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Research Article

Quantum Game-Based Study on the Incentive Mechanism for the Cooperative Distribution of E-Commerce Logistics Alliance

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Motivating active participation in e-commerce logistics alliances to enhance delivery efficiency and customer satisfaction has long been a societal interest. Leveraging the quantum game theory, this paper develops a model for incentivizing collaboration within these alliances. This model enables theoretical and numerical analysis of members' strategies and entanglement levels. The findings show that quantum strategies increase members' profits, achieving Nash equilibriums and Pareto optimal outcomes, outperforming the classical game theory. In addition, the size of quantum entanglement emerges as a critical determinant influencing members' active participation in collaborative distribution. Strengthening information sharing and aligning interests can enhance entanglement levels among members, making them more inclined to adopt strategies promoting active involvement in collaborative distribution. Moreover, members can adapt their strategies based on the initial entanglement in collaborative distribution, thereby incentivizing participation and reducing ethical risks. In conclusion, through numerical analysis, we present relevant strategies and recommendations for incentivizing collaborative distribution within e-commerce logistics alliances.

1. Introduction

The logistics industry faces rising demand driven by e-commerce growth but noticeable inefficiencies and bottlenecks persist. The fourth-party logistics (4PL) platform functions as a coordination centre derived from the third-party logistics (3PL) platform, integrating resources across the supply chain to offer customers efficient and satisfactory services. However, investing extensively in a single 4PL platform for goods delivery is impractical. To improve business efficiency, establishing a close cooperative relationship between the e-commerce platform and the 4PL platform becomes imperative. This led to the formation of e-commerce logistics alliances, effectively consolidating customer information and logistics resources among members while improving overall operational efficiency [1]. Despite these advantages, the practical adoption of the alliances' operational models remains uncommon due to profit-focused members, resulting in issues such as unfair profit distribution, free-riding, and moral hazards [2]. These challenges, arising from conflicts between short-term individual interests and long-term collective interests, are commonly known as social dilemmas. Consequently, motivating active participation in collaborative distribution within e-commerce logistics alliances emerges as a pressing issue in the logistics and e-commerce sectors.

Scholars globally have advanced the study of social dilemmas and supply chain management significantly. Addressing social dilemmas, Ariful Kabir et al. [3, 4] investigated factors influencing network reciprocity's impact, exploring their role in promoting cooperative behavior. Rajib Arefin et al. [5] identified a dual relationship between the dilemma strength and variations in social efficiency deficits. In supply chain management, Zhang et al. [6] examined asymmetric information's effects on retailer incentive contract design, considering manufacturer process innovation costs. Du et al. [7] developed an evolutionary game model for cross-border e-commerce platforms and logistics enterprises' information coordination. He et al. [8] compared logistics integration strategies in e-commerce

platform service supply chains using game models. Niu et al. [9] constructed a game model for logistics sharing alliances among competitive e-commerce companies. Wang et al. [10] explored government dynamic punishment and incentive mechanisms' impact on trust evolution between platform e-commerce and consumers. Du et al. [11] designed an incentive model for cooperative distribution alliances, considering moral risk.

In collaborative distribution within e-commerce logistics alliances, members oversee and share information to enhance service quality, resource integration, and overall alliance efficiency, akin to entanglement in quantum mechanics that denotes correlation between observable values of different subsystems [12]. The quantum game theory, situated at the intersection of the quantum information theory and game theory, emerged as a distinct field with Meyer [13] introducing the concept in 1999. Eisert et al. [14] quantized the prisoner's dilemma model, demonstrating the capability of the quantum game theory to resolve traditional game dilemmas. Subsequently, the quantum game theory has garnered global scholarly attention, with increasing research in the field. Gender game quantization addresses Nash equilibrium point selection issues [15]. Quantum strategies have been found to outperform Nash equilibrium strategies in numerous instances when comparing classical games to quantum games [16]. Experimental evidence supports the practical efficacy of quantum games, not merely a theoretical result [17, 18]. Quantum game models established through EPR-type experiments exhibit a more direct connection with classical games [19]. The stability of the quantum Nash equilibrium increases with the increase of quantum entanglement, as found in the quantization Stackelberg duopoly game model [20]. Recently, quantum games have found application in economic investment, management decision-making, and supply chain management, with a particular emphasis on collaborative cooperation. In economic investment, quantum games have been used to address the risk exit dilemma in the financial investment market, analyze strategic choices for outward investment by venture capitalists and entrepreneurs, and provide new insights into mechanism design, auction, and contract theory [21–23]. In management decision-making, scholars have applied quantum games to examine alliance formation in production competition, collaboration in innovation involving industry, academia, and research, and cooperation among diverse governments in environmental governance [24–26]. Research on supply chain management indicates that quantum game models with distinct characteristics better guide decision-making, pricing, and cooperative incentive issues in the supply chain [27–30].

Based on the analysis of relevant literature, this research, focusing on the e-commerce logistics alliance, extends beyond the classical game theory and adopts the quantum game theory framework. Quantum game theory, distinct from the classical game theory, stands out due to features such as superposition and entanglement and proves more effective in managing cooperation, resolving dilemmas and influencing equilibrium outcomes. Hence, this paper employs the quantum game theory to explore incentive issues in collaborative distribution within e-commerce logistics alliances. This approach is taken to examine the strategic actions of alliance members and assess their expected payoffs. In contrast to prior studies, the main contributions of this paper are as follows:

(1) This paper focuses on e-commerce logistics alliances as its research subject and constructs both classical and quantum game models. These models are used to analyze equilibrium solutions and changes in expected payoffs for alliance members in both scenarios.

(2) The paper investigates the effects of different initial entanglement values in the quantum game model on alliance members' strategic choices and their corresponding changes in expected payoffs, along with an analysis of the corresponding critical conditions.

(3) Through numerical analysis experiments, it examines the impact of different quantum strategies and entanglement values on the profits of members within e-commerce logistics alliances. Based on the analytical results, the paper provides recommendations for effectively incentivizing the active participation of alliance members in collaborative distribution.

The rest of this article is organized as follows. Section 2 introduces the problem description and model assumptions, establishing the classical game model. Section 3 develops the quantum game model, examining the impact of quantum strategies on alliance collaborative distribution in both nonentangled and entangled states. Section 4 carries out numerical analysis, evaluates the parameters of the quantum game model, discusses the research results, and gives corresponding management suggestions. Finally, Section 5 provides a comprehensive summary and identifies directions for future research.

2. Research Background and Fundamental Assumptions

During e-commerce logistics alliances' cooperative distribution, parties' effort levels remain challenging to accurately observe due to implicit investments. For example, gauging e-commerce platforms' costs in customer and merchant maintenance proves difficult, while the 4PL platform struggles to discern the e-commerce platform operators' efforts in handling customer inquiries. Hence, the collaborative distribution process is not a deterministic "full effort-no effort" binary strategy set game; effort levels should be treated as a continuous variable. This concept resembles the quantum mechanics notion of superposition, prompting the adoption of a quantum game analysis framework for studying collaborative distribution in e-commerce logistics alliances. In this cooperative distribution game, the following hypotheses are posited:

Hypothesis 1. the e-commerce logistics alliance is a complete ecosystem where members exhibit bounded rationality and possess learning capabilities. They aim to maximize their interests by selecting and modifying strategies.

Hypothesis 2. this study only considers the benefits and common costs generated by the cooperative allocation process between the alliance parties.

Hypothesis 3. the cooperative distribution benefits and total cooperative distribution costs in the e-commerce logistics alliance remain constant. Alliance parties share a fixed net profit and common costs based on specific allocation coefficients.

Hypothesis 4. the study does not account for the influence of entities outside the e-commerce logistics alliance, which are not the subject of this research.

During the collaborative distribution process within e-commerce logistics alliances, the e-commerce platform is denoted as E, and the 4PL platform is denoted as F. This paper introduces variables e_E, e_F as the effort degree of the e-commerce platform and 4PL platform to participate in collaborative distribution tasks, where $e_i = 0, 1$ ($i = E, F$, 0 indicates no effort, and 1 indicates full effort). Assuming that the final revenue function of the alliance is the Cobb Douglas type [31], i.e.,

$$\pi = Ae_E^{\alpha} e_F^{1-\alpha} \varepsilon, \tag{1}$$

where A is the output coefficient of cooperative distribution, and $\alpha\,(0 < \alpha < 1)$ and $1 - \alpha$ represent the elasticity of effort utility of e-commerce platforms and 4PL platforms, respectively, which are used to measure the contributions of both parties' efforts. ε is the random disturbance term subject to normal distribution. The total cost of cooperative distribution is recorded as C_E and C_F, and the cost coefficients of cooperative distribution are ω_E and ω_F, respectively. As the effort degree increases, the cost also increases. Here, we use the quadratic function model, that is, the cost function is proportional to the square of the degree of effort.

$$\begin{aligned} C_E(e_E) &= \frac{\omega_E}{2} e_E^2, \\ C_F(e_F) &= \frac{\omega_F}{2} e_F^2. \end{aligned} \tag{2}$$

The profits obtained by both parties during the cooperative distribution process are recorded as R_E and R_F, and the distribution of profits is in a linear form, that is, $R_E = (1-\beta)\pi$ and $R_F = \beta\pi$, where β is the profit distribution coefficient. θ_i ($i = E, F$, $\theta_i = 0, 1$) is viewed as the level of effort of e-commerce platforms and 4PL platform, and the corresponding relationship with e_i given by

$$\theta_i = 1 - e_i, \quad i = E, F. \tag{3}$$

The expected payoff function of alliance members (e-commerce platform and 4PL platform) is as follows:

$$ER_E(\theta_E) = (1-\beta)Ae_E^{\alpha} e_F^{1-\alpha}\varepsilon - C_E = (1-\beta)A(1-\theta_E)^{\alpha}(1-\theta_F)^{1-\alpha}\varepsilon - \frac{\omega_E}{2}(1-\theta_E)^2, \tag{4}$$

$$ER_F(\theta_F) = \beta Ae_E^{\alpha} e_F^{1-\alpha}\varepsilon - C_F = \beta A(1-\theta_E)^{\alpha}(1-\theta_F)^{1-\alpha}\varepsilon - \frac{\omega_F}{2}(1-\theta_F)^2. \tag{5}$$

Within the classical game theory framework, this study investigates the game between an e-commerce platform and a 4PL platform based on the aforementioned assumptions. The payoff matrix is presented in Table 1, while Table 2 provides the parameters and symbols for the game model between the two parties within the alliance.

