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Research Article

A Hierarchical Optimal Control of Uncertain and Time-Varying Knowledge Dissemination Model in Complex Network

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In this paper, considering the multifactor influence of the knowledge dissemination process, a new ignorant-knower-spreader-forgetter (IKSF) knowledge dissemination model is proposed, which considers internalization mechanism, degradation mechanism, communication, and willingness, as well as time-varying and uncertain parameters. First, we prove that the knowledge loss equilibrium of the model is globally asymptotically stable when the basic reproduction number $R_0 < 1$, and knowledge is permanent when $R_0 > 1$. Next, improving the willingness of knower individuals and reducing the knowledge degradation function of spreader individuals can make the best effect of propagation; a hierarchical control strategy is designed. At the upper layer, an effective optimal control mechanism of the IKSF knowledge dissemination model is studied to provide optimal control action and minimization costs. At the lower layer, to guarantee robustness control performance and track the control targets, an intervention optimal guaranteed cost control strategy for the IKSF knowledge dissemination system with uncertain parameters is studied. Converting the controller's design problem into a minimization problem with linear matrix inequalities, not only the impact of uncertain parameters is reduced but also the propagation effect of the knowledge dissemination model is guaranteed. Simulation results confirm our method.

1. Introduction

People acquire knowledge mainly to improve their own quality, but knowledge can not only improve economic production but also promote social prosperity. Knowledge is power. With the advent of the information age, knowledge can be flexibly used in our hands to create what we want; at this time, knowledge dissemination is essential. For example, knowledge dissemination plays an important role in social innovation and competition [1–3]. However, the process of knowledge transfer is very difficult [4, 5] to describe in the real world [6]. Therefore, how to describe an accurate transmission process, in reality, has been puzzling to researchers. To solve this problem, many researchers have introduced complex networks. Therefore, how to analyze complex networks becomes particularly important. Jun Hu reviews the latest state estimation schemes for complex dynamical networks, especially those over the networked environment [7]. Rai et al. provide a brief overview of complex networks, as well as measurements of several key related concepts, the structure and social impact of complex networks, and more [8].

In any system, the mechanism plays a fundamental role. In the ideal state, with a suitable mechanism, even when the external conditions are uncertain changes, the system can automatically respond quickly and adjust the original strategy and measures to achieve the optimization goal. Mechanism originally refers to how a machine works, but it can mean different things in different fields. In the area of knowledge dissemination, the mechanism represents the internal law of knowledge dissemination. There are a lot of researchers who have studied this in depth; for example, Cao et al. [9] took the forgetting mechanism into account and developed a completely new model. Rozewski and Jankowski [10] integrated self-learning ability into the model and simulated it with a new method. Furthermore, Wang et al.

found in the course of their research that human self-learning ability can spread knowledge more widely [11]. Chen and Chung [12] found that regular review can indeed preserve knowledge longer. According to the different ways of knowledge existence, Liao and Yi [13] used an internalization mechanism to analyze knowledge transmission in complex networks. So far, many internal mechanisms of knowledge dissemination have been considered, and they are incorporated into the model from the abstract space, which makes the model more vivid and concrete. However, the mechanisms do not exist in isolation. To work, different mechanisms must echo and complement each other. In recent years, many researchers have been studying multiple mechanisms. For example, in the field of biology, Warren et al. [14] found that the invasion of plant species in woodlands is jointly affected by multiple mechanisms. In the field of sociology, Tanner [15] found that language production is influenced by multiple mechanisms.

Knowledge transmission is a complex behavior; most of the existing studies only focus on the influence of a certain mechanism in the dissemination process, such as the dissemination route [16], self-learning [11], forgetting mechanism, forgetting level [9], forgetting review [17], and internalization mechanisms [13]. However, in real life, knowledge dissemination is affected by various mechanisms. For example, knowledge receivers can acquire knowledge in various ways (self-learning, contact, and communication), and knowledge disseminators will also lose knowledge due to forgetting or losing interest. It may happen in real life. Therefore, it is necessary to comprehensively consider the process of knowledge dissemination in complex networks through various mechanisms. We investigate whether the interaction of multiple mechanisms affects the performance of knowledge dissemination according to the existing mechanisms. Specifically, an ignorant-knower-spreader-forgetter (IKSF) knowledge propagation model is proposed, which integrates existing knowledge propagation mechanisms and analyzes the dynamics in scale-free networks.

The uncertainty of the system is generally divided into two kinds, namely, the uncertainty of the parameters and the uncertainty of the unknown function. The uncertainty of the parameters mainly means that the parameters will change over time, and the range of change is known, but the change in the system is unknown, for example, in engineering applications, air resistance, and coefficient of friction. The uncertainty created by these parameters generally does not affect the structure of the system. However, the uncertainty of unknown functions may lead to uncertainty in the system structure because they are affected by the external environment, resulting in their own structure not being determined. In uncertain systems, nonlinear systems tend to have uncertainties. This is because the models' people designs are often different from the real models. In engineering applications, there will be many external interferences or errors that cause uncertainty to occur. Many researchers today are studying the uncertainty of nonlinear systems. Wang et al. [18] studied the tracking control of uncertain nonlinear systems under neural networks and proposed a new adaptive critical framework. Souzanchi-K and Akbarzadeh-T [19] studied the impedance control of emotional learning in the brain with time delay, integrating emotional learning into an uncertain system. Haddad and Mirkin [20] applied adaptive tracking for uncertain nonlinear systems to urban traffic.

However, in the field of knowledge dissemination, the default parameters of the previous knowledge dissemination model are constant, but in real life, their values changed over time. When there are uncertain parameters in the system, whether the system is still stable needs to enter an in-depth study. To remedy this defect, we introduce a feedback control strategy to solve the influence of system parameter disturbance on system stability [21]. The introduction of control will also bring a specific cost and the investment of a large number of charges may not be worth the loss, but the optimal control technology can effectively solve this problem. Optimal control [22] is simply to optimize the system by adjusting parameters on the premise of accurately describing the system. Optimal control has been applied in many fields, such as simulated annealing algorithms, genetic algorithms, predictive control, and chaotic optimal control. In epidemiology, optimal control techniques can be used to reduce viral infections [23–25]. Different from epidemiology, the optimal control in knowledge dissemination [26] is mainly to speed up the dissemination of knowledge while reducing control costs. To deal with uncertain parameters, the guaranteed cost control (GCC) technology [21] can not only solve the stability of the control system but also make the control system have a good performance level. The proposal of GCC solves many practical problems, and in recent years, it has also been deeply studied [27–31]. For example, Chen et al. [31] added cost control to the gene regulation network to make it more precise. Li et al. [30] imposed cost control on the high-speed travel of the train to make its description more realistic. In the study, we found that knowledge dissemination also has the problem of cost control, which is because the current communication efficiency often relies on a large amount of cost input to improve. People often do not consider the cost for the sake of high efficiency, which is effective but not the best choice.

There are other recent works on the robust control of uncertain systems using the LMI approach. Turki et al. [32] adopted a state-feedback controller to robustly control the motion of a 1-DoF double-side impact mechanical oscillator subject to norm-bounded parametric uncertainties. Gritli [33] studied the robust calming problem of a class of continuous-time nonlinear systems and applied it to a simple helicopter model. He also addressed the problem of static output feedback (SOF) stabilization for continuous-time linear systems subject to norm-bounded parameter uncertainties using the LMI approach [34]. Badri and Sojoodi investigated the robust stability and stabilization analysis of interval fractional-order systems with time-varying delay [35]. Moradi et al. proposed an offline robust model predictive control (RMPC) method based on LMIs to solve vehicle suspension design problems [36]. Nodozi and Rahmani proposed a LMI approach to mixed-integer model predictive control (MPC) of uncertain hybrid systems with binary and real-valued control inputs [37].

These studies have great implications for our application in the field of knowledge dissemination.

Through the above description, this paper integrates the existing mechanism, studies the IKSF propagation model under multiple mechanisms, also considers the impact of uncertain parameters on the system, and optimizes the control.

The innovations of this paper are as follows:

(1) The model incorporates self-learning mechanisms, forgetting mechanisms self-learning ability, communication, willingness, internalization, forgetting level, etc. At the same time, the prerequisites for the dissemination of knowledge are derived.

(2) The time-varying and uncertain parameters are taken into account in the IKSF model.

(3) Using the optimal control theory to improve the above given model, the purpose of hierarchical optimal control is to speed up the spread of knowledge with the minimum resources consumed. The optimal guaranteed cost control technology is employed to guarantee the stability and robust performance level of the IKSF model under uncertain parameters.

Other contents in this article are as follows. In Section 2, we mainly describe the propagation mechanism of the IKSF model. In Section 3, we conduct theoretical research on the IKSF model, including the conditions for judging whether knowledge can be disseminated, and the study of the equilibrium point. Section 4 mainly discusses optimal control strategies, namely, optimal control and optimal cost control. Section 5 primarily validates the conclusions we made in the previous sections, including the simulation of the results of the validation theory, the study of the influence of different parameters on the four groups of people, and the application of optimal control and optimal cost control of uncertain parameters. Finally, the summary is contained in Section 6.

2. Description

The design of network topology and the interaction rules [10] that drive knowledge diffusion are two important dimensions of knowledge diffusion. Generally speaking, a knowledge transmission model starts with ignorant and spreader. Ignorant individuals are learning individuals who resemble susceptible populations in the mode of viral transmission. Spreader individuals are communicators who possess the knowledge and share it with their neighbors. Since knowledge exists in different ways, the ignorant individual absorbs and digests the knowledge first after being spread by knowledge, whether he/she is willing to spread it or not. Individuals who become the third type after absorbing knowledge are called knowers. Knower refers to an individual who possesses the knowledge and has the ability to share knowledge. The acquisition of knowledge can bring benefits to people [38]; it can be supposed that ignorant people like to learn knowledge, but knowers may be motivated by external conditions to share knowledge. Spreader individuals may forget knowledge or lose interest in knowledge in the interference of themselves or external factors and then degenerate into forgetters; forgetters can regain knowledge through self-learning and communication.

Next, we will describe in detail the propagation mechanism of the IKSF model. When an ignorant individual contacts a spreader individual, the ignorant individual will change to the state of the knower individual with probability λ_1. Ignorant individuals can acquire knowledge through instinctive self-learning abilities; μ_1 represents the average self-learning ability of ignorant individuals. The knower shares knowledge with probability δ in response to an external stimulus. Since knower individuals will not remember their knowledge forever unless they use knowledge, if knower individuals may lose their willingness to share knowledge, they will degenerate into ignorant individuals with probability ε. The probability that a knowledge disseminator will degenerate into an ignorant individual due to himself/herself or external factors is γ. At the same time, the forgetter individual can regain knowledge through interaction with the neighbors or self-learning and return to the state of the knower individual. Let λ_2 represent the probability of the forgetter individual's secondary interaction and use μ_2 to represent the self-learning rate of the forgetter individual. The specific details of the IKSF model are shown in Figure 1. In general, all parameters defining a procedure are nonnegative and $\lambda_1 \neq \mu_1, \lambda_2 \neq \mu_2$.

Suppose the system is on a scale-free network with degree distribution $p(k)$, and the average network degree is $\langle k \rangle = \sum kp(k), k = 1, 2, \ldots, k_{\max}$. Let $I_k(t), K_k(t), S_k(t), F_k(t)$ denote the density of ignorant, knower, spreader, and forgetter individuals with degrees k at time t, respectively. The probability that an ignorant person is infected by the people around him/her at time t is $\Theta(t)$, and $\Theta(t)$ is represented as

$$\Theta(t) = \frac{\sum_k kp(k)S_k(t)}{\langle k \rangle}, \quad k = 1, 2, \ldots, k_{\max}, \tag{1}$$

where $k_{\max}$ represents the maximum degree of the node.

Applying the mean field theory, the IKSF model can be expressed as follows:

$$\begin{cases} \dfrac{dI_k(t)}{dt} = -(\lambda_1 k + \mu_1)I_k(t)\Theta(t) + \varepsilon K_k(t), \\ \dfrac{dK_k(t)}{dt} = (\lambda_1 k + \mu_1)I_k(t)\Theta(t) + (\lambda_2 k + \mu_2)F_k(t) - \delta K_k(t) - \varepsilon K_k(t), \\ \dfrac{dS_k(t)}{dt} = \delta K_k(t) - \gamma S_k(t), \\ \dfrac{dF_k(t)}{dt} = \gamma S_k(t) - (\lambda_2 k + \mu_2)F_k(t), \end{cases} \tag{2}$$

where $0 \leq I_k(t), K_k(t), S_k(t), F_k(t) \leq 1$.

