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Research Article

Based Fault-Tolerance Consensus of Second-Order Heterogeneous System under Input Saturation with Dynamics and Static Leader

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We focus on fault-tolerant consensus for heterogeneous dynamics systems with static and dynamic leaders under input saturation in this article. We apply theory of finite-time stability to multiagent system cooperative control. Also, we use integral sliding mode to overcome disturbance. The primary goal is a set of second-order linear and second-order nonlinear agents moving along the leader's trajectory. We use topology graph to describe communication between multiple agents. By using integral sliding mode control way, corresponding controller is introduced to make system stable. Finally, correctness of experiment was confirmed by MATLAB numerical simulation.

1. Introduction

Recently, we have witnessed great progress in the development of control systems. Due to characteristics of cooperative control of multiagent systems [1–3], such as its increasingly wide application and high execution efficiency, it is receiving increased attention from researchers. Multiagent systems have a wide range of applications, such as formation of unmanned aerial vehicles, encirclement of unmanned boats at sea, and containment control consisting of leaders who can detect obstacles and followers who cannot detect danger. Consensus [4–8] is the most fundamental research question of cooperative control, which represents that states of all agents reach the same state under the action of the consensus control protocol, i.e., their state errors converge to zero. Consensus issues can be categorized into leaderless consensus [9–11] and leader-following consensus [12–14] depending on the criterion of the number of leaders.

Depending on whether the leader will cause movement, it can be divided into static leader [15] and dynamic leader [16]. The simple understanding is that the dynamic leader has a certain speed, and the state of leader will cause certain changes. Also, the static leader has no speed and will not move. In [15], the authors designed new control protocols and introduced a nonlinear feedback control to solve finite-time containment control with dynamic and static leaders. Multiagent system with model uncertainties was introduced in [16] to address chattering reduction containment control problem.

Convergence time is a key research point in the study of consensus, and based on convergence time of consensus of multiagent systems, consensus can be categorized into asymptotic time consensus [17, 18], finite-time consensus [19, 20], and fixed-time consensus [21, 22]. As the name suggests, asymptotic time consensus is the consensus that converges at an exponential rate over an infinite amount of time. Regarding asymptotic time consensus, finite-time consensus is proposed because convergence time of asymptotic time consensus is not well calculated. In [17], high-order multiagent system with input quantization, actuator faults, unknown nonlinear functions, and directed communication topology were studied, and asymptotic consensus was considered. Leader-following bipartite consensus of Euler–Lagrange systems was investigated in [18] under system uncertain and deception attacks. Finite-time consensus can calculate its corresponding convergence time compared to asymptotic time consensus. Convergence time of finite-time consensus is associated with primary value of state. Controller was designed by considering relative position, and velocity measurements were investigated to deal with finite-time input saturation consensus in work of [19]. Authors considered finite-time output consensus in [20] of dynamics system with directed network and disturbance. According to this limitation, fixed-time consensus is proposed and convergence time of fixed-time consensus is independent on primary value. Because of the advantage of fixed-time consensus, the study of fixed-time consensus is more interested. Fixed-time consensus of heterogeneous dynamics systems consisting of first- and second-order systems was regarded in [21]. Fixed-time consensus of uncertain system was focused on with state constraints and input delay in [22].

In research of dynamics system, a healthy actuator and controller are generally studied. However, in real life, due to the needs of industrial engineering and the age of the actuator, some damage will inevitably occur. Therefore, it is necessary to study the situation of how to maintain stability of system when actuator fails [23–25]. In [23], under the condition of considering actuator fails, authors investigated fuzzy fixed-time consensus of nonlinear dynamics system with adding-a-power-integrator method. In [24], finite-time consensus control of nonlinear discrete-time system with Markov jump parameters and actuator faults was focused on. In [25], leader-following consensus of nonlinear dynamic system under actuator faults was taken into account, and communication graph is directed and connected.

Theoretically, any system can be stabilized if the control is only large enough, but this is not realistic. This is because we study the input saturation of the system [26–28]. In [26], the authors demonstrate in detail that bipartite consensus of linear dynamics system and input control was regarded as saturation. Time-varying formation control of linear dynamics system could be achieved by authors in [27], and input saturation was considered. In the work of [28], consensus control of mixed second-order linear and nonlinear system was studied.

For a general system, each agent has the same model and application environment. With the wide application of dynamics systems, in many cases, the agents will have different models or have different application environments. Therefore, it is important to study heterogeneous systems [29–32]. In [29], the authors explored bipartite output formation containment of heterogeneous linear dynamics system. In [30], bipartite output consensus of heterogeneous linear system was introduced, and finite- and fixed-time could be reached. Finite-time heterogeneous consensus was studied in [31] with integral sliding mode control and pinning control methods. In [32], the author studied tracking consensus for heterogeneous group system based on switching topology and input time delay.

Thus, based on some of the above articles, we study fixed-time consensus control of heterogeneous nonlinear dynamics systems according to input saturation and actuator fault. Main contributions are as follows:

(1) Compared to each agent having the same dynamic, we are studying heterogeneous multiagent systems, which mean that each agent has its own state equation, but they can still satisfy consensus through controller. Compared to homogeneous multiagent systems, heterogeneous systems have more research significance in practical applications.
(2) Compared to general linear or terminal sliding modes, we use integral sliding mode. The integral sliding mode has better robustness and avoids the drawbacks of the conventional sliding mode approach stage. The use of integral sliding mode can not only avoid singular phenomena but also achieve better robustness performance.
(3) Compared to a normal controller, we are studying a faulty system. When the actuator of an agent faults, how to maintain system stability is the focus of our research. When the input of the controller is too large, the method of input saturation can be used to solve this problem.

