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Research Article

Research on a Model for an Empirical Analysis of Inherent Defect Insurance Based on Ruin Theory

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The system of inherent defect insurance is an important measure to serve the real economy through financial means and improve the quality of construction projects, which is the future development direction of China's construction industry. However, the related research is not perfect due to the short implementation of the insurance. This could bring risks to the promotion of insurance for companies. Insurance ruin theory is an important method in risk management theory, so adopting it to manage the risk inherent defect insurance from the perspective of insurance companies is vital. The research starts with the classic insurance ruin theory and determines the coefficient of premium collection from the perspective of claim settlement distribution expectations. Furthermore, the approximate distribution of claim settlement is deduced, and a comprehensive risk assessment model is constructed. Finally, based on the data of insurance actuarial practice in Shanghai, both the ruin probability of inherent defect insurance in each insuring term and its average required initial reserve are calculated, which provides the analyses on the risks and main subitems of inherent defect insurance as well as relevant suggestions. Finally, the sensitivity analysis is used to further analyse the risk of different insurance stages of IDI, and the relevant measures are proposed. The research can provide theoretical assistances for insurance companies to carry out effective risk management and provide model tools to make scientific decisions.

1. Introduction

Construction quality pertains to the safety of people's lives and property, cities' future and heritage, and the level of new-type urbanisation development. Promoting construction insurance is necessary to better facilitate financial services for the real economy, enhance construction works' quality, improve the city's business environment, and reduce complaints from residents.

Inherent defect insurance (IDI) refers to insurance safeguarded by the construction unit within the insurance scope and period stipulated in the contract; the insurance company undertakes the liability of compensation for damage concerning the insured project caused by potential defects in engineering quality [1].

Insurance for construction engineering quality originated in France, became prevalent in Europe, and gradually developed in America, Asia, and Africa. It is an indispensable insurance service in the construction industry that exists in diversified forms and has a history of over 30 years; it features a mature mechanism that guarantees construction quality insurance and is applicable worldwide [2]. However, in China, the implementation of insurance for potentially defective engineering quality is still poor, the market mechanism is immature, the vast majority of insurance projects have not entered the claim settlement period, and no complete insurance project data exist for actuarial analysis. Therefore, the current insurance pricing for potentially defective engineering tends to be empirical; professional institutions, including insurance companies, establish the premium rate. However, the empirical rate does not pass actuarial risk analysis; hence, it may precipitate ruin risk for insurance projects with potential engineering quality defects, thus affecting insurance companies' management and operation. Improving the risk analysis for construction quality insurance—by developing a set of practical and effective models to determine risk probability and quantitative risk insurance—would help address among the most concerning problems faced by risk insurance companies, strengthen these companies' position when providing insurance for potentially defective engineering quality, and develop the theoretical basis for scientific decision-making and modelling tools in this respect.

In the operation of China's insurance companies, insurance premium is the main source of income, and claim settlement is the main risk factor. Insurance companies must fully consider the risks they face in order to ensure the operation of insurance companies, and the research on insurance ruin theory mainly focuses on how insurance companies estimate risks they face. In order to better explore risks faced by IDI, this paper introduces ruin theory to analyse its risks. Ruin theory is a general theory for operators and decision makers to quantitatively analyse and predict risks. Under a series of realistic assumptions, it uses probability statistics, random processes, and other tools to establish a model of random risk in insurance business and study the nature of the model and then analyse the ruin risk quantitatively. It provides theoretical and technical support for insurers to carry out effective risk management and control in reality.

Considering this situation, this study attempts to build a risk analysis model to assess insurance for potentially defective engineering quality based on insurance ruin theory. The remainder of the paper is arranged as follows: Section 2 reviews relevant national and international research. Section 3 introduces the primary theoretical basis used herein, that is, insurance ruin theory. According to the insurance ruin theory, Section 4 constructs a risk model to assess insurance for potentially defective engineering quality through probability analysis from a single-division and subdivisional engineering perspective. Section 5 conducts empirical analysis and sensitivity analysis based on specific insurance companies' insurance practice data in Shanghai using the insurance risk model adopted in this research. Finally, Section 6 elucidates the conclusion regarding the risk analysis of insurance for potentially defective engineering quality, providing a decision-making tool for promoting insurance for potentially defective engineering quality from the insurance companies' perspective.

2. Literature Review

France was the first country to utilise construction quality insurance. France established a relatively complete construction quality insurance framework system in 1978 when the Spinetta statute was enacted to ensure construction engineering quality for ten years with defect insurance [2]. Josephson and Hammarlund collected and sorted nearly 3,000 defects and corresponding root causes by monitoring construction projects and explained the effectiveness of preventive measures based on the development and function of IDI [3]. Georgiou et al. conducted a comparative study on engineering quality defects in houses developed and constructed by real estate developers and built by individual owners in Victoria, Australia, concluding that people should actively participate in insurance for potentially defective engineering quality under different construction conditions [4]. Lavers studied the problem of economic loss allocation from the perspective of the cost bearers of construction failure by considering different loss-sharing patterns among developers, building producers, and end users, who may be purchasers or other occupiers [5]. Considering that litigation owing to defects in construction projects has increased, Manifoff and Randy proposed the inclusion of insurance in the engineering quality assurance system, aiming to improve engineering quality and reduce related litigation [6].

Lavers described the treatment of liability and insurance issues arising after construction in 19 different legal systems, introduced the operational process of the insurance system based on the analysis of relevant cases in various countries, and demonstrated insurance's effectiveness against potential defects in construction projects' quality [7]. Some researchers believe that builders working in residential construction can partially transfer liability for claim settlement pertaining to defects in engineering quality through insurance to improve the maximum obtainable income [8–10].

Knocke described, in detail, insurance systems—for construction engineering quality—in Australia, Spain, and the United Kingdom by conducting a comparative study [11] and, consequently, offering reform suggestions for various countries and establishing a complete model of quality insurance for construction projects [12]. By analysing the French insurance system model's success, Lovergrove suggested that the construction engineering insurance model must be improved to adapt to Australia's national conditions [13]. Du and Qi summarised the typical development experience of insurance for potentially defective engineering quality and the current exploration situation in China and analysed relevant subjects, highlighting that the key to implementing quality insurance for engineering defects is guiding the relevant parties to participate in multiple measures actively and achieve a win-win situation [14]. Some scholars have analysed the causes of potential defects in engineering quality and studied the mechanism from the perspective of real estate contractors and design units [15–17].

Based on an analysis of local quality insurance for potentially defective engineering practices, Smith concluded that quality defects in engineering projects in Las Vegas were caused by contractors, increasing related complaints and triggering the rapid growth of insurance in the region [18]. Ray confirmed this finding by examining the implementation of insurance for potentially defective engineering quality in Oregon, United States, which promoted the development of such insurance [19]. Kane analysed the impact and prevalence of various potential defects in buildings from the design perspective, collected data, and compared potential defects with relevant professionals to develop design standards and codes for major defects [20]. Meng examined performance in 103 construction projects and found that 37 were late, 26 were over budget, and 90 had quality defects, indicating that defects are the main factors precipitating poor project performance [21]. Chong and Low asserted that reducing defects is a key task for construction projects. Therefore, the key to improving engineering quality is understanding the causes of defects and formulating defect prevention and reduction strategies by identifying defects critical to engineering quality [22]. However, potential engineering quality defects are challenging to detect at the delivery and acceptance stage of a completed project. Moreover, compared with other defects, they are more intractable and more likely to cause serious consequences for the project [23].

Insurance against potential defects in construction quality has been piloted in several provinces and cities in China. With the development and promotion of insurance for potentially defective engineering quality in local insurance practice, the insurance and compensation amounts associated with a project with insurance for potentially defective engineering quality will increase, and the corresponding insurance risk will also increase. Therefore, developing a perfect and effective risk assessment model as the theoretical basis for the promotion of insurance for potentially defective engineering quality is necessary.

Recently, scholars have been focusing on analysing insurance risk from the insurance perspective, which entails analysing the influencing factors of insurance to explore the insurance risk distribution further. Tartarini et al. analysed the risk of agricultural insurance and developed a new framework through metamodelling to derive an index characterised by low insurance-based risk. In addition, Tartarini et al. have quantified insurance risk and conducted empirical research on frost and dry heat damage to barley and soft and hard wheat in two regions of Italy to verify the evaluation index's reliability [24]. Fernandes-Hugo amd Ferreira-Fernando measured health insurance risk assessment using cognitive mapping and multicriteria decision analysis and combined cognitive mapping and the measurement of attractiveness to construct a unique nonparametric decision support system for the risk analysis of individual private health insurance [25]. Su and Yu proposed a novel method to estimate the Gerger–Shiu function in classical insurance risk models based on the expansion of the Laguerre series regarding sample claim settlement number and magnitude and verified that the estimator tends towards a normal distribution. Su and Yu quantitatively assessed insurance risk [26], and Guillermo et al. described the risk relationship and common characteristics of flood risk in the industry from the perspective of flood insurance risk. Accordingly, Guillermo et al. constructed a numerical model assessing flood events' potential impact to investigate flood insurance's management, pricing, and transfer risk. In addition, Guillermo et al. used these aspects to analyse and evaluate flood insurance risk [27].

