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Research Article

Impact of Corporate Social Responsibility on Operations of a Live-Streaming Supply Chain

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Corporate social responsibility (CSR) is widely noticed as an essential tool for business operation and sustainable development. Meanwhile, the fiercely competitive external environment and unpredictable events prompt enterprises' cooperation to prevent supply chain collapse. We investigate the cooperative strategy in a live-streaming supply chain (LSC) consisting of a dominant brand owner, a retailer, and a live streamer, where the brand owner considers CSR by considering the welfare of stakeholders. We construct one non-cooperative and three cooperative Stackelberg game models to explore the impact of CSR on cooperative strategy and LSC operations. The results show the following. (1) When the brand owner considers CSR, LSC members and systems are more profitable in the four models than when the brand owner does not consider CSR. (2) When the flow effect is small, the brand owner tends to cooperate with the retailer; otherwise, the brand owner prefers to cooperate with the live streamer. (3) The grand coalition C (the brand owner cooperates with the retailer and live streamer) is the consistent strategy for the LSC system, consumers, and society. These findings help enterprises recognize the importance of CSR and collaboration, thus further providing reference opinions on engaging in CSR and how to achieve collaboration.

1. Introduction

With the upgrade of people's consumption patterns, the live-streaming e-commerce industry has exploded, becoming a "new driver" of economic growth [1, 2]. According to statistics, more than 50% of the brand owners seized the new trend of Taobao Live, and the full-day live revenue during the "Double 11" was nearly 20 billion RMB. The total scale of China's live-streaming e-commerce industry reached 1,201.2 billion RMB in 2021 and will reach 2137.3 billion RMB by 2025 [3]. As an emerging social marketing method, live-streaming e-commerce has real-time interactivity, social shopping attributes, a fan economy, and a two-way inflow of content and e-commerce compared to traditional e-commerce modes [4–6]. This shortens the distance between marketing and trading, improves the shopping experience of consumers [1], and enriches the channels of consumer shopping, attracting more and more enterprises to join the ranks of live-streaming marketing [5, 7]. In practice, the typical live-streaming shopping mode is where a brand owner hires a live streamer [6]. Thus, many brand owners not only started to sell their products through live-streaming channels but also retained their traditional RCs, where brand owners, live streamers, retailers, and consumers constitute the live-streaming supply chains (LSCs) [8].

In the live-streaming supply chain (LSC), the high turnover is inseparable from the "flow" of the live streamer, and the "flow" has a positive impact on the consumers' continued viewing and purchasing desire [9, 10]. On September 20, 2022, "Lipstick One" Li Jiaqi broadcasted live on Taobao. Without preheating and product previews, the live broadcast had 5 million views in a single half-hour session and 21 million views in a single one-hour session, with 4 products sold out in the mall, and the final viewership exceeded 63.5 million (https://www.thepaper.cn/newsDetail_forward_20007107, Accessed on 21 September

2022). Furthermore, we can realize that the competition between traditional retail and live-streaming channels has become increasingly fierce when brand owners hire live streamers to market products. Meanwhile, there is a spillover effect of the "flow" of the live streamers and the product display in the live room on the traditional retail channels [8, 11]. Positive spillover effects drive consumers to be more interested in the products, thereby expanding the demand for traditional channels. Based on this, this paper focuses on the impact of the live streamers' flow effect and the positive spillover effect on the pricing and operation strategies of LSC.

Enterprises are gradually realizing the importance of cooperation in the context of fierce competition between supply chain channels [12, 13]. Existing scholars have confirmed that cooperation can eliminate the negative effects of double marginalization, bring huge benefits to enterprises, and improve their competitiveness and supply chain performance [14, 15]. In practice, a large mobile phone brand, Xiaomi, and Youmi, a Korean smartphone retailer, have allied to expand their mobile phone market in Korea [12]. Changhong, a well-known Chinese home appliance brand, has established a partnership with Suning, a famous downstream retailer [16]. Several beauty brands, such as Tom Ford, YSL, Dior, Givenchy, Flower West, Perfect Diary, and Ukiss, have cooperated with live streamer Li Jiaqi to market their products with the help of live streamer's professional merchandising ability (https://www.sohu.com/a/355523279_100156659, Accessed on 23 November 2019). A more in-depth study of LSC members' cooperation motivations and strategic choices is relevant and necessary, and thus this paper fills in this work.

With the increasingly frequent live-streaming overturning incidents, consumers' awareness of environmental protection, and sustainable development concepts, enterprises have found that consumers' motivation to purchase products should include not only economic factors such as price but also social factors such as corporate social responsibility (CSR) (https://zhuanlan.zhihu.com/p/356506325, Accessed on 16 March 2023) [16–18]. Consequently, more enterprises have released CSR reports gaining consumers' trust and cooperation opportunities with partners [19]. For example, the number of CSR reports released by Chinese enterprises from 2009 to 2019 has shown an increasing trend year by year (https://finance.sina.com.cn/esg/investment/2020-11-12/doc-iiznctke1113520.shtml, Accessed on 11 November 2020). In addition, consumers identify with and are somewhat loyal to companies that undertake CSR and are willing to actively purchase the company's products and pay higher prices [19]. But there are some brand owners, such as Nike and Adidas, who have faced consumer resistance and brand reputation damage due to negatively fulfilling CSR [16]. Thus, brand owners as the dominant players usually implement CSR that benefits stakeholders to improve supply chain performance in LSCs [16]. For instance, Hongxing Erke donated 50 million RMB of materials during the rescue of a rainstorm and flood in Henan Province. As a result, the sales volume of its live-streaming room exceeded 100 million RMB (https://www.cls.cn/detail/797759, Accessed on 24 July 2021). Therefore, this paper investigates whether brand owners that engage in CSR can gain consumer recognition and greater corporate reputation and improve the operational efficiency of LSC.

However, there is little related literature examining enterprises' cooperative motivations and strategic choices under considering enterprises' CSR awareness in an LSC. We consider the live streamers' flow effect and the positive spillover effect, unlike much literature that considers the live streamers' sales efforts. Therefore, this paper aims to explore the brand owner's cooperative strategies in an LSC consisting of a brand owner, a retailer, and a live streamer. In the LSC, we construct four game-theoretic models: the brand owner cooperates with the retailer and live streamer to form a centralized model (Model C), a decentralized model where each member makes their decisions (Model D), the brand owner cooperates with the retailer to form a coalition BR (Model BR), and the brand owner cooperates with the live streamer to form a coalition BL (Model BL). We try to solve the following three questions. (1) When the brand owner has CSR awareness, does the brand owner's CSR benefit each LSC member, coalition BR, coalition BL, and LSC system? (2) For the brand owner, does he prefer to collaborate with the retailer or live streamer under what conditions? (3) What is the impact of CSR on the profits of LSC, consumer surplus, and social welfare? Is there a collaborative strategy to reach a profitable multiparty situation for LSC, consumers, and society?

To solve the above three questions, we first obtain the equilibrium solutions for four game models and analyze whether the brand owner should consider CSR. Then, we analyze the relationship between cooperative strategies and game parameters, including the degree of the flow effect spillover effect, and the flow effect of the live streamer. Meanwhile, we compare the optimal decisions and the members' profits to explore the optimal cooperative strategy for the brand owner in the LSC. Furthermore, the impact of CSR on LSC's profit, consumer surplus, and social welfare was verified through numerical analysis. Finally, we expand the case study to go into more qualitative depth on CSR practices and motivations. We obtain the following findings: when the brand owner considers CSR, the profits of the brand owner, retailer, live streamer, coalition BR, coalition BL, and LSC system in the four models are higher than those when the brand owner does not consider CSR. The brand owner tends to cooperate with the retailer when the flow effect is small. On the contrary, the brand owner prefers to cooperate with the live streamer when the flow effect is large. Moreover, when the brand owner cooperates with the retailer (live streamer), the profit of the live streamer (retailer) in coalition BR (BL) is lower than that of decentralized model D. When the brand owner cooperates with the retailer and the live streamer to form a grand coalition C, the coalition C is the consistent strategy for the LSC system, consumers, and society.

The main contributions are as follows. First, we explore the impact of the live streamers' flow effect and the positive spillover effect considering the brand owner's CSR on the pricing and operation strategies of LSC, which is rarely discussed compared to related literature. Second, we

construct four game-theoretic models to enrich the theoretical study of examining the effect of CSR on cooperation strategies for enterprises. This can provide suitable cooperation strategies for considering CSR and inform suggestions for live-streaming practice operational decisions. Finally, we propose a partial cooperation strategy for the brand owner. We find that the coalition strategy can achieve a win-win-win situation for consumers, LSC, and society. The findings of this paper are not only helpful to expanding research on live-streaming operations and CSR but also consistent with examples of platform operational value in practice, such as the beauty industry we expanded on in Section 5.

The remainder of our work is structured as follows. Section 2 elaborates on the three relevant literature streams: LSC operation, supply chain cooperative strategy, and CSR in supply chains. Section 3 formulates the problem and gives model assumptions. The equilibrium solutions are obtained for Models C, D, BR, and BL in Section 4. Section 5 analyzes and compares the optimal outcomes of the four scenarios and explores the optimal cooperative strategies of the brand owner. The section further investigates the impact of CSR on the LSC, consumer surplus, and social welfare. In addition, a case study—the Beauty Industry—is also studied. Finally, Section 6 concludes this paper's main conclusions.

2. Literature Review

2.1. LSC Operation. LSC as an emerging business model has received widespread attention from enterprises, consumers, and researchers [6, 9, 10, 20]. Many scholars conducted extensive research on whether enterprises establish live-streaming channels [6, 9]. Zhang and Tang [6] examined the unique factors of live-streaming, such as commission rates and the number of live-streaming fans, that affect manufacturers' opening of live-streaming channels. Chen et al. [20] explored the live-streaming channel strategy for an online retailer. Ji et al. [5] gave decision makers a threshold for adopting live-streaming channels. Zhang et al. [21] studied whether to introduce live-streaming services. Pan et al. [2] found that sellers should only open live-streaming channels when the sales ability of the live streamer exceeds the time cost of consumers. Under the establishment of live-streaming channels, scholars developed a strong interest in the selection of sales models between live-streaming channels and other sales channels [1, 22]. Xu et al. [23] studied the manufacturer's sales modes of products directly or indirectly through the platform that provides live-streaming functionality. He et al. [24] investigated the optimal contract design for live-streaming shopping by designing three cooperation methods between retailers and live streamers. In addition, some scholars have focused on live-streaming marketing strategies based on the introduction of live-streaming channels [25]. Under multichannel sales, Xin et al. [4] analyzed three e-commerce live-streaming product showcasing modes (i.e., brand self-live streaming, influencer-led live-streaming mixture, and influencer-led special live-streaming) and explored strategic product showcasing mode. Xie et al. [26] studied the optimal live-streaming service strategy.

The above-existing research studies mainly focus on whether enterprises establish live-streaming channels and the selection of sales models after building live-streaming channels. Few scholars have focused on the flow effect of live streamers and the impact of spillover effects generated by the flow on the operational strategies of LSCs. Therefore, we incorporate the flow effect of a live streamer and the spillover effect into the LSC (i.e., a brand owner, a retailer, and a live streamer) to consider the operation strategy.

2.2. Supply Chain Cooperative Strategy. Supply chain leaders tend to establish partnerships with others to cope with fierce competition and improve the efficiency and competitiveness of the supply chain. Most literature considers cooperative strategies in closed-loop supply chains (CLSCs) [14, 15, 27]. Chen and Chang [28] analyzed under what conditions original equipment manufacturers can adopt cooperative strategies by participating in remanufacturing. Jena and Sarmah [29] indicated that the global cooperative system is the best. Zheng et al. [15] and Zu-Jun et al. [27] found that collaboration can increase alliance profits and supply chain performance. Wei et al. [30] found that the manufacturer's optimal choice is to integrate the retailer and only one collector when the collection competition is intense. In green supply chains, Zhang et al. [13] found that cooperation can help increase the profit of channels or members and improve product greenness. Xie [31] believed that cooperative strategies and parameter adjustments can effectively improve the supply chain's sustainability. In addition to considering cooperative strategies in CLSCs and green supply chains, cooperative strategies in other types of supply chains have attracted scholars' attention. For instance, in a green CLSC, a model of non-cooperation among members and all possible alliance forms among members were examined [32]. Leng and Parlar [33] used cooperative game theory to analyze whether there is a unique allocation scheme to save costs. In a sustainable supply chain, Liu et al. [14] studied how a retailer's fairness concerns affect cooperative strategies.

