Hindawi
Discrete Dynamics in Nature and Society
Volume 2023, Article ID 9570805, 16 pages
https://doi.org/10.1155/2023/9570805

Research Article

Robustness Analysis of BAM Cellular Neural Network with Deviating Arguments of Generalized Type

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Received 6 September 2022; Revised 3 February 2023; Accepted 10 March 2023; Published 22 April 2023

Academic Editor: Junwei Wang

By generating equivalent integral equations, we analyze the existence and uniqueness of solutions of bidirectional associative memory cellular neural network (BAMCNN) with deviating arguments firstly. Secondly, the question of robustness of stability (RoS) of BAMCNN with deviating argument is studied. Using the Gronwall inequality, we calculate the upper bounds of the interference intensities that can maintain the initial stability of system. The perturbed BAMCNN will maintain its original stability if the strength of one or more perturbations is less than the upper bounds that we calculated in this study. To demonstrate the validity of the conjectural values, a variety of numerical illustrations are provided.

1. Introduction

As a kind of nonlinear dynamic system with structure law and infinitely expandable dimension, cellular neural network (CNN) is first proposed by Chua and Yang in 1988 [1]. Cells are the basic architecture of CNN which are essentially similar analog circuits, grouped in any arrays which dimensions larger than 2. CNNs have adjacency, it is one of the most significant properties of those systems, that is, every unit connects with its neighbors in the same way. Its dynamics primarily exhibit chaotic, periodic, almost periodic, and stable properties. We can refer to [2, 3] and any related references.

Bidirectional associative memory cellular neural network (BAMCNN) as a model of supervised learning in artificial neural network, were first proposed by Kosko in [4] in 1988. In application, it needs recurrent neural networks to receive a set of neurons as an input and create a set of outputs that related but different with inputs. Besides, the X-layer and the Y-layer are the two layers of BAMCNNs. Neurons in the same layer are completely linked to those in the other layer, and there are no connections between neurons in one layer. And its dynamical behaviors gain more and more interests in recent decades because of its special properties and usefulness in several domains, such as image identification, the problem of optimization, and other areas [5, 6].

Stability as one of the most important dynamical behaviors has received more extensive attention [7–16]. For example, the global exponential stability (ES) of periodic solutions of delayed BAMNN (DBAMNN) of cohen-crossberg type in [7]. In [8], the ES of delayed impulsive discrete-time stochastic BAMNN (SBAM) is investigated by using Lyapunov theory and contraction mapping principle. Several criteria of delay-dependent asymptotical stability (AS) of uncertain BAMNN are derived in [9]. Liu et al. explore the stabilization of uncertain BAMNN in finite time in [10]. In [11], Chen and Cao et al. discarded the common assumptions and some analytic techniques are used to estimate the exponential convergence rate of DBAMNN. By constructing Lyapunov functions, the ES of stochastic DBAMNN is studied in [12]. AS of a class of SBAMs is investigated in [13]. In [15, 16], Zhu et al.'s further analysis of the stability of delayed SBAM with Markovian jumping is conducted.

It is worth mentioning that all the above researches are all devoted to the stability of BAMNN without deviating argument. However, for some complex physical problems, such as the problem of population dynamics [17], it is affected not only by the delayed states but also by the advanced states. The potential of anticipation in systems may be viewed as a result of the analytical model's complexity. This additional complexity is discovered by treating the time dimension as a degree of freedom accessible to the system rather than as a given. Thus, neural networks with time delays or stochastic disturbances cannot simulate this desired outcome well, so the deviating argument theory is introduced. Differential equation with deviating argument is a hybrid of continuous and discrete equation that combines the traits of differential and difference equations. Meanwhile, noise disturbances are inevitable in actual nervous systems. All of these disturbances have a significant impact on the stability of dynamical systems. Consequently, it is essential to take into account stochastic neural networks with deviating arguments. And many results of neural networks or dynamical systems with deviating arguments are obtained. For instances, the differential dynamical system with deviating argument is discussed in detail in [18]. Wu and Zeng investigate the existence and uniqueness of fuzzy neurodynamic system with deviating argument in [19]. Integral manifold and almost periodic solution of a class of differential equation with deviating argument are discussed in [20, 21], respectively. Akhmet et al. explored the stability of several type neural networks with deviating arguments in [22, 23]. In [24], Li investigates the ES of stochastic CNN with deviating argument by using Lyapunov functional. In [25], the stability of semi-linear stochastic differential equation with deviating argument is discussed. We can find that the majority of the researches included in the aforementioned studies, however, focus on the stability of stochastic dynamical systems or neural networks with deviating arguments and no bidirectional associative memory. Few studies have looked at the robustness of stability.

Robustness is the ability of a control system to maintain a specific level of performance when certain parameters are perturbed, and it is of great significance for the design and application of system. As for the analysis of RoS, there are lots of interesting outcomes, such as, Si et al. further studied RoS of dynamical systems with deviating arguments in [26–28]; Fang et al. analyzed the RoS of fuzzy CNN with deviating argument in [29]. In [30], RoS of recurrent neural network is investigated. But, few publications have explored the robustness of stability for BAMCNN with deviating argument (BAMDA). This inspires us to finish this paper. As far as we know, the literature has never addressed the issue of RoS of BAMDA. The following are the works and contributions in this article:

(i) This research investigates the necessary circumstances for the existence and uniqueness of BAMDA solutions by constructing equivalent integral equations.

(ii) The RoS of BAMDA and stochastic BAMDA (SBAMDA) are investigated by using Gronwall inequality and other inequality techniques, furthermore, the upper bounds of the interval of deviating argument and the intensity of noise before losing its stability are calculated. Furthermore, the mutual constraint between the two disturbances is highlighted.

(iii) With the use of the Gronwall–Bellman lemma and inequality techniques, the academic framework for developing BAMDA that fulfills performance standards is provided.

Finally, we list the organizations of this paper. Section 2 introduces the model as well as the lemmas we utilized to get the key outcomes. In Section 3, we will investigate the robustness of BAMDA we proposed. And not only do we make a research on the robustness of SBAMDA but also we derive the maximum of noise intensity and the max extent of the deviating intervals in Section 4. Besides, several numerical instances are given in Section 5 to demonstrate the theoretical values.

Notations: Denote $\mathbb{N} = \{1, 2, \cdots\}$. $\mathbb{R}$, $\mathbb{R}^+$, and $\mathbb{R}^n$ represent the space of real number, positive real number, and n-dimensional vectors, respectively. $|\cdot|$ denotes the Euclidean norm of a real vector. Complete filtered probability space $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t\geq 0}, P)$ embraces all P-null sets, where filtration $\{\mathscr{F}_t\}_{t\geq 0}$ is right continuous and satisfies the usual conditions. Scalar Brownian movement $\mho(t)$ is defined at $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t\geq 0}, P)$. E represents the mathematical expectation operator.

2. Introduction

The following system is under consideration in this part, it is formed by BAMCNN with deviating arguments:

$$\begin{cases} \dot{\Phi}_\nu(t) = -a_\nu \Phi_\nu(t) + \sum_{\mu=1}^{\varsigma} b_{\nu\mu}\Omega_\mu\left(\Psi_\mu(t)\right) + \sum_{\mu=1}^{\varsigma} c_{\nu\mu}\Omega_\mu\left(\Psi_\mu(\Theta(t))\right), \\ \dot{\Psi}_\mu(t) = -d_\mu \Psi_\mu(t) + \sum_{\nu=1}^{\omega} e_{\mu\nu}\Upsilon_\nu\left(\Phi_\nu(t)\right) + \sum_{\nu=1}^{\omega} h_{\mu\nu}\Upsilon_\nu\left(\Phi_\nu(\Theta(t))\right), \end{cases} \tag{1}$$

where $\Phi_\nu(t) \in \mathbb{R}$, $\nu \in \{1, 2, \cdots, \omega\}$, and $\Psi_\mu(t) \in \mathbb{R}$, $\mu \in \{1, 2, \cdots, \varsigma\}$ represents the ν th cell and μ th cell of time t, respectively; positive numbers a_ν and d_μ represent the rates of the ν th and μ th cell adjust to their electricpotential to the static state when loss connections with other cells and inputs from the exterior at time t. $b_{\nu\mu}$ and $c_{\nu\mu}$ are arbitrary numbers, and they represent the connection strength between the ν th and μ th cell at time t, respectively; $e_{\mu\nu}$ and $h_{\mu\nu}$ are contrary to the definition of $b_{\nu\mu}$ and $c_{\nu\mu}$, which are the connection strengths between the μ th and ν th cell at time t, respectively. Besides, $\Omega_\mu(\cdot)$ and $\Upsilon_\nu(\cdot)$ are nonlinear activation functions of μ th and ν th neuron at time t, respectively, and they satisfy $\Omega_\mu(0)=0$ and $\Upsilon_\nu(0)=0$. $\Theta(t)$ is a deviating function with $\Theta(t)=\Theta_\xi^*$, when $t\in[\mathfrak{Y}_\xi,\mathfrak{Y}_{\xi+1})$, where $\xi\in\mathbb{N}$ and sequences $\{\Theta_\xi^*\}$ and $\{\mathfrak{Y}_\xi\}$ satisfy that $\mathfrak{Y}_\xi<\Theta_\xi^*<\mathfrak{Y}_{\xi+1}$, $\mathfrak{Y}_\xi\longrightarrow\infty$, and $\Theta_\xi^*\longrightarrow\infty$, if $\xi\longrightarrow\infty$.

Furthermore, from (1), we can get the undisturbed system as follows:

$$\begin{cases}\dot{\tilde{\Phi}}_\nu(t)=-a_\nu\tilde{\Phi}_\nu(t)+\sum_{\mu=1}^{\varsigma}b_{\nu\mu}\Omega_\mu\left(\tilde{\Psi}_\mu(t)\right)+\sum_{\mu=1}^{\varsigma}c_{\nu\mu}\Omega_\mu\left(\tilde{\Psi}_\mu(t)\right),\\ \dot{\tilde{\Psi}}_\mu(t)=-d_\mu\tilde{\Psi}_\mu(t)+\sum_{\nu=1}^{\omega}e_{\mu\nu}\Upsilon_\nu\left(\tilde{\Phi}_\nu(t)\right)+\sum_{\nu=1}^{\omega}h_{\mu\nu}\Upsilon_\nu\left(\tilde{\Phi}_\nu(t)\right).\end{cases}\tag{2}$$

Obviously, system (2) has equilibrium point (0, 0). Because of the arguments may change their deviation characteristic during its movement, thus, the system is delayed or advanced. Consider the interval $[\mathfrak{Y}_\xi,\mathfrak{Y}_{\xi+1})$, $\xi\in\mathbb{N}$, then, $\Theta(t)=\Theta_\xi^*$, thus, when t satisfies $\mathfrak{Y}_\xi\le t<\Theta_\xi^*$, that is, $\Theta(t)>t$, then $\Theta(t)$ is an advanced argument. As the same time, if $\Theta_\xi^*<t<\mathfrak{Y}_{\xi+1}$, that is, $\Theta(t)<t$, then $\Theta(t)$ is an delayed argument. Therefore, we call it mixed type.

We use the following notations for the purpose of simplicity:

$$\begin{aligned}m_1&=\max_{1\le\nu\le\omega}\left|a_\nu\right|, m_2=\max_{1\le\mu\le\varsigma}\Lambda_\mu\sum_{\nu=1}^{\omega}\left|b_{\nu\mu}\right|,\\ m_3&=\max_{1\le\mu\le\varsigma}\Lambda_\mu\sum_{\nu=1}^{\omega}\left|c_{\nu\mu}\right|, m_4=\max_{1\le\mu\le\varsigma}\left|d_\mu\right|,\\ m_5&=\max_{1\le\nu\le\omega}\Xi_\nu\sum_{\mu=1}^{\varsigma}\left|e_{\mu\nu}\right|, m_6=\max_{1\le\nu\le\omega}\Xi_\nu\sum_{\mu=1}^{\varsigma}\left|h_{\mu\nu}\right|.\end{aligned}\tag{3}$$

The following are the presumptions we will require for this paper:

(i) A1: There are two positive constants Λ_μ and Ξ_ν such that

$$\begin{aligned}\left|\Omega_\mu(\imath)-\Omega_\mu(\mathcal{J})\right|&\le\Lambda_\mu|\imath-\mathcal{J}|,\\ \left|\Upsilon_\nu(\imath)-\Upsilon_\nu(\mathcal{J})\right|&\le\Xi_\nu|\imath-\mathcal{J}|.\end{aligned}\tag{4}$$

and this assumption implies that $\Omega_\mu(0)=\Upsilon_\nu(0)=0$.

(ii) A2: There is a $\mathfrak{Y}>0$ satisfies $\mathfrak{Y}_\xi-\mathfrak{Y}_{\xi-1}<\mathfrak{Y}$;

(iii) A3: $M<1$;

(iv) A4: $\mathfrak{Y}(2M_1+M_2)\exp(M_2\mathfrak{Y})<1$;

where

$$\begin{aligned}M&=\max\{(m_1+m_5+m_6)\mathfrak{Y},(m_2+m_3+m_4)\mathfrak{Y}\},\\ M_1&=\max\{m_3,m_6\},\\ M_2&=\max\{m_1+m_5,m_2+m_4\}.\end{aligned}\tag{5}$$

Remark 1. If assumptions A3 and A4 hold, then $M_1<1$ and $M_2<1$, and then we can derive that $M_1\mathfrak{Y}\exp(M_2\mathfrak{Y})<1$ and $\mathfrak{Y}[M_1+M_2(1+M_1\mathfrak{Y})\exp(M_2\mathfrak{Y})]<1$.

