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Research Article

Decision Support System for Single-Valued Neutrosophic Aczel–Alsina Aggregation Operators Based on Known Weights

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Multiattribute decision making (MADM) approach is a well-known decision-making process utilized in a variety of fields such as solid waste management, renewable energy resources, air quality assurance, hotel location decision, sustainable supplier selection, partner recognition, green supplier enterprises, game theory, construction development authority, and weapon group target estimation. The aggregation operators (AOs) are essential components of the decision-making process and have a great capability to deal with ambiguous and unpredictable information in the different fields of fuzzy environments. In this article, we expressed the theory concepts of single-valued neutrosophic (SVN) sets (SVNS) and also characterized their basic operations. The power aggregation tools are allowed to input arguments to support each other among different arguments. Recently, Aczel–Alisna aggregation tools conquered great attention from several research scholars. We also exposed some reliable operations of Aczel–Alsina aggregation models under the consideration of SVN information. We established a series of new approaches, including the "single-valued neutrosophic Aczel–Alsina power weighted average" (SVNAAPWA) operator and "single-valued neutrosophic Aczel–Alsina power weighted geometric" (SVNAAPWG) operators. To show the effectiveness and compatibility of derived approaches, some prominent characteristics are also established. We constructed a MADM technique to solve an application of engineering and construction materials under consideration of our derived methodologies. An experimental case study is also presented to determine a suitable optimal option from a group of options. To find the flexibility of our proposed work, we provided a comparative study that compares the results of existing AOs with our proposed work. A comprehensive overview is also presented here.

1. Introduction

In 1965, Zadeh [1] invented the fuzzy set (FS) and introduced a truth value (TV) between 0 and 1 in place of the common crisp value of 0 and 1. In the theory and science of decision-making, the fuzzy theory is a significant and engaging study issue, but FS lacks a falsity value (FV); instead, it is only identified by its TMV, which ranges from 0 to 1. To address the shortcomings of FS, Atanassov [2] developed the idea of an intuitionistic fuzzy set (IFS), which is distinguished by its TV and FV between 0 and 1. Yager [3] extended the theory of IFSs in the framework of Pythagorean FS (PyFS) with the sum of the square of TV and FV lies on the interval [0,1]. An interval-valued IFS (IVIFS), which is distinguished by its interval TV and interval FV in the unit interval $[0, 1]$, was presented by Atanassov and Gargov [4] as a further generalization of an IFS. Smarandache [5, 6] created a neutrosophic set (NS) from a philosophical point of view to express indeterminate and inconsistent information because IFSs and IVIFSs cannot represent it. In an NS B, $[\varphi_A(x)\colon \longrightarrow {}^{-}0, 1^{+}]$ represents TV, $[\xi_A(x)\colon \longrightarrow {}^{-}0, 1^{+}]$ represents the abstinence value (AV), and $[\psi_A(x)\colon \longrightarrow {}^{-}0, 1^{+}]$ represents FV of a set B. The difficulties of practical applications may, therefore, be caused by the nonstandard interval $[{}^{-}0, 1^{+}]$. Consequently, based on the actual standard range [0, 1], Ye [7] introduced a simplified neutrosophic set (SNS), which includes the concepts of a SVNS given by Wang et al. [8] and an interval NS (INS) introduced by the Ye [9], which are the extension of IFS and IVIFS. An innovative idea of SVNNs based on weighted aggregated sum product valuation to solve a MADM technique was developed by Mishra et al. [10]. Seikh and Dutta [11] illustrated the theory of SVN information and developed advanced programming for the solution of matrix game theory. These concepts were first presented as classifications of NS to be easily used for real-time applications. Consequently, SVNS and INS are subclasses of NS, whereas SNS is a subclass of NS. NS is the generalization of FS and IFS; we also studied literature related to current research work seen in [12–14].

The AOs are appropriate mathematical tools to aggregate ambiguous and uncertain information. Recently, several research scientists worked on different AOs in the system of fuzziness such as AOs of IFS given by Xu [15], AOs of PyFSs developed by Rahman et al. [16], AOs of interval-valued PyFSs (IVPyFSs) presented by the Peng and Yang [17], AOs of IV T-spherical FSs given by the Ullah et al. [18], AOs of picture FSs (PFSs) presented by the Garg [19], AOs of q-rung orthopair FSs (q-ROFSs) developed by the Jana et al. [20], AOs of the bipolar valued hesitant fuzzy system given by the Khan et al. [21], and AOs of complex IFSs given by the Hussain et al. [22]. Fan et al. [23] provided some AOs of SVNSs by using operations of linguistic variables and also defined a MADM technique. Sodenkamp et al. [24] provided a list of new AOs to find the reliability of multicriteria decision-making problems under the system of SVNSs. Garg [25] introduced a fresh idea of neutrality AOs by using operations of sum and scalar multiplications based on SVNSs. Saha et al. [26] checked the flexibility and compatibility of hesitant FSs which is the advanced version of FSs and determined a series of new approaches based on Archimedean aggregation tools. A robust theory of Dombi Archimedean aggregation tools based on Hesitant fuzzy was reviewed by Liu et al. [27]. Saha et al. [28] presented the theory of Dombi Bonferroni mean aggregation expressions to evaluate real-life challenges.

The triangular norms are powerful tools to overcome the influence of vague and impression information. Several researchers invented different TNM and TCM to aggregate information in the system of fuzziness. Firstly, the concepts of TNM and TCNM were developed by Klement [29] in 1982. There are such types of TNM and TCNM which include Lukasiewicz TNM and TCNM [30], Nilpotent TNM and TCNM [31], Drastic TNM and TCNM [32], Archimedean TNM and TCNM [33], Einstein TNM and TCNM [34], and Frank TNM and TCNM [35]. Many researchers explored the concepts of TNM and TCNM in the different fuzzy frameworks. Klement et al. [36] worked on a family of TNM and TCNM. Babu and Ahmed [37] invented several parametric TNMs based on the function generator. Seikh and Mandal [38] presented some new AOs of PFSs and their application based on the MADM technique. Seikh and Mandal [39] also elaborated on the theory of Dombi TNM and TCNM under the system of IFS. Seikh and Mahnaz [40] generalized the system of T-spherical FS by using the basic operational laws of frank TNM and TCNM.

A well-known and efficient aggregation model is known as the power average (PA) operator. The theory PA given by Yager [41] is utilized to express support among each other in the aggregation process. Xu and Yager [42] also elaborated on the theory of PA and presented a robust aggregation model of the power geometric (PG) operator. Liu [43] utilized the theoretic concepts of PA and integrated an application of a MADM problem. Jana and Pal [44] constructed some appropriate power aggregation methodologies based on SVNSs under consideration of Dombi operational laws. Ashraf et al. [45] illustrated some trigonometric aggregation models to solve a MADM technique for the selection of a hydrogen power plant. Senapati et al. [46] exposed a series of new aggregation approaches and tried to solve an application of sharing sustainable transportation enterprises.

Aczél and Alsina [47] gave an appropriate mathematical tool to overcome the effect of unreasonable and vague information in a fuzzy framework under condition $0 \leq \mathbb{N} \leq \infty$. Alsina et al. [48] worked on probabilistic metric space and its basic properties. Senapati et al. [49] explored the idea of A-TNM and A-TCNM under the system of IFSs and also established a MADM to select a suitable selection for a multinational company. Senapati et al. [50] also explored the concept of A-TNM and A-TCNM in the framework of IVIFSs with an application under a MADM technique. Naeem et al. [51] developed a series of new AOs of PFSs based on Aczel–Alsina operations. Hussain et al. [52] extended the model of PyFSs by using Aczel–Alsina operations and studied a MADM technique for the selection process of a multinational company. The theory of Aczel–Alsina aggregation tools has been explored by numerous research scholars. Senapati et al. [53] generalized the theory of Aczel–Alsina aggregation tools and developed a class of new approaches in light of PyFS information. Senapati et al. [54] also illustrated the potential of cyclone disaster enterprises under consideration of Hesitant Fuzzy system. Farid et al. [55] constructed an algorithm for evaluation of robust green supplier management. A robust construction of interval-valued PyF information based on Aczel–Alsina aggregation tools was presented by Senapati et al. [56]. Sustainable green supplier enterprises were evaluated by the Riaz et al. [57]. Alcantud [58] published latest research work related to decision-making process and also developed the series of robust methodologies in a book.

The abovementioned fuzzy environments carried restricted information, and decision-makers face a lot of challenges due to insufficient information during the aggregation process. The SVNSs are a more reliable generalization of FSs, IFSs, and IVIFSs and provide freedom to decision-makers for the decision process. The power aggregation tools are well-known and provide a smooth approximation in the decision-making process. Recently, Aczel–Alsina aggregation expressions acquired a lot of extension from multiresearch scholars. We found a research gap in the environments of SVNSs under consideration of the power aggregation models. We extended research mythologies which are presented in [59] and developed some new appropriate aggregation approaches under consideration of power aggregation models. The major purpose of this article is particularized as follows.

(a) To express the notion of SVNSs and their related appropriate aggregation operations.

(b) To study well-known aggregation operators such as power average and power geometric operators under consideration of SVN environment.

(c) We also expressed the theoretic concepts of Aczel–Alsina aggregation tools and some fundamental operational laws under consideration in SVN environments.

(d) We also derived a series of new approaches including SVNAAPWA and SVNAAPWG operators with some special characteristics of our derived approaches.

(e) We construct a MADM technique to solve an application of engineering and construction materials under consideration of our derived methodologies. An experimental case study is also presented to determine a suitable optimal option from a group of options.

(f) To find the flexibility of our proposed work, we provide a comparative study that compares the results of existing AOs with those of our proposed work. A comprehensive overview is also presented here.

The structure of this article is as follows: In Section 1, we recall the previous history of our research work, Section 2 presents the basic notions of TNM and TCNM with the help of an example; furthermore, authors also expose the notion of SVNSs and its fundamental operational laws based on SVNNs. Section 3 presents Aczel–Alsina operations based on SVNSs and we also gave a numerical example to support Aczel–Alsina operations. In Section 4, we develop AOs of SVNAAPWA operator and their characteristics such as idempotency, monotonicity, and boundedness, In Section 5, we also present SVNAAPWG operator based on Aczel–Alsina operations. In Section 6, we study a MADM technique to solve an application of engineering and construction materials with the help of a numerical example. In Section 7, we contrast the results of exiting AOs with the results of our proposed technique. In Section 8, we summarized our research work.

2. Preliminaries

We recall the notions of TNM and TCNM with some examples. Moreover, we also discuss the notion of A-TNM and A-TCNM for further development of this article. Firstly, the concepts of TNM and TCNM were given by Klement [29]. Symbols with their appropriate meanings are listed in Table 1.

Definition 1 (see [29]). A function $\mathbb{T}: [0,1]^2 \longrightarrow [0,1]$ is a TNM, if it satisfies conditions such as symmetry, monotonicity, associativity, and one identity element.

Table 1: Symbols and their meanings in this article.

