Hindawi
Discrete Dynamics in Nature and Society
Volume 2023, Article ID 7656214, 6 pages
https://doi.org/10.1155/2023/7656214

Research Article

Metric Dimension of Line Graphs of Bakelite and Subdivided Bakelite Network

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Received 17 February 2023; Revised 21 May 2023; Accepted 29 July 2023; Published 16 August 2023

Academic Editor: Abdellatif Ben Makhlouf

Graph theory is considered one of the major subjects, and it also plays a significant role in the digital world. It has numerous uses in computer science, robot navigation, and chemistry. Graph theory is employed in forming the structures of different chemical networks, locating robots on a network, and troubleshooting computer networks. Additionally, it finds applications in scheduling airplanes and studying diffusion mechanisms. The current work investigates the metric dimension of the line graphs of the Bakelite and subdivided Bakelite networks. The results prove that these families of graphs do not have a constant metric dimension. The invention of Bakelite was influential in the development of modern plastics, and it has various applications in fields such as jewelry, clocks, toys, kitchenware, electrical, and sports industries.

1. Introduction

The concept of metric dimension has been the subject of research in various fields, including molecular chemistry, communication networks, and social networks. The metric dimension has been studied for various graph models, including the complete graph, cycle graph, and grid graph. The metric dimension is a measure of the resolving power of a graph, which is the ability to uniquely identify the location of its vertices. In order to understand this concept, it is important to first understand the notion of distance in connected graphs. In a connected graph G, the distance between two vertices a and b is defined as the length of the shortest path between them and is denoted by $d(a, b)$. The metric dimension of a graph G is denoted by $\dim(G)$.

The concept of the metric dimension of a graph was introduced by Harary and Melter in 1976 [1], along with a method for its calculation. They also demonstrated that the metric dimension of a graph is at most equal to its diameter and its maximum degree. In 1984, Melter and Harary [2] explored the metric dimension of various graph types, such as trees, cycles, and complete graphs. They also established bounds on the metric dimension of graphs based on their order and size. A survey by Chartrand and Harary in 1985 [3] provided a comprehensive overview of the different findings and applications of the metric dimension of a graph. In 1988, Slater [4] introduced the concept of resolving sets and demonstrated that the minimum cardinality of a resolving set is equivalent to the metric dimension of a graph. A resolving set for a graph G is a set of vertices such that the distance vectors from these vertices to all other vertices of G

are unique. A survey by Oellermann and Pfaff in 1993 [5] gave a detailed and comprehensive overview of the various findings and applications of the metric dimension of a graph. In 1996, Khuller et al. [6] introduced the concept of the k-metric dimension, which is a generalization of the metric dimension to k-resolving sets. In a paper by Pal and Das in 2000 [7], the authors studied the metric dimension of disconnected graphs and provided a method for its calculation. In 2016, a paper by Simon and Sastry [8] wrote an article on the metric dimension of different chemical graphs, including the Star of David network. They also establish bounds on the metric dimension of these graphs in terms of their order and size.

There are many more studies and articles which focus on the metric dimension and its applications; these are just a few examples, but they will give you an idea about the research and studies that have been conducted in this field [9–11].

A path graph is the only graph that has a metric dimension of 1, as stated in [12]. The cycle graph has a metric dimension of 2 for $n \geq 3$. This concept is particularly useful in applications such as space routing and chemistry. In space routing, for example, the goal is to place the least number of robots at certain vertices in such a way that they can visit each and every vertex precisely one time. This problem can be solved using the concept of metric dimension.

In chemistry, many chemical compounds exist that have the same chemical equation but different chemical structures. Chemists need to select the compound that best communicates its leading physical and chemical properties. To do this, they require a scientific labeling system that gives unique labels to specific compounds. The numerical representation of unique chemical compounds is crucial for chemists in drug discovery. Graphs can be used to represent chemical compounds, with vertices representing atoms and edges representing bond types [13, 14]. Theoretical descriptions of graphs and their applications are discussed in the papers [14–16].

The metric dimension is the most famous field in graph theory related to the distances of graphs. After gaining some ideas from the latest study on resolving properties of graphs [9–11], metric dimension of line graphs of Bakelite, and subdivided Bakelite network would be determined.