In this paper, we presume that the solutions of the model (1) we proposed above are continuous. Generically, it is discontinuous for the right-side of (1) at the moment $t=\mathfrak{Y}_{\xi+1}$, $\xi\in\mathbb{N}$, because of the discontinuity of deviating function at the moment. Thus, based on our assumptions above, the solutions of the equations are viewed as continuously differentiable at the interval $[\mathfrak{Y}_\xi,\mathfrak{Y}_{\xi+1})$, $\xi\in\mathbb{N}$. That means (1) is satisfied by $\Phi(t),\Psi(t)$ on $(\mathfrak{Y}_\xi,\mathfrak{Y}_{\xi+1})$, $\xi\in\mathbb{N}$ where $(\Phi(t),\Psi(t))^T=(\Phi_1(t),\cdots,\Phi_\nu(t),\Psi_1(t),\cdots,\Psi_\mu(t))^T$ denotes the solution of equation (1). Besides, $\Phi(t)$ and $\Psi(t)$ exists one side derivative at $\mathfrak{Y}_\xi$ and $\xi\in\mathbb{N}$ which denotes that $\|\Phi\|=\sum_{\nu=1}^{\varsigma}|\Phi_\nu|$, $\mathfrak{Y}_\xi<\mathfrak{Y}_{\xi+1}$, and $\mathfrak{Y}_\xi<\Theta_\xi^*<\mathfrak{Y}_{\xi+1}$.

Lemma 1. *Let assumptions A1 to A4 hold, then, for any initial values* $(\Phi_0,\Psi_0)^T$, *there exists a unique solution* $(\Phi(t),\Psi(t))^T$ *such that* $\Phi(t_0)=\Phi_0$, $\Psi(t_0)=\Psi_0$.

Proof. Existence: For all $\xi\in\mathbb{N}$, we assume that $\mathfrak{Y}_\xi\le\Theta_\xi^*<t_0<\mathfrak{Y}_{\xi+1}$, and denote $\Phi_\nu(t_0)=\Phi_\nu^0$ and $\Psi_\mu(t_0)=\Psi_\mu^0$ for simplicity.

Firstly, for $t\in[\mathfrak{Y}_\xi,\mathfrak{Y}_{\xi+1})$, $t\in\mathbb{R}^+$, then

$$\begin{cases}\Phi_\nu(t)=\Phi_\nu^0+\int_{t_0}^{t}\left[-a_\nu\Phi_\nu(u)+\sum_{\mu=1}^{\varsigma}b_{\nu\mu}\Omega_\mu\left(\Psi_\mu(u)\right)+\sum_{\mu=1}^{\varsigma}c_{\nu\mu}\Omega_\mu\left(\Psi_\mu\left(\Theta_\xi^*\right)\right)\right]\mathrm{d}u,\\ \Psi_\mu(t)=\Psi_\mu^0+\int_{t_0}^{t}\left[-d_\mu\Psi_\mu(u)+\sum_{\nu=1}^{\omega}e_{\mu\nu}\Upsilon_\nu\left(\Phi_\nu(u)\right)+\sum_{\nu=1}^{\omega}h_{\mu\nu}\Upsilon_\nu\left(\Phi_\nu\left(\Theta_\xi^*\right)\right)\right]\mathrm{d}u.\end{cases}\tag{6}$$

We denote 0-norm as $\|\Phi(t)\|_0 = \max_{[\Theta_\xi^*, t_0]}\|\Phi(t)\|$, $\|\Psi(t)\|_0 = \max_{[\Theta_\xi^*, t_0]}\|\Psi(t)\|$, and we let $z(t) = (z_1(t), \cdots, z_\omega(t))$, $v(t) = (v_1(t), \cdots, v_\varsigma(t))$, simplicity. And we transform (1) to the following equation:

$$\begin{cases} z_\nu(t) = z_\nu^0 + \displaystyle\int_{t_0}^t \left[-a_\nu z_\nu(u) + \sum_{\mu=1}^{\varsigma} b_{\nu\mu}\Omega_\mu\left(v_\mu(u)\right) + \sum_{\mu=1}^{\varsigma} c_{\nu\mu}\Omega_\mu\left(v_\mu\left(\Theta_\xi^*\right)\right)\right]\mathrm{d}u, \\ v_\mu(t) = v_\mu^0 + \displaystyle\int_{t_0}^t \left[-d_\mu v_\mu(u) + \sum_{\nu=1}^{\omega} e_{\mu\nu}\Upsilon_\nu(z_\nu(u)) + \sum_{\nu=1}^{\omega} h_{\mu\nu}\Upsilon_\nu\left(z_\nu\left(\Theta_\xi^*\right)\right)\right]\mathrm{d}u. \end{cases} \tag{7}$$

Then, we construct the following sequences $z_\nu^r(t)$ and $v_\mu^r(t)$, where $z_\nu^0(t) \equiv \Phi_\nu^0$, $v_\mu^0(t) \equiv \Psi_\mu^0$ such that

$$\begin{cases} z_\nu^{r+1}(t) - z_\nu^r(t) = \displaystyle\int_{t_0}^t \Big\{-a_\nu\left(z_\nu^r(u) - z_\nu^{r-1}(u)\right) \\ + \displaystyle\sum_{\mu=1}^{\varsigma} b_{\nu\mu}\left[\Omega_\mu\left(v_\mu^r(u)\right) - \Omega_\mu\left(v_\mu^{r-1}(u)\right)\right] \\ + \displaystyle\sum_{\mu=1}^{\varsigma} c_{\nu\mu}\left[\Omega_\mu\left(v_\mu^r\left(\Theta_\xi^*\right)\right) - \Omega_\mu\left(v_\mu^{r-1}\left(\Theta_\xi^*\right)\right)\right]\Big\}\mathrm{d}u, \\ v_\mu^{r+1}(t) - v_\mu^r(t) = \displaystyle\int_{t_0}^t \Big\{-d_\mu\left(v_\mu^r(u) - v_\mu^{r-1}(u)\right) \\ + \displaystyle\sum_{\nu=1}^{\omega} e_{\mu\nu}\left[\Upsilon_\nu(z_\nu^r(u)) - \Upsilon_\nu\left(z_\nu^{r-1}(u)\right)\right] \\ + \displaystyle\sum_{\nu=1}^{\omega} h_{\mu\nu}\left[\Upsilon_\nu\left(z_\nu^r\left(\Theta_\xi^*\right)\right) - \Upsilon_\nu\left(z_\nu^{r-1}\left(\Theta_\xi^*\right)\right)\right]\Big\}\mathrm{d}u. \end{cases} \tag{8}$$

Furthermore,

$$\begin{aligned} \left\|z_\nu^{r+1}(t) - z_\nu^r(t)\right\| \le & \sum_{\nu=1}^{\omega}\left|a_\nu\right|\mathfrak{Y}\left|z_\nu^r(u) - z_\nu^{r-1}(u)\right| \\ & + \sum_{\nu=1}^{\omega}\sum_{\mu=1}^{\varsigma}\mathfrak{Y}\left|b_{\nu\mu}\right|\Lambda_\mu\left|v_\mu^r(u) - v_\mu^{r-1}(u)\right| \\ & + \sum_{\nu=1}^{\omega}\sum_{\mu=1}^{\varsigma}\mathfrak{Y}\left|c_{\nu\mu}\right|\Lambda_\mu\left|v_\mu^r\left(\Theta_\xi^*\right) - v_\mu^{r-1}\left(\Theta_\xi^*\right)\right| \\ \le & \sum_{\nu=1}^{\omega}\left|a_\nu\right|\mathfrak{Y}\left|z_\nu^r(u) - z_\nu^{r-1}(u)\right| \\ & + \sum_{\mu=1}^{\varsigma}\sum_{\nu=1}^{\omega}\mathfrak{Y}\left|b_{\nu\mu}\right|\Lambda_\mu\left|v_\mu^r(u) - v_\mu^{r-1}(u)\right| \\ & + \sum_{\mu=1}^{\varsigma}\sum_{\nu=1}^{\omega}\mathfrak{Y}\left|c_{\nu\mu}\right|\Lambda_\mu\left|v_\mu^r\left(\Theta_\xi^*\right) - v_\mu^{r-1}\left(\Theta_\xi^*\right)\right|. \end{aligned} \tag{9}$$

Similarly, we also have

$$\begin{aligned} & \left\|v_\mu^{r+1}(t) - v_\mu^r(t)\right\| \\ & \le \sum_{\mu=1}^{\varsigma}\mathfrak{Y}\left|d_\mu\right\| v_\mu^r(u) - v_\mu^{r-1}(u)\Big| \\ & \quad + \sum_{\nu=1}^{\omega}\sum_{\mu=1}^{\varsigma}\mathfrak{Y}\left|e_{\mu\nu}\right|\Xi_\nu\left|z_\nu^r(u) - z_\nu^{r-1}(u)\right| \\ & \quad + \sum_{\nu=1}^{\omega}\sum_{\mu=1}^{\varsigma}\mathfrak{Y}\left|h_{\mu\nu}\right|\Xi_\nu\left|z_\nu^r\left(\Theta_\xi^*\right) - z_\nu^{r-1}\left(\Theta_\xi^*\right)\right|. \end{aligned} \tag{10}$$

Then,

$$\begin{cases} \left\|z_\nu^{r+1}(t) - z_\nu^r(t)\right\|_0 \\ \le m_1\mathfrak{Y}\left\|z_\nu^r(u) - z_\nu^{r-1}(u)\right\|_0 \\ +(m_2 + m_3)\mathfrak{Y}\left\|v_\mu^r(u) - v_\mu^{r-1}(u)\right\|_0, \\ \left\|v_\mu^{r+1}(t) - v_\mu^r(t)\right\|_0 \\ \le m_4\mathfrak{Y}\left\|v_\mu^r(u) - v_\mu^{r-1}(u)\right\|_0 \\ +(m_5 + m_6)\mathfrak{Y}\left\|z_\nu^r(u) - z_\nu^{r-1}(u)\right\|_0. \end{cases} \tag{11}$$

Let $M = \max\{(m_1 + m_5 + m_6)\mathfrak{Y}, (m_2 + m_3 + m_4)\mathfrak{Y}\}$, then we can get

$$\begin{aligned} & \left\|z_\nu^{r+1}(t) - z_\nu^r(t)\right\|_0 + \left\|v_\mu^{r+1}(t) - v_\mu^r(t)\right\|_0 \\ & \le M\left[\left\|z_\nu^r(u) - z_\nu^{r-1}(u)\right\|_0 + \left\|v_\mu^r(u) - v_\mu^{r-1}(u)\right\|_0\right]. \end{aligned} \tag{12}$$

Moreover,

$$\begin{aligned} & \left\|z_\nu^{r+1}(t) - z_\nu^r(t)\right\|_0 + \left\|v_\mu^{r+1}(t) - v_\mu^r(t)\right\|_0 \\ & \le M^r\left[\left\|z_\nu^1(u) - z_\nu^0(u)\right\|_0 + \left\|v_\mu^1(u) - v_\mu^0(u)\right\|_0\right] \\ & \le M^r\left[(m_1 + m_5 + m_6)\mathfrak{Y}\left\|\Phi_\nu^0\right\| + (m_2 + m_3 + m_4)\mathfrak{Y}\left\|\Psi_\mu^0\right\|\right] \\ & \le M^{r+1}\left(\left\|\Phi_\nu^0\right\| + \left\|\Psi_\mu^0\right\|\right). \end{aligned} \tag{13}$$

Thus, $(z(t), v(t))^T$ is the solution of (7) on $[\Theta_\xi, t_0]$. Then condition A1 implies that $\Phi(t)$ and $\Psi(t)$ can be continuous on $\mathfrak{Y}_{\xi+1}$.

Using the same method, we can continue $\Phi(t)$ and $\Psi(t)$ from $t = \mathfrak{Y}_{\xi+1}$ to $t = \Theta_{\xi+1}$, and then to $\mathfrak{Y}_{\xi+2}$. Therefore, we complete this proof of existences by mathematical induction.

Uniqueness: For simplicity, we denote $\Phi^1(t) = \Phi(t; t_0, \Phi^1)$, $\Psi^1(t) = \Psi(t; t_0, \Psi^1)$, $\Phi^2(t) = \Phi(t; t_0, \Phi^2)$, $\Psi^2(t) = \Psi(t; t_0, \Psi^2)$, and we assume $(\Phi^1(t), \Psi^1(t))^T$ and $(\Phi^2(t), \Psi^2(t))^T$ are solutions of (1), where Φ^1, Ψ^1, Φ^2, and Ψ^2 are initial values and $t_0 \in \mathbb{R}^+$.

