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Research Article

Epidemiological Characteristics of Generalized COVID-19 Deterministic Disease Model

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Coronavirus disease 2019 (COVID-19) is an infection that can result in lung issues such as pneumonia and, in extreme situations, the most severe acute respiratory syndrome. COVID-19 is widely investigated by researchers through mathematical models from different aspects. Inspired from the literature, in the present paper, the generalized deterministic COVID-19 model is considered and examined. The basic reproduction number is obtained which is a key factor in defining the nonlinear dynamics of biological and physical obstacles in the study of mathematical models of COVID-19 disease. To better comprehend the dynamical behavior of the continuous model, two unconditionally stable schemes, i.e., mixed Euler and nonstandard finite difference (NSFD) schemes are developed for the continuous model. For the discrete NSFD scheme, the boundedness and positivity of solutions are discussed in detail. The local stability of disease-free and endemic equilibria is demonstrated by constructing Jacobian matrices for NSFD scheme; nevertheless, the global stability of aforementioned equilibria is verified by using Lyapunov functions. Numerical simulations are also presented that demonstrate how both the schemes are effective and suitable for solving the continuous model. Consequently, the outcomes obtained through these schemes are completely according to the solutions of the continuous model.

1. Introduction

COVID-19, caused by SARS-CoV-2 (severe acute respiratory syndrome coronavirus 2), appear out of Wuhan city of China at the end of 2019. The epidemic then expanded quickly to more than 210 countries [1, 2] and is still causing severe public health and socioeconomic problems in numerous places throughout the world. In the battle against COVID-19 and the shortfall of an antibody or treatment, the more interest is in setting up equipments that outline the guidelines of its virility and, therefore, control the spread of the infection. To stop COVID-19 from spreading, different governments estimated various preventive measures, including the use of masks, maintaining a six-foot social distance, regularly washing hands, and avoiding sick people [3, 4].

COVID-19 [5–8] is a contagious disease that can be passed on by direct contact with affected people as well as droplets and aerosols. This disease is a major issue for the human society as it has infected many individuals with restricted assets in numerous nations. COVID-19 has been spread quickly all over the world as it has been found to have a higher elevated level of infection and pandemic risk than SARS. On March 11, the WHO proclaimed the COVID-19 outbreak a global pandemic due to the sharp increase in transmission. The symptoms of COVID-19 are fever, cough, malign, fatigue sputum, headache, diarrhea dyspnoea, and lymphopenia [9]. The COVID-19 incubation period can last up to 14 days, with a mean of about 5 days [10].

By employing mathematical modeling, we may concentrate on how an infectious disease transmits all over a population. In order to explain epidemic infectious diseases, the researchers are employing both integer orders [11–15] as well as fractional orders [16–18] mathematical models. These models can be used to improve all of their sources and more effectively implement control measures. Many authors have constructed a number of mathematical models to assess the transmission and dynamical behavior of COVID-19 disease [5, 19, 20]. A deterministic epidemic model for COVID-19 based on the health status of the populations is proposed and analyzed in [19]. To explain the uncertainty or variance observed in the spread of disease, a stochastic extension of the deterministic model is also taken into account. In [5], a COVID-19 mathematical model in which the resistive compartment along with quarantine class has been considered by the authors. The resistive class together with quarantine class makes the model unique in all respect than the previous models existing in the literature. Some models for COVID-19 were introduced in [20] that address important questions about the global health care. The authors suggested three well-known numerical techniques, such as Euler's and Runge–Kutta scheme of order two (RK-2) and of order four (RK-4) for solving such equations.

Recently, Peter et al. [21] studied the COVID-19 disease model by concentrating the real data from Pakistan that evaluates the effect of some management methods on the transmission of COVID-19 in a human population. The author only determined the global stability of disease-free equilibrium point for the continuous model. The aim of present paper is to use more sophisticated mixed Euler and NSFD schemes to evaluate various features of the continuous model to exhibit its endurance and biological sustainability. Multiple ideas and criteria are utilized to discuss the local as well as global stability of disease-free and endemic equilibria for the NSFD scheme. The outcomes show that both the schemes are unconditionally stable which are not only appropriate for the continuous model but also produces exceptionally efficient and precise results.

The paper is arranged as follows: The mathematical model is presented and its parameters are thoroughly discussed in Section 2. In Section 3, the equilibria and basic reproduction number are provided for the model. The reproduction number is the most significant threshold measure used to describe the local and global stability of equilibria. The unconditionally stable mixed Euler and NSFD schemes are developed in Section 4 and Section 5, respectively, which preserve important characteristics of the continuous model. For the NSFD scheme, the Schur–Cohn and Routh–Hurwitz criteria are utilized to assess the local stability of disease-free and endemic equilibria, respectively. However, the global stability of aforementioned equilibria is discussed using Lyapunov functions. To support the analytical outcomes, numerical simulations are illustrated at each step. The conclusions are provided in the final section to summarize the whole article.

2. Mathematical Model and Its Detail Description

The COVID-19 dynamical system [21] including five differential equations is provided as follows. The total population denoted by $N(t)$ is separated into five classes, i.e., susceptible $S(t)$, exposed $E(t)$, infected $I(t)$, quarantined $Q(t)$, and recovered $R(t)$ where $N(t) = S(t) + E(t) + I(t) + Q(t) + R(t)$.

$$\begin{aligned}
\frac{dS}{dt} &= \beta - \frac{\delta_c(1-\mathscr{E})(1-\gamma)(E+I)S}{N} - \theta S + \Sigma R,\\
\frac{dE}{dt} &= \frac{\delta_c(1-\mathscr{E})(1-\gamma)(E+I)S}{N} - (\theta+\alpha)E,\\
\frac{dI}{dt} &= \alpha E - (\theta+\eta+\psi+\Omega)I,\\
\frac{dQ}{dt} &= \psi I - (\theta+\eta+\varphi)Q,\\
\frac{dR}{dt} &= \varphi Q + \Omega I - (\Sigma+\theta)R.
\end{aligned} \tag{1}$$

2.1. Parameters and Their Explanations. Following is a description of the parameters that compose the recently suggested COVID-19 model (1).

β Rate of recruitment into susceptible persons

θ Natural morality rate

η The death rate due to COVID-19

α The rate of evolution from exposed into infectious class

Σ Rate of immunity loss

Ω Treatment rate for infectious persons

φ Treatment rate for quarantine persons

$\mathscr{E}$ The rate of individuals that preserve social distancing

γ A part of the population uses hand sanitizer and face masks.

ψ The rate of recovery from infection

δ_c The rate of transmission efficiency.

The state variables and other parameters of proposed model (1) are thoroughly explained in Figure 1, where the force of infection $(\delta_c(1-\mathscr{E})(1-\gamma)(E+I)S/N)$ is denoted by Γ. In addition, it is assumed that each parameter in model (1) is a constant positive value.

3. Equilibria of Model and Basic Reproduction Number

3.1. Equilibria of Model. By equating the right hand side of model (1) to zero, we get the state known as disease-free equilibrium (DFE) point. If we denote DFE by

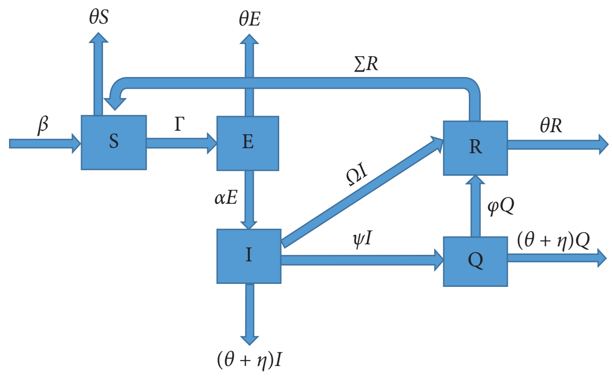

Figure 1: Flowchart for COVID-19 mathematical model (1).

$E_0 = (S^0, E^0, I^0, Q^0, R^0)$, then for model (1), it can easily be obtained that $E_0 = (\beta/\theta, 0, 0, 0, 0)$.

In order to determine the disease endemic equilibrium (DEE) point, the suggested model (1) is simultaneously solved for the state variables S, E, I, Q, and R. If the DEE point be denoted by $E^* (S^*, E^*, I^*, Q^*, R^*)$, then from model (1), we get

$$S^* = \frac{\beta - \delta_c (1 - \mathscr{E})(1 - \gamma)(E^* + I^*)S^* + \Sigma R^*}{N(t)\theta}, E^* = \frac{\delta_c (1 - \mathscr{E})(1 - \gamma)(E^* + I^*)S^*}{N(t)(\theta + \alpha)},$$
$$I^* = \frac{\alpha E^*}{(\theta + \eta + \psi + \Omega)}, Q^* = \frac{\psi I^*}{(\theta + \eta + \varphi)}, R^* = \frac{\varphi Q^* + \Omega I^*}{(\Sigma + \theta)}. \tag{2}$$

3.2. Basic Reproduction Number (R_0). The quantity R_0 [22] is an epidemiological estimate that represents the total number of associated disease results of a single infected individual in a completely exposed community during disease period. To obtain R_0, we use transmission $F(x)$ and translation $V(x)$ matrices, respectively. Let $x = (E, I, Q)$, then it follows from system (1) that

$$\frac{dy}{dx} = F(x) - V(x), \tag{3}$$

where

$$F(x) = \begin{bmatrix} \dfrac{\delta_c (1 - \mathscr{E})(1 - \gamma)(E + I)S}{N} \\ 0 \\ 0 \end{bmatrix}, V(x) = \begin{bmatrix} (\theta + \alpha)E \\ -\alpha E + (\theta + \eta + \psi + \Omega)I \\ -\psi I + (\theta + \eta + \varphi)Q \end{bmatrix}. \tag{4}$$

Simple calculations yields

$$F = \begin{bmatrix} \frac{\delta_c (1-\mathscr{E})(1-\gamma)}{N} S & \frac{\delta_c (1-\mathscr{E})(1-\gamma)}{N} S & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$V = \begin{bmatrix} (\theta+\alpha) & 0 & 0 \\ -\alpha & (\theta+\eta+\psi+\Omega) & 0 \\ 0 & -\psi & (\theta+\eta+\varphi) \end{bmatrix}. \quad (5)$$

After putting DFE point $E^0 = ((\beta/\theta), 0, 0, 0, 0)$, we obtain

$$F = \begin{bmatrix} \frac{\delta_c (1-\mathscr{E})(1-\gamma)}{N} \frac{\beta}{\theta} & \frac{\delta_c (1-\mathscr{E})(1-\gamma)}{N} \frac{\beta}{\theta} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$V = \begin{bmatrix} (\theta+\alpha) & 0 & 0 \\ -\alpha & (\theta+\eta+\psi+\Omega) & 0 \\ 0 & -\psi & (\theta+\eta+\varphi) \end{bmatrix}. \quad (6)$$

