n\times n}$, $y(\overline{A_\delta(t)}) \in \mathbb{C}^n$, $\mathbb{R} \ni l_\epsilon > 0$, $K^\epsilon \in \mathbb{R}^{N\times N}$, $H_\epsilon \in \mathbb{R}^{n\times n}$, and $\widehat{A_\kappa(t)} \in \mathbb{R}^n$ have the same meanings as in Section 4.1.

Similarly, the error vector $z_\delta(t) = A_\delta(t) + A_*(t)$ can be governed by equations as follows:

$$\begin{aligned}
\dot{z}_\delta(t) &= -Dz_\delta(t) + P\left(A_\delta(t)\right) y\left(\overline{A_\delta(t)}\right) + v_\delta(t)\\
&\quad + P\left(A_*(t)\right) y\left(A_*(t)\right)\\
&\quad + \sum_{\epsilon=1}^{\eta}\sum_{\kappa=1}^{N} l_\epsilon K_{\delta\kappa}^{\epsilon} H_\epsilon \widehat{z_\kappa(t)}, \delta = 1, 2, \ldots, N.
\end{aligned} \tag{66}$$

Separating (66) into the following imaginary and real parts:

$$\begin{aligned}\dot{z}_\delta^R(t) &= -Dz_\delta^R(t) + P^R\left(A_\delta^R(t)\right)G^R\left(\overline{z_\delta^R(t)}\right) - P^I\left(A_\delta^I(t)\right)G^I\left(\overline{z_\delta^I(t)}\right) - \left[P^R\left(A_\delta^R(t)\right) - P^R\left(A_*^R(t)\right)\right]y^R\left(A_*^R(t)\right) + v_\delta^R(t) \\ &\quad + \left[P^I\left(A_\delta^I(t)\right) - P^I\left(A_*^I(t)\right)\right]y^I\left(A_*^I(t)\right) + \sum_{\epsilon=1}^{\eta}\sum_{\kappa=1}^{N} l_\epsilon K_{\delta\kappa}^\epsilon H_\epsilon \widehat{z_\kappa^R(t)}, \\ \dot{z}_\delta^I(t) &= -Dz_\delta^I(t) + P^R\left(A_\delta^R(t)\right)G^I\left(\overline{z_\delta^I(t)}\right) + P^I\left(A_\delta^I(t)\right)G^R\left(\overline{z_\delta^R(t)}\right) - \left[P^R\left(A_\delta^R(t)\right) - P^R\left(A_*^R(t)\right)\right]y^I\left(A_*^I(t)\right) + v_\delta^I(t) \\ &\quad - \left[P^I\left(A_\delta^I(t)\right) - P^I\left(A_*^I(t)\right)\right]y^R\left(A_*^R(t)\right) + \sum_{\epsilon=1}^{\eta}\sum_{\kappa=1}^{N} l_\epsilon K_{\delta\kappa}^\epsilon H_\epsilon \widehat{z_\kappa^I(t)},\end{aligned} \tag{67}$$

where $v_\delta^R(t)$ and $v_\delta^I(t)$ are given as in (47).

Theorem 4. *The network (65) is pinning adaptive anti-synchronization if there are matrices $0 < (\widehat{\rho}^\epsilon)^R = diag((\widehat{\rho}_1^\epsilon)^R, (\widehat{\rho}_2^\epsilon)^R, \ldots, (\widehat{\rho}_m^\epsilon)^R, 0, \ldots, 0) \in \mathbb{R}^{N\times N}$, $0 < (\widehat{\rho}^\epsilon)^I = diag((\widehat{\rho}_1^\epsilon)^I, (\widehat{\rho}_2^\epsilon)^I, \ldots, (\widehat{\rho}_m^\epsilon)^I, 0, \ldots, 0) \in \mathbb{R}^{N\times N}$, and $0 < \Psi_\epsilon = diag(b_1^\epsilon, b_2^\epsilon, \ldots, b_{nN}^\epsilon) \in \mathbb{R}^{nN\times nN}$, $\epsilon = 1, 2, \ldots, \eta$, satisfying*

$$\Phi_4^R < 0 \text{ and } \Phi_4^I < 0, \tag{68}$$

where $\Phi_4^R = I_N \otimes (-2D + \widetilde{P}^R + \widetilde{P}^I + 2J^R\Lambda) - 2\Upsilon^R + \sum_{\epsilon=1}^{\eta} l_\epsilon [(K^\epsilon \otimes H_\epsilon)\Psi_\epsilon^{-1}((K^\epsilon)^T \otimes H_\epsilon^T) - (\widehat{\rho}^\epsilon)^R \otimes (H_\epsilon + H_\epsilon^T) + 1/(1-\overline{\varrho}_\epsilon)\Psi_\epsilon]$; $\Phi_4^I = I_N \otimes (-2D + \widetilde{P}^R + \widetilde{P}^I + 2J^I\Lambda) - 2\Upsilon^I + \sum_{\epsilon=1}^{\eta} l_\epsilon [(K^\epsilon \otimes H_\epsilon)\Psi_\epsilon^{-1}((K^\epsilon)^T \otimes H_\epsilon^T) - (\widehat{\rho}^\epsilon)^R \otimes (H_\epsilon + H_\epsilon^T) + 1/(1-\overline{\varrho}_\epsilon)\Psi_\epsilon]$; $\Theta^R = diag(\Theta_1^R, \Theta_2^R, \ldots, \Theta_N^R) \in \mathbb{R}^{nN\times nN}$, $\Theta^I = diag(\Theta_1^I, \Theta_2^I, \ldots, \Theta_N^I) \in \mathbb{R}^{nN\times nN}$; $(\widehat{\rho}_\delta^r)^R > 0$ *and* $(\widehat{\rho}_\delta^r)^I > 0$ *for* $\delta = 1, 2, \ldots, m$.