The research on cooperative strategies for supply chain operation management mainly focuses on CLSCs, green supply chains, sustainable supply chains, etc. However, there is little literature on considering cooperative strategies in LSCs. Accordingly, our research fills this gap and explores the impact of the brand owner's CSR behavior on cooperative strategies and supply chain decisions.

2.3. CSR in Supply Chains. CSR promotes an enterprise's brand image, enhances its social value, and affects the enterprise's supply chain performance [19, 34, 35]. In recent years, many scholars have introduced CSR into supply chain optimization operations and focused on examining the impact of CSR behavior on pricing, collection rate, recycling, member profits, and performance [36–38]. Panda et al. [17] focused on manufacturer exhibits of CSR to analyze the coordination of the supply chain. A contract-bargaining

process was proposed to resolve channel conflict and to allocate surplus profit among the channel members. The manufacturer demonstrates CSR by using reverse channels to recycle used products collected by the retailer [18]. Only when the manufacturer's CSR exceeds a certain threshold, the manufacturer's CSR benefits consumers, retailers, and supply chain systems, but it can harm the profits of leading companies [39]. The above research on CSR aims to improve stakeholders' interests (i.e., employees, customers, consumers, government, competitors, upstream and downstream partners, interest groups, etc.) by setting goals that consider maximizing consumer surplus social welfare. In this case, CSR is considered an exogenous variable. Moreover, CSR investment is an endogenous variable. Modak and Kelle [35] and Modak et al. [40] considered social donation activities, as a CSR investment, and further examined the CSR in CLSCs' operations. Liu et al. [41] examined the impact of CSR cost information asymmetry on supply chain decisions.

The aforementioned literature has examined the impact of CSR on supply chain operations from various aspects, but much literature has not explored the impact of CSR on LSC pricing decisions, profits, and cooperative strategies in the existing literature. Thence, we incorporate the brand owner's CSR into LSC to investigate LSC's cooperative strategies.

3. Problem Description and Model Assumptions

We consider an LSC consisting of a brand owner (B), a retailer (R), and a live streamer (L), where the brand owner sells products through traditional retail and live-streaming channels. In the retail channel (henceforth RC), the retailer purchases products from the brand owner at a unit wholesale price w and sells products to consumers at a unit retail price p_r. In the live-streaming channel (henceforth LC), the brand owner hires a live streamer to market products to consumers at a unit retail price p_l. We examine possible coalitions to explore the optimal cooperative strategy based on common examples from reality. Thus, four models are constructed: the brand owner cooperates with the retailer and live streamer to form a centralized model C (Model C), each member makes their decisions to form a decentralized model D (Model D), the band owner cooperates with the retailer to form a coalition BR (Model BR), and the brand owner cooperates with the live streamer to form a coalition BL (Model BL), which are shown in Figure 1.

In Figure 1, we notice that the "pit fee F + unit commission fee m" is the most common cooperation when the brand owner hires a live streamer for product marketing (https://chinae.com.cn/mobile/art_info.php?id=11881, Accessed on 28 December 2022). For instance, Li Jiaqi and some other famous live steamers cooperate with brand owners, such as Jo Malone, MAC, and Proya (https://www.dsb.cn/199178.html, Accessed on 11 October 2022). We summarize the parameters shown in Table 1.

To make the analysis tractable, we introduce the following assumptions.

3.1. Consumer Preference. Following Ji et al. [5], Zheng et al. [15], and Zhang et al. [42], we assume that consumers'willingness to pay for a product from RC is v, which is assumed to be uniformly distributed in [0,1]. However, compared to RC, consumers buy products from LC with discount value θ, $\theta \in [0,1]$, where the parameter θ denotes the consumers' channel acceptance, and the greater θ represents the degree of competition between RC and LC [7]. This assumption is reasonable because consumers need to participate in the live-streaming show within a fixed time to purchase products [2]. Thence, the utility of consumers from RC and LC is $U_r = v - p_r + bf$ and $U_l = \theta v - p_l + f$, respectively [1, 14, 42]. b represents the positive spillover effect of LC sales on the RC sales, and we assume that $b \in [0,1]$, in which $b = 0$ means no effect, and $b > 0$ indicates that the LC positively affects the RC [8].

3.2. Consumer Demand. If $U_r \geq U_l$ and $U_r \geq 0$, i.e., $v \geq (p_r - p_l + (1-b)f)/(1-\theta)$ and $v \geq p_r - bf$, consumers will purchase a product from RC; if $U_r < U_l$ and $U_l \geq 0$, i.e., $v < (p_r - p_l + (1-b)f)/(1-\theta)$ and $v \geq (p_l - f)/\theta$, consumers will purchase a product from LC [7]. Similar to Zheng et al. [15] and Yan et al. [11], we only pay attention to the coexistence of products in both RC and LC, i.e., $(p_r - p_l + (1-b)f)/(1-\theta) > (p_l - f)/\theta$. Thus, RC's and LC's demand functions are given as $D_r = 1 - (p_r - p_l + (1-b)f)/(1-\theta)$ and $D_l = (\theta(p_r - bf) - (p_l - f))/(\theta(1-\theta))$, respectively.

3.3. CSR. Because manufacturing enterprises usually determine the quality and function of products, fulfilling CSR can establish a brand image, enhance brand value, and gain the trust and loyalty of consumers, ultimately forming a brand advantage [18]. Therefore, brand owners as the dominant player typically have more initiative to undertake CSR [16]. Following Zhang et al. [43], Panda et al. [17], and Panda et al. [18], we assume that brand owner's CSR is reflected through their focus on consumer surplus. Consumer surplus can be expressed as $CS = \int_{(p_r - p_l + (1-b)f)/(1-\theta)}^{1} (v - p_r + bf)dv + \int_{(p_l - f)/\theta}^{(p_r - p_l + (1-b)f)/(1-\theta)} (\theta v - p_l + f)dv$ [7]. When the brand owner considers CSR, the brand owner's total profit consists of his pure profit and a CSR measure, i.e., $V_B = \pi_B + rCS$. Where r∈[0,1], r=0 implies that the brand owner is the pure profit maximizer, r∈(0,1) means that the brand owner undertakes CSR, but it is not a perfect maximization of social welfare, and r=1 represents that the brand owner is the perfect welfare maximizer [17, 18].

3.4. Cost Structure. Since the live streamer or retailer only displays products, we assume that the unit product operating cost of RC and LC is zero [21]. Moreover, the production cost of products is assumed to be zero for convenience of analysis [3, 6, 44].

In the following part, the equilibrium demands in the four models are negative if the flow effect f is sufficiently large (i.e., $f \geq f_0$). Therefore, we focus on the case of $f < f_0$

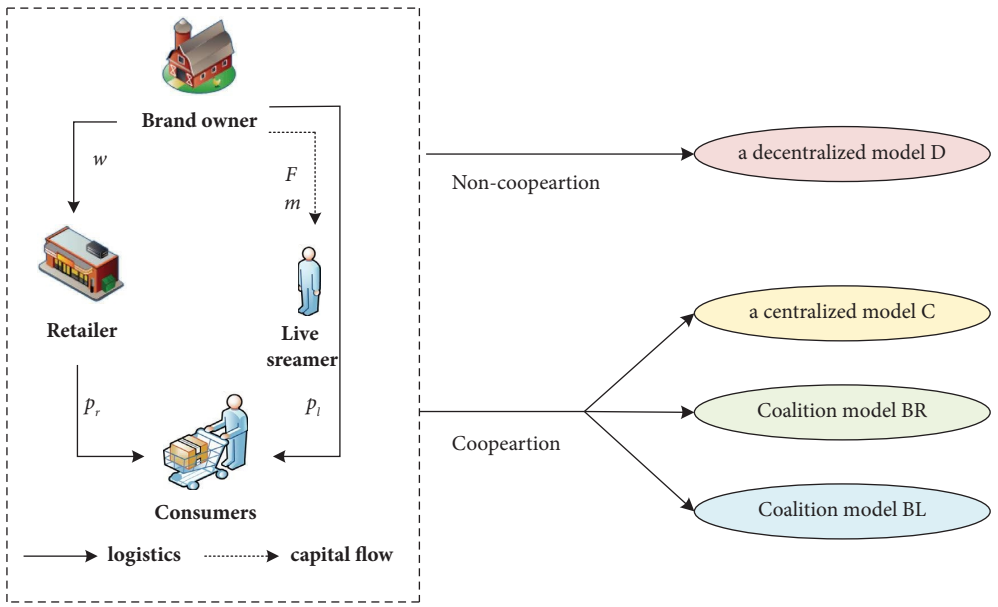

Figure 1: The non-cooperative and cooperative models of the LSC.

Table 1: Parameters and decision variables.

Notations	Definition
v	Consumers' willingness to pay for products
F	Pit fee paid by the brand owner to live streamer
θ	Consumer's channel acceptance of LC
b	Degree of the flow effect spillover effect
f	The flow effect of the live streamer
r	Coefficient of CSR
D_r/D_l	The demand for RC/LC
w	Unit wholesale price of RC
p_r/p_l	Unit retail price of RC/LC
m	Unit commission fee paid by the brand owner to live streamer
π_i^j	Pure profit of LSC members i in model j, $i \in \{B, L, R, \text{BR}, \text{BL}, T\}$ and $j \in \{C, D, \text{BR}, \text{BL}\}$
V^j	Total profit of the decision maker in model j, $j \in \{C, D, \text{BR}, \text{BL}\}$
CS^j	Consumer surplus in model j, $j \in \{C, D, \text{BR}, \text{BL}\}$

throughout our paper to maintain positive equilibrium demands [21, 32], where $f_0 = (1-\theta)/(1-b)$.

4. Models and Equilibrium Result Analysis

4.1. The Centralized Model C. In this model, the brand owner, retailer, and live streamer adopt centralized decision making to maximize the overall benefits of the LSC to decide retail prices (i.e., p_r and p_l). At this time, the objective function of the supply chain to pursue social welfare (total profit) maximization is formulated as

$$\max_{p_r, p_l} V_T^{\text{Ci}} = p_r\left(1 - \frac{p_r - p_l + (1-b)f}{1-\theta}\right) + p_l\frac{\theta(p_r - bf) - (p_l - f)}{\theta(1-\theta)} + r\text{CS}, \tag{1}$$

where $i \in \{N, Y\}$ and model CN and model CY indicate whether the brand owner considers CSR, which means that model CN and model CY, respectively, represent two scenarios: $r = 0$ and $r \in (0,1]$. By first-order conditions, we obtain the equilibrium results for models CN and CY, which are described in Table 2.

Corollary 1. *(1) In models CY and CN, as b increases, p_r increase, while p_l remain unchanged, i.e., $\partial p_r^{Ci*}/\partial b > 0$ and $\partial p_l^{Ci*}/\partial b = 0$. (2) When the brand owner considers CSR, i.e., $r \in (0,1]$, p_r and p_l decrease as r increase, i.e., $\partial p_r^{CY*}/\partial r < 0$ and $\partial p_l^{CY*}/\partial r < 0$. (3) The profit of LSC in model CY is greater than that in model CN, i.e., $V_T^{CY*} > V_T^{CN*}$.*

Corollary 1(1) indicates that as the spillover effect increases, it could increase the retail prices of RC, while the equilibrium retail prices of LC remain unchanged whether

Table 2: Equilibrium results for models CN and CY.