It can easily be shown that

$$V^{-1} = \begin{bmatrix} \frac{1}{(\theta+\alpha)} & 0 & 0 \\ \frac{\alpha}{(\theta+\alpha)(\theta+\eta+\psi+\Omega)} & \frac{1}{(\theta+\eta+\psi+\Omega)} & 0 \\ \frac{\alpha\psi}{(\theta+\alpha)(\theta+\eta+\psi+\Omega)(\theta+\eta+\varphi)} & \frac{\alpha\psi}{(\theta+\alpha)(\theta+\eta+\psi+\Omega)(\theta+\eta+\varphi)} & \frac{1}{(\theta+\eta+\varphi)} \end{bmatrix}. \quad (7)$$

Also, therefore,

$$FV^{-1} = \begin{bmatrix} \frac{\beta\delta_c (1-\mathscr{E})(1-\gamma)(\theta+\eta+\psi+\Omega+\alpha)}{\theta N (\theta+\alpha)(\theta+\eta+\psi+\Omega)} & \frac{\beta\delta_c (1-\mathscr{E})(1-\gamma)}{\theta N (\theta+\eta+\psi+\Omega)} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \quad (8)$$

As $R_0 = \rho(FV^{-1})$, hence from above, we obtain

$$R_0 = \frac{\beta\delta_c (1-\mathscr{E})(1-\gamma)(\theta+\eta+\psi+\Omega+\alpha)}{\theta N (\theta+\alpha)(\theta+\eta+\psi+\Omega)}. \quad (9)$$

4. The Mixed Euler Scheme

The mixed Euler scheme [23] is a technique used for calculating numerical solutions to the system of differential equations. The mixed Euler scheme is a combination of both backward and forward Euler schemes. For system (1), it can be developed as follows:

$$\begin{aligned}
\frac{S_{n+1}-S_n}{h} &= \beta - \frac{\delta_c(1-\mathscr{E})(1-\gamma)(E_n+I_n)S_{n+1}}{N} - \theta S_{n+1} + \Sigma R_n,\\
\frac{E_{n+1}-E_n}{h} &= \frac{\delta_c(1-\mathscr{E})(1-\gamma)(E_{n+1}+I_n)S_{n+1}}{N} - (\theta+\alpha)E_{n+1},\\
\frac{I_{n+1}-I_n}{h} &= \alpha E_{n+1} - (\theta+\eta+\psi+\Omega)I_{n+1},\\
\frac{Q_{n+1}-Q_n}{h} &= \psi I_{n+1} - (\theta+\eta+\varphi)Q_{n+1},\\
\frac{R_{n+1}-R_n}{h} &= \varphi Q_{n+1} + \Omega I_{n+1} - (\Sigma+\theta)R_{n+1}.
\end{aligned} \tag{10}$$

From above, the mixed Euler scheme for system (1) becomes

$$\begin{aligned}
S_{n+1} &= S_n + h\left(\beta - \frac{\delta_c(1-\mathscr{E})(1-\gamma)(E_n+I_n)S_{n+1}}{N} - \theta S_{n+1} + \Sigma R_n\right),\\
E_{n+1} &= E_n + h\left(\frac{\delta_c(1-\mathscr{E})(1-\gamma)(E_{n+1}+I_n)S_{n+1}}{N} - (\theta+\alpha)E_{n+1}\right),\\
I_{n+1} &= I_n + h\left(\alpha E_{n+1} - (\theta+\eta+\psi+\Omega)I_{n+1}\right),\\
Q_{n+1} &= Q_n + h\left(\psi I_{n+1} - (\theta+\eta+\varphi)Q_{n+1}\right),\\
R_{n+1} &= R_n + h\left(\varphi Q_{n+1} + \Omega I_{n+1} - (\Sigma+\theta)R_{n+1}\right).
\end{aligned} \tag{11}$$

The above equations can further be simplified as

$$\begin{aligned}
S_{n+1} &= \frac{Nh(\beta+\Sigma R_n) + NS_n}{\left(N + h\left(\delta_c(1-\mathscr{E})(1-\gamma)(E_n+I_n) + N\theta\right)\right)},\\
E_{n+1} &= \frac{NE_n + h\left(\delta_c(1-\mathscr{E})(1-\gamma)(I_nS_{n+1})\right)}{N(1+h(\theta+\alpha)) - h\left(\delta_c(1-\mathscr{E})(1-\gamma)S_{n+1}\right)},\\
I_{n+1} &= \frac{I_n + h(\alpha E_{n+1})}{1+h(\theta+\eta+\psi+\Omega)},\\
Q_{n+1} &= \frac{h(\psi I_{n+1}) + Q_n}{1+h(\theta+\eta+\varphi)},\\
R_{n+1} &= \frac{R_n + h(\varphi Q_{n+1} + \Omega I_{n+1})}{1+h(\Sigma+\theta)}.
\end{aligned} \tag{12}$$

The numerical solutions offered in Figures 2(a)–2(d) shows that the qualitative properties, such as stability and positivity, are observed by the mixed Euler scheme (12) for all step sizes. Whenever $R_0 \leq 1$, the solutions of the mixed Euler scheme (12) converge to DFE point, as shown in Figures 2(a)–2(c). The solutions of the mixed Euler scheme (12) diverge from the DFE point and converge to DEE point when R_0 surpasses one, as shown in Figure 2(d).

5. The NSFD Scheme

The NSFD scheme is introduced by Mickens in 1994 [24]. One of the important aspects of the discrete-time epidemic models created by the Mickens method is that they share the same traits as the original continuous-time models. The upcoming subsections will demonstrate how the discrete NSFD scheme maintains all the dynamical characteristics of the associated continuous model (1), independent of step size h.

5.1. Construction of the NSFD Scheme. For model (1), we denote S_n, E_n, I_n, Q_n, and R_n as the numerical estimations of $S(t)$, $E(t)$, $I(t)$, $Q(t)$, and $R(t)$ at $t = nh$, where n is a non-

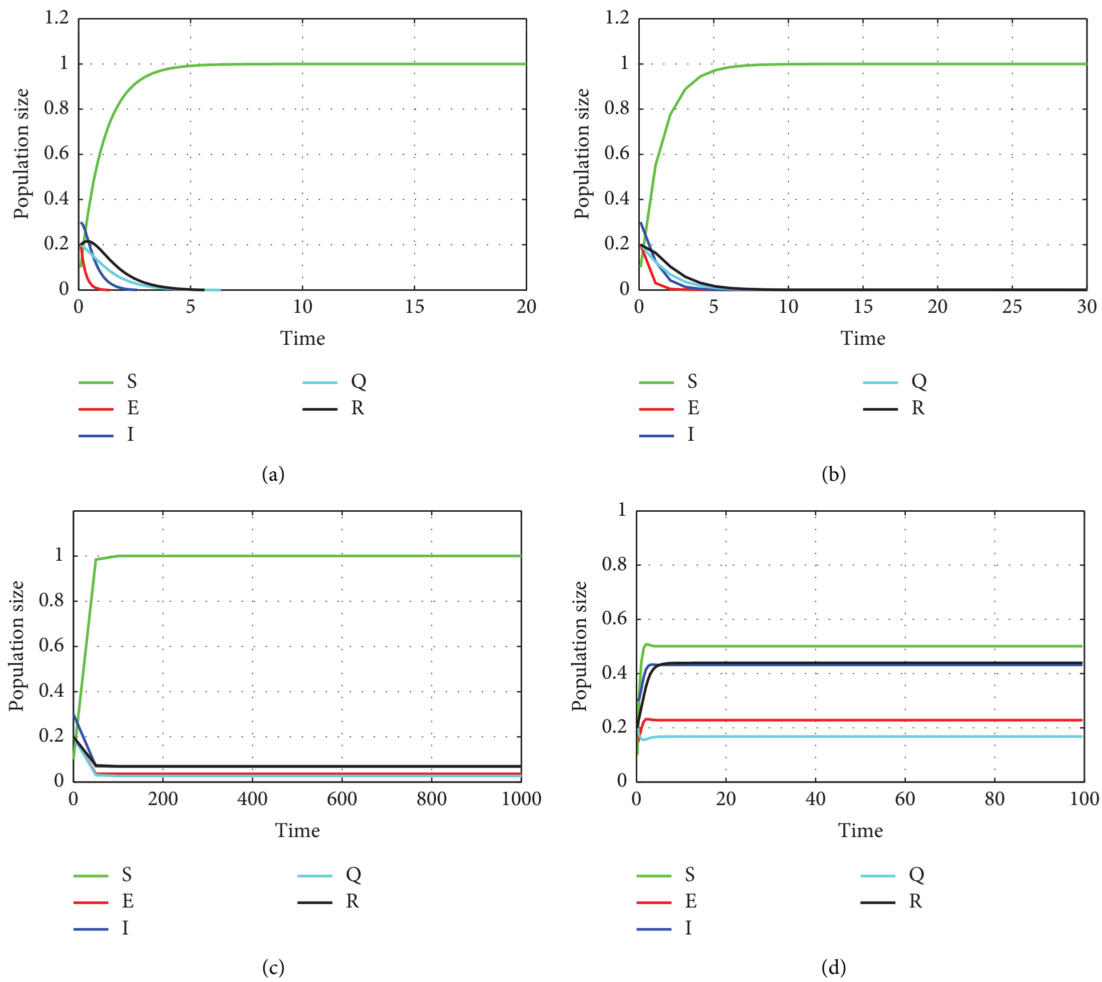

Figure 2: Numerical simulation for model (1) by using mixed Euler scheme with (a) $h = 0.1$, (b) $h = 1$, (c). $h = 0.1$, and (d) $h = 0.5$. (a–c) Stable DFE point with $\gamma = 1.01$, $\delta_c = 0.814715\text{-}01$. (d) Stable DEE point $\gamma = 0.07, \delta_c = 2.814715$. Other parameters remain fixed as $\beta = 1, \theta = 1$, $\eta = 0.05$, $\Sigma = 0.104874e - 01$, $\Omega = 10.270934,$, $\varphi = 0.584931e - 03$, $N = 1, \varepsilon = 0.999373e - 01$, $\alpha = 4.7$, $\psi = 4.0786530e - 01$.

negative integer and h symbolizes the time step size which should also be non-negative [25]. Model (1) then allows us to write the following equation:

$$
\begin{aligned}
\frac{S_{n+1} - S_n}{h} &= \beta - \frac{\delta_c (1 - \mathscr{E})(1 - \gamma)(E_n + I_n) S_{n+1}}{N} - \theta S_{n+1} + \Sigma R_n, \\
\frac{E_{n+1} - E_n}{h} &= \frac{\delta_c (1 - \mathscr{E})(1 - \gamma)(E_n + I_n) S_{n+1}}{N} - (\theta + \alpha) E_{n+1}, \\
\frac{I_{n+1} - I_n}{h} &= \alpha E_{n+1} - (\theta + \eta + \psi + \Omega) I_{n+1}, \\
\frac{Q_{n+1} - Q_n}{h} &= \psi I_{n+1} - (\theta + \eta + \varphi) Q_{n+1}, \\
\frac{R_{n+1} - R_n}{h} &= \varphi Q_{n+1} + \Omega I_{n+1} - (\Sigma + \theta) R_{n+1}.
\end{aligned}
\tag{13}
$$

5.2. Positivity and Boundedness of the NSFD Scheme. We assume that the initial values of discrete SEIQR scheme (13) are non-negative, i.e., $S_0 \geq 0, E_0 \geq 0, I_0 \geq 0, Q_0 \geq 0$, and $R_0 \geq 0$. These variables have estimated quantities which are also non-negative due to the assumptions: $S_n \geq 0, E_n \geq 0, I_n \geq 0, Q_n \geq 0, R_n \geq 0$. Therefore, solutions of the NSFD scheme (13) imply the positivity of scheme (13), i.e., $S_{n+1} \geq 0, E_{n+1} \geq 0, I_{n+1} \geq 0, Q_{n+1} \geq 0, R_{n+1} \geq 0$.