Symbols	Meanings
$\mathbb{T}$	TNM
$\mathbf{S}$	TCNM
$\boldsymbol{\varphi}$	TV
$\boldsymbol{\xi}$	AV
$\boldsymbol{\omega}$	Weight vector
$\mathbf{R}$	Decision matrix
$\mathbf{Y}$	Weighted support
$\mathbf{A}$	Support
ψ	FV
X	Nonempty set
$\mathfrak{R}$	Score function
H	Accuracy
Б	Alternative
Ɇ	Attributes
R	Decision matrix

(i) $\mathbb{T}(\varepsilon, \tau) = \mathbb{T}(\tau, \varepsilon)$

(ii) $\mathbb{T}(\varepsilon, \tau) \leq \mathbb{T}(\varepsilon, \nu)$ if $\tau \leq \nu$

(iii) $\mathbb{T}(\varepsilon, \mathbb{T}(\tau, \nu)) = \mathbb{T}(\mathbb{T}(\varepsilon, \tau), \nu)$

(iv) $\mathbb{T}(\varepsilon, 1) = \varepsilon$

$$\forall, \varepsilon, \tau, \nu \in [0,1]. \tag{1}$$

Example 1. Some examples of TNM are as follows.

(i) Product t-norm: $\mathbb{T}_P(\varepsilon, \tau) = \varepsilon.\tau$

(ii) Minimum t-norm: $\mathbb{T}_M(\varepsilon, \tau) = \min(\varepsilon, \tau)$

(ii) Lukasiewicz t-norm: $\mathbb{T}_L(\varepsilon, \tau) = \max(\varepsilon + \tau - 1, 0)$

(iv) Drastic t-norm:

$$\mathbb{T}_D(\varepsilon, \tau) = \begin{cases} \varepsilon, & \text{if } \tau = 1, \\ \tau, & \text{if } \varepsilon = 1, \\ 0, & \text{otherwise,} \end{cases} \tag{2}$$

for all $\varepsilon, \tau, \nu \in [0,1]$.

Definition 2 (see [29]). A function $S: [0,1]^2 \longrightarrow [0,1]$ is a TCNM, if it satisfies conditions such as symmetry, monotonicity, associativity, and one identity element.

(i) $S(\varepsilon, \tau) = S(\tau, \varepsilon)$

(ii) $S(\varepsilon, \tau) \leq S(\varepsilon, \nu)$ if $\tau \leq \nu$

(iii) $S(\varepsilon, S(\tau, \nu)) = S(S(\varepsilon, \tau), \nu)$

(iv) $S(\varepsilon, 0) = \varepsilon$

for all $\varepsilon, \tau, \nu \in [0,1]$.

Example 2. Some of the examples are given as follows.

(i) Probabilistic sum: $S_P(\varepsilon, \tau) = \varepsilon + \tau - \varepsilon.\tau$

(ii) Minimum t-conorm: $S_M(\varepsilon, \tau) = \max(\varepsilon, \tau)$

(iii) Lukasiewicz t-conorm: $S_L(\varepsilon, \tau) = \min(\varepsilon + \tau, 1)$

(iv) Drastic t-norm:

$$S_D(\varepsilon,\tau)=\begin{cases}\varepsilon, & \text{if } \tau=0,\\ \tau, & \text{if } \varepsilon=0,\\ 1, & \text{otherwise},\end{cases} \quad (3)$$

for all $\varepsilon,\tau,\nu\in[0,1]$.

$$\begin{aligned}&\mathbb{T}(\varepsilon,\tau)\le\min(\varepsilon,\tau),\\ &S(\varepsilon,\tau)\ge\max(\varepsilon,\tau),\end{aligned} \quad (4)$$

where $\mathbb{T}$ is TNM and S is TCNM. For all $\varepsilon,\tau,r\in[0,1]$.

Definition 3 (see [47, 48]). A new category of TNM was presented by Aczel–Alsina in 1980.

The A-TNM $\mathbb{T}_A^{\mathbb{N}}\mathbb{N}\in[0,1]$ is defined as follows:

$$\mathbb{T}_A^{\mathbb{N}}(\varepsilon,\tau)=\begin{cases}\mathbb{T}_D(\varepsilon,\tau), & \text{if } \mathbb{N}=0,\\ \min(\varepsilon,\tau), & \text{if } \mathbb{N}=\infty,\\ e^{-\left((-\mathrm{Ln}\varepsilon)^{\mathbb{N}}+(-\mathrm{Ln}\tau)^{\mathbb{N}}\right)^{1/\mathbb{N}}}, & \text{otherwise}.\end{cases} \quad (5)$$

A-TCNM $S_A^{\mathbb{N}}\mathbb{N}\in[0,1]$ of is defined as follows:

$$S_A^{\mathbb{N}}(\varepsilon,\tau)=\begin{cases}S_D(\varepsilon,\tau), & \text{if } \mathbb{N}=0,\\ \max(\varepsilon,\tau), & \text{if } \mathbb{N}=\infty,\\ 1-e^{-\left(1-(-\mathrm{Ln}(1-\varepsilon))^{\mathbb{N}}+(-\mathrm{Ln}(1-\tau))^{\mathbb{N}}\right)^{1/\mathbb{N}}}, & \text{otherwise}.\end{cases} \quad (6)$$

In limiting cases, $\mathbb{T}_A^0=\mathbb{T}_D, \mathbb{T}_A^1=\mathbb{T}_1, \mathbb{T}_A^{\infty}=\min, S_A^0=S_D, S_A^1=S_P$, and $S_A^{\infty}=\max$. For every $\mathbb{N}\in[0,1]$, the A-TNM $\mathbb{T}_A^{\mathbb{N}}$ and A-TCNM $S_A^{\mathbb{N}}$ are dual for each other. The type of A-TNM and A-TCNM is strictly maximizing.

Now, we will study the notion of NS and SVNS on a universal set X and also discuss some basic operations of SVNS. To compare different SVNNs, we explore the notions of score function and accuracy function.

Definition 4 (see [2]). Let X be a nonempty set, and an IFS A'' over X is defined as follows:

$$A''=\{(\jmath,(\varphi_A(\jmath),\psi_A(\jmath))\mid \jmath\in X)\}, \quad (7)$$

where $\varphi_A(\jmath)\colon \longrightarrow[0,1]$ and $\psi_A(\jmath)\colon \longrightarrow[0,1]$ represent truth value (TV) and falsity value (FV), respectively. The IFS must satisfy the following axiom:

$$0\le\varphi_A(\jmath)+\psi_A(\jmath)\le1. \quad (8)$$

The hesitancy value of an IFS is given by $\mathfrak{D}(\jmath)=1-(\varphi_A(\jmath)+\psi_A(\jmath))$.

Definition 5 (see [60]). Let X be a nonempty set, and a linear Diophantine FS (LDFS) A'' over X is characterized as follows:

$$A''=\{((\jmath,(\varphi_A(\jmath),\psi_A(\jmath)),(\delta,\chi))\mid\jmath\in X)\}, \quad (9)$$

where $\varphi_A(\jmath)\colon \longrightarrow[0,1], \psi_A(\jmath)\colon \longrightarrow[0,1]$, and $(\delta,\chi)\in[0,1]$ represent truth value, falsity value, and reference parameters, respectively. The LDFS must satisfy the following axiom:

$$\begin{aligned}&0\le\delta\varphi_A(\jmath)+\chi\psi_A(\jmath)\le1,\\ &0\le\delta+\chi\le1.\end{aligned} \quad (10)$$

The hesitancy value of an LDFS is given by $\mathfrak{D}(\jmath)=1-(\delta\varphi_A(\jmath)+\chi\psi_A(\jmath))$.

Definition 6 (see [8]). Let X be a nonempty set, and a SVNS A over X is defined as follows:

$$A=\{(\jmath,(\varphi_A(\jmath),\xi_A(\jmath),\psi_A(\jmath))\mid\jmath\in X)\}, \quad (11)$$

where $\varphi_A(\jmath)\colon \longrightarrow[0,1], \xi_A(\jmath)\colon \longrightarrow[0,1]$, and $\psi_A(\jmath)\colon \longrightarrow[0,1]$ represent truth value, abstinence value and falsity value, respectively. The SVNS must satisfy the following axiom:

$$0\le\varphi_A(\jmath)+\xi_A(\jmath)+\psi_A(\jmath)\le3. \quad (12)$$

Furthermore, a single-valued neutrosophic number (SVNN) is denoted by $\alpha=(\varphi_a,\xi_\alpha,\psi_\alpha)$.

Definition 7 (see [61]). Let $A=\{(\jmath,(\varphi_A(\jmath),\xi_A(\jmath),\psi_A(\jmath))\mid\jmath\in X)\}$ and $B=\{(\jmath,(\varphi_B(\jmath),\xi_B(\jmath),\psi_B(\jmath))\mid\jmath\in X)\}$ be two SVNSs. Then, some basic operations of SVNSs such as union, intersection, and compliment are defined as follows.

(i) $A\cup B=\{(\jmath,(\max(\varphi_A(\jmath),\varphi_B(\jmath)),\max(\xi_A(\jmath),\xi_B(\jmath)),\min(\psi_A(\jmath),\psi_B(\jmath)))|\jmath\in X)\}$

(ii) $A\cap B=\{(\jmath,(\min(\varphi_A(\jmath),\varphi_B(\jmath)),\min(\xi_A(\jmath),\xi_B(\jmath)),\max(\psi_A(\jmath),\psi_B(\jmath)))\colon|\jmath\in X)\}$

(iii) $A\subseteq B\ if$ and only if $\varphi_A(\jmath)\le\varphi_B(\jmath)$, $\xi_A(\jmath)\ge\xi_B(\jmath)$, and $\psi_A(\jmath)\ge\psi_B(\jmath), \forall\jmath\in X$

(iv) $A=B\ if$ and only if $A\subseteq B$ and $B\subseteq A$

(v) $A^c=\{(\psi_A(\jmath),\xi_A(\jmath),\varphi_A(\jmath))\}$

Now, we expressed some appropriate comparison rules, which defined how we compare different SVNNs.

Definition 8 (see [61]). Consider $\alpha=(\varphi,\xi,\psi)$ be a SVNN. Then, the score value $\mathfrak{R}(\alpha)$ is defined as follows:

$$\mathfrak{R}(\alpha)=\frac{1}{3}(2+\varphi-\xi-\psi),\quad \mathfrak{R}(\alpha)\in[0,1]. \quad (13)$$

Definition 9 (see [61]). Consider $\alpha=(\varphi,\xi,\psi)$ be a SVNN. Then, the accuracy value $H(\alpha)$ is defined as follows:

$$H(\alpha)=\varphi-\psi,\quad H(\alpha)\in[-1,1]. \quad (14)$$

Remark 10. Let A and B be two SVNNs. Then, the score value of A and B is denoted by $\mathfrak{R}(A)$ and $\mathfrak{R}(B)$, respectively. Similarly, the accuracy value of A and B is denoted by $H(A)$ and $H(B)$, respectively. The relation between the score function and accuracy function is defined as follows.