From the payoff matrix, it is evident that the "full effort" strategy adopted by both parties constitutes the only Pareto optimal outcome in this game. However, there exist two pure strategy Nash equilibrium points in which both parties opt for either "full effort" or "no effort." If one party exerts complete effort while the other party does not exert any effort, the former not only bears the cost of their own effort but also must face a situation where the latter gains no profit due to "betrayal." This poses a great risk for the exerting party, especially in projects that require significant investment (when C_E and C_F are large). The challenge for this paper is to find a solution that enables alliance members to achieve Pareto optimality while avoiding the risk of potential betrayal for the exerting party.

3. Quantum Game Models

Based on the preceding discussion and analysis, this section aims to examine the distinctive features of quantum games compared to classical games. Specifically, the Eisert–Wilkens–Levenstein (EWL) quantum game scheme will be employed to investigate the influence of quantum entanglement on the earnings of alliance members and to highlight the disparities between quantum strategies and classical strategies.

3.1. Model Construction. Under the theoretical framework of quantum computing, the participant game process is described as the process of accepting, manipulating, and measuring quantum bits, which is actually the information processing process in the game, i.e., the state transition process of the game. Alliance members continuously adjust their own strategies based on the observed situations during the game between the two parties to better achieve an

Table 1: The profit matrix from the perspective of classical game.

Strategies		4PL platform	
		Full effort 0	No effort 1
E-commerce platform	Full effort 0	$((1-\beta)A\varepsilon-\omega_E/2, \beta A\varepsilon-\omega_F/2)$	$(-\omega_E/2, 0)$
	No effort 1	$(0, -\omega_F/2)$	$(0, 0)$

Table 2: Main parameters and their meanings.

Parameters $(i = E, F)$	Parameters meanings
e_i	Alliance member's efforts level in cooperative distribution $0 \le e_i \le 1$
R_i	Alliance member's profit in cooperative distribution
C_i	Alliance member's cost in cooperative distribution
ω_i	Coefficient of alliance member's cost $\omega_i > 0$
α	Elastic coefficient $0 < \alpha < 1$
A	Output coefficient in cooperative distribution $A > 0$
ε	Random disturbance term $0 \le \varepsilon \le 1$
β	Distribution coefficient of alliance member's profit $0 < \beta < 1$
θ_i	Alliance member's efforts level in cooperative distribution $\theta_i \in [0, \pi]$
φ_i	The degree of quantum strategy adopted by alliance member's $\varphi_i \in [0, \pi/2]$
γ	The degree of entanglement between alliance members $\gamma \in [0, \pi/2]$

evolutionary stable state. The main idea of the EWL scheme is as follows [14], and the specific quantization process is shown in Figure 1:

Within the framework of EWL quantum games, each member can be described as a qubit in a two-dimensional Hilbert space, represented by state vectors $|0\rangle = (1, 0)^T$ and $|1\rangle = (0, 1)^T$. Initially, each member is in the state represented by $|0\rangle$. Subsequently, the general entanglement gate $\hat{J}$ is applied to achieve the following:

(1) Quantumizing classical game problems, within the framework of the quantum game theory, the two classical game strategies "full effort" and "no effort at all" correspond, respectively, to two polarized states $|0\rangle$ and $|1\rangle$ in a two-dimensional Hilbert space (i.e., $\theta_i = 0$ and $\theta_i = 1$ in classical games). The initial strategies of the game are expressed through tensor product states of quantum bits, denoted as $|00\rangle, |01\rangle, |10\rangle, |11\rangle$, representing four possible combinations (the first digit representing the e-commerce platform, and the second digit representing the 4PL). This paper assumes that both parties initially adopt the strategy of "full effort," denoted as $|0\rangle$, which means that the initial quantum states for both sides are represented as $|\psi_0\rangle = |00\rangle = |0\rangle \otimes |0\rangle$, where $\otimes$ signifies the tensor product.

(2) This paper discusses the EWL quantum game model in the two-parameter case as shown in Figure 1. That is, the strategies of the e-commerce platform and 4PL platform are unitary operator U_E and U_F. The quantum strategies selected by each member of the cooperative distribution alliance are shown as follows:

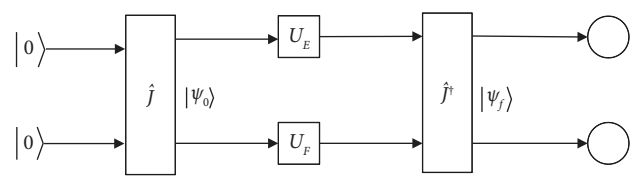


Figure 1: EWL quantum game model.

$$\hat{U}_i(\theta_i, \varphi_i) = \begin{pmatrix} e^{i\varphi_i}\cos\frac{\theta_i}{2} & \sin\frac{\theta_i}{2} \\ -\sin\frac{\theta_i}{2} & e^{-i\varphi_i}\cos\frac{\theta_i}{2} \end{pmatrix}. \quad (6)$$

The strategy space is composed of a two-parameter set ($\theta_i \in [0, \pi]$, $\varphi_i \in [0, \pi/2]$) of a 2×2 matrix, where $\theta_i (i = E, F)$ is the effort degree parameter and φ_i is the cooperative distribution capability parameter displayed of each member. $e^{i\varphi_i}$ is the complex phase, $\cos(\theta_i/2)$ is the amplitude, and the product of the two gives the probability amplitude of the quantum strategy. In this context, it can be understood as the probability amplitude of benefits obtained by alliance members in the cooperative distribution process when they choose the quantum strategy. Strategies $\hat{U}(0, 0)$ and $\hat{U}(\pi, 0)$ are referred to as "full effort" strategies and "no effort" strategies. The general quantum strategy is denoted by $\hat{U}(\theta, \varphi)$ when $0 < \varphi < \pi/2$.

(3) Suppose that the default entanglement operator of alliance members is $\hat{J}$,

$$\hat{J} = \exp\left(i\frac{\gamma}{2}\sigma_x \otimes \sigma_x\right) = \cos\frac{\gamma}{2} \cdot I + i\sin\frac{\gamma}{2} \cdot (\sigma_x \otimes \sigma_x). \quad (7)$$

Here, $\sigma_x' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ is a variant of the Pauli-x matrix, I is the 4×4 identity matrix, and γ represents the entanglement degree between the two players ($\gamma \in [0, \pi/2]$). When $\gamma = 0$, the state is unentangled; in other words, both parties are completely unaffected by each other when they play the game. When $\gamma = \pi/2$, the entanglement is maximized, that is, the strategy chosen by both parties and the action information transparent to each other.

By solving for the entanglement operator $\widehat{J}$, we can obtain the initial state $|\psi_0\rangle$ as follows:

$$|\psi_0\rangle = \widehat{J}(|0\rangle \otimes |0\rangle) = \left(\cos\frac{\gamma}{2} \;\; 0 \;\; 0 \;\; i\sin\frac{\gamma}{2}\right)^T. \tag{8}$$

After one round of the game, the state becomes $(U_E \otimes U_F)\widehat{J}|00\rangle$.

(4) The antientanglement operator $\widehat{J}^\dagger$ can be solved according to the entanglement operator $\widehat{J}$:

$$\widehat{J}^\dagger = \cos\frac{\pi}{2} \cdot I - i\sin\frac{\pi}{2} \cdot (\sigma_x \otimes \sigma_x). \tag{9}$$

The final state is obtained by the antientanglement operator $|\psi_f\rangle = \widehat{J}^\dagger (U_E \otimes U_F)\widehat{J}|00\rangle$.

According to the collapse property of quantum measurement, the final state $|\psi_f\rangle$ is observed, and it will randomly collapse into one of the four basis vectors $|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$. The probabilities of each result are as follows:

$$\begin{cases} P_{00} = \left[\cos^2(\varphi_E + \varphi_F) + \sin^2(\varphi_E + \varphi_F)\cos^2\gamma\right]\cos^2\frac{\theta_E}{2}\cos^2\frac{\theta_F}{2}, \\ P_{01} = \left(\cos^2\varphi_E + \sin^2\varphi_E\cos^2\gamma\right)\cos^2\frac{\theta_E}{2}\sin^2\frac{\theta_F}{2} + \sin^2\varphi_F\sin^2\gamma\sin^2\frac{\theta_E}{2}\cos^2\frac{\theta_F}{2}, \\ P_{10} = \left(\cos^2\varphi_F + \sin^2\varphi_F\cos^2\gamma\right)\sin^2\frac{\theta_E}{2}\cos^2\frac{\theta_F}{2} + \sin^2\varphi_E\sin^2\gamma\cos^2\frac{\theta_E}{2}\sin^2\frac{\theta_F}{2}, \\ P_{11} = \sin^2(\varphi_E + \varphi_F)\sin^2\gamma\cos^2\frac{\theta_E}{2}\cos^2\frac{\theta_F}{2} + \sin^2\frac{\theta_E}{2}\sin^2\frac{\theta_F}{2}, \end{cases} \tag{10}$$

which is calculated as $P_{00} + P_{01} + P_{10} + P_{11} = 1$.

