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Research Article

Local Entropy-Based Coupled Anisotropic Diffusion for Detail-And Edge-Preserving Smoothing

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It is important in image restoration to remove noise while preserving sharp edges and fine details such as blurred thin edges and low-contrast fine feature. The Perona–Malik (P-M) model is a well-known anisotropic diffusion denoising model, which can effectively remove noise while preserving edges. However, its diffusion coefficient only associates with the gradient of each pixel but not with the local region information; thus, the P-M model is not able to effectively preserve the important details of image. To address this problem, this paper proposes an anisotropic diffusion denoising model based on local entropy. The diffusion coefficient of the new model not only depends on the gradient of image but also on the local region information described by local entropy. On this basis, a coupled anisotropic diffusion scheme is proposed for detail-and edge-preserving smoothing. Experimental results show that the proposed model not only can effectively remove noise while preserving the boundaries better but also can maintain important details in an image very well.

1. Introduction

Images are often suffered by noise in acquisition and transmission, which usually degrades the visual quality of the image. Image restoration is an important step in low-level computer vision, especially when the input image is blurred or noisy [1, 2]. An ideal restoration algorithm is expected to remove noise and meanwhile preserve details in the input image. For that, a number of nonlinear methods were proposed in the literature; for example [3–6].

In [3], Perona and Malik first proposed an anisotropic diffusion equation for image smoothing and edge detection, in which the diffusion coefficient depends on the value of the image gradient. Specifically, let $I(x, y)$ represent the original image defined in a convex domain $\Omega \subset R \times R$. In the anisotropic diffusion [3], a family of increasingly smoothed images, $u(x, y, t)$, is derived from the solution of the following partial differential equation:

$$\partial_t u = \operatorname{div}(c(|\nabla u|)\nabla u), \tag{1}$$

with the initial condition $u(x, y, 0) = I(x, y)$. In (1), div and ∇ represent the divergence and gradient operators, respectively, and the diffusion coefficient $c(\cdot)$ is a nonnegative decreasing function of $|\nabla u|$ (the gradient magnitude of u). If $c(\cdot)$ is a constant, the anisotropic diffusion (1) reduces to the classical Gaussian filtering. The desirable diffusion coefficient should be to make (1) diffuse more in flat areas and less around edges. Two such diffusion coefficients suggested by Perona and Malik [3] are as follows:

$$c(|\nabla u|) = \exp\left(-\left(\frac{|\nabla u|}{K}\right)^2\right), \tag{2}$$

and

$$c(|\nabla u|) = \frac{1}{1 + (|\nabla u|/K)^2}, \tag{3}$$

where $K > 0$ is a constant parameter to be tuned for a particular application. The diffusion coefficients by (2) or (3) clearly make the diffusion process perform selectively smoothing, which only depends on the magnitude of image gradient at a point.

The P-M model is essentially a gradient-based filter, in which the diffusion coefficient at a point depends on the gradient magnitude of the diffused point. When the gradient magnitude is large (often happens around the edge), the diffusion coefficient is approximate to zero and then the smoothing process is terminated. Conversely, when the gradient magnitude is small (often happens on the flat region), the diffusion coefficient is close to one, and so the smoothing process is performed. Therefore, the P-M model can reduce noise and preserve edges simultaneously.

Unfortunately, it has been widely noted that the anisotropic diffusion (1) has some limits in applications. In this model, high gradient magnitude is generally considered as a good indication of edges, while low gradient magnitude always points to nonedge regions. However, the low gradient magnitude may not always point to nonedge regions or noise because some important local details along with weak edges may have low gradient magnitude. For example, if the initial image has some important details, but the magnitude of image gradient around the details is not large enough, and then the large diffusion coefficient results in a strong smoothing effect, and hence, the details lose. In other words, the diffusion process by (1) does not perform well for detail-preserving. To alleviate this problem, Chao and Tsai [4] proposed an improved anisotropic diffusion model. This model incorporates both local gradient and gray-level variance in the diffusion coefficient, and so can preserve edges and fine details while removing noise effectively. However, this method cannot preserve weak edges accurately. Recently, Wen et al. [5] proposed a new anisotropic fourth-order diffusion equation model based on image features for image denoising, in which the diffusion coefficient is dependent on the first-order derivatives for preserving edges and second-order derivatives for smoothing image.

