Hindawi
Discrete Dynamics in Nature and Society
Volume 2024, Article ID 2045842, 11 pages
https://doi.org/10.1155/2024/2045842

Research Article

Research on Nash Equilibrium of Dual-Channel Supply Chain Based on Wholesale Price Discrimination and Sales Efforts

Jin Sha[1] and Sisi Zheng[2]

[1]School of Mechanical Engineering, Guangdong Ocean University, Zhanjiang 524088, China
[2]School of Mathematics and Statistics, Huizhou University, Huizhou 516007, China

Correspondence should be addressed to Sisi Zheng; zhengss@hzu.edu.cn

Received 30 October 2023; Revised 7 December 2023; Accepted 6 January 2024; Published 18 January 2024

Academic Editor: Fabio Tramontana

The progress of communication technology and the Internet has made the dual-channel supply chain have great development. Supply chain members have to compete and cooperate with other enterprises on the chain in order to obtain greater self-benefit. In the competitive-cooperative relationship among enterprises, price discrimination and sales effort are common tools of operation, and Nash equilibrium theory is a common strategy in the process of supply chain game. Whether manufacturers can promote retailers to implement sales efforts through price discrimination and ensure that the overall revenue of the supply chain is improved at the Nash equilibrium is great significance to both the supply chain and individual enterprises. This paper constructs a decentralized dual-channel supply chain mathematical model composed of an independent manufacturer, an offline retailer, and an online retailer, taking into account the impact of the offline retailer's sales efforts on his demand and its positive external for the other channel. The revenue matrix of retailers' decisions about whether or not to pay their sales efforts is built under the condition that manufacturers charge different wholesale prices to different retailers and solves the condition of every decision combination becoming the Nash equilibrium using the Nash equilibrium game theory. Finally, the optimal pricing strategy of the manufacturers is analysed to get the constraints of this dual-channel supply chain achieving Pareto improvement when manufacturer uses wholesale price discrimination strategy.

1. Introduction

The increase in the number of personal computers and smartphones, combined with the rapid popularization of the Internet and the increasing perfection of logistics networks, has resulted in a significant shift in consumer behaviour patterns. The penetration rate of online shopping has increased significantly due to the fact that it provides customers with a wealth of information and reduces various restrictions on shopping. Because it effectively reduces the operating costs of sellers, it has greater price competitiveness and is preferred by merchants. To seize the market and gain more benefits, enterprises begin to replan distribution channels, and the dual-channel supply chain system with the coexistence of online and offline sales channels has become a hot spot in theoretical research and enterprise practice.

Supply chain management aims to meet the needs of consumers while maximizing overall benefits by coordinating enterprises' internal and external resources. As an independent economic and decision-making entity, each enterprise comprising the supply chain pursues profit as its primary objective, and supply chain management must solve the issue of how to effectively coordinate the relationships between enterprises. Sales is the only way for enterprises to obtain profits, and price, as the most sensitive factor affecting sales, is crucial to the impact of corporate profits. Due to the fact that the same product is sold through both channels of the dual-channel supply chain, there is intense competition between bricks and mortar and online retailers. To compete for market share and increase profits, sales efforts, such as increasing publicity and enhancing service quality, have become essential. Zhang et al. [1] confirmed the important impact of sales efforts on the profits for supply chain enterprises. However, sales effort has significant positive externalities, and consumers are likely to free ride, enjoying the sales effort of one channel while consuming through another. For example, consumers accept the services provided by offline channels to learn about product information and then choose online channels with lower price for purchase (this phenomenon is especially common in the sale processes of clothing and small household appliances). The free-riding behaviour of consumers and retailers has a significant impact on the willingness to pay for sales efforts, which will eventually lead to the loss of the overall efficiency of supply chain. Therefore, it is of great theoretical and practical value to investigate how to use price to motivate retailers of different channels to make sales efforts and to coordinate the dual-channel supply chain.

2. Literature Review

As a classic game theory, Nash equilibrium was widely used to analyse the behaviour of supply chain members. Lu et al. [2] used a progressive hedging algorithm to find Nash equilibrium of a two-stage supply-chain model constituted by multimanufacturers and multisuppliers. Liu et al. [3] analysed dual-channel supply chain decision making in the presence of market volatility and risk aversion and compared the differences in decision making in centralized, manufacturers or retailers as dominant player and Nash equilibrium. The study by Pal et al. [4] researched the dual-channel supply chain that the sale price set by every player and vertical Nash and manufacturer Stackelberg models were discussed in the decentralized structure. Gou et al. [5] focused on the cooperative advertising program of supply chain that manufacturers help retailers bear the corresponding advertising costs and derived Nash equilibrium advertising investments of manufacturers and retailers. Clempner and Poznyak [6] used Nash equilibrium to maintain the decentralization of departments and used strong Nash equilibrium to achieve the optimal effect of centralized decision-making. Considering two competing supply chains, Mahmoodi [7] transformed internal and external competition of supply chain into a bilevel programming problem by game theory and solved Stackelberg-Nash equilibrium of this problem at the same time. Liu et al. [8] developed a numerical scheme for computation of Nash equilibrium and extended the method to set distributional robust Stackelberg model to study hierarchical competition of supply chain. Rofin and Mahanty [9] studied how the speed of price adjustment affects the Bertrand Nash equilibrium under the dual-channel supply chain environment and how retailers use the speed of price adjustment to maximize their profits. The research by Qian et al. [10] showed that the prisoner's dilemma in technology investment can be avoided effectively and the interest of the social welfare system composed of consumers, manufacturers, and technology suppliers can be maximized by asymmetric Nash equilibrium.

On the other hand, as the main means of competition and cooperation are among supply chain members, price had been widely studied by many scholars. Liu et al. [11] derived the method to reflect the impact of consumer overconfidence on market demand, pricing strategies, and profits of supply chain members based on the analysis of centralized and decentralized pricing strategies in dual-channel supply chain. Liu and Zhang [12] analysed the profits models and its pricing rules of dual-channel green supply chain in the big data environment and verified only when retailers are willing to bear some cost of green production technology, the optimal wholesale price is affected by the cost-sharing parameter. Sun et al. [13] studied optimal decisions in a dual-channel supply chain composed of single supplier and two different duopolistic retailers for three competitive behaviour patterns (Cournot, Collusion, and Stackelberg) and found that supplier can achieve more profit by raising maximum retail price or holding down self-price sensitivity factor. The study by Yang et al. [14] analysed how the innovation input of retailers affects the supply chain operation using Stackelberg game model based on consumer utility theory and drew the conclusion that the optimal efficiency of supply chain is affected by retail prices, wholesale prices, and innovation input levels. The research by Zhang et al. [15] showed that the dual-channel supply chain under decentralized decision-making can get the same optimal benefits as centralized decision-making using price discount and compensation strategy. Hu et al. [16] found that facing to the uncertain market demand, manufacturers and retailers in the dual-channel supply chain are able to achieve higher profits by cooperating than competing, so they have a stronger willingness to make sales efforts. On the basis of analysing the pricing strategy of dual-channel supply chain in different decision models, Sha and Zheng [17] applied modified Shapley value with cost to realize the reasonable distribution of individual benefits under the premise of maximizing the overall revenue of supply chain. Yan et al. [18] incorporated reference price to the design of product line to reflect the influence of reference price on the company and the consumer and it is concluded that the optimal range of quality in product line will decrease with the increasing importance of price comparison. Li et al. [19] developed a duopoly game considering the free-riding cost based on the Hotelling model to discuss the effectiveness of the reference price mechanism and the results showed that the brick-and-mortar/online retailers can benefit from implementing the reference price mechanism separately. Yan et al. [20] demonstrated that manufacturers are more likely to choose a uniform wholesale price strategy rather than price discrimination due to the role of innovation incentives and the optimal pricing level depends on fluctuations in market demand. Wang et al. [21] looked at which is the better choice manufacturers or channels rebate and formulated different rebate strategies using game theory on the premise of considering rebate sensitivity. Yang and Wang [22] took a supply chain consisting of a single manufacturer and duopoly retailers as the target studied information access and three-level price discrimination and analysed how the price strategies of supply chain members are affected by the proportion of high price sensitive consumers and the different price sensitivities of consumers. The research by Zhou et al. [23] investigated the manufacturer's behaviour-based price discrimination strategy with the retailer's information disclosure service in dual-channel supply chain. Chen et al. [24] indicated that the robustness of the business model will enhance with the development of the trade-in program. Cai et al. [25] showed that when cost saving and demand growth are large enough or acceptance of online channel of consumer reaches a certain level, the uniform pricing strategy will be superior to the price discrimination strategy. Matsui [26] investigated the supply chain system consisting of traditional retailers and manufacturers integrating online channels and obtained the optimal wholesale and retail prices using non-cooperative game theory.