Proof. A proper Lyapunov functional for (66) is constructed as follows:

$$\begin{aligned}V_4(t) &= \sum_{\epsilon=1}^{\eta}\frac{l_\epsilon}{1-\overline{\varrho}_\epsilon}\int_{t-w_\epsilon(t)}^{t}\left(z^R(\varepsilon)\right)^T\Psi_\epsilon z^R(\varepsilon)d\varepsilon \\ &\quad + \sum_{\epsilon=1}^{\eta}\frac{l_\epsilon}{1-\overline{\varrho}_\epsilon}\int_{t-w_\epsilon(t)}^{t}\left(z^I(\varepsilon)\right)^T\Psi_\epsilon z^I(\varepsilon)d\varepsilon \\ &\quad + \sum_{\epsilon=1}^{\eta}\sum_{\delta=1}^{N}\frac{l_\epsilon}{\zeta_\delta^\epsilon}\left((\rho_\delta^\epsilon(t))^R - (\widehat{\rho}_\delta^\epsilon)^R\right)^2 \\ &\quad + \sum_{\epsilon=1}^{\eta}\sum_{\delta=1}^{N}\frac{l_\epsilon}{\zeta_\delta^\epsilon}\left((\rho_\delta^\epsilon(t))^I - (\widehat{\rho}_\delta^r)^I\right)^2 \\ &\quad + 2\sum_{\delta=1}^{N}\sum_{h=1}^{n}\int_{t-u_h(t)}^{t}\frac{\left(j_h^R z_{\delta h}^R(\varepsilon)\right)^2}{1-\varrho_h}d\varepsilon \\ &\quad + 2\sum_{\delta=1}^{N}\sum_{h=1}^{n}\int_{t-u_h(t)}^{t}\frac{\left(j_h^I z_{\delta h}^I(\varepsilon)\right)^2}{1-\varrho_h}d\varepsilon \\ &\quad + \sum_{\delta=1}^{N}\left(z_\delta^R(t)\right)^T z_\delta^R(t) + \sum_{\delta=1}^{N}\left(z_\delta^I(t)\right)^T z_\delta^I(t).\end{aligned} \tag{69}$$

Taking the derivative of $V_4(t)$, we have

$$\dot{V}_4(t) \le 2\sum_{\delta=1}^{N}\left(z_\delta^R(t)\right)^T\Big\{-Dz_\delta^R(t) + P^R\left(A_\delta^R(t)\right)G^R\left(\overline{z_\delta^R(t)}\right) - P^I\left(A_\delta^I(t)\right)G^I\left(\overline{z_\delta^I(t)}\right) - \left[P^R\left(A_\delta^R(t)\right) - P^R\left(A_*^R(t)\right)\right]y^R\left(A_*^R(t)\right)$$
$$+\sum_{\epsilon=1}^{\eta}\sum_{\kappa=1}^{N} l_\epsilon K_{\iota\kappa}^\epsilon H_\epsilon \widehat{z_\kappa^R(t)} - \Theta_\delta^R z_\delta^R(t) - \operatorname{sign}\left(z_\delta^R(t)\right)\left(\overline{P}^R\overline{Y}^R + \overline{P}^I\overline{Y}^I\right) - \sum_{\epsilon=1}^{\eta} l_\epsilon(\rho_\delta^\epsilon(t))^R H_\epsilon z_\delta^R(t) + \left[P^I\left(A_\delta^I(t)\right)\right.$$
$$\left.-P^I\left(A_*^I(t)\right)\right]y^I\left(A_*^I(t)\right)\Big\} + 2\sum_{\delta=1}^{N}\left(z_\delta^I(t)\right)^T\Big\{-Dz_\delta^I(t) + P^R\left(A_\delta^R(t)\right)G^I\left(\overline{z_\delta^I(t)}\right) + P^I\left(A_\delta^I(t)\right)G^R\left(\overline{z_\delta^R(t)}\right) - \left[P^R\left(A_\delta^R(t)\right)\right.$$
$$\left.-P^R\left(A_*^R(t)\right)\right]y^I\left(A_*^I(t)\right) + \sum_{\epsilon=1}^{\eta}\sum_{\kappa=1}^{N} l_\epsilon K_{\iota\kappa}^\epsilon H_\epsilon \widehat{z_\kappa^I(t)} - \Theta_\delta^I z_\delta^I(t) - \operatorname{sign}\left(z_\delta^I(t)\right)\left(\overline{P}^R\overline{Y}^I + \overline{P}^I\overline{Y}^R\right) - \sum_{\epsilon=1}^{\eta} l_r(\rho_\delta^\epsilon(t))^I H_\epsilon z_\delta^I(t)$$
$$-\left[P^I\left(A_\delta^I(t)\right) - P^I\left(A_*^I(t)\right)\right]y^R\left(A_*^R(t)\right)\Big\} + \sum_{\epsilon=1}^{\eta}\sum_{\delta=1}^{m} l_\epsilon\left((\rho_\delta^\epsilon(t))^R - (\widehat{\rho}_\delta^\epsilon)^R\right)\left(z_\delta^R(t)\right)^T\left(H_\epsilon + H_\epsilon^T\right)z_\delta^R(t) + \sum_{\epsilon=1}^{\eta}\sum_{\delta=1}^{m} l_\epsilon(\rho_\delta^\epsilon(t))^I$$
$$-(\widehat{\rho}_\delta^\epsilon)^I\left(z_\delta^I(t)\right)^T\left(H_\epsilon + H_\epsilon^T\right)z_\delta^I(t) + 2\left(z^R(t)\right)^T\left(I_N\otimes\left(J^R\Lambda\right)\right)z^R(t) - 2\left(\overline{z^R(t)}\right)^T\left(I_N\otimes J^R\right)\overline{z^R(t)} + 2\left(z^I(t)\right)^T\left(I_N\right.$$
$$\left.\otimes\left(J^I\Lambda\right)\right)z^I(t) - 2\left(\overline{z^I(t)}\right)^T\left(I_N\otimes J^I\right)\overline{z^I(t)} + \sum_{\epsilon=1}^{\eta}\frac{l_\epsilon}{1-\overline{\varrho}_\epsilon}\left(z^R(t)\right)^T\Psi_\epsilon z^R(t) - \sum_{\epsilon=1}^{\eta} l_\epsilon\left(\widehat{z^R(t)}\right)^T\Psi_\epsilon\widehat{z^R(t)}$$
$$+\sum_{\epsilon=1}^{\eta}\frac{l_\epsilon}{1-\overline{\varrho}_\epsilon}\left(z^I(t)\right)^T\Psi_\epsilon z^I(t) - \sum_{\epsilon=1}^{\eta} l_\epsilon\left(\widehat{z^I(t)}\right)^T\Psi_\epsilon\widehat{z^I(t)}. \tag{70}$$