According to the above results, combined with equations (4) and (5), the expected revenue function of the e-commerce platform and 4PL platform can be expressed in the following form:

$$\begin{aligned} ER_E &= \left[(1-\beta)A\varepsilon - \frac{\omega_E}{2}\right]P_{00} - \frac{\omega_E}{2}P_{01} + 0 \cdot P_{10} + 0 \cdot P_{11} \\ &= \left[(1-\beta)A\varepsilon - \frac{\omega_E}{2}\right]\left[\cos^2(\varphi_E + \varphi_F) + \sin^2(\varphi_E + \varphi_F)\cos^2\gamma\right]\cos^2\frac{\theta_E}{2}\cos^2\frac{\theta_F}{2} \\ &\quad - \frac{\omega_E}{2}\left[\left(\cos^2\varphi_E + \sin^2\varphi_E\cos^2\gamma\right)\cos^2\frac{\theta_E}{2}\sin^2\frac{\theta_F}{2} + \sin^2\varphi_F\sin^2\gamma\sin^2\frac{\theta_E}{2}\cos^2\frac{\theta_F}{2}\right], \\ ER_F &= \left(\beta A\varepsilon - \frac{\omega_F}{2}\right)P_{00} + 0 \cdot P_{01} - \frac{\omega_F}{2}P_{10} + 0 \cdot P_{11} \\ &= \left(\beta A\varepsilon - \frac{\omega_F}{2}\right)\left[\cos^2(\varphi_E + \varphi_F) + \sin^2(\varphi_E + \varphi_F)\cos^2\gamma\right]\cos^2\frac{\theta_E}{2}\cos^2\frac{\theta_F}{2} \\ &\quad - \frac{\omega_F}{2}\left[\left(\cos^2\varphi_F + \sin^2\varphi_F\cos^2\gamma\right)\sin^2\frac{\theta_E}{2}\cos^2\frac{\theta_F}{2} + \sin^2\varphi_E\sin^2\gamma\cos^2\frac{\theta_E}{2}\sin^2\frac{\theta_F}{2}\right]. \end{aligned} \tag{11}$$

3.2. Nonentangled State. Under the nonentangled state ($\gamma = 0$), the expected payoff of the e-commerce platform and the 4PL platform are as follows:

$$\mathrm{ER}_E = \left[(1-\beta)A\varepsilon\cos^2\frac{\theta_F}{2} - \frac{\omega_E}{2}\right]\cos^2\frac{\theta_E}{2},$$
$$\mathrm{ER}_F = \left(\beta A\varepsilon\cos^2\frac{\theta_E}{2} - \frac{\omega_F}{2}\right)\cos^2\frac{\theta_F}{2}. \tag{12}$$

From equation (12), it is clear that the e-commerce platform and the 4PL platform's expected benefits rely solely on the parameter θ, implying that the cooperation efforts of each platform directly affect their expected benefits. The following propositions outline how each platform chooses its strategy based on the cooperative efforts of the opponent:

Proposition 5. *If the alliance members are in a nonentangled state, the following can be inferred:*

(1) When $\theta_F \in [0, \theta_F^*)$, the expected revenue ER_E for the e-commerce platform decreases with θ_E, while ER_E increases with θ_E when $\theta_F \in (\theta_F^*, \pi]$. In addition, $\theta_F^* = 2\arccos\sqrt{\omega_E/2(1-\beta)A\varepsilon}$, $\theta_F^* \in [0, \pi]$.

(2) When $\theta_E \in [0, \theta_E^*)$, the expected revenue ER_F for 4PL platform decreases with θ_F, while ER_F increases with θ_F when $\theta_E \in (\theta_E^*, \pi]$. In addition, $\theta_E^* = 2\arccos\sqrt{\omega_E/2(1-\beta)A\varepsilon}$, $\theta_E^* \in [0, \pi]$.

Proof. See Appendix A.1.

Proposition 5 shows that insufficient effort from one party does not harm its own benefits but does not improve them either. Positive correlation between effort and benefits only occurs when one party's effort is substantial. In a nonentangled state, an alliance member might engage in free riding for benefits. Table 3 illustrates this with four strategies and expected benefits for both platforms. Entanglement is introduced to address this, and the discussion in the following explores its impact on the game process. □

3.3. Entangled State. In this situation $(0 < \gamma < \pi/2)$, the members of the e-commerce logistics alliance are in an entangled state. For the convenience of mathematical computation, this paper considers the case of maximum entanglement, denoted as $\gamma = \pi/2$.

Proposition 6. *Under the condition of maximum entanglement $\gamma = \pi/2$, if the e-commerce platform adopts a nonquantum strategy $\widehat{U}_E(\theta_E, 0)$, the sufficient and necessary condition for ER_E will decrease with θ_E, that is, $[(1-\beta)A\varepsilon - \omega_E/2]\cos^2\varphi_F\cos^2\theta_F/2 - \omega_E/2\sin^2\theta_F/2 \geq 0$ holds simultaneously with $\sin^2\varphi_F\cos^2\theta_F/2 \geq 0$ and neither takes the value of "=" at the same time; in other words, ER_E increases with the effort level e_E. Currently, the e-commerce platform's optimal strategy is to make full efforts $\theta_E = 0$. Similarly, if the 4PL platform adopts a nonquantum strategy $\widehat{U}_F(\theta_F, 0)$, the sufficient and necessary condition for ER_F will decrease with θ_F, that is, $(\beta A\varepsilon - \omega_F/2)\cos^2\varphi_E\cos^2\theta_E/2 - \omega_F/2\sin^2\theta_E/2 \geq 0$ holds simultaneously with $\sin^2\varphi_E\cos^2\theta_E/2 \geq 0$ and neither takes the value of "=" at the same time; in other words, ER_F increases with the effort level e_E. Currently, the optimal strategy for the 4PL platform is to make full efforts $\theta_F = 0$.*

Proof. See Appendix A.2.

Proposition 6 indicates that within the collaborative distribution alliance, when the 4PL platform does not make efforts, the e-commerce platform must exhibit specific cooperative distribution capabilities and invest efforts to incentivize the 4PL platform. □

Proposition 7. *Under the condition of maximum entanglement $\gamma = \pi/2$, if the e-commerce platform adopts a fully quantum strategy $\widehat{U}_E(\theta_E, \pi/2)$, then the sufficient and necessary condition for ER_E increase with e_E is $\sin\varphi_F\cos^2\theta_E/2 > 0$, and the optimal strategy for the e-commerce platform is to exert full effort $\widehat{U}_E(0, \pi/2)$. Similarly, if the 4PL platform adopts a fully quantum strategy $\widehat{U}_F(\theta_F, \pi/2)$, then the sufficient and necessary condition for ER_F increase with e_F is $\sin\varphi_E\cos^2\theta_F/2 > 0$, and the optimal strategy for the 4PL platform is also to exert full effort $\widehat{U}_F(0, \pi/2)$.*

Proof. See Appendix A.3.

Proposition 7 shows that when both e-commerce and 4PL platforms collaborate actively within the alliance, and member interests rise with improved collaborative distribution capabilities and efforts. To visually illustrate the impact of quantum strategies on the expected returns of alliance members under entanglement, we further analyze four specific strategies as presented in Table 4.

From Table 4, it can be seen that among the six Nash equilibrium points, only $(\widehat{U}_E(0, 0), \widehat{U}_F(0, 0))$ and $(\widehat{U}_E(0, \pi/2), \widehat{U}_F(0, \pi/2))$ can bring payoffs to both parties, and the strategies $\widehat{U}_E(0, \pi/2)$ and $\widehat{U}_F(0, \pi/2)$ are the Pareto optimal in the scenario of maximum entangled state, that is, the situation where both parties adopt "complete quantum strategy for fully effort." If the e-commerce platform adopts the strategy $\widehat{U}(0, \pi/2)$ in the entangled state, no concern is required regarding the passive stance of the 4PL platform, as it can eliminate the betrayal risk resulting from the no-efforts party. It signifies that the 4PL platform will bear its own losses. This implies that when both platforms employ the quantum strategy in an entangled state, free-riding conduct can be efficiently averted, and alliance members can be encouraged to actively engage in cooperative distribution, resulting in mutually beneficial cooperation. □

4. Numerical Simulations

In this section, numerical analysis was performed using MATLAB R2022b software to investigate the influence of quantum strategies and entanglement on the expected returns of the alliance members and $(A, \beta_i, \varepsilon, \omega_i) = (70, 0.6, 0.3, 6)$. Specifically, the parameter ε was varied to analyze its impact on the members' returns. It was found that when $\varepsilon = 0.3$, the resulting graph provided the clearest depiction and effectively illustrated the underlying dynamics.

Table 3: The profit matrix of alliance members under four strategies in the nonentangled state.

$\widehat{U} = (\theta, \varphi)$	$\widehat{U}_F(0, 0)$	$\widehat{U}_F(0, \pi/2)$	$\widehat{U}_F(\pi, 0)$	$\widehat{U}_F(\pi, \pi/2)$
$\widehat{U}_E(0, 0)$	$((1-\beta)A\varepsilon - \omega_E/2, \beta A\varepsilon - \omega_F/2)$	$((1-\beta)A\varepsilon - \omega_E/2, \beta A\varepsilon - \omega_F/2)$	$(-\omega_E/2, 0)$	$(-\omega_E/2, 0)$
$\widehat{U}_E(0, \pi/2)$	$((1-\beta)A\varepsilon - \omega_E/2, \beta A\varepsilon - \omega_F/2)$	$((1-\beta)A\varepsilon - \omega_E/2, \beta A\varepsilon - \omega_F/2)$	$(-\omega_E/2, 0)$	$(-\omega_E/2, 0)$
$\widehat{U}_E(\pi, 0)$	$(0, -\omega_F/2)$	$(0, -\omega_F/2)$	$(0, 0)$	$(0, 0)$
$\widehat{U}_E(\pi, \pi/2)$	$(0, -\omega_F/2)$	$(0, -\omega_F/2)$	$(0, 0)$	$(0, 0)$

Table 4: The profit matrix of alliance members under four strategies in the entangled state.