A comprehensive analysis of the existing literature reveals that research on the price coordination mechanism of dual-channel supply chains primarily lacks in the following three areas: First, the selection of research objects typically focuses on vertical and partially centralized dual-channel supply chains (i.e., manufacturer-owned network channels) in four typical dual-channel supply chains [27–29], whereas there are few studies on decentralized and horizontally centralized dual-channel supply chains (i.e., manufacturers and network channels are independent of one another). Second, few studies have examined the effect of retailers' sales efforts on the dual-channel supply chain, and even fewer have examined the interaction between channels of positive externalities of sales efforts. Third, most studies focus on the formulation of optimal strategies and the acquisition of optimal values, which minimizes the harshness of the conditions required to achieve the optimal and ignores the possibility of Pareto improvement even if adequate coordination of the dual-channel supply chain is not possible. In light of the aforementioned flaws, this paper selects the distributed dual-channel supply chain in which manufacturers, traditional retailers, and online retailers are independent of one another as the research object, fully considering the sales efforts of each channel and the impact of positive externalities on the supply chain, and employs Nash equilibrium theory to analyse the conditions for manufacturers to ensure the Pareto improvement of dual-channel supply chain through positive externalities.

3. Materials and Methods

3.1. Assumptions and Parameter Settings. This paper constructs a distributed dual-channel supply chain consisting of a single manufacturer, traditional retailers, and online retailers and considers only the sales of a single commodity. Manufacturers adopt the wholesale price discrimination strategy, selling products to various retail channels at varying wholesale prices. Traditional retailers only sell through physical stores, while online retailers only sell through direct online channels. There is no subordination between members of the supply chain, and they make decisions independently and risk-neutral. In order to preserve generality, the manufacturers determine the wholesale price first, followed by the two retailers, who each determine the retail price of the channel.

The key mathematical notations used in this paper are listed in Table 1.

Considering that consumers of network channels are often more sensitive to price in reality, it is assumed that $b_2 > b_1 > \theta_2 > \theta_1, \lambda_2 > \lambda_1 > \mu_2 > \mu_1$.

Based on reference [30], fully considering the impact of sales efforts on the demand of this channel and the impact of its positive externalities on the demand of other channels, the demand function of each channel of the dual-channel supply chain is set as follows:

$$d_1 = (1-\phi)a - b_1 p_1 + \theta_1 p_2 + \lambda_1 c_1 + \mu_1 c_2, \tag{1}$$

$$d_2 = \phi a - b_2 p_2 + \theta_2 p_1 + \lambda_2 c_2 + \mu_2 c_1. \tag{2}$$

3.2. Model Establishment and Analysis. According to the above hypothesis, the respective profits of traditional retailers and online retailers in the dual-channel supply chain are obtained as follows:

$$\pi_1 = (p_1 - \omega_1)\left((1-\phi)a - b_1 p_1 + \theta_1 p_2 + \lambda_1 c_1 + \mu_1 c_2\right) - c_1, \tag{3}$$

$$\pi_2 = (p_2 - \omega_2)\left(\phi a - b_2 p_2 + \theta_2 p_1 + \lambda_2 c_2 + \mu_2 c_1\right) - c_2. \tag{4}$$

Because of the Cournot game between traditional retailers and online retailers, judge $(\partial^2 \pi_1 / \partial p_1^2) = -2b_1 < 0$ and $(\partial^2 \pi_2 / \partial p_2^2) = -2b_2 < 0$. It is known that p_1 and p_2 exist to maximize π_1 and π_2.

Simultaneous and solve equations:

$$\begin{cases} \dfrac{\partial \pi_1}{\partial p_1} = (1-\phi)a - 2b_1 p_1 + \theta_1 p_2 + \lambda_1 c_1 + \mu_1 c_2 + b_1 \omega_1 = 0, \\ \\ \dfrac{\partial \pi_2}{\partial p_2} = \phi a - 2b_2 p_2 + \theta_2 p_1 + \lambda_2 c_2 + \mu_2 c_1 + b_2 \omega_2 = 0. \end{cases} \tag{5}$$

It can be known that

$$p_1^* = \frac{2b_2\left((1-\phi)a + \lambda_1 c_1 + \mu_1 c_2\right) + \theta_1\left(\phi a + \lambda_2 c_2 + \mu_2 c_1\right) + \theta_1 b_2 \omega_2 + 2b_1 b_2 \omega_1}{4b_1 b_2 - \theta_1 \theta_2}, \tag{6}$$

$$p_2^* = \frac{2b_1\left(\phi a + \lambda_2 c_2 + \mu_2 c_1\right) + \theta_2\left((1-\phi)a + \lambda_1 c_1 + \mu_1 c_2\right) + \theta_2 b_1 \omega_1 + 2b_1 b_2 \omega_2}{4b_1 b_2 - \theta_1 \theta_2}. \tag{7}$$

Table 1: Notations used in this paper.