By (20)–(23), (26)–(29), (51)-(52), (60)-(61), and (70), we have

$$\dot{V}_4(t) \le \left(z^R(t)\right)^T\left\{I_N\otimes\left(-2D + \widetilde{P}^R + \widetilde{P}^I + 2J^R\Lambda\right) - 2\Theta^R + \sum_{\epsilon=1}^{\eta} l_\epsilon\left[(K^\epsilon\otimes H_\epsilon)\Psi_\epsilon^{-1}\left((K^\epsilon)^T\otimes H_\epsilon^T\right) - (\widehat{\rho}^\epsilon)^R\otimes\left(H_\epsilon + H_\epsilon^T\right) + \frac{1}{1-\overline{\varrho}_\epsilon}\Psi_\epsilon\right]\right\}z^R(t)$$
$$+\left(z^I(t)\right)^T\left\{I_N\otimes\left(-2D + \widetilde{P}^R + \widetilde{P}^I + 2J^I\Lambda\right) - 2\Theta^I + \sum_{\epsilon=1}^{\eta} l_\epsilon\left[(K^\epsilon\otimes H_\epsilon)\Psi_\epsilon^{-1}\left((K^\epsilon)^T\otimes H_\epsilon^T\right) - (\widehat{\rho}^\epsilon)^I\otimes\left(H_\epsilon + H_\epsilon^T\right) + \frac{1}{1-\overline{\varrho}_\epsilon}\Psi_\epsilon\right]\right\}z^I(t)$$
$$\le \alpha_4\|z(t)\|^2, \tag{71}$$

where $\alpha_4 = \max\{\lambda_M(\Phi_4^R), \lambda_M(\Phi_4^I)\} < 0$. By combining the proofs of Theorems 2 with 3, we can obtain $\lim_{t\longrightarrow+\infty}\|z(t)\| = 0$. Thus, the network (65) is anti-synchronized under the controller (47). □ □

5. Numerical Examples

Example 1. Consider the MWCCVDMNNs illustrated by

$$\dot{A}_\delta(t) = -DA_\delta(t) + P\left(A_\delta(t)\right)y\left(\overline{A_\delta(t)}\right) + c_\delta(t)$$
$$+ l_1\sum_{\kappa=1}^{6} K_{\delta\kappa}^1 H_1 A_\kappa(t) + l_2\sum_{\kappa=1}^{6} K_{\delta\kappa}^2 H_2 A_\kappa(t) + l_3\sum_{\kappa=1}^{6} K_{\delta\kappa}^3 H_3 A_\kappa(t), \tag{72}$$

where$\delta = 1, 2, \ldots, 6$, $y_i^R(\mu) = y_i^I(\mu) = (|\mu+1| - |\mu-1|)/5\,(i = 1, 2, 3)$, $D = \operatorname{diag}(0.4, 0.8, 0.5)$, $l_1 = 0.1$, $l_2 = 0.2$, $l_3 = 0.4$, $u_h(t) = 1 - 1/(2+h)x^{-t}$, $u = 1$, $\varrho_h = 1/(2+h)$, $h = 1, 2, 3$, and the matricesH_ϵ, $K^\epsilon = (K_{\iota\kappa}^\epsilon)_{6\times6}\,(\epsilon = 1, 2, 3)$, and the elements in the matrices $P(A_\delta(t))\,(\delta = 1, 2, \ldots, 6)$ are selected as follows:

$$
\begin{gathered}
H_1=\begin{pmatrix}0.4&0.1&0.2\\0.1&0.4&0.2\\0.4&0.2&0.5\end{pmatrix},H_2=\begin{pmatrix}0.4&0.3&0.4\\0.2&0.7&0.3\\0.4&0.9&0.6\end{pmatrix},H_3=\begin{pmatrix}0.4&0.3&0.2\\0.2&0.8&0.4\\0.2&1.1&0.7\end{pmatrix},\\
p_{11}^R\left(a_{\delta1}^R(t)\right)=\begin{cases}0.42, & \left|a_{\delta1}^R(t)\right|\leqslant 1.7,\\ -0.32, & \left|a_{\delta1}^R(t)\right|>1.7,\end{cases}
p_{12}^R\left(a_{\delta1}^R(t)\right)=\begin{cases}-0.32, & \left|a_{\delta1}^R(t)\right|\leqslant 1.7,\\ -0.36, & \left|a_{\delta1}^R(t)\right|>1.7,\end{cases}\\
p_{13}^R\left(a_{\delta1}^R(t)\right)=\begin{cases}-0.28, & \left|a_{\delta1}^R(t)\right|\leqslant 1.7,\\ -0.40, & \left|a_{\delta1}^R(t)\right|>1.7,\end{cases}
p_{21}^R\left(a_{\delta2}^R(t)\right)=\begin{cases}0.33, & \left|a_{\delta2}^R(t)\right|\leqslant 1.7,\\ -0.28, & \left|a_{\delta2}^R(t)\right|>1.7,\end{cases}\\
p_{22}^R\left(a_{\delta2}^R(t)\right)=\begin{cases}-0.29, & \left|a_{\delta2}^R(t)\right|\leqslant 1.7,\\ 0.36, & \left|a_{\delta2}^R(t)\right|>1.7,\end{cases}
p_{23}^R\left(a_{\delta2}^R(t)\right)=\begin{cases}0.20, & \left|a_{\delta2}^R(t)\right|\leqslant 1.7,\\ -0.19, & \left|a_{\delta2}^R(t)\right|>1.7,\end{cases}\\
p_{31}^R\left(a_{\delta3}^R(t)\right)=\begin{cases}0.24, & \left|a_{\delta3}^R(t)\right|\leqslant 1.7,\\ -0.13, & \left|a_{\delta3}^R(t)\right|>1.7,\end{cases}
p_{32}^R\left(a_{\delta3}^R(t)\right)=\begin{cases}0.39, & \left|a_{\delta3}^R(t)\right|\leqslant 1.7,\\ 0.26, & \left|a_{\delta3}^R(t)\right|>1.7,\end{cases}\\
p_{33}^R\left(a_{\delta3}^R(t)\right)=\begin{cases}-0.34, & \left|a_{\delta3}^R(t)\right|\leqslant 1.7,\\ -0.27, & \left|a_{\delta3}^R(t)\right|>1.7,\end{cases}
p_{11}^I\left(a_{\delta1}^I(t)\right)=\begin{cases}-0.34, & \left|a_{\delta1}^I(t)\right|\leqslant 1.7,\\ 0.25, & \left|a_{\delta1}^I(t)\right|>1.7,\end{cases}\\
p_{12}^I\left(a_{\delta1}^I(t)\right)=\begin{cases}0.24, & \left|a_{\delta1}^I(t)\right|\leqslant 1.7,\\ 0.17, & \left|a_{\delta1}^I(t)\right|>1.7,\end{cases}
p_{13}^I\left(a_{\delta1}^I(t)\right)=\begin{cases}-0.42, & \left|a_{\delta1}^I(t)\right|\leqslant 1.7,\\ 0.36, & \left|a_{\delta1}^I(t)\right|>1.7,\end{cases}\\
p_{21}^I\left(a_{\delta2}^I(t)\right)=\begin{cases}0.27, & \left|a_{\delta2}^I(t)\right|\leqslant 1.7,\\ -0.14, & \left|a_{\delta2}^I(t)\right|>1.7,\end{cases}
p_{22}^I\left(a_{\delta2}^I(t)\right)=\begin{cases}0.26, & \left|a_{\delta2}^I(t)\right|\leqslant 1.7,\\ 0.17, & \left|a_{\delta2}^I(t)\right|>1.7,\end{cases}\\
p_{23}^I\left(a_{\delta2}^I(t)\right)=\begin{cases}-0.19, & \left|a_{\delta2}^I(t)\right|\leqslant 1.7,\\ -0.21, & \left|a_{\delta2}^I(t)\right|>1.7,\end{cases}
p_{31}^I\left(a_{\delta3}^I(t)\right)=\begin{cases}-0.25, & \left|a_{\delta3}^I(t)\right|\leqslant 1.7,\\ 0.31, & \left|a_{\delta3}^I(t)\right|>1.7,\end{cases}\\
p_{32}^I\left(a_{\delta3}^I(t)\right)=\begin{cases}0.27, & \left|a_{\delta3}^I(t)\right|\leqslant 1.7,\\ 0.26, & \left|a_{\delta3}^I(t)\right|>1.7,\end{cases}
p_{33}^I\left(a_{\delta3}^I(t)\right)=\begin{cases}-0.38, & \left|a_{\delta3}^I(t)\right|\leqslant 1.7,\\ 0.40, & \left|a_{\delta3}^I(t)\right|>1.7,\end{cases}\\
K^1=\begin{pmatrix}-0.6&0.2&0.1&0.1&0&0.2\\0.3&-0.7&0.1&0.1&0.1&0.1\\0.1&0.2&-0.8&0.1&0.3&0.1\\0.2&0.1&0.3&-1.1&0.4&0.1\\0.1&0.1&0.2&0.2&-0.9&0.3\\0.2&0.5&0&0&0.3&-1.0\end{pmatrix},\\
K^2=\begin{pmatrix}-0.7&0.1&0.2&0.3&0.1&0\\0.2&-0.9&0&0.3&0.3&0.1\\0.2&0.2&-0.9&0&0.3&0.2\\0&0.4&0.2&-0.8&0.1&0.1\\0.3&0.1&0.4&0.1&-1.1&0.2\\0.1&0.1&0.2&0.2&0&-0.6\end{pmatrix},\\
K^3=\begin{pmatrix}-0.8&0.3&0&0.2&0.1&0.2\\0.4&-0.8&0.1&0.2&0&0.1\\0&0&-1.0&0.5&0.2&0.3\\0.3&0.1&0.4&-1.2&0.2&0.2\\0.2&0.1&0&0.3&-0.9&0.3\\0.2&0.4&0&0.2&0.3&-1.1\end{pmatrix}.
\end{gathered}
\tag{73}
$$

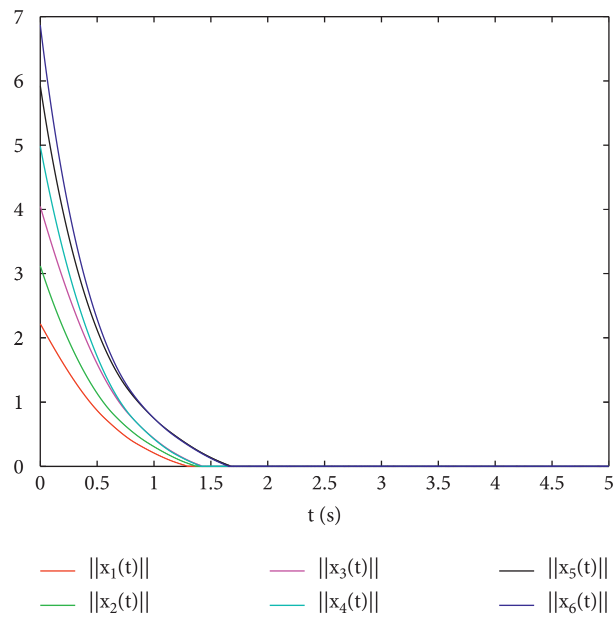

Figure 1: Time evolution of $\|z_\iota(t)\|$, $\iota = 1, 2, \ldots, 6$.