From (1), then

$$\begin{cases} \left\|\Phi^1(t) - \Phi^2(t)\right\| \le \left\|\Phi^1 - \Phi^2\right\| \\ + \int_{t_0}^{t} \Big[m_1\left\|\Phi^1(u) - \Phi^2(u)\right\| + m_2\left\|\Psi^1(u) - \Psi^2(u)\right\| \\ + m_3\left\|\Psi^1\left(\Theta_\xi^*\right) - \Psi^2\left(\Theta_\xi^*\right)\right\| \mathrm{d}s\Big], \\ \left\|\Psi^1(t) - \Psi^2(t)\right\| \le \left\|\Psi^1 - \Psi^2\right\| \\ + \int_{t_0}^{t} \Big[m_4\left\|\Psi^1(u) - \Psi^2(u)\right\| + m_5\left\|\Phi^1(u) - \Phi^2(u)\right\| \\ + m_6\left\|\Phi^1\left(\Theta_\xi^*\right) - \Phi^2\left(\Theta_\xi^*\right)\right\|\Big] \mathrm{d}u. \end{cases} \tag{14}$$

From (14), then

$$\begin{aligned} &\left\|\Phi^1(t) - \Phi^2(t)\right\| + \left\|\Psi^1(t) - \Psi^2(t)\right\| \\ &\le \Big\{\left(\left\|\Phi^1 - \Phi^2\right\| + \left\|\Psi^1 + \Psi^2\right\|\right) \\ &\quad + \Big[m_3\mathfrak{Y}\left\|\Phi^1\left(\Theta_\xi^*\right) - \Phi^2\left(\Theta_\xi^*\right)\right\| \\ &\quad + m_6\mathfrak{Y}\left\|\Psi^1\left(\Theta_\xi^*\right) - \Psi^2\left(\Theta_\xi^*\right)\right\|\Big]\Big\} \\ &\quad + \int_{t_0}^{t} \big[(m_1 + m_5)\left\|\Phi^1(u) - \Phi^2(u)\right\| \\ &\quad + (m_2 + m_4)\left\|\Psi^1(u) - \Psi^2(u)\right\|\big] \mathrm{d}u. \end{aligned} \tag{15}$$

Let $\hbar_1(t) = \|\Phi^1(t) - \Phi^2(t)\| + \|\Psi^1(t) - \Psi^2(t)\|$, $\hbar_2 = \|\Phi^1 - \Phi^2\| + \|\Psi^1 + \Psi^2\|$, and $\hbar_3 = \|\Phi^1(\Theta_\xi^*) - \Phi^2(\Theta_\xi^*)\| + \|\Psi^1(\Theta_\xi^*) - \Psi^2(\Theta_\xi^*)\|$, then (15) can be recorded as

$$\hbar_1(t) \le \{\hbar_2 + M_1\mathfrak{Y}\hbar_3\} + \int_{t_0}^{t} M_2\hbar_1(u)\mathrm{d}u. \tag{16}$$

Applied Gronwall's inequality, we can get

$$\hbar_1(t) \le \{\hbar_2 + M_1\mathfrak{Y}\hbar_3\}\exp\left(M_2\mathfrak{Y}\right), \tag{17}$$

and when $t = \Theta_\xi^*$, we have

$$\hbar_3 \le \{\hbar_2 + M_1\mathfrak{Y}\hbar_3\}\exp\left(M_2\mathfrak{Y}\right), \tag{18}$$

then

$$\hbar_1(t) \le \frac{\exp\left(M_2\mathfrak{Y}\right)}{1 - M_1\mathfrak{Y}\exp\left(M_2\mathfrak{Y}\right)}\hbar_2. \tag{19}$$

On the hand, if $(\Phi^1(t), \Psi^1(t))^T = (\Phi^2(t), \Psi^2(t))^T$ and $t \in [\mathfrak{Y}_\xi, \mathfrak{Y}_{\xi+1}]$, similarly we can get

$$\begin{cases} \left\|\Phi^1 - \Phi^2\right\| \le \int_{t_0}^{t} \Big[m_1\left\|\Phi^1(u) - \Phi^2(u)\right\| \\ + m_2\left\|\Psi^1(u) - \Psi^2(u)\right\| + m_3\left\|\Psi^1\left(\Theta_\xi^*\right) - \Psi^2\left(\Theta_\xi^*\right)\right\|\Big]\mathrm{d}u, \\ \left\|\Psi^1 - \Psi^2\right\| \le \int_{t_0}^{t} \Big[m_4\left\|\Psi^1(u) - \Psi^2(u)\right\| \\ + m_5\left\|\Phi^1(u) - \Phi^2(u)\right\| + m_6\left\|\Phi^1\left(\Theta_\xi^*\right) - \Phi^2\left(\Theta_\xi^*\right)\right\|\Big]\mathrm{d}u. \end{cases} \tag{20}$$

We can obtain (21) as follows

$$\hbar_2 \le M_1\mathfrak{Y}\hbar_3 + \int_{t_0}^{t} M_2\hbar_1(u)\mathrm{d}s. \tag{21}$$

Combine (19)–(21), we can get

$$\hbar_2 \le \mathfrak{Y}\left(2M_1 + M_2\right)\exp\left(M_2\mathfrak{Y}\right)\hbar_2. \tag{22}$$

Obviously, we can see that A5 contradicts (22), and then the uniqueness of solutions of system we proposed is proved for $t \in [\mathfrak{Y}_\xi, \mathfrak{Y}_{\xi+1}]$. And it is evident that the extension of the uniqueness of solutions on $\mathbb{R}^+$ is available. □

3. Deviating Argument Impacts on Stability

Let us introduce the definition of globally exponentially stable (GES) first.

Definition 1. The states of (2) are GES, if for all $t_0 \ge 0$ and $\psi(0) \in \mathbb{R}^{\nu+\mu}$, there are $\mathfrak{A} > 0$, and $\mathfrak{B} > 0$ such that

$$\left\|\widetilde{\Phi}_\nu(t)\right\| + \left\|\widetilde{\Psi}_\mu(t)\right\| \le \mathfrak{A}\|\psi(0)\|\exp\left(-\mathfrak{B}\left(t - t_0\right)\right), t \ge t_0, \tag{23}$$

where $\psi(0) = (\widetilde{\Phi}(t_0), \widetilde{\Psi}(t_0))^T = (\Phi(t_0), \Psi(t_0))^T$ is the initial value of system (2).

Another assumption must be met before we can proceed to our major results.

(i) A5 $2\mathfrak{A}/\mathfrak{B}M_1\exp\left[2\hbar\left(M_2 + M_1\right)\right] + \mathfrak{A}\exp\left(-\mathfrak{B}\hbar\right) < 1$.

To reach our primary results, we first present the following lemma.

Lemma 2. *Let A1 to A4 hold, then there exists a $\mathfrak{Q} > 0$ such that*

$$\|\Phi(\Theta(u))\| + \|\Psi(\Theta(u))\| \le \mathfrak{Q}\left(\|\Phi(t)\| + \|\Psi(t)\|\right), \tag{24}$$

where $\Phi(t) = (\Phi_1(t), \cdots, \Phi_\mu(t))$ and $\Psi(t) = (\Psi_1(t), \cdots, \Psi_\nu(t))$ are the solutions of equation (1), and $\mathfrak{Q} = \{1 - \mathfrak{Y}[M_1 + M_2(1 + M_1\mathfrak{Y})\exp(M_2\mathfrak{Y})]\}^{-1}$.

Proof. Without losing generality, suppose that $\mathfrak{Y}_\xi < \Theta_\xi^* < t < \mathfrak{Y}_{\xi+1}$, then $\Theta(t) = \Theta_\xi^*$.

From (1), then

$$\begin{cases} \|\Phi(t)\| \le \left\|\Phi\left(\Theta_\xi^*\right)\right\| + \sum_{\nu=1}^{\varsigma}\left|\int_{\Theta_\xi^*}^{t}\left[-a_\nu\Phi_\nu(u)\right.\right. \\ \left.\left. + \sum_{\mu=1}^{\varsigma} b_{\nu\mu}\Omega_\mu\left(\Psi_\mu(u)\right) + \sum_{\mu=1}^{\varsigma} c_{\nu\mu}\Omega_\mu\left(\Psi_\mu\left(\Theta_\xi^*\right)\right)\right]du\right|, \\ \|\Psi(t)\| \le \left\|\Psi\left(\Theta_\xi^*\right)\right\| + \sum_{\mu=1}^{\omega}\left|\int_{\Theta_\xi^*}^{t}\left[-d_\mu\Psi_\mu(u)\right.\right. \\ \left.\left. + \sum_{\nu=1}^{\omega} e_{\mu\nu}\Upsilon_\nu\left(\Phi_\nu(u)\right) + \sum_{\nu=1}^{\omega} h_{\mu\nu}\Upsilon_\nu\left(\Phi_\nu\left(\Theta_\xi^*\right)\right)\right]du\right|. \end{cases} \quad (25)$$

From (25),

$$\|\Phi(t)\| \le \left\|\Phi\left(\Theta_\xi^*\right)\right\| + \int_{\Theta_\xi^*}^{t}\left[m_1\|\Phi(u)\| + m_2\|\Psi(u)\| + m_3\left\|\Psi\left(\Theta_\xi^*\right)\right\|\right]du. \quad (26)$$

Similarly,

$$\|\Psi(t)\| \le \left\|\Psi\left(\Theta_\xi^*\right)\right\| + \int_{\Theta_\xi^*}^{t}\left[m_4\|\Psi(u)\| + m_5\|\Phi(u)\| + m_6\left\|\Phi\left(\Theta_\xi^*\right)\right\|\right]du. \quad (27)$$

Then

$$\begin{aligned} \|\Phi(t)\| + \|\Psi(t)\| &\le \left\|\Phi\left(\Theta_\xi^*\right)\right\| + \left\|\Psi\left(\Theta_\xi^*\right)\right\| \\ &+ \int_{\Theta_\xi^*}^{t}\left[(m_1+m_5)\|\Phi(u)\| + (m_4+m_2)\|\Psi(u)\|\right. \\ &\left. + m_6\left\|\Phi\left(\Theta_\xi^*\right)\right\| + m_3\left\|\Psi\left(\Theta_\xi^*\right)\right\|\right]du \\ &\le (M_1\mathfrak{Y}+1)\left[\left\|\Phi\left(\Theta_\xi^*\right)\right\| + \left\|\Psi\left(\Theta_\xi^*\right)\right\|\right] \\ &+ \int_{\Theta_\xi^*}^{t} M_2\left(\|\Phi(u)\| + \|\Psi(u)\|\right)ds. \end{aligned} \quad (28)$$

By applying Gronwall–Bellman Lemma

$$\|\Phi(t)\| + \|\Psi(t)\| \le (1 + M_1\mathfrak{Y})\exp(M_2\mathfrak{Y})\left[\left\|\Phi\left(\Theta_\xi^*\right)\right\| + \left\|\Psi\left(\Theta_\xi^*\right)\right\|\right]. \quad (29)$$

Similarly,

$$\begin{aligned} &\left\|\Phi\left(\Theta_\xi^*\right)\right\| + \left\|\Psi\left(\Theta_\xi^*\right)\right\| \\ &\le \|\Phi(t)\| + \|\Psi(t)\| + M_1\mathfrak{Y}\left(\left\|\Phi\left(\Theta_\xi^*\right)\right\| + \left\|\Psi\left(\Theta_\xi^*\right)\right\|\right) \\ &\quad + M_2\int_{\Theta_\xi^*}^{t}\left[\|\Phi(u)\| + \|\Psi(u)\|\right]du, \end{aligned} \quad (30)$$

therefore,

$$\begin{aligned} &\left\|\Phi\left(\Theta_\xi^*\right)\right\| + \left\|\Psi\left(\Theta_\xi^*\right)\right\| \\ &\le \|\Phi(t)\| + \|\Psi(t)\| + M_1\mathfrak{Y}\left(\left\|\Phi\left(\Theta_\xi^*\right)\right\| + \left\|\Psi\left(\Theta_\xi^*\right)\right\|\right) \\ &\quad + M_2\mathfrak{Y}(1 + M_1\mathfrak{Y})\exp(M_2\mathfrak{Y}) \\ &\quad \times\left(\left\|\Phi\left(\Theta_\xi^*\right)\right\| + \left\|\Psi\left(\Theta_\xi^*\right)\right\|\right). \end{aligned} \quad (31)$$

Substituting (29) and (30)

$$\left\|\Phi\left(\Theta_\xi^*\right)\right\| + \left\|\Psi\left(\Theta_\xi^*\right)\right\| \le \mathfrak{Q}\left(\|\Phi(t)\| + \|\Psi(t)\|\right), \quad (32)$$

where $\mathfrak{Q} = \{1 - \mathfrak{Y}[M_1 + M_2(1 + M_1\mathfrak{Y})\exp(M_2\mathfrak{Y})]\}^{-1}$. □

Theorem 1. *Let assumptions A1–A5 hold, then, BAMNN (2) is GES, when* $\mathfrak{Y} < \min\{\hbar/2, \widehat{\mathfrak{Y}}\}$, *where*

$$\frac{\mathfrak{A}M_1(1+\overline{\mathfrak{Q}})\exp\left[2\hbar\left(M_2 + M_1\overline{\mathfrak{Q}}\right)\right] + \mathfrak{A}\exp(-\mathfrak{B}(\hbar - \widehat{\mathfrak{Y}}))}{\mathfrak{B}} = 1. \quad (33)$$

$$\begin{aligned} &\hbar > \frac{\ln\mathfrak{A}}{\mathfrak{B}}, \\ &\overline{\mathfrak{Q}} = \left\{1 - \widehat{\mathfrak{Y}}\left[M_1 + M_2\left(1 + M_1\widehat{\mathfrak{Y}}\right)\exp\left(M_2\widehat{\mathfrak{Y}}\right)\right]\right\}^{-1}. \end{aligned} \quad (34)$$