Symbol	Description	Additional information
d	Channel demand	$d_1, d_2 \geq 0$
Subscript "1," "2," "3"	Respectively represent the parameters of traditional retail channel, network direct marketing channel, and manufacturer	
a	Total demand of the maximum potential market	$a \geq 0$
φ	Market share of the network direct sales channel	$0 < \varphi < 1$
p	Channel sales price	
b	Price elasticity coefficient	$b \geq 0$
θ	Cross-price elasticity coefficient	$\theta \geq 0$
c	Cost of sales effort	$c \geq 0$
λ	Sales effort demand coefficient (the cost of c sales effort will cause λc demand increase for this channel)	$\lambda \geq 0$
μ	Positive external spillover coefficient of sales efforts (demand increase caused by sales effort of other channels is μc)	$\mu \geq 0$
π	Enterprise profit	
ω	Manufacturer's wholesale price	$\omega \geq 0$

Substituting equations (6) and (7) into equations (1) and (2), we obtain

$$d_1 = \left(\frac{2b_2\left((1-\phi)a + \lambda_1 c_1 + \mu_1 c_2\right) + \theta_1\left(\phi a + \lambda_2 c_2 + \mu_2 c_1\right) + \theta_1 b_2 \omega_2 - \left(2b_1 b_2 - \theta_1\theta_2\right)\omega_1}{4b_1 b_2 - \theta_1\theta_2}\right) b_1, \tag{8}$$

$$d_2 = \left(\frac{2b_1\left(\phi a + \lambda_2 c_2 + \mu_2 c_1\right) + \theta_2\left((1-\phi)a + \lambda_1 c_1 + \mu_1 c_2\right) + \theta_2 b_1 \omega_1 - \left(2b_1 b_2 - \theta_1\theta_2\right)\omega_2}{4b_1 b_2 - \theta_1\theta_2}\right) b_2. \tag{9}$$

To ensure that the market demand of each channel is greater than zero, it is required that $d_1 > 0$ and $d_2 > 0$, and the inequality set is solved as follows:

$$\begin{cases} \omega_1 < \dfrac{b_2\left((1-\phi)a + \lambda_1 c_1 + \mu_1 c_2\right) + \theta_1\left(\phi a + \lambda_2 c_2 + \mu_2 c_1\right)}{b_1 b_2 - \theta_1\theta_2} = \widehat{\omega}_1, \\[2ex] \omega_2 < \dfrac{b_1\left(\phi a + \lambda_2 c_2 + \mu_2 c_1\right) + \theta_2\left((1-\phi)a + \lambda_1 c_1 + \mu_1 c_2\right)}{b_1 b_2 - \theta_1\theta_2} = \widehat{\omega}_2. \end{cases} \tag{10}$$

Substituting equations (6) and (7) into equations (3) and (4), we obtain

$$\pi_1 = \left(\frac{2b_2\left((1-\phi)a + \lambda_1 c_1 + \mu_1 c_2\right) + \theta_1\left(\phi a + \lambda_2 c_2 + \mu_2 c_1\right) + \theta_1 b_2 \omega_2 - \left(2b_1 b_2 - \theta_1\theta_2\right)\omega_1}{4b_1 b_2 - \theta_1\theta_2}\right)^2 b_1 - c_1, \tag{11}$$

$$\pi_2 = \left(\frac{2b_1\left(\phi a + \lambda_2 c_2 + \mu_2 c_1\right) + \theta_2\left((1-\phi)a + \lambda_1 c_1 + \mu_1 c_2\right) + \theta_2 b_1 \omega_1 - \left(2b_1 b_2 - \theta_1\theta_2\right)\omega_2}{4b_1 b_2 - \theta_1\theta_2}\right)^2 b_2 - c_2. \tag{12}$$

4. Results and Discussion

4.1. Nash Equilibrium Analysis of Traditional Retailers and Online Retailers. As shown in Table 2, the income matrix for each retailer in the dual-channel supply chain with and without sales investment was constructed.

When traditional retailers invest sales efforts, $c_1 > 0$, and vice versa, $c_1 = 0$, and the same is true for online retailers. The superscript "I" is used to represent the investment strategy, and the superscript "N" is used to represent the noninvestment strategy. For instance, the superscript (I, N) represents the strategy combination parameters when traditional retailers invest in sales efforts, but online retailers do not. Using equations (11) and (12), we can obtain

$$\begin{aligned}
A &= \left(\frac{2b_2((1-\phi)a + \lambda_1 c_1 + \mu_1 c_2) + \theta_1(\phi a + \lambda_2 c_2 + \mu_2 c_1) + \theta_1 b_2 \omega_2 - (2b_1 b_2 - \theta_1\theta_2)\omega_1}{4b_1 b_2 - \theta_1\theta_2}\right)^2 b_1 - c_1,\\
B &= \left(\frac{2b_1(\phi a + \lambda_2 c_2 + \mu_2 c_1) + \theta_2((1-\phi)a + \lambda_1 c_1 + \mu_1 c_2) + \theta_2 b_1 \omega_1 - (2b_1 b_2 - \theta_1\theta_2)\omega_2}{4b_1 b_2 - \theta_1\theta_2}\right)^2 b_2 - c_2,\\
C &= \left(\frac{2b_2((1-\phi)a + \lambda_1 c_1) + \theta_1(\phi a + \mu_2 c_1) + \theta_1 b_2 \omega_2 - (2b_1 b_2 - \theta_1\theta_2)\omega_1}{4b_1 b_2 - \theta_1\theta_2}\right)^2 b_1 - c_1,\\
D &= \left(\frac{2b_1(\phi a + \mu_2 c_1) + \theta_2((1-\phi)a + \lambda_1 c_1) + \theta_2 b_1 \omega_1 - (2b_1 b_2 - \theta_1\theta_2)\omega_2}{4b_1 b_2 - \theta_1\theta_2}\right)^2 b_2,\\
E &= \left(\frac{2b_2((1-\phi)a + \mu_1 c_2) + \theta_1(\phi a + \lambda_2 c_2) + \theta_1 b_2 \omega_2 - (2b_1 b_2 - \theta_1\theta_2)\omega_1}{4b_1 b_2 - \theta_1\theta_2}\right)^2 b_1,\\
F &= \left(\frac{2b_1(\phi a + \lambda_2 c_2) + \theta_2((1-\phi)a + \mu_1 c_2) + \theta_2 b_1 \omega_1 - (2b_1 b_2 - \theta_1\theta_2)\omega_2}{4b_1 b_2 - \theta_1\theta_2}\right)^2 b_2 - c_2,\\
G &= \left(\frac{2b_2(1-\phi)a + \theta_1 \phi a + \theta_1 b_2 \omega_2 - (2b_1 b_2 - \theta_1\theta_2)\omega_1}{4b_1 b_2 - \theta_1\theta_2}\right)^2 b_1,\\
H &= \left(\frac{2b_1 \phi a + \theta_2(1-\phi)a + \theta_2 b_1 \omega_1 - (2b_1 b_2 - \theta_1\theta_2)\omega_2}{4b_1 b_2 - \theta_1\theta_2}\right)^2 b_2.
\end{aligned} \tag{13}$$

When $[A > E] \cap [B > D]$, the (investment, investment) strategy combination is a Nash equilibrium solution, that is, all channel retailers expand sales efforts. Combining and solving this set of inequalities, we obtain