(i) If $\mathfrak{R}(A)>\mathfrak{R}(B)$, then $A>B$

(ii) If $\mathfrak{R}(A)<\mathfrak{R}(B)$, then $A<B$

(iii) If $\mathfrak{R}(A)=\mathfrak{R}(\beta)$, then

  (a) If $H(A)>H(B)$, then $A>B$

(b) If $H(A) < H(B)$, then $A < B$
(c) If $H(A) = H(B)$, then $A = B$

Definition 11 (see [61]). Consider $\alpha = (\varphi, \xi, \psi), \alpha_1 = (\varphi_1, \xi_1, \psi_1)$, and $\alpha_2 = (\varphi_2, \xi_2, \psi_2)$ be three SVNNs. Then, we have

(i) $\alpha_1 \oplus \alpha_2 = (\varphi_1 + \varphi_2 - \varphi_1\varphi_2, \xi_1\xi_2, \psi_1\psi_2)$
(ii) $\alpha_1 \otimes \alpha_2 = (\varphi_1\varphi_2, \xi_1 + \xi_2 - \xi_1\xi_2, \psi_1 + \psi_2 - \psi_1\psi_2)$
(iii) $\lambda\alpha = (1 - (1 - \varphi_1^\lambda)^\lambda, \xi_1^\lambda, \psi_1^\lambda); \lambda > 0$
(iv) $\alpha_1^\lambda = (\varphi_1^\lambda, 1 - (1 - \xi_1)^\lambda, 1 - (1 - \psi_1)^\lambda); \lambda > 0$

Definition 12 (see [62]). Consider $\alpha_{\jmath} = (\varphi_{\jmath}, \xi_{\jmath}, \psi_{\jmath}), \jmath = 1,2,3,4,\ldots,n$, be the set of SVNNs, with corresponding weights vector $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ such that $\omega_{\jmath} > 0$ and $\sum_{\jmath=1}^n \omega_{\jmath} = 1$. Then, the aggregated value of SVN weighted averaging (SVNWA) and SVN weighted geometric (SVNWG) operators is defined as follows:

$$\begin{aligned} \operatorname{SVNWA}(\alpha_1, \alpha_2, \ldots, \alpha_n) &= \left(1 - \prod_{\jmath=1}^n \left(1 - \varphi_{\jmath}\right)^{\omega_{\jmath}}, \prod_{\jmath=1}^n (\xi)^{\omega_{\jmath}}, \prod_{\jmath=1}^n \left(\psi_{\jmath}\right)^{\omega_{\jmath}}\right), \\ \operatorname{SVNWG}(\alpha_1, \alpha_2, \ldots, \alpha_n) &= \left(\prod_{\jmath=1}^n \left(\varphi_{\jmath}\right)^{\omega_{\jmath}}, 1 - \prod_{\jmath=1}^n \left(1 - \xi_{\jmath}\right)^{\omega_{\jmath}}, 1 - \prod_{\jmath=1}^n \left(1 - \psi_{\jmath}\right)^{\omega_{\jmath}}\right). \end{aligned} \tag{15}$$

Definition 13 (see [41]). Consider $\alpha_{\jmath} = (\varphi_{\jmath}, \xi_{\jmath}, \psi_{\jmath})$, $\jmath = 1, 2, 3, \ldots, n$, be the set of SVNNs, then the power averaging (PA) operators are termed as follows:

$$\operatorname{PA}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \frac{\sum_{\jmath=1}^n \left(1 + \mathfrak{A}\left(\alpha_{\jmath}\right)\right)\alpha_{\jmath}}{\sum_{\tilde{\tilde{e}}=1}^{\rho} \left(1 + \mathfrak{A}\left(\alpha_{\jmath}\right)\right)}, \tag{16}$$

where $\mathfrak{Y}_{\jmath} = ((1 + \mathfrak{A}(\alpha_{\jmath}))/\sum_{\jmath=1}^n (1 + \mathfrak{A}(\alpha_{\jmath}))), \mathfrak{A}(\alpha_{\jmath}) = \sum_{\substack{\jmath=1, \tilde{\tilde{e}}=1 \\ \jmath \neq \tilde{\tilde{e}}}}^n \operatorname{Supp}(\alpha_{\jmath}, \alpha_{\tilde{\tilde{e}}})$, and $\jmath = 1, 2, \ldots, \rho, \tilde{\tilde{e}} = 1, 2, \ldots, n$.

Definition 14 (see [42]). Consider $\alpha_{\jmath} = (\varphi_{\jmath}, \xi_{\jmath}, \psi_{\jmath})$, $\jmath = 1, 2, 3, \ldots, n$, be the set of SVNNs, then the power geometric (PG) operators are termed as follows:

$$\operatorname{PG}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \left(\alpha_{\jmath}\right)^{\left(1 + \mathfrak{A}(\alpha_{\jmath})\right)/\sum_{\jmath=1}^n \left(1 + \mathfrak{A}(\alpha_{\jmath})\right)}, \tag{17}$$

where $\mathfrak{Y}_{\jmath} = (1 + \mathfrak{A}(\alpha_{\jmath}))/\sum_{\jmath=1}^n (1 + \mathfrak{A}(\alpha_{\jmath})), \mathfrak{A}(\alpha_{\jmath}) = \sum_{\substack{\jmath=1, \tilde{\tilde{e}}=1 \\ \jmath \neq \tilde{\tilde{e}}}}^n \operatorname{Supp}(\alpha_{\jmath}, \alpha_{\tilde{\tilde{e}}}), \jmath = 1, 2, \ldots, \rho$, and $\tilde{\tilde{e}} = 1, 2, \ldots, n$.

3. Aczel–Alsina Operation-Based SVNNs

In this section, we will discuss the basic operations of A-TNM and A-TCNM based on SVNNs with some practical examples.

Consider T_A and S_A represent A-TNM and A-TCNM, respectively. We defined some operations of Aczel–Alsina product and Aczel–Alsina sum. Furthermore, we also defined some generalized unions and intersections over two SVNNs A and B as follows:

$$\begin{aligned} A \otimes B &= \left\{\left(\jmath, T_A\left\{\varphi_A(\jmath), \varphi_B(\jmath)\right\}, S_A\left\{\xi_A(\jmath)\xi_B(\jmath)\right\}, S_A\left\{\psi_A(\jmath)\psi_B(\jmath)\right\} \mid \jmath \in X\right)\right\}, \\ A \oplus B &= \left\{\left(\jmath, S_A\left\{\varphi_A(\jmath), \varphi_B(\jmath)\right\}, T_A\left\{\xi_A(\jmath)\xi_B(\jmath)\right\}, T_A\left\{\psi_A(\jmath)\psi_B(\jmath)\right\} \mid \jmath \in X\right)\right\}. \end{aligned} \tag{18}$$

Definition 15. Let $\alpha = (\varphi, \xi, \psi), \alpha_1 = (\varphi_1, \xi_1, \psi_1)$, and $\alpha_2 = (\varphi_2, \xi_2, \psi_2)$ be the three SVNNs. Then, some basic operations of A-TNM and A-TCNM are defined as follows: for $\mathbb{N} \geq 1$ and $\lambda > 0$,

(i) $$\alpha_1 \oplus \alpha_2 = \begin{pmatrix} 1 - e^{-\left(\left(-\operatorname{Ln}(1-\varphi_{\alpha_1})\right)^{\mathbb{N}} + \left(-\operatorname{Ln}(1-\varphi_{\alpha_2})\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ e^{-\left(\left(-\operatorname{Ln}(\xi_{\alpha_1})\right)^{\mathbb{N}} + \left(-\operatorname{Ln}(\xi_{\alpha_2})\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ e^{-\left(\left(-\operatorname{Ln}(\psi_{\alpha_1})\right)^{\mathbb{N}} + \left(-\operatorname{Ln}(\psi_{\alpha_2})\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \end{pmatrix}$$

(ii) $$\alpha_1 \otimes \alpha_2 = \begin{pmatrix} e^{-\left(\left(-\operatorname{Ln}(\varphi_{\alpha_1})\right)^{\mathbb{N}} + \left(-\operatorname{Ln}(\varphi_{\alpha_2})\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ 1 - e^{-\left(\left(-\operatorname{Ln}(1-\xi_{\alpha_1})\right)^{\mathbb{N}} + \left(-\operatorname{Ln}(1-\xi_{\alpha_2})\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ 1 - e^{-\left(\left(-\operatorname{Ln}(1-\psi_{\alpha_1})\right)^{\mathbb{N}} + \left(-\operatorname{Ln}(1-\psi_{\alpha_2})\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \end{pmatrix}$$

(iii) $$\lambda\alpha = \begin{pmatrix} 1 - e^{-\left(\lambda\left(-\operatorname{Ln}(1-\varphi_\alpha)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ e^{-\left(\lambda\left(-\operatorname{Ln}(\xi_\alpha)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ e^{-\left(\lambda\left(-\operatorname{Ln}(\psi_\alpha)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \end{pmatrix}$$

(iv) $$\alpha^\lambda = \begin{pmatrix} e^{-\left(\lambda\left(-\operatorname{Ln}(\varphi_\alpha)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ 1 - e^{-\left(\lambda\left(-\operatorname{Ln}(1-\xi_\alpha)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ 1 - e^{-\left(\lambda\left(-\operatorname{Ln}(1-\psi_\alpha)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \end{pmatrix}$$

Example 3. Let $\alpha = (0.66, 0.55, 0.45), \alpha_1 = (0.87, 0.44, 0.54)$, and $\alpha_2 = (0.59, 0.32, 0.60)$ be three SVNNs. Then, by using the Aczel–Alsina operation, for $\mathbb{N} = 3$ and $\lambda = 4$, we have

(i) $\alpha_1 \oplus \alpha_2 = \begin{pmatrix} 1 - e^{-((-\mathrm{Ln}(1-\varphi_{\alpha_1}))^{\mathbb{N}} + (-\mathrm{Ln}(1-\varphi_{\alpha_2}))^{\mathbb{N}})^{(1/\mathbb{N})}} \\ e^{-((-\mathrm{Ln}(\xi_{\alpha_1}))^{\mathbb{N}} + (-\mathrm{Ln}(\xi_{\alpha_2}))^{\mathbb{N}})^{(1/\mathbb{N})}} \\ e^{-((-\mathrm{Ln}(\psi_{\alpha_1}))^{\mathbb{N}} + (-\mathrm{Ln}(\psi_{\alpha_2}))^{\mathbb{N}})^{(1/\mathbb{N})}} \end{pmatrix},$

$$\alpha_1 \oplus \alpha_2 = \begin{pmatrix} 1 - e^{-((-\mathrm{Ln}(1-0.87))^3 + (-\mathrm{Ln}(1-0.59))^3)^{(1/\mathbb{N})}} \\ e^{-((-\mathrm{Ln}(0.44))^3 + (-\mathrm{Ln}0.32)^3)^{(1/\mathbb{N})}} \\ e^{-((-\mathrm{Ln}(0.54))^3 + (-\mathrm{Ln}0.60)^3)^{(1/\mathbb{N})}} \end{pmatrix},$$

$$\alpha_1 \oplus \alpha_2 = (0.9740,\ 0.4500,\ 0.8840)$$

(ii) $\alpha_1 \otimes \alpha_2 = \begin{pmatrix} e^{-((-\mathrm{Ln}(\varphi_{\alpha_1}))^{\mathbb{N}} + (-\mathrm{Ln}(\varphi_{\alpha_2}))^{\mathbb{N}})^{(1/\mathbb{N})}} \\ 1 - e^{-((-\mathrm{Ln}(1-\xi_{\alpha_1}))^{\mathbb{N}} + (-\mathrm{Ln}(1-\xi_{\alpha_2}))^{\mathbb{N}})^{(1/\mathbb{N})}} \\ 1 - e^{-((-\mathrm{Ln}(1-\psi_{\alpha_1}))^{\mathbb{N}} + (-\mathrm{Ln}(1-\psi_{\alpha_2}))^{\mathbb{N}})^{(1/\mathbb{N})}} \end{pmatrix},$