The literature has predominantly focused on evaluating insurance risk posed by the natural environment, but insurance risk is frequently related to human factors; therefore, insurance risk evaluation must be studied from human factors' perspective. However, an increasing number of scholars have been analysing insurance risk based on the insurance ruin model, which primarily analyses the accumulation of insurance companies' earnings over time [28]. Numerous scholars have analysed the mathematical probability model and explored the characteristic changes in bankruptcy models under different conditions [29–31]. In addition, several researchers have tried applying the ruin model to specific insurance practices and have used mathematical methods to analyse insurance risks [32–37]. Insurance ruin theory—as the basic theory of insurance actuarial science, particularly insurance risk analysis—has broad applicability in actuarial practice. Therefore, this theory can be utilised for the risk analysis of insurance for potentially defective engineering quality; model optimisation can be facilitated based on the insurance ruin model.

3. Methods

3.1. Inherent Defect Insurance. The IDI's insurance period currently implemented in China is generally 10 years, and the coverage scope is specified in the individual contract. However, it includes subprojects—such as foundation, main structure, cracks, and thermal insulation and waterproofing. For quality problems in the engineering division and subdivisions agreed upon during the insurance period, the insurance company provides funds for the purchaser to provide compensation. IDI claim settlement does not involve monetary compensation; it is provided through direct maintenance. Therefore, the insurance company's claim settlement strongly relates to the defects in engineering quality.

IDI is commercial, such as other types of insurance. Insurance companies earn revenue by collecting insurance premiums and making claim settlement against insureds in the event of potential defects in engineering quality. IDI's occurrence involves great uncertainty, and the compensation amount is generally high. Therefore, IDI's implementation entails certain commercial risks, and analysing the risks from insurance projects' perspective scientifically is vital.

3.2. Insurance Ruin Theory. Risk theory is a general theory of quantitative analysis, and its research process generally includes risk identification, risk model establishment, risk analysis, risk decision-making, and risk control. Ruin theory is the core content of risk theory, which studies the influence of risk on insurer's solvency from the perspective of claim settlement by ignoring the influence of return on investment, interest rate, and inflation. Solvency is a variable affected by a variety of uncertain factors and is usually regarded as a random process in risk theory.

Swedish actuary Filip Lundberg proposed insurance ruin theory. In the original insurance ruin model, only the basic initial investment, premium income, and insurance project claims settlement process are considered. The relationship is presented as follows:

$$P(t) = u_0 + \mathrm{it} - S(t). \tag{1}$$

Herein, u_0 represents the initial investment in the insurance project, i represents the fixed premium income per unit of time, and $S(t)$ represents the claim settlement process that occurred within the time $[0, t]$. The claims process is represented as $S(t) = \sum_{j=1}^{N_t} x_j$, where N_t and x_j in the formula represent the number of claim settlement and the claim settlement amount of the j-th claim settlement within the insurance period, respectively.

Figure 1 reflects an example of an insurance ruin process' trajectory. Figure 1 illustrates that the insurance program's surplus $P(t)$ is a function of time. As the premium rate is constant, the solid lines' slopes in the figure are equal. The fault point in the figure represents the claim settlement in the insurance process, and the moment t_0—when $P(t) < 0$ appears for the first time—is the ruin moment of the insurance item, which indicates that the insurance item's surplus is negative. A ruin moment's probability in the insurance period reflects the insurance ruin's probability.

Assuming the derivative of t based on insurance ruin theory, equation (2) can be derived as follows:

$$\frac{\mathrm{dP}(t)}{\mathrm{d}t} = i - \frac{\mathrm{dS}(t)}{\mathrm{d}t}. \tag{2}$$

Equation (2) clearly reveals that the ruin risk of an insurance project is predominantly related to the premium income per unit time and claim settlement process of the insurance project. However, the premium income is frequently a function of the claims settlement process; hence, research on insurance ruin theory is generally conducted from the perspective of the claims settlement process' distribution.

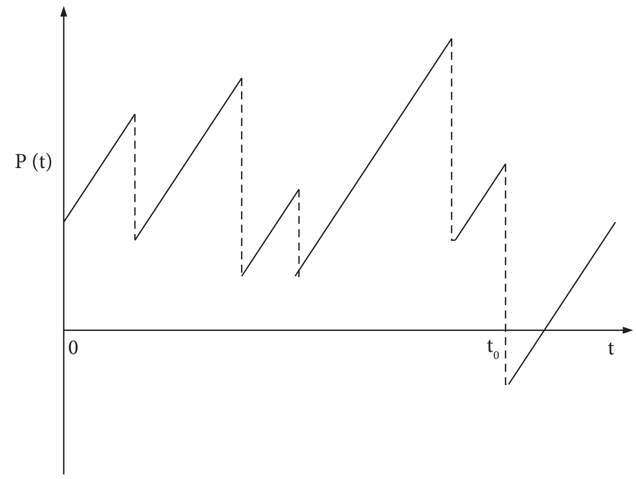

Figure 1: Insurance ruin process.

4. Insurance Risk Model of Inherent Defect Insurance Based on Ruin Theory

Insurance ruin theory has a good effect in the field of risk management, especially in the quantitative analysis of insurance risk. Therefore, insurance ruin theory is used to construct the risk model of IDI to measure the risk of IDI.

In conclusion, the insurance project risks with which insurance companies are concerned predominantly arise from the insurance project claims process. However, the IDI claims process represents the appearance of engineering quality defects. The final distribution of the claims process is related to the frequency and amount of a single claim settlement. Therefore, this study also examines the IDI's insurance risk by considering the frequency of the occurrence of engineering quality defects and the degree of damage. As the insurance underwriting item involves more than one engineering division and subdivision, the method to assess the IDI's insurance risk involves analysing the part first and then analysing the whole. Exploring the risk distribution of each related engineering division and subdivision is finally aggregated into the overall insurance risk distribution. The insurance risk model's operation mechanism is presented in Figure 2.

4.1. Distribution of the Claims Settlement Process under Single-Division and Subdivisional Engineering. In actuarial models, the Poisson distribution is generally used to fit discrete frequency distributions. The Poisson distribution is a discrete distribution based on the binomial distribution, which means the probability of success k times in n trials that satisfy binomial, independent, and repeatable conditions. It can effectively describe the distribution of times.

This study assumes that the occurrence frequency N_t of latent defects in engineering quality obeys the Poisson distribution, which means $N_t \sim P(\lambda)$. According to the properties of the Poisson distribution, we know that the expectation and variance of this distribution are both λ. Therefore, parameter estimation can be performed with the help of historical data on IDI. This research assumes that there are m items of engineering divisions and subdivisions covered by IDI, the number of claims settlement for the k-th engineering divisions and subdivisions in the first unit of time is $N_1(k)$, the total number of insurance policies is N, and the cumulative effect of the project's quality problems is $\delta_k(t)$. From this, we can conclude that the parameter determination formula for the frequency distribution of this subitem engineering problem is expressed as follows:

$$\lambda_k = \frac{N_1(k)}{N} \delta_k(t). \tag{3}$$

The distribution of the claim settlement amount can be considered an independent and identical distribution for specific engineering divisions and subdivisions; hence, it can be considered that the claim settlement process of specific engineering divisions and subdivisions obeys the compound Poisson distribution [38]. Therefore, it is only necessary to determine the distribution of the claim settlement amount of the engineering divisions and subdivisions; thereafter, the expression of the overall distribution of the engineering division and subdivision claim settlement process can be obtained. For certain engineering divisions and subdivisions, the compensation amount can be considered to obey the exponential distribution [39], and its probability density function is $(x) = 1/\theta e^{-x/\theta}$. As per the nature of the exponential distribution, its expectation is the reciprocal of the parameter; hence, the average payout per insurance policy can be used to determine the parameter. This study assumes that the number of claim settlement for the k-