$$\begin{aligned}
\omega_1 &< \frac{b_2 - b_2\phi + \theta_1\phi}{b_1 b_2 - \theta_1\theta_2} a + \frac{2b_1 b_2^2 \lambda_1 + 3\theta_1\mu_2 b_1 b_2 - \left(\theta_1^2\theta_2\mu_2/2\right)}{(4b_1 b_2 - \theta_1\theta_2)(b_1 b_2 - \theta_1\theta_2)} c_1 + \frac{4b_1 b_2^2 \mu_1 + 3\theta_1\lambda_2 b_1 b_2 - (3\theta_1\theta_2\mu_1 b_2/2) - \theta_1^2\theta_2\lambda_2}{(4b_1 b_2 - \theta_1\theta_2)(b_1 b_2 - \theta_1\theta_2)} c_2\\
&\quad - \frac{(4b_1 b_2 - \theta_1\theta_2)(2b_1 b_2 - \theta_1\theta_2)}{2b_1(2b_2\lambda_1 + \theta_1\mu_2)(b_1 b_2 - \theta_1\theta_2)} - \frac{(4b_1 b_2 - \theta_1\theta_2)\theta_1}{2(2b_1\lambda_2 + \theta_2\mu_1)(b_1 b_2 - \theta_1\theta_2)} = \omega_1^{(I,I)},\\
\omega_2 &< \frac{\theta_2 - \theta_2\phi + b_1\phi}{b_1 b_2 - \theta_1\theta_2} a + \frac{4b_1^2 b_2 \mu_2 + 3\theta_2\lambda_1 b_1 b_2 - (3\theta_1\theta_2\mu_2 b_1/2) - \theta_1\theta_2^2\lambda_1}{(4b_1 b_2 - \theta_1\theta_2)(b_1 b_2 - \theta_1\theta_2)} c_1 + \frac{2b_1^2 b_2 \lambda_2 + 3\theta_2\mu_1 b_1 b_2 - \left(\theta_1\theta_2^2\mu_1/2\right)}{(4b_1 b_2 - \theta_1\theta_2)(b_1 b_2 - \theta_1\theta_2)} c_2\\
&\quad - \frac{(4b_1 b_2 - \theta_1\theta_2)\theta_2}{2(2b_2\lambda_1 + \theta_1\mu_2)(b_1 b_2 - \theta_1\theta_2)} - \frac{(4b_1 b_2 - \theta_1\theta_2)(2b_1 b_2 - \theta_1\theta_2)}{2b_2(2b_1\lambda_2 + \theta_2\mu_1)(b_1 b_2 - \theta_1\theta_2)} = \omega_2^{(I,I)}.
\end{aligned} \tag{14}$$

Table 2: Sales effort strategy benefit matrix.

Traditional retailers	Online retailers	
	Investment	Noninvestment
Investment	A, B	C, D
Noninvestment	E, F	G, H

When $[G > C] \cap [H > F]$, the (noninvestment, noninvestment) strategy combination is the Nash equilibrium solution, that is, neither retailer in the two channels expends sales effort. The result of combining and solving this set of inequalities is

$$
\begin{aligned}
\omega_1 &> \frac{b_2 - b_2\phi + \theta_1\phi}{b_1b_2 - \theta_1\theta_2}a + \frac{2b_1b_2^2\lambda_1 + \theta_1\mu_2b_1b_2 - \left(\theta_1^2\theta_2\mu_2/2\right) - \theta_1\theta_2b_2\lambda_1}{\left(4b_1b_2 - \theta_1\theta_2\right)\left(b_1b_2 - \theta_1\theta_2\right)}c_1 + \frac{\theta_1\lambda_2b_1b_2 + \left(\theta_1\theta_2\mu_1b_2/2\right)}{\left(4b_1b_2 - \theta_1\theta_2\right)\left(b_1b_2 - \theta_1\theta_2\right)}c_2 \\
&\quad - \frac{\left(4b_1b_2 - \theta_1\theta_2\right)\left(2b_1b_2 - \theta_1\theta_2\right)}{2b_1\left(2b_2\lambda_1 + \theta_1\mu_2\right)\left(b_1b_2 - \theta_1\theta_2\right)} - \frac{\left(4b_1b_2 - \theta_1\theta_2\right)\theta_1}{2\left(2b_1\lambda_2 + \theta_2\mu_1\right)\left(b_1b_2 - \theta_1\theta_2\right)} = \omega_1^{(N,N)}, \\
\omega_2 &> \frac{\theta_2 - \theta_2\phi + b_1\phi}{b_1b_2 - \theta_1\theta_2}a + \frac{\theta_2\lambda_1b_1b_2 + \left(\theta_1\theta_2\mu_2b_1/2\right)}{\left(4b_1b_2 - \theta_1\theta_2\right)\left(b_1b_2 - \theta_1\theta_2\right)}c_1 + \frac{2b_1^2b_2\lambda_2 + \theta_2\mu_1b_1b_2 - \left(\theta_1\theta_2^2\mu_1/2\right) - \theta_1\theta_2b_1\lambda_2}{\left(4b_1b_2 - \theta_1\theta_2\right)\left(b_1b_2 - \theta_1\theta_2\right)}c_2 \\
&\quad - \frac{\left(4b_1b_2 - \theta_1\theta_2\right)\theta_2}{2\left(2b_2\lambda_1 + \theta_1\mu_2\right)\left(b_1b_2 - \theta_1\theta_2\right)} - \frac{\left(4b_1b_2 - \theta_1\theta_2\right)\left(2b_1b_2 - \theta_1\theta_2\right)}{2b_2\left(2b_1\lambda_2 + \theta_2\mu_1\right)\left(b_1b_2 - \theta_1\theta_2\right)} = \omega_2^{(N,N)}.
\end{aligned}
\tag{15}
$$

When $[C > G] \cap [D > B]$, the (investment, noninvestment) strategy combination is a Nash equilibrium solution, i.e., traditional retailers expand sales efforts while online retailers do not. The result of combining and solving this set of inequalities is

$$
\begin{aligned}
\omega_1 &< \frac{b_2 - b_2\phi + \theta_1\phi}{b_1b_2 - \theta_1\theta_2}a + \frac{2b_1b_2^2\lambda_1 + 3\theta_1\mu_2b_1b_2 - \left(\theta_1^2\theta_2\mu_2/2\right)}{\left(4b_1b_2 - \theta_1\theta_2\right)\left(b_1b_2 - \theta_1\theta_2\right)}c_1 + \frac{\theta_1\lambda_2b_1b_2 + \left(\theta_1\theta_2\mu_1b_2/2\right)}{\left(4b_1b_2 - \theta_1\theta_2\right)\left(b_1b_2 - \theta_1\theta_2\right)}c_2 \\
&\quad - \frac{\left(4b_1b_2 - \theta_1\theta_2\right)\left(2b_1b_2 - \theta_1\theta_2\right)}{2b_1\left(2b_2\lambda_1 + \theta_1\mu_2\right)\left(b_1b_2 - \theta_1\theta_2\right)} - \frac{\left(4b_1b_2 - \theta_1\theta_2\right)\theta_1}{2\left(2b_1\lambda_2 + \theta_2\mu_1\right)\left(b_1b_2 - \theta_1\theta_2\right)} = \omega_1^{(I,N)}, \\
\omega_2 &> \frac{\theta_2 - \theta_2\phi + b_1\phi}{b_1b_2 - \theta_1\theta_2}a + \frac{4b_1^2b_2\mu_2 + 3\theta_2\lambda_1b_1b_2 - \left(3\theta_1\theta_2\mu_2b_1/2\right) - \theta_1\theta_2^2\lambda_1}{\left(4b_1b_2 - \theta_1\theta_2\right)\left(b_1b_2 - \theta_1\theta_2\right)}c_1 + \frac{2b_1^2b_2\lambda_2 + \theta_2\mu_1b_1b_2 - \left(\theta_1\theta_2^2\mu_1/2\right) - \theta_1\theta_2b_1\lambda_2}{\left(4b_1b_2 - \theta_1\theta_2\right)\left(b_1b_2 - \theta_1\theta_2\right)}c_2 \\
&\quad - \frac{\left(4b_1b_2 - \theta_1\theta_2\right)\theta_2}{2\left(2b_2\lambda_1 + \theta_1\mu_2\right)\left(b_1b_2 - \theta_1\theta_2\right)} - \frac{\left(4b_1b_2 - \theta_1\theta_2\right)\left(2b_1b_2 - \theta_1\theta_2\right)}{2b_2\left(2b_1\lambda_2 + \theta_2\mu_1\right)\left(b_1b_2 - \theta_1\theta_2\right)} = \omega_2^{(I,N)}.
\end{aligned}
\tag{16}
$$