Definition 1. A resolving set with the least number of vertices is known as the basis for the graph G [15].

Definition 2. A line graph $L(G)$ for any simple graph G is obtained by associating a vertex with each edge of the graph and connecting two vertices with an edge iff the corresponding edges of G have a vertex in common [17].

Definition 3. Eccentricity of the vertex is the value of the biggest distance between two vertices of a connected graph [18].

Definition 4. The constant metric dimension of the family of connected graphs is stated as if all the graphs in the family have the same metric dimension then such family is said to have constant metric dimension [19].

2. Bakelite Network

In this section, the chemical background of the Bakelite network $(B(n \times m))$ is discussed. After that, we proved it in the form of different theorems that the line graph of a Bakelite network does not have a constant metric dimension.

Bakelite, whose chemical name is phenol-formaldehyde resin or phenolic resin, was invented in 1907. This date marks the beginning of the modern plastics industry. An American chemist, Leo Hendrik Baekeland, who was born in Belgium, was credited with the invention of the Bakelite after he applied for a patent on a phenol-formaldehyde thermoset. The phenol-formaldehyde polymers, also known as plastics, were the first completely synthetic polymers to be commercialized. Even today, almost half of the total production of thermosetting polymers is used as adhesives.

Bakelite is a chemical compound that has a wide range of applications across various fields of life. Some of the key uses of Bakelite include:

(i) Its high resistivity to electricity and heat makes it an ideal material for use in automotive and industrial components.

(ii) It is commonly used in the manufacturing of jewelry articles, clocks, and toys.

(iii) Many kitchenware products, such as frying pans and special types of spoons are also made of this type of plastic.

(iv) In the electrical industry, it is used to make non-conducting parts of various electrical devices such as radios, telephones, sockets, base for electron tubes, light bulbs, supports, and other insulators.

(v) In the sports industry, it is used to make various game pieces such as billiard balls, chess sets, and poker chips.

The invention of the Bakelite is considered to be the foundation of the modern plastics industry. The general Bakelite network is composed of n number of rows and m number of columns, denoted by $B(n \times m)$.

Theorem 5. *For $G \cong L(B(1 \times m))$ where m varies, then G has metric dimension 2.*

Proof. The general vertex set for $L((B(n \times m))$ is as follows:

$$\begin{aligned}V(L(B(n\times m))) = &\{a_u^v\colon 1\le u\le n, 1\le v\le 4m\}\cup\{b_u^v\colon 1\le u\le n, 1\le v\le 2m\}\cup\{c_u^v\colon 1\le u\le n, 1\le v\le m\}\\ &\cup\{d_u^v\colon 1\le u\le n, 1\le v\le m\}\cup\{e_u^v\colon 0\le u\le n-1, 1\le v\le 2m\}\cup\{f_u^v\colon 0\le u\le n-1, 1\le v\le 2m\}\\ &\cup\left\{a_u^{'v}\colon 1\le u\le n, 1\le v\le 4m\right\}\cup\left\{b_u^{'v}\colon 1\le u\le n, 1\le v\le 2m\right\}\cup\left\{c_u^{'v}\colon 1\le u\le n, 1\le v\le m\right\}\\ &\cup\left\{d_u^{'v}\colon 1\le u\le n, 1\le v\le m\right\}\\ &\cup\left\{e_u^{'v}\colon 0\le u\le n-1, 1\le v\le 2m\right\}\cup\left\{f_u^{'v}\colon 0\le u\le n-1, 1\le v\le 2m\right\}.\end{aligned} \tag{1}$$

We can label molecules of line graph of the Bakelite network as shown in Figure 1

