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*Research Article*

# A New Mathematical Model for Economic Ordering with Preventive Maintenance and Reworking in a Supply Chain

**Guolei Ding,[1] Karthikeyan Kaliyaperumal ,[2] and Xiaoguang Wang[3]**

[1]*School of Business Administration, Shanghai Lixin University of Accounting and Finance, Shanghai, China*
[2]*IoT-HH Campus, Ambo University, Ambo, Ethiopia*
[3]*School of Business Administration, Shanghai Lixin University of Accounting and Finance, Shanghai, China*

Correspondence should be addressed to Karthikeyan Kaliyaperumal; kirithicraj@ambou.edu.et

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Cost reduction in production systems is one of the main concerns of most manufacturing industries. Achieving this goal requires the effective use of resources. Nowadays, due to the competitiveness of production and the need to reduce costs and deliver goods on time, equipment availability and prevention of unexpected stops are particularly important in industrial units. In recent years, equipment, machinery, and human resources have been the foundations of any organization. On the other hand, to increase the productivity and efficiency of providing services and achieve global standards, special attention should be paid to increase the efficiency of machinery and reducing downtime costs. This research conducted a mathematical formulation to find the best possible solution for the economic production quantity and inventory ordering by considering preventive maintenance. In this model, the success of organizations in providing services and increasing the quality depends on various reasons, such as maintenance and repair systems. In the proposed model, the defective items are considered, and the goal is to achieve an optimal amount of production in such a way that the costs of the entire system, including production cost, setup, maintenance, inspection, and rework costs, are minimized during a period per unit of time. The numerical results show that increasing the preventive maintenance periods leads to an upward trend in the total cost of the supply chain. Moreover, the sensitivity analysis shows that the longer the preventive maintenance on the machine, the higher reliability of the system as well as total costs will be achieved.

## 1. Introduction

Economical and profitable production requires comprehensive and accurate planning for all stages of the production system, and the actual behavior is compared and controlled promptly with the situation specified in the plan [1, 2]. These controls help to make the necessary decisions to affect the production systems to get the minimum delay in providing customers' orders. If the production work in the industries is carried out without a plan and subsequent control, productivity will decrease. Moreover, there will be no criteria and standards regarding the performance of different departments. It will cause conflicts within the organization and make the customers dissatisfied. The economic production quantity model helps companies and factories in this field determine the optimal production cumulative size by minimizing the total inventory production costs [1].

The quality and cost of materials that flow from raw to final products are selected as the most important characteristic features in the production systems [2]. In order to adjust these characteristics in the production process, several methods have been prepared for their planning and control. For this reason, manufacturing companies consider organizational units and information systems for production planning and control, inventory control, quality control, and cost control [2, 3].

*1.1. Repairs and Maintenance.* One of the most important activities to improve the level of access to a production system is the repair and maintenance of the production system. Maintenance and repairs are defined as "a set of activities to maintain or improve the safety, performance, reliability, and availability of the production site of a system or components to ensure their proper performance when needed" [2].

The cost of maintenance includes the following:

(i) Workforce costs for repairs
(ii) The costs of providing spare parts
(iii) Costs of machinery failure (production stagnation) and material waste.

In the technical management and control system of preventive maintenance (PM) affairs, the costs are divided into two types as follows:

(1) Direct costs, such as the cost of workforce hours for repairs (including workers' wages and overhead costs in maintenance and repairs)
(2) Indirect costs, such as the cost of stagnation in production when machines are stopped for repairs or waiting in line for repairs.

In any case, the cost analysis should also consider the capital costs involved in deploying additional machines.

In this research, the hybrid model that will be presented is the combination of determining the amount of economic production and the level of preventive maintenance and repairs, considering the allowed shortage for a production process. The assessed production process is such that after performing operations on a batch of input materials and producing healthy products, a percentage of defective products is generated. Defective products are divided into two categories of defective products with rework ability and nonrework ability through quality checks. Defective products that can be reworked are re-entered into the process and subjected to machining, but defective products that cannot be reworked are considered waste. A machine whose task is to perform machining, based on the policy of preventive maintenance and repairs in each period of production, is stopped by the operator at a specific time and immediately subjected to a preventive maintenance operation by the maintenance and repair department of the organization during an average period of time and continues the machining process. It will resume in the same way as before.

## 2. Research Background

Zipkin [2] enumerated the assumptions of the Economic Production Quality (EPQ) model as follows: fixed and fixed production rate, fixed and fixed demand rate, fixed preparation and maintenance cost, and unauthorized shortage cost, which are the main and basic assumptions of the model. The purpose of this model is to find the amount of economic production in such a way that the total costs of the system are minimized. Schwaller [3] attributed a certain percentage of a batch of received items to defective parts and also considered an inspection operation with fixed and variable costs.

In 2003, Chiu [4] developed a model by adding the assumption that instead of all defective goods, only a portion of them is reworked to achieve the desired quality, and the remaining quantities are sold at auction prices. He minimized the cost of the system by considering backward demand and a stochastic rate for defective parts, scrap, and rework of defective parts with reworkability in his model. One of the strengths of the abovementioned model is the consideration of rework.