In addition to the nonlinear diffusion models, there are two well-known nonlinear filters to reduce noise and preserve edges and/or details simultaneously. Tomasi and Manduchi [6] proposed the bilateral filter, which is an edge-preserving and noise-reducing smoothing method. This filter is a weighted average of the local neighborhood samples, where the weight depends not only on Euclidean distance but also on the radiometric differences (differences in the range, e.g., color intensity) between the center sample and the neighboring samples. This filter is locally adaptive, and it was shown to give similar and possibly better results to those obtained by the P-M model. However, it works better only on obvious boundaries but fails to preserve small details. Buades et al. [7] introduced the NL-means algorithm which is based on the redundancy property of periodic images, textured images, or natural images to remove noise. This method is essentially a neighborhood filter, in which the noisy grey-values are replaced by a weighted average (mean) of the grey values in the entire noisy image where the weight is determined by the neighborhood similarity of image patches. The NL-mean filter is good at detail-preserving but is very time consuming.

In this paper, inspired by the work in [4, 5], we propose a new detail-and edge-preserving smoothing technique based on the P-M anisotropic diffusion and entropy of the local region histogram (local entropy), which incorporates both local gradient and local entropy in the diffusion coefficient. The local entropy has the properties as follows: a large value is taken around the edges and details, and the small value can be made in the flat region. Besides, the local entropy is robust to noise. Therefore, the proposed model can preserve edges and fine details while removing noise effectively. Experiments show that the proposed method can really filter out noise effectively, while preserving both edges and fine details in the restored image.

The organization of this paper is as follows: Section 2 briefly introduces the local entropy for image analysis and then presents the proposed model combining local entropy with anisotropic diffusion. Section 3 shows some experimental results. This paper is concluded in Section 4.

2. The Proposed Model

2.1. Local Entropy. The entropy of a system as defined by Shannon gives a measure of uncertainty about its actual structure. Entropy is a widely used measure of local information content or uncertainty. Pun [8] used Shannon's concept to define the entropy of an image, assuming that an image is entirely represented by its gray level histogram only.

Let Ω_k be a small neighborhood window of size $M_k \times N_k$, located at the central pixel k. Following Pun's definition [8], the local entropy $E(\Omega_k)$ is defined as

$$E(\Omega_k) = -\sum_{j=0}^{L-1} p_j \log p_j, \; p_j = \frac{n_j}{M_k \times N_k}, \tag{4}$$

where L and n_j are the maximal gray level and the number of pixels with gray level j appearing in the neighborhood Ω_k, respectively. We call $E(\Omega_k)$ the local entropy of the central pixel k. Generally, $M_k = N_k$, $M_k \in \{3, 5, 7\}$; in this paper, we adopt $M_k = N_k = 3$.

By moving the window Ω_k pixel by pixel within the image from left to right and top to bottom, we obtain the entropy image of the original image, which is composed of the local entropy value of each pixel.

Figure 1 shows the entropy images for a synthetic noisy image and real Lena image. From Figure 1, we can see that local entropy has the two properties: (1) local entropy is not sensitive to noise due to common effort of all pixels in a local window; (2) local entropy is larger for a heterogeneous region (e.g., including edge and/or details) but smaller for a homogeneous neighborhood.