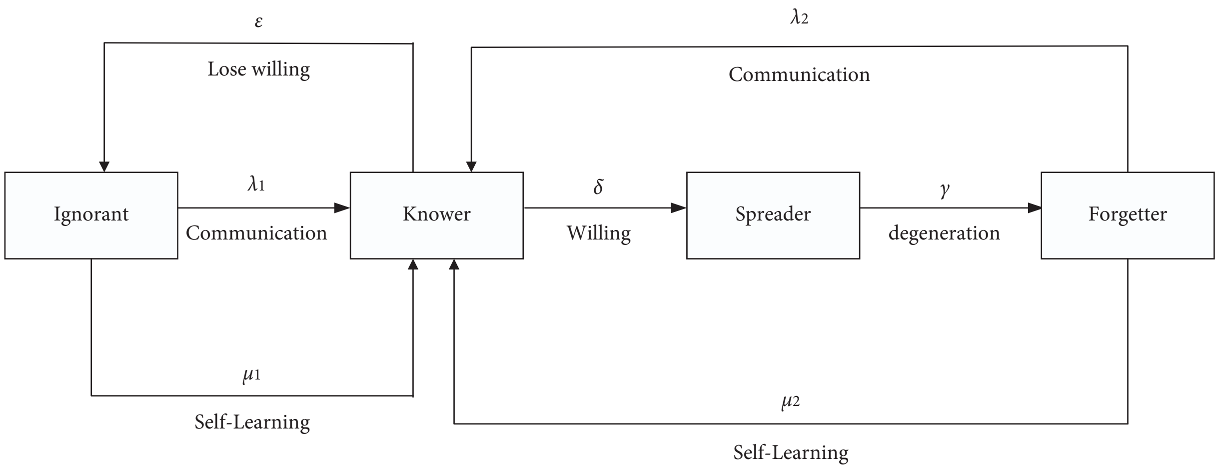


Figure 1: Structure of the IKSF knowledge transmission process.

For the above model, we analyze the multimechanism fusion of the knowledge transfer process and model it, considering the internalization mechanism, degradation mechanism, communication, willingness, and self-learning, respectively.

Remark 1. In the existing research study, knowledge diffusion models mainly focus on the influence of a certain mechanism, such as self-learning [11], internalization [13], forgetting [9], and review [17]. However, they did not study these mechanisms in a single model. Knowledge dissemination is affected by these mechanisms simultaneously. Therefore, we should fully consider the impact of multiple mechanisms on the model.

3. Theoretical Analysis

3.1. Equilibrium Point and Basic Reproduction Number

Theorem 1. *There are two equilibrium points for system (2); if $R_0 < 1$, the system has a knowledge loss equilibrium point $E^0 = (I_1^0, K_1^0, S_1^0, F_1^0, \ldots, I_{k_{\max}}^0, K_{k_{\max}}^0, S_{k_{\max}}^0, F_{k_{\max}}^0)$, where $I_k^0 = 1, K_k^0 = 0, S_k^0 = 0, F_k^0 = 0$; if $R_0 > 1$, there is a knowledge-endemic equilibrium in the system $E^* = (I_1^*, K_1^*, S_1^*, F_1^*, \ldots, I_{k_{\max}}^*, K_{k_{\max}}^*, S_{k_{\max}}^*, F_{k_{\max}}^*)$, where*

$$\begin{aligned}
I_k^* &= \frac{(\lambda_2 k + \mu_2)\varepsilon\gamma}{[(\lambda_2 k + \mu_2)(\gamma + \delta) + \gamma\delta](\lambda_1 k + \mu_1)\Theta^* + (\lambda_2 k + \mu_2)\varepsilon\gamma},\\
K_k^* &= \frac{(\lambda_1 k + \mu_1)(\lambda_2 k + \mu_2)\gamma\Theta^*}{[(\lambda_2 k + \mu_2)(\gamma + \delta) + \gamma\delta](\lambda_1 k + \mu_1)\Theta^* + (\lambda_2 k + \mu_2)\varepsilon\gamma},\\
S_k^* &= \frac{(\lambda_1 k + \mu_1)(\lambda_2 k + \mu_2)\delta\Theta^*}{[(\lambda_2 k + \mu_2)(\gamma + \delta) + \gamma\delta](\lambda_1 k + \mu_1)\Theta^* + (\lambda_2 k + \mu_2)\varepsilon\gamma},\\
F_k^* &= \frac{(\lambda_1 k + \mu_1)\gamma\delta\Theta^*}{[(\lambda_2 k + \mu_2)(\gamma + \delta) + \gamma\delta](\lambda_1 k + \mu_1)\Theta^* + (\lambda_2 k + \mu_2)\varepsilon\gamma},
\end{aligned} \tag{3}$$

and $\Theta^*(t) = (\sum_k kp(k)S_k(t)/\langle k\rangle)$ is the only positive root of the following equation:

$$f(\Theta^*) = \Theta^* - \frac{\sum_k kp(k)}{\langle k\rangle} \frac{(\lambda_1 k + \mu_1)(\lambda_2 k + \mu_2)\delta\Theta^*}{[(\lambda_2 k + \mu_2)(\gamma + \delta) + \gamma\delta](\lambda_1 k + \mu_1)\Theta^* + (\lambda_2 k + \mu_2)\varepsilon\gamma}. \tag{4}$$

Proof. Substituting $E^0 = (1, 0, 0, 0, \ldots, 1, 0, 0, 0)$ into equation (2), it is easy to see that E^0 is the solution to equation (2). Next to prove the equilibrium of E^*, according to the stability condition of system (2), we obtain

$$\begin{cases}
-(\lambda_1 k + \mu_1)I_k(t)\Theta(t) + \varepsilon K_k(t) = 0,\\
(\lambda_1 k + \mu_1)I_k(t)\Theta(t) + (\lambda_2 k + \mu_2)F_k(t) - \delta K_k(t) - \varepsilon K_k(t) = 0,\\
\delta K_k(t) - \gamma S_k(t) = 0,\\
\gamma S_k(t) - (\lambda_2 k + \mu_2)F_k(t) = 0.
\end{cases} \tag{5}$$

The solution in (5) can be obtained as

$$I_k^* = \frac{(\lambda_2 k + \mu_2)\varepsilon\gamma}{[(\lambda_2 k + \mu_2)(\gamma + \delta) + \gamma\delta](\lambda_1 k + \mu_1)\Theta^* + (\lambda_2 k + \mu_2)\varepsilon\gamma},$$

$$K_k^* = \frac{(\lambda_1 k + \mu_1)(\lambda_2 k + \mu_2)\gamma\Theta^*}{[(\lambda_2 k + \mu_2)(\gamma + \delta) + \gamma\delta](\lambda_1 k + \mu_1)\Theta^* + (\lambda_2 k + \mu_2)\varepsilon\gamma},$$

$$S_k^* = \frac{(\lambda_1 k + \mu_1)(\lambda_2 k + \mu_2)\delta\Theta^*}{[(\lambda_2 k + \mu_2)(\gamma + \delta) + \gamma\delta](\lambda_1 k + \mu_1)\Theta^* + (\lambda_2 k + \mu_2)\varepsilon\gamma},$$

$$F_k^* = \frac{(\lambda_1 k + \mu_1)\gamma\delta\Theta^*}{[(\lambda_2 k + \mu_2)(\gamma + \delta) + \gamma\delta](\lambda_1 k + \mu_1)\Theta^* + (\lambda_2 k + \mu_2)\varepsilon\gamma}. \quad (6)$$

Substituting the third equation of (3) into (1), Θ^* can be converted to the following form:

$$\Theta^* = \frac{\sum_k kp(k)}{\langle k\rangle} \frac{(\lambda_1 k + \mu_1)(\lambda_2 k + \mu_2)\delta\Theta^*}{[(\lambda_2 k + \mu_2)(\gamma + \delta) + \gamma\delta](\lambda_1 k + \mu_1)\Theta^* + (\lambda_2 k + \mu_2)\varepsilon\gamma}. \quad (7)$$

It can be seen that (HTML translation failed) is the solution of system (2); next, we discuss the nontrivial solution of $\Theta^* \in (0,1)$ in system (2).

Defining

$$f(\Theta^*) = \Theta^* - \frac{\sum_k kp(k)}{\langle k\rangle} \frac{(\lambda_1 k + \mu_1)(\lambda_2 k + \mu_2)\delta\Theta^*}{[(\lambda_2 k + \mu_2)(\gamma + \delta) + \gamma\delta](\lambda_1 k + \mu_1)\Theta^* + (\lambda_2 k + \mu_2)\varepsilon\gamma}, \quad (8)$$

it is easy to obtain

$$\frac{df(\Theta^*)}{d\Theta^*} = 1 - \frac{1}{\langle k\rangle}\sum kp(k)\frac{AB\delta[(B(\gamma+\delta)+\gamma\delta)A\Theta^* + B\varepsilon\delta] - [B(\gamma+\delta)+\gamma\delta]A^2B\delta\Theta^*}{[(B(\gamma+\delta)+\gamma\delta)A\Theta^* + B\varepsilon\gamma]^2}, \quad (9)$$

where $A = (\lambda_1 k + \mu_1)$ *and* $B = (\lambda_2 k + \mu_2)$.

It is easy to see that $(df^2(\Theta^*)/d\Theta^{*2}) > 0$ and satisfied $f(0) = 0$, while

$$f(1) = 1 - \frac{\sum_k kp(k)}{\langle k\rangle} \frac{(\lambda_1 k + \mu_1)(\lambda_2 k + \mu_2)\delta\Theta^*}{[(\lambda_2 k + \mu_2)(\gamma + \delta) + \gamma\delta](\lambda_1 k + \mu_1)\Theta^* + (\lambda_2 k + \mu_2)\varepsilon\gamma} > 0, k = 1, 2, \ldots, k_{\max}. \quad (10)$$

To allow the existence of an extraordinary solution, the inequality

$$\frac{df(\Theta^*)}{d\Theta^*} = 1 - \frac{1}{\langle k\rangle}\sum kp(k)\frac{A\delta}{\varepsilon\gamma} < 0, \quad (11)$$

must be satisfied.

Therefore,

$$R_0 = \frac{\delta}{\varepsilon\gamma}\left(\lambda_1\left(\frac{\langle k^2\rangle}{\langle k\rangle} - 1\right) + \mu_1\left(1 - \frac{1}{\langle k\rangle}\right)\right). \quad (12)$$

To sum up, when $R_0 < 1$, Θ^* has no nontrivial solution at (0, 1) (see Figure 2(a)). When $R_0 > 1$, Θ^* has only one nontrivial solution at (0, 1) (see Figure 2(b)).

Then, we have the following: when $R_0 < 1$, system (2) has a knowledge-free equilibrium $E^0 = (1, 0, 0, 0, \ldots, 1, 0, 0, 0)$; when $R_0 > 1$, system (2) has a knowledge-endemic equilibrium $E^* = (I_1^*, K_1^*, S_1^*, F_1^*, \ldots, I_{k_{\max}}^*, K_{k_{\max}}^*, S_{k_{\max}}^*, F_{k_{\max}}^*)$. □

3.2. Stability Analysis

Theorem 2. *If $R_0 < 1$, then the knowledge loss equilibrium E^0 of system (2) is globally asymptotically stable.*

Proof. For a sufficiently small $\varepsilon_1 > 0$, there exists $t_1 > 0$ such that when $t > t_1$, there is $I_k(t) \leq I_k^0 + \varepsilon_1$, where $I_k^0 = 1$.

The second formula of system (2) has $(dK_k(t)/dt) \leq (\lambda_1 k + \mu_1)\Theta(t)(I_k^0 + \varepsilon_1) - \varepsilon K + k(t), t \longrightarrow \infty$.

For $\varepsilon_2 > 0$ small enough, there exists $t_2 > 0$ such that when $t > t_2$, $K_k(t) \leq ((\lambda_1 k + \mu_1)\Theta(t)(I_k^0 + \varepsilon_1)/\varepsilon) + \varepsilon_2$.