Equilibrium	Model CN	Model CY
p_r^*	$(1+bf)/2$	$(1-r)(1+bf)/(2-r)$
p_l^*	$(\theta+f)/2$	$(1-r)(\theta+f)/(2-r)$
D_r^*	$(1-\theta-f+bf)/(2(1-\theta))$	$(1-\theta-f+bf)/((1-\theta)(2-r))$
D_l^*	$(1-\theta b)f/(2\theta(1-\theta))$	$(1-\theta b)f/(\theta(1-\theta)(2-r))$
V_T^*	$(\theta(1-\theta)(1+2bf)+((1-\theta b)^2+\theta b^2(1-\theta))f^2)/(4\theta(1-\theta))$	$(\theta(1-\theta)(1+2bf)+((1-\theta b)^2+\theta b^2(1-\theta))f^2)/(2\theta(1-\theta)(2-r))$

the brand owner has CSR behavioral awareness. This means that the flow spillover effect of the live streamer has a positive spillover effect, which has prompted the retailer to increase retail prices. Corollary 1(2) presents that a higher CSR coefficient could decrease the equilibrium retail prices. This is because when the dominant brand owner has CSR behavioral awareness, he actively lowers retail prices to better benefit his stakeholders (i.e., retailer and live streamer), thus maximizing social welfare. Corollary 1(3) shows that when the brand owner considers CSR, the profit of LSC is higher than that of the brand owner as a pure profit earner.

4.2. The Decentralized Model D. In this model, the brand owner, retailer, and live streamer play a Stackelberg game, and the brand owner considers CSR. Specifically, the brand owner first determines the wholesale price w and the retail price p_l, and then the retailer and live streamer decide the retail price p_r and unit commission fee m, respectively. The optimal model is formulated as

$$\begin{aligned}
&\max_{w,p_l} V_B^{Di} = w\left(1 - \frac{p_r - p_l + (1-b)f}{1-\theta}\right) + (p_l - m)\frac{\theta(p_r - bf) - (p_l - f)}{\theta(1-\theta)} - F + r\mathrm{CS},\\
&s.t. \max_{p_r} \pi_R^{Di} = (p_r - w)\left(1 - \frac{p_r - p_l + (1-b)f}{1-\theta}\right),\\
&\max_{m} \pi_L^{Di} = m\left(\frac{\theta(p_r - bf) - (p_l - f)}{\theta(1-\theta)}\right) + F,
\end{aligned} \tag{2}$$

where $i \in \{N, Y\}$ and model DN and model DY indicate whether the brand owner considers CSR. Following Cao et al. [7], we introduce the margin profit $d = p - m$ of the brand owner to solve the game through reverse induction. The optimal results are given in Table 3.

Corollary 2. *(1) In models DY and DN, as b increases, w and p_r increase, while p_l and m decrease (i.e., $\partial w^{Di*}/\partial b > 0$, $\partial p_r^{Di*}/\partial b > 0$, $\partial p_l^{Di*}/\partial b < 0$, and $\partial m^{Di*}/\partial b < 0$). (2) When the brand owner considers CSR, i.e., $r \in (0,1]$, as r increases, w, p_r, and p_l decrease but m increases (i.e., $\partial w^{DY*}/\partial r < 0$, $\partial p_r^{DY*}/\partial r < 0$, $\partial p_l^{DY*}/\partial r < 0$, and $\partial m^{DY*}/\partial r > 0$). (3) The profits of the brand owner, retailer, live streamer, and LSC in model DY are greater than those in model DN (i.e., $V_B^{DY*} > V_B^{DN*}$, $\pi_R^{DY*} > \pi_R^{DN*}$, $\pi_L^{DY*} > \pi_L^{DN*}$, and $V_T^{DY*} > V_T^{DN*}$).*

Corollary 2(1) indicates how the equilibrium decisions change with the spillover effect in the decentralized model. Especially, the spillover effect has a positive effect on the wholesale and retail prices of RC, while it hurts the retail prices of LC and unit commission fees. For the RC, the retailer increases the retail prices due to the positive spillover effect of the live streamer. However, the flow overflow of the live streamer improves the competitiveness of RC while weakening the advantages of LC. Thus, in the LC, the brand owner lowers the retail price to further attract consumers.

Corollary 2(2) presents that a higher CSR coefficient could decrease the wholesale and retail prices, while it increases the commission fee in model DY. When the dominant brand owner has CSR behavioral awareness, he aims to better benefit his stakeholders, rather than pursuing profit maximization. On the one hand, the brand owner lowers wholesale prices and retail prices of LC to benefit the retailer and consumers. In this case, the retailer is willing to lower the retail price of RC to benefit consumers due to the decrease in wholesale prices. On the other hand, live streamer charges higher unit commission fee when the brand owner considers CSR, which indicates that the brand owner cares for the benefits of downstream supply chain members. Similar to Corollary 1(3), Corollary 2(3) indicates that when the brand owner considers CSR, the profits of the brand owner, retailer, live streamer, and LSC are higher than those of the brand owner as a pure profit earner.

4.3. The Brand Owner and Retailer Form a Coalition BR. In this model, the brand owner cooperates with the retailer to form a coalition BR, which first decides retail prices p_r and maximizes its profit. Then, the live streamer sets its unit commission m. The coalition model is given as

$$\begin{aligned}
&\max_{p_r,p_l} V_{\mathrm{BR}}^{\mathrm{BR}i} = p_r\left(1 - \frac{p_r - p_l + (1-b)f}{1-\theta}\right) + (p_l - m)\frac{\theta(p_r - bf) - (p_l - f)}{\theta(1-\theta)} + r\mathrm{CS},\\
&s.t. \max_{m} \pi_L^{\mathrm{BR}i} = m\frac{\theta(p_r - bf) - (p_l - f)}{\theta(1-\theta)},
\end{aligned} \tag{3}$$

Table 3: Equilibrium results for models DN and DY.

Equilibrium	Model DN	Model DY
p_r^*	$(2(1+bf)(3-\theta)-(\theta+f))/(2(4-\theta))$	$((1+bf)(r^2-7r-4\theta+12)-2(\theta+f))/(4(4-2r-\theta)+r^2)$
w^*	$(1+bf)/2$	$((1+bf)(r^2-6r-2\theta+8)-(\theta+f)r)/(4(4-2r-\theta)+r^2)$
p_l^*	$((\theta+f)(5-2\theta)+f(1-\theta b))/(2(4-\theta))$	$((\theta+f)(r^2-7r-4\theta+10)+2f(1-\theta b))/(4(4-2r-\theta)+r^2)$
m^*	$((\theta+f)(1-\theta)+(1-\theta b)f)/(2(4-\theta))$	$(2(\theta+f)(1-\theta)+f(2-r)(1-\theta b))/(4(4-2r-\theta)+r^2)$
D_r^*	$(2(1-\theta-f+bf)+f(1-\theta b))/(2(1-\theta)(4-\theta))$	$((4-r)(1-\theta-f+bf)+2f(1-\theta b))/((1-\theta)(4(4-2r-\theta)+r^2))$
D_l^*	$((\theta+f)(1-\theta)+f(1-\theta b))/(2\theta(1-\theta)(4-\theta))$	$(2(\theta+f)(1-\theta)+f(2-r)(1-\theta b))/(\theta(1-\theta)(4(4-2r-\theta)+r^2))$
V_B^*	$((\theta b^2(1-\theta)+((1-\theta b)^2+\theta b^2(1-\theta))f^2)/(4\theta(1-\theta)(4-\theta))$	$(2\theta b^2(1-\theta)+(((1-\theta b)^2+\theta b^2(1-\theta))(2-r))f^2)f^2/(2\theta(1-\theta)(4(4-2r-\theta)+r^2))$
π_R^*	$+((1-\theta)((\theta+f)^2+2\theta(1+2bf))/(4\theta(1-\theta)(4-\theta))-F$	$+((1-\theta)(2(\theta+f)^2+\theta(4-r)(1+2bf)))/(2\theta(1-\theta)(4(4-2r-\theta)+r^2))-F$
π_L^*	$((2(1-\theta-f+bf)+f(1-\theta b))^2)/(4(1-\theta)(4-\theta)^2)$	$(((4-r)(1-\theta-f+bf)+2f(1-\theta b))^2)/((1-\theta)(4(4-2r-\theta)+r^2)^2)$
V_T^*	$(((\theta+f)(1-\theta)+(1-\theta b)f)^2)/(4\theta(1-\theta)(4-\theta)^2)+F$	$((2(\theta+f)(1-\theta)+f(2-r)(1-\theta b))^2)/(\theta(1-\theta)(4(4-2r-\theta)+r^2)^2)+F$
	$V_B^*+\pi_R^*+\pi_L^*$	$V_B^*+\pi_R^*+\pi_L^*$

where $i \in \{N, Y\}$ and model BRN and model BRY indicate whether the brand owner considers the CSR. We obtain the equilibrium results for models BRN and BRY through reverse induction. We have the optimal outcomes described in Table 4.

Corollary 3. *(1) In models BRY and BRN, as b increases, p_r increase but p_l and m decrease (i.e., $\partial p_r^{BRi*}/\partial b > 0$, $\partial p_l^{BRi*}/\partial b < 0$, and $\partial m^{BRi*}/\partial b < 0$). (2) When the coalition BR considers CSR, i.e., $r \in (0,1]$, as r increases, p_r and p_l decrease but m increases (i.e., $\partial p_r^{BRY*}/\partial r < 0$, $\partial p_l^{BRY*}/\partial r < 0$, and $\partial m^{BRY*}/\partial r > 0$). (3) The profits of the coalition BR, live streamer, and LSC in model BRY are greater than those in model BRN (i.e., $V_{BR}^{BRY*} > V_{BR}^{BRN*}$, $\pi_L^{BRY*} > \pi_L^{BRN*}$, and $V_T^{BRY*} > V_T^{BRN*}$).*

Corollary 3(1) indicates how the equilibrium decisions change with the spillover effect in coalition of the brand owner and the retailer. When the brand owner cooperates with the retailer, the retail prices of RC increase with the increase of the spillover effect, while the retail prices of LC and unit commission fees decrease. Corollary 3(2) presents that a higher CSR coefficient could decrease the equilibrium retail prices, while it increases the commission fee in model BRY. When the brand owners collaborate with the retailer, the alliance BR, as the leading decision maker, has a CSR behavioral awareness that benefits stakeholders, thereby providing higher unit commissions and reducing retail prices. Corollary 3(3) shows that when the brand owner considers CSR, the profits of coalition BR, live streamer, and LSC are higher than those of the brand owner as a pure profit earner.

4.4. The Brand Owner and Live Streamer Form a Coalition BL. In this model, the brand owner cooperates with the live streamer to form a coalition BL, which first decides the wholesale price w and retail price p_l to maximize its profit. Then, the retailer determines the retail price p_r. The coalition optimal model is given as

$$\begin{aligned} &\max_{w,p_l} V_{\mathrm{BL}}^{\mathrm{BL}i} = w\left(1 - \frac{p_r - p_l + (1-b)f}{1-\theta}\right) + p_l\frac{\theta(p_r - bf) - (p_l - f)}{\theta(1-\theta)} + r\mathrm{CS}, \\ &s.t. \max_{p_r} \pi_R^{\mathrm{BL}i} = (p_l - w)\left(1 - \frac{p_r - p_l + (1-b)f}{1-\theta}\right), \end{aligned} \tag{4}$$

where $i \in \{N, Y\}$ and model BLN and model BLY indicate whether the brand owner considers CSR. We obtain the equilibrium results for models BLN and BLY through reverse induction. The optimal results are described in Table 5.

Corollary 4. *(1) In models BLY and BLN, as b increases, w and p_r increase but p_l remain unchanged (i.e., $\partial w^{BLi*}/\partial b > 0$, $\partial p_r^{BLi*}/\partial b > 0$, and $\partial p_l^{BLi*}/\partial b = 0$). (2) When the coalition BL considers CSR, i.e., $r \in (0,1]$, w, p_r, and p_l decrease as r increases (i.e., $\partial w^{BLi*}/\partial r < 0$ $\partial p_r^{BRY*}/\partial r < 0$, and $\partial p_l^{BRY*}/\partial r < 0$). (3) The profits of coalition BL, retailer, and LSC in model BLY are greater than those in model BLN (i.e., $V_{BL}^{BLY*} > V_{BL}^{BLN*}$, $\pi_R^{BLY*} > \pi_R^{BLN*}$, and $V_T^{BLY*} > V_T^{BLN*}$).*

Corollary 4(1) indicates how the equilibrium decisions change with the spillover effect in coalition BL. When the brand owner cooperates with the live streamer, the retail prices of RC increase with the increase of the spillover effect, while the retail prices of LC and unit commission fees decrease. Corollary 4(2) presents that a higher CSR coefficient could decrease the equilibrium wholesale and retail prices, while it increases the commission fee in model BLY. When the dominant decision maker coalition BL has CSR behavioral awareness, he aims to better benefit his stakeholders, rather than pursuing profit maximization. Corollary 4(3) shows that when the brand owner considers CSR, the profits of coalition BL, retailer, and LSC are higher than those of the brand owner as a pure profit earner.