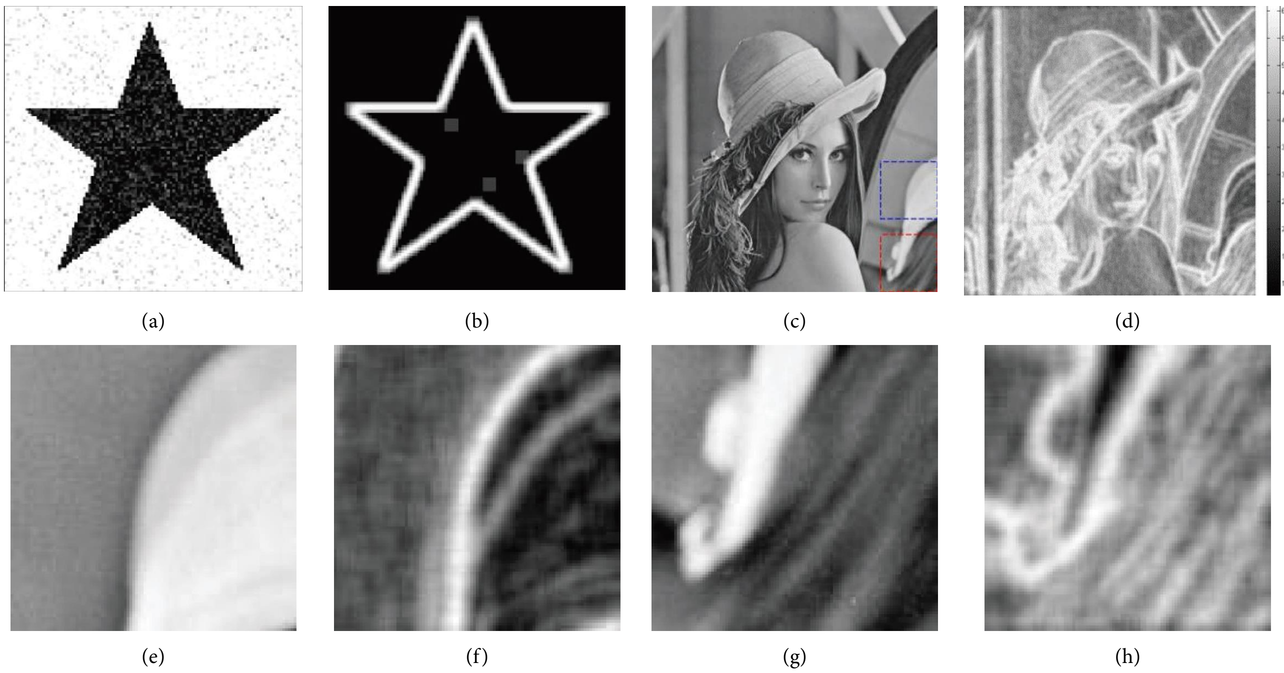


Figure 1: Local entropy images. (a, c): original images (synthetic noisy image and real Lena image); (b, d): local entropy images of (a, c); (e, g): two patches shown in the blue and red bounding boxes for Lena image; (f, h): local entropy images of (e, g).

2.2. Model Description. The deficiencies of the P-M model [3] are mainly caused by the diffusion coefficient, which only considers the magnitude of image gradient at each point. However, the gradient magnitude may be high at a noisy point, so the filtering performance is not good enough. Besides, the gradient magnitudes for some details are small, and therefore, the diffusion coefficient is large; this leads to a strong smoothing effect and the detail losses. Based on this observation, we propose a new diffusion coefficient based on gradient magnitude and local entropy in (4), as follows:

$$g\left(\left|\nabla I\left(x,y\right)\right|,E\left(x,y\right)\right)=\frac{1}{1+\left(\left|\nabla I\left(x,y\right)\right|\cdot f\left(E\left(x,y\right)\right)/k_0\right)^2}, \quad (5)$$

where $|\nabla I(x,y)|$ is the gradient magnitude of the image I at point (x,y), $E(x,y)$ is the local entropy at (x,y) by (4), and K_0 is a positive constant used as an edge strength threshold. The function $f(E(x,y))$ is defined as

$$f\left(E\left(x,y\right)\right)=1+M\frac{E\left(x,y\right)-E_{\min}}{E_{\max}-E_{\min}}, \quad (6)$$

where $E_{\max}$ and $E_{\min}$ represent the maximum and minimum local entropies, respectively, and M is the maximum of $|\nabla I(x,y)|$ over image domain. Clearly, $1\le f(E(x,y))\le M+1$.