The remaining parts of this work are structured as follows. Section 2 introduces preamble and formulation of problem. In Section 3, main results of analysis are provided. Simulations results are provided in Section 4. Section 5 summarizes work done in this paper.

2. Preliminaries

2.1. Notion. $R^{\mathrm{pq}}, R^{\mathrm{pq}\times\mathrm{mn}}$ means set of pq dimensional vectors, and mn $\times$ pq means dimensional matrix, respectively. Denote I_{bm} where bm is the dimensional identity matrix. $\otimes$ is the Kronecker product. Define $\mathrm{sgn}(p) = [\mathrm{sgn}(p_1), \ldots, \mathrm{sgn}(p_n)]^T$, where $\mathrm{sgn}(\bullet)$ is signum function, and $p = (p_1, \ldots, p_n)^T$.

2.2. Graph Theory. What is discussed in this section is a topology with n followers and one leader. That topology is denoted by $G = (V_G, E_G, A)$, and graph G is a directed graph, where $E_G \in V_G \times V_G$ is the set of edges, $V_G = \{v_0, \ldots, v_n\}$ represents the set of nodes, and $A = [a_{ij}] \in R^{(n+1)\times(n+1)}$ is the weighted adjacency matrix of graph G.

Node indexes belong to a nonempty finite index set $\Gamma = \{0, 1, \ldots, n\}$, and followers' nodes belong to $\Gamma_1 = \{1, \ldots, n\}$. A directed edge $(v_j, v_i) \in E_G$ in graph G means that agent i can receive information from agent j, but not conversely. Define $a_{ij} > 0$ if $(v_j, v_i) \in E_G$, and while $a_{ij} = 0$ otherwise. Degree matrix is $D = \mathrm{diag}\,(d_1, \ldots d_n)$, where $d_i = \sum_{j=1}^{n} a_{ij}$. Laplacian matrix $L = [l_{ij}] \in R^{n\times n}$ which is associated with A, defined as $l_{ii} = \sum_{j=1, j\neq i}^{n} a_{ij}$ and $l_{ij} = -a_{ij}$, $i \neq j, i, j = 1, \ldots, n$, and $L = D - A$. If there is a path between any two distinct vertices, then directed graph G is called strongly connected.

Connection weight between any of followers and a leader is displayed by b_i, $i \in \Gamma_1$. If the i-th follower is connected to the leader, then $b_i > 0$; otherwise, $b_i = 0$. Let $B = \mathrm{diag}\,\{b_1, \ldots b_n\}$.

2.3. Some Useful Lemmas and Definitions

Definition 1 (see [33]). Connected graph with leader is connected if there exists one or more agents in G that can connect to the leader via an edge.

Definition 2 (see [34, 35]). System $\dot{y}_s = h(y_s)$ is nonlinear, with $h(0) = 0$, $y_s \in R^n$, where $h(\cdot)$: $R^n \longrightarrow R^n$ is a continuous function. If system's equilibrium point is zero and is Lyapunov stable in a finite time, it is stable for a finite time. Finite-time attractive means that there is a function $T_h(y_{s0})$ such that $\lim_{t\longrightarrow T_h(y_{s0})} y_s(t, y_{s0}) = 0$ that $x(t, y_{s0}) = 0$, $\forall t \geq T_h\ (y_{s0})$, where $y_s(t, y_{s0})$ is the solution of system starting from $y_s(0) = y_{s0}$. If the system is finite-time stable and fixed-time attractive, then it is called fixed-time stable. Fixed-time attractive requires that there is a constant T_h such that pervious finite convergent time $T_h(y_{s0})$ satisfies $T_h(y_{s0}) \leq T$ for all $y_{s0} \in R^n$.

Definition 3 (see [36]). A vector filed $f(y_s) = (f_1(y_s), \ldots, f_n(y_s))^T$ is said to be homogeneous in the 0-limit or ∞-limit with associated triple (r_p, k_p, f_p) where $r_p = [r_{p1}, \ldots, r_{pn}] \in R^n$ is weight, k_p is degree, and f_p is approximating vector field, if $k_p + r_{pj} > 0$, and then, function $f_i(y_s)$ is homogeneous in 0-limit or ∞-limit with associated triple $(r_p, k_p + r_p, f_p)$ for each i.

Lemma 4 (see [36]). *For $\dot{y}_s = f(t, y_s)$, $y_s(0) = y_{s0}$, suppose that $f(y_s)$ is homogeneous in the 0-limit or ∞-limit with associated triples (r_0, k_0, f_0) and $(r_\infty, k_\infty, f_\infty)$. If the origins of system $\dot{y}_s = f(y_s)$, $\dot{y}_{s0} = f_0(y_s)$, and $\dot{y}_{s\infty} = f_\infty(y_s)$ are global asymptotically stable, then the origin of $\dot{y}_s = f(t, y_s)$, $y_s(0) = y_{s0}$ is fixed-time stable when condition $k_\infty > 0 > k_0$ holds.*

Lemma 5 (see [35]). *Nonlinear dynamics system is $\dot{x}_s = h(x_s)$ with $h(0) = 0$, $x_s \in R^n$. If there exists a positive definite continuous function $V_h(x_s)$ such that $\dot{V}_h(x_s) \leq -\alpha_h V_h^{p_h}(x_s) - \beta_h V_h^{q_h}(x_s)$, where $\alpha_h, \beta_h > 0$, $0 < p_h < 1$, $q_h > 1$, then the origin is fixed-time stable equilibrium of system and settling time satisfies $T_h \leq 1/\alpha_h(1 - p_h) + 1/\beta_h(1 - q_h)$, and $V_h(t) = 0$ for arbitrary $t_h > T_h(x_s(0))$.*

Assumption 6. Communication topology among followers is directed. For any follower, there is at least a directed path from leader to follower.