$\widehat{U} = (\theta, \varphi)$	$\widehat{U}_F(0, 0)$	$\widehat{U}_F(0, \pi/2)$	$\widehat{U}_F(\pi, 0)$	$\widehat{U}_F(\pi, \pi/2)$
$\widehat{U}_E(0, 0)$	$((1-\beta)A\varepsilon - \omega_E/2, \beta A\varepsilon - \omega_F/2)$	$(0, 0)$	$(-\omega_E/2, 0)$	$(-\omega_E/2, 0)$
$\widehat{U}_E(0, \pi/2)$	$(0, 0)$	$((1-\beta)A\varepsilon - \omega_E/2, \beta A\varepsilon - \omega_F/2)$	$(0, -\omega_F/2)$	$(0, -\omega_F/2)$
$\widehat{U}_E(\pi, 0)$	$(0, -\omega_F/2)$	$(-\omega_E/2, 0)$	$(0, 0)$	$(0, 0)$
$\widehat{U}_E(\pi, \pi/2)$	$(0, -\omega_F/2)$	$(-\omega_E/2, 0)$	$(0, 0)$	$(0, 0)$

4.1. The Impact of θ on the Alliance Members' Profits Given φ. In this section, we analyze the influence of θ on the alliance members' profits when $\varphi = 0, \pi/2$. Figure 2 shows the influence of θ on the profits of both platforms in the nonentangled state, while Figure 3 presents the impact of θ on the members' profits in the entangled state.

4.1.1. Nonentangled State. Under the nonentangled state ($\gamma = 0$), the members' returns in the alliance are solely dependent on their respective levels of effort, considering the case of the e-commerce platform, as shown in Figure 2. Figure 4 shows the two-dimensional profile of the impact on the payoffs of alliance members when the effort degree θ_i reaches $\theta_i = 0$ to $\theta_i = \pi$, five different values in the nonentangled state. Figure 5 further details the impact on alliance members when the effort degree is $\theta_i = 0$ and $\theta_i = \pi$.

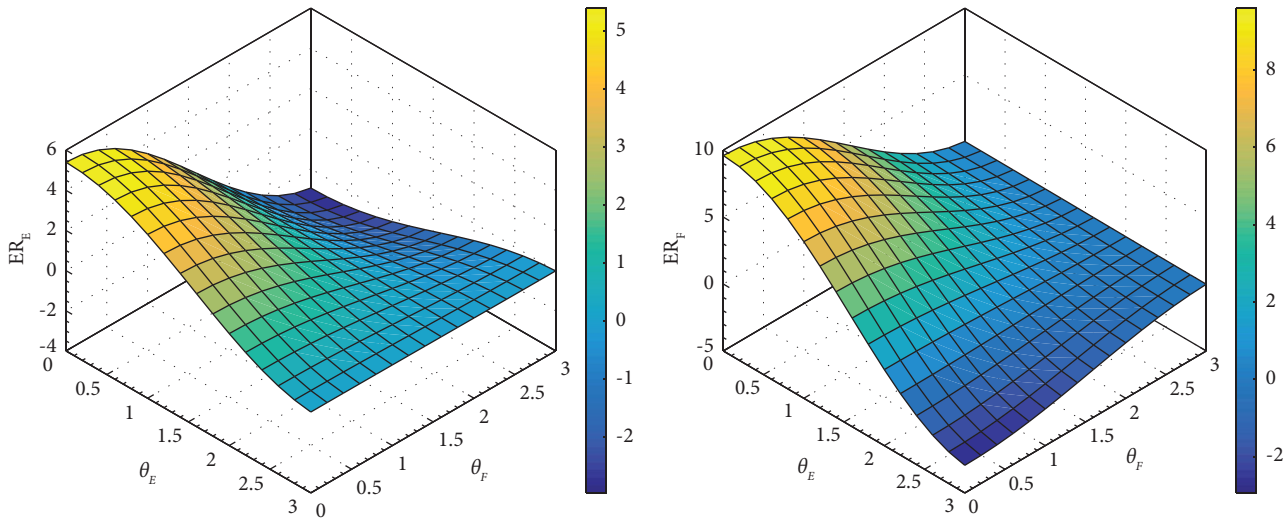
It can be seen from Figures 2, 3, and 5 that (1) in the process of θ_F increasing and approaching $\pi/2$, the maximum value ER_E decreases from 5.4 to 1.2 and decreases with θ_E, that is, the payoff of the e-commerce platform increases with its own efforts; (2) in the process of θ_F increasing and approaching from $3\pi/4$ to π, the minimum value of ER_E decreases from about -1.77 to -3 and increases with θ_E; in other words, the profit of the e-commerce platform will decrease with its own efforts; (3) when θ_F is approaching π, ER_E decreases with θ_F, that is, the payoff of the e-commerce platform increases with the effort of 4PL platform; (4) the trend shown in Figures 4 and 5 accords with the hypothesis of Proposition 5, where the critical point θ_F^* is between $\pi/2$ and $3\pi/4$. In other words, before the critical point, the revenue of the e-commerce platform increases with its effort level. After reaching the critical point, the revenue of the e-commerce platform decreases with its effort level. This critical point is primarily influenced by the effort level of the 4PL platform. The same is true of the relationship between ER_F and θ_F.

4.1.2. Entangled State. Under quantum entanglement ($\gamma = \pi/2$), member's profits are related to their effort level and cooperative distribution capabilities. As in the entangled state, Figures 3, 6, and 7 also show the relationship between θ and φ in the entangled state.

From Figures 3, 6, and 7, it can be observed that (1) in the entangled state, as the effort degree increases from $\theta_F = 0$ to $\theta_F = \pi$, ER_E decreases with θ_E, and the decreasing amplitude of ER_E tends to be gentle with the increase of θ_F; (2) in the process of $\theta_F = 0$ increasing to $\theta_F = \pi$, ER_E from the maximum 5.4 gradually decreased to about 1×10^{-32}, that is, the payoff of the e-commerce platform not only increases with its own efforts but also increases with the efforts of 4PL platform; (3) the e-commerce platform's payoff increases with its own effort, and this increase is positively correlated with the effort of the 4PL platform. In the maximum entangled state, the e-commerce platform is not burdened with the cost of the 4PL platform's lack of effort. The same is true of the relationship between ER_F and θ_F.

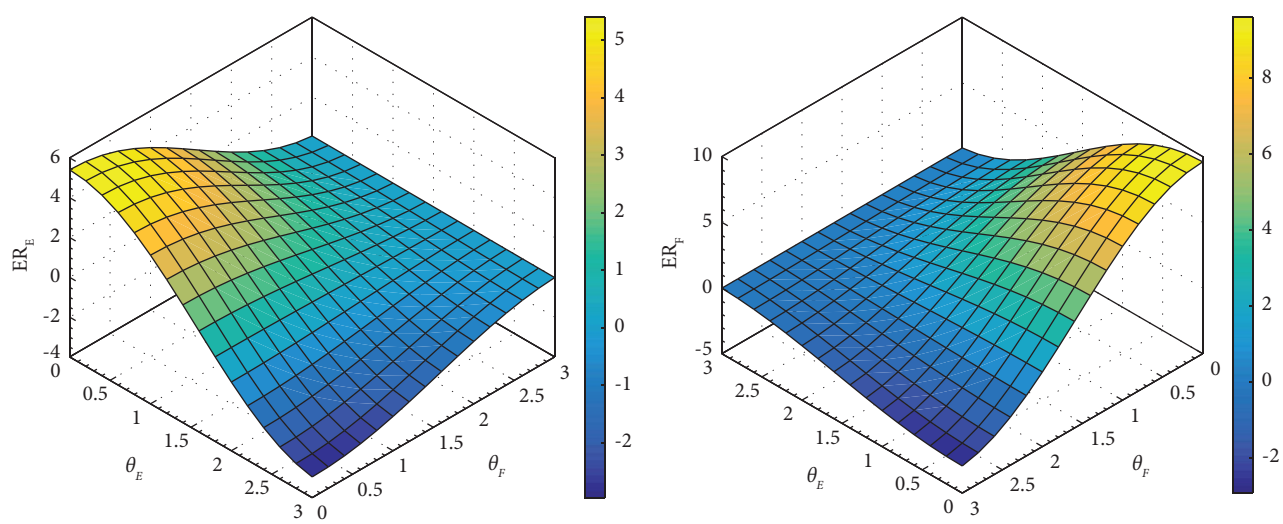
Figure 8 reveals the following observations: (1) when the e-commerce platform adopts the strategy $\widehat{U}_E = (\theta_E, 0)$ (cooperative distribution capacity is 0), the optimal strategy for the 4PL platform is $\widehat{U}_F = (0, \pi/2)$. In this scenario, as θ_E approaches 0, the payoff of the e-commerce platform decreases with θ_F. In other words, the revenue of the e-commerce platform increases with the effort level of the e-commerce platform and 4PL platform. At this point, the optimal strategy for the 4PL platform is to "fully exert effort and demonstrate the maximum collaborative distribution capability; " (2) when the e-commerce platform adopts the strategy $\widehat{U}_E = (\theta_E, \pi/2)$ and the 4PL platform chooses the strategy $\widehat{U}_F = (\theta_F, 0)$, the payoff of the e-commerce platform remains unaffected with the effort level of the e-commerce platform increasing or decreasing; (3) when the e-commerce platform adopts the strategy $\widehat{U}_E = (\theta_E, \pi/2)$ and 4PL platform chooses the strategy $\widehat{U}_F = (\theta_F, \pi/2)$, as θ_F approaches 0, the payoff of the e-commerce platform will decrease with θ_E. In other words, the income of the e-commerce platform will increase with the degree of effort. Therefore, the optimal strategy for the 4PL platform is $\widehat{U}_F = (0, \pi/2)$. Likewise, the optimal strategy for the e-commerce platform is $\widehat{U}_E = (0, \pi/2)$.