In [9], Nordström introduced the fidelity source $I-u$ in diffusion (1), the role of which is to force $u\ (x,t)$ to always remain close to the original image $I\ (x,y)$ during the diffusion, as follows:

$$\partial_t u-\operatorname{div}\left(c\left(|\nabla u|\right)\nabla u\right)=I-u, \quad (7)$$

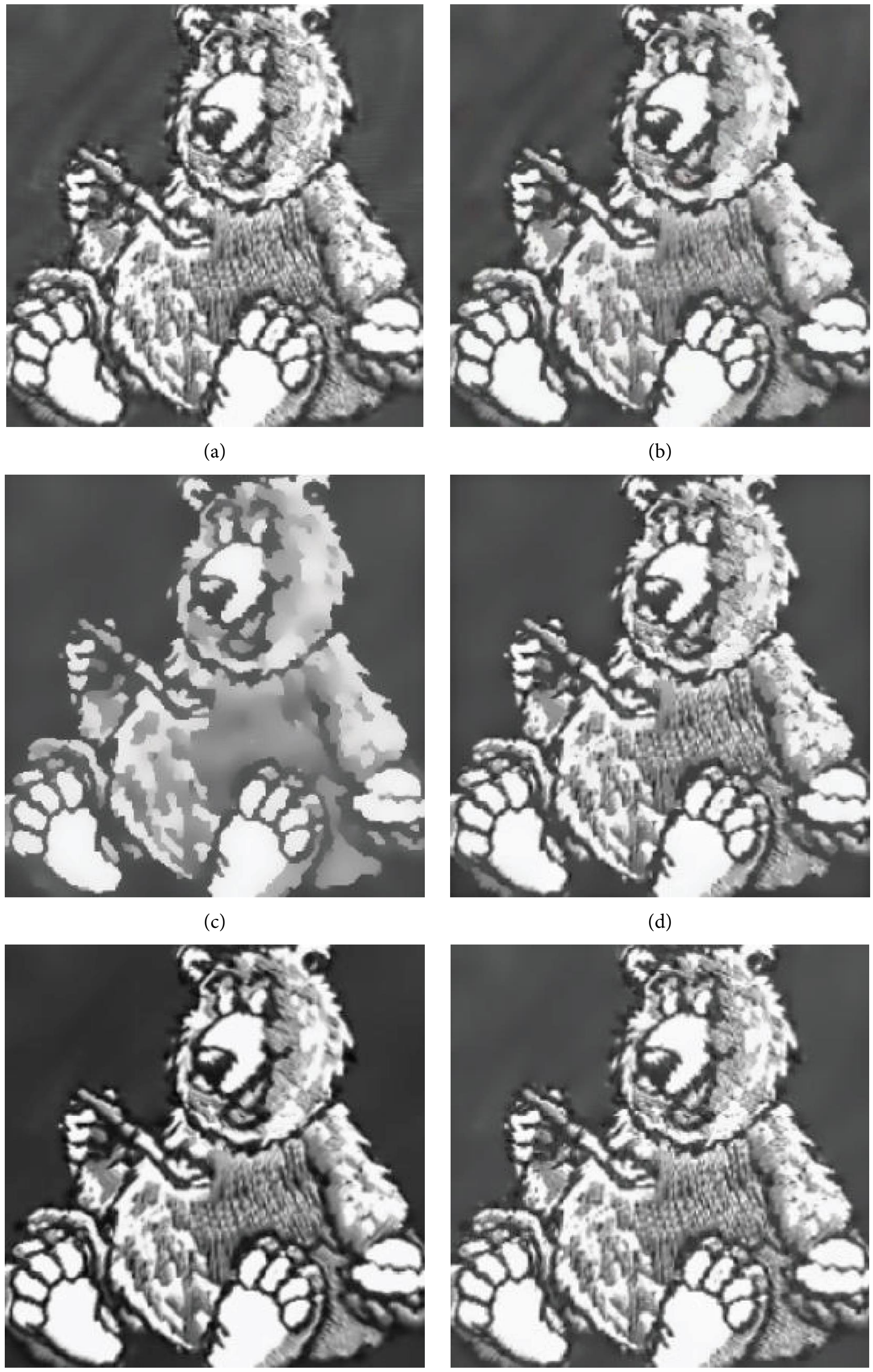
which has a priori advantage of having a nontrivial steady state. The strategy of introducing the fidelity source in the diffusion equation has been adopted in subsequent studies; see for example [5].