Assumption 7. Nonlinear dynamic continuous function $f_i(x_i, v_i, t)$ is assumed to be bounded and satisfied

$$\left|f(x_i, v_i, t)\right| \leq c_1\left|x_i\right| + c_2\left|v_i\right|, \tag{1}$$

where c_1 and c_2 are any nonnegative constants.

Assumption 8. The i-th follower agents bounded unknown disturbance δ_i satisfies

$$\left|\delta_i\right| \leq \varphi < +\infty. \tag{2}$$

2.4. Problem Formulation. Consider a set of dynamics system including $m\,(m < n)$ second-order systems with linear terms and $n - m$ second-order systems with nonlinear terms. Let $F^{s-} = (1, \ldots, m)$, $F^{s+} = (m + 1, \ldots, n)$, and $F^{\mathrm{sn}} = (1, \ldots, n)$.

Dynamics equation of the follower is pictured as

$$\begin{aligned} &\begin{cases} \dot{x}_i = v_i, \\ \dot{v}_i = u_i, \end{cases} \quad i \in \{1, \ldots, m\}, \\ &\begin{cases} \dot{x}_i = v_i, \\ \dot{v}_i = u_i + f_i(x_i, v_i, t) + d_i, \end{cases} \quad i \in \{m + 1, \ldots, n\}, \end{aligned} \tag{3}$$

where $x_i \in R^n$ is position, $v_i \in R^n$ is velocity, $u_i \in R^n$ is control input, $f_i(x_i, v_i, t)$ is nonlinear dynamics, and d_i is external disturbance.

In this paper, loss of effectiveness was concerned about actuator faults. Assume that the leader is not subject to actuator faults. Also, u_i^a is the actual control input, which is expressed by $u_i^a = c_i u_i$, where u_i is ideal input and c_i is unknown faults, where $0 \leq c_i \leq 1$.

Dynamics of leader is

$$\begin{cases} \dot{x}_0 = v_0, \\ \dot{v}_0 = u_0, \end{cases} \tag{4}$$

where $x_0 \in R^n$ is position, $v_0 \in R^n$ is velocity, and $u_0 \in R^n$ is control input, correspondingly.

To organize the above systems (3) and (4), it can obtain

$$\begin{aligned} &\begin{cases} \dot{x}_i = v_i, \\ \dot{v}_i = c_i u_i, \end{cases} \quad i \in \{1, \ldots, m\}, \\ &\begin{cases} \dot{x}_i = v_i, \\ \dot{v}_i = c_i u_i + f_i(x_i, v_i, t) + d_i, \end{cases} \quad i \in \{m + 1, \ldots, n\}. \end{aligned} \tag{5}$$

(5) becomes a matrix of

$$\begin{cases} \dot{x} = v, \\ \dot{v} = cu + f + d = u + (c - I)u + d + f = u + D + f, \end{cases} \tag{6}$$

where $D = (c - I)u + d$ describes so-called lumped faults, in which external disturbances and actuator faults are included.

Also, define

$$\begin{aligned} x &= \left[x_1, x_2, \ldots, x_n\right]^T, \\ v &= \left[v_1, v_2, \ldots, v_n\right]^T, \\ c &= \left[c_1, c_2, \ldots, c_n\right]^T, \\ u &= \left[u_1, u_2, \ldots, u_n\right]^T, \\ f &= \left[0, \cdots 0, f_{m+1}, \ldots, f_n\right]^T, \\ d &= \left[0, \cdots 0, d_{m+1}, \ldots, d_n\right]^T, \\ D &= \left[D_1, D_2, \ldots, D_n\right]^T, \\ s &= \left[s_1, s_2, \ldots, s_n\right]^T. \end{aligned} \tag{7}$$

We can get the following assumption for nonlinear dynamics of this agent.

Assumption 9. Following nonlinear continuous function $f_i(x_i, v_i, t)$ is supposed to be bounded and satisfies

$$\left|f_i(x_i, v_i, t) - f_0(x_0, v_0, t)\right| \leq \alpha_1 \left|x_i - x_0\right| + \alpha_2 \left|v_i - v_0\right|, \tag{8}$$

where α_1 and α_2 are any constants other than negative numbers.

System error is defined as

$$\begin{cases} e_{\mathrm{xi}} = \sum_{j=1}^{n} a_{ij}\left(x_i(t) - x_j(t)\right) + b_i\left(x_i(t) - x_0(t)\right), \\ e_{\mathrm{vi}} = \sum_{j=1}^{n} a_{ij}\left(v_i(t) - v_j(t)\right) + b_i\left(v_i(t) - v_0(t)\right), \end{cases} \quad i \in \{1, \ldots, n\}. \tag{9}$$

Matrix form of the error (9) is

$$\begin{cases} e_x = \left((L+B)\otimes I_m\right)\widetilde{x}, \\ e_v = \left((L+B)\otimes I_m\right)\widetilde{v}, \end{cases} \quad i \in \{1, 2, \ldots, n\}, \tag{10}$$

where $\widetilde{x} = x - 1_n \otimes x_0, \widetilde{v} = v - 1_n \otimes v_0$.