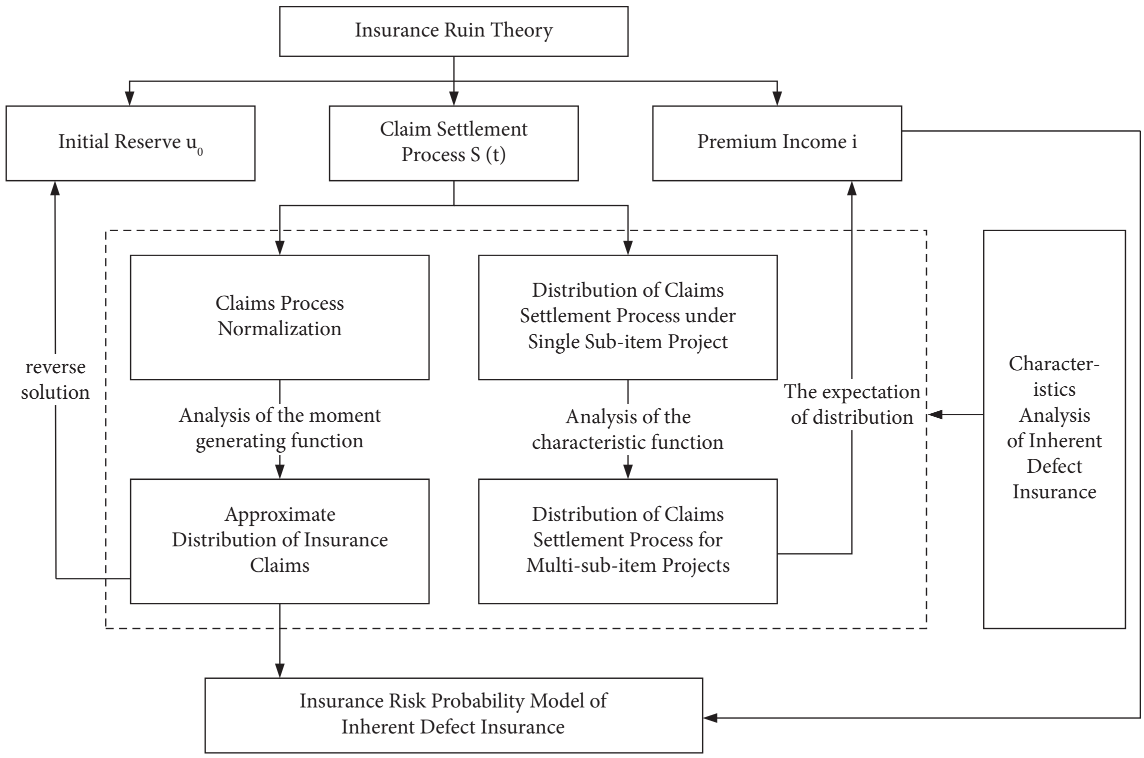

Figure 2: Operation mechanism of the insurance risk model for inherent defect insurance.

thsubitem of IDI in the first unit of time is $N_1(k)$, and the total insurance claim settlement amount of this subitem in the corresponding time is W_k. From this, the probability density function and distribution function of the IDI claim settlement amount can be calculated as per the following equations:

$$f(x)=\frac{N_1(k)}{W_k}e^{\left(-N_1(k)/W_k\right)\cdot x},\tag{4}$$

$$F(x)=\begin{cases}1-e^{\left(-N_1(k)/W_k\right)\cdot x}, & x\geq 0,\\ 0, & x<0.\end{cases}\tag{5}$$

4.2. Aggregate Distribution of the Claims Settlement Process for Multidivisional and Subdivisional Engineering. If multiple composite Poisson distributions are superimposed in the same distribution, the final distribution still obeys the composite Poisson distribution. Its distribution is determined, and the proof process is as follows.

Two mutually independent composite Poisson distributions are denoted as S_1 and S_2, respectively, where $S_1=\sum_{j=1}^{N_1}X_j$, $S_2=\sum_{j=1}^{N_2}Y_j$. Among them, N_1 and N_2 obey the Poisson distribution with parameters λ_1 and λ_2, respectively. The characteristic function corresponding to S_1 is as follows:

$$\varphi_{S_1}(\theta)=E\left(\exp\left(i\theta S_1\right)\right)=E\left\{\exp\left(i\theta\sum_{j=1}^{N_1}X_j\right)\right\}.\tag{6}$$

It can be obtained from the total probability formula as follows:

$$\begin{aligned}\varphi_{S_1}(\theta)&=\sum_{k=0}^{\infty}E\left(\exp\left(i\theta\sum_{j=1}^{N_1}X_j\right)\Big|N_1=k\right)\cdot P\left(N_1=k\right)\\&=\sum_{k=0}^{\infty}E\left(\exp\left(i\theta\sum_{j=1}^{k}X_j\right)\right)\cdot P\left(N_1=k\right)\\&=\sum_{k=0}^{\infty}\left(E\left(\exp\left(i\theta X_j\right)\right)\right)^k\cdot P\left(N_1=k\right)\\&=\sum_{k=0}^{\infty}\left(\varphi_X(\theta)\right)^k\cdot P\left(N_1=k\right)\\&=\exp\left(\lambda_1\left(\varphi_X(\theta)-1\right)\right).\end{aligned}\tag{7}$$

Using the same method, we obtain the characteristic function corresponding to S_2 as $\varphi_{S_2}(\theta)=\exp\left(\lambda_2\left(\varphi_Y(\theta)-1\right)\right)$.

Therefore, the characteristic function of the random variable S obtained by the accumulation of the random variables S_1 and S_2 is as follows:

$$\begin{aligned}\varphi_S(\theta)&=E\left(\exp\left(i\theta S\right)\right)=E\left(\exp\left(i\theta S_1\right)\cdot\exp\left(i\theta S_2\right)\right)\\&=E\left(\exp\left(i\theta S_1\right)\right)\cdot E\left(\exp\left(i\theta S_2\right)\right)\\&=\exp\left(\lambda_1\left(\varphi_X(\theta)-1\right)\right)\cdot\exp\left(\lambda_2\left(\varphi_Y(\theta)-1\right)\right)\\&=\exp\left(\left(\lambda_1+\lambda_2\right)\cdot\left(\frac{\lambda_1}{\lambda_1+\lambda_2}\varphi_X(\theta)+\frac{\lambda_2}{\lambda_1+\lambda_2}\varphi_Y(\theta)-1\right)\right).\end{aligned}\tag{8}$$

Evidently, the derivation of the above formula reveals that with the addition of the two composite Poisson distributions, there is still obeyance to the composite Poisson distribution, and the parameter of the number of occurrences is the sum of the parameters of each sub-Poisson distribution. Therefore, the formula for determining the frequency parameters of insurance claim settlement for IDI is as follows:

$$\lambda = \sum_{k=1}^{m} \lambda_k = \frac{1}{N}\sum_{k=1}^{m} N_1(k)\cdot \delta_k(t). \tag{9}$$

The probability density function of multiple composite Poisson distributions is a linear combination of each sub-distribution, which is expressed as follows:

$$f(x) = \sum_{k=1}^{m} \frac{\lambda_k}{\lambda} f_k(x). \tag{10}$$

In sum, the claim settlement process of IDI obeys the compound Poisson distribution. The number of occurrences obeys the Poisson distribution with parameter λ, and the probability density function of the claim settlement amount is $f(x) = \sum_{k=1}^{m}\lambda_k/\lambda \cdot N_1(k)/W_k e^{-N_1(k)/W_k\cdot x}$.

4.3. Ruin Model of Inherent Defect Insurance Based on Ruin Theory. This study introduces ruin theory to construct an overall insurance risk model of IDI. According to the extant research on the distribution of the claim settlement process, it is known that the claim settlement process $S(t)$ obeys the composite Poisson distribution, with the claim frequency λ and the claim settlement amount distribution expressed as $F(x)$. An insurance premium setting is often determined based on expected claim settlement multiplied by the safety factor. The formula is as follows:

$$\text{it} = (1+\sigma)E[S(t)]. \tag{11}$$

Considering that the claim settlement process $S(t)$ obeys the composite Poisson distribution, as per the properties of the composite Poisson distribution, the expectation of the composite Poisson distribution is equal to the product of its own two distributions, that is, $E(S) = E(X)\cdot E(N)$. Therefore, the expected calculation formula of the IDI claim settlement process is as follows:

$$\begin{aligned} E[S(t)] &= E[N]\cdot E[X] = \lambda E[X] \\ &= \lambda \int_{-\infty}^{+\infty} x f(x)\mathrm{d}x \\ &= \lambda \int_{-\infty}^{+\infty} x \sum_{k=1}^{m} \frac{\lambda_k}{\lambda} f_k(x)\mathrm{d}x \\ &= \lambda \int_{-\infty}^{+\infty} \sum_{k=1}^{m} \frac{\lambda_k}{\lambda} x f_k(x)\mathrm{d}x \\ &= \lambda \sum_{k=1}^{m} \frac{\lambda_k}{\lambda} \int_{-\infty}^{+\infty} x f_k(x)\mathrm{d}x \\ &= \sum_{k=1}^{m} \lambda_k \cdot \frac{W_k}{N_1(k)} \\ &= \sum_{k=1}^{m} \frac{N_1(k)\cdot\delta_k(t)}{N}\cdot\frac{W_k}{N_1(k)} \\ &= \frac{1}{N}\sum_{k=1}^{m} \delta_k(t)\cdot W_k. \end{aligned} \tag{12}$$

The IDI risk model can be obtained by integrating the claim settlement process with insurance ruin theory, as in the following equation:

$$\begin{aligned} P(t) &= u_0 + \text{it} - S(t) \\ &= u_0 + \frac{1+\sigma}{N}\sum_{k=1}^{m} \delta_k(t)\cdot W_k - S(t). \end{aligned} \tag{13}$$