$$\begin{aligned}\lambda(a_u^v) &= \{v-1, 8m-v \quad \text{for } 1\le v\le 4m, m=1,2,3,\ldots\},\\ \lambda(b_u^v) &= \{2v-1, 8m-2v+1 \quad \text{for } 1\le v\le 2m, m=1,2,3,\ldots\},\\ \lambda(c_u^v) &= \{4v-2, 8m-4v+3 \quad \text{for } 1\le v\le m, m=1,2,3,\ldots\},\\ \lambda(d_u^v) &= \{4v-1, 8m-4v+2 \quad \text{for } 1\le v\le m, m=1,2,3,\ldots\},\\ \lambda\left(a_u^{'v}\right) &= \{8m-v, v-1 \quad \text{for } 1\le v\le 4m, m=1,2,3,\ldots\},\\ \lambda\left(b_u^{'v}\right) &= \{8m-2v+1, 2v-1 \quad \text{for } 1\le v\le 2m, m=1,2,3,\ldots\},\\ \lambda\left(c_u^{'v}\right) &= \{8m-4v+3, 4v-2 \quad \text{for } 1\le v\le m, m=1,2,3,\ldots\},\\ \lambda\left(d_u^{'v}\right) &= \{8m-4v+2, 4v-1 \quad \text{for } 1\le v\le m, m=1,2,3,\ldots\}.\end{aligned} \tag{2}$$

$L(B(1\times m))$ has metric dimension 2. Only a path graph has metric dimension one [12], as $L(B(1\times m))$ is not the path so its metric dimension is not 1. $L(B(1\times m))$ has a resolving set $H=\left\{a_1^1, a_1^{'1}\right\}$. The metric dimension of $L(B(1x1))$ is shown in Figure 2. □

Theorem 6. *For $G\cong L(B(2\times m))$ if $m=1,2$, then G has metric dimension 3.*

Proof. We will prove this theorem by showing that there does not exist any resolving set H of graph G with two vertices. Contradictorily, we suppose that graph G has metric dimension 2.

Let $H=\left\{a_1^1, a_1^{'1}\right\}$ be the resolving set for $L(B(2\times 1))$, then $r(c_2^1\,|\,H)=r(d_2^1\,|\,H)$ is obtained which implies that H is not a resolving set for the graph. Now, consider a resolving set $H=\{a_1^1, a_2^1\}$ for $L(B(2\times 2))$ which results in $r(a_2^6\,|\,H)=r(b_2^3\,|\,H)$, and therefore, H is not a resolving set. Suppose that $H=\left\{a_1^1, a_2^{'1}\right\}$ and $H=\left\{a_1^{'1}, b_1^{'2}\right\}$ are the resolving sets, then we will have $r(a_1^{'1}\,|\,H)=r(b_1^{'1}\,|\,H)$ and $r(c_2^{'1}\,|\,H)=r(d_2^{'1}\,|\,H)$, respectively. So, H is not a resolving set. If $H=\{b_1^1, b_1^2\}$ and $H=\left\{e_1^1, e_1^{'1}\right\}$ are the resolving sets, then $r(c_2^1\,|\,H)=r(d_2^1\,|\,H)$ and $r(a_1^1\,|\,H)=r(b_1^1\,|\,H)$ can easily be observed. Therefore, H is not a resolving set.

In a similar way, if we suppose that $H=\left\{e_1^1, f_1^1\right\}$, $\left\{a_1^{'1}, e_1^1\right\}$, $\left\{b_2^{'2}, b_2^2\right\}$, $\{a_2^4, a_1^1\}$ are the resolving sets, then we can observe that $r(a_1^2\mid H)=r(a_1^3|H)$, $r(a_1^1\mid H)=b(b_1^1\,|\,H)$, $r(a_1^1\mid H)=r(b_1^1\,|\,H)$, and $r(a_1^{'1}\,|\,H)=r(b_1^{'1}\,|\,H)$, respectively. Hence, H is not a resolving set.

In general, there does not exist any resolving set having two vertices for the graph $L(B(2\times m))$ where $m=1,2$. So, its metric dimension is 3 and its resolving set is $H=\left\{a_1^1, a_1^{'1}, a_2^{'1}\right\}$. □

Theorem 7. *The metric dimension for graph $G\cong L(B(n\times m))$ is as follows:*

$$\text{Metric Dimension}(G)=\begin{cases}4 & \text{for } n=2 \text{ and } m>2,\\ 4 & \text{for } n\ge 3 \text{ and } m>1.\end{cases} \tag{3}$$

Proof. We will prove this theorem by using contradiction process. As we mentioned above that the graph G has metric dimension 4. Contradictorily, we suppose that above statement is not true and metric dimension of graph G is 3. Let $H=\left\{a_1^1, a_1^{'1}, f_1^{'1}\right\}$ be a resolving set, then it implies that $r(a_2^7|H)=r(b_2^4|H)$. Therefore, H is not a resolving set.