In 2009, Monami et al. [5] presented an economic ordering model for defective quality items under backlog. That paper presented an economic order model for items with a certain percentage of defective items. The percentage of defective items follows a uniform distribution. Moreover, they did not investigate the causes of defective items and maintenance and repairs. Weinstein and Chung [6] presented a model at the production planning level that considered maintenance and repairs. They presented a hierarchical model of production planning considering the cost of maintenance and repairs. In the presented model, side contracts, deficit, and limitation of machinery resources are ignored.

Aghezzaf et al. [7] presented an integrated production and maintenance and repair model for a single production line that minimizes the total cost of emergency repairs. In their model, it is assumed that shortages are not allowed. Aghezzaf and Najid [8] presented an integrated production and maintenance and repair model for multiple production lines that minimize the total cost of emergency repairs with the help of two mathematical models, but this model does not include sales returns. Leung and Chan [9] stated that maintenance is a set of activities to maintain or improve the safety, performance, reliability, and availability of a system or components to ensure proper performance when needed in a production facility.

In 2012, Krishnamoorthi and Panayappan [10] presented an economic production model for a defective manufacturing system with sales returns and reworking. In this model, it is assumed that defective production items enter the rework process. The purpose of that model was to determine the economic order in such a way that the total costs are minimized. Taleizadeh et al. [11] developed an economic production model with random defective items and rework failures, which aimed to obtain the optimal production cycle time and production quantity and backorder quantity for the product.

In 2015, Li et al. [12] investigated the simultaneous deterioration of the product and the production system by reworking the total costs of the production system, which is allowed in this product return model, but the effect of shortage on the total costs of the system was not investigated. Hsu and Hsu [13] presented two economic production models considering incomplete production process, inspection error, shortage, and sales return. They assumed the received items included defective items identified during the inspection. This model aimed to find the optimal amount of production and the maximum amount of shortage. Two numerical examples for the stated model, in the first example, the probability distribution function of the first and second types of inspection error follows a uniform distribution, and in the second example, a beta distribution. Moreover, the model also considers the sensitivity analysis of the effect of the defective probability, the inspection error, maintenance cost, and the shortage.

Shah et al. [14] investigated the effect of the EPQ model for propensity demand with random rework and maintenance time on total system costs, but they did not consider return items in this model and did not allow for shortages. Sadeghi Rad and Nahavandi [15] have introduced a multi-objective mathematical model for order allocation in a multilevel supply chain. In this research, the order discount between different levels is considered. The goals of this model include reducing total costs and reducing the harmful effect on the environment. An exact solution approach has been used to solve this model.

Chen et al. [16] have developed a 2-level robustness model to optimize energy hub inventory considering the economic order model. In the first level, the location of the hub and the energy level are optimized. Rajeswari et al. [17] have developed an economic ordering model for substitute products. In the first step, the life cycle of the products is analyzed, and in the second step, the level of order and maintenance of the products is determined. In this research, the demand is considered as a fuzzy parameter, and the fuzzification error is investigated and analyzed.

Recently, Alimohammadi and Behnamian [18] proposed a scheduling model for preventive maintenance scheduling. This model was optimized using GAMS software. It was claimed that the results of this research could be used to reduce the amount of undistributed energy in the network and subsequently increase subscriber satisfaction. Liu et al. [19] investigated carbon emissions in construction supply chains. In this research, the asphalt supply chain in China was considered, and a scenario-based sensitivity analysis on different parameters with various increasing degrees of abatement levels was implemented. Gholizadeh et al. [20] presented a preventive maintenance model for disposable appliance supply chains under uncertainty. They applied the robust optimization approach to deal with uncertainty in the supply chain. In order to optimize this model, they applied a genetic algorithm and particle swarm optimization.

After reviewing the most important research papers in the field of production and preventive maintenance optimization, the main contribution of this research can be summarized as follows:

(1) Investigating the production, inventory, and preventive maintenance optimization simultaneously
(2) Considering the reworkable and nonreworkable products in the inventory planning
(3) Considering the inventory and shortage in each period of time
(4) Evaluating the effect of parameters on the total cost of the production system.

## 3. Research Methodology

In this section, a proposed mathematical model is presented in detail. For this purpose, first, the assumptions of the model are introduced. Next, the elements of the proposed model (indices, parameters, and variables) are provided. Finally, the mathematical formulations of the proposed model are presented.

*3.1. Model Assumptions.* To model any problem, the conditions governing that problem must be known. Therefore, the assumptions of the problem are as follows:

(1) There is a single product system, and the demand is deterministic
(2) The production rate is fixed and is more than the demand
(3) The setup time for the rework process is considered to be zero
(4) The proportion of waste is lower than that of defective goods
(5) The time to perform preventive repairs is considered as a time average in model calculations
(6) Deficiency is considered as a backdrop
(7) In this model, $x$% of items are defective and are produced at $d$ rate
(8) $\theta$% of low-quality items are considered waste and are not included in the rework process
(9) There is a space limitation for holding items.