4.2. The Impact of φ on the Alliance Members' Profits Given θ. This section considers the impact of parameters φ on the revenue of alliance members when $\theta_i = 0, \pi$, as shown in Figure 9. Since when $(\theta_E, \theta_F) = (\pi, \pi)$, the income of both



(a) (b)

Figure 2: The effect of θ on the payoffs of alliance members in the nonentangled state. (a) The impact of different effort levels of the e-commerce platform and the 4PL platform on the revenue of the e-commerce platform under the nonentangled state. (b) The influence of different effort levels of the e-commerce platform and the 4PL platform on the revenue of the 4PL platform under the nonentangled state. In the figures, the size of member revenue can be observed from the color scale (shifting from blue to yellow indicates an increase in the value).



(a) (b)

Figure 3: The effect of θ on the payoffs of alliance members in the entangled state. (a) The impact of different effort levels of the e-commerce platform and the 4PL platform on the revenue of the e-commerce platform under the entangled state. (b) The influence of different effort levels of the e-commerce platform and the 4PL platform on the revenue of the 4PL platform under the entangled state. In the figures, the size of member revenue can be observed from the color scale (shifting from blue to yellow indicates an increase in the value).

members is 0, the discussion of this case is omitted in this paper. It can be seen from Figure 9 that (1) when both members adopt strategies $\widehat{U} = (0, \varphi)$, if one member chooses strategy $\widehat{U} = (0, 0)$, the other member's returns will decrease with φ; and if one member chooses strategy $\widehat{U} = (0, \pi/2)$, the other member's returns will increase with φ. (2) Specifically, when the e-commerce platform chooses strategy $\widehat{U}_E = (0, \varphi)$, the revenue of the e-commerce platform will increase with φ_E as φ_F approaches $\pi/2$. In this scenario, the optimal strategy for the 4PL platform is "fully exert effort and demonstrate the maximum collaborative distribution capability $\widehat{U}_F = (0, \pi/2)$." (3) If the e-commerce platform chooses strategy $\widehat{U}_E = (\pi, \varphi)$, φ_E does not have an influence on the revenue of the e-commerce platform as φ_F approaches $\pi/2$. At this point, the optimal strategy for the 4PL platform is $\widehat{U}_F = (0, \pi/2)$. Similarly, it can be concluded that the optimal strategy for the e-commerce platform is $\widehat{U}_E = (0, \pi/2)$.

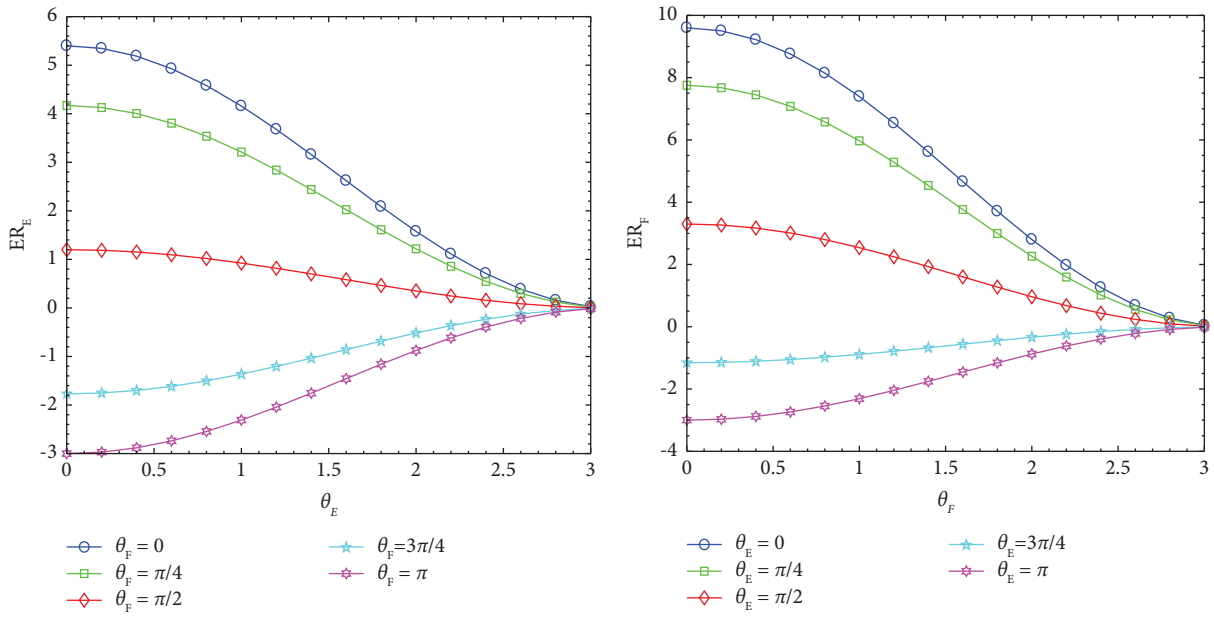


Figure 4: The effect of $\theta_i = 0$ to $\theta_i = \pi$ on the payoffs of alliance members in the nonentangled state. (a) The impact of variations in the effort level of the e-commerce platform on its revenue under the nonentangled state, considering different effort levels of the 4PL platform (represented by curves with distinct colors and shapes corresponding to values $0, \pi/4, \pi/2, 3\pi/4, \pi$). In (b), under the nonentangled state, the influence of changes in the effort level of the 4PL platform on its revenue is illustrated, while keeping the effort level of the e-commerce platform constant (with values $0, \pi/4, \pi/2, 3\pi/4, \pi$, represented by curves with different colors and shapes).

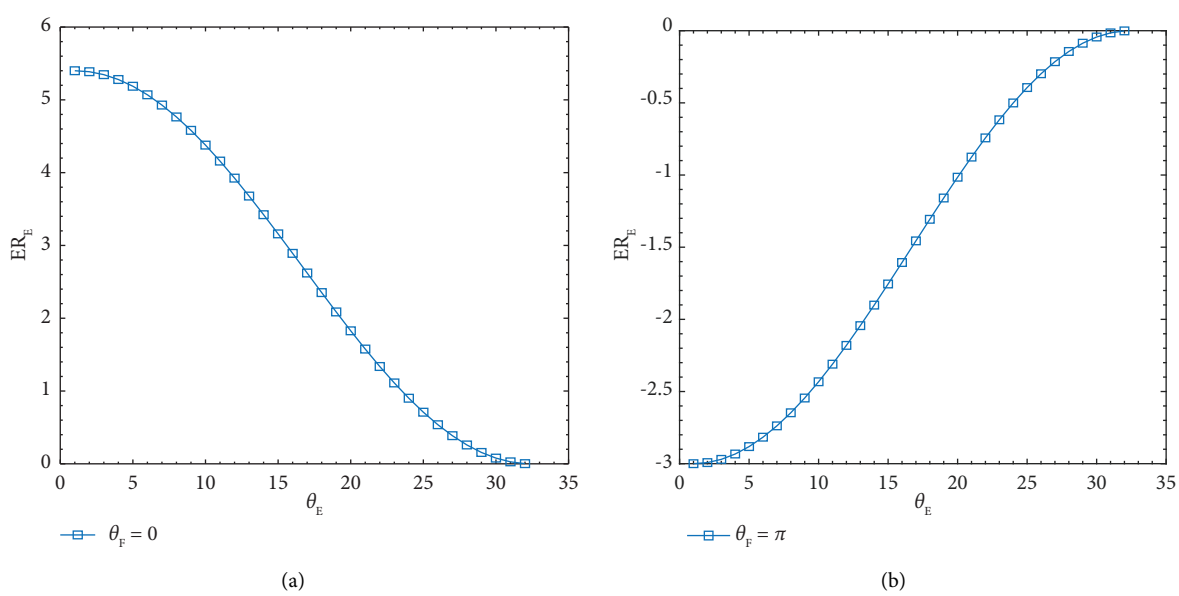


Figure 5: Continued.

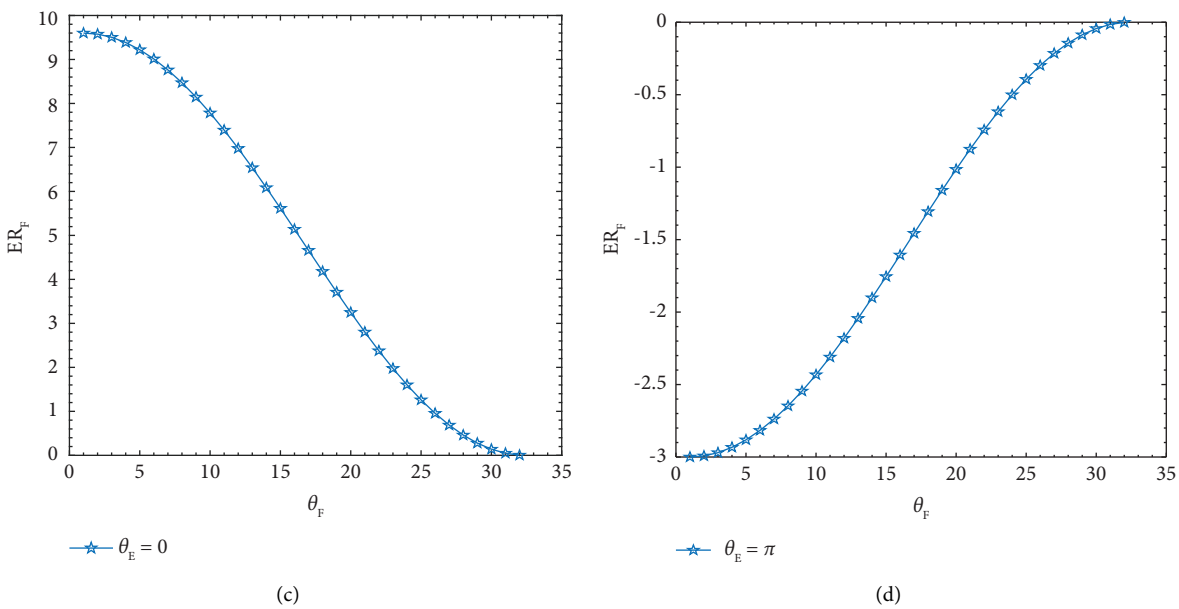


Figure 5: The effect of $\theta_i = 0$ and $\theta_i = \pi$ on the returns of alliance members in the nonentangled state. (a-b) The impact of changes in the effort level of the e-commerce platform on its revenue under the nonentangled state, considering 4PL platform values of 0 and π, respectively. In (c-d), the specific depiction is provided for the influence of variations in the effort level of the 4PL platform on its revenue under the nonentangled state, with the e-commerce platform values set at 0 and π, respectively.