Similarly, if we consider $H=\{a_1^1, a_1^4, c_1^1\}$, $\left\{b_1^2, b_1^{'2}, f_1^1\right\}$, $\left\{a_1^1, a_1^{'1}, d_1^5\right\}$, $\left\{a_1^1, a_1^{'1}, e_1^{'3}\right\}$, $\left\{a_1^1, a_1^{'1}, c_1^1\right\}$ as resolving sets, then we will have $r(c_2^1\,|\,H)=r(d_2^1\,|\,H)$, $r(a_2^1\,|\,H)=r(a_2^2\,|\,H)$, $r(d_2^3\,|\,H)=r(d_2^4\,|\,H)$, $r(c_2^{'3}\,|\,H)=r(d_2^{'3}\,|\,H)$, and $r(c_2^1\,|\,H)=r(d_2^1\,|\,H)$ respectively. Hence, H is not a resolving set.

In general, there does not exist any resolving set H having three vertices in it for the graph G. So, its metric dimension is 4 and its resolving set is $\left\{a_1^1, d_1^{'1}, a_n^1, a_n^{'1}\right\}$. □

Theorem 8. *For $G\cong B(n\times 1)$ if n varies, then graph G has the metric dimension $n+1$.*

Proof

The metric dimension of the line graph of the Bakelite unit network $L(B(1\times 1))$ is 2.

The metric dimension of the line graph of the Bakelite network for $L(B(2\times 1))$ is 3.

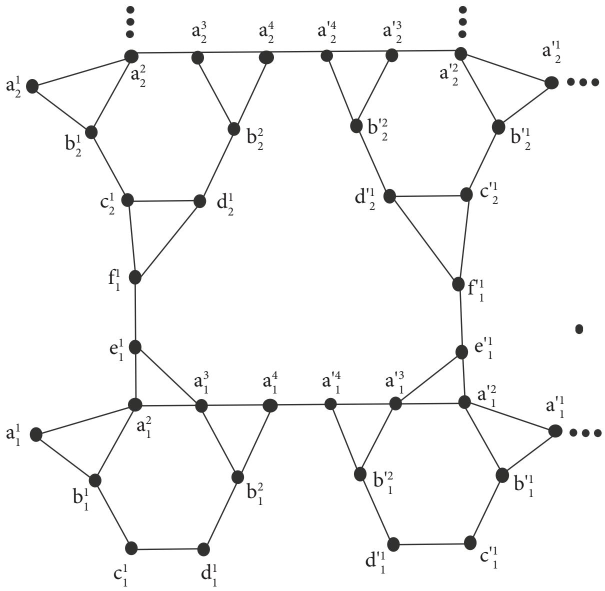

Figure 1: Labeling for molecule of $L(B(n\times m))$.

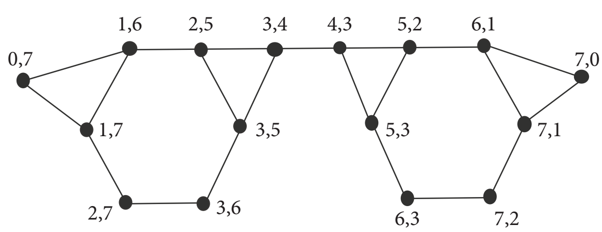

Figure 2: Metric dimension of $L(B(1\times 1))$.

The metric dimension of the line graph of the Bakelite network for $L(B(3\times 1))$ is 4.

The metric dimension of the line graph of the Bakelite network for $L(B(4\times 1))$ is 5.

Proceeding this way, the metric dimension of the line graph of the Bakelite network for $L(B(n\times 1))$ is $n+1$. So, it is proved that for $G\cong L(B(n\times 1))$ if n varies, then graph G has the metric dimension $n+1$. □

3. Subdivided Bakelite Network

In this section, the subdivision of the Bakelite network $(B(n\times m))$ is discussed, as is the structure of the subdivided Bakelite network $(SB(n\times m))$ is elaborated with the help of Figure 3. After that, it is proved that the line graph of a subdivided Bakelite network $(SB(n\times m))$ do not have a constant metric dimension.