When $t > \max(t_1, t_2)$, we have $(S_k(t)/dt) \leq \delta((\lambda_1 k + \mu_1)\Theta(t)(I_k^0 + \varepsilon_1)/\varepsilon) + \varepsilon_2\delta - \gamma S_k(t)$.

This is enough to prove that the positive solution of the following auxiliary system tends to zero as t tends to infinity:

$$\frac{d\widetilde{S}_k(t)}{dt} = \delta\frac{(\lambda_1 k + \mu_1)\Theta(t)\left(I_k^0 + \varepsilon_1\right)}{\varepsilon} + \varepsilon_2\delta - \gamma S_k(t), \tag{13}$$

where $\widetilde{\Theta}(t) = (\sum_k kp(k)\widetilde{S}_k(t)/\langle k\rangle)$.

Let $\widetilde{S}_k(0) = S_k(0)$; there is $\widetilde{S}_k(0) \leq S_k(0)$.

The Lyapunov function is defined as follows: $L(t) = (\sum_k kp(k)\widetilde{S}_k(t)/\langle k\rangle)$.

$L(t)$ is derived along system (13) to obtain

$$\begin{aligned} L'(t) &= \frac{1}{\langle k\rangle}\sum_k kp(k)\left[\delta\frac{(\lambda_1 k + \mu_1)\Theta(t)\left(I_k^0 + \varepsilon_1\right)}{\varepsilon} + \varepsilon_2\delta - \gamma\right] \\ &\leq \gamma\widetilde{\Theta}(t)\left[\frac{1}{\langle k\rangle}\sum_k kp(k)\frac{\delta(\lambda_1 k + \mu_1)(1 + \varepsilon_1)}{\varepsilon} - 1\right] + \varepsilon_2\delta\left(1 - \frac{1}{\langle k\rangle}\right) \\ &= \gamma\widetilde{\Theta}(t)\left[R_0(1 + \varepsilon_1) - 1\right] + \varepsilon_2\delta\left(1 - \frac{1}{\langle k\rangle}\right). \end{aligned} \tag{14}$$

For $R_0 < 1$, when $t \longrightarrow \infty$, $\varepsilon_1, \varepsilon_2$ is small enough to make $L'(t) < 0$. Guaranteed $(dL(t)/dt) \leq 0$ for all $\widetilde{S}_k(t) \geq 0$ and $\widetilde{S}_k(t) = 0$ only if $(dL(t)/dt) = 0$. It can be known from the comparison theorem that when t tends to infinity, the solution of system (2) tends to 0, that is,

$$\lim_{t\longrightarrow +\infty} S_k(t) = 0. \tag{15}$$

The proof of Theorem 2 is completed. □

Theorem 3. *If $R_0 > 1$, knowledge will always be preserved; i.e., there is a constant $\zeta > 0$ such that*

$$\lim_{t\longrightarrow +\infty}\inf S(t) = \lim_{t\longrightarrow +\infty}\inf\sum_k kp(k)S_k(t) > \zeta. \tag{16}$$

Proof. Using the results in [39], we can get the knowledge is permanent; Liu and Zhang [40] provide the relevant theorem.

Defining

$$\begin{aligned} X &= \left\{\left(I_1, K_1, S_1, F_1, \ldots, I_{k_{\max}}, K_{k_{\max}}, S_{k_{\max}}, F_{k_{\max}}\right): I_k, K_k, S_k, F_k \geq 0, I_k + K_k + S_k + F_k = 1\right\}, \\ X_0 &= \left\{\left(I_1, K_1, S_1, F_1, \ldots, I_{k_{\max}}, K_{k_{\max}}, S_{k_{\max}}, F_{k_{\max}}\right) \in X: \sum_k p(k)S_k > 0\right\}, \\ \partial X_0 &= X \backslash X_0, \end{aligned} \tag{17}$$

where $k = 1, 2, .., k_{\max}$, X is the feasible region of system (2).

Clearly, X is positively invariant to system (2). From the third equation of system (2), $(\sum_k p(k)S_k(t))' \geq -\gamma\sum_k p(k)S_k(t)$ is easily obtained.

Because of $\sum_k kp(k)S_k(t) > 0$, it can be seen that $\sum_k p(k)S_k(t) \geq \sum_k p(k)S_k(0)e^{-\gamma t} > 0$ indicates that X_0 is positive and invariant, and there exists a compact set B where all solutions initiated by system (2) in X go into B and

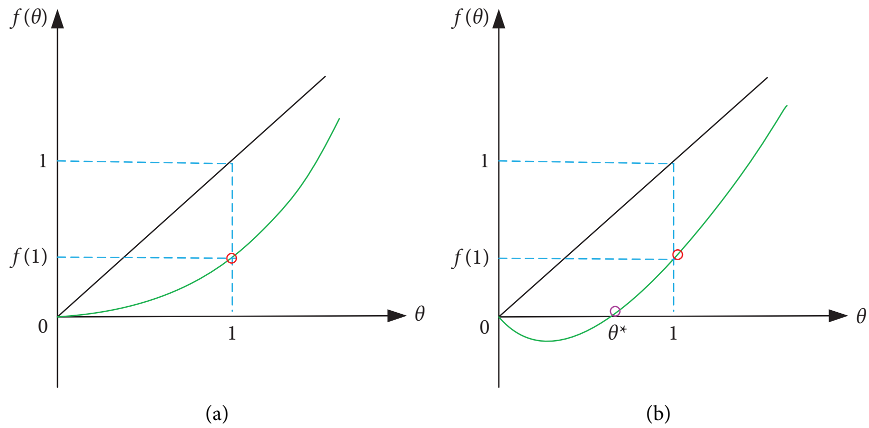


Figure 2: Diagram of solutions for $f(\Theta)$ in different cases.

exist forever; for this set B, this condition is easily verified in [39].

Defining the following set,

$$M_\partial = \{(I_k(0), K_k(0), S_k(0), F_k(0)):\ (I_k(t), K_k(t), S_k(t), F_k(t)) \in \partial X_0, k = 1, 2, \ldots, k_{\max}, t \geq 0\},$$
$$\Omega = \cup\{\omega(y)\}, y \in M_\partial, \tag{18}$$

where $\omega(y)$ is the limit set of model (2).

According to M_∂, we rewrite the system:

$$\begin{aligned} I_k(t) &= \varepsilon K_k(t),\\ K_k(t) &= (\lambda_2 k + \mu_2)F_k(t) - \delta K_k(t) - \varepsilon K_k(t),\\ S_k(t) &= \delta K_k(t) - \gamma S_k(t),\\ F_k(t) &= \gamma S_k(t) - (\lambda_2 k + \mu_2)F_k(t). \end{aligned} \tag{19}$$

It is easy to show that system (19) has a unique equilibrium E^0 on X. Hence, E^0 is the only equilibrium of system (2) in M_∂. Since (19) is a linear system, it is easy to prove that E^0 is globally asymptotically stable. Hence, $\Omega = \{E^0\}$, where E^0 is a cover of Ω, and E^0 is isolated acyclic. Next, we proved that E^0 is the weak repulsion force of X_0, that is,

$$\lim_{t \longrightarrow +\infty} \sup \operatorname{dist}\left((I_k(t), K_k(t), S_k(t), F_k(t)), E^0\right) > 0, \tag{20}$$

where $((I_k(t), K_k(t), S_k(t), F_k(t)), E^0)$ is any solution with the initial value on X_0. According to Lemma 3.5 in [41], it is only necessary to prove $W^s(E^0) \cap X_0 = \varnothing$, where $W^s(E^0)$ is the stable manifold of E^0. By contradiction, assuming it is incorrect, then there is a solution $((I_k(t), K_k(t), S_k(t), F_k(t)), E^0) \in X_0$, such that

$$I_k(t) \longrightarrow 1, K_k(t) \longrightarrow 0, S_k(t) \longrightarrow 0, F_k(t) \longrightarrow 0, \text{as} \quad t \longrightarrow \infty. \tag{21}$$

For $R_0 = (\delta/\varepsilon\gamma)(\lambda_1((\langle k^2\rangle/\langle k\rangle) - 1) + \mu_1(1 - (1/\langle k\rangle))) > 1$, we choose a sufficiently small $\delta > 0, \eta > 0$, such that

$$\frac{\delta}{\varepsilon\gamma}\left(\lambda_1\left(\frac{\langle k^2\rangle}{\langle k\rangle} - 1\right) + \mu_1\left(1 - \frac{1}{\langle k\rangle}\right)\right)(1 - \sigma) - \eta > 1. \tag{22}$$

From (21), we know that there is $t > t_3$, such that

$$1 - \sigma < I_k(t) < 1 + \sigma, 0 \leq K_k(t) < \sigma, 0 \leq S_k(t) < \sigma, 0 \leq F_k(t) < \sigma, k = 1, 2, \ldots, k_{\max}. \tag{23}$$

From the second equation of system (2), we can obtain

$$\frac{\mathrm{d}K_k(t)}{\mathrm{d}t} \geq (\lambda_1 k + \mu_1)\Theta(t)I_k(t) + (\lambda_2 k + \mu_2)F_k(t) - \varepsilon K_k(t). \tag{24}$$

When $t \longrightarrow \infty$, from (21), we obtain

$$\frac{\mathrm{d}K_k(t)}{\mathrm{d}t} \geq (\lambda_1 k + \mu_1)\Theta(t)I_k(t) - \varepsilon K_k(t). \tag{25}$$

For $t > t_4$, there exists $t_4 > 0$ such that

$$K_k(t) \geq \left[\frac{(\lambda_1 k + \mu_1)}{\varepsilon} - \varepsilon_3\right]\Theta(t)I_k(t), \tag{26}$$

where ε_3 is an arbitrary constant.

Let $W(t) = \sum_k kp(k)S_k(t)$; we have

$$\begin{aligned} W'(t) &= \sum_k kp(k)\left\{\delta\left[\frac{(\lambda_1 k + \mu_1)}{\varepsilon} - \varepsilon_3\right]\Theta(t)I_k(t) - \gamma S_k(t)\right\} \\ &\geq \sum_k kp(k)S_k(t)\left\{\delta\left[\frac{(\lambda_1 k + \mu_1)}{\varepsilon} - \varepsilon_3\right]\frac{1}{\langle k\rangle}\sum_k kp(k)(1-\sigma) - \gamma\right\} \\ &= \gamma\sum_k kp(k)S_k(t)\left[R_0(1-\eta) - 1 - \left(1 - \frac{1}{\langle k\rangle}\right)\frac{\delta\varepsilon_3(1-\sigma)}{\gamma}\right] \\ &= \gamma\sum_k kp(k)S_k(t)\left[R_0(1-\sigma) - 1 - \eta\right] \\ &= \psi W(t). \end{aligned} \tag{27}$$

For all $t > \max(t_3, t_4)$, there is

$$\psi = \gamma\left[\frac{\delta}{\varepsilon\gamma}\left(\lambda_1\left(\frac{\langle k^2\rangle}{\langle k\rangle} - 1\right) + \mu_1\left(1 - \frac{1}{\langle k\rangle}\right)\right)(1-\sigma) - \eta\right], \tag{28}$$

where

$$\eta = \left(1 - \frac{1}{\langle k\rangle}\right)\frac{\delta\varepsilon_3(1-\sigma)}{\gamma}. \tag{29}$$

Therefore, when $t \longrightarrow \infty$, $W(t) \longrightarrow \infty$, which contradicts the assumption; hence, the theorem holds. □

4. Optimal Control Strategy

4.1. Analysis of Optimal Control. As we all know, knowledge dissemination has a positive impact on enterprise development and social innovation. Therefore, intervention control strategies can be introduced into the knowledge diffusion system inspired by the infectious disease control model (2). The benefits of control strategies are capable of increasing the number of spreader individuals to speed up knowledge diffusion, but the introduction of the control strategy will also increase a lot of costs inevitably, so the optimal control is considered here; that is, the objective function should not only maximize the effect of knowledge diffusion but also reduce business costs as much as possible.

This section mainly modulates the knowledge dissemination process by introducing an intervention controller. The specific details of the IKSF model with two controllers are shown in Figure 3. Specific descriptions of the interventions are as follows.