Proof. For simplicity, we denote $\Phi(t;t_0,\Phi_0)$ and $\Psi(t;t_0,\Phi_0)$ as $\Phi(t)$ and $\Psi(t)$. From BAMNN (1) and (2),

$$\begin{cases} \sum_{\nu=1}^{\omega}\left|\Phi_\nu(t) - \widetilde{\Phi}_\nu(t)\right| = \sum_{\nu=1}^{\omega}\left|\int_{t_0}^{t}\left\{-a_\nu\left[\Phi_\nu(u) - \widetilde{\Phi}_\nu(u)\right]\right.\right. \\ + \sum_{\mu=1}^{\varsigma} b_{\nu\mu}\left[\Omega_\mu\left(\Psi_\mu(u)\right) - \Omega_\mu\left(\widetilde{\Psi}_\mu(u)\right)\right] \\ \left.\left. + \sum_{\mu=1}^{\varsigma} c_{\nu\mu}\left[\Omega_\mu\left(\Psi_\mu(\Theta(u))\right) - \Omega_\mu\left(\widetilde{\Psi}_\mu(u)\right)\right]\right\}du\right|, \\ \sum_{\mu=1}^{\varsigma}\left|\Psi_\mu(t) - \widetilde{\Psi}_\mu(t)\right| = \sum_{\mu=1}^{\varsigma}\left|\int_{t_0}^{t}\left\{-d_\mu\left[\Psi_\mu(u) - \widetilde{\Psi}_\mu(u)\right]\right.\right. \\ + \sum_{\nu=1}^{\omega} e_{\mu\nu}\left[\Upsilon_\nu\left(\Phi_\nu(u)\right) - \Upsilon_\nu\left(\widetilde{\Phi}_\nu(u)\right)\right] \\ \left.\left. + \sum_{\nu=1}^{\omega} h_{\mu\nu}\left[\Upsilon_\nu\left(\Phi_\nu(\Theta(u))\right) - \Upsilon_\nu\left(\widetilde{\Phi}_\nu(u)\right)\right]\right\}du\right|. \end{cases} \quad (35)$$

Then,

$$\begin{cases} \|\Phi(t)-\widetilde{\Phi}(t)\| \le \int_{t_0}^{t}\left\{\sum_{\nu=1}^{\omega}\left|a_\nu\right|\left|\Phi_\nu(u)-\widetilde{\Phi}_\nu(u)\right| + \sum_{\mu=1}^{\varsigma}\sum_{\nu=1}^{\omega}\left|b_{\nu\mu}\right|\Lambda_\mu\left|\Psi_\mu(u)-\widetilde{\Psi}_\mu(u)\right| + \sum_{\mu=1}^{\varsigma}\sum_{\nu=1}^{\omega}\left|c_{\nu\mu}\right|\Lambda_\mu\left|\Psi_\mu(\Theta(u))-\widetilde{\Psi}_\mu(u)\right|\right\}du, \\ \|\Psi(t)-\widetilde{\Psi}(t)\| \le \int_{t_0}^{t}\left\{\sum_{\mu=1}^{\varsigma}\left|d_\mu\right|\left|\Psi_\mu(u)-\widetilde{\Psi}_\mu(u)\right| + \sum_{\nu=1}^{\omega}\sum_{\mu=1}^{\varsigma}\left|e_{\mu\nu}\right|\Xi_\nu\left|\Phi_\nu(u)-\widetilde{\Phi}_\nu(u)\right| + \sum_{\nu=1}^{\omega}\sum_{\mu=1}^{\varsigma}\left|h_{\mu\nu}\right|\Xi_\nu\left|\Phi_\nu(\Theta(u))-\widetilde{\Phi}_\nu(u)\right|\right\}du. \end{cases} \tag{36}$$

In what follows,

$$\begin{aligned} &\|\Phi(t)-\widetilde{\Phi}(t)\|+\|\Psi(t)-\widetilde{\Psi}(t)\| \\ &\le \int_{t_0}^{t}\left\{\sum_{\nu=1}^{\omega}\left[\left|a_\nu\right|+\sum_{\mu=1}^{\varsigma}\left|e_{\mu\nu}\right|\Xi_\nu\right]\left|\Phi_\nu(u)-\widetilde{\Phi}_\nu(u)\right| + \sum_{\mu=1}^{\varsigma}\left[\left|d_\mu\right|+\sum_{\nu=1}^{\omega}\left|b_{\nu\mu}\right|\Lambda_\mu\right]\left|\Psi_\mu(u)-\widetilde{\Psi}_\mu(u)\right| \right. \\ &\quad + \sum_{\mu=1}^{\varsigma}\sum_{\nu=1}^{\omega}\left|c_{\nu\mu}\right|\Lambda_\mu\left|\Psi_\mu(\Theta(u))-\widetilde{\Psi}_\mu(u)\right| \left. + \sum_{\nu=1}^{\omega}\sum_{\mu=1}^{\varsigma}\left|h_{\mu\nu}\right|\Xi_\nu\left|\Phi_\nu(\Theta(u))-\widetilde{\Phi}_\nu(u)\right|\right\}du. \end{aligned} \tag{37}$$

Then,

$$\begin{aligned} &\|\Phi(t)-\widetilde{\Phi}(t)\|+\|\Psi(t)-\widetilde{\Psi}(t)\| \\ &\le \int_{t_0}^{t}\{M_2[\|\Phi(u)-\widetilde{\Phi}(u)\|+\|\Psi(u)-\widetilde{\Psi}(u)\|] \\ &\quad +M_1[\|\Psi(\Theta(u))-\widetilde{\Psi}(u)\|+\|\Phi(\Theta(u))-\widetilde{\Phi}(u)\|]\}du. \end{aligned} \tag{38}$$

From (24) and Definition 1,

$$\begin{aligned} &\|\Phi(t)-\widetilde{\Phi}(t)\|+\|\Psi(t)-\widetilde{\Psi}(t)\| \\ &\le \int_{t_0}^{t}\{(M_2+M_1\mathfrak{Q})[\|\Phi(u)-\widetilde{\Phi}(u)\|+\|\Psi(u)-\widetilde{\Psi}(u)\|]\}du, \\ &\quad + \int_{t_0}^{t}\mathfrak{A}M_1(1+\mathfrak{Q})\|\psi(0)\|\exp(-\mathfrak{B}(s-t_0))ds \\ &\le \int_{t_0}^{t}\{(M_2+M_1\mathfrak{Q})[\|\Phi(u)-\widetilde{\Phi}(u)\|+\|\Psi(u)-\widetilde{\Psi}(u)\|]\}du \\ &\quad + \mathfrak{A}/\mathfrak{B}M_1(1+\mathfrak{Q})\|\psi(0)\|. \end{aligned} \tag{39}$$

Using the Gronwall–Bellman Lemma, for $t_0+\mathfrak{Y}\le t\le t_0+2\hbar$,

$$\begin{aligned} &\|\Phi(t)-\widetilde{\Phi}(t)\|+\|\Psi(t)-\widetilde{\Psi}(t)\| \\ &\le \frac{\mathfrak{A}M_1(1+\mathfrak{Q})\exp[2\hbar(M_2+M_1\mathfrak{Q})]\|\psi(0)\|}{\mathfrak{B}}, \end{aligned} \tag{40}$$

then for $t_0+\hbar-\mathfrak{Y}\le t\le t_0+2\hbar-\mathfrak{Y}$, we have

$$\begin{aligned} &\|\Phi\|+\|\Psi\| \\ &\le \|\Phi(t)-\widetilde{\Phi}(t)\|+\|\Psi(t)-\widetilde{\Psi}(t)\|+\|\widetilde{\Phi}(t)\|+\|\widetilde{\Psi}(t)\| \\ &\le \mathfrak{A}/\mathfrak{B}M_1(1+\mathfrak{Q})\exp[2\hbar(M_2+M_1\mathfrak{Q})]\|\psi(0)\| \\ &\quad + \mathfrak{A}\exp(-\mathfrak{B}(\hbar-\mathfrak{Y}))\|\psi(0)\| \\ &= \aleph\|\psi(0)\|, \end{aligned} \tag{41}$$

where

$$\begin{aligned} \aleph = {}& \frac{\mathfrak{A}M_1(1+\mathfrak{Q})\exp[2\hbar(M_2+M_1\mathfrak{Q})]}{\mathfrak{B}} \\ &+ \mathfrak{A}\exp(-\mathfrak{B}(\hbar-\mathfrak{Y})). \end{aligned} \tag{42}$$

From $Q(\mathfrak{Q})=\mathfrak{A}/\mathfrak{B}M_1(1+\mathfrak{Q})\exp[2\hbar(M_2+M_1\mathfrak{Q})]+\mathfrak{A}\exp(-\mathfrak{B}(\hbar-\mathfrak{Y}))$, it is obvious that $Q(\infty)>1$ and $Q(1)<1$, besides, for parameter $\mathfrak{Q}$, $Q(\mathfrak{Q})$ is increasing, thus, undoubtedly, we can find a unique $\widehat{\mathfrak{Q}}\in(1,+\infty)$ to satisfy $Q(\widehat{\mathfrak{Q}})=1$.

From $P(\mathfrak{Y})=\mathfrak{Y}[M_1+M_2(1+M_1\mathfrak{Y})\exp(M_2\mathfrak{Y})]$, we can easily obtain $P(\mathfrak{Y})$ which is also strictly increasing, thus a unique $\overline{\mathfrak{Y}}$ can be fonded such that $P(\overline{\mathfrak{Y}})=1$, then $\mathfrak{Q}=\{1-\mathfrak{Y}[M_1+M_2(1+M_1\mathfrak{Y})\exp(M_2\mathfrak{Y})]\}^{-1}\in(1,+\infty)$ and $\forall\mathfrak{Y}\in(0,\overline{\mathfrak{Y}})$, so there must be a unique $\widehat{\mathfrak{Y}}$ such that $\mathfrak{Q}(\widehat{\mathfrak{Y}})=\widehat{\mathfrak{Q}}$, hence, when $\mathfrak{Y}<\widehat{\mathfrak{Y}}$, we can obtain $\aleph<1$.

So when $\mathfrak{Y}<\min(\hbar/2,\widehat{\mathfrak{Y}})$, setting $\widehat{\aleph}=-\ln(\aleph)/\hbar$, we obtain

$$\|\Phi\|+\|\Psi\|\le\exp(-\hbar\widehat{\aleph})\|\psi(0)\|. \tag{43}$$

From the uniqueness of the solutions we have proved before, we get

$$\begin{aligned} &\left\|\Phi(t;t_0,\Phi_0)\right\|+\left\|\Psi(t;t_0,\Psi_0)\right\| \\ &=\left\|\Phi(t;t_0+(\mathcal{S}-1)\hbar,\Phi(t_0+(\mathcal{S}-1)\hbar;t_0,\Phi_0))\right\| \\ &\quad +\left\|\Psi(t;t_0+(\mathcal{S}-1)\hbar,\Psi(t_0+(\mathcal{S}-1)\hbar;t_0,\Psi_0))\right\|, \end{aligned} \tag{44}$$

where $\mathcal{S}\in\mathbb{N}$. Thus, by (43) and (44) we can be obtained for $t>t_0+\mathcal{S}\hbar-\mathfrak{Y}$,

$$\begin{aligned} &\left\|\Phi(t;t_0,\Phi_0)\right\|+\left\|\Psi(t;t_0,\Psi_0)\right\| \\ &=\left\|\Phi(t;t_0+(\mathcal{S}-1)\hbar,\Phi(t_0+(\mathcal{S}-1)\hbar;t_0,\Phi_0))\right\| \\ &\quad +\left\|\Psi(t;t_0+(\mathcal{S}-1)\hbar,\Psi(t_0+(\mathcal{S}-1)\hbar;t_0,\Psi_0))\right\| \\ &\le\exp(-\hbar\widehat{\aleph})[\left\|\Phi(t_0+(\mathcal{S}-1)\hbar;t_0,\Phi_0)\right\| \\ &\quad +\left\|\Psi(t_0+(\mathcal{S}-1)\hbar;t_0,\Psi_0)\right\|] \\ &=\exp(-\hbar\widehat{\aleph}) \\ &\quad \times[\|\Phi(t;t_0+(\mathcal{S}-2)\hbar,\Phi(t_0+(\mathcal{S}-2)\hbar;t_0,\Phi_0))\| \end{aligned}$$

$$+\left\|\Psi\left(t;t_0+(\mathcal{S}-2)\hbar,y\left(t_0+(\mathcal{S}-2)\hbar;t_0,\Psi_0\right)\right)\right\|\Big]$$
$$\leq\cdots \tag{45}$$
$$\leq\exp(-\mathcal{S}\hbar\widehat{\aleph})\|\psi(0)\|.$$