Let G be a graph. The subdivided graph G' of G can be obtained by replacing each edge (u,v) in G with a new vertex w, and adding edges (u,w), and (v,w) to G'. In other words, for every edge (u,v) in G, we create a new vertex w and connect it to the vertices u and v. This results in a new graph G' with more vertices and fewer edges than the original graph G.

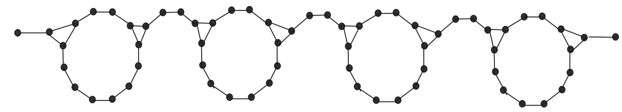

Figure 3: $L(SB(2\times 1))$.

Theorem 9. *For $G\cong L(SB(1\times m))$ where m varies, G has metric dimension 2.*

Proof. The general vertex set for $L((SB(n\times m))$ is as follows: $V(L(SB(n\times m)) = \{a_u^v\colon 1\le u\le n, 1\le v\le 8m\}\cup\{b_u^v\colon 1\le u\le n, 1\le v\le 2m\}\cup\{c_u^v\colon 1\le u\le n, 1\le v\le 2m\}\cup\{d_u^v\colon 1\le u\le n, 1\le v\le m\}\cup\{e_u^v\colon 1\le u\le n, 1\le v\le m\}\cup\{f_u^v\colon 1\le u\le n, 1\le v\le m\}\cup\{g_u^v\}\colon 1\le u\le n, 1\le v\le m\}\cup\{h_u^v\colon 1\le u\le n-1$ where $n\ge 2, 1\le v\le m\}\cup\{i_u^v\colon 1\le u\le n-1$ where $n\ge 2, 1\le v\le m\}\cup\{j_u^v\colon 1\le u\le n-1$ where $n\ge 2, 1\le v\le m\}\cup\{k_u^v\colon 1\le u\le n-1$ where $n\ge 2, 1\le v\le m\}\cup\{a'^v_u\colon 1\le u\le n, 1\le v\le 8m\}\cup\{b'^v_u\colon 1\le u\le n, 1\le v\le 2m\}\cup\{c'^v_u\colon 1\le u\le n, 1\le v\le 2m\}\cup\{d'^v_u\colon 1\le u\le n, 1\le v\le m\}\cup\{e'^v_u\colon 1\le u\le n, 1\le v\le m\}\cup\{f'^v_u\colon 1\le u\le n, 1\le v\le m\}\cup\{g'^v_u\colon 1\le u\le n, 1\le v\le m\}\cup\{h'^v_u\colon 1\le u\le n-1$ where $n\ge 2, 1\le v\le m\}\cup\{i'^v_u\colon 1\le u\le n-1$ where $n\ge 2, 1\le v\le m\}\cup\{j'^v_u\colon 1\le u\le n-1$ where $n\ge 2, 1\le v\le m\}\cup\{k'^v_u\colon 1\le u\le n-1$ where $n\ge 2, 1\le v\le m\}$

$$
\begin{aligned}
\lambda(a_u^v) &= \{v-1, 16m-v \quad \text{for } 1\le v\le 8m\, m = 1,2,3,\ldots\},\\
\lambda(b_u^v) &= \{4v-2, 16m-4v+2 \quad \text{for } 1\le v\le 2m\, m = 1,2,3,\ldots\},\\
\lambda(c_u^v) &= \{4v-1, 16m-4v+3 \quad \text{for } 1\le v\le 2m\, m = 1,2,3,\ldots\},\\
\lambda(d_u^v) &= \{8v-4, 16m-8v+7 \quad \text{for } 1\le v\le m\, m = 1,2,3,\ldots\},\\
\lambda(e_u^v) &= \{8v-3, 16m-8v+6 \quad \text{for } 1\le v\le m\, m = 1,2,3,\ldots\},\\
\lambda(f_u^v) &= \{8v-2, 16m-8v+5 \quad \text{for } 1\le v\le m\, m = 1,2,3,\ldots\},\\
\lambda(g_u^v) &= \{8v-1, 16m-8v+4 \quad \text{for } 1\le v\le m\, m = 1,2,3,\ldots\},\\
\lambda\left(a'^v_u\right) &= \{16m-v, v-1 \quad \text{for } 1\le v\le 8m\, m = 1,2,3,\ldots\},\\
\lambda\left(b'^v_u\right) &= \{16m-4v+2, 4v-2 \quad \text{for } 1\le v\le 2m\, m = 1,2,3,\ldots\},\\
\lambda\left(c'^v_u\right) &= \{16m-4v+3, 4v-1 \quad \text{for } 1\le v\le 2m\, m = 1,2,3,\ldots\},\\
\lambda\left(d'^v_u\right) &= \{16m-8v+7, 8v-4 \quad \text{for } 1\le v\le m\, m = 1,2,3,\ldots\},\\
\lambda\left(e'^v_u\right) &= \{16m-8v+6, 8v-3 \quad \text{for } 1\le v\le m\, m = 1,2,3,\ldots\},\\
\lambda\left(f'^v_u\right) &= \{16m-8v+5, 8v-2 \quad \text{for } 1\le v\le m\, m = 1,2,3,\ldots\},\\
\lambda\left(g'^v_u\right) &= \{16m-8v+4, 8v-1 \quad \text{for } 1\le v\le m\, m = 1,2,3,\ldots\}.
\end{aligned}
\tag{4}
$$