*3.2. Parameters and Decision Variables.* The indices and parameters used in the model include input parameters and decision variables as follows:

*3.2.1. Input Parameters*

$D$: Demand rate per unit time

$P$: Production rate per unit of time

$d$: Production rate of defective items during the normal process (d = Px)

$W$: Rate of defective items from the customer ($W = Dy$)

$C0$: Preparation cost per time unit

$Ch$: Maintenance cost per time unit

$Cq$: Cost of quality improvement

$CR$: Rework cost for each unit

$Cr$: The cost of disposing of each item of waste produced

$Cp$: Production cost per unit

$Cb$: Shortage cost for each unit

$\theta$: The ratio of low-quality and defective items that cannot be reworked (waste)

$X$: Proportion of defective items during the normal process ($0 < x < 0.1$)

$y$: Proportion of defective items from the customer ($0 < y < 0.1$)

$T$: Cycle length

$t1$: Production time

$t2$: Rework time

$t3$: The period of time when the production is stopped

$t4$: shortage time

MTTR: Average time required to perform preventive maintenance and repairs (minutes)

$\gamma$: Machine maintenance and repair cost rate per unit of time

$F$: Total available volume or surface

$f$: Level or volume of each product unit.

*3.2.2. Decision Variable*

$Q^*$: Optimum production rate

$B^*$: Maximum allowed deficiency

*3.3. Mathematical Model.* The assessed production system has applied PM regarding the machine situations, technical specifications of each machine, and the standards used in its construction. In this case, the machine is put under maintenance operations based on a specific schedule before it has an accidental breakdown during a production period. In this model, It is assumed that during a production period, the machine is stopped once by the operator in the average MTTR time to take the necessary actions.

As shown in Figure 1, $t1$ is the production time, $t2$ is the rework time, $t3$ is the consumption time, and $t4$ is the shortage time. Defective items are identified in each cycle and reworked. In the production process, good and defective goods are produced (at the rate of $x$) during time $t1$. The line AO represents the slope $P - D - d - W$.

Inventory increases at the rate of $P$ and momentarily decreases at the rate of $D + d$ as the demand and W is the return from the sale by the customer. Therefore, inventory accumulates at the rate $P - D - d - W$. The number of defective items produced at time $t1$ will be equal to the amount of $xQ$. In this model, it is assumed that $\theta$ percent of defective items are considered waste.

Therefore, at the end of time $t1$, the waste ($x\theta Q$) is identified and separated from the main inventory, and the $jK$ represents this value. The remaining defective items ($Qx(1-\theta)$) will be reworked at the rate of $P$. $Q1$ is the number of healthy goods that remains after consumption during time $t1$. In fact, $Q1$ and $Q2$ are the inventory on hand after production time ($t1$) and rework time ($t2$), respectively which is shown in equations (1) and (2).

$$t_1 = \left(\frac{Q}{P}\right) = \frac{Q_1}{P - D - d - W}, \tag{1}$$

$$Q_1 = (P - D - d - W)t_1 = (P - D - d - W)\frac{Q}{P} - B. \tag{2}$$

The total number of defective items produced at time $t_1$ is calculated based on equation (3)

$$dt_1 = (Px)\left(\frac{Q}{P}\right) = QX. \tag{3}$$

Rework of low-quality items starts immediately after the end of time $t_1$ is calculated in equations (4)–(8).

$$t_2 = \frac{MS}{P} = \frac{OJ - JK}{P} = \frac{xQ - xQ\theta}{P} = \frac{(1-\theta)xQ}{P}, \tag{4}$$

$$Q_2 = Q_1 + NS = Q_1 + (P - D - d - W)t_2 = (P - D - d - W)\left(\frac{Q}{P}\right) - B + \frac{(P - D - W)xQ(1-\theta)}{P}, \tag{5}$$

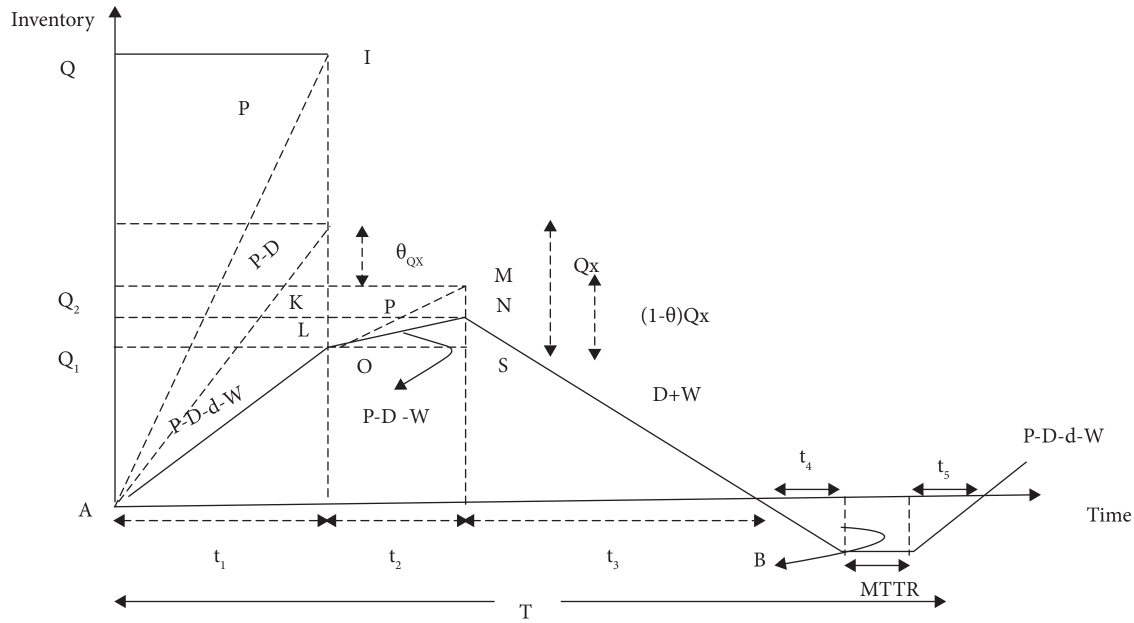


Figure 1: The inventory position of the EPQ model with rework, lack of backlog and the state of PM application.