4.4. Analysing the Approximate Distribution of Insurance Claim Settlement. According to the idea of normalisation, this study constructs a new process function as $Z = S - E(S)/\sqrt{\mathrm{Var}(S)} = S - \lambda E(X)/\sqrt{\lambda E(X^2)}$. As the moment-generating function of the probability distribution is unique, the approximate distribution of the new process function can be derived using the moment-generating function, as in the following equation:

$$M_Z(t) = E\left[\exp\left(\frac{S-\lambda E(X)}{\sqrt{\lambda E(X^2)}}\,t\right)\right] = M_s\left(\frac{t}{\sqrt{\lambda E(X^2)}}\right)\exp\left(\frac{-\lambda E(X)t}{\sqrt{\lambda E(X^2)}}\right). \tag{14}$$

The claim settlement process $S(t)$ obeys the composite Poisson distribution, so there is $M_S(t) = \exp[\lambda(M_x(t) - 1)]$, and equation (15) is obtained as follows:

$$M_Z(t) = \exp\left\{\lambda\left[M_X\left(\frac{t}{\sqrt{\lambda E\left(X^2\right)}}\right) - 1\right] - \frac{\lambda E(X)t}{\sqrt{\lambda E\left(X^2\right)}}\right\}. \tag{15}$$

The Taylor function is used to expand $M_X(t)$ and obtain the following formula:

$$M_Z(t) = \exp\left(\frac{1}{2}t^2 + \frac{1}{6\sqrt{\lambda E\left(X^2\right)}}t^3 + \ldots\right). \tag{16}$$

If the conditions $\lambda \longrightarrow \infty$ are satisfied, this research can obtain the result $\lim_{\lambda\longrightarrow\infty} M_Z(t) = e^{t^2/2}$. From this, it can be concluded that the distribution of the new process $Z(t)$ is close to the standard normal distribution when the parameter λ is sufficiently large.

4.5. Construction of an Insurance Risk Probability Model of Inherent Defect Insurance. The initial deposit u_0 on an insurance company's insurance project is determined according to the relevant requirement ratio. The derivative function of ruin theory evidently reveals that the introduction of the initial margin only raises the lower limit of bankruptcy, which is unrelated to the time change. When the final surplus of the insurance project is less than u_0, the project is still in a state of loss, though the insurance project is notbankrupt. Therefore, in insurance risk analysis, u_0 in ruin theory can be omitted, and only the probability of $S(t) \geq \text{it}$ in insurance ruin theory can be analysed using the following equation:

$$\begin{aligned}
p\{S(t) \geq \text{it}\} &= p\left\{S(t) \geq \frac{1+\sigma}{N}\sum_{k=1}^{m}\delta_k(t)\cdot W_k\right\} \\
&= p\left\{\frac{S(t) - \lambda E(X)}{\sqrt{\lambda E\left(X^2\right)}} \geq \frac{(1+\sigma)\sum_{k=1}^{m}\delta_k(t)\cdot W_k - \lambda E(X)\cdot N}{N\sqrt{\lambda E\left(X^2\right)}}\right\} \\
&= 1 - \Phi\left(\frac{(1+\sigma)\sum_{k=1}^{m}\delta_k(t)\cdot W_k - \lambda E(X)\cdot N}{N\sqrt{\lambda E\left(X^2\right)}}\right) \\
&= 1 - \Phi\left(\frac{(1+\sigma)\sum_{k=1}^{m}\delta_k(t)\cdot W_k - \lambda\cdot\theta\cdot N}{N\sqrt{2\lambda\theta}}\right).
\end{aligned} \tag{17}$$

The derived model formula reveals that the risk model reported in this paper relies on the standard normal distribution; hence, its expectation and variance can be obtained according to the standard normal distribution. Since the expectation of the standard normal distribution is 0 and the variance is 1, the expectation and variance of the risk model constructed in this study are both 1.

4.6. Testing Insurance Risk Models. This study uses parameter estimation to test the risk model from unbiasedness, effectiveness, and consistency perspectives. As the risk model constructed in this research is based on the claim settlement process, the estimation of the parameters in the model is determined by real claim settlement data. Therefore, the model must be unbiased; hence, analysing its effectiveness and consistency is vital.

4.6.1. Effectiveness Test. Effectiveness pertains to the variance value of the estimator. Variance implies volatility, and the smaller the volatility, the more effective the model. The conditional tail expectation (CTE) model, a risk analysis model, commonly used in statistical actuarial studies, is compared with the risk model constructed in this research. The CTE model measures the expected value of the loss when the loss exceeds VaR, which represents the average loss size above a certain quantile. It is defined as follows [40]:

$$\text{CTE}_\alpha = E\left(L | L > Q_\alpha\right), \tag{18}$$

where Q_α means the α quantile of the probability distribution of the loss random variable L. For the insurance whose loss is a continuous random variable, the premium expectation that it exceeds a certain value is as follows:

$$E\left(\left(L - Q_\alpha\right)_+\right) = \int_{Q_\alpha}^{\infty}\left(t - Q_\alpha\right)f_L(t)\mathrm{d}t = \int_{Q_\alpha}^{\infty}\left(1 - F_L(t)\right)\mathrm{d}t = E(L) - E\left(\max\left(L, Q_\alpha\right)\right), \tag{19}$$

where $(L-Q_\alpha)_+ = \max(L-Q_\alpha, 0)$, and the actual payment expectation for each claim settlement can be further obtained as follows:

$$E\left((L-Q_\alpha)_+ \middle| L>Q_\alpha\right) = \frac{E\left((L-Q_\alpha)_+\right)}{1-F_L(Q_\alpha)} = \frac{E(L)-E\left(\max(L,Q_\alpha)\right)}{1-F_L(Q_\alpha)}. \tag{20}$$

According to the definition of the quantile, there is $F_L(Q_\alpha)=\alpha$. Therefore, the calculation formula of the CTE model can be determined as follows:

$$\mathrm{CTE}_\alpha = E(L|L>Q_\alpha) = E\left(L-Q_\alpha \middle| L>Q_\alpha\right) + Q_\alpha = \frac{E(L)-E\left(\max(L,Q_\alpha)\right)}{1-\alpha} + Q_\alpha. \tag{21}$$

Assuming that the loss distribution also follows an exponential distribution with parameter θ, equation (22) can be derived:

$$\mathrm{CTE}_\alpha = \theta + Q_\alpha = \theta(1-\ln(1-\alpha)). \tag{22}$$

Furthermore, the variance of the CTE model can be calculated using the following equation:

$$D\left(\mathrm{CTE}_\alpha\right) = (1-\ln(1-\alpha))^2. \tag{23}$$

Considering $\alpha<1$, it can be concluded that $\ln(1-\alpha)<0$ and $D(\mathrm{CTE}_\alpha)>1$. The risk model constructed in this study relies on the standard normal distribution, and its variance is 1. In sum, the variance of the insurance risk model constructed in this study is smaller, and the overall model is more effective.

4.6.2. Consistency Test. Consistency entails the fit of the model in the case of large samples, meaning that when the number of samples is sufficiently large, the distribution of the results remains consistent and gradually approaches a constant [41]. Let the number of model samples be $\longrightarrow +\infty$, and we obtain the following equation:

$$\begin{aligned}
&\lim_{N\longrightarrow+\infty} p\{S(t)\geq \mathrm{it}\} \\
&= 1-\Phi\left(\lim_{N\longrightarrow+\infty}\frac{(1+\sigma)\sum_{k=1}^{m}\delta_k(t)\cdot W_k - \lambda\cdot\theta\cdot N}{N\sqrt{2\lambda\theta}}\right) \\
&= 1-\Phi\left(\frac{\sqrt{\lambda\cdot\theta}}{\sqrt{2}}\right).
\end{aligned} \tag{24}$$

The model still obeys the standard normal distribution when the number of model samples is $N\longrightarrow+\infty$. The risk result will approach a constant when the claim frequency parameter λ and the claim settlement amount parameter θ are determined. Therefore, the model constructed in this study is consistent.

5. Empirically Analysing Insurance Risks of Inherent Defect Insurance Based on Insurance Practice

5.1. Insurance Practice Statistics in Shanghai. IDI has not been implemented in China for a considerable period, and several projects have not yet entered the insurance claim settlement term, so there are few corresponding claim settlements. Shanghai is the city with the earliest implementation of IDI and represents China's most experienced and complete insurance system. Therefore, the insurance practice data of Shanghai are selected to comprise the sample for risk analysis. The company will underwrite about 800 insurance policies in Shanghai for the year 2021, according to the claim settlement statistics of an insurance company there. In the same year, 1,243 claim settlement cases were filed and closed. Among them, the quantity and proportion of claim settlement involved in each relevant engineering division and subdivision are presented in Table 1.