Corollaries 1(3)–4(3) mean that in both non-cooperative strategy and three cooperative strategies, the profits of LSC members and the system are greater than those when the brand owner does not consider CSR. That is to say, whether the brand owner has formed a new coalition with other LSC members, LSC members and the system can always benefit from the brand owner considering CSR. Therefore, we further examine our research in the context of the brand owner's CSR behavioral awareness in the following text. For further analysis, we simplify models CY, DY, BRY, and BLY as C, D, BR, and BL, respectively.

When the brand owner considers CSR, to ensure that the brand owner's profits in models D and BR are all non-negative, we assume that

$$\begin{aligned} 0 \le F \le \min\Bigg\{ &\frac{\left(2\theta b^2(1-\theta) + \left(\left((1-\theta b)^2 + \theta b^2(1-\theta)\right)(2-r)\right)f^2\left((1-\theta b)^2 + \theta b^2(1-\theta)\right)(2-r)\right)f^2 2\theta b^2(1-\theta) +}{2\theta(1-\theta)\left(4(4-2r-\theta) + r^2\right)} \\ &+ \frac{(1-\theta)\left(2(\theta+f)^2 + \theta(4-r)(1+2bf)\right)}{2\theta(1-\theta)\left(4(4-2r-\theta) + r^2\right)}, \frac{\left(\left((1-\theta b)^2 + \theta b^2(1-\theta)\right)(2-r)\right)f^2}{2\theta(1-\theta)(2-r)(4-r)} + \frac{\theta(1-\theta)\left(2b^2 + (4-r)(1+2bf)\right)}{2\theta(1-\theta)(2-r)(4-r)}\Bigg\}. \end{aligned} \tag{5}$$

Table 4: Equilibrium results for models BRN and BRY.

Equilibrium	Model BRN	Model BRY
p_r^*	$(1+bf)/2$	$(1-r)(1+bf)/(2-r)$
p_l^*	$(f(1-\theta b)+2(\theta+f))/4$	$(2f(1-\theta b)+(\theta+f)(r^2-5r+4))/((2-r)(4-r))$
m^*	$(1-\theta b)f/4$	$(1-\theta b)f/(4-r)$
D_r^*	$(1-\theta-f+bf)/(2(1-\theta))$	$((4-r)(1-\theta-f+bf)+2f(1-\theta b))/((1-\theta)(2-r)(4-r))$
D_l^*	$(1-\theta b)f/(2\theta(1-\theta))$	$(1-\theta b)f/(\theta(1-\theta)(4-r))$
V_{BR}^*	$((1-\theta b)^2+\theta b^2(1-\theta))f^2/(8\theta(1-\theta))$ $+\theta(1-\theta)(b^2+2(1+2bf))/(8\theta(1-\theta))-F$	$((1-\theta b^2+\theta b^2(1-\theta))(2-r))f^2/(2\theta(1-\theta)(2-r)(4-r))$ $+\theta(1-\theta)(2b^2+(4-r)(1+2bf))/(2\theta(1-\theta)(2-r)(4-r))-F$
π_L^*	$f^2(1-\theta b)^2/(16\theta(1-\theta))+F$	$f^2(1-\theta b)^2/(\theta(1-\theta)(4-r)^2)+F$
V_T^*	$V_{BR}^*+\pi_L^*$	$V_{BR}^*+\pi_L^*$

Table 5: Equilibrium results for models BLN and BLY.

Equilibrium	Model BLN	Model BLY
w^*	$(1+bf)/2$	$((1+bf)(4-4r+r^2)-r(\theta+f))/((2-r)(4-r))$
p_l^*	$(\theta+f)/2$	$(1-r)(\theta+f)/(2-r)$
p_r^*	$(3(1+bf)-(\theta+f))/4$	$((1+bf)(6-5r+r^2)-2(\theta+f))/((2-r)(4-r))$
D_r^*	$(1-\theta-f+bf)^2/(16(1-\theta))$	$(1-\theta-f+bf)/((1-\theta)(4-r))$
D_l^*	$((\theta+f)(1-\theta)+f(1-\theta b))/(4\theta(1-\theta))$	$(2(\theta+f)(1-\theta)+f(1-\theta b)(2-r))/(\theta(1-\theta)(2-r)(4-r))$
V_{BL}^*	$((1-\theta)((\theta+f)^2+\theta(1+2bf)))/(8\theta(1-\theta))$	$((1-\theta)(2(\theta+f)^2+\theta(2-r)(1+2bf))/(2\theta(1-\theta)(2-r)(4-r))$
π_R^*	$+(((1-\theta b)^2+\theta b^2(1-\theta))f^2)/(8\theta(1-\theta))$	$+((((1-\theta b)^2+\theta b^2(1-\theta))(2-r))f^2)/(2\theta(1-\theta)(2-r)(4-r))$
V_T^*	$((1-\theta-f+bf)^2)/(16(1-\theta))$	$((1-\theta-f+bf)^2)/((1-\theta)(4-r)^2)$
	$V_{BL}^*+\pi_R^*$	$V_{BL}^*+\pi_R^*$

This shows that when the pit fee is too high, the profits of the brand owner are negative. Against the backdrop of fierce competition from the world, many brand owners are pinning their hopes on live-streaming shopping to increase sales. However, the high pit fees are eroding the profits of enterprises, as they need to pay the pit fees in advance, and it is unknown whether the live streamers can achieve a certain number of sales, which means that the risk is entirely borne by the enterprise. Therefore, to better expand LCs, live streamers should control their pit fees and collaborate with brand owners [24].

5. Equilibrium Result Analysis

5.1. Comparison Analysis. In this section, by defining $f_1 = (2\theta^2 - (6-r)\theta + 4 - r)/((1-b)(4-r-\theta r) + (1-\theta)(2-r))$, $f_2 = \theta(1-\theta)(2-r)/((4-r-\theta r) - 2\theta(3-\theta-r)b)$, and $f_3 = \sqrt{(2\theta(1-b))^2 + 4\theta(1-\theta)(1-\theta b^2)}/(2(1-\theta b^2)) - \theta(1-b)/(1-\theta b^2)$, we compare the optimal solutions of four models and obtain the following conclusions.

Proposition 5. *(1) The wholesale price in model D is higher than that in model BL, i.e., $w^{D*} > w^{BL*}$. (2) The retail price of the RC in model D is the highest, followed by model BL, and then those under models BR and C are the lowest, i.e., $p_r^{D*} > p_r^{BL*} > p_r^{BR*} = p_r^{C*}$. (3) The retail price of the LC in model D is the highest, followed by model BR, and then those under models BL and C are the lowest, i.e., $p_l^{D*} > p_l^{BR*} > p_l^{BL*} = p_l^{C*}$. (4) The unit commission fee in model D is higher than that in model BR, i.e., $m^{D*} > m^{BR*}$.*

Proposition 5(1) indicates that in LSC, when the brand owner adopts a non-cooperative strategy, the wholesale price is higher than that when the brand owner cooperates with the live streamer (i.e., $w^{D*} > w^{\mathrm{ML}*}$). When the brand owner collaborates with the live streamer, they no longer need to pay a unit commission fee to the live streamer, thus increasing the brand owner's marginal profit in the LC. In this case, the brand owner strategically reduces wholesale prices in the RC, meaning that the costs saved by the brand owner in live-streaming channels will be used for the RC to stabilize its market dominance and increase its total profits. On the other hand, when the brand owner adopts a non-cooperative strategy, the brand owner, retailer, and live streamer aim to maximize their benefits, and thus the brand owner sets higher wholesale prices to strive for more profits. Similarly, when the brand owner collaborates with the retailer, from Proposition 5(4), we can find that the unit commission fee is lower than that when the brand owner adopts a non-cooperative strategy.

From Propositions 5(2) and 5(3), we find that the brand owner's cooperation with LSC members is beneficial for consumers because of the lower retail prices. Especially, the retail price in the grand coalition C is the lowest. When the brand owner does not cooperate with the retailer and live streamer, the retail price is the highest. In addition, compared to the coalition between the brand owner and the live streamer (retailer), when the brand owner cooperates with the retailer (live streamer), it is more conducive to reducing the retail price of the RC (LC). This is because when the m brand owner collaborates with a retailer or live streamer, or with both of them, he saves the retailer's unit profit or the anchor's unit commission fee. Thus, coalition BR, coalition BL, and grand coalition C lower the retail price of the RC to attract consumers, and coalition BR lowers the retail price more significantly, due to the coalition directly deciding on the retail price. Similarly, in the LC, coalition BL lowers the retail price more significantly, as coalition BL saves the commission fee.

Proposition 6. *(1) In the RC, the demand in coalition BR is the highest, while that in coalition BL is the lowest, i.e., $D_r^{MR*} > \max\{D_r^{C*}, D_r^{D*}\} > \min\{D_r^{C*}, D_r^{D*}\} > D_r^{BL*}$. If $f_1 < f_0$ and $0 < f \le f_1$, or $f_1 > f_0$, the demand of RC in grand coalition C is greater than that in model D, i.e., $D_r^{C*} \ge D_r^{D*}$; otherwise, we have $D_r^{C*} < D_r^{D*}$. (2) In the LC, the demand in coalition BL is the highest, while that in coalition BR is the lowest, i.e., $D_l^{BL*} > \max\{D_l^{C*}, D_l^{D*}\} > \min\{D_l^{C*}, D_l^{D*}\} > D_l^{BR*}$. If $f_2 < f_0$ and $0 < f \le f_2$, or $f_2 > f_0$, the demand of LC in model D is greater than that in grand coalition C, i.e., $D_l^{D*} \ge D_l^{C*}$; otherwise, we have $D_l^{D*} < D_l^{C*}$.*

Proposition 6 indicates that in LSC, cooperation between the brand owner and retailer or live streamer, as well as third-party cooperation, does not always improve market demand. Especially, when the brand owner cooperates with the retailer (live streamer), the demand of RC (LC) is highest; otherwise, the demand is lowest when the brand owner and live streamer (retailer) form a coalition. This means that for the RC (LC), the cooperation between the brand owner and the retailer (live streamer) is the best because the cooperation between the two parties improves the competitiveness of the channel.

From Proposition 6, when the brand owner, retailer, and live streamer form a grand coalition C, we also find an interesting phenomenon that the market demand is lower than that when the brand owner adopts a non-cooperative strategy. Specifically, the collaboration among LSC members cannot attract more consumers to purchase products, and the flow effect affects the relationship between model C and model D. For the RC, when the flow effect threshold f_1 is less than the threshold f_0 and the flow effect is relatively large, the demand of RC in grand coalition C is higher than that in non-cooperative model D. If the three party members cooperate to attract more consumers to purchase products, brand owners should evaluate whether the live streamers' flow effect is within a certain range when selecting live streamers. Similar to the RC, the demand of LC has a threshold between model C and model D.

5.2. Optimal Coalition Strategy for the Brand Owner. In this section, we first analyze whether the brand owner has a motivation to coalition with the retailer or live streamer and how the coalition affects the profits of third-party supply chain members by comparing coalitions BR and BL with non-cooperative model D.

Proposition 7. *Comparing coalitions BR and BL with non-cooperative model D, we have the following. (1) The brand owner always has the motivation to collaborate with the retailer or live streamer:* $V_{BR}^{BR*} > V_B^{D*} + \pi_R^{D*}$, $V_{BL}^{BL*} > V_B^{D*} + \pi_L^{D*}$. *(2) The third-party supply chain member (i.e., member outside the coalition group) prefers a non-cooperative model D:* $\pi_L^{D*} > \pi_L^{BR*}$, $\pi_R^{D*} > \pi_R^{BL*}$.