$$t_3 = \frac{Q_2}{D+W} = \frac{1}{D+W}\left[(P-D-d-W)\left(\frac{Q}{P}\right) - B + \frac{(P-D-W)xQ(1-\theta)}{P}\right], \tag{6}$$

$$t_4 = \frac{B}{D+W}, \tag{7}$$

$$t_5 = \frac{B}{P-D-d-W}. \tag{8}$$

The time of a period (the interval between two operations starts) is obtained from the total production time, rework, product consumption, and shortage, as well as the average time spent on maintenance and repairs, which is calculated in equation (9).

$$T = \sum_{i}^{5} t_i + \text{MTTR} = \frac{Q}{D}\left[1 - X + X(1-\theta)\right] + \text{MTTR} = \frac{Q}{D}(1 - x\theta) + \text{MTTR}. \tag{9}$$

It should be noted that when $\theta = X = 0$, then $T = Q/D + \text{MTTR}$ becomes.

*3.4. Cost Calculation.* The following mathematical formulation is presented to calculate the system's total costs, including the cost of startup and maintenance.

*3.4.1. Setup Cost.* In each period, the production process starts with a fixed cost of $C_0$. Therefore, the cost of setting up the process during a period per unit of time is equal to equation (10).

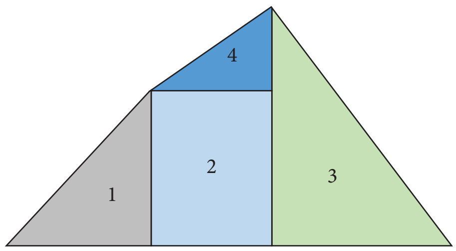


Figure 2: Inventory level.

$$SC = \frac{C_0}{T}. \tag{10}$$

3.4.2. *Maintenance Cost.* The average inventory per unit of time is calculated from the sum of the areas in Figure 2. Accordingly, the inventory is divided into four segments, and the amount of inventory is calculated in equations (11)–(14), and the total inventory level is equal to equation (15)

$$S_1 = \frac{1}{2} Q_1 t_1, \tag{11}$$

$$S_2 = Q_1 t_2, \tag{12}$$

$$S_3 = \frac{1}{2} t_1 (Q_2 - Q_1), \tag{13}$$

$$S_4 = \frac{1}{2} Q_2 t_3, \tag{14}$$

$$\begin{aligned} S_1 + S_2 + S_3 + S_4 &= \left[\frac{1}{2} Q_1 t_1 + Q_1 t_2 + \frac{1}{2} t_1 (Q_2 - Q_1) + \frac{1}{2} Q_2 t_3\right] \\ &= \frac{1}{2}\Bigg[\left\{(P - D - d - w)\frac{Q}{P} - B\right\}\frac{Q}{P} + 2\left\{(P - D - d - w)\frac{Q}{P} - B\right\} \\ &\quad \cdot \frac{xQ(1-\theta)}{P} + \frac{(P - D - w)x^2 Q^2 (1-\theta)^2}{P^2} \\ &\quad + \frac{1}{D + W}\left\{(P - D - d - w)\frac{Q}{P} - B + \frac{(P - D - w)x(1-\theta)Q}{P}\right\}^2\Bigg] \\ &\quad - \frac{1}{2P^2(D+W)}\big[P(D+W)BQ + 2P(D+W)BxQ(1-\theta) - P^2B^2 \\ &\quad + 2P(P - D - d - w)QB + 2P(P - D - w)xQB(1-\theta)\big] \\ &= \frac{Q^2}{2PD}\left[P(1-\theta X)^2 - D(1+y)\left(1 + x - 2x\theta + X^2(1-\theta)^2\right)\right] \\ &\quad - \frac{BQ}{2PD}\left[2P(1-\theta x) - D(1+y) + \frac{B^2}{2D}\right]. \end{aligned} \tag{15}$$

As a result, the cost of maintaining inventory during a period per unit of time is calculated based on equation (16).

$$HC = C_h \cdot \frac{C_h}{T}\left[\frac{Q^2}{2PD}\left[P(1-\theta x)^2 - D(1+y)\left(1 + x - 2x\theta + x^2(1-\theta)^2\right)\right] - \frac{BQ}{2PD}\left[2P(1-\theta x) - D(1+y)\right] + \frac{B^2}{2D}\right]. \tag{16}$$

3.4.3. *The Cost of Shortage per Unit of Time.* According to Figure 1, in order to calculate the shortage cost, it is first necessary to calculate the average shortage in the cycle. Since at time $t1$ or $t3$, the inventory level in the hands of the system is positive. In this period, the production system will not face a shortage. Only at times $t5$ and $t4$ the demand faces a shortage. The average shortage during a period per time unit is calculated from the sum of the areas in Fig. 3 as follows:

The shortage level is obtained from the sum of the three levels and is calculated based on equations (17)–(19).