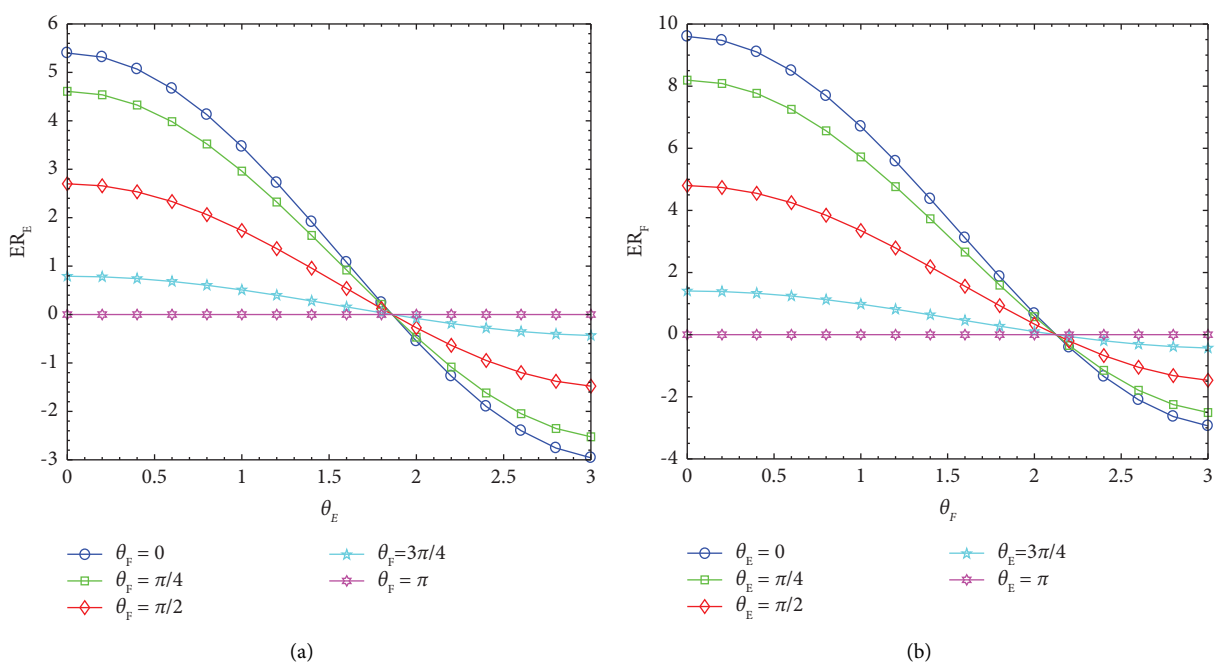


Figure 6: The effect of $\theta_i = 0$ to $\theta_i = \pi$ on the payoffs of alliance members in the entangled state. (a) The impact of changes in the effort level of the e-commerce platform on its revenue under the entangled state, with a given effort level for the 4PL platform (represented by curves with distinct colors and shapes corresponding to values $0, \pi/4, \pi/2, 3\pi/4, \pi$). In (b), the illustration focuses on the influence of variations in the effort level of the 4PL platform on its revenue under the entangled state, with a given effort level for the e-commerce platform (with values $0, \pi/4, \pi/2, 3\pi/4, \pi$, represented by curves with different colors and shapes).

As can be seen from Figures 8 and 9, if a member chooses the strategy $\widehat{U} = (0, \varphi)$, when φ tends to 0, it means that the member's attitude towards cooperative distribution is "efforts but insufficient ability." In general, regardless of the opponent's strategy, the optimal strategy for alliance members is $\widehat{U} = (0, \pi/2)$. This means that in the entangled

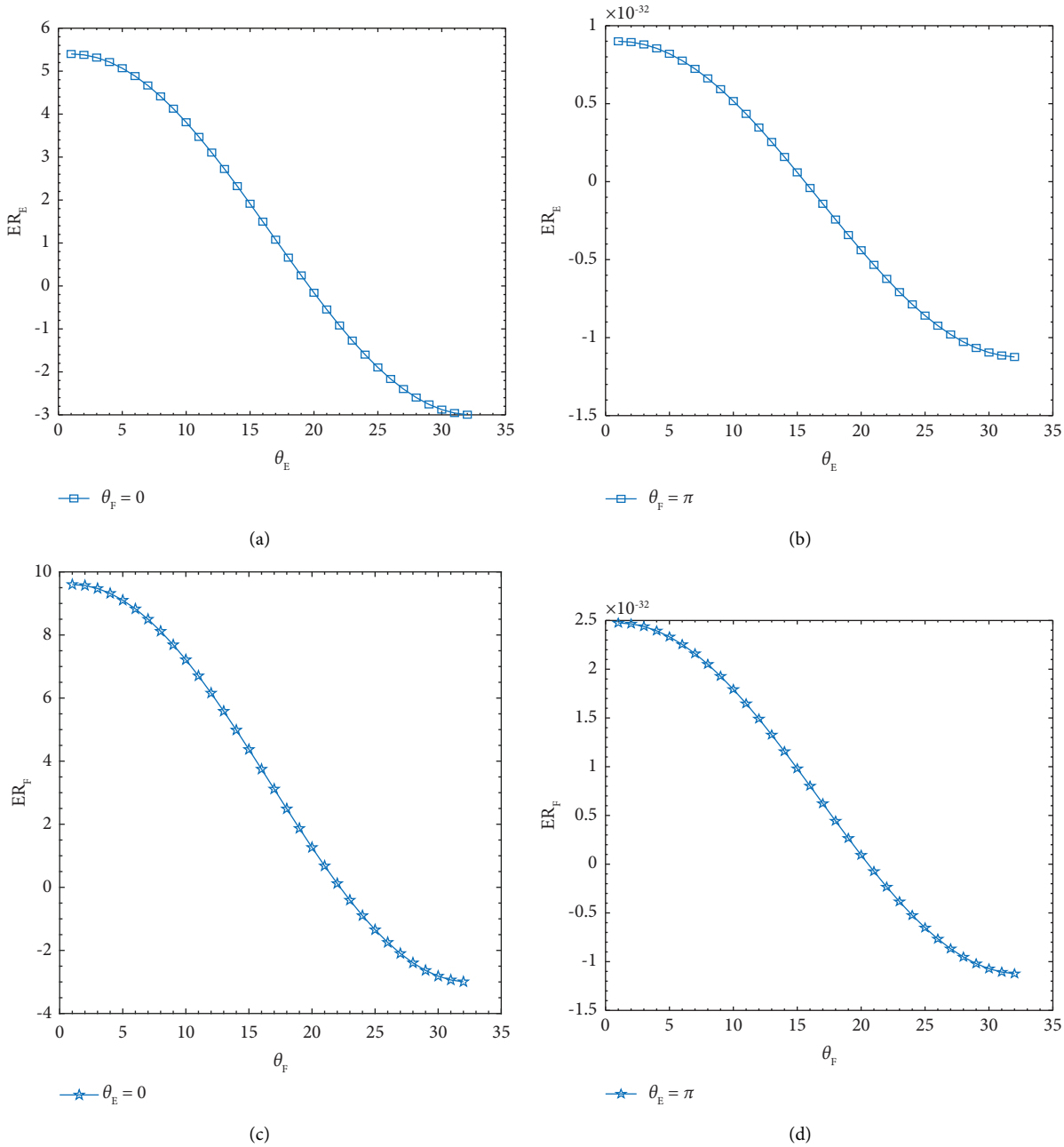


FIGURE 7: The effect of $\theta_i = 0$ and $\theta_i = \pi$ on the payoffs of alliance members in the entangled state. (a-b) A detailed showcase of the impact of changes in the effort level of the e-commerce platform on its revenue under the entangled state, considering the 4PL platform with values of 0 and π. In (c-d), the specific demonstration focuses on the influence of variations in the effort level of the 4PL on its revenue under the entangled state, considering the e-commerce platform with values of 0 and π.

state, the Nash equilibrium of the quantum game is $(\widehat{U}_E = (0, \pi/2), \widehat{U}_F = (0, \pi/2))$ which is also the Pareto optimum in this game.

4.3. The Impact of γ on the Alliance Members' Profits Given Some Specific Strategies. In this section, the impact of entanglement on the profits of alliance members is examined under specific strategies. By analyzing Table 5, it is evident that when one platform adopts the strategy "fully exert effort and demonstrate the maximum collaborative distribution capability $\widehat{U} = (0, \pi/2)$" while the other platform does not choose same strategy, the profit of the latter platform decreases with increasing levels of entanglement.