$$\alpha_1 \otimes \alpha_2 = \begin{pmatrix} e^{-((-\mathrm{Ln}(O.87))^3 + (-\mathrm{Ln}0.59)^3)^{(1/3)}} \\ 1 - e^{-((-\mathrm{Ln}(1-0.44))^3 + (-\mathrm{Ln}(1-0.32))^3)^{(1/3)}} \\ 1 - e^{-((-\mathrm{Ln}(1-0.54))^3 + (-\mathrm{Ln}(1-0.60))^3)^{(1/3)}} \end{pmatrix},$$

$$\alpha_1 \otimes \alpha_2 = (0.4500,\ 0.1696,\ 0.9610)$$

(iii) $\lambda\alpha = \begin{pmatrix} 1 - e^{-(\lambda(-\mathrm{Ln}(1-\varphi_{\alpha}))^{\mathbb{N}})^{(1/\mathbb{N})}} \\ e^{-(\lambda(-\mathrm{Ln}(\xi_{\alpha}))^{\mathbb{N}})^{(1/\mathbb{N})}} \\ e^{-(\lambda(-\mathrm{Ln}(\psi_{\alpha}))^{\mathbb{N}})^{(1/\mathbb{N})}} \end{pmatrix},$

$$4\alpha = \begin{pmatrix} 1 - e^{-((4)(-\mathrm{Ln}(1-0.66))^3)^{(1/3)}} \\ e^{-((4)(-\mathrm{Ln}(0.55))^3)^{(1/3)}} \\ e^{-((4)(-\mathrm{Ln}(0.45))^3)^{(1/3)}} \end{pmatrix},$$

$$4\alpha = (0.7151,\ 0.8076,\ 0.6010)$$

(iv) $\alpha^{\lambda} = \begin{pmatrix} e^{-(\lambda(-\mathrm{Ln}(\varphi_{\alpha}))^{\mathbb{N}})^{(1/\mathbb{N})}} \\ 1 - e^{-(\lambda(-\mathrm{Ln}(1-\xi_{\alpha}))^{\mathbb{N}})^{(1/\mathbb{N})}} \\ 1 - e^{-(\lambda(-\mathrm{Ln}(1-\psi_{\alpha}))^{\mathbb{N}})^{(1/\mathbb{N})}} \end{pmatrix},$

$$\alpha^4 = \begin{pmatrix} e^{-((4)(-\mathrm{Ln}(0.66))^3)^{(1/3)}} \\ 1 - e^{-((4)(-\mathrm{Ln}(1-0.55))^3)^{(1/3)}} \\ 1 - e^{-((4)(-\mathrm{Ln}(1-0.45))^3)^{(1/3)}} \end{pmatrix},$$

$$\alpha^4 = (0.9308,\ 0.3990,\ 0.1924)$$

Definition 16 (see [59]). Let $\alpha_{\mathcal{J}} = (\varphi_{\alpha_{\mathcal{J}}}, \xi_{\alpha_{\mathcal{J}}}, \psi_{\alpha_{\mathcal{J}}})$, $\mathcal{J} = 1,2,3,\ldots,n$, be the set of SVNNs, with corresponding weight vectors $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ of $\alpha_{\mathcal{J}}$, $\mathcal{J} = 1,2,3,\ldots,n$ such that $\omega_{\mathcal{J}} > 0$ and $\sum_{\mathcal{J}=1}^{n} \omega_{\mathcal{J}} = 1$. Then, the SVNAAWA operator is given by the following equation:

$$\begin{aligned} \mathrm{SVNAAWA}(\alpha_1, \alpha_2, \ldots, \alpha_n) &= \overset{n}{\underset{\mathcal{J}=1}{\oplus}} \left(\omega_{\mathcal{J}} \alpha_{\mathcal{J}}\right) \\ &= \omega_1\alpha_1 \oplus \omega_2\alpha_2 \oplus, \ldots, \oplus \omega_n\alpha_n \\ &= \begin{pmatrix} 1 - e^{-\left(\sum_{\mathcal{J}=1}^{n} (\omega_{\mathcal{J}})\left(-\mathrm{Ln}\left(1-\varphi_{\alpha_{\mathcal{J}}}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ e^{-\left(\sum_{\mathcal{J}=1}^{n} (\omega_{\mathcal{J}})\left(-\mathrm{Ln}\left(\xi_{\alpha_{\mathcal{J}}}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ e^{-\left(\sum_{\mathcal{J}=1}^{n} (\omega_{\mathcal{J}})\left(-\mathrm{Ln}\left(\psi_{\alpha_{\mathcal{J}}}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \end{pmatrix}. \end{aligned} \tag{19}$$

Definition 17 (see [59]). Let $\alpha_{\mathcal{J}} = (\varphi_{\alpha_{\mathcal{J}}}, \xi_{\alpha_{\mathcal{J}}}, \psi_{\alpha_{\mathcal{J}}})$, $\mathcal{J} = 1,2,3,\ldots,n$, be the set of SVNNs, with corresponding weight vector $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ of $\alpha_{\mathcal{J}}$, ($\mathcal{J} = 1,2,3,\ldots,n$) such that $\omega_{\mathcal{J}} > 0$ and $\sum_{\mathcal{J}=1}^{n} \omega_{\mathcal{J}} = 1$. Then, the SVNAAWG operator is given by the following equation:

$$\begin{aligned} \mathrm{SVNAAWG}(\alpha_1, \alpha_2, \ldots, \alpha_n) &= \overset{n}{\underset{\mathcal{J}=1}{\otimes}} \left(\alpha_{\mathcal{J}}^{\ \omega_{\mathcal{J}}}\right) = \alpha_1^{\omega_1} \otimes \alpha_2^{\omega_2} \otimes, \ldots, \otimes \alpha_n^{\omega_n} \\ &= \begin{pmatrix} e^{-\left(\sum_{\mathcal{J}=1}^{n} (\omega_{\mathcal{J}})\left(-\mathrm{Ln}\left(\varphi_{\alpha_{\mathcal{J}}}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ 1 - e^{-\left(\sum_{\mathcal{J}=1}^{n} (\omega_{\mathcal{J}})\left(1-\mathrm{Ln}\left(\xi_{\alpha_{\mathcal{J}}}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ 1 - e^{-\left(\sum_{\mathcal{J}=1}^{n} (\omega_{\mathcal{J}})\left(1-\mathrm{Ln}\left(\psi_{\alpha_{\mathcal{J}}}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \end{pmatrix}. \end{aligned} \tag{20}$$

Theorem 18. *Let $\alpha = (\varphi, \xi, \psi), \alpha_1 = (\varphi_1, \xi_1, \psi_1)$, and $\alpha_2 = (\varphi_2, \xi_2, \psi_2)$ be three SVNNs. Then, we have*

(i) $\alpha_1 \oplus \alpha_2 = \alpha_2 \oplus \alpha_1$
(ii) $\alpha_1 \otimes \alpha_2 = \alpha_2 \otimes \alpha_1$
(iii) $\lambda(\alpha_1 \oplus \alpha_2) = \lambda\alpha_1 \oplus \lambda\alpha_2, \lambda > 0$
(iv) $(\lambda_1 + \lambda_2)\alpha = \lambda_1\alpha + \lambda_2\alpha, \lambda_1, \lambda_2 > 0$
(v) $(\alpha_1 \otimes \alpha_2)^\lambda = \alpha_1^\lambda \otimes \alpha_2^\lambda, \lambda > 0$
(vi) $\alpha^{\lambda_1} \otimes \alpha^{\lambda_2} = \alpha^{(\lambda_1+\lambda_2)}, \lambda_1, \lambda_2 > 0$

Proof. Let $\alpha = (\varphi, \xi, \psi), \alpha_1 = (\varphi_1, \xi_1, \psi_1)$, and $\alpha_2 = (\varphi_2, \xi_2, \psi_2)$ be three SVNNs and $\lambda, \lambda_1, \lambda_2, > 0$. By proving Theorem 18 and by using Definition 15, we get

(i) $$\alpha_1 \oplus \alpha_2 = \begin{pmatrix} 1 - e^{-((-\mathrm{Ln}(1-\varphi_{\alpha_1}))^{\mathbb{N}} + (-\mathrm{Ln}(1-\varphi_{\alpha_2}))^{\mathbb{N}})^{(1/\mathbb{N})}} \\ e^{-((-\mathrm{Ln}(\xi_{\alpha_1}))^{\mathbb{N}} + (-\mathrm{Ln}(\xi_{\alpha_2}))^{\mathbb{N}})^{(1/\mathbb{N})}} \\ e^{-((-\mathrm{Ln}(\psi_{\alpha_1}))^{\mathbb{N}} + (-\mathrm{Ln}(\psi_{\alpha_2}))^{\mathbb{N}})^{(1/\mathbb{N})}} \end{pmatrix},$$

$$\alpha_1 \oplus \alpha_2 = \begin{pmatrix} 1 - e^{-((-\mathrm{Ln}(1-\varphi_{\alpha_2}))^{\mathbb{N}} + (-\mathrm{Ln}(1-\varphi_{\alpha_1}))^{\mathbb{N}})^{(1/\mathbb{N})}} \\ e^{-((-\mathrm{Ln}(\xi_{\alpha_2}))^{\mathbb{N}} + (-\mathrm{Ln}(\xi_{\alpha_1}))^{\mathbb{N}})^{(1/\mathbb{N})}} \\ e^{-((-\mathrm{Ln}(\psi_{\alpha_2}))^{\mathbb{N}} + (-\mathrm{Ln}(\psi_{\alpha_1}))^{\mathbb{N}})^{(1/\mathbb{N})}} \end{pmatrix},$$

$$\alpha_1 \oplus \alpha_2 = \alpha_2 \oplus \alpha_1$$

(ii) $$\alpha_1 \otimes \alpha_2 = \begin{pmatrix} e^{-((-\mathrm{Ln}(\varphi_{\alpha_1}))^{\mathbb{N}} + (-\mathrm{Ln}(\varphi_{\alpha_2}))^{\mathbb{N}})^{(1/\mathbb{N})}} \\ 1 - e^{-((-\mathrm{Ln}(1-\xi_{\alpha_1}))^{\mathbb{N}} + (-\mathrm{Ln}(1-\xi_{\alpha_2}))^{\mathbb{N}})^{(1/\mathbb{N})}} \\ 1 - e^{-((-\mathrm{Ln}(1-\psi_{\alpha_1}))^{\mathbb{N}} + (-\mathrm{Ln}(1-\psi_{\alpha_2}))^{\mathbb{N}})^{(1/\mathbb{N})}} \end{pmatrix},$$

$$= \begin{pmatrix} e^{-((-\mathrm{Ln}(\varphi_{\alpha_2}))^{\mathbb{N}} + (-\mathrm{Ln}(\varphi_{\alpha_1}))^{\mathbb{N}})^{(1/\mathbb{N})}} \\ 1 - e^{-((-\mathrm{Ln}(1-\xi_{\alpha_2}))^{\mathbb{N}} + (-\mathrm{Ln}(1-\xi_{\alpha_1}))^{\mathbb{N}})^{(1/\mathbb{N})}} \\ 1 - e^{-((-\mathrm{Ln}(1-\psi_{\alpha_2}))^{\mathbb{N}} + (-\mathrm{Ln}(1-\psi_{\alpha_1}))^{\mathbb{N}})^{(1/\mathbb{N})}} \end{pmatrix},$$

$$\alpha_1 \otimes \alpha_2 = \alpha_2 \otimes \alpha_1$$

(iii) Let $s = 1 - e^{-((-\mathrm{Ln}(1-\varphi_1(\alpha)))^{\mathbb{N}} + (-\mathrm{Ln}(1-\varphi_2(\alpha)))^{\mathbb{N}})^{(1/\mathbb{N})}}$