$$S_1 = \frac{1}{2} B t_4, \quad (17)$$

$$s_2 = \text{MTTTR}.B, \quad (18)$$

$$S_3 = \frac{1}{2} B t_5, \quad (19)$$

$$\begin{aligned} B_S &= S_1 + S_2 + S_3 \\ &= \frac{1}{2} B t_4 + \text{MTTTR}.B + \frac{1}{2} B t_5 \\ &= \frac{B^2 (P - D)}{2 (P - D - d)} + \text{MTTR} * B. \end{aligned} \quad (20)$$

Therefore, the cost of backlog shortage per unit of time is calculated as equation (21).

$$\begin{aligned} BA &= C_b.\overline{B_s} \\ &= \frac{C_b}{T} \left[ \frac{B^2 \ (P - D)}{P - D - d} + \text{MTTR} * B \right]. \end{aligned} \quad (21)$$

*3.4.4. Production Cost per Unit of Time.* The cost of producing goods during a period of time is calculated using equation (22).

$$PC = \frac{C_p Q}{T}. \quad (22)$$

*3.4.5. PM Const and Repairs per Unit of Time.* The cost of performing preventive maintenance and repairs on the machine during a period per unit of time is equal to equation (23).

$$MT = \frac{\gamma \text{MTTR}}{T}. \quad (23)$$

*3.4.6. Quality Check Cost for Defective Items.* The quality of defective items is checked and classified into two groups: waste and reworkable. The cost of quality inspection for defective items is equal to equation (24).

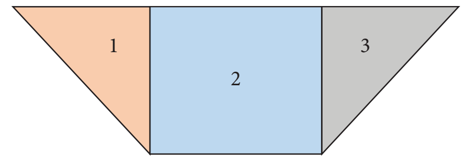


FIGURE 3: Shortage level.

TABLE 1: The value of each effective parameter.

| Parameter | Value |
|---|---|
| $D$ | 4500 unit/year |
| $C_h$ | \$10 |
| $C_o$ | \$100 |
| CQ | \$5 |
| $C_R$ | \$5 |
| $C_r$ | \$1 |
| CP | \$50 |
| $C_b$ | \$10 |
| $\theta$ | 0.2 |
| $X$ | 0.001 |
| $\gamma$ | 20 |
| $Y$ | 0.001 |
| $f$ | 0.9 $m^3$ |
| MTTR | 0.222 |
| $F$ | 900 $m^3$ |

$$\begin{aligned} QC &= \frac{\theta C_Q d t_1}{T} \\ &= \frac{\theta C_Q Q x}{T}. \end{aligned} \quad (24)$$

The rework cost for low-quality (defective) items that can be reworked is equal to equation (25).

$$RC = \frac{(1 - \theta) Q x C_R}{T}. \quad (25)$$

Poor quality and defective items that are placed in the waste group will not be able to be reworked. The cost of disposal of items that are considered waste will be as equation (26).

$$rc = \frac{\theta Q x C_r}{T}. \quad (26)$$

Therefore, the total cost of the system per unit of time is obtained from the sum of the costs of setup, maintenance, shortage, inspection, preventive maintenance, rework, waste disposal and the quality cost of defective items, which is shown in equation (27).

$$\begin{aligned} TC = \frac{1}{T} \Bigg[ & C_0 + C_h \left[ \frac{Q^2}{2PD} \left[ P (1 - \theta x)^2 - D (1 + y) \left( 1 + x - 2x\theta + x^2 (1 - \theta x)^2 \right) \right] - \frac{BQ}{2PD} (1 - \theta x) - D\ (1 + y) \right] + \frac{B^2}{2D} \Bigg] \\ & + C_b \left[ \frac{B^2 (P - D)}{2 (P - D - d)} + \text{MTTR} * B \right] + C_p Q + \gamma \text{MTTR} + \theta C_Q Q x + (1 - \theta)\ Q x C_R + \theta Q x C_r. \end{aligned} \quad (27)$$

Table 2: The optimal solution of the proposed mathematical model.

| | $Q^*$ | $T^*$ | $TC^*$ |
|---|---|---|---|
| LINGO | 1000 | 0.44 | 227014.1 |

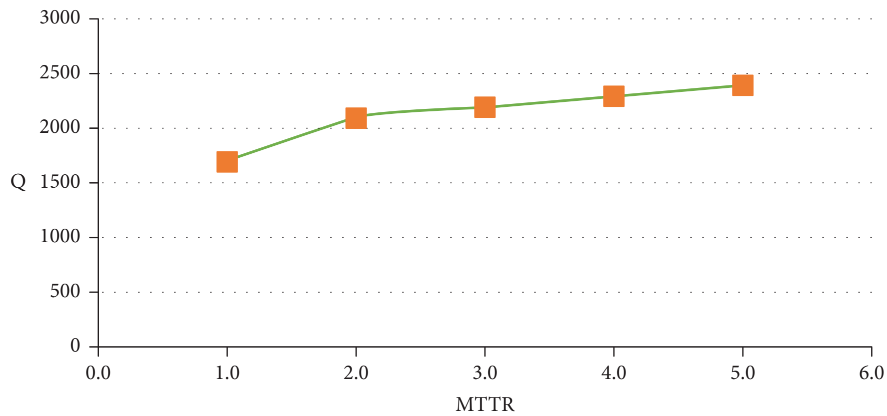


Figure 4: The effect of MTTR on the amount of economic production order.