4.4. Discussion and Managerial Insights. This section unveils intriguing discoveries. First, in the classical game scenario, where one party contributes maximum effort while the other does not, the fully committed party not only bears its own effort costs but also faces an unprofitable outcome due to the other party's "betrayal." In addition, within quantum nonentanglement scenarios, the strategy space for

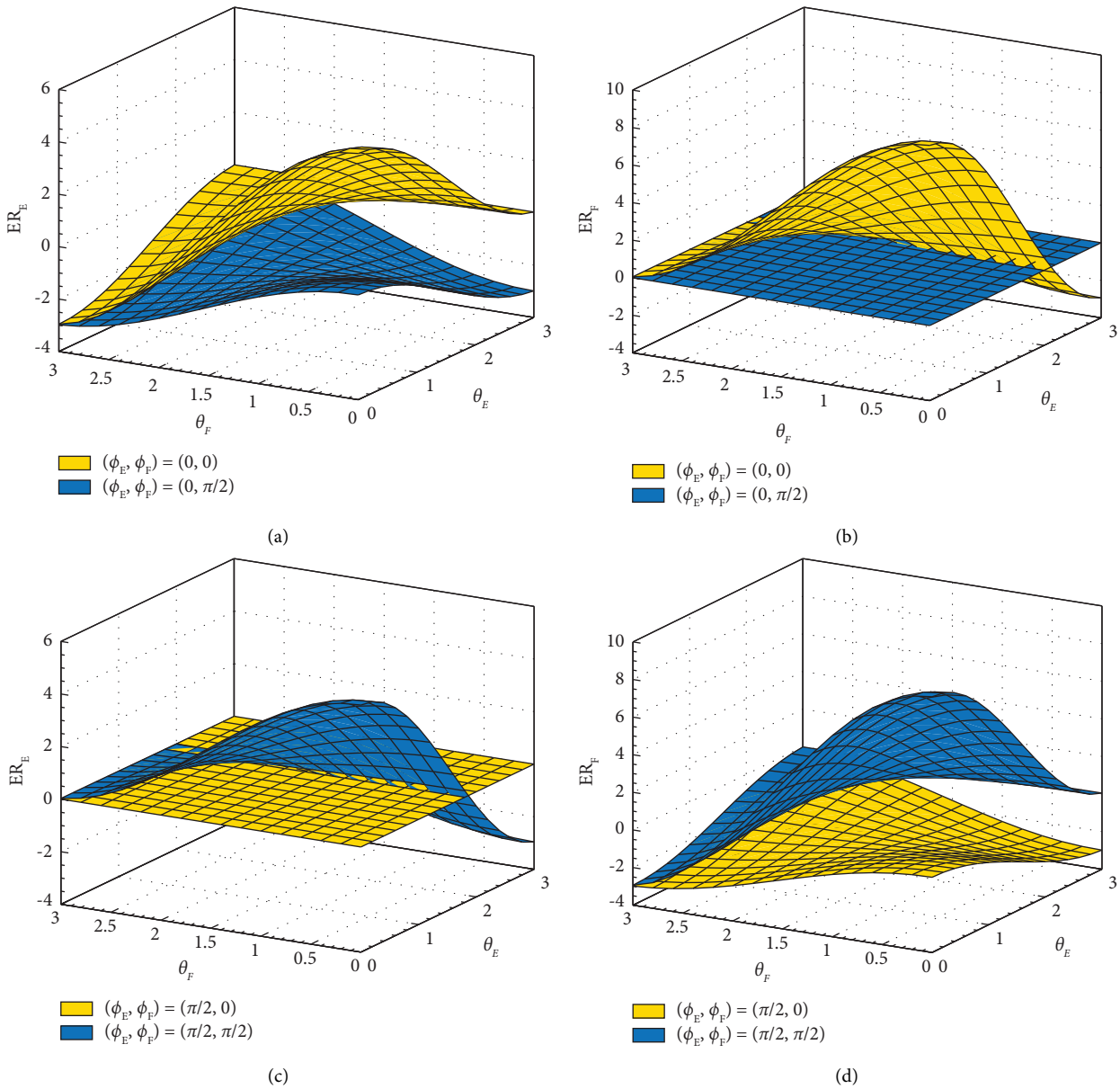


Figure 8: The effect of θ_i on the payoffs of alliance members in specific φ_i. Under the entangled state, given the values of φ (represented by $0, \pi/2$), Figure 8 illustrates the 3D impact of variations in θ on the alliance members' revenue. In (a-b), under the entangled state, when the e-commerce platform exhibits a collaborative distribution capacity of 0 and the 4PL platform exhibits a collaborative distribution capacity of 0 and $\pi/2$, and the different effort levels of the e-commerce platform and the 4PL platform, respectively, influence the revenue of both parties. In (c-d), under the entangled state, when the e-commerce platform exhibits a collaborative distribution capacity of $\pi/2$ and the 4PL platform exhibits a collaborative distribution capacity of 0 and $\pi/2$, the different effort levels of the e-commerce platform and the 4PL platform, respectively, influence the revenue of both parties.

e-commerce platforms and 4PL platforms expands. While results align in both classical and quantum non-entanglement scenarios, they lay the groundwork for analyzing the quantum maximum entanglement state. Consequently, under the quantum maximum entanglement state, the party refraining from effort bears the cost itself rather than shifting it to the fully committed party. The risk of noneffort-based betrayal can be entirely avoided, effectively reducing free-rider behavior. Ultimately, all alliance members tend to choose the "fully committed complete quantum strategy," benefiting both parties and leading to a win-win situation. In contrast, our research indicates that the quantum game theory can yield optimal results. This approach posits that the states of the game players are continually evolving, expanding the strategy space for both parties, rendering it more aligned with practical scenarios. Consequently, we conclude that quantum gaming holds certain advantages over classical gaming, as its strategy set is shaped by its unique characteristics of superposition and entanglement.

Based on these findings, some managerial insights are summarized as follows:

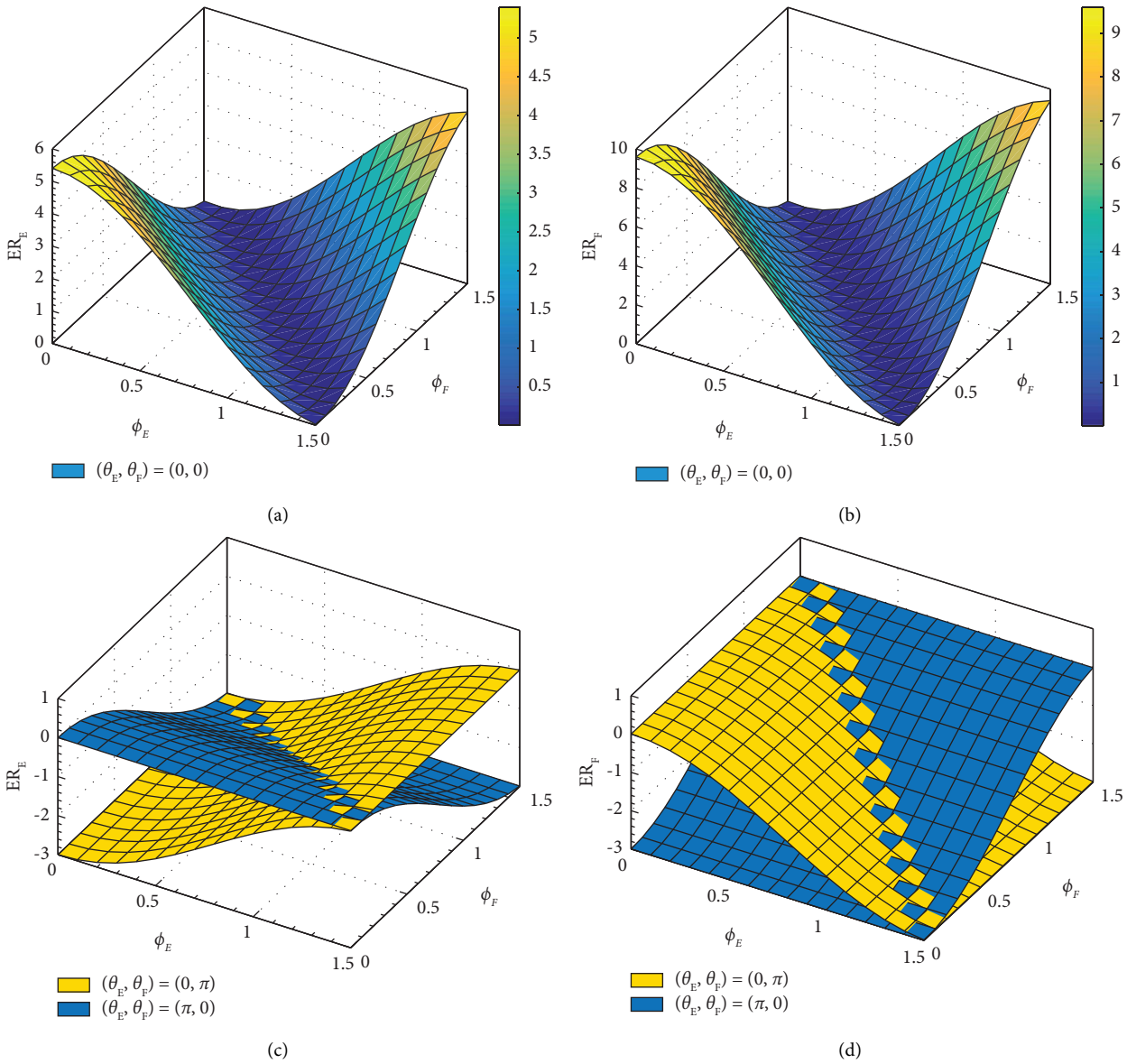


Figure 9: The influence of φ on the payoffs of alliance members in specific θ_i. (a-b) The impact of varying collaborative distribution capacities exhibited by the e-commerce platform and the 4PL platform on their respective revenues under the entangled state when $(\theta_E, \theta_F) = (0, 0)$. In (c-d), under the entangled state, when $(\theta_E, \theta_F) = (0, 0)$ and $(\theta_E, \theta_F) = (\pi, 0)$, the diverse collaborative distribution capacities demonstrated by the e-commerce platform and the 4PL platform affect their respective revenues.