$L(SB(1\times m))$ has metric dimension 2. Only path graph has metric dimension 1 [12], as $L(SB(1\times m))$ is not the path so its metric dimension is not 1. $L(SB(1\times m))$ has a resolving set $H=\left\{a_1^1, a'^1_1\right\}$. □

Theorem 10. *For $G\cong L(SB(2\times m))$ if $m=1,2$, then G has metric dimension 3.*

Proof. We will prove this theorem by showing that ∄ any resolving set M of graph G with two vertices. Contradictorily, we suppose that graph G has metric dimension 2.

Let $M = \{a_1^1, a_1^{\prime 1}\}$ be the resolving set, then $r(e_2^1 \mid M) = r(f_2^1 \mid M)$, which shows M is not a resolving set for the graph.

Now, consider a resolving set $M = \{a_1^1, b_1^1\}$, then its results are $r(b_1^2 \mid M) = r(a_1^7 \mid M)$. Hence, M is not a resolving set. Consider another resolving set $M = \{a_1^1, e_1^{\prime 1}\}$, then $r(e_2^{\prime 1} \mid M) = r(f_2^{\prime 1} \mid M)$; therefore, M is not a resolving set. Let a resolving set $M = \{a_2^{\prime 1}, a_1^1\}$, then $r(a_1^{\prime 2} \mid M) = r(b_1^{\prime 1} \mid M)$, which leads to the result that M is not a resolving set. Suppose a resolving set $M = \{d_1^1, a_2^{\prime 1}\}$, then $r(a_1^{\prime 2} \mid M) = r(b_1^1 \mid M)$, which implies that B is not a resolving set. Let suppose a resolving set $M = \{a_1^7, a_1^{\prime 7}\}$, then it is observed that $r(e_1^{\prime 1} \mid M) = r(f_1^{\prime 1} \mid M)$. Hence, we can say that M is not a resolving set. Consider resolving sets $H = \{j_1^1, j_1^{\prime 1}\}, \{h_1^1, k_1^1\}, \{h_1^1, h_1^{\prime 1}\}, \{g_1^1, k_1^1\}$, then $r(a_1^1 \mid M) = r(b_1^1 \mid M)$, $r(e_1^1 \mid M) = r(f_1^1 \mid M)$, $r(a_1^1 \mid M) = r(b_1^1 \mid M)$, $r(a_1^{\prime 2} \mid M) = r(b_1^{\prime 1} \mid M)$. So, M is not a resolving set.

Similarly, there does not exist any resolving set M that has two vertices in it for graph G. So, its metric dimension is 3 and its resolving set is $M = \{a_1^1, a_1^{\prime 1}, a_2^1\}$. □

Theorem 11. *For* $G \cong L(SB(n \times m))$

$$\text{Metric Dimension}(G) = \begin{cases} 4 & \text{for } n = 2 \text{ and } m > 2, \\ 4 & \text{for } n \geq 3 \text{ and } m > 1. \end{cases} \quad (5)$$

Proof. We will prove this theorem by using contradiction process. As we mentioned above, graph G has metric dimension 4. Contradictorily, we suppose that above-mentioned statement is not true and metric dimension of graph G is 3.