In order to discuss the boundedness of the solutions of the discrete system (13), we consider $P_n = S_n + E_n + I_n + Q_n + R_n$. Then,

$$\frac{P_{n+1} - P_n}{h} = \beta - \theta P_{n+1}, \tag{14}$$

i.e.,

$$(1 + h\theta)P_{n+1} = h\beta + P_n. \tag{15}$$

Therefore, we get

$$P_{n+1} \leq \frac{h\beta}{(1 + h\theta)} + \frac{P_n}{(1 + h\theta)} \Longleftrightarrow h\beta \sum_{j+1}^{n} \left(\frac{1}{(1 + h\theta)}\right)^j + P_0\left(\frac{1}{1 + h\theta}\right)^n. \tag{16}$$

By using Gronwall's inequality [26–28], if $0 < P(0) < \beta/\theta$, then

$$P_n \leq \frac{\beta}{\theta}\left(1 - \frac{1}{(1 + h\theta)^n}\right) + P_0\left(\frac{1}{1 + h\theta}\right)^n = \frac{\beta}{\theta} + \left(P_0 - \frac{\beta}{\theta}\right)\left(\frac{1}{1 + h\theta}\right)^n. \tag{17}$$

Since $(1/1 + h\theta)^n < 1$, so we obtain $P_n \longrightarrow (\beta/\theta)$ as $n \longrightarrow \infty$. This shows that the solutions of system (13) are bounded and the feasible region becomes

$$B = \left\{(S_n + E_n + I_n + Q_n + R_n)\colon 0 \leq S_n + E_n + I_n + Q_n + R_n \leq \frac{\beta}{\theta}\right\}. \tag{18}$$

From (13), one can obtain the explicit discrete form of the NSFD scheme (13) as follows:

$$\begin{aligned}
S_{n+1} &= \frac{N\left(h\left(\beta + \Sigma R_n\right) + S_n\right)}{\left(N + h\left(\delta_c\left(1 - \mathscr{E}\right)\left(1 - \gamma\right)\left(E_n + I_n\right) + N\theta\right)\right)},\\
E_{n+1} &= \frac{NE_n + h\left(\delta_c\left(1 - \mathscr{E}\right)\left(1 - \gamma\right)\left(E_n + I_n\right)S_{n+1}\right)}{N\left(1 + h\left(\theta + \alpha\right)\right)},\\
I_{n+1} &= \frac{I_n + h\left(\alpha E_{n+1}\right)}{1 + h\left(\theta + \eta + \psi + \Omega\right)},\\
Q_{n+1} &= \frac{h\left(\psi I_{n+1}\right) + Q_n}{1 + h\left(\theta + \eta + \varphi\right)},\\
R_{n+1} &= \frac{R_n + h\left(\varphi Q_{n+1} + \Omega I_{n+1}\right)}{1 + h\left(\Sigma + \theta\right)}.
\end{aligned} \tag{19}$$

In the following, we now discuss the conditions under which the DFE and DEE points of the discrete NSFD scheme (19) are stable or unstable. To achieve this goal, we first verify the local stability of DFE point, and we consider

$$\begin{aligned}
S_{n+1} &= \frac{N\left(h\left(\beta + \Sigma R_n\right) + S_n\right)}{\left(N + h\left(\delta_c\left(1 - \mathscr{E}\right)\left(1 - \gamma\right)\left(E_n + I_n\right) + N\theta\right)\right)} = f_1,\\
E_{n+1} &= \frac{NE_n + h\left(\delta_c\left(1 - \mathscr{E}\right)\left(1 - \gamma\right)\left(E_n + I_n\right)S_{n+1}\right)}{N\left(1 + h\left(\theta + \alpha\right)\right)} = f_2,\\
I_{n+1} &= \frac{I_n + h\left(\alpha E_{n+1}\right)}{1 + h\left(\theta + \eta + \psi + \Omega\right)} = f_3,\\
Q_{n+1} &= \frac{h\left(\psi I_{n+1}\right) + Q_n}{1 + h\left(\theta + \eta + \varphi\right)} = f_4,\\
R_{n+1} &= \frac{R_n + h\left(\varphi Q_{n+1} + \Omega I_{n+1}\right)}{1 + h\left(\Sigma + \theta\right)} = f_5.
\end{aligned} \tag{20}$$

5.3. Local Stability of Equilibria for NSFD Scheme. We will apply the Schur–Cohn criterion [29, 30] given in the following lemma to demonstrate the local stability of the DFE point.

Lemma 1. *The roots of* $T^2 - T\lambda + D = 0$ *assure* $|\lambda_i| < 1$ for $i = 1, 2$, *if and only if the following requirements are fulfilled:*

(1) $D < 1$,

(2) $1 + T + D > 0$,

(3) $1 - T + D > 0$,

where T denotes trace and D indicates determinant of the Jacobian matrix.

Theorem 2. *If* $R_0 < 1$, *then DFE point for the NSFD model (19) is locally asymptotically stable for all* $h > 0$.

Proof. based on above informations, the Jacobian matrix can be written as

$$J(S,E,I,Q,R) = \begin{bmatrix} \frac{\partial f_1}{\partial S} & \frac{\partial f_1}{\partial E} & \frac{\partial f_1}{\partial I} & \frac{\partial f_1}{\partial Q} & \frac{\partial f_1}{\partial R} \\ \frac{\partial f_2}{\partial S} & \frac{\partial f_2}{\partial E} & \frac{\partial f_2}{\partial I} & \frac{\partial f_2}{\partial Q} & \frac{\partial f_2}{\partial R} \\ \frac{\partial f_3}{\partial S} & \frac{\partial f_3}{\partial E} & \frac{\partial f_3}{\partial I} & \frac{\partial f_3}{\partial Q} & \frac{\partial f_3}{\partial R} \\ \frac{\partial f_4}{\partial S} & \frac{\partial f_4}{\partial E} & \frac{\partial f_4}{\partial I} & \frac{\partial f_4}{\partial Q} & \frac{\partial f_4}{\partial R} \\ \frac{\partial f_5}{\partial S} & \frac{\partial f_5}{\partial E} & \frac{\partial f_5}{\partial I} & \frac{\partial f_5}{\partial Q} & \frac{\partial f_5}{\partial R} \end{bmatrix}, \quad (21)$$

where f_1, f_2, f_3, f_4, and f_5 are provided in (20). We first find out all the derivatives employed in (21) as follows:

$$\begin{aligned}
&\frac{\partial f_1}{\partial S} = \frac{N}{(N + h(\delta_c(1-\mathscr{E})(1-\gamma)(E+I) + N\theta))}, \frac{\partial f_1}{\partial E} = \frac{-N(h(\beta + \Sigma R_n) + S)h\delta_c(1-\mathscr{E})(1-\gamma)}{(N + h(\delta_c(1-\mathscr{E})(1-\gamma)(E_n + I_n) + N\theta))^2}, \\
&\frac{\partial f_1}{\partial I} = \frac{-N(h(\beta + \Sigma R_n) + S)h\delta_c(1-\mathscr{E})(1-\gamma)}{(N + h(\delta_c(1-\mathscr{E})(1-\gamma)(E_n + I_n) + N\theta))^2}, \frac{\partial f_1}{\partial Q} = 0, \frac{\partial f_1}{\partial R} = \frac{hN\Sigma}{(N + h(\delta_c(1-\mathscr{E})(1-\gamma)(E+I) + N\theta))}, \\
&\frac{\partial f_2}{\partial S} = \frac{h\delta_c(1-\mathscr{E})(1-\gamma)(E+I)}{N(1 + h(\theta+\alpha))}, \frac{\partial f_2}{\partial E} = \frac{N + h(\delta_c(1-\mathscr{E})(1-\gamma)S_{n+1})}{N(1 + h(\theta+\alpha))}, \frac{\partial f_2}{\partial I} = \frac{h(\delta_c(1-\mathscr{E})(1-\gamma)S_{n+1})}{N(1 + h(\theta+\alpha))}, \\
&\frac{\partial f_2}{\partial Q} = 0, \frac{\partial f_2}{\partial R} = 0, \frac{\partial f_3}{\partial S} = 0, \frac{\partial f_3}{\partial E} = \frac{h\alpha}{1 + h(\theta+\eta+\psi+\Omega)}, \frac{\partial f_3}{\partial I} = \frac{1}{1 + h(\theta+\eta+\psi+\Omega)}, \frac{\partial f_3}{\partial Q} = 0, \frac{\partial f_3}{\partial R} = 0, \frac{\partial f_4}{\partial S} = 0, \frac{\partial f_4}{\partial E} = 0, \\
&\frac{\partial f_4}{\partial I} = \frac{h\psi}{1 + h(\theta+\eta+\varphi)}, \frac{\partial f_4}{\partial Q} = \frac{1}{1 + h(\theta+\eta+\varphi)}, \frac{\partial f_4}{\partial R} = 0, \frac{\partial f_5}{\partial S} = 0, \frac{\partial f_5}{\partial E} = 0, \\
&\frac{\partial f_5}{\partial I} = \frac{h\Omega}{1 + h(\Sigma+\theta)}, \frac{\partial f_5}{\partial Q} = \frac{h\varphi}{1 + h(\Sigma+\theta)}, \frac{\partial f_5}{\partial R} = \frac{1}{1 + h(\Sigma+\theta)}.
\end{aligned} \quad (22)$$