Then, $\mathrm{Ln}(1-s) = -((-\mathrm{Ln}(1-\varphi_{\alpha_1}))^{\mathbb{N}} + (-\mathrm{Ln}(1-\varphi_{\alpha_2}))^{\mathbb{N}})^{(1/\mathbb{N})}$. Using this equation, we get the following equation:

$$\lambda(\alpha_1 \oplus \alpha_2) = \lambda \begin{pmatrix} 1 - e^{-\left(\left(-\mathrm{Ln}\left(-\mathrm{Ln}\left(1-\varphi_{\alpha_1}\right)\right)\right)^{\mathbb{N}} + \left(-\mathrm{Ln}-\mathrm{Ln}\left(1-\varphi_{\alpha_2}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ e^{-\left(\left(-\mathrm{Ln}\left(\xi_{\alpha_1}\right)\right)^{\mathbb{N}} + \left(-\mathrm{Ln}\left(\xi_{\alpha_2}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ e^{-\left(\left(-\mathrm{Ln}\left(\psi_{\alpha_1}\right)\right)^{\mathbb{N}} + \left(-\mathrm{Ln}\left(\psi_{\alpha_2}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \end{pmatrix},$$

$$\lambda(\alpha_1 \oplus \alpha_2) = \begin{pmatrix} 1 - e^{-\left(\lambda\left(\left(-\mathrm{Ln}\left(1-\varphi_{\alpha_1}\right)\right)^{\mathbb{N}} + \left(-\mathrm{Ln}\left(1-\varphi_{\alpha_2}\right)\right)^{\mathbb{N}}\right)\right)^{(1/\mathbb{N})}} \\ e^{-\left(\lambda\left(\left(-\mathrm{Ln}\left(\xi_{\alpha_1}\right)\right)^{\mathbb{N}} + \left(-\mathrm{Ln}\left(\xi_{\alpha_2}\right)\right)^{\mathbb{N}}\right)\right)^{(1/\mathbb{N})}} \\ e^{-\left(\lambda\left(\left(-\mathrm{Ln}\left(\psi_{\alpha_1}\right)\right)^{\mathbb{N}} + \left(-\mathrm{Ln}\left(\psi_{\alpha_2}\right)\right)^{\mathbb{N}}\right)\right)^{(1/\mathbb{N})}} \end{pmatrix}, \tag{21}$$

$$\lambda(\alpha_1 \oplus \alpha_2) = \begin{pmatrix} \left(1 - e^{-\left(\lambda\left(-\mathrm{Ln}\left(1-\varphi_{\alpha_1}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}}, e^{-\left(\lambda\left(-\mathrm{Ln}\left(\xi_{\alpha_1}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}}, e^{-\left(\lambda\left(-\mathrm{Ln}\left(\psi_{\alpha_1}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}}\right) \oplus \\ \left(1 - e^{-\left(\lambda\left(-\mathrm{Ln}\left(1-\varphi_{\alpha_2}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}}, e^{-\left(\lambda\left(-\mathrm{Ln}\left(\xi_{\alpha_2}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}}, e^{-\left(\lambda\left(-\mathrm{Ln}\left(\psi_{\alpha_2}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}}\right) \end{pmatrix},$$

$$\lambda(\alpha_1 \oplus \alpha_2) = \lambda\alpha_1 \oplus \lambda\alpha_2.$$

$$\lambda_1\alpha+\lambda_2\alpha=\left(\begin{array}{c}\left(1-e^{-\left(\lambda_1\left(-\mathrm{Ln}\left(1-\varphi_\alpha\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}},e^{-\left(\lambda_1\left(-\mathrm{Ln}\left(\xi_\alpha\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}},e^{-\left(\lambda_1\left(-\mathrm{Ln}\left(\psi_\alpha\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}}\right)\oplus\\ \left(1-e^{-\left(\lambda_2\left(-\mathrm{Ln}\left(1-\varphi_\alpha\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}},e^{-\left(\lambda_2\left(-\mathrm{Ln}\left(\xi_\alpha\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}},e^{-\left(\lambda_2\left(-\mathrm{Ln}\left(\psi_\alpha\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}}\right)\end{array}\right)$$

$$\lambda_1\alpha+\lambda_2\alpha=\left(\begin{array}{c}1-e^{-\left(\left(\lambda_1+\lambda_2\right)\left(-\mathrm{Ln}\left(1-\varphi_\alpha\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}},\\ e^{-\left(\left(\lambda_1+\lambda_2\right)\left(-\mathrm{Ln}\left(\xi_\alpha\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}},\\ e^{-\left(\left(\lambda_1+\lambda_2\right)\left(-\mathrm{Ln}\left(\psi_\alpha\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}}\end{array}\right) \tag{22}$$

$$\lambda_1\alpha+\lambda_2\alpha=\left(\lambda_1+\lambda_2\right)\alpha$$

$$\left(\alpha_1\otimes\alpha_2\right)^\lambda=\left(\begin{array}{c}e^{-\left(\left(-\mathrm{Ln}\left(\varphi_{\alpha_1}\right)\right)^{\mathbb{N}}+\left(-\mathrm{Ln}\left(\varphi_{\alpha_2}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}}\\ 1-e^{-\left(\lambda\left(-\mathrm{Ln}\left(1-\xi_{\alpha_1}\right)\right)^{\mathbb{N}}+\left(-\mathrm{Ln}\left(1-\xi_{\alpha_2}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}}\\ 1-e^{-\left(\lambda\left(-\mathrm{Ln}\left(1-\psi_{\alpha_1}\right)\right)^{\mathbb{N}}+\left(-\mathrm{Ln}\left(1-\psi_{\alpha_2}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}}\end{array}\right)^\lambda,$$

$$\left(\alpha_1\otimes\alpha_2\right)^\lambda=\left(\begin{array}{c}e^{-\left(\lambda\left(\left(-\mathrm{Ln}\left(\varphi_{\alpha_1}\right)\right)^{\mathbb{N}}+\left(-\mathrm{Ln}\left(\varphi_{\alpha_2}\right)\right)^{\mathbb{N}}\right)\right)^{(1/\mathbb{N})}}\\ 1-e^{-\left(\lambda\left(\left(-\mathrm{Ln}\left(1-\xi_{\alpha_1}\right)\right)^{\mathbb{N}}+\left(-\mathrm{Ln}\left(1-\xi_{\alpha_2}\right)\right)^{\mathbb{N}}\right)\right)^{(1/\mathbb{N})}}\\ 1-e^{-\left(\lambda\left(\left(-\mathrm{Ln}\left(1-\psi_{\alpha_1}\right)\right)^{\mathbb{N}}+\left(-\mathrm{Ln}\left(1-\psi_{\alpha_2}\right)\right)^{\mathbb{N}}\right)\right)^{(1/\mathbb{N})}}\end{array}\right),$$

$$\left(\alpha_1\otimes\alpha_2\right)^\lambda=\left(\begin{array}{c}\left(\begin{array}{c}e^{-\left(\lambda\left(\left(-\mathrm{Ln}\left(\varphi_{\alpha_1}\right)\right)^{\mathbb{N}}\right)\right)^{(1/\mathbb{N})}}\\ 1-e^{-\left(\lambda\left(\left(-\mathrm{Ln}\left(1-\xi_{\alpha_1}\right)\right)^{\mathbb{N}}\right)\right)^{(1/\mathbb{N})}}\\ 1-e^{-\left(\lambda\left(\left(-\mathrm{Ln}\left(1-\psi_{\alpha_1}\right)\right)^{\mathbb{N}}\right)\right)^{(1/\mathbb{N})}}\end{array}\right)\otimes\\ \left(\begin{array}{c}e^{-\left(\lambda\left(\left(-\mathrm{Ln}\left(\varphi_{\alpha_2}\right)\right)^{\mathbb{N}}\right)\right)^{(1/\mathbb{N})}}\\ 1-e^{-\left(\lambda\left(\left(-\mathrm{Ln}\left(1-\xi_{\alpha_2}\right)\right)^{\mathbb{N}}\right)\right)^{(1/\mathbb{N})}}\\ 1-e^{-\left(\lambda\left(\left(-\mathrm{Ln}\left(1-\psi_{\alpha_2}\right)\right)^{\mathbb{N}}\right)\right)^{(1/\mathbb{N})}}\end{array}\right)\end{array}\right), \tag{23}$$

$$\left(\alpha_1\otimes\alpha_2\right)^\lambda=\alpha_1^\lambda\otimes\alpha_2^\lambda.$$

$$\alpha^{\lambda_1}\otimes\alpha^{\lambda_2}=\left(\begin{array}{c}\left(\begin{array}{c}e^{-\left(\lambda_1\left(\left(-\mathrm{Ln}\left(\varphi_\alpha\right)\right)^{\mathbb{N}}\right)\right)^{(1/\mathbb{N})}}\\ 1-e^{-\left(\lambda_1\left(\left(-\mathrm{Ln}\left(1-\xi_\alpha\right)\right)^{\mathbb{N}}\right)\right)^{(1/\mathbb{N})}}\\ 1-e^{-\left(\lambda_1\left(\left(-\mathrm{Ln}\left(1-\psi_\alpha\right)\right)^{\mathbb{N}}\right)\right)^{(1/\mathbb{N})}}\end{array}\right)\otimes\\ \left(\begin{array}{c}e^{-\left(\lambda_2\left(\left(-\mathrm{Ln}\left(\varphi_\alpha\right)\right)^{\mathbb{N}}\right)\right)^{(1/\mathbb{N})}}\\ 1-e^{-\left(\lambda_2\left(\left(-\mathrm{Ln}\left(1-\xi_\alpha\right)\right)^{\mathbb{N}}\right)\right)^{(1/\mathbb{N})}}\\ 1-e^{-\left(\lambda_2\left(\left(-\mathrm{Ln}\left(1-\psi_\alpha\right)\right)^{\mathbb{N}}\right)\right)^{(1/\mathbb{N})}}\end{array}\right)\end{array}\right)$$

$$=\left(\begin{array}{c}e^{-\left(\left(\lambda_1+\lambda_2\right)\left(-\mathrm{Ln}\left(\varphi_\alpha\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}}\\ 1-e^{-\left(\left(\lambda_1+\lambda_2\right)\left(-\mathrm{Ln}\left(1-\xi_\alpha\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}}\\ 1-e^{-\left(\left(\lambda_1+\lambda_2\right)\left(-\mathrm{Ln}\left(1-\psi_\alpha\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}}\end{array}\right), \tag{24}$$

$$\alpha^{\lambda_1}\otimes\alpha^{\lambda_2}=\alpha^{\left(\lambda_1+\lambda_2\right)}.$$

□

4. Single-Value Neutrosophic Aczel–Alsina Power Average Aggregation Operators

Now, by using the basic operations of Aczel–Alsina aggregation tools, we derived appropriate methodologies such as SVNAAPWA operators with some reliable properties under consideration of SVNNs. We also utilized a degree of weighted support throughout this article by using the following equation: $\mathfrak{Y}_{\jmath} = (\omega_{\jmath}(1+\mathfrak{A}(\alpha_{\jmath}))/\sum_{\jmath=1}^{n}\omega_{\jmath}(1+\mathfrak{A}(\alpha_{\jmath})))$, where the support of $\alpha_{\jmath}$ is denoted by $\mathfrak{A}(\alpha_{\jmath}) = \sum_{\jmath=1,\tilde{\hat{e}}=1, \jmath\neq\tilde{\hat{e}}}^{n}\omega_{\jmath} Supp(\alpha_{\jmath},\alpha_{\tilde{\hat{e}}})$, $\jmath = 1,2,\ldots,\rho$, $\tilde{\hat{e}} = 1,2,\ldots, n$ and associated weight vector of $\alpha_{\jmath}$ is $\omega = (\omega_1,\omega_2,\ldots\omega_n)^T$, $(\jmath = 1,2,\ldots n)$, $\omega_{\jmath} > 0$, and $\sum_{\jmath=1}^{n}\omega_{\jmath} = 1$.