5.2. Insurance Risk Analysis of Inherent Defect Insurance Based on Insurance Practice. Dividing the overall insurance term considering the inconsistency of the IDI terms across various engineering divisions and subdivisions is necessary. Presently, China's IDI does not cover the entire life cycle of construction projects. Therefore, the cumulative effect of quality problems in construction projects after a certain number of years can be ignored, which means that the possibility of quality defects in construction projects at different times is the same by default, that is, $\delta_k(t)=t$. The insurance coefficient σ is calculated according to China's actuarial practice by taking 1% simultaneously.

Since all the subitem works in the table are involved in the first two years of IDI coverage, the insurance claim frequency parameter λ and the claim settlement amount parameter θ can be calculated, respectively, as $\lambda = 1/N\sum_{k=1}^{m}N_1(k)\cdot\delta_k(t)\approx 3.1$ and $\theta=\sum_{k=1}^{m}W_k/N_1(k)\approx 8629.17$. Therefore, the insurance risk for the first two years of underwriting can be calculated as follows:

Table 1: Claim settlement details of each engineering division and subdivision in Shanghai in 2021.

Scope of claim settlement	Claim settlement number	Proportion (%)	Claim settlement amount (yuan)	Proportion (%)	Average claim settlement amount (yuan)
10-year term foundation and main structure	10	0.80	134444.00	1.25	13444.40
5-year term waterproofing and insulation	817	65.73	7219099.04	67.30	8836.11
5-year term window and door	1	0.08	6150.00	0.06	6150.00
5-year term wall and ceiling plastering layer	2	0.16	8366.00	0.08	4183.00
2-year term electrical, water supply, and drainage	145	11.67	1842486.87	17.18	12706.81
2-year term window and door	83	6.68	241773.00	2.25	2912.93
2-year term wall and ceiling plastering layer	44	3.54	282834.99	2.64	6428.07
2-year term renovation	141	11.34	990910.30	9.24	7027.73
Total	1243	100.00	10726064.20	100.00	8629.17

$$p\{S(t)\geq \mathrm{it}|t\in[0,2]\}=1-\Phi\left(\frac{(1+\sigma)\sum_{k=1}^{m}\delta_k(t)\cdot W_k-\lambda\cdot\theta\cdot N}{N\sqrt{2\lambda\theta}}\right)\approx 1-\Phi(1.44)\approx 0.0749. \tag{25}$$

Similarly, the claim frequency parameter λ of underwriting for three to five years and six to ten years can be calculated as 3.1125 and 0.0625, respectively, and the claim settlement amount parameter θ is 8877.18 and 13444.4, respectively. Furthermore, equations (26) and (27) can be used to calculate the insurance risks associated with underwriting with respect to three to five years and six to ten years as follows:

$$p\{S(t)\geq \mathrm{it}|t\in[3,5]\}=1-\Phi\left(\frac{(1+\sigma)\sum_{k=1}^{m}\delta_k(t)\cdot W_k-\lambda\cdot\theta\cdot N}{N\sqrt{2\lambda\theta}}\right)\approx 1-\Phi(1.18)=0.119, \tag{26}$$

$$p\{S(t)\geq \mathrm{it}|t\in[6,10]\}=1-\Phi\left(\frac{(1+\sigma)\sum_{k=1}^{m}\delta_k(t)\cdot W_k-\lambda\cdot\theta\cdot N}{N\sqrt{2\lambda\theta}}\right)\approx 1-\Phi(0.21)=0.4168. \tag{27}$$

Figure 3 reflects the risk changes of IDI according to different insuring terms.

The analysis of insurance ruin probability shows that the insurance ruin probability for the 2-year and 5-year periods is not high. However, after the end of the remaining subprojects' insuring term, the insurance ruin probability in six to ten years has increased significantly, meaning there is a relatively large insurance risk. On the one hand, the reason may be that the average compensation amount for 10-year subprojects is relatively high. On the other hand, due to the low frequency of claim settlement for its corresponding engineering divisions and subdivisions, approximating the distribution of samples by the law of large numbers is difficult, thus precipitating high insurance risks. Comprehensively analysing the risks associated with each insuring term of IDI from the initial reserve setting's perspective is necessary to explore the internal reasons for the 10-year insurance risk increase.

5.3. Initial Reserve Setting of Inherent Defect Insurance Based on Insurance Practice. According to the aforementioned constructed insurance risk model and insurance ruin probability analysis, it can be concluded that the insurance risk associated with IDI is the highest in six to ten years. Insurance companies need to set reasonable initial reserves to control insurance risks better. Therefore, according to the constructed insurance risk model, the ruin probability can be set as 0.01 so that the initial reserve required of IDI can be obtained through the reverse solution. Based on the aforementioned points, the risk model for introducing the initial insurance margin is as follows:

$$\begin{aligned}
p\{S(t)\geq \mathrm{it}+u_0\}&=p\left\{S(t)\geq\frac{1+\sigma}{N}\sum_{k=1}^{m}\delta_k(t)\cdot W_k+u_0\right\}\\
&=p\left\{\frac{S(t)-\lambda E(X)}{\sqrt{\lambda E(X^2)}}\geq\frac{(1+\sigma)\sum_{k=1}^{m}\delta_k(t)\cdot W_k+u_0\cdot N-\lambda E(X)\cdot N}{N\sqrt{\lambda E(X^2)}}\right\}\\
&=1-\Phi\left(\frac{(1+\sigma)\sum_{k=1}^{m}\delta_k(t)\cdot W_k+u_0\cdot N-\lambda E(X)\cdot N}{N\sqrt{\lambda E(X^2)}}\right)\\
&=1-\Phi\left(\frac{(1+\sigma)\sum_{k=1}^{m}\delta_k(t)\cdot W_k+u_0\cdot N-\lambda\cdot\theta\cdot N}{N\sqrt{2\lambda\theta}}\right).
\end{aligned} \tag{28}$$

If the final calculation result is 0.01, it can be obtained from the standard normal distribution table that the value of the standard normal distribution is 2.33. The calculation formula of the average initial reserve required for each order can be obtained using the inverse solution. The calculation formula is as follows:

$$u_0=2.33\sqrt{2\lambda\theta}+\lambda\theta-\frac{1+\sigma}{N}\sum_{k=1}^{m}\delta_k(t)\cdot W_k. \tag{29}$$

The insurance practice data for Shanghai show that the frequency parameters of claim settlement for 0~2 years of IDI are $\lambda\approx 3.1$, and the claim settlement amount parameters are $\theta \approx 8629.17$. It can be calculated that the average initial reserve required for each order for zero to two years of IDI is about 206.05, and the ratio to the average claim settlement amount of related engineering divisions and subdivisions is about 0.024. The average initial reserve required for each order for IDI in three to five years and six to ten years and its ratio to the average claim settlement amount of relevant engineering divisions and subdivisions can be calculated from this. The average initial reserve required for each order for three to five years and six to ten years of IDI is 271.42 and 87.11, respectively. The ratio to the average claim settlement amount of related engineering divisions and subdivisions is 0.03 and 0.0066, respectively. The calculation result is shown in Figure 4.

In Figure 4, the bar chart represents the average initial reserve required for each order of IDI with different insuring terms. The higher the initial reserve required per order, the greater the insurance risk. The line graph represents the ratio of the average initial deposit required for each order to the average claim settlement amount. This ratio eliminates inconsistency caused by different insuring contents of different insuring terms and can clearly and intuitively reflect insurance risks associated with different insuring terms. The insurance risk associated with IDI increases sharply in six to ten years because the average claim settlement amount is high. The main reason is that the claim settlement sample is too small and does not satisfy the law of large numbers. Therefore, to effectively prevent insurance risks for six to ten years, the initial insurance premium only needs to be increased by a small percentage. However, the main risk of IDI exists in engineering divisions and subdivisions corresponding to three to five years, particularly thermal insulation and waterproofing. They have the largest proportion of claim settlement and the highest average claim settlement amount. In the practice of IDI, it is necessary to implement detailed risk prevention and control to ensure construction projects' thermal insulation and waterproofing elements.

5.4. Model Analysis of Ruin Probability Function of Inherent Defect Insurance. It can be seen from the calculation formula of the final average initial reserve that the claim frequency parameter λ and claim settlement amount parameter θ can be regarded as a whole, and the product of the claim frequency parameter and the claim settlement amount parameter can be denoted as ω, that is, $\omega = \lambda\theta$. Therefore, the average initial reserve per order can be expressed as a function of ω, that is, $u_0 = u_0(\omega)$, and these parameters have the same influence on insurance risk of IDI. By taking the derivative of function $u_0(\omega)$, we can see that the derivative function is always greater than 0, so it is impossible to find a certain initial reserve guarantee to reduce the risk to a certain value, and the corresponding initial reserve must be determined according to the claim frequency and claim settlement amount expectation corresponding to the specific coverage.