Proposition 7 indicates that the cooperative strategy increases the profit of coalition BR or BL but is disadvantageous for enterprises that do not participate in LSC cooperation. Specifically, when the brand owner collaborates with the retailer or live streamer, the profit and social welfare of coalition BR or BL are higher than when they do not cooperate. But the retailer or live streamer cannot benefit from a cooperative strategy. This is because when the brand owner and retailer or live streamer cooperate, they weaken the double marginal utility but increase the intensity of competition between retail and LCs, which cannot effectively increase the demand of the whole market. Thus, the profit of enterprises not involved in cooperation is lower than that of the non-cooperative strategy.

Next, we explore from the perspective of the brand owner who is more inclined to cooperate. Here, $\pi_{\mathrm{BR}}^{\mathrm{BR}*} - V_B^{D*} - \pi_R^{D*}$ and $\pi_{\mathrm{BL}}^{\mathrm{BL}*} - V_B^{D*} - \pi_L^{D*}$ represent the cooperation additional profit when the brand owner and retailer or live streamer form a coalition, respectively.

Proposition 8. *For the brand owner: if* $f_3 < f_0$ *and* $0 < f \le f_3$, *or* $f_3 > f_0$, $\pi_{BR}^{BR*} - V_B^{D*} - \pi_R^{D*} \ge \pi_{BL}^{BL*} - V_B^{D*} - \pi_L^{D*}$; *otherwise,* $\pi_{BR}^{BR*} - V_B^{D*} - \pi_R^{D*} < \pi_{BL}^{BL*} - V_B^{D*} - \pi_L^{D*}$.

Proposition 8 indicates that the brand owner always has the motivation to ally with the retailer or live streamer. From Proposition 8, we can know that the value of this incentive is related to the spillover effect. When the flow effect threshold f_3 is less than the threshold f_0 and the flow effect is relatively small or large, the brand owner and live streamer have stronger motivation to cooperate due to achieving higher profits and social welfare; otherwise, the brand owner is more inclined to collaborate with retailer. As a result, it is clear that whether the brand owner allies with a retailer or a live streamer, coalition BR or BL reduces retail prices and drives consumers to the coalition channel, resulting in higher market demand and profits. Figure 2 intuitively illustrates the decision of whom the brand owner tends to cooperate with.

It can be seen that the brand owner always tends to cooperate with the retailer or live streamer, but the coalition is unfavorable for the third-party member, and the three parties cannot achieve a win-win-win situation. Therefore, we further compare the profits of LSC in the four models.

Proposition 9. *In the LSC, we have* $V_T^{C*} > \max\{V_T^{BR*}, V_T^{BL*}\}$, $V_T^{C*} > V_T^{D*}$. *Moreover, if* $f_3 < f_0$ *and* $0 < f \le f_3$, *or* $f_3 > f_0$, $\pi_T^{BR*} \ge \pi_T^{BL*}$; *otherwise,* $\pi_T^{BR*} \ge \pi_T^{BL*}$.

Proposition 9 indicates that in LSC, the brand owner's grand cooperative strategy is beneficial for increasing consumer surplus compared to the decentralized model D, coalition cooperative strategy BR, and coalition cooperative strategy BL, which improves the profit of LSC. Specifically, when the brand owner, retailer, and live streamer form a grand coalition C, the profit of LSC is the largest. In addition, when the brand owner collaborates with the retailer or live streamer, we can draw the same conclusion as the brand owner's cooperative strategy. When the flow effect threshold f_3 is less than the threshold f_0 and the flow effect is relatively large, the profit of the LSC under coalition BL is higher than that in coalition BR; otherwise, the profit of the LSC under coalition BR is higher than that in coalition BL. Based on Proposition 7, the brand owner and retailer or live streamer have the motivation to form coalitions to increase profits and social welfare, but only through tripartite cooperation among LSC members can the benefits of the LSC be maximized. Thence, the government can formulate some incentive measures to promote tripartite cooperation, thereby achieving a win-win situation in both economic and social benefits.

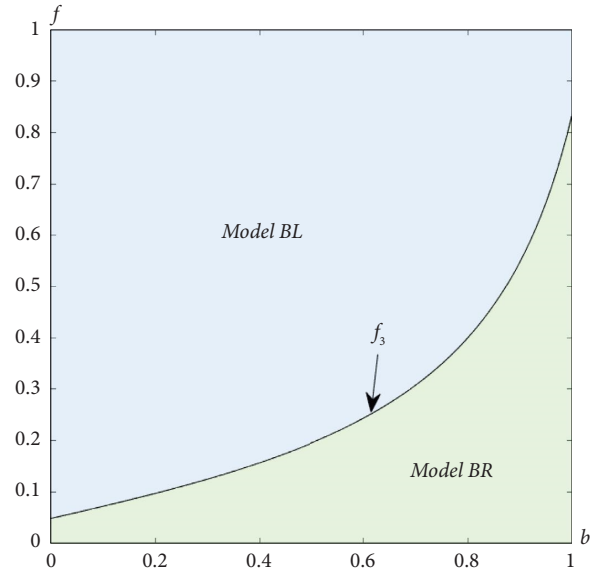

Figure 2: Decision of whom to cooperate with for brand owner ($\theta = 0.7$).

5.3. Numerical Analysis of LSC, Consumer Surplus, and Social Welfare. In this section, we further examine the impact of CSR on the profit of LSC and consumer surplus to provide a reference for enterprises and consumers by using numerical analysis. Following Cao et al. [7], Zhang et al. [21], and Zhang et al. [45], we set $\theta = 0.7$, $\beta = 0.5$, $F = 0$, and $f < f_0 = 0.6$. Under the above parameters' settings, r varied with the range of [0,1], and the profits of LSC are shown in Figure 3.

Figure 3 shows that as the brand owner's CSR awareness increases, the profits of LSC in the four models increase. Figure 3(a) indicates that the LSC's profit in grand coalition C is always highest, followed by coalition BR when the flow effect is relatively small (i.e., $f = 0.2$). When the flow effect is relatively large (i.e., $f = 0.4$), Figure 3(b) implies that the LSC's profit in grand coalition C is always highest, followed by coalition BL. This means that the brand owner should form a grand coalition with the retailer and live streamer to

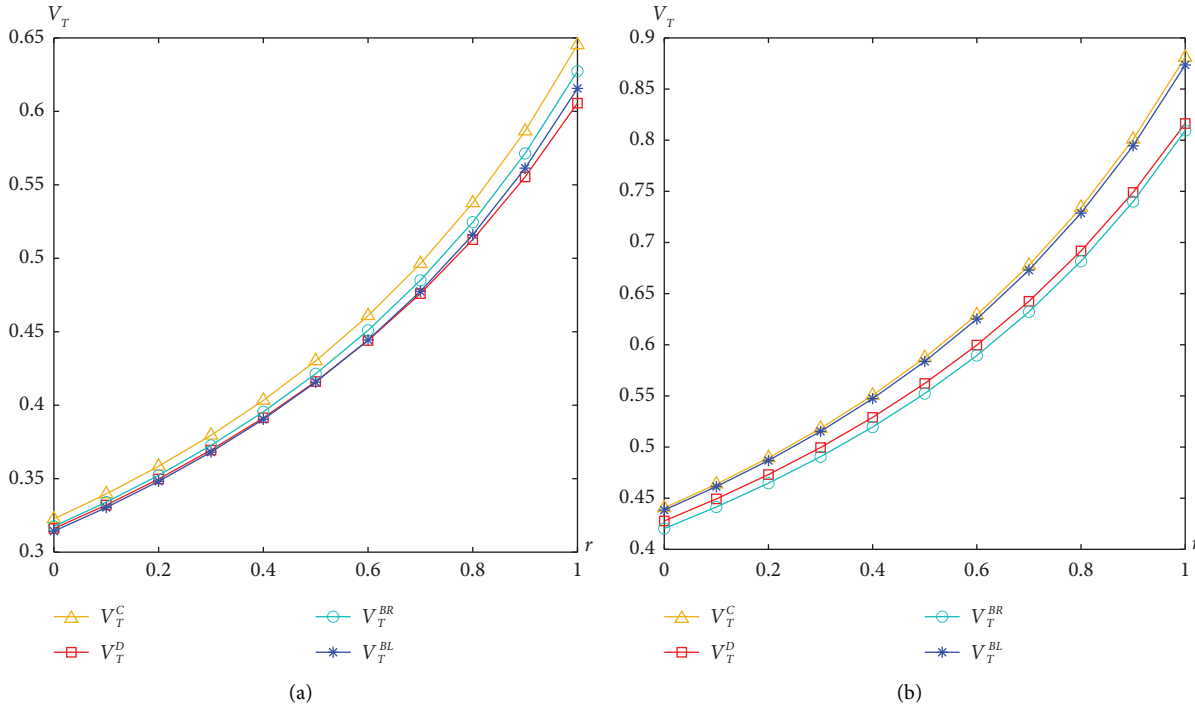

Figure 3: Impact of CSR on the profits of LSC. (a) $f = 0.2$. (b) $f = 0.4$.

maximize profits for LSC, followed by forming a coalition BR with the retailer when the flow effect is small or a coalition BL with the live streamer when the flow effect is large. In summary, as the leader in the LSC, brand owners should pay attention to the interests of other members of the LSC and enhance their CSR awareness. Moreover, they should establish cooperative relationships with retailers and live streamers to maximize supply chain profits.

The consumer surpluses are shown in Figure 4.

Figure 4 shows that as the brand owner's CSR awareness increases, the consumer surpluses in the four models increase. From Figure 4(a), we can find that the consumer surplus in model C is always highest, followed by coalition BR and then the coalition BL, and it is lowest in model D when the flow effect is relatively small (i.e., $f = 0.2$). When the flow effect is relatively large (i.e., $f = 0.4$), Figure 4(b) implies that the consumer surplus in model C is always highest, followed by coalition BL and then the coalition BR, and it is lowest in model D. The reasons are as follows. The CSR behavior of the brand owner lowers the retail prices to stimulate consumers to purchase products, which in turn increases the market demand. Additionally, by combining Proposition 8, we can find that the demands for retail and LCS and consumer surpluses in grand coalition C are highest and lowest in model D. In summary, the LSC tripartite cooperation is beneficial for consumers.

Figures 3 and 4 indicate that when the brand owner considers CSR, he cooperates with the retailer and live streamer to form a grand coalition C which is beneficial for the LSC system and consumers. Following Zhang et al. [42] and Liu and Xiao [37], social welfare can be calculated as $SW = V_T + CS$. From Figures 3 and 4, we can find that social welfare is highest in grand coalition C. Therefore, it is recommended that LSC members actively cooperate in the face of fierce external environments to achieve a multiparty win-win situation for the supply chain system, consumers, and society.