Definition 19. Let $\alpha_{\jmath} = (\varphi_{\alpha_{\jmath}},\xi_{\alpha_{\jmath}},\psi_{\alpha_{\jmath}})$, $\jmath = 1,2,3,\ldots,n$, be the set of SVNNs, with corresponding weight vectors $\mathfrak{Y} = (\mathfrak{Y}_1,\mathfrak{Y}_2,\mathfrak{Y}_3,\ldots,\mathfrak{Y}_n)^T$ of $\alpha_{\jmath}$, $\jmath = 1,2,3,\ldots,n$ such that $\mathfrak{Y}_{\jmath} > 0$ and $\sum_{\jmath=1}^{n}\mathfrak{Y}_{\jmath} = 1$. Then, the SVNAAPWA operator is a function as follows:

$$\mathrm{SVNAAPWA}(\alpha_1,\alpha_2,\ldots,\alpha_n) = \bigoplus_{\jmath=1}^{n}\left(\mathfrak{Y}_{\jmath}\alpha_{\jmath}\right) = \mathfrak{Y}_1\alpha_1 \oplus \mathfrak{Y}_2\alpha_2 \oplus,\ldots,\oplus \mathfrak{Y}_n\alpha_n. \quad (25)$$

Theorem 20. *Let $\alpha_{\jmath} = (\varphi_{\alpha_{\jmath}},\xi_{\alpha_{\jmath}},\psi_{\alpha_{\jmath}})$, $\jmath = 1,2,3,\ldots,n$, be the set of SVNNs. Then, the aggregated value of the SVNAAPWA operator is also a SVNN as follows:*

$$\mathrm{SVNAAPWA}(\alpha_1,\alpha_2,\ldots,\alpha_n) = \begin{pmatrix} 1 - e^{-\left(\sum_{\jmath=1}^{n}(\mathfrak{Y}_{\jmath})\left(-\mathrm{Ln}\left(1-\varphi_{\alpha_{\jmath}}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ e^{-\left(\sum_{\jmath=1}^{n}(\mathfrak{Y}_{\jmath})\left(-\mathrm{Ln}\left(\xi_{\alpha_{\jmath}}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ e^{-\left(\sum_{\jmath=1}^{n}(\mathfrak{Y}_{\jmath})\left(-\mathrm{Ln}\left(\psi_{\alpha_{\jmath}}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \end{pmatrix}. \quad (26)$$

Proof. We will prove Theorem 20 with the help of a mathematical induction technique by using the following method.

(i) For $\jmath = 2$ depending on Aczel–Alsina operations of SVNNs, we get the following equation:

$$\begin{aligned} \mathfrak{Y}_1\alpha_1 &= \begin{pmatrix} 1 - e^{-\left((\mathfrak{Y}_1)\left(-\mathrm{Ln}\left(1-\varphi_{\alpha_1}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ e^{-\left((\mathfrak{Y}_1)\left(-\mathrm{Ln}\left(\xi_{\alpha_1}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ e^{-\left((\mathfrak{Y}_1)\left(-\mathrm{Ln}\left(\psi_{\alpha_1}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \end{pmatrix}, \\ \mathfrak{Y}_2\alpha_2 &= \begin{pmatrix} 1 - e^{-\left((\mathfrak{Y}_2)\left(-\mathrm{Ln}\left(1-\varphi_{\alpha_2}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ e^{-\left((\mathfrak{Y}_2)\left(-\mathrm{Ln}\left(\xi_{\alpha_2}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ e^{-\left((\mathfrak{Y}_2)\left(-\mathrm{Ln}\left(\psi_{\alpha_2}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \end{pmatrix}. \end{aligned} \quad (27)$$

Using abovementioned Definition 19, we have the following equation:

$$
\begin{aligned}
\mathrm{SVNAAPWA}\left(\alpha_1,\alpha_2\right) = \mathfrak{Y}_1\alpha_1 \oplus \mathfrak{Y}_2\alpha_2 &= \begin{pmatrix} \begin{pmatrix} 1-e^{-\left(\left(\mathfrak{Y}_1\right)\left(-\mathrm{Ln}\left(1-\varphi_{\alpha_1}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ e^{-\left(\left(\mathfrak{Y}_1\right)\left(-\mathrm{Ln}\left(\xi_{\alpha_1}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ e^{-\left(\left(\mathfrak{Y}_1\right)\left(-\mathrm{Ln}\left(\psi_{\alpha_1}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \end{pmatrix} \oplus \\ \begin{pmatrix} 1-e^{-\left(\left(\mathfrak{Y}_2\right)\left(-\mathrm{Ln}\left(1-\varphi_{\alpha_2}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ e^{-\left(\left(\mathfrak{Y}_2\right)\left(-\mathrm{Ln}\left(\xi_{\alpha_2}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ e^{-\left(\left(\mathfrak{Y}_2\right)\left(-\mathrm{Ln}\left(\psi_{\alpha_2}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \end{pmatrix} \end{pmatrix} \\
&= \begin{pmatrix} 1-e^{-\left(\left(\mathfrak{Y}_1\right)\left(-\mathrm{Ln}\left(1-\varphi_{\alpha_1}\right)\right)^{\mathbb{N}}+\left(\mathfrak{Y}_2\right)\left(-\mathrm{Ln}\left(1-\varphi_{\alpha_2}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ e^{-\left(\left(\mathfrak{Y}_1\right)\left(-\mathrm{Ln}\left(\xi_{\alpha_1}\right)\right)^{\mathbb{N}}+\left(\mathfrak{Y}_2\right)\left(-\mathrm{Ln}\left(\xi_{\alpha_2}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ e^{-\left(\left(\mathfrak{Y}_1\right)\left(-\mathrm{Ln}\left(\psi_{\alpha_1}\right)\right)^{\mathbb{N}}+\left(\mathfrak{Y}_2\right)\left(-\mathrm{Ln}\left(\psi_{\alpha_2}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \end{pmatrix} \\
&= \begin{pmatrix} 1-e^{-\left(\sum_{j=1}^{2}\left(\mathfrak{Y}_j\right)\left(\mathrm{Ln}\left(1-\varphi_{\alpha_j}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ e^{-\left(\sum_{j=1}^{2}\left(\mathfrak{Y}_j\right)\left(-\mathrm{Ln}\left(\xi_{\alpha_j}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ e^{-\left(\sum_{j=1}^{2}\left(\mathfrak{Y}_j\right)\left(-\mathrm{Ln}\left(\psi_{\alpha_j}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \end{pmatrix}.
\end{aligned} \tag{28}
$$

Hence, this is true for $j = 2$.

(ii) Now, suppose that this will be true for $j = k$. Then, we have the following equation:

$$
\begin{aligned}
\mathrm{SVNAAPWA}\left(\alpha_1,\alpha_2,\ldots,\alpha_k\right) &= \underset{j=1}{\overset{k}{\oplus}} \mathfrak{Y}_k\alpha_k \\
&= \begin{pmatrix} 1-e^{-\left(\sum_{j=1}^{k}\left(\mathfrak{Y}_k\right)\left(\mathrm{Ln}\left(1-\varphi_{\alpha_k}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ e^{-\left(\sum_{j=1}^{k}\left(\mathfrak{Y}_k\right)\left(-\mathrm{Ln}\left(\xi_{\alpha_k}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ e^{-s\left(\sum_{j=1}^{k}\left(\mathfrak{Y}_k\right)\left(-\mathrm{Ln}\left(\psi_{\alpha_k}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \end{pmatrix}.
\end{aligned} \tag{29}
$$

Now, we have to show that it also holds for $j = k + 1$ as follows:

$$\mathrm{SVNAAPWA}\left(\alpha_1+\alpha_k,\alpha_{k+1}\right)=\overset{k}{\underset{\jmath=1}{\oplus}}\left(\mathfrak{Y}_k\alpha_k\oplus\mathfrak{Y}_{k+1}\alpha_{k+1}\right)=\left(\begin{pmatrix}1-e^{-\left(\sum_{\jmath=1}^{k}(\mathfrak{Y}_\jmath)\left(\mathrm{Ln}\left(1-\varphi_{\alpha_\jmath}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}}\\ e^{-\left(\sum_{\jmath=1}^{k}(\mathfrak{Y}_\jmath)\left(-\mathrm{Ln}\left(\xi_{\alpha_\jmath}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}}\\ e^{-\left(\sum_{\jmath=1}^{k}(\mathfrak{Y}_\jmath)\left(-\mathrm{Ln}\left(\psi_{\alpha_\jmath}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}}\end{pmatrix}\oplus \\ \begin{pmatrix}1-e^{-\left((\mathfrak{Y}_{k+1})\left(\mathrm{Ln}\left(1-\varphi_{k+1}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}}\\ e^{-\left(\mathfrak{Y}_{k+1}\left(-\mathrm{Ln}\left(\xi_{k+1}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}}\\ e^{-\left(\mathfrak{Y}_{k+1}\left(-\mathrm{Ln}\left(\psi_{k+1}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}}\end{pmatrix}\right),$$

$$=\begin{pmatrix}1-e^{-\left(\sum_{\jmath=1}^{k+1}(\mathfrak{Y}_\jmath)\left(\mathrm{Ln}\left(1-\varphi_{\alpha_\jmath}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}}\\ e^{-\left(\sum_{\jmath=1}^{k+1}(\mathfrak{Y}_\jmath)\left(-\mathrm{Ln}\left(\xi_{\alpha_\jmath}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}}\\ e^{-\left(\sum_{\jmath=1}^{k+1}(\mathfrak{Y}_\jmath)\left(-\mathrm{Ln}\left(\psi_{\alpha_\jmath}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}}\end{pmatrix}, \tag{30}$$

which is true for $\jmath=k+1$. □

Example 4. Let $\alpha_1=(0.99,0.67,0.76)$, $\alpha_2=(0.87,0.56,0.35)$, $\alpha_3=(0.45,0.66,0.89)$, and $\alpha_4=(0.56,0.45,0.76)$ be the four SVNNs with weight vector $(0.20,0.35,0.30,0.15)$.