In order to better analyse the impact of initial reserves on IDI ruin probability, this study further analyses and finds out the impact of initial reserves on insurance project ruin probability. As in the previous part of this study, the insurance ruin probability is 0.01, and the expected ruin probability is denoted as p_α. The function of p_α and u_0 can be inversely solved based on equation (28), and the calculation formula is as follows:

$$p_\alpha = 1 - \Phi\left(\frac{(1+\sigma)\sum_{k=1}^{m}\delta_k(t)\cdot W_k + u_0\cdot N - \lambda\cdot\theta\cdot N}{N\sqrt{2\lambda\theta}}\right). \tag{30}$$

This study simulated the functional relationship between the initial reserve of IDI and the insurance ruin probability in different insurance periods to better display the inner relationship, as shown in Figure 5.

Based on the property of the standard normal distribution, the analysis of equation (30) shows that the range of this function is a subset of [0, 1]. It can be seen from Figure 5, the X-axis is the asymptote of the function, and the ruin probability gradually approaches 0 when u_0 is large enough. The 6–10-year period leads to a higher ruin probability when the initial reserve is small, but with the increase in the initial reserve, the ruin probability decreases rapidly and is at a low level. Compared with the 0–2-year period, the ruin probability of the 3–5-year period is always higher. Therefore, it can be considered that the insurance items involved in the 3–5-year period are a part of IDI that needs to be emphasized. To further analyse the weak part of insurance risk, equations (31) and (32) can be obtained by differentiating the function:

$$\frac{dp_\alpha}{du_0} = \frac{-1}{\sqrt{2\lambda\theta}}\cdot\varphi\left[\frac{(1+\sigma)\cdot\sum_{k=1}^{m}\delta_k(t)\cdot W_k + u_0\cdot N - \lambda\cdot\theta\cdot N}{N\sqrt{2\lambda\theta}}\right], \tag{31}$$

$$\frac{d^2p_\alpha}{du_0^2} = \frac{(1+\sigma)\sum_{k=1}^{m}\delta_k(t)W_k + u_0N - \lambda\theta N}{N(2\lambda\theta)^{3/2}}\cdot\varphi\left[\frac{(1+\sigma)\cdot\sum_{k=1}^{m}\delta_k(t)\cdot W_k + u_0\cdot N - \lambda\cdot\theta\cdot N}{N\sqrt{2\lambda\theta}}\right]. \tag{32}$$

Since the probability density function of the standard normal distribution is always greater than 0, the derivative function represented by equation (31) is always less than 0, and the function image is monotonically decreasing, so there is no maximum value. However, it can be seen from the zero point of the second derivative expressed in equation (32)

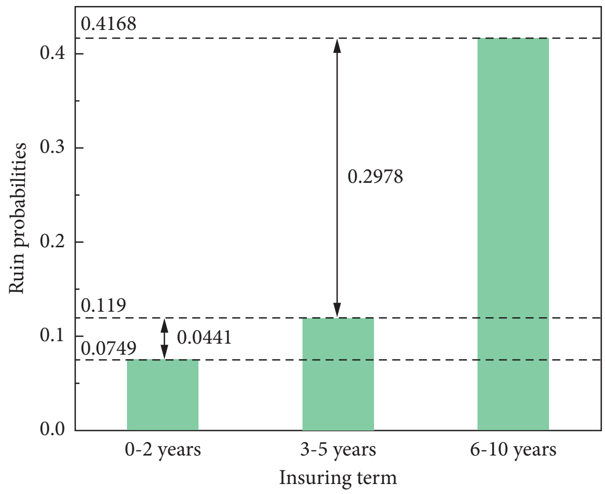

FIGURE 3: Ruin probability of inherent defect insurance with respect to each insuring term.

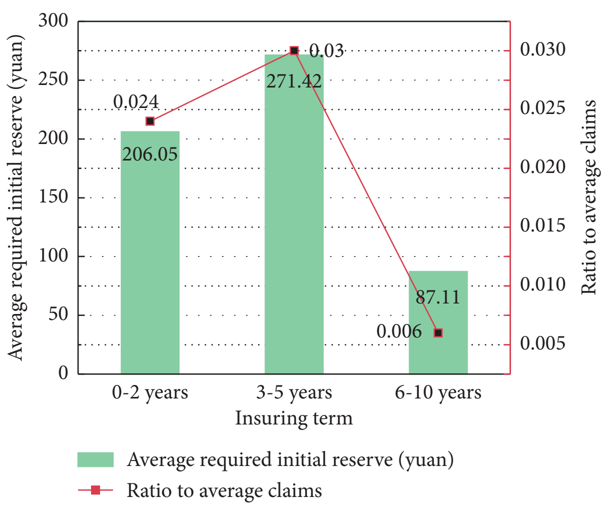

FIGURE 4: Average required initial reserve and ratio to average claim settlement amount for each insuring term.

that the function has an inflection point at which the convexity of the function changes. The optimal initial reserve is determined as follows:

$$u_0^* = \lambda\theta - \frac{1+\sigma}{N} \cdot \sum_{k=1}^{m} \delta_k(t) \cdot W_k. \tag{33}$$

By introducing the actuarial data of Shanghai into the model, the average optimal initial reserve of each order at each insurance stage of IDI in the analysis data can be calculated, as shown in Table 2.

Due to the shapes of different curves being very different, u_0^* represents the initial reserve corresponding to the inflection point of the curve which corresponds to different ruin probabilities. Therefore, highest u_0^* in the 0–2-year period does not mean that the insurance risk is the greatest. For the average initial reserve per order u_0 can be expressed as a function of the product of the claim frequency parameter λ and claim settlement amount parameter θ, and

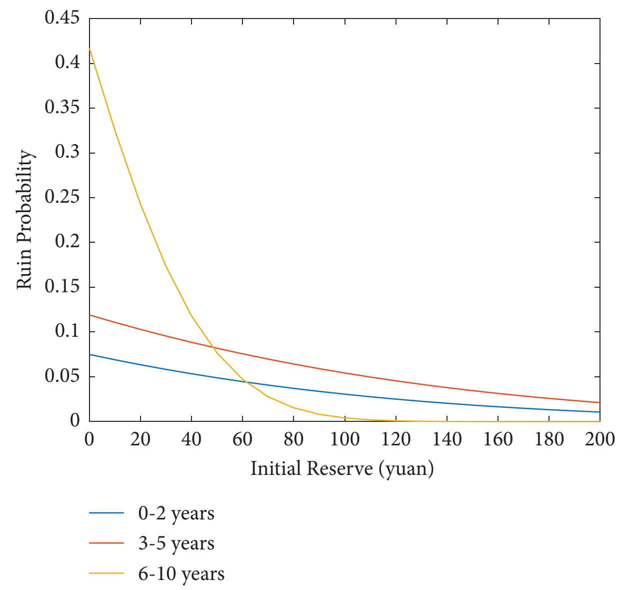

FIGURE 5: The function relation between initial reserve and insurance ruin probability.

TABLE 2: Average optimal initial reserve for each insurance period.

Insurance period	0–2 years	3–5 years	6–10 years
u_0^* (yuan)	333.08	277.39	8.6

the degree of the denominator is higher in equation (32), the function is inversely proportional. Since the claim frequency parameter λ and claim settlement amount parameter θ of the 3–5-year period are both slightly larger than the values of the 0–2-year period, the optimal initial reserve u_0^* of the 3–5-year period will be slightly smaller than the value of the 0–2-year period. This method can calculate and analyse the optimal initial reserves of IDI according to the existing data of insurance companies, which can effectively provide decision-making reference for insurance companies.

It can be seen from the model results that the risk of IDI mainly occurs in the stages of 0–2 years and 3–5 years. In these stages, the average required initial reserve and the ratio to average claim settlement of the subitem project are significantly higher than the value of 6–10 years. Comparing the subprojects corresponding to different underwriting periods in Table 1, it can be seen that the subprojects corresponding to the 6–10-year underwriting period are the foundation and main structure. It shows that with the rapid development of construction technology, the application of new materials, new technology, new structure, and other new technologies in construction engineering has improved the durability of the foundation and main structure, which makes the foundation and the main structure of the number of claims appear less [42]. Therefore, the risk of IDI mainly lies in the subprojects outside the foundation and main structure, especially the subprojects such as thermal insulation and waterproof engineering corresponding to the 3–5-year stage. Because these subprojects are greatly affected by the external environment and are prone to loss, the

TABLE 3: The initial reserve required for different confidence levels of IDI insurance periods.

α	−30%	−20%	−10%	0.01	10%	20%	30%
0–2 years	235.486	224.303	214.289	206.05	196.883	189.195	182.044
3–5 years	301.337	289.971	279.793	271.42	262.104	254.29	247.022
6–10 years	92.327	90.345	88.57	87.11	85.485	84.123	82.855

frequency of claims in IDI practice is high, which has become the focus of IDI risk management [42, 43].