The consensus error can be obtained by (10):

$$\begin{cases} \dot{e}_x = e_v, \\ \dot{e}_v = \left((L+B)\otimes I_m\right)\left(u + f + D - 1_n \otimes u_0\right). \end{cases} \tag{11}$$

According to error, integral sliding mode is designed as

$$s_i = e_{\mathrm{vi}} + \int_0^t \left(p_1 e_{\mathrm{xi}}(s)^{\rho_1/\pi_1} + p_2 e_{\mathrm{xi}}(s)^{\delta_1/\sigma_1} + p_3 e_{\mathrm{vi}}(s)^{\rho_2/\pi_2} + p_4 e_{\mathrm{vi}}(s)^{\delta_2/\sigma_2}\right)\mathrm{d}s, \tag{12}$$

where $p_i > 0, \rho_i > \pi_i, \delta_i < \sigma_i, \rho_1/\pi_1 = \rho_2/2\pi_2 - \rho_2, \delta_1/\sigma_1 = \delta_2/2\sigma_2 - \delta_2$.

According to (12), $\dot{s}_i$ can be obtained that

$$\dot{s}_i = \dot{e}_{\mathrm{vi}} + p_1 e_{\mathrm{xi}}(t)^{\rho_1/\pi_1} + p_2 e_{\mathrm{xi}}(t)^{\delta_1/\sigma_1} + p_3 e_{\mathrm{vi}}(t)^{\rho_2/\pi_2} + p_4 e_{\mathrm{vi}}(t)^{\delta_2/\sigma_2}. \tag{13}$$

The saturated protocol is written as

$$\begin{aligned} u = &\left(\left(L+B\right)^{-1}\otimes I_m\right)\left[-\tanh\left(\mathrm{sig}(s)^{\kappa_1}\right) - \tanh\left(\mathrm{sig}(s)^{\kappa_2}\right) - p_1 e_x(t)^{\rho_1/\pi_1} \right. \\ &\left. - p_2 e_x(t)^{\delta_1/\sigma_1} - p_3 e_v(t)^{\rho_2/\pi_2} - p_4 e_v(t)^{\delta_2/\sigma_2}\left((L+B)\otimes I_m\right)\left(1_n\otimes u_0 - \alpha_1|x| - \alpha_2|v| - \widetilde{D}\right)\right], \end{aligned} \tag{14}$$

where $\kappa_1 > 1, 0 < \kappa_2 < 1$.

3. Main Result

Theorem 10. *According to Assumptions 6 and 9 hold. Introducing (12) as sliding mode, achievement of $s = 0$ makes e_x and e_v converge to 0 in fixed time.*

Proof. When $s = 0$, one can get

$$\begin{aligned} &\dot{e}_{\mathrm{xi}} = e_{\mathrm{vi}}, \\ &\dot{e}_{\mathrm{vi}} = -p_1 e_{\mathrm{xi}}(t)^{\rho_1/\pi_1} - p_2 e_{\mathrm{xi}}(t)^{\delta_1/\sigma_1} - p_3 e_{\mathrm{vi}}(t)^{\rho_2/\pi_2} - p_4 e_{\mathrm{vi}}(t)^{\delta_2/\sigma_2}. \end{aligned} \tag{15}$$

Consider candidate Lyapunov function

$$V_S = \frac{1}{1+\rho_1/\pi_1}\left|e_x\right|^{1+\rho_1/\pi_1} + \frac{1}{1+\delta_1/\sigma_1}\left|e_x\right|^{1+\delta_1/\sigma_1} + \left|e_v\right|^2. \tag{16}$$

Taking derivative of V_S yields

$$\begin{aligned}\dot{V}_S &= 2p_1\left|e_x\right|^{\rho_1/\pi_1}\left|e_v\right| + 2p_2\left|e_x\right|^{\delta_1/\sigma_1}\left|e_v\right| + 2\left|e_v\right|\left|\dot{e}_v\right| \\ &= 2p_1\left|e_x\right|^{\rho_1/\pi_1}\left|e_v\right| + 2p_2\left|e_x\right|^{\delta_1/\sigma_1}\left|e_v\right| + 2\left|e_v\right|\left|\dot{e}_v\right| \\ &= 2p_1\left|e_x\right|^{\rho_1/\pi_1}\left|e_v\right| + 2p_2\left|e_x\right|^{\delta_1/\sigma_1}\left|e_v\right| \\ &\quad + 2\left|e_v\right|\left(-p_1e_x(t)^{\rho_1/\pi_1} - p_2e_x(t)^{\delta_1/\sigma_1} - p_3e_v(t)^{\rho_2/\pi_2} - p_4e_v(t)^{\delta_2/\sigma_2}\right) \\ &= -2p_3e_v(t)^{\rho_2/\pi_2+1} - 2p_4e_v(t)^{\delta_2/\sigma_2+1}.\end{aligned} \tag{17}$$

Therefore, e_{vi} converges to 0 asymptotically. Moreover, based on equation (15), it has e_{xi} converged to 0 asymptotically.

Considering $p_i > 0, \rho_i > \pi_i, \delta_i < \sigma_i, \rho_1/\pi_1 = \rho_2/2\pi_2 - \rho_2, \delta_1/\sigma_1 = \delta_2/2\sigma_2 - \delta_2$, error system (9) in 0-limit is written as follows:

$$\begin{aligned}\dot{e}_{\mathrm{xi}} &= e_{\mathrm{vi}}, \\ \dot{e}_{\mathrm{vi}} &= -p_2e_{\mathrm{xi}}(t)^{\delta_1/\sigma_1} - p_4e_{\mathrm{vi}}(t)^{\delta_2/\sigma_2}.\end{aligned} \tag{18}$$

So $V_0 = 2p_2 1/1 + \delta_2/\sigma_2|e_x|^{1+\delta_2/\sigma_2} + |e_v|^2$, and its derivative is

$$\begin{aligned}\dot{V}_0 &= 2p_2\left|e_x\right|^{\delta_2/\sigma_2}\left|e_v\right| + 2\left|e_v\right|\left|\dot{e}_v\right| \\ &= 2p_2\left|e_x\right|^{\delta_2/\sigma_2}\left|e_v\right| + 2\left|e_v\right|\left(-p_2e_x(t)^{\delta_1/\sigma_1} - p_4e_v(t)^{\delta_2/\sigma_2}\right) \\ &= -2p_4e_v(t)^{1+\delta_2/\sigma_2}.\end{aligned} \tag{19}$$