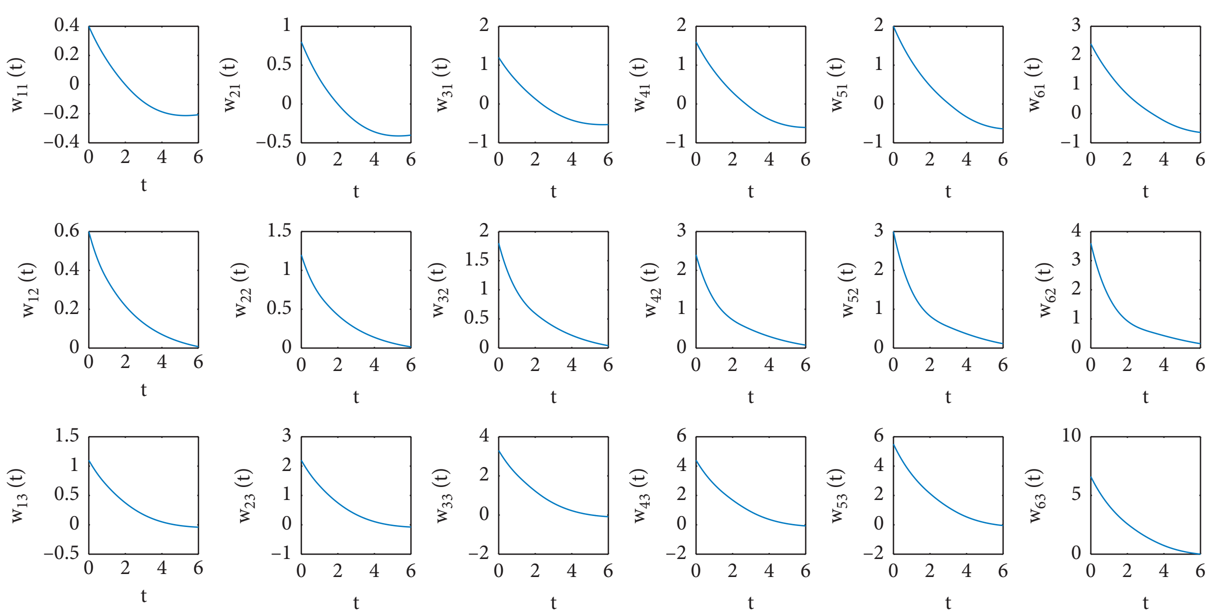

Figure 2: The single dynamical change process of uncoupled NN in (72).

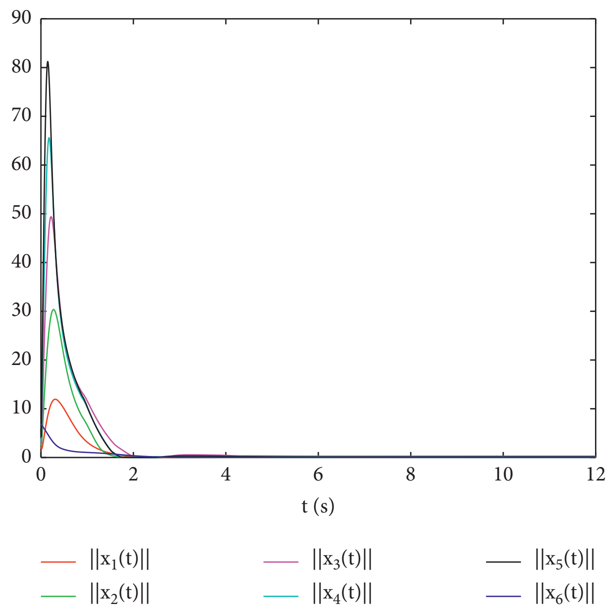

FIGURE 3: Time evolution of $\|z_\iota(t)\|$ under pinning controller (47), where $\iota = 1, 2, \ldots, 6$.

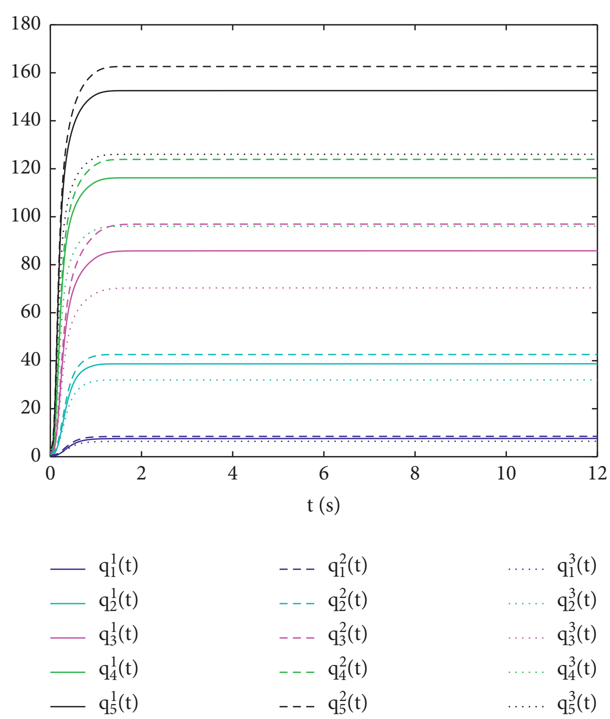

FIGURE 4: $\rho_\iota^r(t)$ when $\zeta_\iota^r = 0.5$, $\iota = 1, 2, \ldots, 5$, $r = 1, 2, 3$.