Now, we investigate the value of the SVNAAPWA operator for $\mathbb{N}=3$ and $\alpha=4$.

The degree of weighted support associated with SVNNs is as follows: $\mathfrak{Y}_\jmath=(\omega_\jmath(1+\mathfrak{A}(\alpha_\jmath))/\sum_{\jmath=1}^{n}\omega_\jmath(1+\mathfrak{A}(\alpha_\jmath)))$.

$$\mathfrak{Y}_1=0.2113,\mathfrak{Y}_2=0.3322,\mathfrak{Y}_3=0.2942,\mathfrak{Y}_1=0.1624,$$

$$\mathrm{SVNAAPWA}\left(\alpha_1,\alpha_2,\ldots,\alpha_n\right)=\begin{pmatrix}1-e^{-\left(\sum_{\jmath=1}^{n}(\mathfrak{Y}_\jmath)\left(-\mathrm{Ln}\left(1-\varphi_{\alpha_\jmath}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}}\\ e^{-\left(\sum_{\jmath=1}^{n}(\mathfrak{Y}_\jmath)\left(-\mathrm{Ln}\xi_{\alpha_\jmath}\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}}\\ e^{-\left(\sum_{\jmath=1}^{n}(\mathfrak{Y}_\jmath)\left(-\mathrm{Ln}\psi_{\alpha_\jmath}\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}}\end{pmatrix},$$

$$\mathrm{SVNAAPWA}\left(\alpha_1,\alpha_2,\alpha_3,\alpha_4\right)=\begin{pmatrix}1-e^{-\left(\sum_{\jmath=1}^{4}(\mathfrak{Y}_\jmath)\left(-\mathrm{Ln}\left(1-\varphi_{\alpha_\jmath}\right)\right)^{3}\right)^{(1/3)}}\\ e^{-\left(\sum_{\jmath=1}^{4}(\mathfrak{Y}_\jmath)\left(-\mathrm{Ln}\left(\xi_{\alpha_\jmath}\right)\right)^{3}\right)^{(1/3)}}\\ e^{-\left(\sum_{\jmath=1}^{4}(\mathfrak{Y}_\jmath)\left(-\mathrm{Ln}\left(\psi_{\alpha_\jmath}\right)\right)^{3}\right)^{(1/3)}}\end{pmatrix}$$

$$=\begin{pmatrix}1-e^{-\left((\mathfrak{Y}_1)\left(-\mathrm{Ln}\left(1-\varphi_{\alpha_1}\right)\right)^3+(\mathfrak{Y}_2)\left(-\mathrm{Ln}\left(1-\varphi_{\alpha_2}\right)\right)^3+(\mathfrak{Y}_3)\left(-\mathrm{Ln}\left(1-\varphi_{\alpha_3}\right)\right)^3+(\mathfrak{Y}_4)\left(-\mathrm{Ln}\left(1-\varphi_{\alpha_4}\right)\right)^3\right)^{(1/3)}}\\ e^{-\left((\mathfrak{Y}_1)\left(-\mathrm{Ln}\left(\xi_{\alpha_1}\right)\right)^3+(\mathfrak{Y}_2)\left(-\mathrm{Ln}\left(\xi_{\alpha_2}\right)\right)^3+(\mathfrak{Y}_3)\left(-\mathrm{Ln}\left(\xi_{\alpha_3}\right)\right)^3+(\mathfrak{Y}_4)\left(-\mathrm{Ln}\left(\xi_{\alpha_4}\right)\right)^3\right)^{(1/3)}}\\ e^{-\left((\mathfrak{Y}_1)\left(-\mathrm{Ln}\left(\psi_{\alpha_1}\right)\right)^3+(\mathfrak{Y}_2)\left(-\mathrm{Ln}\left(\psi_{\alpha_1}\right)\right)^3+(\mathfrak{Y}_3)\left(-\mathrm{Ln}\left(\psi_{\alpha_3}\right)\right)^3+(\mathfrak{Y}_4)\left(-\mathrm{Ln}\left(\psi_{\alpha_4}\right)\right)^3\right)^{(1/3)}}\end{pmatrix}$$

$$=\begin{pmatrix}1-e^{-\left((0.2113)(-\mathrm{Ln}(1-0.99))^3+(0.3322)(-\mathrm{Ln}(1-0.87))^3+(0.2942)(-\mathrm{Ln}(1-0.45))^3+(0.1624)(-\mathrm{Ln}(1-0.56))^3\right)^{(1/3)}}\\ e^{-\left((0.2113)(-\mathrm{Ln}(0.67))^3+(0.3322)(-\mathrm{Ln}(0.56))^3+(0.2942)(-\mathrm{Ln}(0.66))^3+(0.1624)(-\mathrm{Ln}(0.45))^3\right)^{(1/3)}}\\ e^{-\left((0.2113)(-\mathrm{Ln}(0.76))^3+(0.3322)(-\mathrm{Ln}(0.35))^3+(0.2942)(-\mathrm{Ln}(0.89))^3+(0.1624)(-\mathrm{Ln}(0.76))^3\right)^{(1/3)}}\end{pmatrix}$$

$$=(0.9432,\ 0.5773,\ 0.4809). \tag{31}$$

Theorem 21 (idempotency). *Consider $\alpha_{\jmath} = (\varphi_{\alpha_{\jmath}}, \xi_{\alpha_{\jmath}}, \psi_{\alpha_{\jmath}})$, $\jmath = 1,2,3,\ldots,n$, be the set of identical SVNNs, if $\alpha_n = \alpha$ for all α then we have the following equation:*

$$\text{SVNAAPWA}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \alpha. \tag{32}$$

Proof. Since $\alpha_{\jmath} = (\varphi_{\alpha_{\jmath}}, \xi_{\alpha_{\jmath}}, \psi_{\alpha_{\jmath}})$, $\jmath = 1,2,3,\ldots,n$, be the set of SVNNs, we can get the following equation:

$$\begin{aligned}\text{SVNAAPWA}(\alpha_1, \alpha_2, \ldots, \alpha_n) &= \begin{pmatrix} 1 - e^{-\left(\sum_{\jmath=1}^{n}(\mathfrak{Y}_{\jmath})\left(\text{Ln}\left(1-\varphi_{\alpha_{\jmath}}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ e^{-\left(\sum_{\jmath=1}^{n}(\mathfrak{Y}_{\jmath})\left(-\text{Ln}\left(\xi_{\alpha_{\jmath}}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ e^{-\left(\sum_{\jmath=1}^{n}(\mathfrak{Y}_{\jmath})\left(-\text{Ln}\left(\psi_{\alpha_{\jmath}}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \end{pmatrix} \\ &= \left(1 - e^{-\left(\left(\text{Ln}\left(1-\varphi_{\alpha}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}}, e^{-\left(\left(-\text{Ln}\xi_{\alpha}\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}}, e^{-\left(\left(-\text{Ln}\left(\psi_{\alpha}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}}\right) \\ &= (\varphi_{\alpha}, \xi_{\alpha}, \psi_{\alpha}) = \alpha.\end{aligned} \tag{33}$$

Thus, we can say that $\text{SVNAAPWA}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \alpha$ holds. □

Theorem 22 (boundedness). *Consider $\alpha_{\jmath} = (\varphi_{\alpha_{\jmath}}, \xi_{\alpha_{\jmath}}, \psi_{\alpha_{\jmath}})$, $\jmath = 1,2,3,\ldots,n$, be the set of SVNNs if $\alpha^- = \min(\alpha_1, \alpha_2, \ldots, \alpha_n)$ and $\alpha^+ = \max(\alpha_1, \alpha_2, \ldots, \alpha_n)$, and then we have the following equation:*

$$\alpha^- \leq \text{SVNAAPWA}(\alpha_1, \alpha_2, \ldots, \alpha_n) \leq \alpha^+. \tag{34}$$

Proof. Consider $\alpha_{\jmath} = (\varphi_{\alpha_{\jmath}}, \xi_{\alpha_{\jmath}}, \psi_{\alpha_{\jmath}})$, $\jmath = 1,2,3,\ldots,n$, be the set of SVNNs. Let $\alpha^- = \min(\alpha_1, \alpha_2, \ldots, \alpha_n) = (\varphi_{\alpha}^-, \xi_{\alpha}^-, \psi_{\alpha}^-)$ and $\alpha^+ = \max(\alpha_1, \alpha_2, \ldots, \alpha_n) = (\varphi_{\alpha}^+, \xi_{\alpha}^+, \psi_{\alpha}^+)$. We have $\varphi_{\alpha}^- = \min_{\jmath}\left\{\varphi_{\alpha_{\jmath}}\right\}, \xi_{\alpha}^- = \max_{\jmath}\left\{\xi_{\alpha_{\jmath}}\right\}$, $\psi_{\alpha}^- = \max_{\jmath}\left\{\psi_{\alpha_{\jmath}}\right\}$, $\varphi_{\alpha}^+ = \max_{\jmath}\left\{\xi_{\alpha_{\jmath}}\right\}, \xi_{\alpha}^+ = \min_{\jmath}\left\{\xi_{\alpha_{\jmath}}\right\}$, and $\psi_{\alpha}^+ = \min_{\jmath}\left\{\psi_{\alpha_{\jmath}}\right\}$. Hence, there we have the following subsequent inequalities:

$$\begin{aligned} &1 - e^{-\left(\sum_{\jmath=1}^{n}(\mathfrak{Y}_{\jmath})\left(-\text{Ln}\left(1-\varphi_{\alpha}^-\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \leq 1 - e^{-\left(\sum_{\jmath=1}^{n}(\mathfrak{Y}_{\jmath})\left(-\text{Ln}\left(1-\varphi_{\alpha_{\jmath}}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \leq 1 - e^{-\left(\sum_{\jmath=1}^{n}(\mathfrak{Y}_{\jmath})\left(-\text{Ln}\left(1-\varphi_{\alpha}^+\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}}, \\ &e^{-\left(\sum_{\jmath=1}^{n}(\mathfrak{Y}_{\jmath})\left(-\text{Ln}\left(\xi_{\alpha}^-\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \leq e^{-\left(\sum_{\jmath=1}^{n}(\mathfrak{Y}_{\jmath})\left(-\text{Ln}\left(\xi_{\alpha}^-\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \leq e^{-\left(\sum_{\jmath=1}^{n}(\mathfrak{Y}_{\jmath})\left(-\text{Ln}\left(\xi_{\alpha}^+\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}}, \\ &e^{-\left(\sum_{\jmath=1}^{n}(\mathfrak{Y}_{\jmath})\left(-\text{Ln}\left(\psi_{\alpha}^-\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \leq e^{-\left(\sum_{\jmath=1}^{n}(\mathfrak{Y}_{\jmath})\left(-\text{Ln}\left(\psi_{\alpha_{\jmath}}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \leq e^{-\left(\sum_{\jmath=1}^{n}(\mathfrak{Y}_{\jmath})\left(-\text{Ln}\left(\psi_{\alpha}^+\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}}. \end{aligned} \tag{35}$$

Therefore, $\alpha^- \leq \text{SVNAAPWA}(\alpha_1, \alpha_2, \ldots, \alpha_n) \leq \alpha^+$. □

Theorem 23 (Monotonicity). *Let $\alpha_{\jmath}$ and $\alpha_{\jmath}'$, $\jmath = 1,2,3,\ldots,n$, be two sets of SVNNs, if $\alpha_{\jmath} \leq \alpha_{\jmath}'$ for all α. Then, we have the following equation:*

$$\text{SVNAAPWA}(\alpha_1, \alpha_2, \ldots, \alpha_n) \leq \text{SVNAAPWA}(\alpha_1', \alpha_2', \ldots, \alpha_n'). \tag{36}$$

Now, we will represent SVN Aczel–Alsina ordered weighted averaging (SVNAAOPWA) operator by using the basic operations of Aczel–Alsina operations.