Following but different from [9], we present a new anisotropic diffusion for image smoothing as follows:

$$\partial_t u=\operatorname{div}\left(g\left(|\nabla v|,E\right)\nabla u\right)+\left(1-g\left(|\nabla v|,E\right)\right)\left(I-u\right), \quad (8)$$

with the initial condition $u(x,y,0)=I(x,y)$, where the function v is determined at each iteration by

$$\begin{cases}\partial_t v=g\left(|\nabla v|,E\right)\left(G_\sigma * u-v\right),\\ v|_{t=0}=u.\end{cases} \quad (9)$$



(a) (b)

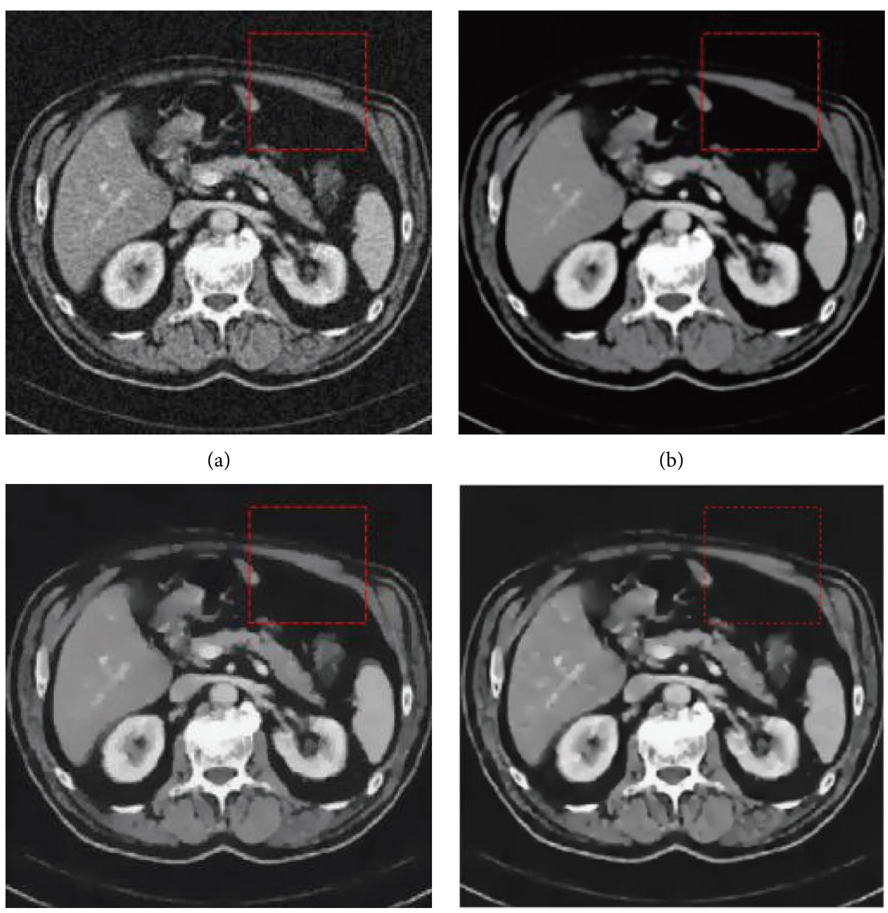
(c) (d)

(e) (f)

Figure 2: Continued.

(g)

Figure 2: Comparison with six filtering methods applied to the noisy bear image. (a) Original image. (b) P-M. (c) BF. (d) NL. (e) Chao et al.' method. (f) Wen et al.' method. (g) Ours.



(a) (b) (c) (d)

Figure 3: Continued.