Thus, for all $t>t_0+\hbar-\mathfrak{Y}$ and $\mathcal{S}\in\mathbb{N}$ can be found to satisfy $t_0+(\mathcal{S}-1)\hbar-\mathfrak{Y}\leq t\leq t_0+\mathcal{S}\hbar-\mathfrak{Y}$, and

$$\begin{aligned}&\|\Phi(t)\|+\|\Psi(t)\|\\&\quad\leq\exp\left(-\widehat{\aleph}(t-t_0)\right)\exp(\widehat{\aleph}(\hbar-\mathfrak{Y}))\|\psi(0)\|.\end{aligned}\tag{46}$$

Clearly, (46) is also hold for $t_0\leq t\leq t_0+\hbar-\mathfrak{Y}$, then, the perturbed system (1) is GES. □

4. Noise and Deviating Arguments Impacts on Stability

We will analyze the stability of SBAMDA in this part, the system is expressed as follows:

$$\begin{cases}d\Phi_\nu(t)=\left[-a_\nu\Phi_\nu(t)+\sum_{\mu=1}^{\varsigma}b_{\nu\mu}\Omega_\mu\left(\Psi_\mu(t)\right)\right.\\\left.+\sum_{\mu=1}^{\varsigma}c_{\nu\mu}\Omega_\mu\left(\Psi_\mu(\Theta(t))\right)\right]dt+\mathfrak{T}\Phi_\nu(t)d\mho(t),\\d\Psi_\mu(t)=\left[-d_\mu\Psi_\mu(t)+\sum_{\nu=1}^{\omega}e_{\mu\nu}\Upsilon_\nu(\Phi_\nu(t))\right.\\\left.+\sum_{\nu=1}^{\omega}h_{\mu\nu}\Upsilon_\nu(\Phi_\nu(\Theta(t)))\right]dt+\mathfrak{T}\Psi_\mu(t)d\mho(t),\end{cases}\tag{47}$$

where $\Phi_\nu(t),\Psi_\mu(t),a_i,d_\mu,b_{\nu\mu},c_{\nu\mu},e_{\mu\nu},h_{\mu\nu},\Omega(\cdot)$, and $\Upsilon(\cdot)$ are same which is defined in (1). $\mathfrak{T}$ is the intensity of disturbances. Based on Section 2, (47) has an unique state $(\Phi(t),\Psi(t))^T$ for any initial value $(\Phi_0,\Psi_0)^T$.

Then, we give the definition of MSES (MSES) of system (47).

Definition 2. System (32) is said to be MSES if there exists $\widehat{\mathfrak{A}}>1$ and $\widehat{\mathfrak{B}}>0$ such that

$$\begin{aligned}&E\left(\|\Phi(t)^2+\|\Psi(t)^2\right)\\&\quad\leq\widehat{\mathfrak{A}}E\left(\|\Phi_0{}^2+\|\Psi_0{}^2\right)\exp\left(-\widehat{\mathfrak{B}}(t-t_0)\right),\end{aligned}\tag{48}$$

where $(\Phi(t),\Psi(t))^T$ and $(\Phi_0,\Psi_0)^T$ are the state and initial value of the SBAMDA (47), respectively.

Definition 3. System (32) is called almost GES, if for any $t_0\in\mathbb{R}^+$, $\Phi_0\in\mathbb{R}^\nu$, $\Psi_0\in\mathbb{R}^\mu$, the Lyapunov exponent $\limsup_{t\longrightarrow\infty}\ln(\|\Phi(t;t_0,\Phi_0)\|+\|\Psi(t;t_0,\Psi_0)\|)/t<0$.

Remark 2. It is obvious that, from the definitions above, the mean square GES follows from the almost sure GES, but the opposite is not true. On the other hand, if A1 holds means that the MSES implies almost sure exponential stable (ASES).

Therefore, we give some assumptions we need in this part as follows:

(i) A6

$$M_3+\mathfrak{Y}(3+M_3)(M_4+M_5)\exp\{\mathfrak{Y}(M_4+M_5)\}<1.\tag{49}$$

(ii) A7

$$2\widehat{\mathfrak{A}}^2\exp(-2\widehat{\mathfrak{B}}h)+\frac{84Q_2\widehat{\mathfrak{A}}^2\hbar^{2\exp\{8\hbar(3Q_1\hbar+36Q_2\hbar)\}}}{\widehat{\mathfrak{B}}}<1,\tag{50}$$

where

$$\begin{aligned}M_3&=\max\left\{9\mathfrak{Y}^2m_3^2,9\mathfrak{Y}^2m_6^2\right\},\\M_4&=\max\left\{9\mathfrak{Y}m_1^2+3\mathfrak{T}^2,9\mathfrak{Y}m_4^2+3\mathfrak{T}^2\right\},\\M_5&=\max\left\{9\mathfrak{Y}m_2^2,9\mathfrak{Y}m_5^2\right\},\\Q_1&=\max\left\{m_1^2+m_5^2,m_2^2+m_4^2\right\},\\Q_2&=\max\left\{m_3^2,m_6^2\right\}.\end{aligned}\tag{51}$$

In the next lemma, we will explore the relationships of state of system (47) and deviating function $\Theta(t)$.

Lemma 3. *Let assumption A1–A6 hold, then exists $\mathcal{Q}>0$ such that*

$$\begin{aligned}&E\left(\|\Phi(\Theta(t))^2+\|\Psi(\Theta(t))^2\right)\\&\quad\leq\mathcal{Q}E\left(\|\Phi(t)^2+\|\Psi(t)^2\right),\end{aligned}\tag{52}$$

where

$$\begin{aligned}&\mathcal{Q}=3(1-\varpi)^{-1},\\&\varpi=M_3+\mathfrak{Y}(3+M_3)(M_4+M_5)\exp\{\mathfrak{Y}(M_4+M_5)\}.\end{aligned}\tag{53}$$

Proof. Since $t\in\mathbb{R}^+$, thus, there must be a positive constant $\xi\in\mathbb{N}$, satisfied $t\in[\mathfrak{Y}_\xi,\mathfrak{Y}_{\xi+1})$, then $\Theta(t)=\Theta_\xi^*$, from (32),

$$\begin{cases}\|\Phi(t)\|\leq\left\|\Phi\left(\Theta_\xi^*\right)\right\|+\int_{\Theta_\xi^*}^t\left[m_1\|\Phi(u)\|+m_2\|\Psi(u)\|\right.\\\left.+m_3\left\|\Psi\left(\Theta_\xi^*\right)\right\|\right]du+\left\|\int_{\Theta_\xi^*}^t\mathfrak{T}\Phi(u)d\mho(u)\right\|,\\\|\Psi(t)\|\leq\left\|\Psi\left(\Theta_\xi^*\right)\right\|+\int_{\Theta_\xi^*}^t\left[m_4\|\Psi(u)\|+m_5\|\Phi(u)\|\right.\\\left.+m_6\left\|\Phi\left(\Theta_\xi^*\right)\right\|\right]du+\left\|\int_{\Theta_\xi^*}^t\mathfrak{T}\Psi(u)d\mho(u)\right\|.\end{cases}\tag{54}$$

Furthermore,

$$\begin{aligned}
&E\|\Phi(t)\|^2\\
&\leq 3E\left\|\Phi\left(\Theta_\xi^*\right)\right\|^2 + 9\mathfrak{Y}\int_{\Theta_\xi^*}^t E\left[m_1^2\|\Phi(u)^2 + m_2^2\|\Psi(u)^2\right.\\
&\quad\left.+m_3^2\left\|\Psi\left(\Theta_\xi^*\right)\right\|^2\right]du + 3\mathfrak{T}^2\int_{\Theta_\xi^*}^t E\|\Phi(u)\|^2 du\\
&\leq 3E\left\|\Phi\left(\Theta_\xi^*\right)^2 + 3\left(3\mathfrak{Y}m_1^2 + \mathfrak{T}^2\right)\int_{\Theta_\xi^*}^t E\right\|\Phi(u)^2 du\\
&\quad+9\mathfrak{Y}\int_{\Theta_\xi^*}^t E\left[m_2^2\|\Psi(u)^2 + m_3^2\|\Psi\left(\Theta_\xi^*\right)^2\right]du\\
&\leq 3E\left\|\Phi\left(\Theta_\xi^*\right)^2 + 9\mathfrak{Y}^2 m_3^2 E\right\|\Psi\left(\Theta_\xi^*\right)^2\\
&\quad+3\left(3\mathfrak{Y}m_1^2 + \mathfrak{T}^2\right)\int_{\Theta_\xi^*}^t E\|\Phi(u)\|^2 du\\
&\quad+9\mathfrak{Y}m_2^2\int_{\Theta_\xi^*}^t E\|\Psi(u)\|^2 du.
\end{aligned} \tag{55}$$

Similarly,

$$\begin{aligned}
&E\|\Psi(t)\|^2\\
&\leq 3E\left\|\Psi\left(\Theta_\xi^*\right)\right\|^2 + 9\mathfrak{Y}\int_{\Theta_\xi^*}^t E\left[m_4^2\|\Psi(u)^2 + m_5^2\|\Phi(u)^2\right.\\
&\quad\left.+m_6^2 E\left\|\Phi\left(\Theta_\xi^*\right)\right\|^2\right]du + 3\mathfrak{T}^2\int_{\Theta_\xi^*}^t E\|\Psi(u)\|^2 du\\
&\leq 3E\left\|\Psi\left(\Theta_\xi^*\right)^2 + 9\mathfrak{Y}^2 m_6^2 E\right\|\Phi\left(\Theta_\xi^*\right)^2\\
&\quad+3\left(3\mathfrak{Y}m_4^2 + \mathfrak{T}^2\right)\int_{\Theta_\xi^*}^t E\|\Psi(u)\|^2 du\\
&\quad+9\mathfrak{Y}m_5^2\int_{\Theta_\xi^*}^t E\|\Phi(u)\|^2 du.
\end{aligned} \tag{56}$$

Then,

$$\begin{aligned}
&E\left\|\Phi(t)^2 + E\right\|\Psi(t)^2\\
&\leq (3+M_3)\left(E\left\|\Phi\left(\Theta_\xi^*\right)^2 + E\right\|\Psi\left(\Theta_\xi^*\right)^2\right)\\
&\quad+(M_4+M_5)\int_{\Theta_\xi^*}^t\left(E\left\|\Phi(u)^2 + E\right\|\Psi(u)^2\right)du.
\end{aligned} \tag{57}$$

Applying the Gronwall–Bellman Lemma,

$$\begin{aligned}
&E\left\|\Phi(t)^2 + E\right\|\Psi(t)^2\\
&\leq (3+M_3)\exp\{\mathfrak{Y}(M_4+M_5)\}\\
&\quad\times\left(E\left\|\Phi\left(\Theta_\xi^*\right)^2 + E\right\|\Psi\left(\Theta_\xi^*\right)^2\right).
\end{aligned} \tag{58}$$

At the same time,

$$\begin{cases}
\left\|\Phi\left(\Theta_\xi^*\right)\right\| \leq \|\Phi(t)\| + \int_{\Theta_\xi^*}^t\left[m_1\|\Phi(u)\| + m_2\|\Psi(u)\|\right.\\
\quad\left.+m_3\left\|\Psi\left(\Theta_\xi^*\right)\right\|\right]du + \left\|\int_{\Theta_\xi^*}^t \mathfrak{T}\Phi(u)d\mho(u)\right\|,\\
\left\|\Psi\left(\Theta_\xi^*\right)\right\| \leq \|\Psi(t)\| + \int_{\Theta_\xi^*}^t\left[m_4\|\Psi(u)\| + m_5\|\Phi(u)\|\right.\\
\quad\left.+m_6\left\|\Phi\left(\Theta_\xi^*\right)\right\|\right]du + \left\|\int_{\Theta_\xi^*}^t \mathfrak{T}\Psi(u)d\mho(u)\right\|.
\end{cases} \tag{59}$$

Similarly,

$$\begin{aligned}
E\left\|\Phi\left(\Theta_\xi^*\right)\right\|^2 &\leq 3E\|\Phi(t)\|^2 + 9\mathfrak{Y}^2 m_3^2 E\left\|\Psi\left(\Theta_\xi^*\right)\right\|^2\\
&\quad+\left(9\mathfrak{Y}m_1^2 + 3\mathfrak{T}^2\right)\int_{\Theta_\xi^*}^t E\|\Phi(u)\|^2 du\\
&\quad+9\mathfrak{Y}m_2^2\int_{\Theta_\xi^*}^t E\|\Psi(u)\|^2 du,
\end{aligned} \tag{60}$$

$$\begin{aligned}
E\left\|\Psi\left(\Theta_\xi^*\right)\right\|^2 &\leq 3E\|\Psi(t)\|^2 + 9\mathfrak{Y}^2 m_6^2 E\left\|\Phi\left(\Theta_\xi^*\right)\right\|^2\\
&\quad+\left(9\mathfrak{Y}m_4^2 + 3\mathfrak{T}^2\right)\int_{\Theta_\xi^*}^t E\|\Psi(u)\|^2 du\\
&\quad+9\mathfrak{Y}m_5^2\int_{\Theta_\xi^*}^t E\|\Phi(u)\|^2 du.
\end{aligned} \tag{61}$$

Then, from (58) we can have

$$\begin{aligned}
&E\left\|\Phi\left(\Theta_\xi^*\right)^2 + E\right\|\Psi\left(\Theta_\xi^*\right)^2\\
&\leq 3\left(E\left\|\Phi(t)^2 + E\right\|\Psi(t)^2\right)\\
&\quad+M_3\left(E\left\|\Phi\left(\Theta_\xi^*\right)^2 + E\right\|\Psi\left(\Theta_\xi^*\right)^2\right)\\
&\quad+(M_4+M_5)\int_{\Theta_\xi^*}^t\left(E\left\|\Phi(u)^2 + E\right\|\Psi(u)^2\right)du\\
&\leq 3\left(E\left\|\Phi(t)^2 + E\right\|\Psi(t)^2\right)\\
&\quad+\varpi\left(E\left\|\Phi\left(\Theta_\xi^*\right)^2 + E\right\|\Psi\left(\Theta_\xi^*\right)^2\right),
\end{aligned} \tag{62}$$

where $\varpi = M_3 + \mathfrak{Y}(3+M_3)(M_4+M_5)\exp\{\mathfrak{Y}(M_4+M_5)\}$.