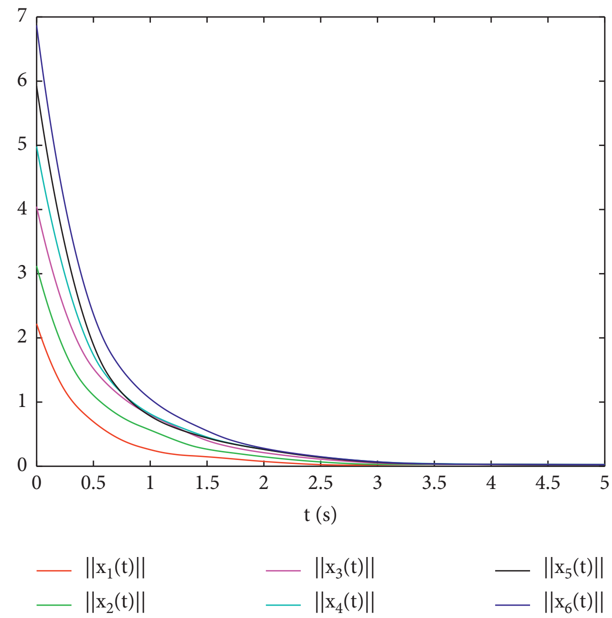

FIGURE 5: Time evolution of $\|z_\iota(t)\|$ without pinned nodes, where $\iota = 1, 2, \ldots, 6$.

It is readily seen that Assumption 1 is satisfied with $Y_\delta^R = Y_\delta^I = 0.4$ and $j_\delta^R = j_\delta^I = 0.4$. Choose the parameters in the controller (15) as follows: $\Theta^R = \mathrm{diag}(\Theta_1^R, \Theta_2^R, \ldots, \Theta_6^R) = \mathrm{diag}(1.01, 0.98, 0.76, 0.69, 1.10, 1.05, 0.86, 0.79, 0.91, 1.23, 1.54, 1.71, 1.82, 1.79, 1.79, 1.06, 0.86, 0.78)$, $\Theta^I = \mathrm{diag}(\Theta_1^I, \Theta_2^I, \ldots, \Theta_6^I) = \mathrm{diag}(0.92, 0.76, 0.87, 0.92, 0.71, 0.78, 0.86, 0.99, 0.94, 0.85, 0.69, 0.76, 0.59, 0.71, 0.77, 0.89, 0.83, 0.90)$.

By making use of the YALMIP Toolbox in MATLAB, we can obtain

$$\begin{aligned}\lambda\left(\Phi_1^R\right) &= \{-6.2147, -5.8330, -4.5881, -4.2615, -3.9606, -3.8739, -3.6252, -3.5277, -2.7759, -2.5151, -2.3761, -2.1608, -2.0750, -1.9064, -1.8040, -1.6740, -1.5948, -1.1972\},\\ \lambda\left(\Phi_1^I\right) &= \{-4.7743, -4.3922, -4.0962, -4.0088, -3.5890, -2.3394, -2.1504, -2.0480, -1.9877, -1.9329, -1.8667, -1.7628, -1.7475, -1.7018, -1.5975, -1.5615, -1.4166, -1.0108\},\end{aligned} \tag{74}$$

which meet condition (17). By Theorem 1, it implies that the network (72) has the capability of realizing anti-synchronization under the pinning adaptive controller (15). Figure 1 depicts the antisynchronization simulation result of network (72). In order to display the effect of multiple coupling matrices, Figure 2 depicts the single dynamical change process of uncoupled NN in (72).

Example 2. Take into account the following MWCCVDMNNs with coupling delays:

$$\begin{aligned}\dot{A}_\delta(t) &= -DA_\delta(t) + P\left(A_\delta(t)\right)y\left(\overline{A_\delta(t)}\right) + c_\delta(t)\\ &\quad + l_1 \sum_{\kappa=1}^{N} K_{\iota\kappa}^1 H_1 \widehat{A_\kappa(t)} + l_2 \sum_{\kappa=1}^{N} K_{\iota\kappa}^2 H_2 \widehat{A_\kappa(t)}\\ &\quad + l_3 \sum_{\kappa=1}^{N} K_{\iota\kappa}^3 H_3 \widehat{A_\kappa(t)},\end{aligned} \tag{75}$$

where $\delta = 1, 2, \ldots, 6$, $y_i^R(\mu) = y_i^I(\mu) = (|\mu + 1| - |\mu - 1|)/4\,(i = 1, 2, 3)$, $D = \mathrm{diag}(0.8, 1.0, 0.7)$, $l_1 = 0.4$, $l_2 = 0.3$, $l_3 = 0.1$, $u_h(t) = 1 - 1/(2 + h)z^{-t}$, $u = 1$, $\varrho_h = 1/(2 + h)$, $w_h(t) = (1/20) - (1/10(h + 4))z^{-t}$, $\overline{\varrho}_h$

$= (1/10(h+4))$, $h = 1,2,3$, and the matrices H_r, $K^r = (K^r_{\delta\kappa})_{6\times 6}\,(r = 1,2,3)$ and the elements in the matrices $P(A_\delta(t))\,(\delta = 1,2,\ldots,6)$ are selected as follows:

$$H_1 = \begin{pmatrix} 0.18 & 0 & 0 \\ 0.04 & 0.19 & 0.06 \\ 0 & 0 & 0.18 \end{pmatrix}, H_2 = \begin{pmatrix} 0.27 & 0 & 0.09 \\ 0 & 0.07 & 0 \\ 0.04 & 0 & 0.12 \end{pmatrix}, H_3 = \begin{pmatrix} 0.13 & 0 & 0 \\ 0.04 & 0.15 & 0.06 \\ 0 & 0 & 0.15 \end{pmatrix},$$

$$p^R_{11}(a^R_{\delta 1}(t)) = \begin{cases} 0.53, & |a^R_{\delta 1}(t)| \leqslant 1.5, \\ 0.47, & |a^R_{\delta 1}(t)| > 1.5, \end{cases} p^R_{12}(a^R_{\delta 1}(t)) = \begin{cases} 0.23, & |a^R_{\delta 1}(t)| \leqslant 1.5, \\ -0.53, & |a^R_{\delta 1}(t)| > 1.5, \end{cases}$$

$$p^R_{13}(a^R_{\delta 1}(t)) = \begin{cases} 0.32, & |a^R_{\delta 1}(t)| \leqslant 1.5, \\ 0.42, & |a^R_{\delta 1}(t)| > 1.5, \end{cases} p^R_{21}(a^R_{\delta 2}(t)) = \begin{cases} -0.47, & |a^R_{\delta 2}(t)| \leqslant 1.5, \\ -0.53, & |a^R_{\delta 2}(t)| > 1.5, \end{cases}$$