When $[E > A] \cap [F > H]$, the (noninvestment, investment) strategy combination is Nash equilibrium solution, i.e., traditional retailers do not invest in sales efforts, while online retailers invest in sales efforts. The result of combining and solving this set of inequalities is

$$\begin{aligned}
\omega_1 &> \frac{b_2 - b_2\phi + \theta_1\phi}{b_1b_2 - \theta_1\theta_2}a + \frac{2b_1b_2^2\lambda_1 + \theta_1\mu_2b_1b_2 - \left(\theta_1^2\theta_2\mu_2/2\right) - \theta_1\theta_2\lambda_1b_2}{(4b_1b_2 - \theta_1\theta_2)(b_1b_2 - \theta_1\theta_2)}c_1 + \frac{4b_1b_2^2\mu_1 + 3\theta_1\lambda_2b_1b_2 - (3\theta_1\theta_2\mu_1b_2/2) - \theta_1^2\theta_2\lambda_2}{(4b_1b_2 - \theta_1\theta_2)(b_1b_2 - \theta_1\theta_2)}c_2 \\
&\quad - \frac{(4b_1b_2 - \theta_1\theta_2)(2b_1b_2 - \theta_1\theta_2)}{2b_1(2b_2\lambda_1 + \theta_1\mu_2)(b_1b_2 - \theta_1\theta_2)} - \frac{(4b_1b_2 - \theta_1\theta_2)\theta_1}{2(2b_1\lambda_2 + \theta_2\mu_1)(b_1b_2 - \theta_1\theta_2)} = \omega_1^{(N,I)}, \\
\omega_2 &< \frac{\theta_2 - \theta_2\phi + b_1\phi}{b_1b_2 - \theta_1\theta_2}a + \frac{\theta_2\lambda_1b_1b_2 + (\theta_1\theta_2\mu_2b_1/2)}{(4b_1b_2 - \theta_1\theta_2)(b_1b_2 - \theta_1\theta_2)}c_1 + \frac{2b_1^2b_2\lambda_2 + 3\theta_2\mu_1b_1b_2 - \left(\theta_1\theta_2^2\mu_1/2\right)}{(4b_1b_2 - \theta_1\theta_2)(b_1b_2 - \theta_1\theta_2)}c_2 \\
&\quad - \frac{(4b_1b_2 - \theta_1\theta_2)\theta_2}{2(2b_2\lambda_1 + \theta_1\mu_2)(b_1b_2 - \theta_1\theta_2)} - \frac{(4b_1b_2 - \theta_1\theta_2)(2b_1b_2 - \theta_1\theta_2)}{2b_2(2b_1\lambda_2 + \theta_2\mu_1)(b_1b_2 - \theta_1\theta_2)} = \omega_2^{(N,I)}.
\end{aligned} \tag{17}$$

Because $[A > E] \cap [B > D]$, $[G > C] \cap [H > F]$, $[C > G] \cap [D > B]$, and $[E > A] \cap [F > H]$ are all strategy combinations, (investment, investment), (noninvestment, noninvestment), (investment, noninvestment), and (noninvestment, investment) are sufficient conditions for the Nash equilibrium solution. For this purpose, consider the following situation:

$$\begin{aligned}
\omega_1^{(I,I)} - \omega_1^{(N,N)} &= \frac{2\theta_1\mu_2b_1b_2 + \theta_1\theta_2b_2\lambda_1}{(4b_1b_2 - \theta_1\theta_2)(b_1b_2 - \theta_1\theta_2)}c_1 + \frac{(2b_2\mu_1 + \theta_1\lambda_2)(2b_1b_2 - \theta_1\theta_2)}{(4b_1b_2 - \theta_1\theta_2)(b_1b_2 - \theta_1\theta_2)}c_2, \\
\omega_2^{(I,I)} - \omega_2^{(N,N)} &= \frac{(2b_1\mu_2 + \theta_2\lambda_1)(2b_1b_2 - \theta_1\theta_2)}{(4b_1b_2 - \theta_1\theta_2)(b_1b_2 - \theta_1\theta_2)}c_1 + \frac{2\theta_2\mu_1b_1b_2 + \theta_1\theta_2b_1\lambda_2}{(4b_1b_2 - \theta_1\theta_2)(b_1b_2 - \theta_1\theta_2)}c_2.
\end{aligned} \tag{18}$$

Because $b_2 > b_1 > \theta_2 > \theta_1$, $c_1 \geq 0$, and $c_2 \geq 0$, then $\omega_1^{(I,I)} \geq \omega_1^{(N,N)}$ and $\omega_2^{(I,I)} \geq \omega_2^{(N,N)}$.

For this reason, we further consider the following situation:

$$\begin{aligned}
\widehat{\omega}_1 - \omega_1^{(I,I)} &= \frac{b_2\lambda_1(2b_1b_2 - \theta_1\theta_2) + \theta_1\mu_2(b_1b_2 - (\theta_1\theta_2/2))}{(4b_1b_2 - \theta_1\theta_2)(b_1b_2 - \theta_1\theta_2)}c_1 + \frac{\theta_1\lambda_2b_1b_2 + (\theta_1\theta_2\mu_1b_2/2)}{(4b_1b_2 - \theta_1\theta_2)(b_1b_2 - \theta_1\theta_2)}c_2 \\
&\quad + \frac{(4b_1b_2 - \theta_1\theta_2)(2b_1b_2 - \theta_1\theta_2)}{2b_1(2b_2\lambda_1 + \theta_1\mu_2)(b_1b_2 - \theta_1\theta_2)} + \frac{(4b_1b_2 - \theta_1\theta_2)\theta_1}{2(2b_1\lambda_2 + \theta_2\mu_1)(b_1b_2 - \theta_1\theta_2)} > 0, \\
\widehat{\omega}_2 - \omega_2^{(I,I)} &= \frac{\theta_2\lambda_1b_1b_2 + (\theta_1\theta_2\mu_2b_1/2)}{(4b_1b_2 - \theta_1\theta_2)(b_1b_2 - \theta_1\theta_2)}c_1 + \frac{b_1\lambda_2(2b_1b_2 - \theta_1\theta_2) + \theta_2\mu_1(b_1b_2 - (\theta_1\theta_2/2))}{(4b_1b_2 - \theta_1\theta_2)(b_1b_2 - \theta_1\theta_2)}c_2 \\
&\quad + \frac{(4b_1b_2 - \theta_1\theta_2)\theta_2}{2(2b_2\lambda_1 + \theta_1\mu_2)(b_1b_2 - \theta_1\theta_2)} + \frac{(4b_1b_2 - \theta_1\theta_2)(2b_1b_2 - \theta_1\theta_2)}{2b_2(2b_1\lambda_2 + \theta_2\mu_1)(b_1b_2 - \theta_1\theta_2)} > 0.
\end{aligned} \tag{19}$$

So, we can come to a conclusion: $\widehat{\omega}_1 > \omega_1^{(I,I)}$ and $\widehat{\omega}_2 > \omega_2^{(I,I)}$.