Then combine with A6, we have

$$\begin{aligned}
&E\left\|\Phi\left(\Theta_\xi^*\right)\right\|^2 + E\left\|\Psi\left(\Theta_\xi^*\right)\right\|^2\\
&\quad\leq 3(1-\varpi)^{-1}\left(E\left\|\Phi(t)^2 + E\right\|\Psi(t)^2\right)\\
&\quad= \mathcal{Q}\left(E\left\|\Phi(t)^2 + E\right\|\Psi(t)^2\right).
\end{aligned} \tag{63}$$

Therefore, (63) holds on the interval $[\mathfrak{Y}_\xi, \mathfrak{Y}_{\xi+1})$, since the randommicities of t and ξ, thus, $\forall t \in \mathbb{R}^+$, $\forall \xi \in \mathbb{N}$, (44) also holds. Therefore, the proof is completed.

In what follows, the implications of both deviating arguments and stochastic disturbances on the stability of BAMNN (2) is examined. □

Theorem 2. *Let A1 to A7 hold and BAMNN (2) is GES, then, SBAMDA (32) is MSES and also ASES if* $|\check{\mathfrak{T}}| < \overline{\mathfrak{T}}/\sqrt{2}$, $\check{\mathfrak{Y}} < \min\{\hbar/2, \overline{\mathfrak{Y}}\}$, $\mathfrak{D}(\check{\mathfrak{Y}}, \check{\mathfrak{T}}) < 1$, *where* $\overline{\mathfrak{T}}$ *and* $\overline{\mathfrak{Y}}$ *are the unique positive solutions of the following two transcendental equations, respectively,*

$$2\widehat{\mathfrak{A}}^2 \exp(-2\mathfrak{B}\hbar) + \frac{4\widehat{\mathfrak{A}}^2}{\widehat{\mathfrak{B}}}\hbar\left(\check{\mathfrak{T}}^2 + 42Q_2\hbar\right) \times \exp\left\{8\hbar\left(\check{\mathfrak{T}}^2 + 3Q_1\hbar + 36Q_2\hbar\right)\right\} = 1, \tag{64}$$

$$2\overline{\Omega}_1 \exp\left(2\hbar\overline{\Omega}_2\right) + 4\widehat{\mathfrak{A}}^2 \exp\left(-2\widehat{\mathfrak{B}}\left(\hbar - \overline{\mathfrak{Y}}\right)\right) = 1,$$

where

$$\begin{aligned}
&\frac{\hbar > \ln\left(2\widehat{\mathfrak{A}}^2\right)}{2\widehat{\mathfrak{B}}},\\
&\overline{\Omega}_1 = \frac{2\hbar\widehat{\mathfrak{A}}^2\left[12\hbar Q_2 + 24\hbar Q_2\overline{\mathcal{Q}} + \check{\mathfrak{T}}^2\right]}{\widehat{\mathfrak{B}}},\\
&\overline{\Omega}_2 = 12\hbar Q_1 + 48\hbar Q_2\overline{\mathcal{Q}} + 2\check{\mathfrak{T}}^2,\\
&\overline{\mathcal{Q}} = 3\left(1 - \check{\omega}\right)^{-1},\\
&\check{\omega} = \overline{M}_3 + \overline{\mathfrak{Y}}\left(3 + \overline{M}_3\right)\left(\overline{M}_4 + \overline{M}_5\right)\exp\left\{\overline{\mathfrak{Y}}\left(\overline{M}_4 + \overline{M}_5\right)\right\},\\
&\overline{M}_3 = \max\left\{9\overline{\mathfrak{Y}}^2 m_3^2, 9\overline{\mathfrak{Y}}^2 m_6^2\right\},\\
&\overline{M}_4 = \max\left\{9\overline{\mathfrak{Y}} m_1^2 + 1.5\check{\mathfrak{T}}^2, 9\overline{\mathfrak{Y}} m_4^2 + 1.5\check{\mathfrak{T}}^2\right\},\\
&\overline{M}_5 = \max\left\{9\overline{\mathfrak{Y}} m_2^2, 9\overline{\mathfrak{Y}} m_5^2\right\}.
\end{aligned} \tag{65}$$

Proof. From (2), (24), (25), and (47),

$$\begin{cases}
E\|\Phi(t) - \widetilde{\Phi}(t)\|^2\\
\le 6\left(t - t_0\right)\int_{t_0}^{t}\left\{m_1^2 E\|\Phi(u) - \widetilde{\Phi}(u)\|^2\right.\\
+m_2^2 E\left\|\Psi(u) - \widetilde{\Psi}(u)^2 + m_3^2 E\right\|\Psi(\Theta(u))\\
\left.-\widetilde{\Psi}(u)^2\right\}du + 2\mathfrak{T}^2\int_{t_0}^{t} E\|\Phi(u)\|^2 du\\
\left.-\widetilde{\Psi}(u)^2\right\}du + 2\mathfrak{T}^2\int_{t_0}^{t} E\|\Phi(u)\|^2 du\\
\le 6\left(t - t_0\right)\int_{t_0}^{t}\left\{m_4^2 E\|\Psi(u) - \widetilde{\Psi}(u)\|^2\right.\\
+m_5^2 E\left\|\Phi(u) - \widetilde{\Phi}(u)^2 + m_6^2\right\|\Phi(\Theta(u))\\
\left.-\widetilde{\Phi}(u)^2\right\}du + 2\mathfrak{T}^2\int_{t_0}^{t} E\|\Psi(u)\|^2 du.
\end{cases} \tag{66}$$

Furthermore, we can obtain

$$\begin{aligned}
&E\left\|\Phi(t) - \widetilde{\Phi}(t)^2 + E\right\|\Psi(t) - \widetilde{\Psi}(t)^2\\
&\le 6\left(t - t_0\right)\int_{t_0}^{t}\left[\left(m_1^2 + m_5^2\right)E\|\Phi(u) - \widetilde{\Phi}(u)\|^2\right.\\
&\left.+\left(m_2^2 + m_4^2\right)E\|\Psi(u) - \widetilde{\Psi}(u)\|^2\right]du\\
&+ 6\left(t - t_0\right)\int_{t_0}^{t} m_3^2 E\|\Psi(\Theta(u)) - \widetilde{\Psi}(u)\|^2 du\\
&+ 6\left(t - t_0\right)\int_{t_0}^{t} m_6^2 E\|\Phi(\Theta(u)) - \widetilde{\Phi}(u)\|^2 du\\
&+ 2\mathfrak{T}^2\int_{t_0}^{t}\left[E\left\|\Phi(u)^2 + E\right\|\Psi(u)^2\right]du.
\end{aligned} \tag{67}$$

Then

$$\begin{aligned}
&E\left\|\Phi(t) - \widetilde{\Phi}(t)^2 + E\right\|\Psi(t) - \widetilde{\Psi}(t)^2\\
&\le\left[6\left(t - t_0\right)Q_1 + 4\mathfrak{T}^2\right]\int_{t_0}^{t}\left[E\|\Phi(u) - \widetilde{\Phi}(u)\|^2\right.\\
&\left.+E\|\Psi(u) - \widetilde{\Psi}(u)\|^2\right]du + 12\left(t - t_0\right)Q_2\mathcal{Q}\\
&\times\int_{t_0}^{t}\left[E\left\|\Phi(u)^2 + E\right\|\Psi(u)^2\right]du\\
&+\left[12\left(t - t_0\right)Q_2 + 4\mathfrak{T}^2\right]\\
&\times\int_{t_0}^{t}\left[E\left\|\widetilde{\Phi}(u)^2 + E\right\|\widetilde{\Psi}(u)^2\right]du\\
&\le\left[6\left(t - t_0\right)Q_1 + 24\left(t - t_0\right)Q_2\mathcal{Q} + 4\mathfrak{T}^2\right]\\
&\times\int_{t_0}^{t}\left[E\left\|\Phi(u) - \widetilde{\Phi}(u)^2 + E\right\|\Psi(u) - \widetilde{\Psi}(u)^2\right]du\\
&+\widehat{\mathfrak{A}}^2/\widehat{\mathfrak{B}}\left(t - t_0\right)\left[6\left(t - t_0\right)Q_2 + 12\left(t - t_0\right)Q_2\mathcal{Q} + 2\mathfrak{T}^2\right]\\
&\times\left(E\left\|\Phi_0{}^2 + E\right\|\Psi_0{}^2\right).
\end{aligned} \tag{68}$$

Applying Gronwall–Bellman Lemma, for $t_0 + \mathfrak{Y} \le t_0 \le t + 2\hbar$, we can get that

$$\begin{aligned}
&E\|\Phi(t) - \widetilde{\Phi}(t)\|^2 + E\|\Psi(t) - \widetilde{\Psi}(t)\|^2\\
&\le\frac{2\hbar\widehat{\mathfrak{A}}^2}{\mathfrak{B}\left[12\hbar Q_2 + 24\hbar Q_2\mathcal{Q} + 2\mathfrak{T}^2\right]}\\
&\times\exp\left\{2\hbar\left[12\hbar Q_1 + 48\hbar Q_2\mathcal{Q} + 4\mathfrak{T}^2\right]\right\}\\
&\times\left(E\left\|\Phi_0\right\|^2 + E\left\|\Psi_0\right\|^2\right)\\
&\coloneqq \Omega_1\exp\left(2\hbar\Omega_2\right)\left(E\left\|\Phi_0\right\|^2 + E\left\|\Psi_0\right\|^2\right).
\end{aligned} \tag{69}$$

Thus, for $t_0 + \hbar - \mathfrak{Y} \le t \le t_0 + 2\hbar - \mathfrak{Y}$,

$$\begin{aligned}&E\|\Phi(t)^2+E\|\Psi(t)^2\\&\le 2\Omega_1\exp(2\hbar\Omega_2)\left(E\|\Phi_0{}^2+E\|\Psi_0{}^2\right)\\&\quad+2\widehat{\mathfrak{A}}^2\left(E\|\Phi_0{}^2+E\|\Psi_0{}^2\right)\exp\left(-2\widehat{\mathfrak{B}}(t-t_0)\right)\\&\le\left\{2\Omega_1\exp(2\hbar\Omega_2)+2\widehat{\mathfrak{A}}^2\exp(-2\widehat{\mathfrak{B}}(\hbar-\mathfrak{Y}))\right\}\\&\quad\times E\left(\|\Phi_0{}^2+\|\Psi_0{}^2\right)\\&=\mathfrak{D}(\mathfrak{Y},\mathfrak{T})E\left(\|\Phi_0{}^2+\|\Psi_0{}^2\right),\end{aligned}\tag{70}$$

where

$$\begin{aligned}&\mathfrak{D}(\mathfrak{Y},\mathfrak{T})=2\Omega_1\exp(2\hbar\Omega_2)+2\widehat{\mathfrak{A}}^2\exp(-2\widehat{\mathfrak{B}}(\hbar-\mathfrak{Y})),\\&\Omega_1=\frac{2\hbar\widehat{\mathfrak{A}}^2}{\widehat{\mathfrak{B}}\left[12\hbar Q_2+24\hbar Q_2\mathcal{Q}+2\mathfrak{T}^2\right]},\\&\Omega_2=12\hbar Q_1+48\hbar Q_2\mathcal{Q}+4\mathfrak{T}^2.\end{aligned}\tag{71}$$

Therefore, we have

$$\begin{aligned}\mathfrak{D}(0,\mathfrak{T})&=2\widehat{\mathfrak{A}}^2\exp(-2\widehat{\mathfrak{B}}\hbar)+\frac{4\widehat{\mathfrak{A}}^2}{\widehat{\mathfrak{B}}}\hbar\left(\mathfrak{T}^2+42Q_2\hbar\right)\\&\quad\times\exp\left\{8\hbar\left(\mathfrak{T}^2+3Q_1\hbar+36Q_2\hbar\right)\right\},\end{aligned}\tag{72}$$

and we can easily get (72) is strictly increase for $\mathfrak{T}$.