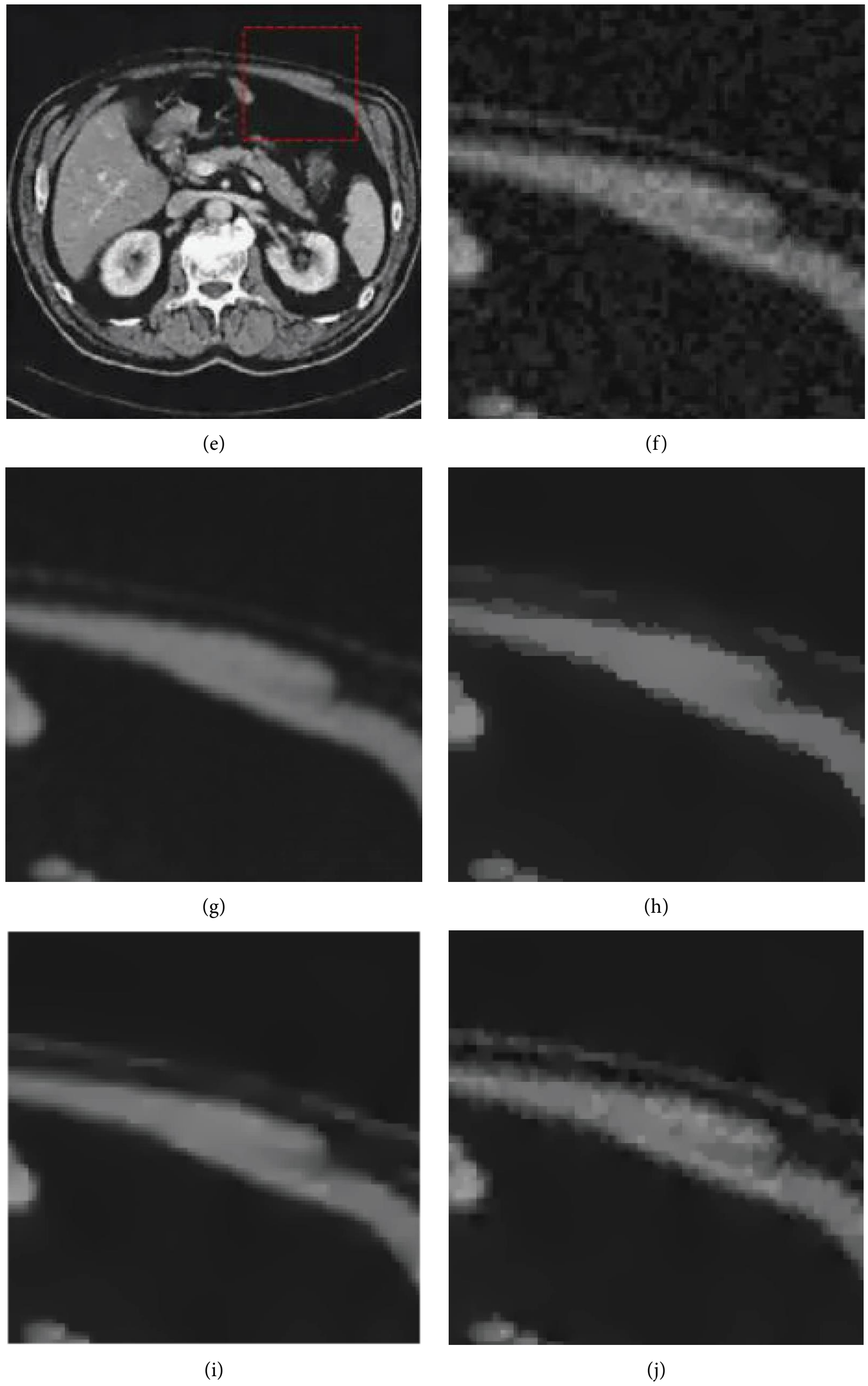


Figure 3: Details preserving comparison of the restoration results used four filtering methods. Top row: (a) original CT image with noise; (b) NL-means; (c) Chao et al.' method; (d) Wen et al.' method; (e) our method. Bottom row: (f)–(j) the zoomed portions corresponding to (a)–(e), respectively.