5.4. Case Study. In the 21st century, the Chinese government began to promote the CSR, and Article 5 of the new Company Law implemented in 2006 clearly stipulates that companies should "fulfill social responsibility." Thus, numerous companies started to publish CSR report under the promotion of society and government. Figure 5 shows the number of CSR reports released by Chinese enterprises. The "green line" and "blue line" represent the number of CSR reports released in each year and linear prediction, respectively. It can be seen that the number of CSR reports by Chinese enterprises is gradually increasing. Meanwhile, the epidemic affected the sales of offline channels of multiple categories but opened up e-commerce sales with the help of live-streaming, an online channel with strong experience and interactivity. It is worth noting that the beauty industry with strong seeding attributes has been developing rapidly by utilizing live-streaming e-commerce. Along with the rise of social e-commerce such as Xiaohongshu and TikTok, many beauty brands carry out content marketing through short videos, graphics, and other forms to effectively enhance brand awareness and traffic conversion rate. According to the Net Society, the transaction scale of China's beauty industry was around 27.8 billion RMB in 2021, with a compound annual growth rate of about 40.3%, and it is estimated that by 2027, the transaction scale will exceed 200 billion RMB (https://bg.qianzhan.com/trends/detail/506/220629-e05a3ef2.html, Accessed on 29 June 2020). These

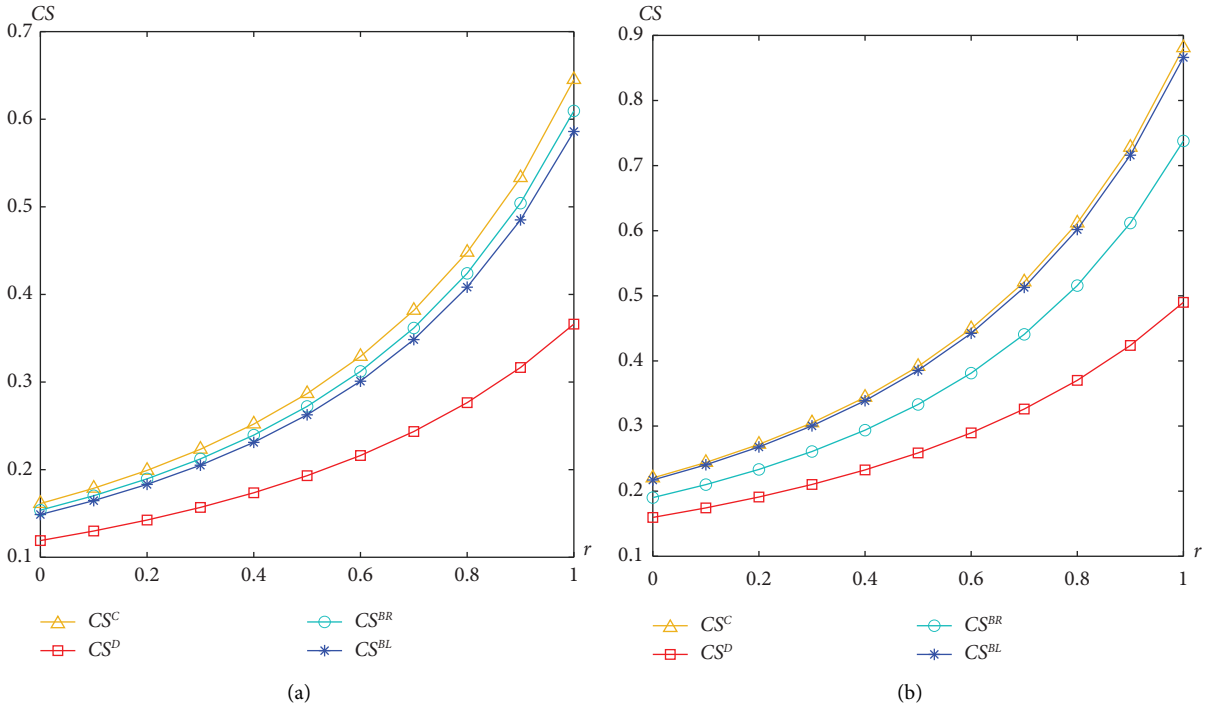

Figure 4: Impact of CSR on the consumer surplus. (a) $f = 0.2$. (b) $f = 0.4$.

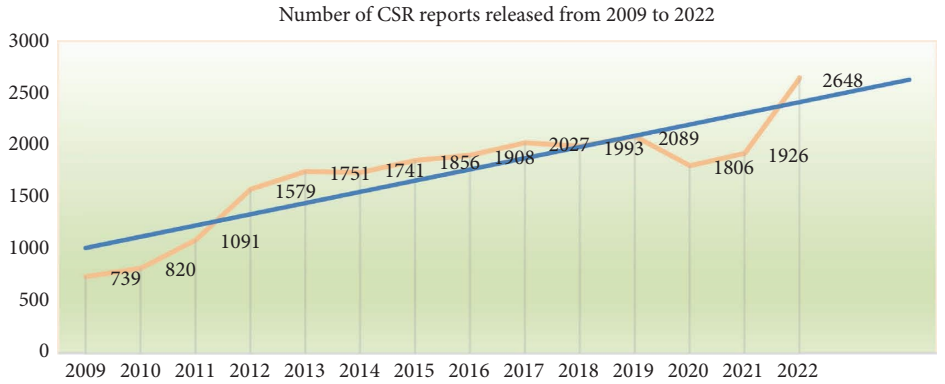

Figure 5: Number of CSR reports released from 2009 to 2022.

data indicate that the new live-streaming e-commerce sales model has opened a "New World" for beauty brands.

Live-streaming e-commerce brings great success to beauty brands, and the reasons are divided into the following two aspects. On the one hand, a successful beauty brand pays attention to corporate CSR, such as focusing on the product itself, adhering to the concept of serving consumers first, and paying attention to the interests of employees, the environment, society, etc. In practice, beauty brands participate in CSR strategies, as shown in Table 6. In addition, against the backdrop of global climate change, sustainable consumption and green products are gradually deepening consumer awareness. According to a survey report conducted in November 2021 by First Insight and the Baker Retail Center at the Wharton School of Business at the University of Pennsylvania, approximately 73% of Generation Z (born in 1995–2009) consumers are willing to pay a premium of over 10% for sustainable products. The representative track of sustainable beauty, "pure beauty," is gradually moving from niche to mainstream. Moreover, the main focus of consumers on "pure beauty" is on safety and risk-free, environmental protection and low-carbon, and humanitarianism, with a proportion of 89.3%, 59.7%, and 41.1%, respectively. According to Brand Essence Market Research, the compound annual growth rate of "pure beauty" from 2020 to 2027 will reach 12%, far higher than the global beauty market growth rate. In addition, top beauty companies are increasingly paying attention to the disclosure of Environmental, Social, and Governance-related reports. From a longitudinal historical perspective, the CSR report disclosure rate of A-share beauty companies has rapidly increased from 27% in 2015 to 73% in 2021, consistently higher than the

Table 6: CSR strategies of several beauty brands.

Brands	CSR strategies
Estée Lauder	Carry out the "origins return to green source packaging recycling" project
L'Oreal Paris	Launch of product environmental impact information and rating system (PIL) Allocate 100 million euros for restoring ecosystems and developing a circular economy
Perfect Diary	Cooperate with SEE foundation to protect endangered animal—red-crowned crane
Origins	Collaborate with climate organizations to launch the "China ecological forest planting" project
Proya	Proya rinsing-free hands sanitizer and alcohol disinfectant spray were designed during the epidemic The 3R principles in packaging design include reduce, replace, and recycle

overall market disclosure rate of A-shares (https://www.vzkoo.com/read/202303029fc901fcde15cf606641ddea.html, Accessed on 2 March 2023). The brand image bonus brought by the CSR strategy continuously enhances the brand's appeal. In the process of carrying out CSR projects that deeply align with the brand image, beauty brands have achieved their public welfare aspirations while winning the emotional recognition of consumers. This emotional identification has also been transformed into stickiness towards various brands, ultimately achieving the sustainable development of the enterprises. On the other hand, beauty brands seize the opportunity of live-streaming and actively collaborate with major retailers and anchors to market products in a fierce environment. Numerous beauty brands (i.e., Estée Lauder, Lamer, 3CE, Sulwhasoo, SK-II, Elixir, Girlcult, Nars, etc.) actively cooperate with retailers, such as JD, Tmall, Taobao, and Pinduoduo, to sell products. In addition, these beauty brands would hire a live streamer to promote products. The live streamers not only describe the characteristics and advantages of each product to consumers but also interact with viewers in the live-streaming room, thus conveying the value of the product and increasing consumers' willingness to purchase beauty products. The beauty brands formed a coalition with live streamers, greatly increasing the brands' profits. In summary, the success of beauty brands stems from their attention to CSR and collaboration with other supply chain members, which fully demonstrates the rationality of this paper.

Overall, the focus of enterprises on CSR has become the mainstream in promoting the sustainable development of live-streaming supply chains and meeting consumers' social responsibility awareness. For example, L'Oreal's 2030 Sustainable Development Commitment proposes a strategy based on three pillars to achieve its vision of corporate social responsibility. First, it proposes self-transformation to respect the boundaries of the earth, aimed at limiting the impact of company operations on climate, water resources, biodiversity, and natural resources. Second, an empowered business ecosystem helps customers, suppliers, and consumers participate in completing the transformation, developing a labeling system for the environmental and social impact of products and allowing consumers to make purchasing choices that align with their values with sufficient information. Finally, it is necessary to provide support for the urgent needs of society and the environment, such as allocating 100 million euros for innovative investment, restoring ecosystems, developing a circular economy, and creating a 50-million-euro charity and donation fund to support vulnerable women. As we can see, many beauty brands are transforming to create more sustainable business models while contributing to addressing the challenges facing the world. To meet consumers' needs and face the transformation of consumers' shopping methods, live-streaming e-commerce is highly favored by enterprises and consumers. In the process of interacting with live streamers, consumers are more likely to recognize products with CSR attributes. In addition, it is inevitable for enterprises to actively cooperate with upstream and downstream members of the supply chain to adapt to the fierce competition context. Therefore, the CSR awareness and cooperation strategies of enterprises have become a powerful tool to achieve economic and social benefits.

6. Conclusions

6.1. Main Findings. In the fiercely competitive environment, strengthening cooperation between enterprises and incorporating CSR into the supply chain has become an important means for enterprises to obtain high profits and improve the environment and social welfare. Thence, we consider an LSC composed of a brand owner, retailer, and live streamer. In the LSC, we build a non-cooperative strategy and three different cooperative strategies, i.e., the brand owner and retailer form a coalition BR, the brand owner and live streamer form a coalition BL, and LSC tripartite members form a grand coalition C. Then we solve the four models, analyze how the flow effect and CSR parameters affect the LSC members' decisions and demands, and explore the brand owner's cooperative strategies. The results show the following. (1) The brand owner considers CSR to be beneficial to each LSC member, coalition BR, coalition BL, and LSC in the four models. (2) The brand owner tends to cooperate with the retailer when the flow effect is small. Otherwise, the brand owner prefers to cooperate with the live streamer. (3) When the brand owner cooperates with the retailer and the live streamer to form a grand coalition C, coalition C is the consistent strategy for the LSC system, consumers, and society.

6.2. Managerial Implications. Our research generates several managerial insights as follows. First, brand owners as the LSC leader should enhance CSR awareness to benefit stakeholders (i.e., consumers, upstream and downstream enterprises, the supply chain system, society, etc.). Second, it is recommended that enterprises actively cooperate with retailers or live streamers according to the flow of the live streamers. In addition, when enterprises cooperate with retailers (live streamers), the alliance party should take measures to compensate live streamers (retailers) for its lower profits compared to the non-cooperative situation. Last, enterprises should actively cooperate with downstream members such as retailers and live streamers to form a grand coalition to provide an optimal cooperative strategy for enterprises.

6.3. Future Research Directions. Although this paper provides some insights into brand owners' cooperative strategies and LSC operations, there are still several limitations. The demand function we mainly consider is linear deterministic demand related to price, flow effect, and spillover effect. However, in practice, the market demand for products often exhibits uncertainty. Thus, future research can consider situations where demand is stochastic. Moreover, we only consider the situation where a brand owner only hires a live streamer to sell products. But some brand owners may collaborate with multiple live streamers. Therefore, when brand merchants hire multiple live streamers to sell products, exploring brand owners' cooperative strategies can gain certain practical significance.

Appendix

A. Proof of Models CN and CY

In model CY, the Hessian matrix of V_T^{CY} in terms of p_r^{CY} and p_l^{CY} is $H^{CY} = \begin{bmatrix} -(2-r)/(1-\theta) & (2-r)/(1-\theta) \\ (2-r)/(1-\theta) & -(2-r)/(\theta(1-\theta)) \end{bmatrix}$. From the assumptions $0 \le \theta < 1$ and $0 \le r \le 1$, we have $|H_1^{\mathrm{CY}}| = -(2-r)/(1-\theta) < 0$ and $|H_2^{\mathrm{CY}}| = (2-r)^2/(\theta(1-\theta)) > 0$. Hence, V_T^{CY} is strictly concave with respect to p_r^{CY} and p_l^{CY}. From $\partial V_T^{\mathrm{CY}}/\partial p_r^{\mathrm{CY}} = 0$ and $\partial V_T^{\mathrm{CY}}/\partial p_l^{\mathrm{CY}} = 0$, we have $p_r^{\mathrm{CY}*} = (1+bf)(1-r)/(2-r)$ and $p_l^{\mathrm{CY}*} = (\theta+f)(1-r)/(2-r)$.

Similar to the proof of model CY, the equilibrium outcomes of model CN can be easily proved.