From the perspective of risk management, the ruin probability of insurance obtained by ruin theory can effectively quantify the risk of insurance and comprehensively examine the risk of insurance on the average level. The average required initial reserve and the ratio to average claim settlement obtained by the reverse solution of ruin theory can provide a richer theoretical basis for the risk management of insurance.

5.5. Sensitivity Analysis of the Risk of Inherent Defect Insurance. Because the ruin probability obtained by the ruin model is similar to the VaR value in risk management, it is considered using its method for further sensitivity analysis of IDI. VaR is a technical method to measure the risk comprehensively, which combines the size of the future loss with the probability of the loss. VaR is widely used as a risk measure for financial markets, which is also conducive to better risk management of IDI.

Since VaR is the maximum loss that may occur at a given confidence level, VaR can be regarded as a function of the confidence level α. Similarly, the ruin probability of IDI is also obtained under a given confidence interval. When the selected confidence interval changes, the corresponding ruin probability will also change. Therefore, the risk of insurance can also be regarded as a function of the confidence interval α, and the sensitivity analysis can be carried out through the change of α, so as to obtain the risk sensitivity of different insurance periods. According to the above derivation, it can be seen that the IDI initial reserve can also reflect the risk of insurance. Compared with the value range of the ruin probability of IDI only on $[0, 1]$, the initial reserve of IDI has a wider value range. Therefore, the IDI initial reserve can be used to represent the insurance risk in the sensitivity analysis. As known from equation (29), under the premise that the claim frequency parameter and the claim amount parameter are determined, the functional relationship between the IDI initial reserve and the confidence interval α is as follows:

$$u_0 = \Phi^{(-1)}(\alpha)\sqrt{2\lambda\theta} + \lambda\theta - \frac{1+\sigma}{N}\sum_{k=1}^{m}\delta_k(t)\cdot W_k. \qquad (34)$$

Taking 0–2-year IDI insurance as an example, the sensitivity analysis is carried out. According to the previous calculation, the claim frequency parameters $\lambda \approx 3.1$ and claim amount parameters $\theta \approx 8629.17$. Similarly, the claim frequency parameters λ and claim amount parameters θ of 3–5 years and 6–10 years can be, respectively, brought into equation (34), and the initial reserve required by IDI can be calculated under different confidence levels of 3–5 years and 6–10 years. Some initial reserves required at the main confidence levels can be calculated, as shown in Table 3.

It can be seen from Table 3 that the risk sensitivity of IDI in 0–2 years and 3–5 years is significantly higher than that in 6–10 years, which indicates that IDI is more affected by the influencing factors in 0–2 years and 3–5 years. In order to better analyse the impact of the confidence interval on insurance risk, the relationship between the confidence interval and the IDI initial reserve can be established to a wider extent. Figure 6 shows the change of the initial reserve required by IDI with the confidence interval α. Taking the actual confidence interval $\alpha = 0.01$ as the origin, the change range of 100% on both sides is taken as the investigation interval to better reflect the change trend of the IDI initial reserve with the confidence interval α during each underwriting period.

It can be seen from Figure 6 that the curve of the confidence interval α in 0–2 years and 3–5 years is steeper, which indicates that the change of the confidence interval α is more sensitive to the initial reserve of IDI in 0–2 years and 3–5 years and has less impact on the initial reserve of IDI in 6–10 years. From the perspective of risk management, IDI in 0–2 years and 3–5 years is more susceptible to other factors, and the relative risk level is higher. In addition, it can be seen that the initial reserve required for 3–5 years is always higher than the initial reserve required for 0–2 year IDI, which indicates that the risk of IDI in the 3–5-year stage is higher than that in the 0–2-year stage. In insurance practice, it is necessary to take corresponding risk prevention measures for relevant subprojects to reduce risks.

In view of the above analysis, in order to better manage the risk of IDI, so as to reduce the insurance risk, better play the role of insurance, and achieve high-quality development, the following measures should be taken. First of all, the underwriting loss mechanism of insurance should be improved. Clarifying the specific scope of insurance underwriting and establishing a more complete claims system to reduce the overall degree of claims can reduce the risk of insurance operation from the level of claims. Furthermore, the insurance actuarial system and related databases should be improved. It is best to develop a more intelligent determination of IDI insurance rates from the perspective of big data. At the same time, it comprehensively sorts out the possible impact of relevant influencing factors on insurance premium rates, constructs a more comprehensive and accurate IDI ratemaking model, and reduces the possibility of insurance bankruptcy from the level of premium income. Finally, the insurance risk prevention system with the initial reserve should be established. It is necessary to set an appropriate initial insurance reserve so as to effectively improve the ability of IDI to resist risks while ensuring the profits of insurance companies and comprehensively reduce IDI risks.

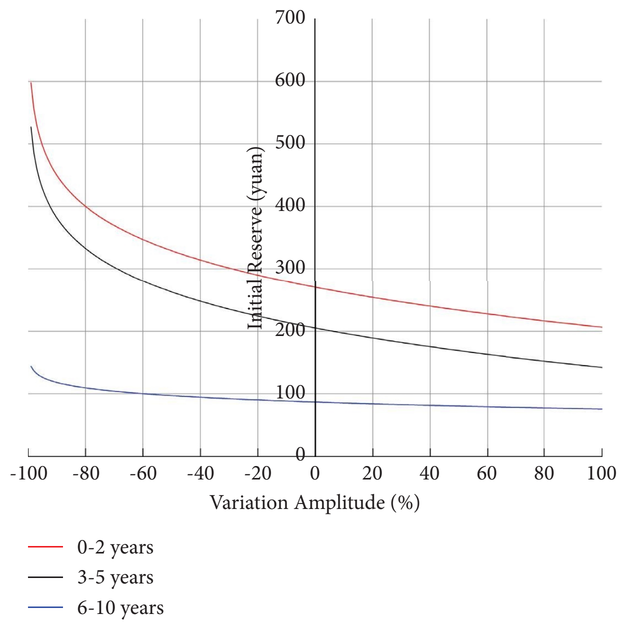

Figure 6: Sensitivity analysis of IDI.

6. Conclusions

The characteristics of IDI clearly reveal that the claim frequency distribution of single-division and subdivisional engineering follows the Poisson distribution; hence, the claim settlement distribution of single-division and subdivisional engineering follows the compound Poisson distribution. The derivation of the characteristic function of the probability distribution proves that the compound Poisson distribution is additive, so the claim settlement distribution of multidivisional engineering synthesis still obeys the compound Poisson distribution.

Therefore, the comprehensive claim settlement distribution of multicomponent projects still follows the compound Poisson distribution. Meanwhile, due to the introduction of a large volume of sample data in insurance practice, the claim settlement distribution will have an approximate distribution. By deriving the moment-generating functions of probability distributions of the insurance claim settlement process for IDI, it is proved that when the parameter λ is sufficiently large, the distribution of the new process $Z(t)$ approaches the standard normal distribution. In this case, the claim settlement process of IDI can be fitted by the standard normal distribution to measure the ruin probability of IDI.

The insurance practice data of IDI in Shanghai are brought into the insurance risk model in this research. This study finds that the ruin risk of zero to two years of insurance is 0.0749, three to five years of insurance is 0.119, and six to ten years of insurance rises to 0.4168. However, based on analysis of the setting of initial insurance benefits, the ratio of the average initial reserve required for each order in six to ten years to the average claim settlement amount is only 0.006, indicating that the insurance risk in six to ten years mainly comes from the fact that the law of large numbers is not satisfied because the claim settlement sample is too small. The risk of IDI is still within three to five years. At this stage, the proportion of building projects' thermal insulation and waterproofing works is larger. Therefore, it can be seen from the above analysis, combined with the deduction, that the risks involved in IDI are mainly concentrated in thermal insulation and waterproofing engineering for five years. At this stage, the requirements for the initial reserve setting of insurance projects are the highest, and the probability of ruin is also high. From the insurance practice data, thermal insulation and waterproofing projects up to five years old account for about two-thirds of the prime minister's claim settlement, a crucial part of the risk control of IDI. On this basis, this study analyses the functional relationship between the initial reserve of IDI and insurance ruin probability, which further analyses the simulation results of IDI in different insurance periods. At the same time, the confidence interval is the variable of insurance risk based on VaR, and the initial reserve is a more extensive measure of insurance risk, so this is a good choice to analyse the sensitivity of IDI by exploring the change of the initial reserve with a confidence interval. The results of sensitivity analysis of IDI also show that IDI in 0–2 years and 3–5 years is more affected by variables obviously and more vulnerable to risk in insurance practice. It is considered that the 3–5-year insurance period is the key stage to control IDI insurance risk. According to the decomposition of the relation between the two functions, the optimal initial reserves of IDI in different periods are found, which provides the basis for scientific decision-making for insurance companies.