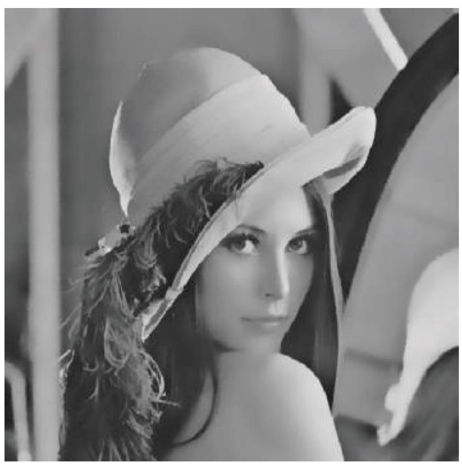
where G_σ is the Gaussian function with standard deviation σ. In (8) and (9), the function $g(\cdot)$ is given by (5).

It should be noted that the convolution $G_\sigma * u$ was first used in the diffusion coefficient of Perona and Malik equation in Catte et al.' work [10]. Theoretical analysis and experimental results in [10] have shown that the convolution $G_\sigma * u$ plays an essential role in the recovered quality of images contaminated by noise.

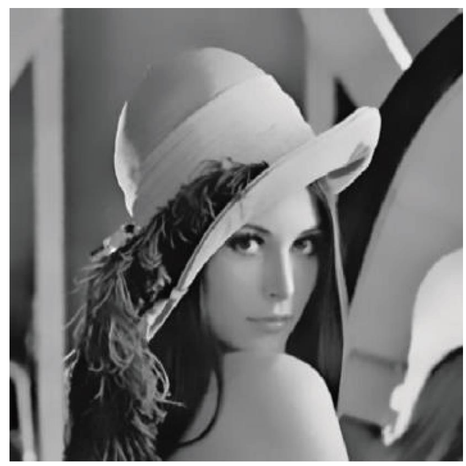
In (8), the balance between the fidelity source term (2nd term) and the diffusion term is made by $1 - g$, which works as a moderated selector of the diffusion process. Due to the fidelity source term, the smoothed image u remains close to the initial image u in the edge areas where $g \approx 0$. On the other hand, in homogeneous areas $g \approx 1$ and therefore, the source term will have an inexpressive effect, which allows for a better renovate of the image.

(a)

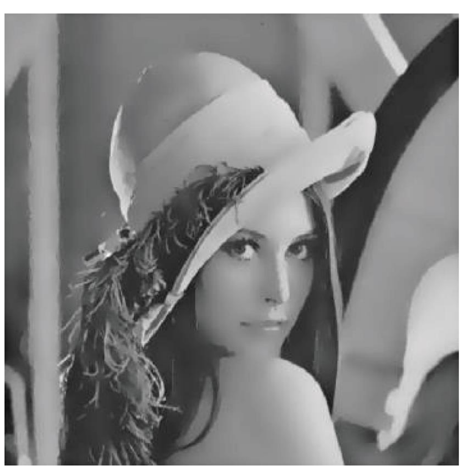
(b)



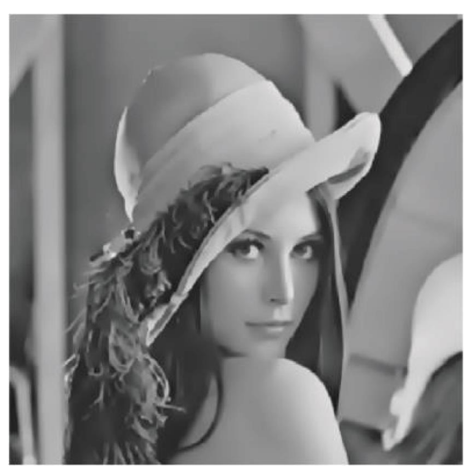
(c)



(d)



(e)



(f)

FIGURE 4: Continued.

(g)

Figure 4: Details preserving comparison of the restoration results used various filtering methods. (a) Original image. (b) P-M. (c) BF. (d) NL. (e) Chao et al.' method. (f) Wen et al.' method. (g) Our model.

Table 1: PSNR values with different filtering methods for the noisy Lena image.

Method	P-M	Bilateral	NL	Chao	Wen	Ours
PSNR (dB)	28.59	32.67	31.47	30.47	31.72	**35.39**

The larger the PSNR, the better the denoising effect. As shown in Table 1, the higher the PSNR of our method, the better the denoising effect of our method. Therefore, the PSNR of our algorithm is shown in bold.