From the assumption A7, we can easily obtain that

$$\begin{aligned}\mathfrak{D}(0,0)&=2\widehat{\mathfrak{A}}^2\exp(-2\widehat{\mathfrak{B}}\hbar)\\&\quad+\frac{168Q_2\widehat{\mathfrak{A}}^2\hbar^2\exp\{8\hbar(3Q_1\hbar+36Q_2\hbar)\}}{\widehat{\mathfrak{B}}}<1.\end{aligned}\tag{73}$$

On the other hand, we can easily get $\mathfrak{D}(0,\infty)>1$, thus, suppose that $\mathfrak{D}(0,\overline{\mathfrak{T}})=1$. So, when $|\mathfrak{T}|<\overline{\mathfrak{T}}$, $\mathfrak{D}(\infty,\mathfrak{T})>1$, and $\mathfrak{D}(0,\mathfrak{T})<1$. Besides, we denote $\varpi(\mathfrak{Y},\mathfrak{T})=M_3+\mathfrak{Y}(3+M_3)(M_4+M_5)\exp\{\mathfrak{Y}(M_4+M_5)\}$, we can easily get $\varpi(\mathfrak{Y},\mathfrak{T})$ is increasing for $\mathfrak{Y}$. Then, from A6, we know that $\varpi(\mathfrak{Y},\overline{\mathfrak{T}})<1$, so exists a $\overline{\mathfrak{Y}}$ such that $\varpi(\overline{\mathfrak{Y}},\overline{\mathfrak{T}})=1$, so, when $\mathfrak{Y}<\overline{\mathfrak{Y}}$, $\varpi(\overline{\mathfrak{Y}},\overline{\mathfrak{T}})<1$ holds.

And because $\mathfrak{D}(\mathfrak{Y},\mathfrak{T})$ is strictly increasing for $\mathfrak{Y}$, so exists $\breve{\mathfrak{Y}}$, $\mathfrak{D}(\breve{\mathfrak{Y}},\breve{\mathfrak{T}})<1$ holds, when $|\breve{\mathfrak{T}}|<\overline{\mathfrak{T}}/\sqrt{2}$, $\breve{\mathfrak{Y}}<\min\{\hbar/2,\overline{\mathfrak{Y}}\}$.

Setting $\mathfrak{B}=-\ln\mathfrak{D}/\hbar$, thus

$$\begin{aligned}&E\|\Phi(t)\|^2+\mathrm{E}\|\Psi(t)\|^2\\&\le\exp(-\mathfrak{B}\hbar)E\left(\|\Phi_0\|^2+\|\Psi_0\|^2\right).\end{aligned}\tag{74}$$

By Theorem 1, the solution of (47) is unique, thus, there is a $\mathcal{K}\in\mathbb{N}$, such that

$$\begin{aligned}&E\|\Phi(t)\|^2+E\|\Psi(t)\|^2\\&=E\left\|\Phi\left(t;t_0+(\mathcal{K}-1)\hbar,\Phi\left(t_0+t_0+(\mathcal{K}-1)\hbar;t_0,\Phi_0\right)\right)\right\|^2\\&\quad+E\left\|\Psi\left(t;t_0+(\mathcal{K}-1)\hbar,\Psi\left(t_0+t_0+(\mathcal{K}-1)\hbar;t_0,\Psi_0\right)\right)\right\|^2\\&\le\exp(-\mathfrak{B}\hbar)\left\{E\left\|\Phi\left(t_0+t_0+(\mathcal{K}-1)\hbar;t_0,\Phi_0\right)\right\|^2\right.\\&\quad\left.+E\left\|\Psi\left(t_0+t_0+(\mathcal{K}-1)\hbar;t_0,\Psi_0\right)\right\|^2\right\}\\&\le\cdots\\&\le\exp(-\mathcal{K}\mathfrak{B}\hbar)E\left(\|\Phi_0\|^2+\|\Psi_0\|^2\right),\end{aligned}\tag{75}$$

holds for $t\ge t_0-\mathfrak{A}+\mathcal{K}\hbar$.

Thus, for any $t>t_0-\mathfrak{Y}+\hbar$, there is a $\mathcal{K}\in\mathbb{N}$ such that $t_0-\mathfrak{Y}+(\mathcal{K}-1)\hbar\le t\le t_0-\mathfrak{Y}+\mathcal{K}\hbar$,

$$\begin{aligned}&E\|\Phi(t)\|^2+E\|\Psi(t)\|^2\\&\le\exp\left(-\mathfrak{B}\left(t-t_0\right)\right)\exp(\mathfrak{B}(\hbar-\mathfrak{Y}))\\&\quad\times E\left(\|\Phi_0\|^2+\|\Psi_0\|^2\right).\end{aligned}\tag{76}$$

Clearly, we can easily get (76) is also hold for $t_0\le t\le t_0-\mathfrak{Y}+\hbar$. Thus, system (47) is satisfies Definition 2. According to Remark 2, (47) is also ASES. □

Remark 3. Table 1 is a brief comparison between this paper and existing literature. The factors we compare are RoS, BAM, deviating argument (DA), the existence and uniqueness of solutions (EU), ES, AS.

5. Examples

We provide a few instances in this section to demonstrate the viability of the results in the earlier sections.

Example 1. Consider the following BAMDA, let $\omega=\varsigma=2$, the parameters and the model of our example are as follows:

Table 1: The brief comparison between this paper and existing literature.

	RoS	BAM	DA	EU	ES	AS
Xiang and Cao [7]		✓			✓	
Sowmiya et al. [8]		✓			✓	
Sowmiya et al. [9]		✓				✓
Balasubramania and Vidhya [13]		✓				✓
Akhmet et al. [23]			✓	✓	✓	✓
Wenxiang et al. [29]	✓		✓		✓	
This paper	✓	✓	✓	✓	✓	

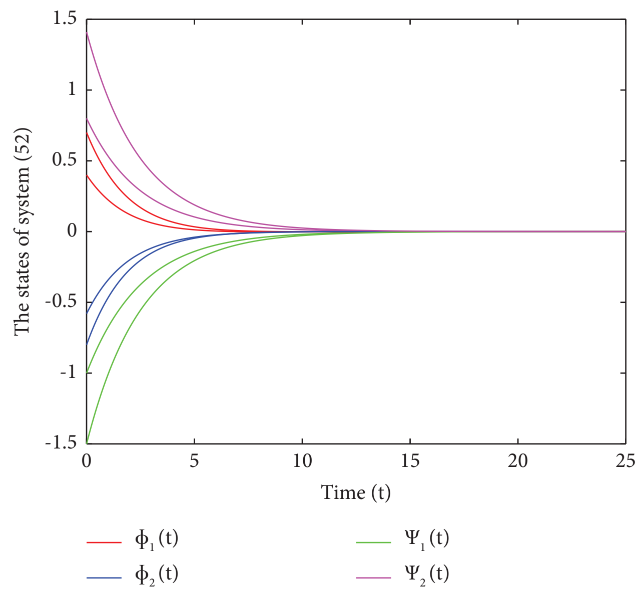

Figure 1: The states of (77) with $\{\mathfrak{Y}_\xi\} = \xi/10$, $\{\Theta_\xi\} = 2\xi + 1/20$ in different initial values.

$$A = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, B = \begin{bmatrix} 0.004 & 0.003 \\ 0.003 & 0.004 \end{bmatrix},$$
$$C = \begin{bmatrix} 0.003 & 0.001 \\ 0.001 & 0.003 \end{bmatrix}, D = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix},$$
$$E = \begin{bmatrix} 0.002 & 0.001 \\ 0.001 & 0.002 \end{bmatrix}, H = \begin{bmatrix} 0.002 & 0.003 \\ 0.003 & 0.002 \end{bmatrix},$$
$$\begin{cases} \dot{\Phi}_1(t) = -0.5\Phi_1(t) + 0.004\tanh(\Psi_1(t)) + 0.003\tanh(\Psi_2(t)) \\ +0.003\tanh(\Psi_1(\Theta(t))) + 0.001\tanh(\Psi_2(\Theta(t))), \\ \dot{\Phi}_2(t) = -0.5\Phi_2(t) + 0.003\tanh(\Psi_1(t)) + 0.004\tanh(\Psi_2(t)) \\ +0.001\tanh(\Psi_1(\Theta(t))) + 0.003\tanh(\Psi_2(\Theta(t))), \\ \dot{\Psi}_1(t) = -0.4\Psi_1(t) + 0.002\tanh(\Phi_1(t)) + 0.001\tanh(\Phi_2(t)) \\ +0.002\tanh(\Phi_1(\Theta(t))) + 0.003\tanh(\Phi_2(\Theta(t))), \\ \dot{\Psi}_2(t) = -0.3\Psi_2(t) + 0.001\tanh(\Phi_1(t)) + 0.002\tanh(\Phi_2(t)) \\ +0.003\tanh(\Phi_1(\Theta(t))) + 0.002\tanh(\Phi_2(\Theta(t))), \end{cases} \tag{77}$$

where $\{\mathfrak{Y}_\xi\} = \xi/10$ and $\{\Theta_\xi\} = (2\xi + 1)/20$.

From the parameters, we have $m_1 = 0.5$, $m_2 = 0.007$, $m_3 = 0.004$, $m_4 = 0.4$, $m_5 = 0.003$, $m_6 = 0.0015$, $M_1 = 0.016$, $M_2 = 0.5030$, and $\hbar = 0.3$.

Moreover, we have

$$0.0333\exp(0.3048) + \exp(-0.09) = 0.9591 < 1. \tag{78}$$

Besides, according to the comparison theorem, we know that the system without deviating argument

$$\begin{cases} \dot{\Phi}_1(t) = -0.5\Phi_1(t) + 0.04\tanh(\Psi_1(t)) + 0.03\tanh(\Psi_2(t)) \\ +0.03\tanh(\Psi_1(t)) + 0.01\tanh(\Psi_2(t)), \\ \dot{\Phi}_2(t) = -0.5\Phi_2(t) + 0.03\tanh(\Psi_1(t)) + 0.04\tanh(\Psi_2(t)) \\ +0.01\tanh(\Psi_1(t)) + 0.03\tanh(\Psi_2(t)), \\ \dot{\Psi}_1(t) = -0.4\Psi_1(t) + 0.02\tanh(\Phi_1(t)) + 0.01\tanh(\Phi_2(t)) \\ +0.02\tanh(\Phi_1(t)) + 0.03\tanh(\Phi_2(t)), \\ \dot{\Psi}_2(t) = -0.3\Psi_2(t) + 0.01\tanh(\Phi_1(t)) + 0.02\tanh(\Phi_2(t)) \\ +0.03\tanh(\Phi_1(t)) + 0.02\tanh(\Phi_2(t)), \end{cases} \tag{79}$$

is GES with $\mathfrak{A} = 1, \mathfrak{B} = 0.3$.

Let $\Lambda_\mu = 1$, $\Xi_\nu = 1$, then we can get $\widehat{\mathfrak{Y}} = 0.1393$ from the equation as follows:

$$\begin{aligned}&\exp(0.3\widehat{\mathfrak{Y}}-0.09)+\left\{0.0167+0.0167/[1-\widehat{\mathfrak{Y}}[(0.503(1+0.0005\widehat{\mathfrak{Y}})+\exp(0.503\widehat{\mathfrak{Y}})0.005]]\right\}\\&\exp\left\{0.6(0.503+0.005\left\{0.0167+0.0167/[1-\widehat{\mathfrak{Y}}[(0.503(1+0.0005\widehat{\mathfrak{Y}})\times\exp(0.503\widehat{\mathfrak{Y}})0.005]]\right\}=1.\end{aligned}\tag{80}$$

Therefore, when $\mathfrak{Y}<\widehat{\mathfrak{Y}}=0.2476$, according to Theorem 1, (47) is GES.

Figure 1 depicts the states of system (77) with $\{\mathfrak{Y}_\xi\}=\xi/10$, $\{\Theta_\xi\}=(2\xi+1)/20$ in different initial values. Since the length of interval of deviating argument we take in system (77) is less than 0.1393, thus, BAMDA (77) is GES.

Example 2. Consider the following SBAMDA:

$$A=\begin{bmatrix}0.5&0\\0&0.5\end{bmatrix},B=\begin{bmatrix}0.001&-0.001\\-0.001&0.001\end{bmatrix},$$

$$C=\begin{bmatrix}0.001&-0.001\\-0.001&0.001\end{bmatrix},D=\begin{bmatrix}0.5&0\\0&0.5\end{bmatrix},$$

$$E=\begin{bmatrix}0.001&-0.001\\-0.001&0.001\end{bmatrix},H=\begin{bmatrix}0.001&-0.001\\-0.001&0.001\end{bmatrix},$$

$$\begin{cases}d\Phi_1(t)=-0.5\Phi_1(t)+0.001\tanh(\Psi_1(t))\\-0.001\tanh(\Psi_2(t))+0.001\tanh(\Psi_1(\Theta(t)))\\-0.001\tanh(\Psi_2(\Theta(t)))+\mathfrak{T}\Phi_1(t)d\mho(t),\\d\Phi_2(t)=-0.5\Phi_2(t)-0.001\tanh(\Psi_1(t))\\+0.001\tanh(\Psi_2(t))-0.001\tanh(\Psi_1(\Theta(t)))\\+0.001\tanh(\Psi_2(\Theta(t)))+\mathfrak{T}\Phi_2(t)d\mho(t),\\d\Psi_1(t)=-0.5\Psi_1(t)+0.001\tanh(\Phi_1(t))\\-0.001\tanh(\Phi_2(t))+0.001\tanh(\Phi_1(\Theta(t)))\\-0.001\tanh(\Phi_2(\Theta(t)))+\mathfrak{T}\Psi_1(t)d\mho(t),\\d\Psi_2(t)=-0.5\Psi_2(t)-0.001\tanh(\Phi_1(t))\\+0.001\tanh(\Phi_2(t))-0.001\tanh(\Phi_1(\Theta(t)))\\+0.001\tanh(\Phi_2(\Theta(t)))+\mathfrak{T}\Psi_2(t)d\mho(t),\end{cases}\tag{81}$$

where the deviating argument $\Theta(t)=\Theta_\xi^*$, when $t\in[\mathfrak{Y}_\xi,\mathfrak{Y}_{\xi+1})$, $\xi\in N$.