Putting all the above derivatives in (21), we obtain

$$J=\begin{bmatrix} \frac{N}{(N+h(\delta_c(1-\mathscr{E})(1-\gamma)(E+I)+N\theta))} & \frac{-N(h(\beta+\Sigma R_n)+S)h\delta_c(1-\mathscr{E})(1-\gamma)}{(N+h(\delta_c(1-\mathscr{E})(1-\gamma)(E_n+I_n)+N\theta))^2} & \frac{-N(h(\beta+\Sigma R_n)+S)h\delta_c(1-\mathscr{E})(1-\gamma)}{(N+h(\delta_c(1-\mathscr{E})(1-\gamma)(E_n+I_n)+N\theta))^2} & 0 & \frac{hN(\Sigma)}{(N+h(\delta_c(1-\mathscr{E})(1-\gamma)(E+I)+N\theta))} \\ \frac{h(\delta_c(1-\mathscr{E})(1-\gamma)(E+I))}{N(1+h(\theta+\alpha))} & \frac{N+h(\delta_c(1-\mathscr{E})(1-\gamma)S)}{N(1+h(\theta+\alpha))} & \frac{h(\delta_c(1-\mathscr{E})(1-\gamma)S)}{N(1+h(\theta+\alpha))} & 0 & 0 \\ 0 & \frac{h\alpha}{1+h(\theta+\eta+\psi+\Omega)} & \frac{1}{1+h(\theta+\eta+\psi+\Omega)} & 0 & 0 \\ 0 & 0 & \frac{h\psi}{1+h(\theta+\eta+\varphi)} & \frac{1}{1+h(\theta+\eta+\varphi)} & 0 \\ 0 & 0 & \frac{h\Omega}{1+h(\Sigma+\theta)} & \frac{h\varphi}{1+h(\Sigma+\theta)} & \frac{1}{1+h(\Sigma+\theta)} \end{bmatrix}. \tag{23}$$

At DFE point $E_0=((\beta/\theta),0,0,0,0)$, the matrix (23) becomes

$$J(E_0)=\begin{bmatrix} \frac{1}{1+h(\theta)} & \frac{-((Nh\beta+(\beta/\theta))h\delta_c(1-\mathscr{E})(1-\gamma))}{(N+N\theta)^2} & \frac{-((Nh\beta+(\beta/\theta))h\delta_c(1-\mathscr{E})(1-\gamma))}{(N+N\theta)^2} & 0 & \frac{h(\Sigma)}{1+(\theta)} \\ 0 & \frac{N+h(\delta_c(1-\mathscr{E})(1-\gamma))(\beta/\theta)}{N(1+h(\theta+\alpha))} & \frac{h(\delta_c(1-\mathscr{E})(1-\gamma))(\beta/\theta)}{N(1+h(\theta+\alpha))} & 0 & 0 \\ 0 & \frac{h\alpha}{1+h(\theta+\eta+\psi+\Omega)} & \frac{1}{1+h(\theta+\eta+\psi+\Omega)} & 0 & 0 \\ 0 & 0 & \frac{h\psi}{1+h(\theta+\eta+\varphi)} & \frac{1}{1+h(\theta+\eta+\varphi)} & 0 \\ 0 & 0 & \frac{h\Omega}{1+h(\Sigma+\theta)} & \frac{h\varphi}{1+h(\Sigma+\theta)} & \frac{1}{1+h(\Sigma+\theta)} \end{bmatrix}. \tag{24}$$

To discuss the eigenvalues, we suppose

$$|J(E_0)-\lambda I|=0, \tag{25}$$

i.e.,

$$\begin{vmatrix} \frac{1}{1+h(\theta)}-\lambda & \frac{-((Nh\beta+(\beta/\theta))h\delta_c(1-\mathscr{E})(1-\gamma))}{(N+N\theta)^2} & \frac{-((Nh\beta+(\beta/\theta))h\delta_c(1-\mathscr{E})(1-\gamma))}{(N+N\theta)^2} & 0 & \frac{h(\Sigma)}{1+h(\theta)} \\ 0 & \frac{N+(\delta_c(1-\mathscr{E})(1-\gamma))(\beta/\theta)}{N(1+h(\theta+\alpha))}-\lambda & \frac{h(\delta_c(1-\mathscr{E})(1-\gamma))(\beta/\theta)}{N(1+h(\theta+\alpha))} & 0 & 0 \\ 0 & \frac{h\alpha}{1+h(\theta+\eta+\psi+\Omega)} & \frac{1}{1+h(\theta+\eta+\psi+\Omega)}-\lambda & 0 & 0 \\ 0 & 0 & \frac{h\psi}{1+h(\theta+\eta+\varphi)} & \frac{1}{1+h(\theta+\eta+\varphi)}-\lambda & 0 \\ 0 & 0 & \frac{h\Omega}{1+h(\Sigma+\theta)} & \frac{h\varphi}{1+h(\Sigma+\theta)} & \frac{1}{1+h(\Sigma+\theta)}-\lambda \end{vmatrix}=0. \tag{26}$$

After simple calculations, from (26), we obtain

$$\left(\frac{1}{1+h(\theta)}-\lambda\right)\left(\frac{1}{1+h(\Sigma+\theta)}-\lambda\right)\left(\frac{1}{1+h(\theta+\eta+\varphi)}-\lambda\right)\begin{vmatrix}\frac{N+h\left(\delta_c(1-\mathscr{E})(1-\gamma)\right)(\beta/\theta)}{N(1+h(\theta+\alpha))}-\lambda & \frac{h\left(\delta_c(1-\mathscr{E})(1-\gamma)\right)(\beta/\theta)}{N(1+h(\theta+\alpha))} \\ \frac{h\alpha}{1+h(\theta+\eta+\psi+\Omega)} & \frac{1}{1+h(\theta+\eta+\psi+\Omega)}-\lambda\end{vmatrix}=0. \tag{27}$$

Equation (27) gives $\lambda_1 = (1/1+h(\theta))<1$, $\lambda_2 = (1/1+h(\Sigma+\theta))<1, \lambda_3 = (1/1+h(\theta+\eta+\varphi))<1$. To find the remaining two eigenvalues, we take

$$\begin{vmatrix}\frac{N+h\left(\delta_c(1-\mathscr{E})(1-\gamma)\right)(\beta/\theta)}{N(1+h(\theta+\alpha))}-\lambda & \frac{h\left(\delta_c(1-\mathscr{E})(1-\gamma)\right)(\beta/\theta)}{N(1+h(\theta+\alpha))} \\ \frac{h\alpha}{1+h(\theta+\eta+\psi+\Omega)} & \frac{1}{1+h(\theta+\eta+\psi+\Omega)}-\lambda\end{vmatrix}=0. \tag{28}$$

i.e.,

$$\lambda^2-\lambda\left(\left(\frac{N+h\left(\delta_c(1-\mathscr{E})(1-\gamma)\right)(\beta/\theta)}{N(1+h(\theta+\alpha))}\right)+\left(\frac{1}{1+h(\theta+\eta+\psi+\Omega)}\right)\right)$$
$$+\left(\left(\frac{N+h\left(\delta_c(1-\mathscr{E})(1-\gamma)\right)(\beta/\theta)}{N(1+h(\theta+\alpha))}\right)\left(\frac{1}{1+h(\theta+\eta+\psi+\Omega)}\right)-\left(\frac{h\alpha}{1+h(\theta+\eta+\psi+\Omega)}\right)\left(\frac{h\left(\delta_c(1-\mathscr{E})(1-\gamma)\right)(\beta/\theta)}{N(1+h(\theta+\alpha))}\right)\right)=0. \tag{29}$$

Comparing (29) with $\lambda^2-T\lambda+D=0$, we get $T = N+h\,\delta_c(1-\mathscr{E})(1-\gamma)(\beta/\theta)/N(1+h(\theta+\alpha)) + (1/1+h(\theta+\eta+\psi+\Omega))$ and $D = (N+h(\delta_c(1-\mathscr{E})(1-\gamma)(\beta/\theta)))/(N((1+h(\theta+\alpha))(1+h(\theta+\eta+\psi+\Omega)))) - h\delta_c(1-\mathscr{E})(1-\gamma)\beta/(N(\theta(1+h(\theta+\alpha))(1+h(\theta+\eta+\psi+\Omega))))$. If $R_0<1$, i.e., $\delta_c(1-\mathscr{E})(1-\gamma)(\theta+\eta+\psi+\Omega+\alpha)\beta<\theta(\theta+\alpha)(\theta+\eta+\psi+\Omega)$ then.

(1) $D = (N+h(\delta_c(1-\mathscr{E})(1-\gamma))(\beta/\theta))/(N((1+h(\theta+\alpha))(1+h(\theta+\eta+\psi+\Omega)))) - (h\delta_c(1-\mathscr{E})(1-\gamma)\beta)/(N(\theta(1+h(\theta+\alpha))(1+h(\theta+\eta+\psi+\Omega))))<1$.

(2) $1+T+D = 1+N+h(\delta_c(1-\mathscr{E})(1-\gamma)(\beta/\theta))/(N(1+h(\theta+\alpha))) + (1/1+h(\theta+\eta+\psi+\Omega)) + (N+h(\delta_c(1-\mathscr{E})(1-\gamma)(\beta/\theta))/N((1+h(\theta+\alpha))(1+h(\theta+\eta+\psi+\Omega))))- (h(\delta_c(1-\mathscr{E})(1-\gamma)\beta))/(N(\theta(1+h(\theta+\alpha))(1+h(\theta+\eta+\psi+\Omega))))>0$.

(3) $1-T+D = 1-N+h(\delta_c(1-\mathscr{E})(1-\gamma)(\beta/\theta))/(N(1+h(\theta+\alpha))) - (1/1+h(\theta+\eta+\psi+\Omega)) + (N+h(\delta_c(1-\mathscr{E})(1-\gamma)(\beta/\theta))/(N((1+h(\theta+\alpha))(1+h(\theta+\eta+\psi+\Omega)))))- (h(\delta_c(1-\mathscr{E})(1-\gamma))\beta)/(N(\theta(1+h(\theta+\alpha))(1+h(\theta+\eta+\psi+\Omega))))>0$.

As a result, whenever $R_0<1$, then all the requirements of Schur-Cohn criterion mentioned in Lemma 1 are satisfied. Therefore, provided that $R_0<1$, the DFE point E_0 of the discrete NSFD scheme (19) is locally asymptotically stable.