$$p^R_{22}(a^R_{\delta 2}(t)) = \begin{cases} 0.31, & |a^R_{\delta 2}(t)| \leqslant 1.5, \\ 0.24, & |a^R_{\delta 2}(t)| > 1.5, \end{cases} p^R_{23}(a^R_{\delta 2}(t)) = \begin{cases} -0.41, & |a^R_{\delta 2}(t)| \leqslant 1.5, \\ 0.51, & |a^R_{\delta 2}(t)| > 1.5, \end{cases}$$

$$p^R_{31}(a^R_{\delta 3}(t)) = \begin{cases} -0.27, & |a^R_{\delta 3}(t)| \leqslant 1.5, \\ -0.43, & |a^R_{\delta 3}(t)| > 1.5, \end{cases} p^R_{32}(a^R_{\delta 3}(t)) = \begin{cases} 0.37, & |a^R_{\delta 3}(t)| \leqslant 1.5, \\ 0.25, & |a^R_{\delta 3}(t)| > 1.5, \end{cases}$$

$$p^R_{33}(a^R_{\delta 3}(t)) = \begin{cases} -0.38, & |a^R_{\delta 3}(t)| \leqslant 1.5, \\ 0.29, & |a^R_{\delta 3}(t)| > 1.5, \end{cases} p^I_{11}(a^I_{\delta 1}(t)) = \begin{cases} -0.43, & |a^R_{\delta 1}(t)| \leqslant 1.5, \\ 0.32, & |a^R_{\delta 1}(t)| > 1.5, \end{cases}$$

$$p^I_{12}(a^I_{\delta 1}(t)) = \begin{cases} 0.35, & |a^R_{\delta 1}(t)| \leqslant 1.5, \\ -0.27, & |a^R_{\delta 1}(t)| > 1.5, \end{cases} p^I_{13}(a^I_{\delta 1}(t)) = \begin{cases} 0.41, & |a^R_{\delta 1}(t)| \leqslant 1.5, \\ 0.40, & |a^R_{\delta 1}(t)| > 1.5, \end{cases}$$

$$p^I_{21}(a^I_{\delta 2}(t)) = \begin{cases} 0.32, & |a^R_{\delta 2}(t)| \leqslant 1.5, \\ -0.38, & |a^R_{\delta 2}(t)| > 1.5, \end{cases} p^I_{22}(a^I_{\delta 2}(t)) = \begin{cases} 0.42, & |a^R_{\delta 2}(t)| \leqslant 1.5, \\ 0.45, & |a^R_{\delta 2}(t)| > 1.5, \end{cases}$$

$$p^I_{23}(a^I_{\delta 2}(t)) = \begin{cases} -0.52, & |a^R_{\delta 2}(t)| \leqslant 1.5, \\ -0.71, & |a^R_{\delta 2}(t)| > 1.5, \end{cases} p^I_{31}(a^I_{\delta 3}(t)) = \begin{cases} -0.32, & |a^R_{\delta 3}(t)| \leqslant 1.5, \\ 0.41, & |a^R_{\delta 3}(t)| > 1.5, \end{cases}$$

$$p^I_{32}(a^I_{\delta 3}(t)) = \begin{cases} 0.71, & |a^R_{\delta 3}(t)| \leqslant 1.5, \\ 0.54, & |a^R_{\delta 3}(t)| > 1.5, \end{cases} p^I_{33}(a^I_{\delta 3}(t)) = \begin{cases} 0.60, & |a^R_{\delta 3}(t)| \leqslant 1.5, \\ 0.57, & |a^R_{\delta 3}(t)| > 1.5, \end{cases}$$

$$\begin{aligned} K^1 &= \begin{pmatrix} -0.7 & 0.1 & 0.3 & 0 & 0.1 & 0.2 \\ 0.3 & -0.8 & 0.1 & 0.2 & 0.1 & 0.1 \\ 0.1 & 0.2 & -1.1 & 0.5 & 0.1 & 0.2 \\ 0 & 0.2 & 0.4 & -1.1 & 0.2 & 0.3 \\ 0.2 & 0.1 & 0 & 0.2 & -1.0 & 0.5 \\ 0.1 & 0.2 & 0.4 & 0.1 & 0 & -0.8 \end{pmatrix}, \\ K^2 &= \begin{pmatrix} -0.8 & 0.2 & 0.2 & 0.3 & 0.1 & 0 \\ 0.1 & -1.0 & 0.2 & 0.2 & 0.3 & 0.2 \\ 0.3 & 0 & -0.9 & 0 & 0.3 & 0.3 \\ 0.2 & 0.1 & 0.1 & -0.8 & 0.3 & 0.1 \\ 0 & 0.3 & 0.4 & 0.2 & -1.1 & 0.2 \\ 0.2 & 0.1 & 0.3 & 0.1 & 0.2 & -0.9 \end{pmatrix}, \\ K^3 &= \begin{pmatrix} -0.9 & 0.3 & 0.1 & 0.1 & 0.2 & 0.2 \\ 0.2 & -0.8 & 0 & 0.4 & 0.1 & 0.1 \\ 0 & 0 & -1.0 & 0.5 & 0.2 & 0.3 \\ 0.2 & 0.1 & 0.2 & -0.7 & 0 & 0.2 \\ 0.1 & 0.1 & 0.2 & 0.2 & -0.9 & 0.3 \\ 0.3 & 0 & 0.1 & 0.2 & 0.4 & -1.0 \end{pmatrix}. \end{aligned} \tag{76}$$

It is readily seen that Assumption 1 is satisfied with $Y^R_\delta = Y^I_\delta = 0.5$ and $j^R_\delta = j^I_\delta = 0.5$. Select the parameters in the controller (47) as follows: $\Theta^R = \mathrm{diag}(\Theta^R_1, \Theta^R_2, \ldots, \Theta^R_6) = \mathrm{diag}(0.89, 0.78, 0.81, 0.64, 0.71, 0.90, 0.89, 0.65, 0.71, 0.67, 0.74, 0.57, 0.73, 0.77, 0.89, 0.75, 0.88, 0.73)$ and $\Theta^I = \mathrm{diag}(\Theta^I_1, \Theta^I_2, \ldots, \Theta^I_6) = \mathrm{diag}(0.74, 0.65, 0.54, 0.71, 0.93, 0.73, 0.89, 0.65, 0.52, 0.67, 0.51, 0.49, 0.81, 0.60, 0.83, 0.77, 0.81, 0.74)$.