4.1.1. Motivation. By establishing a reward mechanism, the willingness of knower individuals can be increased, which in turn motivates them to continue learning. When they keep learning and receiving material rewards, the probability of knower individuals choosing to become knowledge spreaders is ι_1 $(0 \leq \iota_1 \leq 1)$ under a predefined period of time. Furthermore, continuing learning intensity is denoted by u_1. The minimum sleep time for people is 6 h [42]. We assume that the time spent engaging in activities necessary for survival is 1 h and that people are fully engaged in learning. Therefore, the maximum learning intensity $u_{1\max}$ is 17 h per day, and u_1 can be expressed as

$$u_1 = \frac{\overline{u_1}}{u_{1\max}}, \tag{30}$$

where $\overline{u_1}$ is the current learning intensity.

4.1.2. Enhancement. We reduce the knowledge degradation of individual spreaders by establishing a supervision mechanism and formulating excellent learning methods. This can be achieved by increasing the frequency of review. Similarly, the probability of a knowledge spreader becoming a knowledge forgetter after completing the above behavior is ι_2 $(0 \leq \iota_2 \leq 1)$ under a predefined period of time. In addition, the review frequency is denoted by u_2. The shortest memory

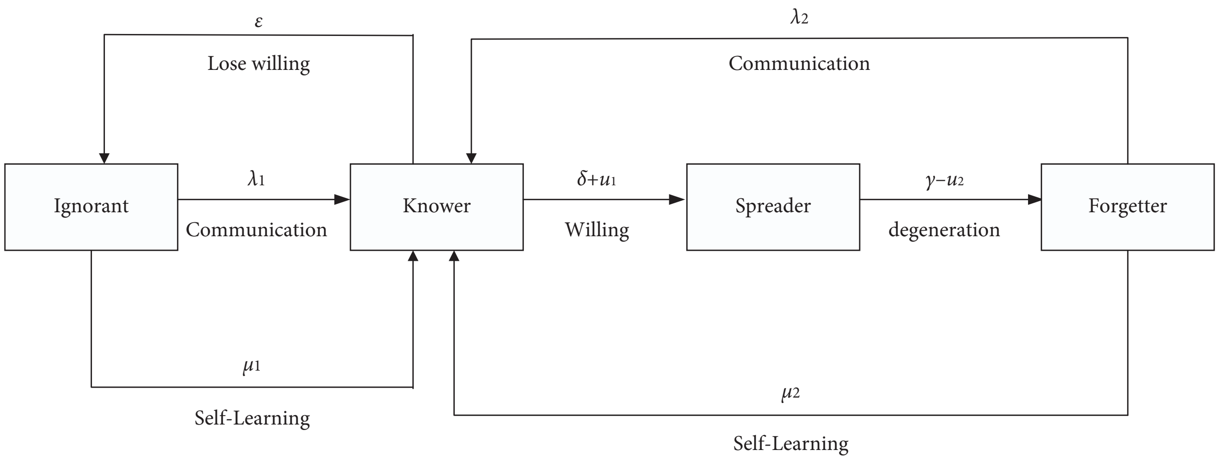


Figure 3: Schematic diagram of knowledge dissemination with controllers.

cycle is 5 min [43]. Therefore, the maximum revision frequency $u_{2\max}$ is 288 per day, and u_2 can be expressed as

$$u_2 = \frac{\overline{u_2}}{u_{2\max}}, \tag{31}$$

where $\overline{u_2}$ is the current review frequency.

According to the above description, when we add the two intervention controllers into model (2), the system is written as

$$\begin{cases} \dfrac{dI_k(t)}{dt} = -(\lambda_1 k + \mu_1) I_k(t)\Theta(t) + \varepsilon K_k(t), \\ \dfrac{dK_k(t)}{dt} = (\lambda_1 k + \mu_1) I_k(t)\Theta(t) + (\lambda_2 k + \mu_2) F_k(t) - (\delta + \iota_1 u_1) K_k(t) - \varepsilon K_k(t), \\ \dfrac{dS_k(t)}{dt} = (\delta + \iota_1 u_1) K_k(t) - (\gamma - \iota_2 u_2) S_k(t), \\ \dfrac{dF_k(t)}{dt} = (\gamma - \iota_2 u_2) S_k(t) - (\lambda_2 k + \mu_2) F_k(t). \end{cases} \tag{32}$$

According to the above description, the cost function should be set to the number of knowledge disseminators minus the cost of the controller. The objective function of the control model (32) is defined as follows:

$$J(u_1, u_2) = \int_0^{t_f} \left(S_k - \frac{c_1}{2} u_1^2 - \frac{c_2}{2} u_2^2 \right) dt, \tag{33}$$

where t_f represents the final time, c_1 and c_2 are the weight constants to increase the willingness and reduce the degradation of knowledge, respectively, and $(c_1/2)u_1^2$ and $(c_2/2)u_2^2$ are represented the cost of two control strategies. The objective function is as follows:

$$J(u_1^*, u_2^*) = \max_{u_1, u_2 \in \Psi} J(u_1, u_2), \tag{34}$$

where u_1 and u_2 are unknown control rate, and the control range is as follows:

$$\Psi = \{(u_1, u_2) \mid 0 \leq u_i \leq 1, u_i \text{ is lebesgue measurable}, i = 1, 2.\}. \tag{35}$$

The existence of the optimal control pair can be obtained using a result by Fleming and Rishel in [44] and by Lukes in [45]. When $u_i = 0, i = 1, 2$, it means that there is no investment in the control strategy. When $u_i = 1, i = 1, 2$, it means that the investment cost is the largest, and the control strategy has the best effect. For the maximization problem

(34) of system (32), it can be transformed into a Hamiltonian problem of maximization by applying Pontryagin's maximum value theorem. Here, the Hamiltonian of the control system (32) is denoted as H:

$$\begin{aligned} H &= S_k - \frac{c_1}{2}u_1^2 - \frac{c_2}{2}u_2^2 + \eta_1\frac{\mathrm{d}I_k}{\mathrm{d}t} + \eta_2\frac{\mathrm{d}K_k}{\mathrm{d}t} + \eta_3\frac{\mathrm{d}S_k}{\mathrm{d}t} + \eta_4\frac{\mathrm{d}F_k}{\mathrm{d}t} \\ &= S_k - \frac{c_1}{2}u_1^2 - \frac{c_2}{2}u_2^2 \\ &\quad + \eta_1\left[-(\lambda_1 k + \mu_1)I_k(t)\Theta(t) + \varepsilon K_k(t)\right] \\ &\quad + \eta_2\left[(\lambda_1 k + \mu_1)I_k(t)\Theta(t) + (\lambda_2 k + \mu_2)F_k(t) - (\delta + \iota_1 u_1)K_k(t) - \varepsilon K_k(t)\right] \\ &\quad + \eta_3\left[(\delta + \iota_1 u_1)K_k(t) - (\gamma - \iota_2 u_2)S_k(t)\right] \\ &\quad + \eta_4\left[(\gamma - \iota_2 u_2)S_k(t) - (\lambda_2 k + \mu_2)F_k(t)\right]. \end{aligned} \tag{36}$$

The optimal solution of the control system (32) is discussed next.

Theorem 4. *Under the initial condition $(I_k(0), K_k(0), S_k(0), F_k(0))$, problem (32) has a unique solution $(I_k^*(0), K_k^*(0), S_k^*(0), F_k^*(0))$ and (u_1^*, u_2^*) on $[0, t_f]$, where*

$$\begin{aligned} u_1^* &= \max\left\{0, \min\left\{\frac{(\eta_3 - \eta_2)\iota_1 K_k}{c_1}, 1\right\}\right\}, \\ u_2^* &= \max\left\{0, \min\left\{\frac{(\eta_3 - \eta_4)\iota_2 S_k}{c_2}, 1\right\}\right\}. \end{aligned} \tag{37}$$

Furthermore, there exists an adjoint function satisfied as

$$\begin{cases} \dfrac{\mathrm{d}\eta_1}{\mathrm{d}t} = -\dfrac{\partial H(t,x,u,\eta)}{\partial I_k} = (\eta_1 - \eta_2)(\lambda_1 k + \mu_1)\Theta(t), \\ \dfrac{d\eta_2}{dt} = -\dfrac{\partial H(t,x,u,\eta)}{\partial K_k} = (\delta + \iota_1 u_1 + \varepsilon)\eta_2 - \eta_1\varepsilon - \eta_3(\delta + \iota_1 u_1), \\ \dfrac{d\eta_3}{dt} = -\dfrac{\partial H(t,x,u,\eta)}{\partial S_k} = -1 + (\eta_1 - \eta_2)(\lambda_1 k + \mu_1)I_k(t)\dfrac{\sum_k kp(k)}{\langle k\rangle} + (\eta_3 - \eta_4)(\gamma - \iota_2 u_2), \\ \dfrac{d\eta_4}{dt} = -\dfrac{\partial H(t,x,u,\eta)}{\partial F_k} = (\eta_4 - \eta_2)(\lambda_2 k + \mu_2), \end{cases} \tag{38}$$

where the transversal conditions are $\eta_1(t_f) = 0, \eta_2(t_f) = 0, \eta_3(t_f) = 0,$ *and* $\eta_4(t_f) = 0$.

Proof. We apply Pontryagin's maximum value theorem to Hamiltonian to find the optimal solution of the system. If

(x, u) is the optimal solution to an optimal control problem, where $x = (I_k, K_k, S_k, F_k)^T$ and $u = (u_1, u_2)^T$, there is a function $\eta = (\eta_1, \eta_2, \eta_3, \eta_4)$ that satisfies the following equation:

$$\frac{d\eta_1}{dt} = -\frac{\partial H(t, x, u, \eta)}{\partial \eta_1},$$
$$\frac{d\eta_2}{dt} = -\frac{\partial H(t, x, u, \eta)}{\partial \eta_2},$$
$$\frac{d\eta_3}{dt} = -\frac{\partial H(t, x, u, \eta)}{\partial \eta_3},$$
$$\frac{d\eta_4}{dt} = -\frac{\partial H(t, x, u, \eta)}{\partial \eta_4},$$
$$\begin{cases} \dfrac{d\eta_1}{dt} = -\dfrac{\partial H(t, x, u, \eta)}{\partial I_k} = (\eta_1 - \eta_2)(\lambda_1 k + \mu_1)\Theta(t), \\ \dfrac{d\eta_2}{dt} = -\dfrac{\partial H(t, x, u, \eta)}{\partial K_k} = (\delta + \iota_1 u_1 + \varepsilon)\eta_2 - \eta_1 \varepsilon - \eta_3(\delta + \iota_1 u_1), \\ \dfrac{d\eta_3}{dt} = -\dfrac{\partial H(t, x, u, \eta)}{\partial S_k} = -1 + (\eta_1 - \eta_2)(\lambda_1 k + \mu_1) I_k(t) \dfrac{\sum_k k p(k)}{\langle k \rangle} + (\eta_3 - \eta_4)(\gamma - \iota_2 u_2), \\ \dfrac{d\eta_4}{dt} = -\dfrac{\partial H(t, x, u, \eta)}{\partial F_k} = (\eta_4 - \eta_2)(\lambda_2 k + \mu_2), \end{cases} \tag{39}$$

where the transversal conditions are $\eta_1(t_f) = 0, \eta_2(t_f) = 0, \eta_3(t_f) = 0, and\ \eta_4(t_f) = 0$.

Next, we will solve the maximization problem of equation (33); let

$$0 = \frac{\partial H(t, x, u, \eta)}{\partial u_1},$$
$$0 = \frac{\partial H(t, x, u, \eta)}{\partial u_2}. \tag{40}$$

We obtain

$$u_1^* = \begin{cases} 0, & u_1 \le 0, \\ \dfrac{(\eta_3 - \eta_2)\iota_1 K_k}{c_1}, & 0 \le u_1 \le 1, \\ 1, & u_1 \ge 1, \end{cases}$$
$$u_2^* = \begin{cases} 0, & u_2 \le 0, \\ \dfrac{(\eta_3 - \eta_4)\iota_2 S_k}{c_1}, & 0 \le u_2 \le 1, \\ 1, & u_2 \ge 1. \end{cases} \tag{41}$$

Then, we get that the optimal control problem (32) has a unique optimal solution $(I_k^*(t), K_k^*(t), S_k^*(t), F_k^*(t))$ and (u_1^*, u_2^*) on $[0, t_f]$ to maximize the objective function (32).

Since the purpose of introducing the control strategy in this paper is to promote faster knowledge diffusion and increase the number of knowledge disseminators, so default here $R_0 > 1$; we do not discuss when $R_0 < 1$. □

Remark 2. When $R_0 > 1$, the stability of the knowledge-endemic equilibrium of the control system (32) based on optimal control is globally asymptotically stable.