In order to discuss the local stability of DEE point, we replace R_n by $(\beta/\theta) - S_n - E_n - I_n - Q_n$ in the first equation of system (13). Then, obviously the system (13) reduced to following four dimensional model.

$$
\begin{aligned}
\frac{S_{n+1} - S_n}{h} &= \beta - \frac{\delta_c (1 - \mathscr{E})(1 - \gamma)(E_n + I_n) S_{n+1}}{N} - \theta S_{n+1} + \Sigma\left(\frac{\beta}{\theta} - S_{n+1} - E_n - I_n - Q_n\right), \\
\frac{E_{n+1} - E_n}{h} &= \frac{\delta_c (1 - \mathscr{E})(1 - \gamma)(E_n + I_n) S_{n+1}}{N} - (\theta + \alpha) E_{n+1}, \\
\frac{I_{n+1} - I_n}{h} &= \alpha E_{n+1} - (\theta + \eta + \psi + \Omega) I_{n+1}, \\
\frac{Q_{n+1} - Q_n}{h} &= \psi I_{n+1} - (\theta + \eta + \varphi) Q_{n+1}.
\end{aligned}
\tag{30}
$$

Therefore, the stability of DEE point for system (30) implies the stability of DEE point for system (19). From (30), we obtain

$$
\begin{aligned}
S_{n+1} &= \frac{N\left(S_n + h\left(\beta + \Sigma(\beta/\theta) - \Sigma E_n - \Sigma I_n - \Sigma Q_n\right)\right)}{\left(N + h\left(\delta_c (1 - \mathscr{E})(1 - \gamma)(E_n + I_n)\right) + N\theta + N\Sigma\right)} = g_1, \\
E_{n+1} &= \frac{\left(N E_n + h\left(\delta_c (1 - \mathscr{E})(1 - \gamma)(E_n + I_n) S_{n+1}\right)\right)}{N(1 + h(\theta + \alpha))} = g_2, \\
I_{n+1} &= \frac{I_n + h(\alpha E_{n+1})}{1 + h(\theta + \eta + \psi + \Omega)} = g_3, \\
Q_{n+1} &= \frac{h\psi I_{n+1} + Q_n}{1 + h(\theta + \eta + \varphi)} = g_4.
\end{aligned}
\tag{31}
$$

□

Theorem 3. *If $R_0 > 1$, then DEE point for NSFD model (30) is locally asymptotically stable for all $h > 0$.*

Proof. The Jacobian matrix becomes

$$
J(S, E, I, Q) = \begin{vmatrix}
\frac{\partial g_1}{\partial S} & \frac{\partial g_1}{\partial E} & \frac{\partial g_1}{\partial I} & \frac{\partial g_1}{\partial Q} \\
\frac{\partial g_2}{\partial S} & \frac{\partial g_2}{\partial E} & \frac{\partial g_2}{\partial I} & \frac{\partial g_2}{\partial Q} \\
\frac{\partial g_3}{\partial S} & \frac{\partial g_3}{\partial E} & \frac{\partial g_3}{\partial I} & \frac{\partial g_3}{\partial Q} \\
\frac{\partial g_4}{\partial S} & \frac{\partial g_4}{\partial E} & \frac{\partial g_4}{\partial I} & \frac{\partial g_4}{\partial Q}
\end{vmatrix},
\tag{32}
$$

where g_1, g_2, g_3, and g_4 are provided in (31). We first find out all the derivatives employed in (32) as follows:

$$\frac{\partial g_1}{\partial S} = \frac{N}{\left(N + h\left(\delta_c\left(1 - \mathscr{E}\right)\left(1 - \gamma\right)\left(E_n + I_n\right)\right) + N\theta + N\Sigma\right)},$$
$$\frac{\partial g_1}{\partial E} = \frac{-\left(\left(N + h\left(\delta_c\left(1 - \mathscr{E}\right)\left(1 - \gamma\right)\left(E_n + I_n\right)\right) + N\theta + N\Sigma\right)\right)\right)Nh\Sigma - N\left(S_n + h\left(\beta + \Sigma\left(\beta/\theta\right) - \Sigma E_n - \Sigma I_n - \Sigma Q_n\right)\right)h\delta_c\left(1 - \mathscr{E}\right)\left(1 - \gamma\right)}{\left(\left(N + h\left(\delta_c\left(1 - \mathscr{E}\right)\left(1 - \gamma\right)\left(E_n + I_n\right)\right) + N\theta + N\Sigma\right)\right)\right)^2},$$
$$\frac{\partial g_1}{\partial I} = \frac{-\left(\left(N + h\left(\delta_c\left(1 - \mathscr{E}\right)\left(1 - \gamma\right)\left(E_n + I_n\right) + N\right)\theta + N\Sigma\right)\right)\right)Nh\Sigma - N\left(S_n + h\left(\beta + \Sigma\left(\beta/\theta\right) - \Sigma E_n - \Sigma I_n - \Sigma Q_n\right)\right)h\delta_c\left(1 - \mathscr{E}\right)\left(1 - \gamma\right)}{\left(\left(N + h\left(\delta_c\left(1 - \mathscr{E}\right)\left(1 - \gamma\right)\left(E_n + I_n\right)\right) + N\theta + N\Sigma\right)\right)\right)^2},$$
$$\frac{\partial g_1}{\partial Q} = \frac{-hN\Sigma}{\left(\left(N + h\left(\delta_c\left(1 - \mathscr{E}\right)\left(1 - \gamma\right)\left(E_n + I_n\right)\right) + N\theta + N\Sigma\right)\right)\right)}, \frac{\partial g_2}{\partial S} = \frac{\left(h\left(\delta_c\left(1 - \mathscr{E}\right)\left(1 - \gamma\right)\left(E + I\right)\right)\right.}{N\left(1 + h\left(\theta + \alpha\right)\right)},$$
$$\frac{\partial g_2}{\partial E} = \frac{\left(N + h\left(\delta_c\left(1 - \mathscr{E}\right)\left(1 - \gamma\right)S_{n+1}\right)\right.}{N\left(1 + h\left(\theta + \alpha\right)\right)}, \frac{\partial g_2}{\partial I} = \frac{\left(h\left(\delta_c\left(1 - \mathscr{E}\right)\left(1 - \gamma\right)S_{n+1}\right)\right.}{N\left(1 + h\left(\theta + \alpha\right)\right)}, \frac{\partial g_2}{\partial Q} = 0, \frac{\partial g_3}{\partial S} = 0, \frac{\partial g_3}{\partial E} = \frac{h\alpha}{1 + h\left(\theta + \eta + \psi + \Omega\right)},$$
$$\frac{\partial g_3}{\partial I} = \frac{1}{1 + h\left(\theta + \eta + \psi + \Omega\right)}, \frac{\partial g_3}{\partial Q} = 0, \frac{\partial g_4}{\partial S} = 0, \frac{\partial g_4}{\partial E} = 0, \frac{\partial g_4}{\partial I} = \frac{h\psi}{1 + h\left(\theta + \eta + \varphi\right)}, \frac{\partial g_4}{\partial Q} = \frac{1}{1 + h\left(\theta + \eta + \varphi\right)}. \tag{33}$$

By replacing DEE point E^* in above derivatives, we get

$$\frac{\partial g_1}{\partial S} = \frac{N}{\left(N + h\left(\delta_c\left(1 - \mathscr{E}\right)\left(1 - \gamma\right)\left(E_n^* + I_n^*\right)\right) + N\theta + N\Sigma\right)},$$
$$\frac{\partial g_1}{\partial E} = \frac{-\left(\left(N + h\left(\delta_c\left(1 - \mathscr{E}\right)\left(1 - \gamma\right)\left(E_n^* + I_n^*\right)\right) + N\theta + N\Sigma\right)\right)\right)Nh\Sigma - N\left(S_n^* + h\left(\beta + \Sigma\left(\beta/\theta\right) - \Sigma E_n^* - \Sigma I_n^* - \Sigma Q_n^*\right)\right)h\delta_c\left(1 - \mathscr{E}\right)\left(1 - \gamma\right)}{\left(\left(N + h\left(\delta_c\left(1 - \mathscr{E}\right)\left(1 - \gamma\right)\left(E_n^* + I_n^*\right)\right) + N\theta + N\Sigma\right)\right)\right)^2},$$
$$\frac{\partial g_1}{\partial I} = \frac{-\left(\left(N + h\left(\delta_c\left(1 - \mathscr{E}\right)\left(1 - \gamma\right)\left(E_n^* + I_n^*\right)\right) + N\theta + N\Sigma\right)\right)\right)Nh\Sigma - N\left(S_n^* + h\left(\beta + \Sigma\left(\beta/\theta\right) - \Sigma E_n^* - \Sigma I_n^* - \Sigma Q_n^*\right)\right)h\delta_c\left(1 - \mathscr{E}\right)\left(1 - \gamma\right)}{\left(\left(N + h\left(\delta_c\left(1 - \mathscr{E}\right)\left(1 - \gamma\right)\left(E_n^* + I_n^*\right)\right) + N\theta + N\Sigma\right)\right)\right)^2},$$
$$\frac{\partial g_1}{\partial Q} = \frac{-hN\Sigma}{\left(\left(N + h\left(\delta_c\left(1 - \mathscr{E}\right)\left(1 - \gamma\right)\left(E_n^* + I_n^*\right)\right) + N\theta + N\Sigma\right)\right)\right)^2}, \frac{\partial g_2}{\partial S} = \frac{\left(h\left(\delta_c\left(1 - \mathscr{E}\right)\left(1 - \gamma\right)\left(E^* + I^*\right)\right)\right.}{N\left(1 + h\left(\theta + \alpha\right)\right)},$$
$$\frac{\partial g_2}{\partial E} = \frac{N + h\left(\delta_c\left(1 - \mathscr{E}\right)\left(1 - \gamma\right)S^*\right)}{N\left(1 + h\left(\theta + \alpha\right)\right)}, \frac{\partial g_2}{\partial I} = \frac{\left(h\left(\delta_c\left(1 - \mathscr{E}\right)\left(1 - \gamma\right)S\right)\right.}{N\left(1 + h\left(\theta + \alpha\right)\right)}, \frac{\partial g_2}{\partial Q} = 0, \frac{\partial g_3}{\partial S} = 0,$$
$$\frac{\partial g_3}{\partial E} = \frac{h\alpha}{1 + h\left(\theta + \eta + \psi + \Omega\right)}, \frac{\partial g_3}{\partial I} = \frac{1}{1 + h\left(\theta + \eta + \psi + \Omega\right)}, \frac{\partial g_3}{\partial Q} = 0,$$
$$\frac{\partial g_4}{\partial S} = 0, \frac{\partial g_4}{\partial E} = 0, \frac{\partial g_4}{\partial I} = \frac{h\psi}{1 + h\left(\theta + \eta + \varphi\right)}, \frac{\partial g_4}{\partial Q} = \frac{1}{1 + h\left(\theta + \eta + \varphi\right)}. \tag{34}$$