So, similarly, $[\widehat{\omega}_1 > \omega_1^{(I,I)} \geq \omega_1^{(I,N)} \geq \omega_1^{(N,N)}] \cap [\widehat{\omega}_1 > \omega_1^{(I,I)} \geq \omega_1^{(N,I)} \geq \omega_1^{(N,N)}]$ and $[\widehat{\omega}_2 > \omega_2^{(I,I)} \geq \omega_2^{(I,N)} \geq \omega_2^{(N,N)}] \cap [\widehat{\omega}_2 > \omega_2^{(I,I)} \geq \omega_2^{(N,I)} \geq \omega_2^{(N,N)}]$. However, the size relationship between $\omega_1^{(I,N)}$ and $\omega_1^{(N,I)}$, $\omega_2^{(I,N)}$ and $\omega_2^{(N,I)}$ cannot be determined. Therefore, we have the following conclusions:

(1) The wholesale price range that prompts the emergence of the (investment, investment) strategy combination is

$$\begin{aligned}&\left[\omega_1 \leq \mathrm{MAX}\left(\omega_1^{(I,N)}, \omega_1^{(N,I)}\right)\right] \cap \left[\omega_2 \leq \omega_2^{(N,N)}\right],\\&\left[\omega_2 \leq \mathrm{MAX}\left(\omega_2^{(I,N)}, \omega_2^{(N,I)}\right)\right] \cap \left[\omega_1 \leq \omega_1^{(N,N)}\right].\end{aligned} \tag{20}$$

(2) The wholesale price range that prompts the emergence of the (investment, noninvestment) strategy combination is

$$\begin{aligned}&\left[\omega_1 \leq \mathrm{MIN}\left(\omega_1^{(I,N)}, \omega_1^{(N,I)}\right)\right]\\&\quad \cap \left[\mathrm{MAX}\left(\omega_2^{(I,N)}, \omega_2^{(N,I)}\right) < \omega_2 < \widehat{\omega}_2\right].\end{aligned} \tag{21}$$

(3) The wholesale price range that prompts the emergence of the (noninvestment, investment) strategy combination is

$$\begin{aligned}&\left[\mathrm{MAX}\left(\omega_1^{(I,N)}, \omega_1^{(N,I)}\right) < \omega_1 < \widehat{\omega}_1\right]\\&\quad \cap \left[\omega_2 \leq \mathrm{MIN}\left(\omega_2^{(I,N)}, \omega_2^{(N,I)}\right)\right].\end{aligned} \tag{22}$$

(4) The wholesale price range that prompts the emergence of the (noninvestment, noninvestment) strategy combination is

$$\begin{aligned}&\left[\mathrm{MIN}\left(\omega_1^{(I,N)}, \omega_1^{(N,I)}\right) < \omega_1 < \widehat{\omega}_1\right] \cap \left[\omega_2^{(I,I)} \leq \omega_2 < \widehat{\omega}_2\right],\\&\left[\omega_1^{(I,I)} \leq \omega_1 < \widehat{\omega}_1\right] \cap \left[\mathrm{MIN}\left(\omega_2^{(I,N)}, \omega_2^{(N,I)}\right) < \omega_2 < \widehat{\omega}_2\right].\end{aligned} \tag{23}$$

When $[\omega_1^{(N,N)} < \omega_1 < \mathrm{MIN}(\omega_1^{(I,N)}, \ \omega_1^{(N,I)})] \cap [\omega_2^{(N,N)} < \omega_2 < \mathrm{MAX}(\omega_2^{(I,N)}, \omega_2^{(N,I)})]$ and $[\mathrm{MIN}(\omega_1^{(I,N)}, \omega_1^{(N,I)}) < \omega_1 < \mathrm{MAX}(\omega_1^{(I,N)}, \omega_1^{(N,I)})] \cap [\omega_2^{(N,N)} < \omega_2 < \omega_2^{(I,I)}]$, or $[\mathrm{MAX}(\omega_1^{(I,N)}, \omega_1^{(N,I)}) < \omega_1 < \omega_1^{(I,I)}] \cap [\mathrm{MIN}(\omega_2^{(I,N)}, \omega_2^{(N,I)}) < \omega_2 < \omega_2^{(I,I)}]$, the (investment, investment) and (noninvestment, noninvestment) strategy combinations will both be Nash equilibrium solutions. Each channel retailer will choose the same strategy as the other channel retailer. Figure 1 illustrates the sales effort strategy combinations of conventional retailers and online retailers at varying wholesale prices.

4.2. Determining the Manufacturer's Optimal Wholesale Price. The manufacturer's sales revenue is

$$\pi_3 = \omega_1 d_1 + \omega_2 d_2. \tag{24}$$

Substituting equations (8) and (9) into equation (24) and calculating the Hessian Matrix, we obtain

$$\begin{bmatrix} \dfrac{-2b_1\left(2b_1b_2 - \theta_1\theta_2\right)}{4b_1b_2 - \theta_1\theta_2} & \dfrac{b_1b_2\left(\theta_1 + \theta_2\right)}{4b_1b_2 - \theta_1\theta_2} \\[2ex] \dfrac{b_1b_2\left(\theta_1 + \theta_2\right)}{4b_1b_2 - \theta_1\theta_2} & \dfrac{-2b_2\left(2b_1b_2 - \theta_1\theta_2\right)}{4b_1b_2 - \theta_1\theta_2} \end{bmatrix}. \tag{25}$$

It can be seen from the analysis

$$\frac{-2b_1\left(2b_1b_2 - \theta_1\theta_2\right)}{4b_1b_2 - \theta_1\theta_2} < 0, \frac{-2b_2\left(2b_1b_2 - \theta_1\theta_2\right)}{4b_1b_2 - \theta_1\theta_2} < 0. \tag{26}$$

Assuming $b_2 > b_1 > \theta_2 > \theta_1$, there is

$$\begin{aligned}H &= \frac{b_1b_2\left(4\left(2b_1b_2 - \theta_1\theta_2\right)^2 - b_1b_2\left(\theta_1 + \theta_2\right)^2\right)}{\left(4b_1b_2 - \theta_1\theta_2\right)^2} > \frac{b_1b_2}{\left(4b_1b_2 - \theta_1\theta_2\right)^2}\left(4\left(2b_1b_2 - b_1^2\right)^2 - b_1b_2\left(b_1 + b_1\right)^2\right)\\&= \frac{b_1^{\,3}b_2}{\left(4b_1b_2 - \theta_1\theta_2\right)^2}\left(16b_2 - 4b_1\right)\left(b_2 - b_1\right) > 0.\end{aligned} \tag{27}$$

Therefore, π_3 is a joint strictly concave function with respect to ω_1 and ω_2. Simultaneous $(\partial\pi_3/\partial\omega_1) = 0$ and $(\partial\pi_3/\partial\omega_2) = 0$, we can solve the system of equations:

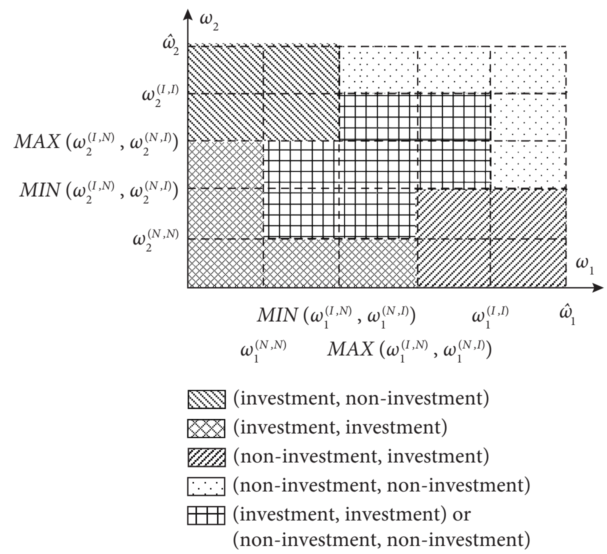

Figure 1: A combination diagram of two-channel retailers' sales effort strategies.