Also, error system (9) in ∞-limit is written as follows:

$$\begin{aligned}\dot{e}_{\mathrm{xi}} &= e_{\mathrm{vi}}, \\ \dot{e}_{\mathrm{vi}} &= -p_1e_{\mathrm{xi}}(t)^{\rho_1/\pi_1} - p_3e_{\mathrm{vi}}(t)^{\rho_2/\pi_2}.\end{aligned} \tag{20}$$

So $V_\infty = 2p_2 1/1 + \rho_1/\pi_1|e_x|^{1+\rho_1/\pi_1} + |e_v|^2$, and its derivative is

$$\begin{aligned}\dot{V}_\infty &= 2p_2\left|e_x\right|^{\delta_2/\sigma_2}\left|e_v\right| + 2\left|e_v\right|\left|\dot{e}_v\right| \\ &= 2p_1\left|e_x\right|^{\rho_1/\pi_1}\left|e_v\right| + 2\left|e_v\right|\left(-p_1e_x(t)^{\rho_1/\pi_1} - p_3e_v(t)^{\rho_2/\pi_2}\right) \\ &= -2p_3e_v(t)^{\rho_2/\pi_2+1}.\end{aligned} \tag{21}$$

Therefore, it is obtained that both the 0-limit and ∞-limit systems (18) and (20) are globally asymptotically stable.

Finally, homogeneity of bilimit systems (18) and (20) will be involved.

For 0-limit, and according to Definition 3, one can obtain $r_1 + k_0 = r_2, r_2 + k_0 = r_1\delta_2/\sigma_2 = r_2\delta_1/\sigma_1$, and let $r_2 = 1$, one can get $k_0 = \delta_1/\sigma_1 - 1 < 0, \delta_1/\sigma_1 = (1-k_0)\delta_2/\sigma_2, \delta_2/\sigma_2 = \delta_1/2\sigma_1 - \delta_1$. Similarly, for ∞-limit case, it can obtain that $k_\infty = \delta_1/\sigma_1 - 1 > 0, \rho_1/\pi_1 = (1-k_\infty)\rho_2/\pi_2, \rho_2/\pi_2 = \rho_1/2\pi_1 - \rho_1$. Thus, fixed-time stability is achieved by Lemma 4. □

3.1. Dynamic Leader. In this part, the main research is about dynamics leader.

Theorem 11. *Suppose Assumptions 6 and 9 hold. For systems (4) and (3), controller is designed as (14), and sliding mode is shown as (8), and then, system will reach sliding mode surface $s = 0$ in fixed time.*

Proof. Choose a Lyapunov candidate function as follows:

$$V_1 = \frac{1}{2}S^TS. \tag{22}$$

Differentiating V_1, we have

$$\begin{aligned}\dot{V}_1 &= s^T\dot{s} \\ &= s^T\left(\dot{e}_v + p_1e_x(t)^{\rho_1/\pi_1} + p_2e_x(t)^{\delta_1/\sigma_1} + p_3e_v(t)^{\rho_2/\pi_2} + p_4e_v(t)^{\delta_2/\sigma_2}\right) \\ &= s^T\left((L+B)\otimes I_m\right)\left(u + f + D - 1_n\otimes u_0\right) \\ &\quad + s^T\left(p_1e_x(t)^{\rho_1/\pi_1} + p_2e_x(t)^{\delta_1/\sigma_1} + p_3e_v(t)^{\rho_2/\pi_2} + p_4e_v(t)^{\delta_2/\sigma_2}\right).\end{aligned} \tag{23}$$

Combining (14), one gets

$$\dot{V}_1 \leq s^T \left(-\tanh\left(\mathrm{sig}(s)^{\kappa_1}\right) - \tanh\left(\mathrm{sig}(s)^{\kappa_2}\right)\right). \tag{24}$$

Because of the fact that $\tanh\left(\mathrm{sig}(x)^{\kappa}\right) = \mathrm{sig}(x)^{\kappa} + o\left(\mathrm{sig}(x)^{\kappa}\right)$,

$$\begin{aligned}\dot{V}_1 &\leq s^T \left(-\mathrm{sig}(s)^{\kappa_1} - \mathrm{sig}(s)^{\kappa_2}\right)\\ &\leq -\mathrm{sig}(s)^{\kappa_1+1} - \mathrm{sig}(s)^{\kappa_2+1}\\ &\leq -2^{\kappa_1+1/2}V_1^{\kappa_1+1/2} - 2^{\kappa_2+1/2}V_1^{\kappa_2+1/2}.\end{aligned} \tag{25}$$

According to Lemma 5, sliding mode (12) for systems (3) and (4) with controller (14) is fixed-time stable. Because of $V_1(s) > 0$ and $\dot{V}_1(s) < 0$, (22) converges to 0. Also, $s = 0$ will remain.

The proof is completed. □

3.2. Static Leader. In the above, the main study is on dynamic leaders. Within this, there is a special case of a static leader, where the speed of the leader is zero.