Let

$$
\begin{aligned}
m_1 &= \frac{N}{\left(N + h\left(\delta_c(1-\mathscr{E})(1-\gamma)\left(E_n^* + I_n^*\right)\right) + N\theta + N\Sigma\right)},\\
m_2 &= \frac{-\left(\left(N + h\left(\delta_c(1-\mathscr{E})(1-\gamma)\left(E_n^* + I_n^*\right)\right) + N\theta + N\Sigma\right)\right)Nh\Sigma - N\left(S_n^* + h\left(\beta + \Sigma(\beta/\theta) - \Sigma E_n^* - \Sigma I_n^* - \Sigma Q_n^*\right)\right)h\delta_c(1-\mathscr{E})(1-\gamma)}{\left(\left(N + h\left(\delta_c(1-\mathscr{E})(1-\gamma)\left(E_n^* + I_n^*\right)\right) + N\theta + N\Sigma\right)\right)^2},\\
m_3 &= \frac{-\left(\left(N + h\left(\delta_c(1-\mathscr{E})(1-\gamma)\left(E_n^* + I_n^*\right) + \right)N\theta + N\Sigma\right)\right)Nh\Sigma - N\left(S_n^* + h\left(\beta + \Sigma(\beta/\theta) - \Sigma E_n^* - \Sigma I_n^* - \Sigma Q_n^*\right)\right)h\delta_c(1-\mathscr{E})(1-\gamma)}{\left(\left(N + h\left(\delta_c(1-\mathscr{E})(1-\gamma)\left(E_n^* + I_n^*\right)\right) + N\theta + N\Sigma\right)\right)^2},\\
m_4 &= \frac{-hN^2\left(\beta + \Sigma(\beta/\theta) - E_n^* - I_n^*\right)}{\left(\left(N + h\left(\delta_c(1-\mathscr{E})(1-\gamma)\left(E_n^* + I_n^*\right)\right) + N\theta + N\Sigma\right)\right)^2}, m_5 = \frac{\left(h\left(\delta_c(1-\mathscr{E})(1-\gamma)\left(E^* + I^*\right)\right)\right.}{N(1 + h(\theta + \alpha))},\\
m_6 &= \frac{N + h\left(\delta_c(1-\mathscr{E})(1-\gamma)S^*\right)}{N(1 + h(\theta + \alpha))}, m_7 = \frac{\left(h\left(\delta_c(1-\mathscr{E})(1-\gamma)S\right)\right.}{N(1 + h(\theta + \alpha))}, m_8 = \frac{h\alpha}{1 + h(\theta + \eta + \psi + \Omega)}, m_9 = \frac{1}{1 + h(\theta + \eta + \psi + \Omega)},\\
m_{10} &= \frac{h(\psi)}{1 + h(\theta + \eta + \varphi)}, m_{11} = \frac{1}{1 + h(\theta + \eta + \varphi).}
\end{aligned}
\tag{35}
$$

By replacing the above quantities in (32), we obtain

$$
J\left(E^*\right) = \begin{bmatrix} m_1 & m_2 & m_3 & m_4 \\ m_5 & m_6 & m_7 & 0 \\ 0 & m_8 & m_9 & 0 \\ 0 & 0 & m_{10} & m_{11} \end{bmatrix}. \tag{36}
$$

To discuss the eigenvalues, we take

$$
\left|J\left(E^*\right) - \lambda I\right| = 0, \tag{37}
$$

i.e.,

$$
\begin{vmatrix} m_1 - \lambda & m_2 & m_3 & m_4 \\ m_5 & m_6 - \lambda & m_7 & 0 \\ 0 & m_8 & m_9 - \lambda & 0 \\ 0 & 0 & m_{10} & m_{11} - \lambda \end{vmatrix} = 0. \tag{38}
$$

After simplifications, we get the following characteristic equation:

$$
\lambda^4 + U_1\lambda^3 + U_2\lambda^2 + U_3\lambda + U_4 = 0, \tag{39}
$$

where

$$
\begin{aligned}
U_1 &= m_1 + m_6 + m_9 + m_{11} > 0,\\
U_2 &= m_1m_6 + m_1m_9 + m_1m_{11} + m_2m_5 + m_6m_9 + m_6m_{11} + m_7m_8 + m_9m_{11} > 0,\\
U_3 &= m_{11}m_5m_2 + m_5m_2m_9 + m_1m_7m_8 + m_1m_9m_{11} + m_1m_6m_{11} + m_1m_9m_6 + m_{11}m_6m_9 + m_{11}m_7m_8 - m_5m_3m_8 > 0,\\
U_4 &= m_1m_9m_6m_{11} + m_1m_8m_7m_{11} + m_2m_5m_9m_{11} + m_4m_5m_8m_{10} - m_5m_3m_8m_{11} > 0.
\end{aligned}
\tag{40}
$$

It is clear that

$$\begin{aligned}
G_1 &= U_1 > 0,\\
G_2 &= U_1U_2 - U_3 = m_1^2m_6 + m_1^2m_9 + m_1m_{11}^2 + m_1m_2m_5 + m_1m_6m_9 + m_1m_6m_{11}\\
&\quad + m_1m_7m_8 + m_1m_9m_{11} + m_1m_6^2 + m_1m_6m_9 + m_1m_6m_{11} + m_2m_5m_6\\
&\quad + m_6^2m_9 + m_{11}m_6^2 + m_6m_7m_8 + m_6m_9m_{11} + m_1m_6m_9 + m_1m_9^2\\
&\quad + m_1m_9m_{11} + m_2m_5m_9 + m_6m_9^2 + m_6m_9m_{11} + m_7m_8m_9 + m_9^2m_{11}\\
&\quad + m_1m_6m_{11} + m_1m_9m_{11} + m_1m_{11}^2 + m_2m_5m_{11} + m_6m_9m_{11} + m_6m_{11}^2 + m_7m_8m_{11} + m_9m_{11}^2\\
&\quad + m_5m_3m_8 - m_{11}m_5m_2 - m_5m_2m_9 - m_1m_7m_8\\
&\quad - m_1m_9m_{11} - m_1m_6m_{11} - m_1m_9m_6 - m_{11}m_6m_9 - m_{11}m_7m_8 > 0,\\
G_3 &= \begin{vmatrix} U_1 & U_3 & 0,\\ 1 & U_2 & U_4,\\ 0 & U_1 & U_3 \end{vmatrix} = -U_3^{\,2} + U_1U_2U_3 - U_1^{\,2}U_4 = U_3G_2 - U_1^{\,2}U_4 > 0,\\
G_4 &= U_4G_3 > 0.
\end{aligned} \tag{41}$$

So, by employing Routh–Hurwitz criterion [31, 32], all the eigenvalues of (39) must have negative real parts whenever $R_0 > 1$. Consequently, the DEE point E^* of the discrete NSFD scheme (19) is locally asymptotically stable. □

5.4. Global Stability of Equilibria for the NSFD Scheme. We shall describe the global stability of equilibria by constructing the Lyapunov function in the same manner as developed by Elaiw and Alshaikh in [33]. To discuss the global stability, we define the function $H(x) \geq 0$ such that $H(x) = x - \ln x - 1$, and, therefore, $\ln x \leq x - 1$.

Theorem 4. *If $R_0 \leq 1$, then DFE point for the NSFD model (19) is globally asymptotically stable for all $h > 0$.*

Proof. Construct a discrete Lyapunov function

$$T_n(S_n, E_n, I_n, Q_n, R_n) = S^0H\left(\frac{S_n}{S^0}\right) + \phi_1E_n + \phi_2I_n + \phi_3Q_n + \phi_4(\Sigma + \theta)R_n, \tag{42}$$

where $\phi_i > 0$ for all $[ii] = 1, 2, 3, 4$ which can be chosen later. Hence, $T_n > 0$ for all $S_n > 0, E_n > 0, I_n > 0, Q_n > 0, R_n > 0$. In addition, $T_n = 0$ if and only if $S_n = S^0, E_n = E^0, I_n = I^0, Q_n = Q^0$ and $R_n = R^0$.

Let us consider

$$\Delta T_n = T_{n+1} - T_n, \tag{43}$$

i.e.,

$$\begin{aligned}
\Delta T_n &= S^0H\left(\frac{S_{n+1}}{S^0}\right) + \phi_1E_{n+1} + \phi_2I_{n+1} + \phi_3Q_{n+1} + \phi_4(\Sigma+\theta)R_{n+1} - \left(S^0H\left(\frac{S_n}{S^0}\right) + \phi_1E_n + \phi_2I_n + \phi_3Q_n + \phi_4(\Sigma+\theta)R_n\right)\\
&= S^0\left(\frac{S_{n+1}}{S^0} - \frac{S_n}{S^0} + \ln\frac{S_n}{S_{n+1}}\right) + \phi_1(E_{n+1} - E_n) + \phi_2(I_{n+1} - I_n) + \phi_3(Q_{n+1} - Q_n) + \phi_4(\Sigma+\theta)(R_{n+1} - R_n).
\end{aligned} \tag{44}$$

Using the inequality $\ln x \leq x - 1$, we obtain

$$\Delta T_n \leq S_{n+1} - S_n + S^0\left(\frac{S_n}{S_{n+1}} - 1\right) + \phi_1(E_{n+1} - E_n) + \phi_2(I_{n+1} - I_n) + \phi_3(Q_{n+1} - Q_n) + \phi_4(\Sigma+\theta)(R_{n+1} - R_n), \tag{45}$$

i.e.,

$$= \left(1 - \frac{S^0}{S_{n+1}}\right)(S_{n+1} - S_n) + \phi_1 (E_{n+1} - E_n) + \phi_2 (I_{n+1} - I_n) + \phi_3 (Q_{n+1} - Q_n) + \phi_4 (\Sigma + \theta)(R_{n+1} - R_n). \tag{46}$$

By employing (13), (46) becomes

$$\begin{aligned}
\Delta T_n \le & \left(1 - \frac{S^0}{S_{n+1}}\right)\left(\beta - \frac{\delta_c (1-\mathscr{E})(1-\gamma)(E_n + I_n)S_{n+1}}{N} - \theta S_{n+1} + \Sigma R_n\right) \\
& + \phi_1 \left(\frac{\delta_c (1-\mathscr{E})(1-\gamma)(E_{n+1} + I_n)S_{n+1}}{N} - (\theta+\alpha)E_{n+1}\right) + \phi_2 (\alpha E_{n+1} - (\theta+\eta+\psi)I_{n+1} - \Omega I_{n+1}) \\
& + \phi_3 (\psi I_{n+1} - (\theta+\eta+\varphi)Q_{n+1}) + \phi_4 (\varphi Q_{n+1} + \Omega I_{n+1} - (\Sigma+\theta)R_{n+1}) + \phi_4 (\Sigma+\theta)(R_{n+1} - R_n).
\end{aligned} \tag{47}$$

Let ϕ_i for $i = 1, 2, 3, 4$ be chosen such that

$$\begin{aligned}
\frac{\delta_c (1-\mathscr{E})(1-\gamma)(E_n + I_n)S_{n+1}}{N} & = \phi_1 \left(\frac{\delta_c (1-\mathscr{E})(1-\gamma)(E_{n+1} + I_n)S_{n+1}}{N}\right), \phi_1 (\theta+\alpha) \\
& = \phi_2 \alpha, \phi_2 (\theta+\eta+\psi) = \phi_3 \psi, \phi_2 \Omega = \phi_4 \Omega, \phi_3 (\theta+\eta+\varphi) = \phi_4 \varphi.
\end{aligned} \tag{48}$$

By replacing the values from (48) into (47), we get

$$\begin{aligned}
\Delta T_n \le & \left(1 - \frac{S^0}{S_{n+1}}\right)\left(\beta - \phi_1 \frac{\delta_c (1-\mathscr{E})(1-\gamma)(E_{n+1} + I_n)S_{n+1}}{N} - \theta S_{n+1} + \Sigma R_n\right) \\
& + \phi_1 \left(\frac{\delta_c (1-\mathscr{E})(1-\gamma)(E_{n+1} + I_n)S_{n+1}}{N} - \phi_2 \alpha E_{n+1}\right) + (\phi_2 \alpha E_{n+1} - \phi_3 \psi I_{n+1} - \phi_4 \Omega I_{n+1}) \\
& + \phi_3 (\psi I_{n+1} - \phi_4 \varphi Q_{n+1}) + \phi_4 (\varphi Q_{n+1} + \Omega I_{n+1} - (\Sigma+\theta)R_{n+1}) + \phi_4 (\Sigma+\theta)(R_{n+1} - R_n).
\end{aligned} \tag{49}$$