B. Proof of Models DN and DY

In model DY, since $\partial^2 \pi_R^{\mathrm{DY}}/\partial (p_r^{\mathrm{DY}})^2 = -2/(1-\theta) < 0$ and $\partial^2 \pi_L^{\mathrm{DY}}/\partial (m^{\mathrm{DY}})^2 = -2/(\theta(1-\theta)) < 0$, π_R^{DY} and π_L^{DY} are concave on p_r^{DY} and m^{DY}, respectively. It can be obtained from $\partial \pi_R^{\mathrm{DY}}/\partial p_r^{\mathrm{DY}} = 0$ and $\partial \pi_L^{\mathrm{DY}}/\partial m^{\mathrm{DY}} = 0$ that $p_r^{\mathrm{DY}} = (p_l^{\mathrm{DY}} - \theta + w^{\mathrm{DY}} - (1-b)f + 1)/2$ and $m^{DY} = (\theta - 2p_l^{DY} + 2f - \theta f + \theta p_l^{DY} + \theta w^{DY} - \theta^2 - \theta b f)/2$.

By substituting p_r^{DY} and m^{DY} into V_B^{DY}, the Hessian matrix of V_B^{DY} in terms of w^{DY} and p_l^{DY} is $H^{\mathrm{DY}} = \begin{bmatrix} -(4-r+2\theta)/(4(1-\theta)) & (8-r-2\theta)/(4(1-\theta)) \\ (8-r-2\theta)/(4(1-\theta)) & -(4(4-3\theta-r)+\theta(\theta+3r))/(4\theta(1-\theta)) \end{bmatrix}$. From the assumptions $0 \le \theta < 1$ and $0 \le r \le 1$, we have $|H_1^{\mathrm{DY}}| = -(4-r+2\theta)/(4(1-\theta)) < 0$ and $|H_2^{\mathrm{DY}}| = (4(4-2r-\theta)+r^2)/(4\theta(1-\theta)) > 0$. Therefore, V_B^{DY} is strictly concave with respect to w^{DY} and p_l^{DY}. It can be obtained from $\partial V_B^{\mathrm{DY}}/\partial w^{\mathrm{DY}} = 0$ and $\partial V_B^{\mathrm{DY}}/\partial p_l^{\mathrm{DY}} = 0$ that $w^{\mathrm{DY}*} = ((1+bf)(r^2-6r-2\theta+8) - (\theta+f)r)/(4(4-2r-\theta)+r^2)$ and $p_l^{DY*} = ((\theta+f)(r^2-7r-4\theta) + 2(5\theta + f(6-\theta b)))/(4(4-2r-\theta)+r^2)$. Substituting $w^{\mathrm{DY}*}$ and $p_l^{\mathrm{DY}*}$ into p_r^{DY} and m^{DY}, we have $p_r^{\mathrm{DY}*} = ((1+bf)(r^2-7r-4\theta+12) - 2(\theta+f))/(4(4-2r-\theta)+r^2)$ and $m^{\mathrm{DY}*} = (2\theta(1-\theta) + f(4-2\theta-r) - \theta f b(2-r))/(4(4-2r-\theta)+r^2)$.

Similar to the proof of model DY, the equilibrium outcomes of model DN can be easily proved. Proofs of models BRN, BRY, BLN, and BLY are similar to the proof of model DY, so we omit it.

C. Proof of Corollary 1

From the assumptions $0 < \theta \le 1$, $0 \le r \le 1$, $0 \le b \le 1$, and $0 < f < (1-\theta)/(1-b)$, by examining the equilibrium results, we can verify that

(1) $\partial p_r^{\mathrm{CN}*}/\partial b = f/2 > 0$, $\partial p_r^{\mathrm{CY}*}/\partial b = f(1-r)/(2-r) > 0$, $\partial p_l^{\mathrm{CN}*}/\partial b = \partial p_l^{\mathrm{CY}*}/\partial b = 0$.

(2) $\partial p_r^{\mathrm{CY}*}/\partial r = -(1+bf)/(2-r)^2 < 0$, $\partial p_l^{\mathrm{CY}*}/\partial r = -(\theta+f)/(2-r)^2 < 0$.

(3) $V_T^{CY*} - V_T^{CN*} = (r(\theta(1-\theta) + 2\theta b f(1-\theta) + ((1-\theta b^2 + \theta b^2(1-\theta))f^2))/(4\theta(1-\theta)(2-r)) > 0$.

D. Proof of Corollary 2

From the assumptions $0 < \theta \le 1$, $0 \le r \le 1$, $0 \le b \le 1$, and $0 < f < (1-\theta)/(1-b)$, by examining the equilibrium results, we can verify that

(1) $\partial w^{\mathrm{DN}*}/\partial b = f/2 > 0$, $\partial p_r^{\mathrm{DN}*}/\partial b = (3-\theta)f/(4-\theta) > 0$, $\partial p_l^{\mathrm{DN}*}/\partial b = -\theta f/(2(4-\theta)) < 0$, $\partial m^{\mathrm{DN}*}/\partial b = -\theta f/(2(4-\theta)) < 0$, $\partial w^{\mathrm{DY}*}/\partial b = (8+r^2-6r-2\theta)f/(4(4-2r-\theta)+r^2) > 0$, $\partial p_r^{\mathrm{DY}*}/\partial b = f(12+r^2-7r-4\theta)/(4(4-2r-\theta)+r^2) > 0$, $\partial p_l^{\mathrm{DY}*}/\partial b = -2\theta f/(4(4-2r-\theta)+r^2) < 0$, $\partial m^{DY*}/\partial b = -\theta f(2-r)/(4(4-2r-\theta)+r^2) < 0$.

(2) $\partial w^{\mathrm{DY}*}/\partial r = -(4(4-\theta)((1+bf)(2-r) + (\theta+f)) + (2-\theta-f+2bf)r^2)/(4(4-2r-\theta)+r^2)^2 < 0$, $\partial p_r^{\mathrm{DY}*}/\partial r = -(4(4-r)(a+f) + (1+bf)(16+4a-8r+r^2))/(2-r)^2 < 0$, $\partial p_l^{\mathrm{DY}*}/\partial r = -((4(1-\theta)+r^2)(\theta+f) + 4(4\theta+f) + 4(3\theta+2f)(1-r) + 4\theta b f(4-r))/(4(4-2r-\theta)+r^2)^2 < 0$, $\partial m^{\mathrm{DY}*}/\partial r = (4\theta(4-r)(1-\theta) + f((1-\theta b)r^2 - 4(2-\theta-\theta b)r + 4(4-3\theta-\theta^2 b)))/(4(4-2r-\theta)+r^2)^2 > 0$.

(3) $V_B^{\mathrm{DY}*} - V_B^{\mathrm{DN}*} = (f^2(2r\theta(((1-\theta b)^2 + \theta b^2(1-\theta)) + 4(1-\theta)b^2) - r^2(\theta b^2(1-\theta) + ((1-\theta b)^2 + \theta b^2(1-\theta)))))/(4\theta(1-\theta)(4-\theta)(4(4-2r-\theta)+r^2)) + (2r(1+2bf)(\theta^2 + \theta(4-r)) + r(8-r)(\theta+f)^2)/(4\theta(1-\theta)(4-\theta)(4(4-2r-\theta)+r^2))$,

$\pi_R^{DY*} - \pi_R^{DN*} = (4((4-r)(1-\theta-f+bf)+2f(1-\theta b))^2(4-\theta)^2 - (2(1-\theta-f+bf)+f(1-\theta b))^2(4(4-2r-\theta)+r^2)^2)/(4(1-\theta)(4-\theta)^2(4(4-2r-\theta)+r^2)^2)$, since $2((4-r)(1-\theta-f+bf)+2f(1-\theta b))(4-\theta) - (2(1-\theta-f+bf)+f(1-\theta b))(4-\theta) = 2r(4-r+\theta)(1-\theta-f+bf)+r(1-\theta b)(8-r)f > 0$, thus $\pi_R^{DY*} - \pi_R^{DN*} = (4((4-r)(1-\theta-f+bf)+2f(1-\theta b))^2(4-\theta)^2 - (2(1-\theta-f+bf)+f(1-\theta b))^2(4(4-2r-\theta)+r^2)^2)/(4(1-\theta)(4-\theta)^2(4(4-2r-\theta)+r^2)^2) > 0$; $\pi_L^{DY*} - \pi_L^{DN*} = (4(2(\theta+f)(1-\theta)+f(2-r)(1-\theta b))^2(4-\theta)^2 - ((\theta+f)(1-\theta)+(1-\theta b)f)^2(4(4-2r-\theta)+r^2)^2)/(4\theta(1-\theta)(4(4-2r-\theta)+r^2)^2)$, since $2(2(\theta+f)(1-\theta)+(2-r)(1-\theta b)f)(4-\theta) - ((\theta+f)(1-\theta)+(1-\theta b)f)(4(4-\theta)-r(8-r)) = r(2f(4-r)+(8-6f-r+fr+bfr)\theta-(8+2bf-r)\theta^2) > 0$, thus $\pi_L^{DY*} - \pi_L^{DN*} = (4(2(\theta+f)(1-\theta)+f(2-r)(1-\theta b))^2(4-\theta)^2 - ((\theta+f)(1-\theta)+(1-\theta b)f)^2(4(4-2r-\theta)+r^2)^2)/(4\theta(1-\theta)(4(4-2r-\theta)+r^2)^2) > 0$, since $V_B^{DY*} > V_B^{DN*}$, $\pi_R^{DY*} > \pi_R^{DN*}$, $\pi_L^{DY*} > \pi_L^{DN*}$, $V_T^{DN*} = V_B^{DN*} + \pi_R^{DN*} + \pi_L^{DN*}$, and $V_T^{DY*} = V_B^{DY*} + \pi_R^{DY*} + \pi_L^{DY*}$, thus, $V_T^{DY*} > V_T^{DN*}$.

E. Proof of Corollary 3

From the assumptions $0 < \theta \le 1$, $0 \le r \le 1$, $0 \le b \le 1$, and $0 < f < (1-\theta)/(1-b)$, by examining the equilibrium results, we can verify that

(1) $\partial p_r^{BRN*}/\partial b = f/2 > 0$, $\partial p_l^{BRN*}/\partial b = -\theta f/4 < 0$, $\partial m^{BRN*}/\partial b = -\theta f/4 < 0$. $\partial p_r^{BRY*}/\partial b = f(1-r)f/(2-r) > 0$, $\partial p_l^{BRY*}/\partial b = -2\theta f/((2-r)(4-r)) < 0$, $\partial m^{BRY*}/\partial b = -\theta f/(4-r) < 0$.

(2) $\partial p_r^{BRY*}/\partial r = -(1+bf)/(2-r)^2 < 0$, $\partial p_l^{BRY*}/\partial r = -(8\theta(2-r)+4f(1-r)+(\theta+f)r^2+4\theta fb(3-r))/((2-r)^2(4-r)^2) < 0$, $\partial m^{BRY*}/\partial r = ((1-\theta b)f/(4-r)^2) > 0$.

(3) $V_{BR}^{BRY*} - V_{BR}^{BRN*} = (r(2-r)((1-\theta b)^2+\theta(1-\theta)b^2)f^2 + r\theta(1-\theta)((6-r)b^2+2(4-r)(1+2bf)))/(8\theta(1-\theta)(2-r)(4-r)) > 0$, $\pi_L^{BRY*} - \pi_L^{BRN*} = r(8-r)(1-\theta b)^2 f^2/(16(1-\theta)(r-4)^2) > 0$, since $V_{BR}^{BRY*} > V_{BR}^{BRN*}$, $\pi_L^{BRY*} > \pi_L^{BRN*}$, $V_T^{BRN*} = V_{BR}^{BRN*} + \pi_L^{BRN*}$, and $V_T^{BRY*} = V_{BR}^{BRY*} + \pi_L^{BRY*}$, thus $V_T^{BRY*} > V_T^{BRN*}$.

F. Proof of Corollary 4

From the assumptions $0 < \theta \le 1$, $0 \le r \le 1$, $0 \le b \le 1$, and $0 < f < (1-\theta)/(1-b)$, by examining the equilibrium results, we can verify that

(1) $\partial w^{BLN*}/\partial b = f/2 > 0$, $\partial p_r^{BLY*}/\partial b = 3f/4 > 0$, $\partial p_l^{BLY*}/\partial b = 0$, $\partial w^{BLY*}/\partial b = f(2-r)/(4-r) > 0$, $\partial p_r^{BLY*}/\partial b = (3-r)f/(4-r) > 0$, $\partial p_l^{BLY*}/\partial b = 0$.