By computing the parameters, we can obtain $\Lambda_\mu=1$, $\Xi_\nu=1$, $m_1=0.5$, $m_2=0.002$, $m_3=0.002$, $m_4=0.5$, $m_5=0.002$, $m_6=0.002$, $Q_1=0.25$, and $Q_2=4\times10^{-6}$. Besides, we let $\hbar=0.8$, by the comparison theorem, we can obtain that the origin BAMCNN of (81).

$$\begin{cases}\dot\Phi_1(t)=-0.5\Phi_1(t)+0.001\tanh(\Psi_1(t))\\-0.001\tanh(\Psi_2(t))+0.001\tanh(\Psi_1(t))\\-0.001\tanh(\Psi_2(t)),\\\dot\Phi_2(t)=-0.5\Phi_2(t)+0.001\tanh(\Psi_1(t))\\-0.001\tanh(\Psi_2(t))+0.001\tanh(\Psi_1(t))\\-0.001\tanh(\Psi_2(t)),\\\dot\Psi_1(t)=-0.5\Psi_1(t)-0.001\tanh(\Phi_1(t))\\+0.001\tanh(\Phi_2(t))-0.001\tanh(\Phi_1(t))\\+0.001\tanh(\Phi_2(t)),\\\dot\Psi_2(t)=-0.5\Psi_2(t)-0.001\tanh(\Phi_1(t))\\+0.001\tanh(\Phi_2(t))-0.001\tanh(\Phi_1(t))\\+0.001\tanh(\Phi_2(t)),\end{cases}\tag{82}$$

is GES with $\widehat{\mathfrak{A}}=1$, $\widehat{\mathfrak{B}}=0.5$.

Moreover, we have

$$2\exp(-0.8)+0.0017\exp(3.8408)=0.9788<1.\tag{83}$$

Substituting the parameters into (72), we have

$$\mathfrak{D}(0,\mathfrak{T})=2\exp(-0.8)+6.4\left(\mathfrak{T}^2+1.344\times10^{-4}\right)\exp\left(6.4\left(\mathfrak{T}^2+0.6001\right)\right).\tag{84}$$

Let $\mathfrak{D}(0,\mathfrak{T})=1$, thus, we have $\overline{\mathfrak{T}}=0.006$. Note that $\breve{\mathfrak{T}}<\overline{\mathfrak{T}}/\sqrt{2}$, that is $\breve{\mathfrak{T}}<0.0042$, then substituting the other parameters into $\mathfrak{D}(\mathfrak{Y},\overline{\mathfrak{T}}/\sqrt{2})$, then we can get

$$\mathfrak{D}(\overline{\mathfrak{Y}},\overline{\mathfrak{T}}/\sqrt{2})=2\overline{\Omega}_1\exp(2\hbar\overline{\Omega}_2)+4\widehat{\mathfrak{A}}^2\exp(-2\widehat{\mathfrak{B}}(\hbar-\overline{\mathfrak{Y}})),\tag{85}$$

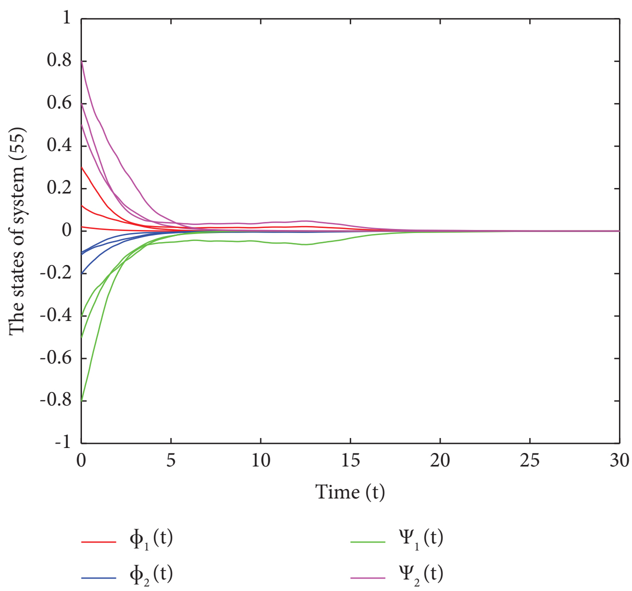

Figure 2: The states of (81) with $\mathfrak{T} = 0.002$, $\{\mathfrak{Y}_\xi\} = \xi/35$, and $\{\Theta_\xi\} = 2\xi + 1/70$ in different initial values.

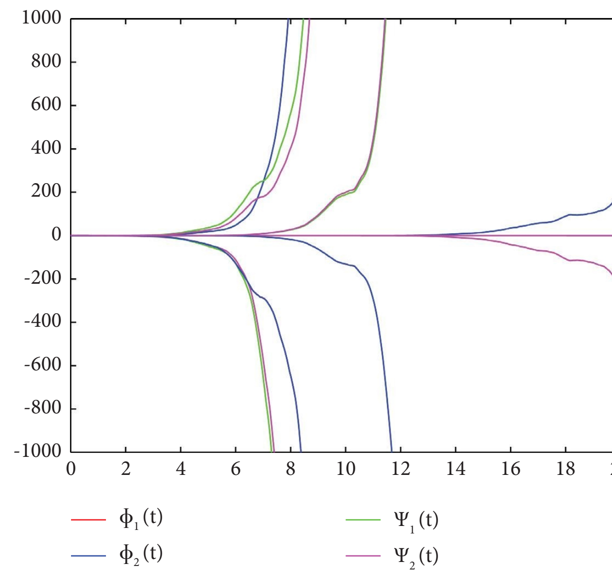

Figure 3: The state of system (81) with $\mathfrak{T} = 0.006$, $\{\mathfrak{Y}_\xi\} = \xi/105$, and $\{\Theta_\xi\} = 2\xi + 1/210$.

where

$$\begin{aligned}
\overline{\Omega}_1 &= \frac{2\hbar\hat{\mathfrak{A}}^2}{\hat{\mathfrak{B}}\left[12\hbar Q_2 + 24\hbar Q_2\overline{\mathcal{Q}} + \check{\mathfrak{T}}^2\right]},\\
\overline{\Omega}_2 &= 12\hbar Q_1 + 48\hbar Q_2\overline{\mathcal{Q}} + 2\check{\mathfrak{T}}^2,\\
Q_1 &= \max\left\{m_1^2 + m_5^2, m_2^2 + m_4^2\right\},\\
Q_2 &= \max\left\{m_3^2, m_6^2\right\},\\
\overline{\mathcal{Q}} &= 3\left(1 - \check{\omega}\right)^{-1},\\
\check{\omega} &= \overline{M}_3 + \mathfrak{Y}\left(3 + \overline{M}_3\right)\left(\overline{M}_4 + \overline{M}_5\right)\exp\left\{\mathfrak{Y}\left(\overline{M}_4 + \overline{M}_5\right)\right\},\\
\overline{M}_3 &= \max\left\{9\mathfrak{Y}^2 m_3^2, 9\mathfrak{Y}^2 m_6^2\right\},\\
\overline{M}_4 &= \max\left\{9\mathfrak{Y}m_1^2 + 1.5\check{\mathfrak{T}}^2, 9\mathfrak{Y}m_4^2 + 1.5\check{\mathfrak{T}}^2\right\},\\
\overline{M}_5 &= \max\left\{9\mathfrak{Y}m_2^2, 9\mathfrak{Y}m_5^2\right\}.
\end{aligned} \tag{86}$$

Thus, we can obtain $\overline{\mathfrak{Y}} = 0.0117$, recalling that $\mathfrak{Y} < \min\{\hbar/2, \overline{\mathfrak{Y}}\}$, and therefore, $\mathfrak{Y} < 0.0117$.

Hence, we can easily get for given stable system, when $\check{\mathfrak{T}} < |\overline{\mathfrak{T}}|/\sqrt{2}$, $\mathfrak{Y} < \min\{\hbar/2, \overline{\mathfrak{Y}}\}$, the perturbed system will also be stable.

The states in Figure 2 is the states under the condition of system (81) with $\mathfrak{T} = 2/1000$, $\{\mathfrak{Y}_\xi\} = \xi/35$, and $\{\Theta_\xi\} = (2\xi + 1)/70$. It shows that if the $\mathfrak{T}$ and $\mathfrak{Y}$ are both lower than we derived in this paper, then (81) will be exponential stable.

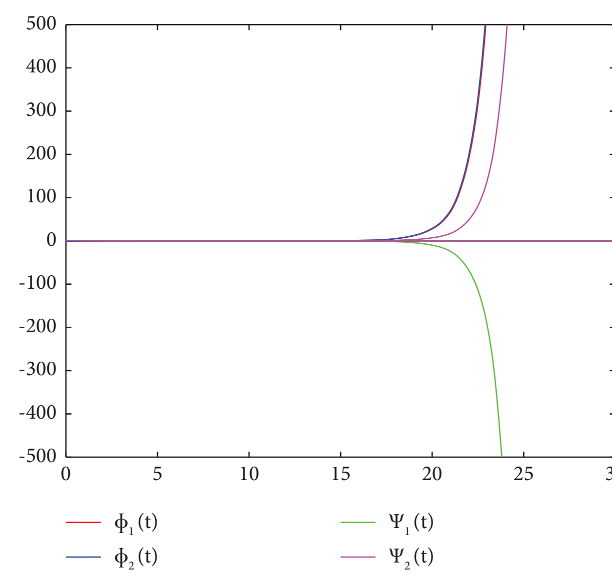

Figure 4: The state of system (81) with $\mathfrak{T} = 0.002$, $\{\mathfrak{Y}_\xi\} = \xi/50$, and $\{\Theta_\xi\} = 2\xi + 1/100$.

In Figure 3, $\mathfrak{T} = 0.006 > 0.0042$, $\mathfrak{Y} = 1/105 < 0.0117$, thus, the states in Figure 3 is not exponentially stable. And Figure 4 shows the states of (81), when $\mathfrak{T} = 0.002 < 0.0042$ and $\mathfrak{Y} = 0.02 > 0.0117$ is bigger than we derived in this paper, we can easily find that the states in Figure 4 has lost its origin stability.

In Figure 5, we choose $\mathfrak{T} = 0.007 > 0.0042$ and $\mathfrak{Y} = 1/20 > 0.0117$, they are both bigger than the results we derived in this paper, hence, the states of SBAMDA (81) is not GES.

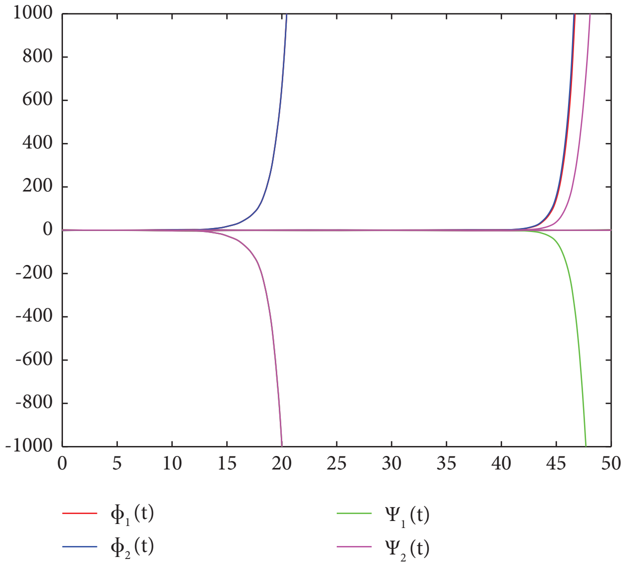

FIGURE 5: The state of system (81) with $\mathfrak{T} = 0.007$, $\{\mathfrak{Y}_\xi\} = \xi/20$, and $\{\Theta_\xi\} = 2\xi + 1/40$.

6. Conclusion

The robustness of the stability of BAMCNN with deviating argument and noise is analyzed. The upper bounds of noise and the deviating intervals must be limited in order to retain the original stability. We may estimate these upper bounds that we derived by resolving transcendental equations. Our findings provide a theoretical underpinning for BAMCNN designs and implementations. Future study may focus on enhancing the upper limits and considering employing classical approaches to optimize the computation process, such as the LMI method and the Lyapunov function method. Furthermore, more sophisticated structural disturbances, such as Markov jump, impulses, state-dependent delays, and so on, can be taken into account.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported by the Natural Science Foundation of China under Grant nos. 62072164 and 12074111.

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