Simple calculations yields

$$\begin{aligned}
\Delta T_n \le & \left(1 - \frac{S^0}{S_{n+1}}\right)\beta - \phi_1 \left(\frac{\delta_c (1-\mathscr{E})(1-\gamma)(E_{n+1} + I_n)S_{n+1}}{N} - \theta S_{n+1} + \Sigma R_n\right) \\
& + \frac{\phi_1 \delta_c (1-\mathscr{E})(1-\gamma)(E_{n+1} + I_n)S_{n+1}}{N} - \phi_2 \alpha E_{n+1} + \phi_2 \alpha E_{n+1} - \phi_3 \psi I_{n+1} - \phi_4 \Omega I_{n+1} + \phi_3 \psi I_{n+1} \\
& - \phi_4 \varphi Q_{n+1} + \phi_4 \varphi Q_{n+1} + \phi_4 \Omega I_{n+1} - \phi_4 (\Sigma+\theta)R_{n+1} + \phi_4 (\Sigma+\theta)R_{n+1} - \phi_4 (\Sigma+\theta)R_n) \\
\le & \left(1 - \frac{S_0}{S_{n+1}}\right)(\beta - \theta S_{n+1}) - \Sigma(\phi_4 - 1)R_n - \phi_4 \theta R_n.
\end{aligned} \tag{50}$$

As we know from DFE point that $S^0 = (\beta/\theta)$ which implies $S^0\theta = \beta$. By replacing β in (50), we obtain

$$\begin{aligned}\Delta T_n &\leq \left(1 - \frac{S^0}{S_{n+1}}\right)\left(S^0\theta - \theta S_{n+1}\right) - \Sigma(\phi_4 - 1)R_n - \phi_4\theta R_n\\ &= \frac{-\theta}{S_{n+1}}\left(S_{n+1} - S^0\right)^2 - \Sigma(\phi_4 - 1)R_n - \phi_4\theta R_n.\end{aligned} \tag{51}$$

From the value of ϕ_4, it is clear that $\phi_4 \geq 1$ whenever $R_0 \leq 1$. Hence, if $R_0 \leq 1$ then from (51), we can write $\Delta T_n \leq 0$ for all $n \geq 0$. Consequently, T_n is a nonincreasing sequence. Therefore, there exists a constant T such that $\lim_{n\longrightarrow\infty} T_n = T$ which implies that $\lim_{n\longrightarrow\infty}(T_{n+1} - T_n) = 0$. From system (13) and $\lim_{n\longrightarrow\infty}\Delta T_n = 0$, we have $\lim_{n\longrightarrow\infty} S_n = S^0$. For the case $R_0 < 1$, we have $\lim_{n\longrightarrow\infty} S_{n+1} = S^0$ and $\lim_{n\longrightarrow\infty} E_n = 0, \lim_{n\longrightarrow\infty} I_n = 0$. From system (13), we obtain $\lim_{n\longrightarrow\infty} E_n = 0, \lim_{n\longrightarrow\infty} I_n = 0$ and $\lim_{n\longrightarrow\infty} Q_n = 0$. For the case $R_0 = 1$, we have $\lim_{n\longrightarrow\infty} S_{n+1} = S^0$. Consequently, from system (13), we obtain $\lim_{n\longrightarrow\infty} R_n = 0, \lim_{n\longrightarrow\infty} Q_n = 0, \lim_{n\longrightarrow\infty} E_n = 0$ and $\lim_{n\longrightarrow\infty} I_n = 0$. Hence, E_0 is globally asymptotically stable. □

Theorem 5. *If $R_0 > 1$, then DEE point for NSFD model (19) is globally asymptotically stable for all $h > 0$.*

Proof. Let us define

$$T_n(S_n, E_n, I_n, Q_n, R_n) = S^* H\left(\frac{S_n}{S^*}\right) + \phi_1 E^* H\left(\frac{E_n}{E^*}\right) + \phi_2 I^* H\left(\frac{I_n}{I^*}\right) + \phi_3 Q^* H\left(\frac{Q_n}{Q^*}\right) + (\Sigma + \theta)\phi_4 R^* H\left(\frac{R_n}{R^*}\right), \tag{52}$$

where $\phi_i > 0$, $[ii] = 1, 2, 3, 4$ which we will chose later. It is clear that $T_n(S_n, E_n, I_n, Q_n, R_n) > 0$ for all $S_n > 0$, $E_n > 0, I_n > 0, Q_n > 0, R_n > 0$ and $T_n(S^*, E^*, I^*, Q^*, R^*) = 0$.

By considering

$$\Delta T_n = T_{n+1} - T_n, \tag{53}$$

we get

$$\begin{aligned}\Delta T_n &= S^* H\left(\frac{S_{n+1}}{S^*}\right) + \phi_1 E^* H\left(\frac{E_{n+1}}{E^*}\right) + \phi_2 I^* H\left(\frac{I_{n+1}}{I^*}\right) + \phi_3 Q^* H\left(\frac{Q_{n+1}}{Q^*}\right) + (\Sigma + \theta)\phi_4 R^* H\left(\frac{R_{n+1}}{R^*}\right)\\ &\quad - \left[S^* H\left(\frac{S_n}{S^*}\right) + \phi_1 E^* H\left(\frac{E_n}{E^*}\right) + \phi_2 I^* H\left(\frac{I_n}{I^*}\right) + \phi_3 Q^* H\left(\frac{Q_n}{Q^*}\right) + (\Sigma + \theta)\phi_4 R^* H\left(\frac{R_n}{R^*}\right)\right]\\ &= S^*\left(\frac{S_{n+1}}{S^*} - \frac{S_n}{S^*} + \ln\frac{S_n}{S_{n+1}}\right) + \phi_1 E^*\left(\frac{E_{n+1}}{E^*} - \frac{E_n}{E^*} + \ln\frac{E_n}{E_{n+1}}\right) + \phi_2 I^*\left(\frac{I_{n+1}}{I^*} - \frac{I_n}{I^*} + \ln\frac{I_n}{I_{n+1}}\right) + \phi_3 Q^*\left(\frac{Q_{n+1}}{Q^*} - \frac{Q_n}{Q^*} + \ln\frac{Q_n}{Q_{n+1}}\right)\\ &\quad + \phi_4 R^*\left(\frac{R_{n+1}}{R^*} - \frac{R_n}{R^*} + \ln\frac{R_n}{R_{n+1}}\right) + (\Sigma + \theta)\phi_4 R^*\left[H\left(\frac{R_{n+1}}{R^*}\right) - H\left(\frac{R_n}{R^*}\right)\right].\end{aligned} \tag{54}$$

Using the inequality $\ln x \leq x - 1$, we obtain

$$\begin{aligned}\Delta T_n &\leq S^*\left(\frac{S_{n+1} - S_n}{S^*} + \frac{S_n}{S_{n+1}} - 1\right) + \phi_1 E^*\left(\frac{E_{n+1} - E_n}{E^*} + \frac{E_n}{E_{n+1}} - 1\right) + \phi_2 I^*\left(\frac{I_{n+1} - I_n}{I^*} + \frac{I_n}{I_{n+1}} - 1\right)\\ &\quad + \phi_3 Q^*\left(\frac{Q_{n+1} - Q_n}{Q^*} + \frac{Q_n}{Q_{n+1}} - 1\right) + \phi_4 R^*\left(\frac{R_{n+1} - R_n}{R^*} + \frac{R_n}{R_{n+1}}\right) + (\Sigma + \theta)\phi_4 R^*\left[H\left(\frac{R_{n+1}}{R^*}\right) - H\left(\frac{R_n}{R^*}\right)\right]\\ &= \left(1 - \frac{S^*}{S_{n+1}}\right)(S_{n+1} - S_n) + \phi_1\left(1 - \frac{E^*}{E_{n+1}}\right)(E_{n+1} - E_n) + \phi_2\left(1 - \frac{I^*}{I_{n+1}}\right)(I_{n+1} - I_n)\\ &\quad + \phi_3\left(1 - \frac{Q^*}{Q_{n+1}}\right)(Q_{n+1} - Q_n) + \phi_4\left(1 - \frac{R^*}{R_{n+1}}\right)(R_{n+1} - R_n) + (\Sigma + \theta)\phi_4 R^*\left[H\left(\frac{R_{n+1}}{R^*}\right) - H\left(\frac{R_n}{R^*}\right)\right].\end{aligned} \tag{55}$$

By employing system (13), (55) becomes

$$\begin{aligned}\Delta T_n \leq & \left(1-\frac{S^*}{S_{n+1}}\right)\left(\beta-\frac{\delta_c(1-\mathscr{E})(1-\gamma)(E_n+I_n)S_{n+1}}{N}-\theta S_{n+1}+\Sigma R_n\right)+\phi_1\left(1-\frac{E^*}{E_{n+1}}\right)\\ &\cdot\left(\frac{\delta_c(1-\mathscr{E})(1-\gamma)(E_{n+1}+I_n)S_{n+1}}{N}-(\theta+\alpha)E_{n+1}\right)+\phi_2\left(1-\frac{I^*}{I_{n+1}}\right)(\alpha E_{n+1}-(\theta+\eta+\psi+\Omega)I_{n+1})\\ &+\phi_3\left(1-\frac{Q^*}{Q_{n+1}}\right)(\psi I_{n+1}-(\theta+\eta+\varphi)Q_{n+1})+\phi_4\left(1-\frac{R^*}{R_{n+1}}\right)\varphi Q_{n+1}+\Omega I_{n+1}-(\Sigma+\theta)R_{n+1}+(\Sigma+\theta)\phi_4R^*\left[H\left(\frac{R_{n+1}}{R^*}\right)-H\left(\frac{R_n}{R^*}\right)\right].\end{aligned} \tag{56}$$