We select the first 5 nodes as pinned nodes. Choose $(\hat{\rho}^1)^R = \mathrm{diag}((\hat{\rho}^1_1)^R, (\hat{\rho}^1_2)^R, \ldots, (\hat{\rho}^1_5)^R, 0) = \mathrm{diag}(0.1, 0.2, 0.3, 0.4, 0.5, 0)$, $(\hat{\rho}^2)^R = \mathrm{diag}((\hat{\rho}^2_1)^R, (\hat{\rho}^2_2)^R, \ldots, (\hat{\rho}^2_5)^R, 0) = \mathrm{diag}(0.3, 0.6, 0.9, 1.2, 1.5, 0)$, $(\hat{\rho}^3)^R = \mathrm{diag}((\hat{\rho}^3_1)^R, (\hat{\rho}^3_2)^R, \ldots, (\hat{\rho}^3_5)^R, 0) = \mathrm{diag}(0.2, 0.4, 0.6, 0.8, 1.0, 0)$, $(\hat{\rho}^1)^I = \mathrm{diag}((\hat{\rho}^1_1)^I, (\hat{\rho}^1_2)^I, \ldots, (\hat{\rho}^1_5)^I, 0) = \mathrm{diag}(0.2, 0.4, 0.6, 0.8, 1.0, 0)$, $(\hat{\rho}^2)^I = \mathrm{diag}((\hat{\rho}^2_1)^I, (\hat{\rho}^2_2)^I, \cdots, (\hat{\rho}^2_5)^I, 0) = \mathrm{diag}(0.4, 0.8, 1.2, 1.6, 2.0, 0)$, and $(\hat{\rho}^3)^I = \mathrm{diag}((\hat{\rho}^3_1)^I, (\hat{\rho}^3_2)^I, \ldots, (\hat{\rho}^3_5)^I, 0) = \mathrm{diag}(0.3, 0.6, 0.9, 1.2, 1.5, 0)$.

By making use of the YALMIP Toolbox of MATLAB, the following $\Psi_\epsilon\,(\epsilon = 1,2,3)$ satisfying (68) can be computed: $\Psi_1 = \mathrm{diag}(0.5033, 0.5033, 0.4082, 0.4840, 0.5203, 0.4739, 0.5319, 0.5095, 0.4281, 0.5077, 0.5072, 0.4086, 0.5230, 0.5148, 0.5041, 0.4876, 0.5213, 0.4381)$, $\Psi_2 = \mathrm{diag}(0.3954, 0.3820, 0.3388, 0.3965, 0.3908, 0.3726, 0.4044, 0.3796, 0.3394, 0.3947, 0.3783, 0.3282, 0.4068, 0.3867, 0.3864, 0.3933, 0.3897, 0.3518)$, and $\Psi_3 = \mathrm{diag}(0.1188, 0.1191, 0.1194, 0.1181, 0.1184, 0.1190, 0.1186, 0.1195, 0.1200, 0.1186, 0.1190, 0.1190, 0.1187, 0.1191, 0.1201, 0.1192, 0.1194, 0.1209)$.

Based on Theorem 4, we conclude that the pinning antisynchronization of network (75) is realized with the controller (47). Figures 3 and 4 show the simulation results. For comparison, Figure 5 shows the variation trajectory of error variables $x_\iota(t)$ without pinned nodes.

6. Conclusions

This paper has investigated the antisynchronization of MWCCVDMNNs without and with coupling delays. On the one hand, we have presented some sufficient conditions for reaching antisynchronization of the proposed network models. On the other hand, some generalized pinning antisynchronization criteria on the basis of the designed pinning control strategy have been established to ensure that the considered MWCCVDMNNs with and without coupling delays realize generalized pinning antisynchronization, respectively. Furthermore, two numerical examples have been shown to verify the correctness of the derived results. This paper is an extended version of our previous work published in [37]. Based on the derived antisynchronization results of MWCCVDMNNs with and without coupling delays in [37], we have further investigated the generalized pinning antisynchronization of the considered networks by designing a novel generalized pinning adaptive controller in this paper. More specifically, the pinning controller we designed in this paper consists of two parts, one part controls all the nodes, and the other part controls the first m nodes of the considered network, which is different from the classical pinning control. To the best of the authors' knowledge, this is the first paper toward to researching the pinning antisynchronization of the MWCCVDMNNs. However, there are several interesting problems for further study. On the one hand, some scholars have investigated the dynamical behavior of a new type of coupled complex-valued networks with intermittent coupling recently [38]. In contrast to the common continuous coupling in this paper, intermittent coupling is a discontinuous form of communication which has greater flexibility for nodes because they are not constrained by communication requirements in decoupling time, which unavoidably result in the difference of dynamics of nodes between the coupling time and decoupling period. Therefore, it would be very interesting to take the intermittent coupling into consideration when studying the pinning antisynchronization of MWCCVDMNNs in our future work. On the other hand, it is known to all that fractional calculus is a theory that generalizes the concept of calculus from the integer order to arbitrary order. Up to now, fractional-order systems have been applied in some new mechanical models due to their non-Markovian and non-Gaussian properties during the studying of dynamical systems. In [39], a novel criterion for achieving synchronization of fractional-order chaotic and hyperchaotic systems was proposed. Motivated by this work, it would be also a very interesting problem of research to insert fractional operators into the proposed MWCCVDMNNs in this paper and study the antisynchronization of this kind of the fractional-order network model.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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