Table 3: Income statement of each party when both traditional retailers and online retailers pay for sales.

ω_1	ω_2	d_1	d_2	p_1	p_2	π_1	π_2	π_3
492	471	6.30	7.66	503.45	483.77	62.13	89.87	6708.21
442	421	10.44	12.05	460.99	441.08	188.26	233.87	9687.18
392	371	14.59	16.43	418.52	398.38	376.84	441.92	11813.41
342	321	18.73	20.81	376.05	355.69	627.85	714.02	13086.90
292	271	22.87	25.20	333.59	313.00	941.31	1050.17	13507.66
242	221	27.02	29.58	291.12	270.30	1317.20	1450.38	13075.67
192	171	31.16	33.96	248.66	227.61	1755.54	1914.63	11790.95
142	121	35.31	38.35	206.19	184.91	2256.32	2442.94	9653.48
92	71	39.45	42.73	163.73	142.22	2819.54	3035.30	6663.27

$$\begin{aligned}
\omega_1^* &= \frac{\begin{array}{c} b_2(\theta_1+\theta_2)\left(2b_1(\phi a+\lambda_2 c_2+\mu_2 c_1)+\theta_2((1-\phi)a+\lambda_1 c_1+\mu_1 c_2)\right) \\ +2(2b_1 b_2-\theta_1\theta_2)\left(\theta_1(\phi a+\lambda_2 c_2+\mu_2 c_1)+2b_2((1-\phi)a+\lambda_1 c_1+\mu_1 c_2)\right)\end{array}}{4(2b_1 b_2-\theta_1\theta_2)^2-b_1 b_2(\theta_1+\theta_2)^2}, \\
\omega_2^* &= \frac{\begin{array}{c} b_1(\theta_1+\theta_2)\left(\theta_1(\phi a+\lambda_2 c_2+\mu_2 c_1)+2b_2((1-\phi)a+\lambda_1 c_1+\mu_1 c_2)\right) \\ +2(2b_1 b_2-\theta_1\theta_2)\left(2b_1(\phi a+\lambda_2 c_2+\mu_2 c_1)+\theta_2((1-\phi)a+\lambda_1 c_1+\mu_1 c_2)\right)\end{array}}{4(2b_1 b_2-\theta_1\theta_2)^2-b_1 b_2(\theta_1+\theta_2)^2}.
\end{aligned} \tag{28}$$

As can be seen from equations (8), (9), and (24), when ω_1 and ω_2 are constant, π_3 is an increasing function of c_1 and c_2, i.e., an increase in the retailer's sales effort level can increase the manufacturer's income.

5. Numerical Analysis

According to the assumptions in Subsection 3.1., let $a = 100$, $\varphi = 0.4$, $b_1 = 0.55$, $b_2 = 0.6$, $\theta_1 = 0.45$, $\theta_2 = 0.5$, $\lambda_1 = 0.35$, $\lambda_2 = 0.4$, $\mu_1 = 0.25$, $\mu_2 = 0.3$, $c_1 = 10$, and $c_2 = 8$. The calculation results are as follows: $\omega_1^{(N,N)} = 518.51$, $\omega_2^{(N,N)} = 499.04$, $\omega_1^{(I,I)} = 544.90$, $\omega_2^{(I,I)} = 527.33$, $\omega_1^{(I,N)} = 530.37$, $\omega_2^{(I,N)} = 518.14$, $\omega_1^{(N,I)} = 533.04$, $\omega_2^{(N,I)} = 508.22$, $\widehat{\omega}_1 = 572.29$, $\widehat{\omega}_2 = 553.90$, $\omega_1^* = 292.12$, and $\omega_2^* = 271.50$.

When $[\omega_1 \leq 533.04 \cap \omega_2 \leq 499.04]$ and $[\omega_2 \leq 518.14 \cap \omega_1 \leq 518.51]$, both channel retailers expend sales efforts. As shown in Table 3, as the wholesale price of the manufacturers decreases, the profits of traditional retailers and online retailers increase, whereas the manufacturer's income demonstrates a trend of first increasing and then decreasing, reaching a maximum value at ω_1^* and ω_2^*. When the dual-channel supply chain manufacturers lower the wholesale price of each channel within the range $[\omega_1^* \leq \omega_1 \leq \mathrm{MAX}(\omega_1^{(I,N)}, \omega_1^{(N,I)})] \cap [\omega_2^* \leq \omega_2 \leq \omega_2^{(N,N)}]$ and $[\omega_2^* \leq \omega_2 \leq \mathrm{MAX}(\omega_2^{(I,N)}, \omega_2^{(N,I)})] \cap [\omega_1^* \leq \omega_1 \leq \omega_1^{(N,N)}]$, the Pareto improvement of the dual-channel supply chain will be achieved. The results indicate that in order to ensure the Pareto improvement of dual-channel supply chain with the sales efforts of retailers, manufacturers should reasonably reduce wholesale prices to a certain range, so as to benefit retailers and achieve increased returns for all members of supply chain.

6. Conclusions

This paper investigates a decentralized dual-channel supply chain system comprised of independent manufacturer, traditional retailer, and online retailer, fully considering the impact of each channel's retailer's sales efforts on this channel's demand as well as its positive externalities on the demand of other channels. A mathematical model of the manufacturer's influence is established, and the profit matrix is constructed for each retailer's decision regarding whether or not to make sales efforts when the manufacturers employ a price discrimination strategy to charge different wholesale prices to different retailers. The Nash equilibrium game theory is used to solve the conditions of each decision combination becoming Nash equilibrium. The results of the study reflect more clearly how wholesale price discrimination by wholesaler will affect the sales efforts of retailers in different channels. Additionally, the optimal pricing strategy for the manufacturers is solved, as well as the conditions for the manufacturers to use wholesale price discrimination to achieve Pareto improvement in the dual-channel supply chain. Although this paper analyses the impact of manufacturers' wholesale prices on retailers' sales efforts, it did not carry out in-depth analysis of the interaction between retailers' sales efforts of different channels and their impact on wholesale prices, which will be the direction of further research.

Data Availability

The data used to support the findings of this study are all derived from computer simulation, so no real data were used in this study.

Conflicts of Interest

All authors declare that they have no conflicts of interest.

Acknowledgments

This paper was supported by the 2022 Provincial and Municipal Joint Fund Project of Guangdong Basic and Applied Basic Research Foundation (2022A1515110437), the 2018 Higher Education of Guangdong Key Platforms and Scientific Research Projects: Young Innovative Talents Program (2018KQNCX251), and the 2017 Scientific Research Project of Guangdong Ocean University (101502/R18014).