By placing $T$ in equation(27), the cost function will be in the form of equation (28).

$$\begin{aligned}TC =& \frac{DC_0}{Q(1-x\theta)+(\text{MTTR}*D)} + \frac{DC_h}{Q(1-x\theta)+(\text{MTTR}*D)}\left(\frac{Q^2}{2PD}\left[P(1-\theta x)^2 - D(1+y)\left(1+x-2x\theta+x^2\right)(1-\theta x)^2\right]\right)\\ &- \frac{BQ\,DC_h}{2PD(Q(1-x\theta)+(\text{MTTR}*D))}\left[2P(1-\theta x)-D(1+y)\right] + \frac{B^2C_h}{2(Q(1-x\theta)+(\text{MTTR}*D))} + \frac{B^2(P-D)C_b}{2(P-D-d)(Q(1-x\theta)+(\text{MTTR}*D))}\\ &+ \frac{D(\text{MTTR}*B)}{Q(1-x\theta)+(\text{MTTR}*D)} + \frac{C_PQD}{Q(1-x\theta)+(\text{MTTR}*D)} + \frac{D\gamma\text{MTTR}}{Q(1-x\theta)+(\text{MTTR}*D)} + \frac{D\theta C_QQx}{Q(1-x\theta)+(\text{MTTR}*D)}\\ &+ \frac{D(1-\theta)QxC_R}{Q(1-x\theta)+(\text{MTTR}*D)} + \frac{D\theta QxC_r}{Q(1-x\theta)+(\text{MTTR}*D)}.\end{aligned} \tag{28}$$

*3.5. Constraints.* In most inventory models, it is assumed that there are no restrictions such as storage capacity, holding time, etc., while any of these constraints may exist in the real world. For example, it is possible that the storage capacity is limited; in this case, constraints are placed on the storage level or otherwise. Equation (29). shows the limitation of warehouse capacity to store items.

$$F \times Q \leq F. \tag{29}$$

It should be noted that the optimal solution of the proposed mathematical model can be found by finding the extreme point of the cost function, which has been proved in Appendix.

## 4. Numerical Results

In order to analyze, better understand and validate the presented model, a numerical example whose parameter values are determined based on Krishnamoorthi and Panayappan [10] with slight changes is solved and analyzed.

In this regard, the selected value for each effective parameter is provided in Table 1.

According to the solution of the proposed model based on the above data, the obtained results are according to Table 2.

The results in Table 2 shows that the best ordering quality is 1000 unit of products, and the period of ordering is 0.44 of a year. In this solution, the optimal value of the objective function is 224014.1, which is the least total cost of the production system.

*4.1. Sensitivity Analysis.* The sensitivity analysis is applied to analyze the model and investigate the effect of some important and influential parameters on the optimal solution of the proposed model.

*4.1.1. Sensitivity Analysis of Maintenance Time (MTTR).* In order to study the effect of repair and maintenance time on the optimal solution of the presented model, we use the sensitivity analysis of the parameter MTTR. The model increases

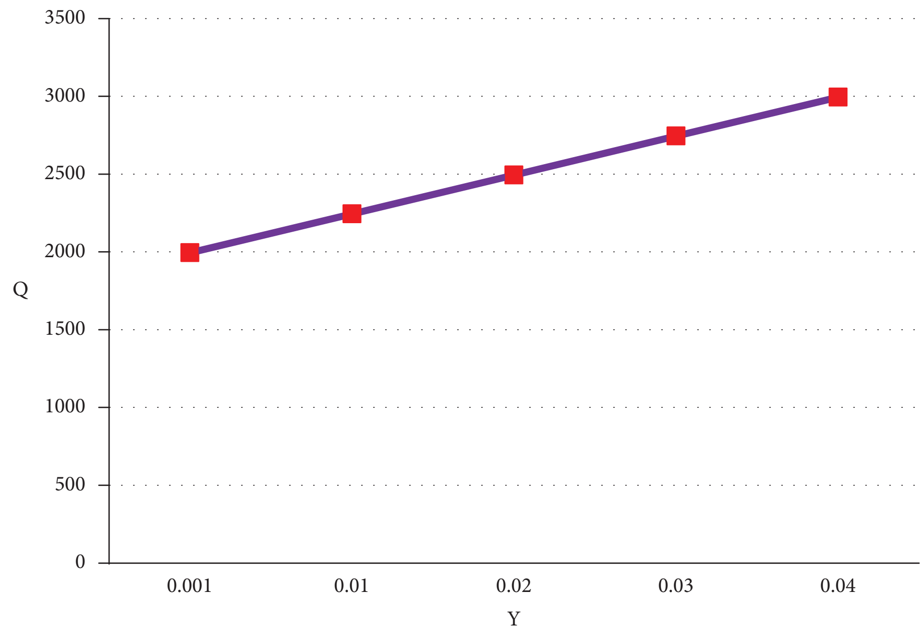


Figure 5: The effect of parameter Y on the amount of economic production order.