4.2. Guaranteed Cost Control under Uncertain Parameters. When there are multiple time-varying parameters in the system, the stability of the optimal controller of the system may not be guaranteed. As shown in Figure 4, when the willingness rate δ changes with time, the number of knowledge disseminators will no longer be stable, and the stability of the system may not be guaranteed. However, the control strategy of guaranteed cost control can effectively solve this problem.

Taking into account the uncertainty of the parameters, we get system (42), and the uncertain parameters of the control system (42) are expressed as follows:

$$\begin{cases} \frac{\mathrm{d}I_k(t)}{dt} = -(\lambda_1(t)k + \mu_1(t))I_k(t)\Theta(t) + \varepsilon(t)K_k(t), \\ \frac{\mathrm{d}K_k(t)}{\mathrm{d}t} = (\lambda_1(t)k + \mu_1(t))I_k(t)\Theta(t) + (\lambda_2(t)k + \mu_2(t))F_k(t) - (\delta(t) + \iota_1 u_1)K_k(t) - \varepsilon(t)K_k(t), \\ \frac{\mathrm{d}S_k(t)}{\mathrm{d}t} = (\delta(t) + \iota_1 u_1)K_k(t) - (\gamma(t) - \iota_2 u_2)S_k(t), \\ \frac{\mathrm{d}F_k(t)}{\mathrm{d}t} = (\gamma(t) - \iota_2 u_2)S_k(t) - (\lambda_2(t)k + \mu_2)F_k(t). \end{cases} \tag{42}$$

Unlike most existing knowledge propagation models with constant parameters, the time-varying uncertainty parameters $\lambda_1(t), \lambda_2(t), \mu_1(t), \mu_2(t), \varepsilon(t), \delta(t), \gamma(t)$ in the above control system are expressed as follows:

$$\begin{cases} \lambda_1(t) = \lambda_1 + \Delta\lambda_1, \lambda_2(t) = \lambda_2 + \Delta\lambda_2, \\ \mu_1(t) = \mu_1 + \Delta\mu_1, \mu_2(t) = \mu_2 + \Delta\mu_2, \\ \varepsilon(t) = \varepsilon + \Delta\varepsilon, \delta(t) = \delta + \Delta\delta, \gamma(t) = \gamma + \Delta\gamma, \end{cases} \tag{43}$$

where $\lambda_1, \lambda_2, \mu_1, \mu_2, \varepsilon, \delta, \gamma$ are constant, representing the normal state of knowledge disseminating individuals without external intervention. And $\Delta\lambda_1, \Delta\lambda_2, \Delta\mu_1, \Delta\mu_2, \Delta\varepsilon, \Delta\delta, \Delta\gamma$ are uncertain parameters, which can be time-varying, representing the fluctuation of knowledge dissemination to individuals around the range of constant parameters under the intervention of external conditions. And satisfying norm-bounded uncertainty, u_1 and u_2 are the control input. By considering the individual's time-varying willingness parameter, time-varying self-learning parameter, time-varying communication parameter, and time-varying degradation parameter, the system can be rewritten to obtain

$$\begin{cases} \frac{dI_k(t)}{dt} = -((\lambda_1 + \Delta\lambda_1)k + (\mu_1 + \Delta\mu_1))I_k(t)\Theta(t) + (\varepsilon + \Delta\varepsilon)K_k(t), \\ \frac{dK_k(t)}{dt} = -((\lambda_1 + \Delta\lambda_1)k + (\mu_1 + \Delta\mu_1))I_k(t)\Theta(t) + ((\lambda_2 + \Delta\lambda_2)k \\ +(\mu_2 + \Delta\mu_2))F_k(t) - ((\delta + \Delta\delta) + \iota_1 u_1)K_k(t) - (\varepsilon + \Delta\varepsilon)K_k(t), \\ \frac{dS_k(t)}{dt} = ((\delta + \Delta\delta) + \iota_1 u_1)K_k(t) - ((\gamma + \Delta\gamma) - \iota_2 u_2)S_k(t), \\ \frac{dF_k(t)}{dt} = ((\gamma + \Delta\gamma) - \iota_2 u_2)S_k(t) - ((\lambda_2 + \Delta\lambda_2)k + (\mu_2 + \Delta\mu_2))F_k(t). \end{cases} \tag{44}$$

Let $u(t) = (u_1, u_2)^T$ and $x(t) = (I_k, K_k, S_k, F_k)^T$; we rewrite system (44) as the following state feedback control system:

$$\dot{x}(t) = A(x)x(t) + \Delta A(x)x(t) + \mathrm{Bu}(t), \tag{45}$$

where

$$\begin{aligned} A(x) &= \begin{pmatrix} -(\lambda_1 k + \mu_1)\Theta & \varepsilon & 0 & 0 \\ (\lambda_1 k + \mu_1)\Theta & -(\delta + \varepsilon) & 0 & (\lambda_2 k + \mu_2) \\ 0 & \delta & -\gamma & 0 \\ 0 & 0 & \gamma & -(\lambda_2 k + \mu_2) \end{pmatrix}, \\ \Delta A(x) &= \begin{pmatrix} -(\Delta\lambda_1 k + \Delta\mu_1)\Theta & \Delta\varepsilon & 0 & 0 \\ (\Delta\lambda_1 k + \Delta\mu_1)\Theta & -(\Delta\delta + \Delta\varepsilon) & 0 & (\Delta\lambda_2 k + \Delta\mu_2) \\ 0 & \Delta\delta & -\Delta\gamma & 0 \\ 0 & 0 & \Delta\gamma & -(\Delta\lambda_2 k + \Delta\mu_2) \end{pmatrix}, \\ B &= \begin{pmatrix} 0 & 0 \\ -\iota_1 K_k & 0 \\ \iota_1 K_k & \iota_2 S_k \\ 0 & \iota_2 S_k \end{pmatrix}. \end{aligned} \tag{46}$$

Considering an uncertain continuous nonlinear system, $\dot{x}(t) = A(x)x(t) + \Delta A(x)x(t) + \mathrm{Bu}(t)$, where $A(x)$ is a polynomial matrix with respect to x.

Because $0 \le \lambda_1, \lambda_2, \mu_1, \mu_2, \varepsilon, \delta, \gamma \le 1$ and $0 \le \lambda_1(t), \lambda_2(t), \mu_1(t), \mu_2(t), \varepsilon(t), \delta(t), \gamma(t) \le 1$, then $0 \le \Delta\lambda_1, \Delta\lambda_2, \Delta\mu_1, \Delta\mu_2, \Delta\varepsilon, \Delta\delta, \Delta\gamma \le 1$. $\Delta A(x)$ is the norm-bounded uncertainty caused by the time-varying parameters of the system, satisfying the following formula:

$$\Delta A(x) = DF(x(t))E, \tag{47}$$

where

$$D = \begin{pmatrix} -k\frac{\sum_k kp(k)}{\langle k\rangle} & 0 & -\frac{\sum_k kp(k)}{\langle k\rangle} & 0 & 1 & 0 & 0 \\ k\frac{\sum_k kp(k)}{\langle k\rangle} & k & \frac{\sum_k kp(k)}{\langle k\rangle} & 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & -k & 0 & -1 & 0 & 0 & 1 \end{pmatrix},$$

$$E = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix},$$

$$F(x(t)) = \begin{pmatrix} \Delta\lambda_1 S_k(t) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \Delta\lambda_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \Delta\mu_1 S_k(t) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Delta\mu_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Delta\varepsilon & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \Delta\delta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \Delta\gamma \end{pmatrix}. \tag{48}$$

Because of $0 \le \Delta\lambda_1 S_k(t)$ and $\Delta\lambda_2, \Delta\mu_1 S_k(t), \Delta\mu_2, \Delta\varepsilon, \Delta\delta, \Delta\gamma \le 1$, the elements of the uncertain matrix $F(x(t))$ are Lebegue measurable and satisfy

$$F^T(x(t))F(x(t)) \le I, \tag{49}$$

where I is the identity matrix of the appropriate dimension and D and E are the known matrices of the appropriate dimension. For the uncertain item $\Delta A(x)$, when it satisfies the above conditions, it is said to be admissible.

For the guaranteed cost control of knowledge dissemination, a lower cost should be guaranteed when the level of knowledge dissemination is high. The requirement for knowledge dissemination becomes even more important when investment costs are low. We consider the number of knowledge disseminators in the system as the performance of knowledge dissemination. The cost functions that are defined as related to knowledge propagation control are as follows:

$$J = \int_0^\infty \left(\left(x(t) - x^*\right)^T Q\left(x(t) - x^*\right) + \left(u(t) - u^*\right)^T R\left(u(t) - u^*\right)\right)\mathrm{d}t, \tag{50}$$

where Q and R are positive definite weighting matrices and x^* and u^* are the optimal state and control action calculated by Theorem 4. The first part of the integral in (50) represents the performance level of knowledge in the process of dissemination and the second part represents the cost input in the process of knowledge dissemination. To solve the optimal control of knowledge dissemination, we design the control law as

$$u(t) = Kx(t), \tag{51}$$

where K is the income matrix, such that the closed-loop system is cost-guaranteed for all admissible uncertainties (47), and the corresponding performance metric does not exceed a certain upper bound:

$$\dot{x}(t) = (A(x) + \mathrm{DFE} + \mathrm{BK})x(t). \tag{52}$$

Lemma 1. (Schur complementary lemma [46]). For a given symmetric matrix,

$$S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}, \tag{53}$$

with $S_{11}^T = S_{11}, S_{22}^T = S_{22}$, and the following three conditions are equivalent:

(1) $S < 0$

(2) $S_{11} < 0, S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$

(3) $S_{22} < 0, S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$

Lemma 2 (see [47]). Given matrices Y, D, and E of appropriate dimensions, where Y is symmetric, then there is the following formula: $Y + \mathrm{DFE} + E^T F^T D^T < 0$. For the matrix F, there is a condition for $F^T(t)F(t) \le I$ to be established; the necessary and sufficient condition for the $Y + \mathrm{DFE} + E^T F^T D^T < 0$ condition to be established is that there is a constant $\varepsilon > 0$ that satisfies $Y + \varepsilon DD^T + \varepsilon^{-1} E^T E < 0$.

Definition 1. For the knowledge propagation control model (44) and the cost function (50), if there is a controller $u(t)$ and a positive value J^* such that the input cost conforms to the performance level in the knowledge propagation process and the system is asymptotically stable, then J^* is called the

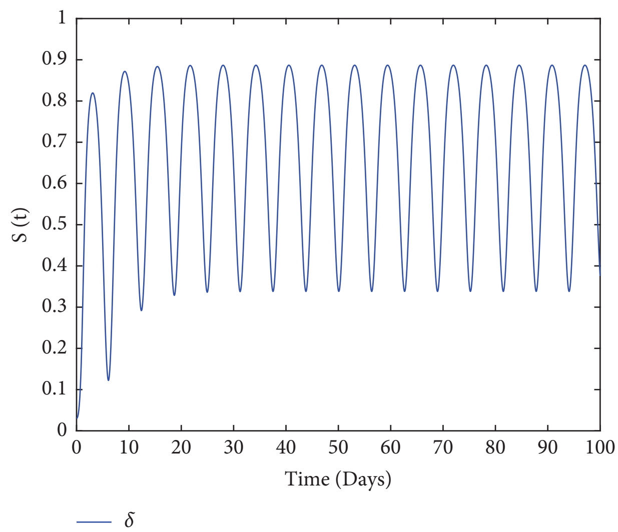


Figure 4: When δ changes over time, the density changes the curve of $S_k(t)$.

upper bound of the performance of system (45) and $u(t)$ is the state feedback guaranteed cost controller of the uncertain control system, where the value of its cost function (50) satisfies $J \leq J^*$.

Theorem 5. *For uncertain continuous systems (45) and performance indicators (50), if there is a symmetric positive definite matrix L, the positive definite matrix P and matrix K are such that, for all allowable uncertainties, inequality (54) is satisfied:*

$$Q + K^T RK + P(\mathrm{DFE} + \mathrm{BK}) + (\mathrm{DFE} + \mathrm{BK})^T P + L^T P + PL < 0. \tag{54}$$

Then, (51) is called the guaranteed cost control law of system (45); $J = x^T(0)Px(0)$ is the corresponding upper bound of system performance at this time.