Let us assume $Q = \{a_1^1, b_1^1, c_1^1\}$ to be a resolving set, then $r(e_2^1 | Q) = r(f_2^1 | Q)$. Hence, this result can be drawn that Q is not a resolving set. Consider a resolving set $Q = \{a_1^1, a_1^{\prime 1}, h_1^1\}$, then $r(a_1^7 | Q) = r(b_1^2 \mid Q)$. Hence, Q is not a resolving set. Now, let a resolving set $Q = \{a_1^8, b_1^2, k_1^1\}$, then $r(e_2^1 | Q) = r(f_2^1 \mid Q)$. Hence, it shows Q is not a resolving set. Consider a resolving set $Q = \{a_1^1, a_1^{\prime 1}, c_1^{\prime 1}\}$, then $r(e_2^{\prime 1} | Q) = r(f_2^{\prime 1} | Q)$. Hence, it implies Q is not a resolving set. Let a resolving set $Q = \{a_2^{\prime 1}, b_2^{\prime 1}, a_1^{\prime 1}\}$, then we see $r(a_1^{\prime 7} | Q) = r(b_1^{\prime 2} | Q)$. Therefore, Q is not a resolving set. Suppose another resolving set $Q = \{h_1^{\prime 1}, h_1^1, j_1^1\}$, then $r(a_1^2 | Q) = r(b_1^1 | Q)$. For this reason, Q is not a resolving set.

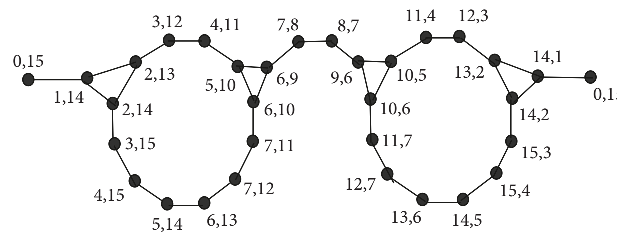

FIGURE 4: Metric dimension of $L(SB(1 \times 1))$.

Similarly, if we proceed in this way, there does not exist any resolving set Q having three vertices in it for the graph G. So, its metric dimension is 4 and its resolving set is $\{a_1^1, a_1^{\prime 1}, a_n^1, a_n^{\prime 1}\}$. □

Theorem 12. *For* $G \cong L(SB(n \times 1))$ *if n varies, then graph G has metric dimension* $n + 1$.

Proof

Metric dimension of the line graph of a subdivided Bakelite unit network $L(SB(1 \times 1))$ is 2.

Metric dimension of the line graph of a subdivided Bakelite network for $L(SB(2 \times 1))$ is 3.

Metric dimension of the line graph of a subdivided Bakelite network for $L(SB(3 \times 1))$ is 4.

Metric dimension of the line graph of a subdivided Bakelite network for $L(SB(4 \times 1))$ is 5.

Similarly,

Proceeding this way metric dimension of the line graph of a subdivided Bakelite network $L(SB(n \times 1))$ is $n + 1$. So proved that for $G \cong L(SB(n \times m))$ if n varies and $m = 1$ then graph G has metric dimension $n + 1$.

We can find metric dimension of line graph of SBN as shown in Figure 4. □

4. Concluding Remarks

Metric dimension instinctively is a very simple idea. However, determining the exact metric dimension of a graph is an NP-complete problem. The metric dimension is very closely related to the Global Positioning System (GPS) and trilateration. It is also used in source localization. In this manifesto, we define the line graph of the Bakelite network and subdivided the Bakelite network. The Bakelite network is being used in the making of automotive and industrial parts due to its outclassed high resistivity against electricity and heat. It has also many uses in the electrical and sports industries as it is used in the making of nonconducting parts of many electrical appliances. The metric dimension of the line graph of these graphs is computed too. Line graphs of the Bakelite network and subdivided Bakelite network do not have a constant and bounded metric dimension.

Consequently, this research leads to some open problems.

Open problem 1. Find the edge metric dimension of the line graph of the Bakelite network.

Open problem 2. Determine the edge metric dimension of the line graph of a subdivided Bakelite network.

Data Availability

No data were used for this work.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

All authors contributed equally.

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