3. Experiments

The proposed diffusion model is implemented using a simple finite differencing, in which the divergence term is discretized via an 8-nearest neighbors, inspiring from [3, 9], namely,

$$\mathrm{div}\left(g\left(|\nabla v|, E\right)\nabla u\right) \approx \sum_{i=1}^{8}\left|g\left(\left|\nabla^i v\right|, E\right)\cdot \nabla^i u\right|, \tag{10}$$

where

$$\begin{aligned}
\nabla^1 u &= u(x+1, y) - u(x, y),\\
\nabla^2 u &= u(x, y+1) - u(x, y),\\
\nabla^3 u &= u(x-1, y) - u(x, y),\\
\nabla^4 u &= u(x, y-1) - u(x, y),\\
\nabla^5 u &= \frac{(u(x+1, y+1) - u(x, y))}{\sqrt{2}},\\
\nabla^6 u &= \frac{(u(x+1, y+1) - u(x, y))}{\sqrt{2}},\\
\nabla^7 u &= \frac{(u(x+1, y+1) - u(x, y))}{\sqrt{2}},\\
\nabla^8 u &= \frac{(u(x+1, y+1) - u(x, y))}{\sqrt{2}},
\end{aligned} \tag{11}$$

where the symbol ∇^i (not to be confused with ∇, which we use for the gradient operator) indicates 8-nearest neighbors differences, similarly for $\nabla^i v$ $(1 \le i \le 8)$.

We present and discuss the experimental results to verify the proposed model. In all experiments, we set $K_0 = 250$ and the iteration number as 100.

Figure 2 shows the comparison among the P-M method [3], bilateral filter (BF) [4], NL-means [7], Chao et al.' method [6], Wen et al.' method [5], and the proposed method when applied to the noisy image with details. From Figure 3, we can see (1) the P-M model can effectively remove noise, but it also removes some important details; (2) the bilateral filtering cannot get rid of noise as much as we need (see background), whereas it can preserve details good; (3) NL-means, Chao et al.' method, Wen et al.' method, and our method have similar visual quality.

In order to further verify the detail-preserving performance of our method, some methods are used for comparison, as shown in Figure 3. Figure 3(g) presents the enlarged details portion by the NL-means method. It can be observed that most noise can be eliminated, but the weak line of the image is reduced. The enlarged details portion from Chao et al.' method is shown in Figure 3(h), in which the edges cannot be effectively preserved and some important edge is fused with the background. Figure 3(i) presents the enlarged details' portion by Wen et al.' method. It can be observed that the weak line of the image is reduced. Figure 3(j) presents the enlarged details portion by our method. It is clearly seen that our method effectively preserves the fine details, while smoothing out noise in the image.

Figure 4 shows another comparison of the restoration results by six filtering methods; all of the results are obtained from the 512×512 Lena image with random noise which ranges from 0 to 5. By visual comparison, it is clearly seen that our method effectively preserves the fine details, while it smooths out noise in the noisy Lena image. This fact can be further validated by quantitative comparison. The metric adopted in this study for comparison is the peak signal-to-noise ratio (PSNR). Table 1 gives the PSNR values for the restored Lena images using the six methods, and our method has the highest PSNR. In most cases, when the max of random noise is less than 10, the PSNR of our model is the highest than others which are compared in this paper.

4. Conclusions

In this paper, we propose a coupled anisotropic diffusion scheme which combines with the local entropy for detail-and edge-preserving smoothing. It can effectively improve the quality of a noisy image and also well preserve the sharp edges and fine details in an image. The main contribution of this study is to exploit entropy of the local region histogram to improve the P-M anisotropic diffusion in terms of detail-and edge-preserving smoothing.

It should be pointed out that because the computing cost of local entropy is very high, our method is time-consuming, especially for large-size images. Besides, the strong noise will decline the filtering performance of the proposed method. But considering its good performance in preserving the sharp edges and fine details of the image, the proposed algorithm is still one of competitive denoising methods for detail-and edge-preserving smoothing.

Data Availability

The data can be obtained from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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