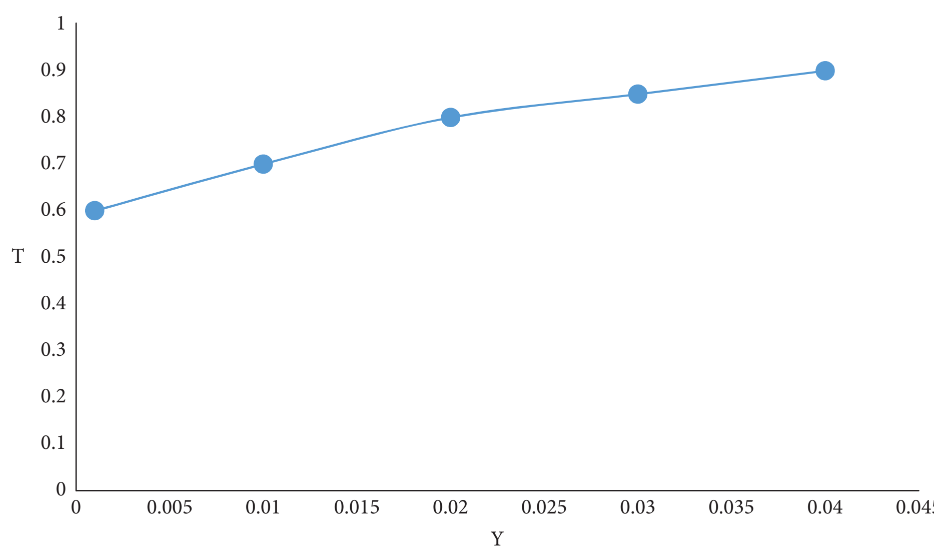


Figure 6: The effect of parameter Y on the time of economic production order.

and decreases the maintenance time by a fixed amount. The results of this sensitivity analysis are shown in Figure 4.

As it is clear from Figure 4, with the increase in the time of repairs and preventive maintenance, the curve first has a steep slope and then has an upward trend with a gentler slope. This curve shows that the longer the time of preventive maintenance on the machine, the more accurately and at a higher level it will lead to an increase in the optimal production rate.

*4.1.2. Sensitivity Analysis for Parameter ? (Proportion of Defective Items from the Customer).* The sensitivity analysis is implemented in order to investigate the effect of the proportion of defective items from the customer on the economic order and the optimal production time.

As can be seen in Figures 5 and 6, with the increase in the proportion of defective goods from the customer ($Y$), the optimal amount of the order ($Q^*$) and the optimal time ($T^*$) will also increase.

## 5. Discussion and Conclusion

Determining the optimal size of the production batch in such a way that the total costs are minimized has always been one of the main topics of scientific and industrial research [21–23]. In addition, nowadays, in industrial units, due to the competitiveness of production and the need to reduce costs and deliver goods on time, equipment availability and the prevention of unexpected stops are of particular importance.

Investigation and research on the number of economic production, minimization of production costs and related issues have continued from the past to the present. So far, lots of optimization models have been presented in this field. Among the issues that have not received much attention is the issue of production machine breakdowns. It is evident that the production machine may encounter a breakdown during production, which justifies the necessity of applying a proper maintenance and repair policy.

In recent years, one of the basic foundations in any organization is equipment, machinery and human resource [24]. On the other hand, to increase productivity and efficiency, provide services and achieve global standards, special attention should be paid to increasing the efficiency of machines and reducing the costs of repairing and stopping machines. Therefore, research in this field is receiving more attention.

In real-world conditions, the success of organizations in providing services and increasing their quality depends on various reasons for having a maintenance and repair system. It is suitable for one of the essential topics of every organization.

The presented mathematical model combines the determination of the economic production amount and the level of preventive maintenance and repairs, taking into account the allowed shortage for a production process. A production system is intended to produce a product item. The production process is such that after performing operations on a batch of input materials, in addition to producing healthy products, a percentage of defective products are also produced. In this research, defective products are divided into two categories through quality inspection: defective products with reworkability and non-reworkability (waste). Defective products with reworkability are reentered into the process and subjected to machining, but defective products are not reworkable and are considered waste.

In order to show the managerial insights of this research, it can be indicated that based on the preventive maintenance policy, the repairs in each period of production are stopped by the maintenance and repairs department of the organization during an average time. This stoppage of production leads to disruption in the process of meeting the needs of customers. Accordingly, determining the appropriate time for preventive maintenance, in addition to reducing the costs of the production system, can have the least issue in meeting the demand.

In order to extend this research for future research, the following developments are suggested.

(1) Increase the number of supply chain echelons and consider the supplier, producer and buyer together.
(2) Considering several suppliers and buyers simultaneously, which in this case makes the model closer to the real world.
(3) Considering the production of the multi-product in the preventive maintenance planning
(4) Considering the fuzzy state for production rate and demand parameters.
(5) In addition to the limitation of warehouse space, other constraints, such as budget and raw material supply, can also be considered to be closer to the real world.

## Data Availability

The data used in this study are available from the corresponding author upon request.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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## Supplementary Materials

In the provided Appendix, proof of the convexity of the cost function is provided. In other words, in order to ensure that a local solution obtained is also its absolute minimum solution, the convexity of this function will be checked. (*Supplementary Materials*)

## References


[1] E. B. Tirkolaee, A. Goli, and A. Mardani, "A novel two-echelon hierarchical location-allocation-routing optimization for green energy-efficient logistics systems," *Annals of Operations Research*, pp. 1–29, 2021.

[2] H. P. Zipkin, *Foundations of Inventory Systems*, McGraw Hill, New York, NY, USA, 2000.