Table 5: The profit matrix of alliance members under four strategies in the entangled state.

$\hat{U} = (\theta, \varphi)$	$\hat{U}_F(0, 0)$	$\hat{U}_F(0, \pi/2)$	$\hat{U}_F(\pi, 0)$	$\hat{U}_F(\pi, \pi/2)$
$\hat{U}_E(0, 0)$	(5.4, 9.6)	(0, 0)	(−3, 0)	(−3, 0)
$\hat{U}_E(0, \pi/2)$	$\begin{pmatrix} 5.4\cos^2\gamma\downarrow, \\ 9.6\sin^2\gamma\downarrow \end{pmatrix}$	(5.4, 9.6)	$\begin{pmatrix} -3\cos^2\gamma\uparrow, \\ -3\sin^2\gamma\downarrow \end{pmatrix}$	$\begin{pmatrix} -3\cos^2\gamma\uparrow, \\ -3\sin^2\gamma\downarrow \end{pmatrix}$
$\hat{U}_E(\pi, 0)$	(0, −3)	$\begin{pmatrix} -3\sin^2\gamma\downarrow, \\ -3\cos^2\gamma\uparrow \end{pmatrix}$	(0, 0)	(0, 0)
$\hat{U}_E(\pi, \pi/2)$	(0, −3)	$\begin{pmatrix} -3\sin^2\gamma\downarrow, \\ -3\cos^2\gamma\uparrow \end{pmatrix}$	(0, 0)	(0, 0)

Note. $\uparrow(\downarrow)$ represents the increase or decrease of each member of the e-commerce logistics alliance with γ.

(1) Within the collaborative distribution process of e-commerce logistics alliances, establishing several observable and quantifiable evaluation metrics or delegating a third-party institution to define assessment criteria, such as order completion volume, delivery time, and customer satisfaction rate, can transform implicit efforts into tangible indicators. This reduces information asymmetry among alliance members, minimizing the potential occurrence of bilateral moral hazards and reinforcing mutual trust.

(2) In addressing the incentive problem of collaborative distribution within e-commerce logistics alliances through the quantum game theory, the key lies in whether due consideration has been given to the effort levels, collaborative distribution capabilities, and entanglement among all alliance members. To ensure sufficient effort and efficient collaboration, an "entanglement contract" can be implemented before the collaborative distribution process. This protocol binds the interests of members, enhancing their interconnectedness. However, practical applications also require consideration of other factors, including trust levels among members and the prevailing market conditions, to formulate more comprehensive and rational quantitative metrics and "entanglement contract."

5. Conclusion

This paper investigated the incentive problem in collaborative distribution within an e-commerce logistics alliance. First, by analyzing the costs and benefits of alliance members in the context of collaborative distribution, the quantum game theory was introduced to quantumize the classical game model, achieving Pareto optimality in collaborative distribution within the e-commerce logistics alliance, thus reducing bilateral moral risks. Second, numerical simulations discussed the impact of different levels of quantum strategies and various quantum entanglement states on alliance members' strategic choices, providing critical conditions for the quantum game system. Finally, according to the abovementioned analysis, the research results were discussed about previous studies, and some management opinions were put forward. Based on the research content of this paper, the following main conclusions are drawn:

(1) The quantum game theory enhances the classical game theory by expanding the binary strategy sets, introducing quantum entanglement and potentially increasing the earnings of alliance members. It effectively addresses the "prisoner's dilemma" issue within the alliance, achieving consistency between Nash equilibrium and Pareto optimality. The benefits acquired by both parties in the game are superior when employing quantum strategies compared to classical game. Consequently, alliance members are more motivated to adopt quantum strategies to maximize their individual gains.

(2) According to the simulation results, as entanglement emerges, the likelihood of choosing complete effort strategies increases and also enhancing returns based on effort levels. It is evident that the profits obtained by alliance members in quantum entanglement states during collaborative distribution vary with the levels of effort and collaborative distribution capability. This significantly mitigates the "free-rider" issue and bilateral moral hazard. These findings highlight the importance of entanglement in promoting cooperative behavior and the advantages of quantum strategies in e-commerce logistics alliances.

While the quantum game model developed in this paper effectively promotes active participation in collaborative distribution among e-commerce logistics alliance participants, it does have certain limitations. First, as highlighted by the research conducted by Khoobkar et al. [32], a comprehensive analysis of stability equilibrium in game theory studies is essential, as it can unveil significant advancements of the proposed method over other approaches. Due to the constraints of our study, this paper temporarily examines the influence of different parameters in the quantum game model on participants' interests and decisions. However, a detailed numerical refinement and analysis of stability equilibrium are still part of our forthcoming series of research. Second, within the context of collaborative distribution in e-commerce logistics alliances, exploring alternative quantum game mechanisms can provide a more comprehensive assessment of the performance of quantum games in the collaborative distribution process.

Appendix

A. The proof of Propositions 5–7

A.1. The Proof of Proposition 5

$$ER_E = \left[(1-\beta)A\varepsilon\cos^2\frac{\theta_F}{2} - \frac{\omega_E}{2}\right]\cos^2\frac{\theta_E}{2},$$
$$ER_F = \left(\beta A\varepsilon\cos^2\frac{\theta_E}{2} - \frac{\omega_F}{2}\right)\cos^2\frac{\theta_F}{2}. \tag{A.1}$$

When $(1-\beta)A\varepsilon\cos^2\theta_F/2 - \omega_E/2 > 0$, that is, $\theta_F < 2\arccos\sqrt{\omega_E/2(1-\beta)A\varepsilon}$, ER_E decreases with θ_E. Therefore, when $\theta_F \in [0, \theta_F^*)$, $\theta_F^* = 2\arccos\sqrt{\omega_E/2(1-\beta)A\varepsilon}$ is a necessary and sufficient condition for ER_E to decrease with θ_E. When $(1-\beta)A\varepsilon\cos^2\theta_F/2 - \omega_E/2 < 0$, that is, $\theta_F > 2\arccos\sqrt{\omega_E/2(1-\beta)A\varepsilon}$, ER_E increases with θ_E. Therefore, when $\theta_F \in (\theta_F^*, \pi]$, $\theta_F^* = 2\arccos\sqrt{\omega_E/2(1-\beta)A\varepsilon}$ is a necessary and sufficient condition for ER_E to increase with θ_E. Similarly, the same applies to the 4PL e-commerce platform. Proposition 5 is proven.

A.2. The Proof of Proposition 6. When $\varphi_E = 0$, $ER_E = \{[(1-\beta)A\varepsilon - \omega_E/2]\cos^2\varphi_F\cos^2\theta_F/2 - \omega_E/2\sin^2\theta_F/2\}\cos^2\theta_E/2 - \omega_F/2\sin^2\varphi_F\sin^2\theta_E/2\cos^2\theta_F/2$, it is observed that when $[(1-\beta)A\varepsilon - \omega_E/2]\cos^2\varphi_F\cos^2\theta_F/2 - \omega_E/2\sin^2\theta_F/2 > 0$, the

first term on the right-hand side of the equation for ER_E decreases with θ_E, and when $\sin^2 \varphi_F \cos^2 \theta_F/2 > 0$, the second term on the right-hand side of the equation for ER_E decreases with θ_E. As a result, when $[(1-\beta)A\varepsilon - \omega_E/2] \cos^2 \varphi_F \cos^2 \theta_F/2 - \omega_E/2 \sin^2 \theta_F/2 \geq 0$ and $\sin^2 \varphi_F \cos^2 \theta_F/2 \geq 0$ hold simultaneously and they are not simultaneously equal to "=," ER_E decreases with θ_E, that is, increases with the effort level θ_E. Similarly, the same applies to the 4PL e-commerce platform. Proposition 6 is proven.

A.3. The Proof of Proposition 7. Let us examine the variations of the e-commerce platform's revenue ER_E when θ_E and θ_F change.

$$\begin{aligned} ER_E = \Bigg\{ & \left[(1-\beta)A\varepsilon - \frac{\omega_E}{2}\right]\cos^2(\varphi_E + \varphi_F)\cos^2\frac{\theta_F}{2} \\ & - \frac{\omega_E}{2}\cos^2\varphi_E \sin^2\frac{\theta_F}{2}\Bigg\}\cos^2\frac{\theta_E}{2} \\ & - \frac{\omega_E}{2}\sin^2\varphi_F \sin^2\frac{\theta_E}{2}\cos^2\frac{\theta_F}{2}. \end{aligned} \tag{A.2}$$

When $\varphi_E = \pi/2$, $ER_E = [(1-\beta)A\varepsilon \cos^2 \theta_E/2 - \omega_E/2 \sin^2 \theta_E/2]\sin^2 \varphi_F \cos^2 \theta_F/2$, it is observed that $(1-\beta)A\varepsilon \cos^2 \theta_E/2 - \omega_E/2 \sin^2 \theta_E/2$ decreases with θ_E. Therefore, when $\sin^2 \varphi_F \cos^2 \theta_F/2 > 0$, it guarantees that the revenue ER_E of the e-commerce platform decreases with θ_E, indicating an increase in effort level e_E. The same holds true for the 4PL platform. Thus, Proposition 7 is verified.

Data Availability

No data were used to support the findings of this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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