For a static leader, the velocity of the leader is zero, and then the leader's dynamic is

$$\begin{cases}\dot{x}_0 = 0,\\ \dot{v}_0 = 0.\end{cases} \tag{26}$$

Then, one can get state error

$$\begin{cases} e_{\mathrm{xi}} = \displaystyle\sum_{j=1}^{n} a_{ij}\left(x_i(t) - x_j(t)\right) + b_i x_i(t),\\ e_{\mathrm{vi}} = \displaystyle\sum_{j=1}^{n} a_{ij}\left(v_i(t) - v_j(t)\right) + b_i v_i(t),\end{cases} \quad i \in \{1, \ldots, n\}. \tag{27}$$

Matrix form of error is

$$\begin{cases} e_x = \left((L+B)\otimes I_m\right)x,\\ e_v = \left((L+B)\otimes I_m\right)v,\end{cases} \quad i \in \{1, 2, \ldots, n\}. \tag{28}$$

Consensus error is obtained by (28):

$$\begin{cases} \dot{e}_x = e_v,\\ \dot{e}_v = \left((L+B)\otimes I_m\right)(u + f + D).\end{cases} \tag{29}$$

The same sliding mode control is used as (12), and the consensus control topology can be designed as

$$\begin{aligned} u = &\left(\left(L+B\right)^{-1}\otimes I_m\right)\Big[-\tanh\left(\mathrm{sig}(s)^{\kappa_3}\right) - \tanh\left(\mathrm{sig}(s)^{\kappa_4}\right) - p_1 e_x(t)^{\rho_1/\pi_1} - p_2 e_x(t)^{\delta_1/\sigma_1}\\ &- p_3 e_v(t)^{\rho_2/\pi_2} - p_4 e_v(t)^{\delta_2/\sigma_2}\left((L+B)\otimes I_m\right)\left(-\alpha_1|x| - \alpha_2|v| - \widetilde{D}\right)\Big],\end{aligned} \tag{30}$$

where $\kappa_3 > 1, 0 < \kappa_4 < 1$.

Theorem 12. *Suppose Assumptions 6 and 9 hold. For systems (26) and (3), controller is designed as (30), and sliding mode is shown as 8, and then, system will reach sliding mode surface $s = 0$ in fixed time.*

Proof. Choose a Lyapunov candidate function $V_2 = 1/2s^T s$.

Differentiating V_2, we have

$$\begin{aligned}\dot{V}_2 &= s^T \dot{s}\\ &= s^T\left(\dot{e}_v + p_1 e_x(t)^{\rho_1/\pi_1} + p_2 e_x(t)^{\delta_1/\sigma_1} + p_3 e_v(t)^{\rho_2/\pi_2} + p_4 e_v(t)^{\delta_2/\sigma_2}\right)\\ &= s^T\left((L+B)\otimes I_m\right)(u + f + D)\\ &\quad + s^T\left(p_1 e_x(t)^{\rho_1/\pi_1} + p_2 e_x(t)^{\delta_1/\sigma_1} + p_3 e_v(t)^{\rho_2/\pi_2} + p_4 e_v(t)^{\delta_2/\sigma_2}\right)\\ &\leq s^T\left(-\tanh\left(\mathrm{sig}(s)^{\kappa_3}\right) - \tanh\left(\mathrm{sig}(s)^{\kappa_4}\right)\right)\\ &\leq -\mathrm{sig}(s)^{\kappa_3+1} - \mathrm{sig}(s)^{\kappa_4+1}\\ &\leq -2^{\kappa_3+1/2}V_2^{\kappa_3+1/2} - 2^{\kappa_4+1/2}V_2^{\kappa_4+1/2}.\end{aligned} \tag{31}$$

According to Lemma 5, sliding mode (12) for systems (3) and (4) with controller (30) is fixed-time stable. Because of $V_2(s) > 0$ and $\dot{V}_2(s) < 0$, (22) converges to 0. Also, $s = 0$ will remain.

The proof is completed. □

4. Simulations

To verify validity of the proposed controller, we give two examples to verify its validity. In this section, we give a topology graph consisting of six followers and a leader whose topology is satisfying the assumed conditions. Topology is designed by Figure 1, where agents labeled as 1, 2, 3, 4, 5, and 6 are followers and labeled as 0 is the leader.

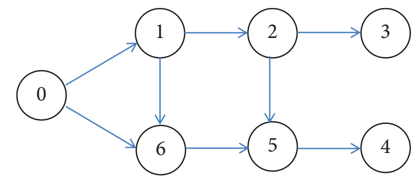

FIGURE 1: The interaction communication topology of 7 agents.

Example 1. First, we give the initial values of system. The initial value of followers position is $x(0) = [4, 0.3, 2, -0.3, 1.1, 0.04]^T$, and the initial value of followers velocity is $v(0) = [-10, 1.8, -1, 5, -0.05, -1.8]^T$. $x_0(0) = 3, v_0(0) = 1$ are the initial values of leader's state. Then, the nonlinear function is $f_i = -\sin(t), i = 3, 4, 5, 6$, and then, the model of the system can be written as

$$\begin{cases} \dot{x}_i = v_i, \\ \dot{v}_i = c_i u_i, \end{cases} \quad i = 1, 2,$$
$$\begin{cases} \dot{x}_i = v_i, \\ \dot{v}_i = c_i u_i - \sin(t) + d_i, \end{cases} \quad i = 3, 4, 5, 6. \tag{32}$$

After having these data above, we simulate it by performing simulation on it. We can get several pictures as shown below.

Figure 2 shows the error graph of position and velocity, and it can be seen that state error converges to zero in picture. According to definition, when state errors converge to zero, then consensus is satisfied. According to the trajectory of state and the trajectory of state error, we can know that it can be seen our controller is efficient and correct.

Figure 3 shows the position and velocity error change. We can see that the position and velocity error can converge to 0. When state errors converge to zero, then it is to meet consensus. According to the position and velocity change of agents and error, we can see that the consensus can be achieved. Our controller is effective and correct.