[3] R. L. Schwaller, "EOQ under inspection costs," *Production and Inventory Management Journal*, vol. 29, no. 3, p. 22, 1988.

[4] Y. P. Chiu, "Determining the optimal lot size for the finite production model with random defective rate, the rework process, and backlogging," *Engineering Optimization*, vol. 35, no. 4, pp. 427–437, 2003.

[5] M. D. Roy, S. S. Sana, and K. Chaudhuri, "An economic order quantity model of imperfect quality items with partial backlogging," *International Journal of Systems Science*, vol. 42, no. 8, pp. 1409–1419, 2011.

[6] L. Weinstein and C. H. Chung, "Integrating maintenance and production decisions in a hierarchical production planning environment," *Computers and Operations Research*, vol. 26, no. 10-11, pp. 1059–1074, 1999.

[7] E. H. Aghezzaf, M. A. Jamali, and D. Ait-Kadi, "An integrated production and preventive maintenance planning model," *European Journal of Operational Research*, vol. 181, no. 2, pp. 679–685, 2007.

[8] E. H. Aghezzaf and N. M. Najid, "Integrated production planning and preventive maintenance in deteriorating production systems," *Information Sciences*, vol. 178, no. 17, pp. 3382–3392, 2008.

[9] S. C. Leung and S. S. Chan, "A goal programming model for aggregate production planning with resource utilization constraint," *Computers and Industrial Engineering*, vol. 56, no. 3, pp. 1053–1064, 2009.

[10] C. Krishnamoorthi and S. Panayappan, "An EPQ model with imperfect production systems with rework of regular production and sales return," *American Journal of Operations Research*, vol. 2, 2012.

[11] A. A. Taleizadeh, H. M. Wee, and S. G. Jalali-Naini, "Economic production quantity model with repair failure and limited capacity," *Applied Mathematical Modelling*, vol. 37, no. 5, pp. 2765–2774, 2013.

[12] N. Li, F. T. S. Chan, S. H. Chung, and A. H. Tai, "An EPQ model for deteriorating production system and items with rework," *Mathematical Problems in Engineering*, vol. 2015, Article ID 957970, 10 pages, 2015.

[13] J. T. Hsu and L. F. Hsu, "Two EPQ models with imperfect production processes, inspection errors, planned backorders, and sales returns," *Computers and Industrial Engineering*, vol. 64, no. 1, pp. 389–402, 2013.

[14] N. H. Shah, D. G. Patel, and D. B. Shah, "EPQ model for imperfect production processes with rework and random preventive machine time for deteriorating items and trended demand," *Yugoslav Journal of Operations Research*, vol. 25, no. 3, pp. 425–443, 2015.

[15] R. Sadeghi Rad and N. Nahavandi, "A novel multi-objective optimization model for integrated problem of green closed loop supply chain network design and quantity discount," *Journal of Cleaner Production*, vol. 196, pp. 1549–1565, 2018.

[16] C. Chen, H. Sun, X. Shen, Y. Guo, Q. Guo, and T. Xia, "Two-stage robust planning-operationco-optimization of energy hub considering precise energy storage economic model," *Applied Energy*, vol. 252, Article ID 113372, 2019.

[17] S. Rajeswari, C. Sugapriya, D. Nagarajan, and J. Kavikumar, "Optimization in fuzzy economic order quantity model involving pentagonal fuzzy parameter," *International Journal of Fuzzy Systems*, vol. 24, no. 1, pp. 44–56, 2022.

[18] M. Alimohammadi and J. Behnamian, "Preventive maintenance scheduling of electricity distribution network feeders to reduce undistributed energy: a case study in Iran," *Electric Power Systems Research*, vol. 201, Article ID 107509, 2021.

[19] Y. Liu, Y. Wang, P. Lyu, S. Hu, L. Yang, and G. Gao, "Rethinking the carbon dioxide emissions of road sector: integrating advanced vehicle technologies and construction supply chains mitigation options under decarbonization plans," *Journal of Cleaner Production*, vol. 321, Article ID 128769, 2021.

[20] H. Gholizadeh, M. Chaleshigar, and H. Fazlollahtabar, "Robust optimization of uncertainty-based preventive maintenance model for scheduling series–parallel production systems (real case: disposable appliances production)," *ISA Transactions*, vol. 128, pp. 54–67, 2022.

[21] A. Goli and H. Mohammadi, "Developing a sustainable operational management system using hybrid Shapley value and Multimoora method: case study petrochemical supply chain," *Environment, Development and Sustainability*, vol. 24, no. 9, pp. 10540–10569, 2022.

[22] A. Goli, A. Ala, and S. Mirjalili, "A robust possibilistic programming framework for designing an organ transplant supply chain under uncertainty," *Annals of Operations Research*, pp. 1–38, 2022.

[23] A. Goli, A. M. Golmohammadi, and J. L. Verdegay, "Two-echelon electric vehicle routing problem with a developed moth-flame meta-heuristic algorithm," *Operations Management Research*, vol. 15, no. 3-4, pp. 891–912, 2022.

[24] Y. Nemati, M. Madhoushi, and A. Safaei Ghadikolaei, "Towards supply chain planning integration: uncertainty analysis using fuzzy mathematical programming approach in a plastic forming company," *Iranian Journal of Management Studies*, vol. 10, no. 2, pp. 335–364, 2017.