Proof. Suppose there are matrices P and K that satisfy the conditions; for uncertainties, inequality (54) will hold, and let $u(t) = Kx(t)$.

We change system (52) to

$$\dot{x}(t) = f(t,x) + (\mathrm{DFE} + \mathrm{BK})x(t). \tag{55}$$

According to the strategy of [48], where $A(x)$ in the function is a piecewise continuous function with respect to t in any finite time interval $[t_0, t_1]$, the elements of $A(x)$ are bounded, which can be simplified by the elementary transformation of the matrix as follows:

$$A(x) = \begin{pmatrix} -(\lambda_1 k + \mu_1)\Theta & \varepsilon & 0 & 0 \\ 0 & -\delta & \gamma & 0 \\ 0 & 0 & -\gamma & \lambda_2 k + \mu_2 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{56}$$

where

$$\|A(x)\| = \max\{-(\lambda_1 k + \mu_1)\Theta, \varepsilon - \delta, 0, \lambda_2 k + \mu_2\}. \tag{57}$$

Therefore, $\|A(x)\| \leq a$; for all $x, y \in R^n$ and $t \in [t_0, t_1]$, there is $\|f(t,x) - f(t,y)\| = \|A(x)x - A(y)y\| \leq \|a(x-y)\| = L\|(x-y)\|$. So, $f(t,x) = A(x)x$ is a nonlinear function satisfying the Lipschitz condition and $f(t,x) \leq Lx$.

We define the Lyapunov function $V(x(t)) = x^T(t)Px(t)$.

Taking the derivation of the above formula along system (52), we obtain

$$\begin{aligned} \dot{V}(x(t)) &= \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) \\ &= [f + (\mathrm{DFE} + \mathrm{BK})x]^T Px + x^T P[f + (\mathrm{DFE} + \mathrm{BK})x] \\ &= f^T Px + x^T(\mathrm{DFE} + \mathrm{BK})^T Px + x^T Pf + x^T P(\mathrm{DFE} + \mathrm{BK})x \\ &\leq x^T L^T Px + x^T(\mathrm{DFE} + \mathrm{BK})^T Px + x^T PLx + x^T P(\mathrm{DFE} + \mathrm{BK})x \\ &= x^T\left[L^T P + (\mathrm{DFE} + \mathrm{BK})^T P + PL + P(\mathrm{DFE} + \mathrm{BK})\right]x. \end{aligned} \tag{58}$$

From (54), we obtain

$$\dot{V}(x(t)) < x^T(t)\left[-K^T \mathrm{RK} - \mathrm{Q}\right]x(t). \tag{59}$$

Furthermore, we integrate the time t from 0 to ∞ on both sides of the above formula, since system (52) is stable, so $V(x(\infty)) = 0$; then, we obtain

$$\int_0^\infty x^T(t)\left[-K^T RK - Q\right]x(t)dt \leq V(x(0)) = x^T(0)Px(0), \tag{60}$$

where $Q + K^T \mathrm{RK}$ is the income brought by the weight of the performance indicator function to the state quantity plus the weight of the control quantity.

Defined by Definition 1, the proof is completed.

The condition in Theorem 5 contains the uncertainty matrix F, so all allowable uncertainty matrices F are tested. We give an equivalent characterization of this condition by the following theorem. □

Theorem 6. *There exist matrices P, L, and K such that, for all admissible uncertainties, matrix inequality (54) holds if and only if there exists a scalar* $\varepsilon > 0$; *symmetric positive definite matrices X and W such that*

$$\begin{pmatrix} BW + (BW)^T + XL^T + LX + \varepsilon DD^T & * & * & * \\ W & -R^{-1} & * & * \\ EX & 0 & -\varepsilon I & * \\ X & 0 & 0 & -Q^{-1} \end{pmatrix} < 0. \tag{61}$$

If matrix inequality (61) has a feasible solution (ε, W, X), then

$$u(t) = WX^{-1}x(t), \tag{62}$$

is a guaranteed cost control law of system (45), and the maximum of system performance is

$$\overline{J} = tr\left(X^{-1}\right) = \overline{J}^*. \tag{63}$$

Proof. We define $Y = Q + K^T \mathrm{RK} + \mathrm{PBK} + (\mathrm{BK})^T P + L^T P + \mathrm{PL}$.

Then, the matrix inequality (54) can be written as $Y + \mathrm{PDFE} + E^T F^T (\mathrm{PD})^T < 0$. According to Lemma 2, the matrix inequality above holds for all indeterminate matrices F satisfying $F^T(t)F(t) \le I$ if there is a scalar $\varepsilon > 0$, such that $Y + \varepsilon \mathrm{PDD}^T P^T + \varepsilon^{-1} E^T E < 0$. Further applying the matrix Schur complement property, we obtain

$$\begin{pmatrix} \mathrm{PBK} + (\mathrm{BK})^T P + L^T P + \mathrm{PL} + \varepsilon \mathrm{PDD}^T P & * & * & * \\ K & -R^{-1} & * & * \\ E & 0 & -\varepsilon I & * \\ I & 0 & 0 & -Q^{-1} \end{pmatrix} < 0. \tag{64}$$

Multiplying left and right by $\mathrm{diag}(P^{-1}, I, I, I)$, respectively, in matrix inequality (64), let $X = P^{-1}, W = KP^{-1}$, then (61) can be obtained; the matrix inequality (64) is a linear matrix inequality about the variable ε, X, W, so the solver feasp in the LMI toolbox can be used to judge the feasibility of the linear matrix inequality. Furthermore, if the inequality has a feasible solution (ε, X, W), then (62) gives a parametric representation of the guaranteed cost control law of system (42). □

5. Numerical Simulation

The proposed theoretical results are verified by numerical simulations, where the effects of model parameters are investigated. Supposing there are 2000 nodes in the network, the degree distribution of the vertices is a power-law distribution, the initial number of vertices in the network is 10, the vertices are randomly connected, and we add new nodes with 5 new edges in turn. At the same time, the effectiveness of optimal control and guaranteed cost control of the system is also verified.

5.1. Theoretical Verification. Figures 5 and 6 show different degrees of density evolution when $R_0 < 1$ and $R_0 > 1$. We show that the evolution under $R_0 < 1$ and $R_0 > 1$ is different, although the two given R_0 are nearly identical, which verifies the setting in Theorem 1. When $R_0 < 1$, the parameter selects $\lambda_1 = 0.01, \lambda_2 = 0.5, \mu\ 1 = 0.01, \mu_2 = 0.6,\ \varepsilon = 0.2,\ \delta = 0.01$, and $\gamma = 0.05$; then, there is $R_0 = 9.9760e - 04 < 1$; when $R_0 > 1$, we choose parameter $\lambda_1 = 0.4, \lambda_2 = 0.5, \mu_1 = 0.3, \mu\ 2 = 0.6, \varepsilon = 0.2,\ \delta = 0.6$, and $\gamma = 0.05$; then, there is $R_0 = 2.2446 > 1$. It can be seen from Figure 5 that when $R_0 < 1$, the density of Spreader will drop to 0; when $R_0 > 1$ in Figure 6, the knowledge will exist forever, which can verify Theorems 2 and 3.

5.2. Parameter Influence. Since we take many factors into account in this model, firstly, the willingness rate δ is studied. Figure 7 shows that as the value of δ increases, so does the number of knowledge disseminators, indicating that the willingness rate δ has a positive impact on knowledge dissemination.

Secondly, the initial communication rate λ_1 is studied. Figure 8 shows that the larger the value of λ_1 in the early stage of knowledge dissemination, the greater the density of knower individuals, indicating that the value of the communication rate λ_1 has a good promotion for the early knowledge dissemination process.

Finally, the degradation rate is studied. Figure 9 shows that, as the value of γ increases, the density of knowledge disseminators is decreasing, indicating that the degradation rate γ hurts knowledge dissemination.

5.3. Optimal Control. When $R_0 > 1$, we conduct knowledge dissemination simulations under various control measures. The performance of the knowledge diffusion model is then validated with three different control strategies. For simplicity, let $\iota_1 = 0.5$ and $\iota_2 = 0.5$.

5.3.1. $u_1 \neq 0$ *and* $u_2 = 0$. We used control u_1, which increases the rate of willingness by increasing learning intensity and material rewards (u_1). The optimal control curve u_1^* is shown in Figure 10. u_1^* rapidly decreases to near 0 until 0. Under this control measure, knower individuals become knowledge disseminators by sharing knowledge stimulated by a reward mechanism. Figure 11 mainly describes the changes in the four groups of people under the control strategy u_1, in which the spreader individuals increased slightly and the other three groups decreased slightly. Furthermore, the maximum rate of change in individual spreaders compared to the noncontrol system is about 15%, which is consistent with our expected results. When increasing the willingness of knower individuals, the knowledge dissemination model performs better.

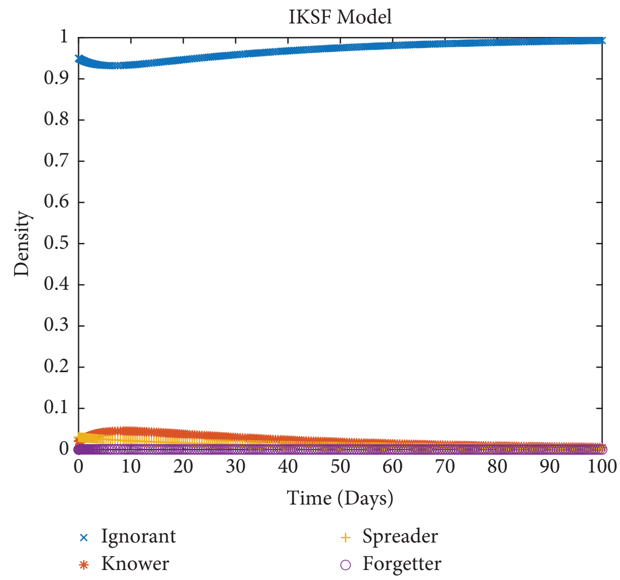


FIGURE 5: Diagram of density change of four types of population when $R_0 < 1$.

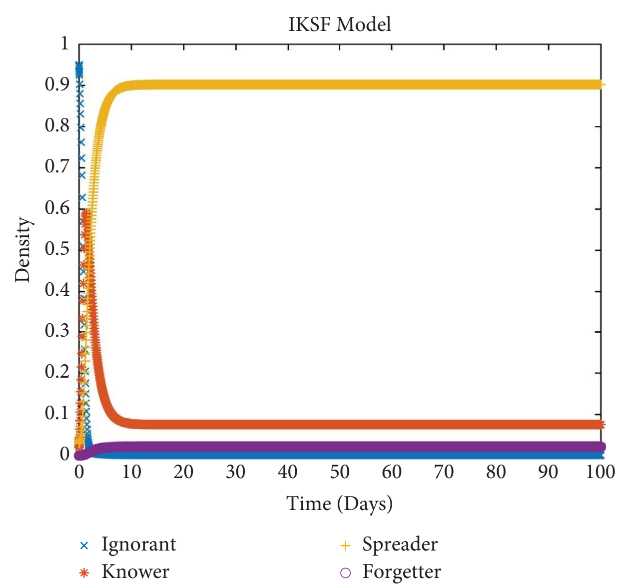


FIGURE 6: Diagram of density change of four types of population when $R_0 > 1$.

5.3.2. $u_1 = 0$ and $u_2 \neq 0$. We used control u_2, which reduces the probability of degradation by increasing the frequency of review. The optimal control curve u_2^* is shown in Figure 12. u_2^* rapidly decreases to near 0 until 0. Under this control, the individual spreader continues to maintain the identity of the knowledge disseminator through regular review of knowledge under a supervision mechanism and excellent learning methods. Figure 13 mainly describes the changes in the four types of populations under the control strategy u_2, in which the spreader individuals and knower individuals slightly increased, while the other two populations slightly decreased. Furthermore, the maximum rate of change in individual spreaders compared to the noncontrol system was about 16%, which is consistent with our expected results. When reducing the knowledge degradation of the individual spreader, the knowledge dissemination model performs better.