References

[1] X. M. Zhang, H. R. Chen, and Z. Liu, "Operation strategy in an E-commerce platform supply chain: whether and how to introduce live streaming services?" *International Transactions in Operational Research*, vol. 8, 2022.

[2] Y. Lu, J. Sun, M. Zhang, and Y. Zhang, "A stochastic variational inequality approach to the nash equilibrium model of a manufacturer-supplier game under uncertainty," *Pacific Journal of Optimization*, vol. 18, no. 3, pp. 635–660, 2022.

[3] G. D. Liu, T. J. Yang, Y. Wei, and X. M. Zhang, "Decisions on dual-channel supply chains under market fluctuations and dual-risk aversion," *Discrete Dynamics in Nature and Society*, vol. 2020, Article ID 2612357, 13 pages, 2020.

[4] B. Pal, L. E. Cardenas-Barron, and K. S. Chaudhuri, "Price, Delivery time, and retail service sensitive dual-channel supply chain," *Scientia Iranica*, vol. 28, no. 3, pp. 1765–1779, 2021.

[5] Q. L. Gou, J. Z. Zhang, J. Zhang, and L. Liang, "Nash equilibria of co-operative advertising programs with advertising threshold effects," *International Journal of Information Technology and Decision Making*, vol. 16, no. 04, pp. 981–1004, 2017.

[6] J. B. Clempner and A. S. Poznyak, "Solving transfer pricing involving collaborative and non-cooperative equilibria in Nash and Stackelberg games: centralized-decentralized decision making," *Computational Economics*, vol. 54, no. 2, pp. 477–505, 2019.

[7] A. Mahmoodi, "Stackelberg-Nash equilibrium of pricing and inventory decisions in duopoly supply chains using a nested evolutionary algorithm," *Applied Soft Computing*, vol. 86, Article ID 105922, 2020.

[8] Y. C. Liu, H. F. Xu, S. J. S. Yang, and J. Zhang, "Distributionally robust equilibrium for continuous games: nash and Stackelberg models," *European Journal of Operational Research*, vol. 265, no. 2, pp. 631–643, 2018.

[9] T. M. Rofin and B. Mahanty, "Impact of price adjustment speed on the stability of Bertrand-Nash equilibrium and profit of the retailers," *Kybernetes*, vol. 47, no. 8, pp. 1494–1523, 2018.

[10] X. D. Qian, W. Liu, and J. Q. Yang, "Game theory analysis of technology adoption timing and pricing decision in supply chain system under asymmetric Nash equilibrium," *Journal of Intelligent and Fuzzy Systems*, vol. 35, no. 3, pp. 3101–3111, 2018.

[11] C. L. Liu, C. K. M. Lee, and L. L. Zhang, "Pricing strategy in a dual-channel supply chain with overconfident consumers," *Computers and Industrial Engineering*, vol. 172, Article ID 108515, 2022.

[12] P. Liu and F. J. Zhang, "Pricing strategies of dual-channel green supply chain considering Big Data information inputs," *Soft Computing*, vol. 26, no. 6, pp. 2981–2999, 2022.

[13] H. X. Sun, Y. Wan, Y. Li, L. L. Zhang, and Z. Zhou, "Competition in a dual-channel supply chain considering duopolistic retailers with different behaviours," *Journal of Industrial and Management Optimization*, vol. 17, no. 2, pp. 601–631, 2021.

[14] W. S. Yang, Y. Y. Si, J. X. Zhang, S. Liu, and A. Appolloni, "Coordination mechanism of dual-channel supply chains considering retailer innovation inputs," *Sustainability*, vol. 13, no. 2, p. 813, 2021.

[15] X. L. Zhang, H. Wang, X. Z. Zhao, and D. D. Wu, "Return decision model of the manufacturer-leading dual-channel supply chain," *Mathematical Problems in Engineering*, vol. 2020, Article ID 8864672, 15 pages, 2020.

[16] H. Hu, Q. Wu, S. Han, and Z. Zhang, "Coordination of dual-channel supply chain with perfect product considering sales effort," *Advances in Production Engineering & Management*, vol. 15, no. 2, pp. 192–203, 2020.
[17] J. Sha and S. S. Zheng, "Revenue distribution of hybrid channel supply chain based on modified Shapley value with cost," *Discrete Dynamics in Nature and Society*, vol. 2021, Article ID 4405185, 8 pages, 2021.
[18] X. M. Yan, W. H. Zhao, and Y. G. Yu, "Optimal product line design with reference price effects," *European Journal of Operational Research*, vol. 302, no. 3, pp. 1045–1062, 2022.
[19] H. Li, Y. Qian, and T. Peng, "How does the reference price mechanism affect competition?" *IEEE Access*, vol. 10, pp. 26637–26653, 2022.
[20] S. Q. Yan, Y. Xiong, Z. B. Lin, and Y. Zhou, "Flexible versus committed and specific versus uniform: wholesale price contracting in a supply chain with downstream process innovation," *International Transactions in Operational Research*, vol. 31, no. 1, pp. 346–369, 2022.
[21] Z. L. Wang, R. Zhang, and B. Liu, "Rebate strategy selection and channel coordination of competing two-echelon supply chains," *Complexity*, vol. 2021, Article ID 8839218, 20 pages, 2021.
[22] H. M. Yang and W. Wang, "Endogenous third-degree price discrimination in a supply chain with one common manufacturer and duopoly retailers," *Discrete Dynamics in Nature and Society*, vol. 2020, Article ID 664271, 15 pages, 2020.
[23] J. H. Zhou, R. J. Zhao, and B. Wang, "Behavior-based price discrimination in a dual-channel supply chain with retailer's information disclosure," *Electronic Commerce Research and Applications*, vol. 39, Article ID 100916, 2020.
[24] H. T. Chen, Z. H. Dong, G. D. Li, and H. T. Zhao, "Joint advertisement and trade-in marketing strategy in closed-loop supply chain," *Sustainability*, vol. 12, no. 6, p. 2188, 2020.
[25] Q. Cai, C. L. Luo, X. Tian, and S. Y. Wang, "Uniform pricing strategy vs. price differentiation strategy in the presence of cost saving and demand increasing," *Journal of Systems Science and Complexity*, vol. 32, no. 3, pp. 932–946, 2019.
[26] K. Matsui, "When and what wholesale and retail prices should be set in multi-channel supply chains?" *European Journal of Operational Research*, vol. 267, no. 2, pp. 540–554, 2018.
[27] W. S. Yoo and E. Lee, "Internet channel entry: a strategic analysis of mixed channel structures," *Marketing Science*, vol. 30, no. 1, pp. 29–41, 2011.
[28] H. M. Li, S. W. Xiang, S. Y. Xia, and S. G. Huang, "Finding the Nash equilibria of $ n $-person noncooperative games via solving the system of equations," *AIMS Mathematics*, vol. 8, no. 6, pp. 13984–14007, 2023.
[29] C. W. Liu, S. W. Xiang, and Y. L. Yang, "Existence and essential stability of Nash equilibria for biform games with Shapley allocation functions," *AIMS Mathematics*, vol. 7, no. 5, pp. 7706–7719, 2022.
[30] X. H. Yue and J. Liu, "Demand forecast sharing in a dual-channel supply chain," *European Journal of Operational Research*, vol. 174, no. 1, pp. 646–667, 2006.