(2) $\partial w^{BLY*}/\partial r = -((8-r^2)(\theta+f)+2(1+bf)(4-4r+r^2))/((2-r)^2(4-r)^2) < 0$, $\partial p_l^{BLY*}/\partial r = -(\theta+f)/(2-r)^2 < 0$, $\partial p_r^{BLY*}/\partial r = -(4(3-r)(\theta+f)+(1+bf)(4-4r+r^2))/((2-r)^2(4-r)^2) < 0$.

(3) $V_{BL}^{BLY*} - V_{BL}^{BLN*} = r(1-\theta)(6-r)((\theta+f)^2+\theta(2-r)(1+2bf)) + r(2-r)((1-\theta b)^2+\theta(1-\theta)b^2)f^2/(8\theta(1-\theta)(2-r)(4-r)) > 0$ $\pi_R^{BLY*} - \pi_R^{BLN*} = r(8-r)(1-\theta-f+bf)^2/(16(1-\theta)(r-4)^2) > 0$, since $V_{BL}^{BLY*} > V_{BL}^{BLN*}$, $\pi_R^{BLY*} > \pi_R^{BLN*}$, $V_T^{BLY*} = V_{BL}^{BLY*} + \pi_R^{BLY*}$, and $V_{BL}^{BLN*} = V_{BL}^{BLN*} + \pi_R^{BLN*}$, thus $V_T^{BLY*} > V_T^{BLN*}$.

G. Proof of Proposition 5

From the assumptions $0 < \theta \le 1$, $0 \le r \le 1$, $0 \le b \le 1$, and $0 < f < (1-\theta)/(1-b)$, it can be easily verified that

(1) $w^{D*} - w^{BL*} = 2r(2(\theta+f)(1-\theta)+f(2-r)(1-\theta b))/((2-r)(4-r)(4(4-2r-\theta)+r^2)) > 0$.

(2) $p_r^{D*} - p_r^{BL*} = 4(2(\theta+f)(1-\theta)+f(2-r)(1-\theta b))/((2-r)(4-r)(4(4-2r-\theta)+r^2)) > 0$, $p_r^{BL*} - p_r^{BR*} = 2(1-\theta-f+bf)/((2-r)(4-r)) > 0$, $p_r^{D*} - p_r^{BR*} = 2((4-r)(1-\theta-f+bf)+2f(1-\theta b))/((2-r)(4(4-2r-\theta)+r^2)) > 0$, $p_r^{BR*} - p_r^{C*} = 0$;

(3) $p_l^{D*} - p_l^{BL*} = 2(2(\theta+f)(1-\theta)+f(2-r)(1-\theta b))/((2-r)(4(4-2r-\theta)+r^2)) > 0$, $p_l^{BR*} - p_l^{BL*} = 2(1-\theta b)/((2-r)(4-r)) > 0$, $p_l^{D*} - p_l^{BR*} = 4\theta((4-r)(1-\theta-f+bf)+2f(1-\theta b))/((2-r)(4-r)(4(4-2r-\theta)+r^2)) > 0$, $p_l^{BL*} - p_l^{C*} = 0$;

(4) $m^{D*} - m^{BR*} = 2\theta((4-r)(1-\theta-f+bf)+2f(1-\theta b))/((4-r)(4(4-2r-\theta)+r^2)) > 0$.

H. Proof of Proposition 6

Denoting $f_0 = (1-\theta)/(1-b)$, $f_1 = (2\theta^2-(6-r)\theta+4-r)/((1-b)(4-r-\theta r)+(1-\theta)(2-r))$, and $f_2 = \theta(1-\theta)(2-r)/((4-r-\theta r)-2\theta(3-\theta-r)b)$, from the assumptions $0 < \theta \le 1$, $0 \le r \le 1$, $0 \le b \le 1$, and $0 < f < f_0$, it can be easily verified that

(1) $D_r^{C*} - D_r^{BR*} = -2f(1-\theta b)/((1-\theta)(2-r)(4-r)) < 0$, $D_r^{C*} - D_r^{BL*} = 2f(1-\theta-f+bf)/((1-\theta)(2-r)(4-r)) > 0$, $D_r^{BL*} - D_r^{BR*} = -2(1+bf)/((2-r)(4-r)) < 0$, $D_r^{D*} - D_r^{BL*} = 2(2(\theta+f)(1-\theta)+f(2-r)(1-\theta b))/((1-\theta)(4-r)(4(4-2r-\theta)+r^2)) > 0$, $D_r^{D*} - D_r^{BR*} = -2(4-2\theta-r)((4-r)(1-\theta-f+bf)+2f(1-\theta b))/((1-\theta)(2-r)(4-r)(4(4-2r-\theta)+r^2)) < 0$, $D_r^{C*} - D_r^{D*} = (2(2\theta^2-(6-r)\theta+4-r-((1-b)(4-r-\theta r)+(1-\theta)(2-r))f))/((1-\theta)(2-r)(4(4-2r-\theta)+r^2))$.

If $f_1 < f_0$, when $0 < f \le f_1$, $D_r^{C*} \ge D_r^{D*}$, when $f_1 < f < f_0$, $D_r^{C*} < D_r^{D*}$; if $f_1 > f_0$, when $0 < f < f_0$, $D_r^{C*} \ge D_r^{D*}$.

(2) $D_l^{C*} - D_l^{BL*} = -2(1-\theta-f+bf)/((1-\theta)(2-r)(4-r)) < 0$, $D_l^{C*} - D_l^{BR*} = 2f(1-\theta b)/(\theta(1-\theta)(2-r)(4-r)) > 0$, $D_l^{BL*} - D_l^{BR*} = (2(\theta+f)/\theta(2-r)(4-r)) > 0$, $D_l^{D*} - D_l^{BR*} = 2((4-r)(1-\theta-f+bf)+2f(1-\theta b))/((1-\theta)(4-r)(4(4-2r-\theta)$

$+r^2))>0\ D_l^{D*}-D_l^{\mathrm{BL}*}=-2(4-2\theta+r)\quad(2(\theta+f)(1-\theta)+f(2-r)(1-\theta b))/(\theta(1-\theta)(4-r)\quad(4(4-2r-\theta)+r^2))<0, D_l^{C*}-D_l^{D*}=(2(((4-r-\theta r)-2\theta(3-\theta-r)b)f-\theta(1-\theta)(2-r)))/(\theta(1-\theta)\quad(2-r)(4(4-2r-\theta)+r^2))$.

If $f_2<f_0$, when $0<f\le f_2$, $D_l^{D*}\ge D_l^{C*}$, when $f_2<f<f_0$, $D_l^{D*}<D_l^{C*}$; if $f_2>f_0$, when $0<f<f_0$, $D_l^{D*}\ge D_l^{C*}$.

I. Proof of Proposition 7

From the assumptions $0<\theta\le 1$, $0\le r\le 1$, $0\le b\le 1$, and $0<f<(1-\theta)/(1-b)$, it can be easily verified that

(1) $V_{\mathrm{BR}}^{\mathrm{BR}*}-V_B^{D*}-\pi_R^{D*}=(2(4-2\theta-r)\quad((4-r)(1-\theta-f+bf)+2f(1-\theta b))^2)/((1-\theta)(2-r)\quad(4-r)\quad(4(4-2r-\theta)+r^2)^2)>0$, $V_{\mathrm{BL}}^{\mathrm{BL}*}-V_B^{D*}-\pi_L^{D*}=(2(4-2\theta-r)(2(\theta+f)(1-\theta)+f(2-r)\quad(1-\theta b))^2)/(\theta(1-\theta)(2-r)(4-r)(4(4-2r-\theta)+r^2)^2)>0$.

(2) $\pi_R^{D*}-\pi_R^{\mathrm{BL}*}=(4(2(\theta+f)\quad(1-\theta)+f(1-\theta b)(2-r))((1-\theta-f+bf)\quad(12-2\theta-7r+r^2)+(1+bf)(4-r)(1-\theta)))/((1-\theta)\quad(4-r)^2(4(4-2r-\theta)+r^2)^2)>0$, $\pi_L^{D*}-\pi_L^{\mathrm{BR}*}=4((4-r)(1-\theta-\quad f+bf)+2f(1-\theta b))((4-r)(\theta+f)(1-\theta)+f(1-\theta b)(12-2\quad\theta-7r+r^2)))/((1-\theta)\quad(4-r)^2(4(4-2r-\theta)+r^2)^2)>0$.

J. Proof of Proposition 8

Denoting $f_0=(1-\theta)/(1-b)$ and $f_3=\sqrt{(2\theta(1-b))^2+4\theta(1-\theta)(1-\theta b^2)}/(2(1-\theta b^2))-\theta(1-b)/(1-\theta b^2)$, from the assumptions $0<\theta\le 1$, $0\le r\le 1$, $0\le b\le 1$, and $0<f<f_0$, it can be easily verified that $V_{\mathrm{BL}}^{\mathrm{BL}*}-V_B^{D*}-\pi_L^{D*}-(V_{\mathrm{BR}}^{\mathrm{BR}*}-V_B^{D*}-\pi_R^{D*})=(2(4-2\theta-r)\quad((1-\theta b^2)f^2+2\theta(1-b)f-\theta(1-\theta)))/(\theta(2-r)(4-r)(4\quad(4-2r-\theta)+r^2))$; if $f_3<f_0$, when $0<f\le f_3$, $V_{\mathrm{BR}}^{\mathrm{BR}*}-V_B^{D*}-\pi_R^{D*}\ge V_{\mathrm{BL}}^{\mathrm{BL}*}-V_B^{D*}-\pi_L^{D*}$, when $f_3<f<f_0$, $V_{\mathrm{BR}}^{\mathrm{BR}*}-V_B^{D*}-\pi_R^{D*}<V_{\mathrm{BL}}^{\mathrm{BL}*}-V_B^{D*}-\pi_L^{D*}$; if $f_3>f_0$, when $0<f<f_0$, $V_{\mathrm{BR}}^{\mathrm{BR}*}-V_B^{D*}-\pi_R^{D*}\ge V_{\mathrm{BL}}^{\mathrm{BL}*}-V_B^{D*}-\pi_L^{D*}$.

K. Proof of Proposition 9

From the assumptions $0<\theta\le 1$, $0\le r\le 1$, $0\le b\le 1$, and $0<f<(1-\theta)/(1-b)$, it can be easily verified that

(1) $V_T^{C*}-V_T^{BR*}=2f^2(1-\theta b^2/(\theta(1-\theta)(2-r)(4-r^2)>0$, $V_T^{C*}-V_T^{BL*}=2(1-\theta+f-bf^2/((1-\theta)(2-r)(4-r^2)>0$, $V_T^{C*}-V_T^{D*}=\left[\begin{array}{l}(2[(4\theta(4-11\theta+10\theta^2-3\theta^3)+4\theta(-2+5\theta-4\theta^2+\theta^3)r+\theta(1-\theta^2r^2)+f(8\theta(1-\theta)(-4((1-b)+\theta(2-3b)\\+b\theta^2)+(8\theta(1-\theta)(3-\theta-2b))r-(2\theta(1-\theta)(2-b(1+\theta)))r^2)+f^2((4(4+\theta-\theta^2)(1+\theta b^2)\\-8\theta(8-5\theta+\theta^2)b+(8\theta(5-\theta)b-4(2+3\theta-\theta^2)(1+\theta b^2))r+((1+3\theta)((1+\theta b^2))-2\theta(3+\theta)b)r^2))\end{array}\right]/(\theta(1-\theta)(2-r)\quad(4(4-2r-\theta)+r^2)^2)>0, V_T^{BL*}-V_T^{BR*}=(2((1-\theta b^2)f^2+2\theta(1-b)f-\theta(1-\theta)))/(\theta(2-r)(4-r)^2)$; if $f_3<f_0$, when $0<f\le f_3$, $V_T^{\mathrm{BR}*}\ge V_T^{\mathrm{BL}*}$, when $f_3<f<f_0$, $V_T^{\mathrm{BR}*}<V_T^{\mathrm{BL}*}$; if $f_3>f_0$, when $0<f<f_0$, $V_T^{\mathrm{BR}*}\ge V_T^{\mathrm{BL}*}$.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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