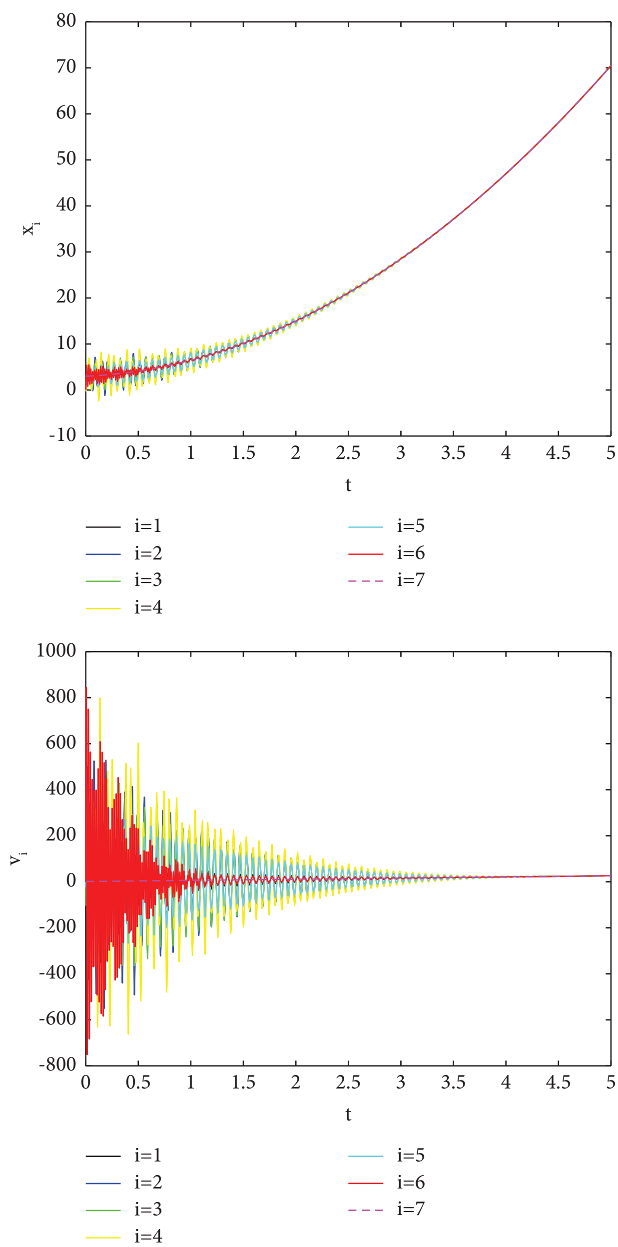

FIGURE 2: State change of agents.

Example 2. In this section, we consider the case of a static leader. Figures 4 and 5 are obtained through simulation.

As shown in Figure 4, it is known that position and velocity of the followers converge to leader's position and speed, consensus with leader's dynamics. Again, because it is a static leader, position and velocity of the followers converge to 0. Actual is consensus with the theory.

As shown in Figure 5, error between followers' position and velocity and leader's position and velocity converges to 0. According to definition of consensus, the same conclusion as in Figure 5 can be obtained. Therefore, it can be obtained from Figures 4 and 5 that the system will realize to fixed-time consensus in the case of a static leader and our controller is efficient and correct.

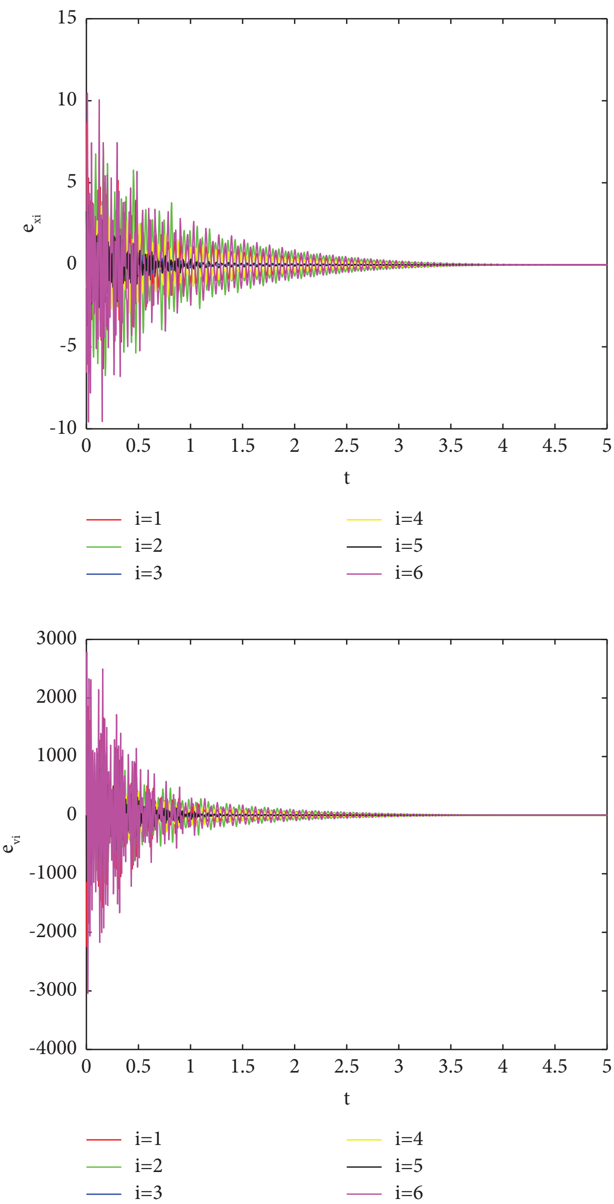

Figure 3: Position and velocity error change of multiagent system with dynamics leader.