5.3.3. $u_1 \neq 0$ and $u_2 \neq 0$. We improve the willingness of knower individuals (u_1) while reducing the degradation of spreader individuals (u_2). The optimal control curve u_1^*, u_2^* is shown in Figure 14. u_1^*, u_2^* rapidly decreases to near 0 until 0. Under this control, the individual spreader continues to maintain the identity of the knowledge disseminator through regular review of knowledge under a supervision mechanism and excellent learning methods. Figure 15 mainly describes the changes in the four groups of people under the two control strategies u_1, u_2, in which the spreader

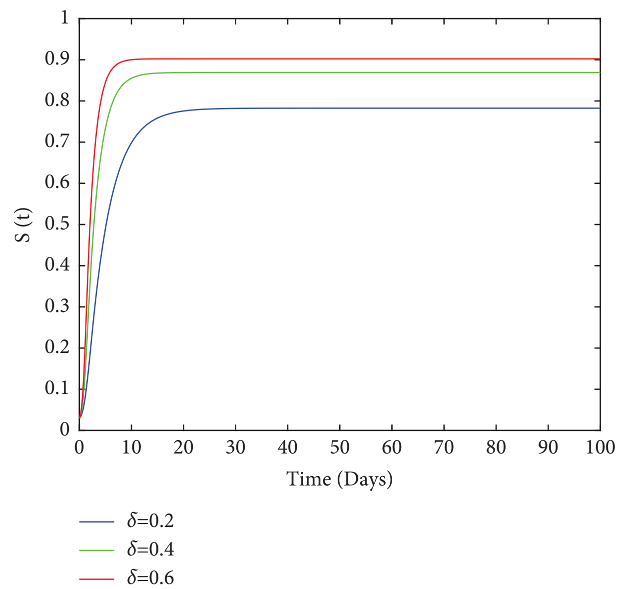


FIGURE 7: The density of knowledge disseminators varies with the value of δ.

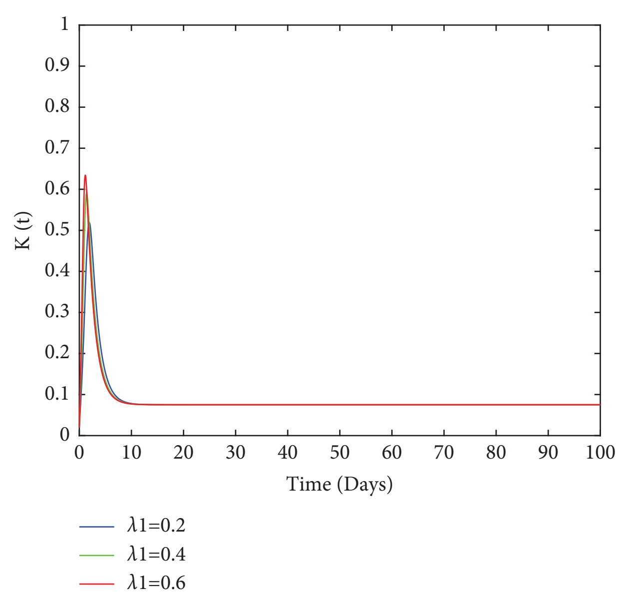


FIGURE 8: The density of knower individuals changes with the value of λ_1.

and knower individuals increased slightly and the other two groups decreased slightly. Furthermore, the maximum rate of change of individual spreaders compared to the non-control system is about 25%, which is consistent with our expected results. When increasing the willingness of knower individuals and reducing the degradation of spreader individuals, the knowledge dissemination model performs best.

5.4. Robustness of Uncertain Parameter Control System. According to the characteristics of knowledge dissemination, the parameters of system (45) are as follows:

$$L = \begin{pmatrix} 200 & 0 & 0 & 0 \\ 0 & 300 & 0 & 0 \\ 0 & 0 & -500 & 0 \\ 0 & 0 & 0 & -200 \end{pmatrix},\quad B = \begin{pmatrix} 0 & 0 \\ -1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix}. \tag{65}$$

Uncertain structure selects

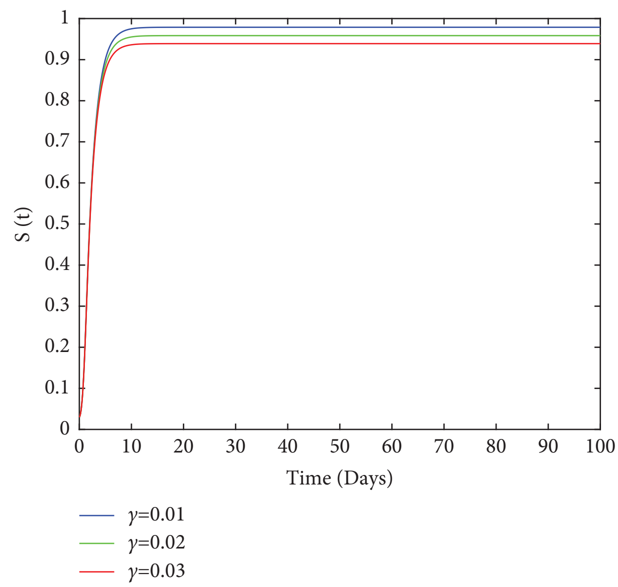


FIGURE 9: The density of knowledge disseminators varies with the value of γ.

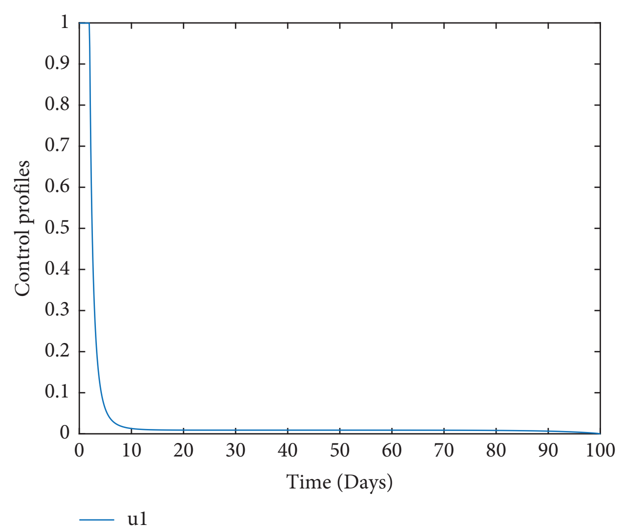
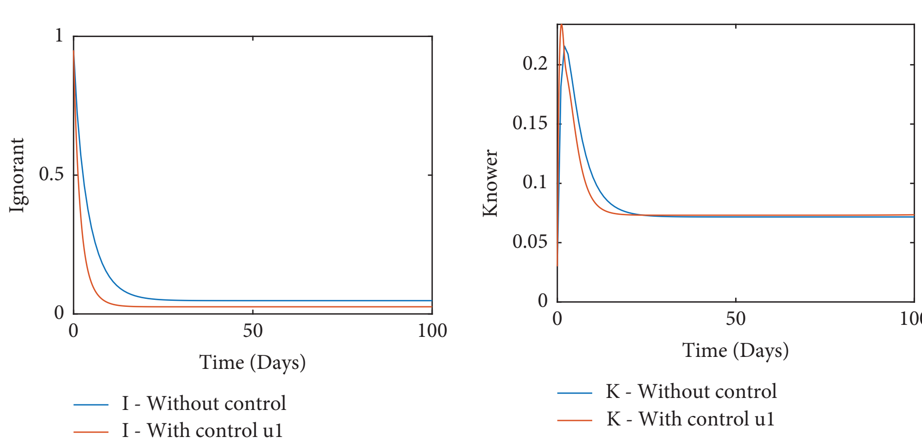
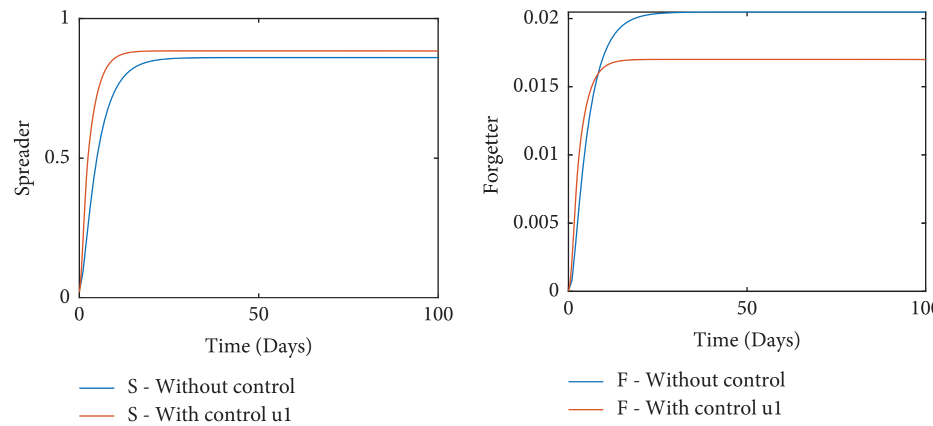


FIGURE 10: Diagram of the optimal solution u_1^* of knowledge propagation.



(a) (b)

FIGURE 11: Continued.



(c) (d)

Figure 11: Comparison of population density with u_1.

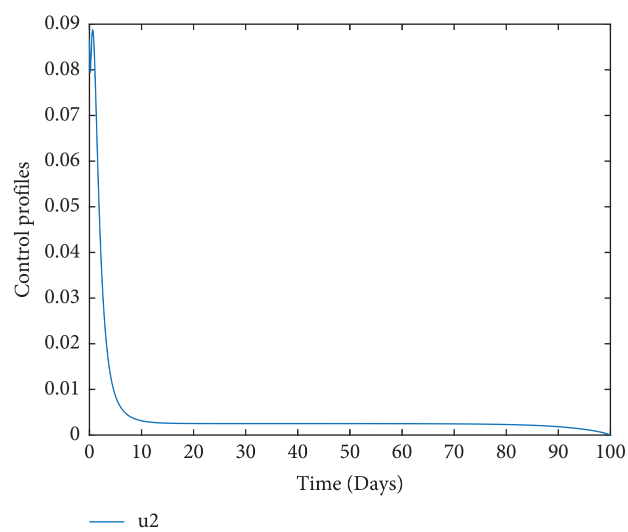
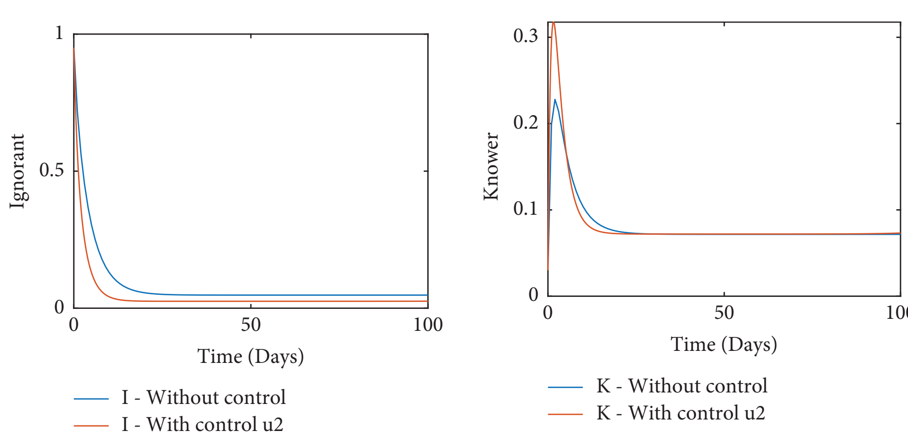
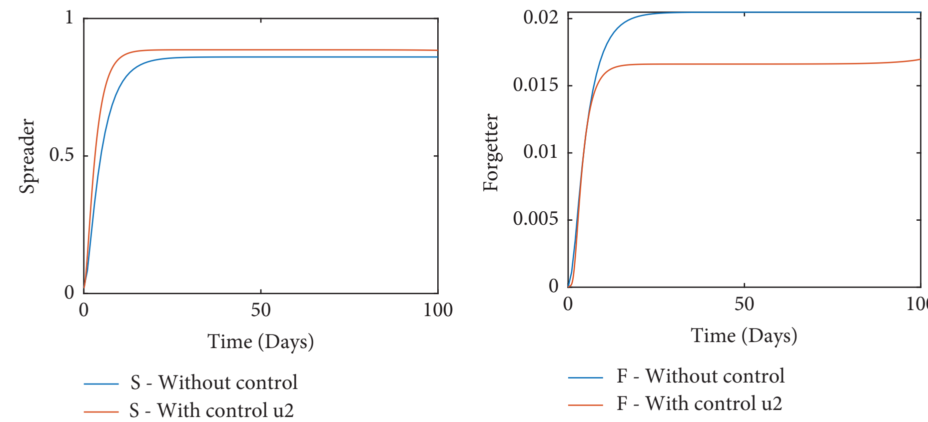


Figure 12: Diagram of the optimal solution u_2^* of knowledge propagation.