As $\beta=\delta_c(1-\mathscr{E})(1-\gamma)(E^*+I^*)S^*/N+\theta S^*-\Sigma R^*$, so by replacing it in (56), we obtain

$$\begin{aligned}\Delta T_n \leq & \left(1-\frac{S^*}{S_{n+1}}\right)\left(\frac{\delta_c(1-\mathscr{E})(1-\gamma)(E^*+I^*)S^*}{N}+\theta S^*-\Sigma R^*-\delta_c(1-\mathscr{E})(1-\gamma)(E_n+I_n)S_{n+1}-\theta S_{n+1}+\Sigma R_n\right)\\ &+\phi_1\left(1-\frac{E^*}{E_{n+1}}\right)\left(\frac{\delta_c(1-\mathscr{E})(1-\gamma)(E_{n+1}+I_n)S_{n+1}}{N}-(\theta+\alpha)E_{n+1}\right)+\phi_2\left(1-\frac{I^*}{I_{n+1}}\right)(\alpha E_{n+1}-(\theta+\eta+\psi+\Omega)I_{n+1})\\ &+\phi_3\left(1-\frac{Q^*}{Q_{n+1}}\right)(\psi I_{n+1}-(\theta+\eta+\varphi)Q_{n+1})+\phi_4\left(1-\frac{R^*}{R_{n+1}}\right)\varphi Q_{n+1}+\Omega I_{n+1}-(\Sigma+\theta)R_{n+1}\\ &+(\Sigma+\theta)\phi_4R^*\left[\frac{R_{n+1}}{R^*}-\frac{R_n}{R^*}+\ln\frac{R_n}{R_{n+1}}\right]\end{aligned} \tag{57}$$

$$\begin{aligned}=& \left(1-\frac{S^*}{S_{n+1}}\right)(\theta S^*-\theta S_{n+1})+\left(1-\frac{S^*}{S_{n+1}}\right)\left(\frac{\delta_c(1-\mathscr{E})(1-\gamma)(E^*+I^*)S^*}{N}-\frac{\delta_c(1-\mathscr{E})(1-\gamma)(E_n+I_n)S_{n+1}}{N}\right)\\ &+\Sigma R^*-\phi_1\frac{E^*}{E_{n+1}}\,\frac{\delta_c(1-\mathscr{E})(1-\gamma)(E_{n+1}+I_n)S_{n+1}}{N}+\phi_1(\theta+\alpha)E^*-\phi_2\frac{I^*}{I_{n+1}}\alpha E_{n+1}\\ &+\phi_2(\theta+\eta+\psi+\Omega)I_{n+1}-\phi_3\frac{Q^*}{Q_{n+1}}\psi I_{n+1}+\phi_3(\theta+\eta+\varphi)Q_{n+1}-\phi_4\frac{R^*}{R_{n+1}}(\varphi Q_{n+1}+\Omega I_{n+1})\\ &+\phi_4(\Upsilon+\tau)R^*+\phi_4(\Sigma+\theta)R^*\left(-\frac{R_n}{R^*}+\ln\frac{R_n}{R_{n+1}}\right).\end{aligned} \tag{58}$$

By replacing $\delta_c(1-\mathscr{E})(1-\gamma)(E^*+I^*)S^*=N\phi_1(\theta+\alpha)E^*$, $\alpha E^*=\phi_2(\theta+\eta+\psi+\Omega)I^*$, $\psi I^*=\phi_3(\theta+\eta+\varphi)Q^*$, $\varphi Q^*+\Lambda I^*=\phi_4(\Sigma+\theta)R^*$ in (58), we can easily get

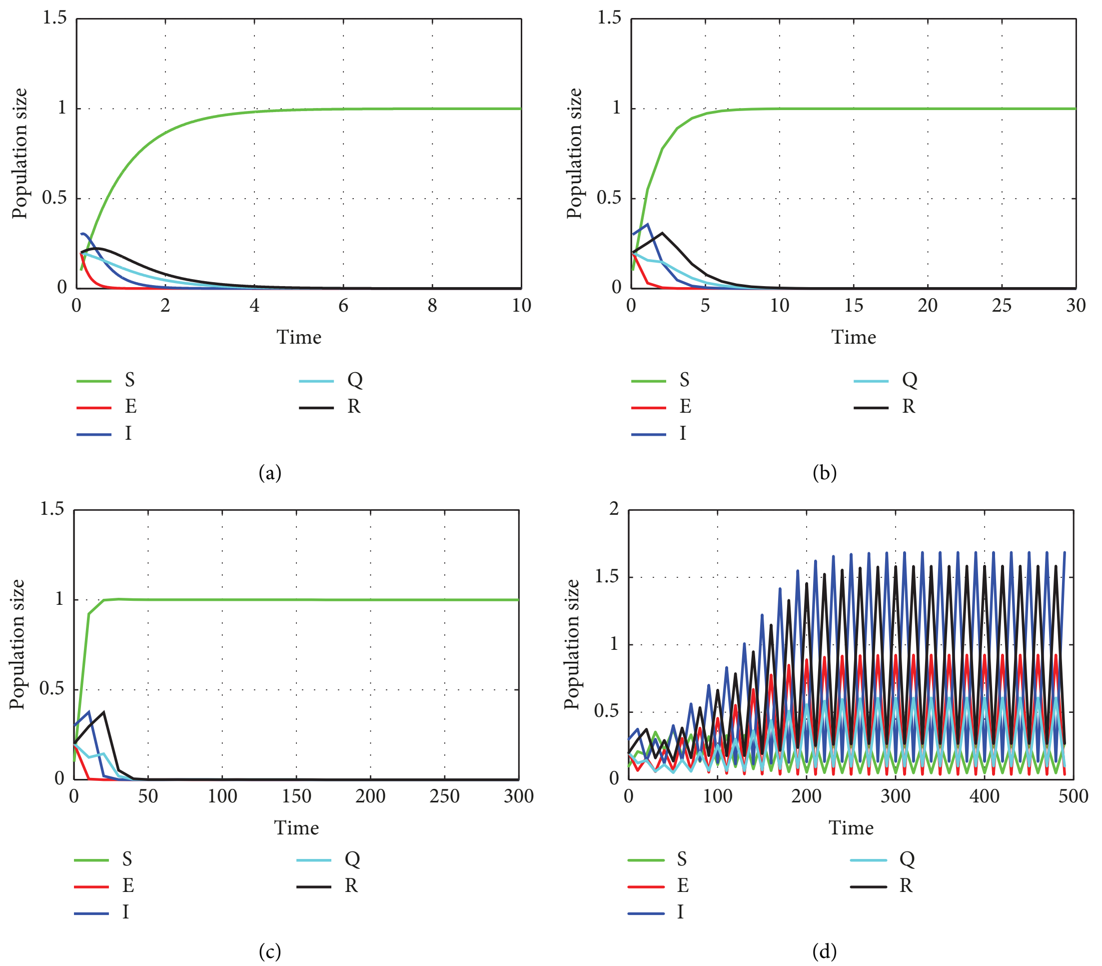

FIGURE 3: Numerical simulation for model (1) by using NSFD scheme with (a)$h = 0.01$, (b)$h = 1$, (c)$h = 10$, and (d)$h = 10$. (a–c) Stable DFE point with $\gamma = 1.01$, $\delta_c = 0.814715e-01$. (d) Stable DEE point $\gamma = 0.7, \delta_c = 4.314715$. Other parameters remain fixed as $\beta = 1, \theta = 1$, $\eta = 0.05$, $\Sigma = 0.104874e-01$, $\Omega = 10.270934$, , $\varphi = 0.584931e-03$, $N = 1, \varepsilon = 0.999373e-01$, $\alpha = 4.7$, $\psi = 4.0786530e-01$.

$$\begin{aligned}\Delta T_n \leq & \frac{-\theta}{S_{n+1}}(S_{n+1}-S^*)^2 + \left(1-\frac{S^*}{S_{n+1}}\right)(\theta+\alpha)E^* - N\phi_1(\theta+\alpha)E^*\frac{S_{n+1}R_nE^*}{S^*R^*E_{n+1}} + \phi_1\Sigma R^* - \phi_2\alpha E^*\frac{S_{n+1}R_nI^*}{S^*R^*I_{n+1}} \\ & - \delta_c(1-\mathscr{E})(1-\gamma)(E^*+I^*)S^*\frac{I^*E_{n+1}}{I_{n+1}E^*} + \phi_2\alpha E^* - \phi_3\psi I^*\frac{S_{n+1}R_nQ^*}{S^*R^*Q_{n+1}} \\ & + \phi_3\psi I^* - \phi_2\alpha E^*\frac{R^*I_{n+1}}{R_{n+1}I^*} - \phi_3\psi I^*\frac{R^*Q^*}{R_{n+1}} + \phi_4(\Sigma+\theta)R^* + \phi_4(\Sigma+\theta)R^*\ln\frac{R_n}{R_{n+1}} \\ = & \frac{-\theta}{S_{n+1}}(S_{n+1}-S^*)^2 - \phi_1\Sigma S^*R^*\left(H\left(\frac{S^*}{S_{n+1}}\right) + H\left(\frac{S_{n+1}R_nE^*}{S^*R^*E_{n+1}}\right) + H\left(\frac{I^*E_{n+1}}{I_{n+1}E^*}\right) + H\left(\frac{R^*I_{n+1}}{R_{n+1}I^*}\right)\right) \\ & - \phi_2\alpha E^*\left(H\left(\frac{S^*}{S_{n+1}}\right) + H\left(\frac{S_{n+1}R_nI^*}{S^*R^*I_{n+1}}\right) + H\left(\frac{R^*I_{n+1}}{R_{n+1}I^*}\right)\right) - \phi_3\psi I^*\left(H\left(\frac{S^*}{S_{n+1}}\right) + H\left(\frac{S_{n+1}R_nQ^*}{S^*R^*Q_{n+1}}\right) + H\left(\frac{Q_{n+1}R^*}{Q^*R_{n+1}}\right)\right).\end{aligned} \tag{59}$$

Thus, T_n is a nonincreasing sequence and there exists a constant T such that $\lim_{n\longrightarrow\infty}T_n = T$ Therefore, $\lim_{n\longrightarrow\infty}\Delta T_n = 0$ which implies $\lim_{n\longrightarrow\infty}S_n = S^*$, $\lim_{n\longrightarrow\infty}E_n = E^*$, $\lim_{n\longrightarrow\infty}I_n = I^*$, $\lim_{n\longrightarrow\infty}Q_n = Q^*$, and $\lim_{n\longrightarrow\infty}R_n = R^*$.

The numerical simulations depicted in Figure 3(a)–3(c) also demonstrate that, for any step size, the solutions of the NSFD scheme (19) converge to the DFE point whenever $R_0 \leq 1$. This demonstrates that the DFE point is unconditionally convergent for the discrete NSFD scheme. The solutions of the NSFD scheme (19) diverge from the DFE point and converge to DEE point for any step size when R_0 exceeds one, as presented in Figure 3(d). □

6. Conclusions

In the present work, a mathematical model enlightening the spread mechanism of COVID-19 is discussed and analyzed. The fundamental reproduction number is estimated, which is crucial in examining the local and global stability of DFE and DEE points. The reproduction number presents that COVID-19 is either under control or growing worse over time. The mixed Euler and NSFD schemes are developed to assess various properties of the continuous model. By using different criteria and conditions, the positivity and boundedness of solutions as well as local and global stability of DFE and DEE points are discussed in detail for the NSFD scheme. It has been revealed that mixed Euler and NSFD schemes are not only unconditionally convergent but also produces findings that are accurate and mathematically as well as biologically feasible for the continuous model. The validity of theoretical results has been verified by numerical simulations.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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