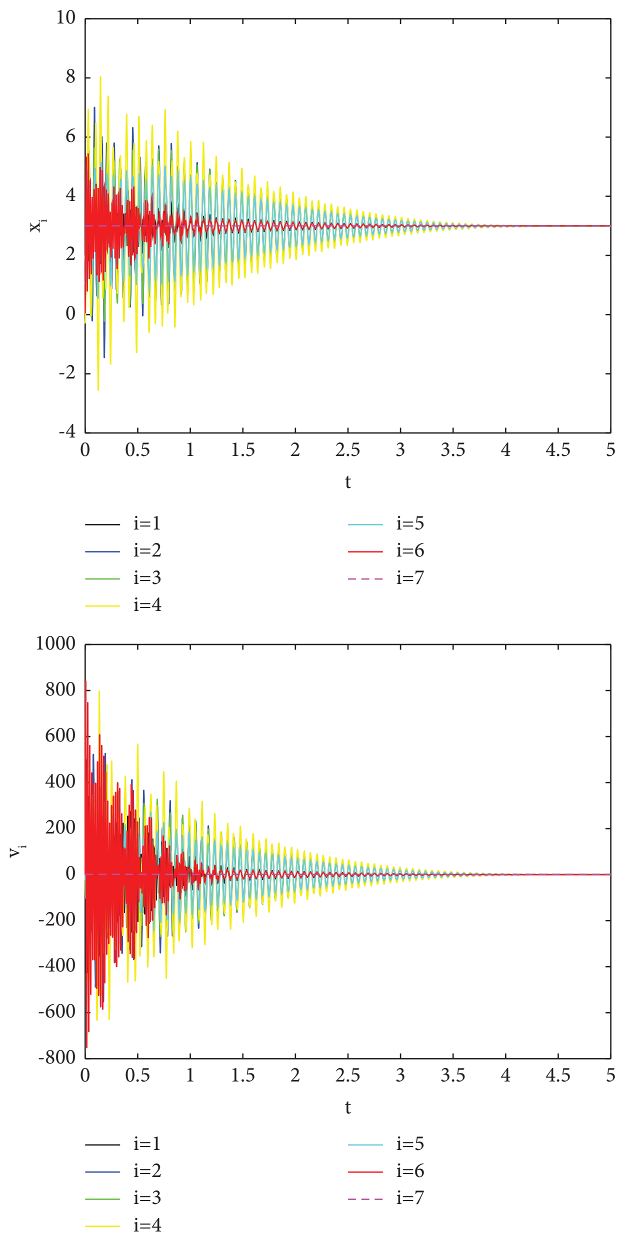

Figure 4: Position and velocity change.

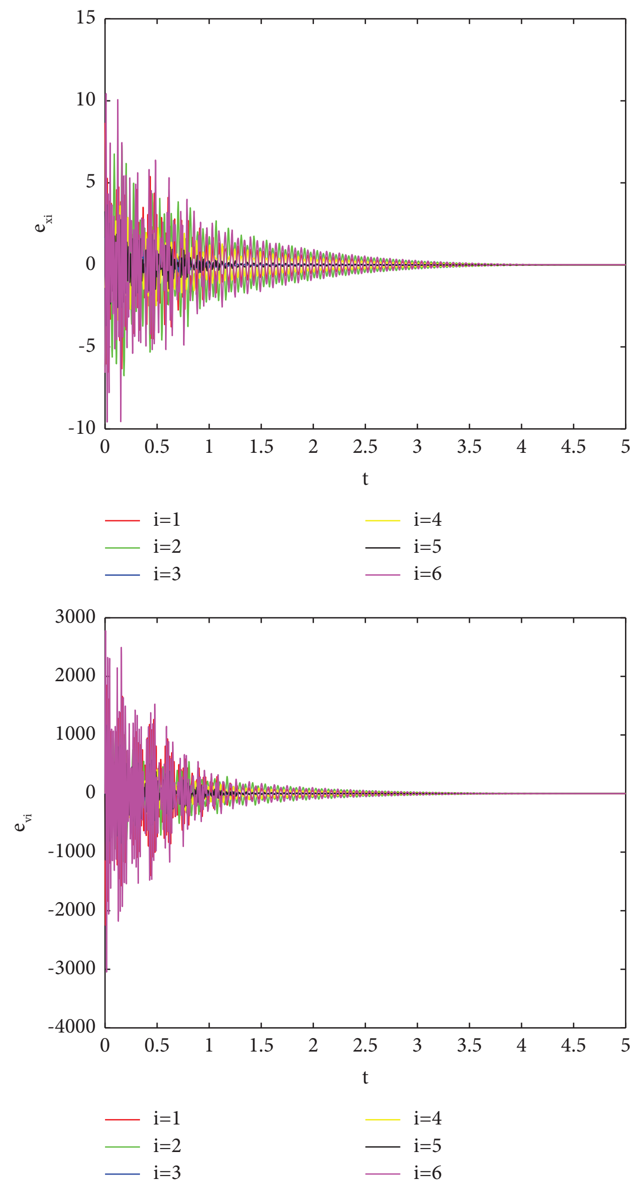

FIGURE 5: Position and velocity error change of multiagent system with static leader.

5. Conclusion

In this section, we focus on a summary of the work done in the whole article, as well as an outlook on what we would like to do in the future. In this paper, we focus on fixed-time consensus problem for nonlinear heterogeneous systems under input saturation and actuator faults. In this article, we mainly use a heterogeneous second-order system, which mainly contains second-order linear system and second-order nonlinear system. Fixed-time consensus with actuator faults is addressed by using a sliding mode control approach. Finally, simulations are used to verify that the proposed controller is effective. In future work, one would like to study heterogeneous systems that are in different locations on land, sea, and air and study how these agents perform tasks such as consensus and containment.

Data Availability

The data that support the findings of this study can be obtained from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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