(a) (b)

Figure 13: Continued.



(c) (d)

Figure 13: Comparison of population density with u_2.

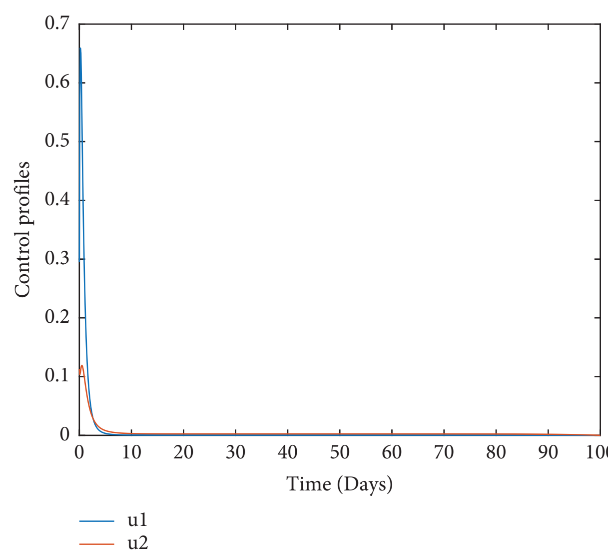
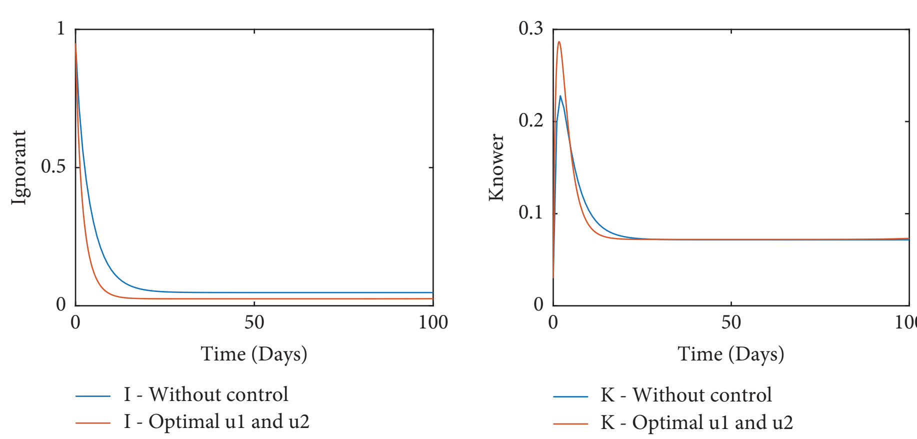
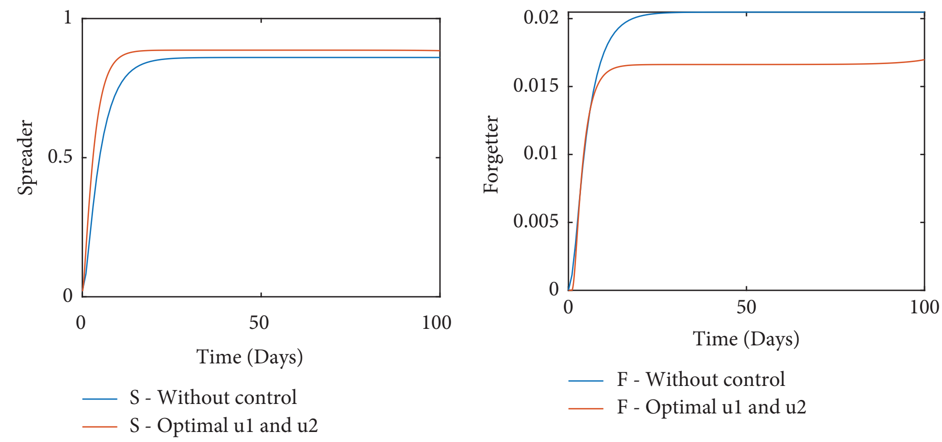


Figure 14: Diagram of the optimal solution u_1^* and u_2^* of knowledge propagation.



(a) (b)

Figure 15: Continued.



(c) (d)

Figure 15: Comparison of population density with u_1, u_2.

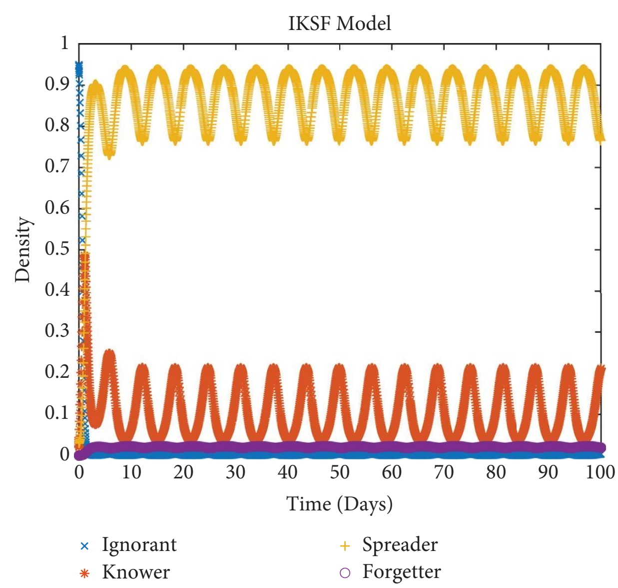


Figure 16: Diagram of knowledge transmission without guaranteed cost control.

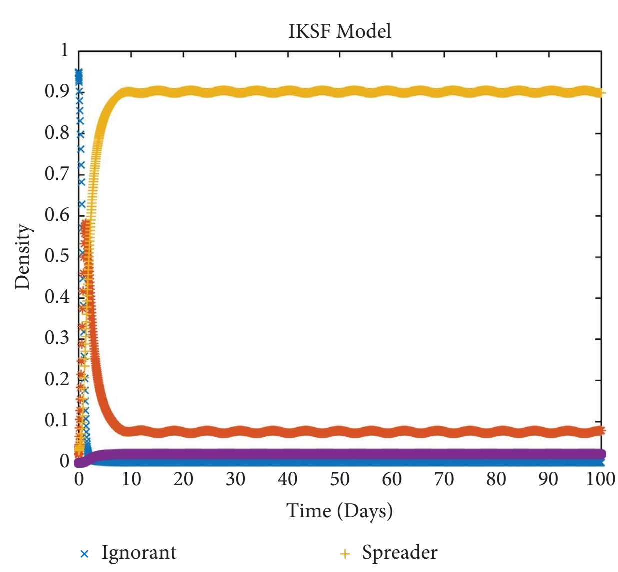


Figure 17: Diagram of knowledge transmission with guaranteed cost control.

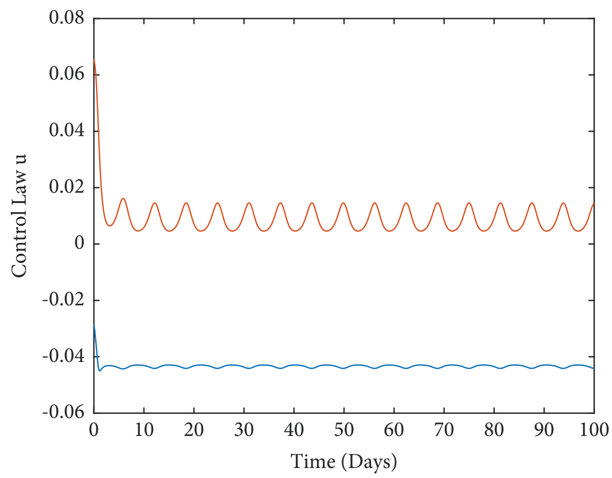


Figure 18: The temporal variation of the control law.

$$D = \begin{pmatrix} -29.928 & 0 & -9.9760 & 0 & 1 & 0 & 0 \\ 29.928 & 3 & 9.9760 & 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & -3 & 0 & -1 & 0 & 0 & 1 \end{pmatrix},$$
$$E = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad (66)$$

given

$$Q = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$
$$R = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (67)$$

Applying Theorem 6, the corresponding linear matrix inequality (61) is available from the LMI toolbox, and a feasible solution is obtained:

$$X = \begin{pmatrix} -1.9292 & 1.5244 & -1.2840e-04 & 0.0209 \\ 1.5244 & -1.2884 & -0.0037 & 0.0764 \\ -1.2840e-04 & -0.0037 & 0.0072 & -0.0011 \\ 0.0209 & 0.0764 & -0.0011 & 0.0348 \end{pmatrix},$$
$$W = \begin{pmatrix} -0.0264 & 0.0198 & -5.5194e-05 & -0.0063 \\ -0.0466 & 0.0171 & -1.3533e-05 & 4.2526e-04 \end{pmatrix},$$
$$\varepsilon = 0.7486. \quad (68)$$

Then, we get the guaranteed cost controller and guaranteed cost index of the system:

$$u^*(t) = \begin{bmatrix} -0.0263 & -0.0497 & -0.0423 & -0.0580 \\ 0.0692 & 0.0591 & 0.0062 & -0.1589 \end{bmatrix} x(t),$$
$$\bar{J}^* = 142.3200. \quad (69)$$

We choose the willingness rate as an uncertain parameter and the time-varying part $\Delta\delta(t) = \sin(t)$. Figure 16 is the simulation diagram of the four types of people in the IKSF dissemination system under the unguaranteed cost control. Figure 17 is the guaranteed cost control effect comparison of the IKSF knowledge dissemination system picture.

Figure 16 shows that when the system parameters fluctuate within the range, the density fluctuations of the four groups of people will be caused. In Figure 17, compared with no controller, the guaranteed cost controller can well control the density fluctuations of the four groups of people brought by the uncertainty parameters of the system so that it can be maintained at the desired given value.

Figure 18 shows the change of control laws over time when the system is affected by time-varying uncertainty. As can be seen from the figure, the control law initially declines, then continues to fluctuate due to uncertainty, and stabilizes at a specific value, which is in line with our expectations.

Remark 3. The aforementioned guaranteed cost control example of uncertain parameters is simulated under the premise of $R_0 > 1$, and the simulation is meaningless when $R_0 < 1$.

6. Conclusion

In this paper, the knowledge transfer process in complex networks is modeled and dynamically analyzed by considering the interaction effects of multiple mechanisms of knowledge transfer. Specifically, we established a new IKSF knowledge dissemination model, and the model simultaneously considers multiple factors such as internalization mechanism, degradation mechanism, communication, and willingness. We obtain the basic reproduction number of the scale-free network that depends on the multimechanism and determine the knowledge loss equilibrium E^0 and the unique endemic equilibrium E^*. Furthermore, we studied the stability of the equilibrium point. Through numerical simulation, enhancing the willingness of knower individuals and reducing the degradation rate of spreader individuals will help to promote the dissemination of knowledge.

In terms of optimal control, to increase the number of knowledge disseminators, we establish incentive mechanisms to stimulate willingness and monitoring mechanisms to reduce knowledge degradation. Experimental results show that the effect is the most noticeable when multiple controls are applied.

To study the process of knowledge propagation more precisely, we study the optimal guaranteed cost control problem of knowledge propagation with uncertain parameters and control constraints. Using the Lyapunov stability theory and guarantee cost control technology, the control law of optimal guarantee cost is designed in the form of linear matrix inequality so that the four groups of people in the IKSF model are stabilized in an ideal state, and the optimal guarantee cost of knowledge dissemination is obtained. Experimental results show that the proposed method has a significant effect on dealing with parameter uncertainty problems.

However, there are still many shortcomings that need further study.

(1) There are many ways to access knowledge, but it is not clear exactly how knowledge diffusion can be carried out in reality because people always have different ways of acquiring knowledge. This prevents us from thinking about access to knowledge in a particular way.

(2) This paper only conducts numerical simulations on scale-free networks and does not compare experiments with other heterogeneous networks.

(3) System parameters may be affected by various factors. We find that when system parameters change over time, the system may be in a chaotic or bifurcated state.

(4) The robust control of uncertain nonlinear systems based on the LMI method is studied, and the influence of adding different control constraints to the system needs to be further studied.

These problems will be considered in future work.

Data Availability

The data is in the simulation of this paper.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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