Therefore, to better control the insurance risk of IDI, it is necessary to focus on thermal insulation and waterproofing works within five years and set a higher underwriting rate for this item to ensure the smooth progress of the insurance project. In addition, the initial margin should be set well to improve insurance projects' risk resistance ability and reduce the probability of ruin of the projects. The insurance risk model proposed in this paper is helpful for insurance companies in terms of making scientific decisions on insurance projects with IDI. However, owing to the small number of claim settlement samples of IDI in China, coupled with the fact that model construction and distribution fitting are mainly reliant on the law of large numbers in probability theory, the small number of samples may affect the accuracy of the model. The model may be further optimised and improved in future studies based on updated actuarial practice data.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

[1] Insurance Association of China, *Specification of Technical Inspection Service of Inherent Defect Insurance for Buildings*, Insurance Association of China, China, 2018.
[2] Z. Zhao and W. Ling, "Comparative study on Chinese and foreign construction quality insurance system and strategies for China," *Construction Economy*, vol. 2010, no. 8, pp. 5–8, 2010.
[3] P. E. Josephson and Y. Hammarlund, "The causes and costs of defects in construction:A study of seven building projects," *Automation in Construction*, vol. 8, no. 6, pp. 681–687, 1999.
[4] J. Georgiou, P. E. D. Love, and J. Smith, "A comparison of defects in houses constructed by owners and registered builders in the Australian state of Victoria," *Structural Survey*, vol. 17, no. 3, pp. 160–169, 1999.
[5] L. Anthony, "Protection of real estate developers and users against economic loss arising from defects in construction," *Structural Survey*, vol. 2000, no. 1, pp. 3–17, 2000.
[6] J. R. Maniloff, "Construction defect litigation and the mysterious insurance crisis," *Insurance Advocate*, vol. 113, no. 15, pp. 30–37, 2002.
[7] A. Lavers, *A. Case Studies in post-construction Liability and Insurance*, Crc Press, Boca Raton, FL, USA, 2005.
[8] M. R. Shusterman, "Liability insurance coverage for construction defect claims," *FDCC Quarterly*, vol. 55, no. 4, pp. 493–534, 2005.
[9] E. R. Frankel, "Insurance coverage for construction defect claims," *Real Estate Finance*, vol. 22, no. 4, pp. 20-21, 2005.
[10] C. Tyils and E. A. Cathleen, "Securing insurance coverage for construction defect claims," *Real Estate Finance Journal*, vol. 20, no. 3, pp. 77-78, 2005.
[11] J. Knocke, *Post-Construction Liability and Insurance System*, Routledge, Oxfordshire, UK, 1993.
[12] J. Knocke, D. Barclay, and G. Blachère, *A Model post-construction Liability and Insurance System*, International Council for Building Research, Studies and Documentation, Paris, France, 1996.
[13] K. Lovergrove, "The new building act : a revolution in building regulations and liability reform," *Law Institute Journal*, vol. 68, no. 7, pp. 604-605, 1994.
[14] J. Du and F. Qi, "Foreign experience and domestic exploration of inherent defects insurance in construction projects," *Journal of Engineering Management*, vol. 34, no. 2, pp. 6–10, 2020.
[15] R. E. Melehers, "Human error in structural design tasks," *ASCE Journal of Structural Engineers*, vol. 115, no. 7, pp. 35–37, 1989.
[16] A. R. Atkinson, "The pathology of building defects;a human error approach," *Engineering Construction and Architectural Management*, vol. 9, no. 1, pp. 53–61, 2002.
[17] A. Sunyoto and T. Minato, "Representing casual mechanism of defective designs:A System approach considering human errors," *Construction Management andEconomies*, vol. 21, no. 3, pp. 297–305, 2003.
[18] S. Smith, *Insurance Not the Answer to Defect Claims*, LasVegas Business Press, Las Vegas, NV, USA, 2002.
[19] H. Ray, "Rise in construction defect lawsuits causes increase in Oregon contractors' Insurance rates," *Engineering Construction and Architectural Management*, vol. 28, no. 11, pp. 1-2, 2005.
[20] M. M. Kane, "A study of latent building defects-the nature and extent of the contribution by the design team involved in the building process," *National Defence University CarolI*, vol. 2013, no. 1, p. 6, 2013.
[21] X. Meng, "The effect of relationship management on project performance in construction," *International Journal of Project Management*, vol. 30, no. 2, pp. 188–198, 2012.
[22] W. K. Chong and S. P. Low, "Assessment of defects at construction and occupancy stages," *Journal of Performance of Constructed Facilities*, vol. 19, no. 4, pp. 283–289, 2005.
[23] Y. S. Cheng, W. D. Yu, and Q. M. Li, "GA-based multi-level association rule mining approach for defect analysis in the construction industry," *Automation in Construction*, vol. 51, no. 51, pp. 78–91, 2015.
[24] S. Tartarini, F. Vesely, and E. Movedi, "Biophysical models and meta-modelling to reduce the basis risk in index-based insurance: a case study on winter cereals in Italy," *Agricultural and Forest Meteorology*, vol. 300, Article ID 108320, 2021.
[25] E. M. Fernandes-Hugo and A. F. Ferreira-Fernando, "Health insurance risk assessment using cognitive mapping and multiple-criteria decision analysis," *International Transactions in Operational Research*, vol. 30, no. 1, Article ID 12895, 2020.
[26] W. Su and W. Yu, "Asymptotically normal estimators of the gerber-shiu function in classical insurance risk model," *Mathematics*, vol. 8, no. 10, p. 1638, 2020.
[27] F. Guillermo, F. B. Joseph, and A. Nuria, "Arguimbau. Evaluation methods of flood risk models in the (re)insurance industry," *Water Security*, vol. 11, Article ID 100069, 2020.
[28] A. Søren and A. Hansjörg, *Ruin Probabilities*, World Scientific, Singapore, 2010.
[29] M. Eling and K. Jung, "Risk aggregation in non-life insurance: standard models vs. internal models," *Insurance: Mathematics and Economics*, vol. 95, no. 95, pp. 183–198, 2020.
[30] C. Li and J. Jin, "A scalar expected value of intuitionistic fuzzy random individuals and its application to risk evaluation in insurance companies," *Mathematical Problems in Engineering*, vol. 2018, no. 1, Article ID 8319859, 16 pages, 2018.
[31] Z. Wang, D. Landriault, and S. Li, "An insurance risk process with a generalized income process: a solvency analysis," *Insurance: Mathematics and Economics*, vol. 98, no. 98, pp. 133–146, 2021.
[32] D. Leppert, T. Dalhaus, and C. J. Lagerkvist, "Accounting for geographic basis risk in heat index insurance: how spatial interpolation can reduce the cost of risk," *WEATHER CLIMATE AND SOCIETY*, vol. 13, no. 2, pp. 273–286, 2021.
[33] P. Jing, C. Chang, H. Zhu, and Q. Hu, "Financial imbalance risk and its control strategy of China's pension insurance contribution rate reduction," *Mathematical Problems in Engineering*, vol. 2021, no. 1, Article ID 5558757, 12 pages, 2021.
[34] M. Bee, J. Hambuckers, and L. Trapin, "Estimating large losses in insurance analytics and operational risk using the g-and-h distribution," *Quantitative Finance*, vol. 21, no. 7, pp. 1207–1221, 2021.

[35] R. M. Verschuren, "Predictive claim scores for dynamic multi-product risk classification in insurance," *Astin Bulletin*, vol. 51, no. 1, pp. 1–25, 2021.
[36] S. Ki Kang, L. Peng, and A. Golub, "Two-step risk analysis in insurance ratemaking," *Scandinavian Actuarial Journal*, vol. 2021, no. 6, pp. 532–542, 2021.
[37] Y. Huang, J. Li, H. Liu, and W. Yu, "Estimating ruin probability in an insurance risk model with stochastic premium income based on the CFS method," *Mathematics*, vol. 9, no. 9, p. 982, 2021.
[38] R. M. Adelson, "Compound POISSON distributions," *Journal of the Operational Research Society*, vol. 17, no. 1, pp. 73–75, 1966.
[39] H. Guo and Y. Liu, "Analysis of quality problems and their traits of main structure in construction engineering," *Journal of Chongqing Jianzhu University*, vol. 27, no. 3, pp. 111–115, 2005.
[40] J. Huang, Y. Jin, and Y. Xiao, *Statistical Models Commonly Used in Actuarial Science*, China Renmin University Press, Beijing, China, 2009.
[41] A. L. Richard and L. M. Morris, *An Introduction to Mathematical Statistics and its Applications*, Prentice Hall, Upper Saddle River, NJ, USA, 2005.
[42] X. L. Hou, L. L. Zhu, and G. Guan, "Evidence-management method and confirmatory analysis for quality problems of residential engineering in China (1)," *China Civil Engineering Journal*, vol. 2008, no. 7, pp. 92–97, 2008.
[43] A. B. R. Peruchi, F. F. Zuchinali, and A. M. Bernardin, "Development of a water-based acrylic paint with resistance to efflorescence and test method to determine the appearance of stains," *Journal of Building Engineering*, vol. 35, no. 35, Article ID 102005, 2021.