Definition 24. Consider $\alpha_{\jmath} = (\varphi_{\alpha_{\jmath}}, \xi_{\alpha_{\jmath}}, \psi_{\alpha_{\jmath}})$, $\jmath = 1,2,3,\ldots,n$, be the set of SVNNs, with corresponding weight vectors of $a_{\jmath}$. Then, an SVNAAOPWA operator of dimension $\jmath$ is a mapping SVNAAOPWA: $(L^*)^n \longrightarrow L^*$ as follows:

$$\text{SVNAAOPWA}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \bigoplus_{\jmath=1}^{n}\left(\mathfrak{Y}_{\jmath}\alpha_{\vartheta(\jmath)}\right) = \mathfrak{Y}_1\alpha_{\vartheta(1)} \oplus \mathfrak{Y}_2\alpha_{\vartheta(2)} \oplus, \ldots, \mathfrak{Y}_n\alpha_{\vartheta(\jmath)}. \tag{37}$$

Theorem 25. *Consider* $\alpha_{\jmath} = (\varphi_{\alpha_{\jmath}}, \xi_{\alpha_{\jmath}}, \psi_{\alpha_{\jmath}})$, $\jmath = 1,2,3,\ldots,n$, *be the set of SVNNs, with the corresponding weight vector* $\mathfrak{Y} = (\mathfrak{Y}_1, \mathfrak{Y}_2, \ldots, \mathfrak{Y}_n)^T$ *such that* $\mathfrak{Y}_n > 0$ *and* $\sum_{\jmath=1}^{n} \mathfrak{Y}_{\jmath} = 1$. *A SVNAAOPWA operator of dimension* $\jmath$ *is a mapping* SVNAAOPWA: $(L^*)^n \longrightarrow L^*$ *as follows:*

$$\text{SVNAAOPWA}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \begin{pmatrix} 1 - e^{-\left(\sum_{\jmath=1}^{n} (\mathfrak{Y}_{\jmath})\left(-\text{Ln}\left(1-\varphi_{\alpha_{\vartheta(\jmath)}}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ e^{-\left(\sum_{\jmath=1}^{n} (\mathfrak{Y}_{\jmath})\left(-\text{Ln}\left(1-\xi_{\alpha_{\vartheta(\jmath)}}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ e^{-\left(\sum_{\jmath=1}^{n} (\mathfrak{Y}_{\jmath})\left(-\text{Ln}\left(1-\psi_{\alpha_{\vartheta(\jmath)}}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \end{pmatrix}. \tag{38}$$

Theorem 26 (Idempotency). *Consider* $\alpha_{\jmath} = (\varphi_{\alpha_{\jmath}}, \xi_{\alpha_{\jmath}}, \psi_{\alpha_{\jmath}})$, $\jmath = 1,2,3,\ldots,n$, *to be the set of identical SVNNs, for all* $\alpha_{\jmath} = \alpha$. *Then, we have the following equation:*

$$\text{SVNAAOPWA}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \alpha. \tag{39}$$

Theorem 27 (boundedness). *Consider* $\alpha_{\jmath} = (\varphi_{\alpha_{\jmath}}, \xi_{\alpha_{\jmath}}, \psi_{\alpha_{\jmath}})$, $\jmath = 1,2,3,\ldots,n$, *to be the set of SVNNs. Let* $\alpha^- = \min x_{\jmath}\alpha_{\jmath} = \max x_{\jmath}\alpha_{\jmath}$. *Then, we have the following equation:*

$$\alpha^- \leq \text{SVNAAOPWA}(\alpha_1, \alpha_2, \ldots, \alpha_n) \leq \text{SVNAAOPWA}(\alpha_1', \alpha_2', \ldots, \alpha_n') \leq \alpha^+. \tag{40}$$

Theorem 28 (monotonicity). *Let* $\alpha_{\jmath}$ *and* $\alpha_{\jmath}'$, $\jmath = 1, 2, 3, \ldots, n$, *be any two sets of SVNNs, if* $\alpha_{\jmath} \leq \alpha_{\jmath}'$ *for all* α. *Then, we have the following equation:*

$$\text{SVNAAOPWA}(\alpha_1, \alpha_2, \ldots, \alpha_n) \leq \text{SVNAAOPWA}(\alpha_1', \alpha_2', \ldots, \alpha_n'). \tag{41}$$

Theorem 29 (commutativity). *Let* $\alpha_{\jmath}$ *and* $\alpha_{\jmath}'$, $\jmath = 1, 2, 3, \ldots, n$, *be any two sets of SVNNs. Then, we have the following equation:*

$$\text{SVNAAOPWA}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \text{SVNAAOPWA}(\alpha_1', \alpha_2', \ldots, \alpha_n'), \tag{42}$$

where $\alpha_{\jmath}'$, $\jmath = 1, 2, 3, \ldots, n$, *is any permutation of* $\alpha_{\jmath}'$, $\jmath = 1, 2, 3, \ldots, n$.

5. Single-Value Neutrosophic Aczel–Alsina Power Weighted Geometric Aggregation Operators

In this section, we will study new AOs of SVN weighted geometric aggregation operator by following the Aczel–Alsina operations.

Definition 30. Let $\alpha_{\jmath} = (\varphi_{\alpha_{\jmath}}, \xi_{\alpha_{\jmath}}, \psi_{\alpha_{\jmath}})$, $\jmath = 1,2,3,\ldots,n$, be the set of SVNNs, with corresponding weight vector $\mathfrak{Y} = (\mathfrak{Y}_1, \mathfrak{Y}_2, \ldots, \mathfrak{Y}_n)^T$ of $\alpha_{\jmath}$, ($\jmath = 1,2,3,\ldots,n$) such that $\mathfrak{Y}_{\jmath} > 0$ and $\sum_{\jmath=1}^{n} \mathfrak{Y}_{\jmath} = 1$. Then, a SVNAAPWG operator is a function SVNAAPWG: $(L^*)^{\jmath} \longrightarrow L^*$ as follows:

$$\text{SVNAAPWG}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \bigotimes_{\jmath=1}^{n} \left(\alpha_{\jmath}^{\mathfrak{Y}_{\jmath}}\right) = \alpha_1^{\mathfrak{Y}_1} \bigotimes \alpha_2^{\mathfrak{Y}_2} \bigotimes, \ldots, \bigotimes \alpha_n^{\mathfrak{Y}_n}. \tag{43}$$

Theorem 31. *Let* $\alpha_{\jmath} = (\varphi_{\alpha_{\jmath}}, \xi_{\alpha_{\jmath}}, \psi_{\alpha_{\jmath}})$ $(\jmath = 1, 2, 3, \ldots, n)$ *be the set of SVNNs. Then, the aggregated value of the SVNAAPWG operator is also a SNNN as follows:*

$$\text{SVNAAPWG}(\alpha_1,\alpha_2,\ldots,\alpha_n)=\begin{pmatrix} e^{-\left(\sum_{\jmath=1}^{n}(\mathfrak{Y}_{\jmath})\left(-\text{Ln}\left(\varphi_{\alpha_{\jmath}}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ 1-e^{-\left(\sum_{\jmath=1}^{n}(\mathfrak{Y}_{\jmath})\left(1-\text{Ln}\left(\xi_{\alpha_{\jmath}}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ 1-e^{-\left(\sum_{\jmath=1}^{n}(\mathfrak{Y}_{\jmath})\left(1-\text{Ln}\left(\psi_{\alpha_{\jmath}}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \end{pmatrix}. \tag{44}$$

Example 5. Let $\alpha_1=(0.78,0.89,0.56)$, $\alpha_2=(0.56,0.78,0.98)$, $\alpha_3=(0.69,0.43,0.51)$, and $\alpha_4=(0.49,0.77,0.45)$ be the four SVNNs, with weight vectors (0.15,0.35,0.30,0.20). We investigate the value of the SVNAAPWG operator for $\mathbb{N}=3$ and $\alpha=4$:

The degree of weighted support of SVNNs is $\mathfrak{Y}_{\jmath}=(\omega_{\jmath}(1+\mathfrak{A}(\alpha_{\jmath}))/\sum_{\jmath=1}^{n}\omega_{\jmath}(1+\mathfrak{A}(\alpha_{\jmath})))$.

$$\mathfrak{Y}_1=0.1613, \mathfrak{Y}_2=0.3332, \mathfrak{Y}_3=0.2938, \mathfrak{Y}_1=0.2118,$$

$$\text{SVNAAPWG}(\alpha_1,\alpha_2,\alpha_3,\ldots,\alpha_n)=\begin{pmatrix} e^{-\left(\sum_{\jmath=1}^{n}(\mathfrak{Y}_{\jmath})\left(-\text{Ln}\left(\varphi_{\alpha_{\jmath}}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ 1-e^{-\left(\sum_{\jmath=1}^{n}(\mathfrak{Y}_{\jmath})\left(-\text{Ln}\left(1-\xi_{\alpha_{\jmath}}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ 1-e^{-\left(\sum_{\jmath=1}^{n}(\mathfrak{Y}_{\jmath})\left(-\text{Ln}\left(1-\psi_{\alpha_{\jmath}}\right)\right)^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \end{pmatrix},$$

$$\begin{aligned}\text{SVNAAPWG}(\alpha_1,\alpha_2,\alpha_3,\alpha_4)&=\begin{pmatrix} e^{-\left(\sum_{\jmath=1}^{4}(\mathfrak{Y}_{\jmath})\left(-\text{Ln}\left(\varphi_{\alpha_{\jmath}}\right)\right)^{3}\right)^{(1/3)}} \\ 1-e^{-\left(\sum_{\jmath=1}^{4}(\mathfrak{Y}_{\jmath})\left(-\text{Ln}\left(1-\xi_{\alpha_{\jmath}}\right)\right)^{3}\right)^{(1/3)}} \\ 1-e^{-\left(\sum_{\jmath=1}^{4}(\mathfrak{Y}_{\jmath})\left(-\text{Ln}\left(1-\psi_{\alpha_{\jmath}}\right)\right)^{3}\right)^{(1/3)}} \end{pmatrix} \\ &=\begin{pmatrix} e^{-\left((\mathfrak{Y}_1)\left(-\text{Ln}\left(\varphi_{\alpha_1}\right)\right)^3+(\mathfrak{Y}_2)\left(-\text{Ln}\left(\varphi_{\alpha_2}\right)\right)^3+(\mathfrak{Y}_3)\left(-\text{Ln}\left(\varphi_{\alpha_3}\right)\right)^3+(\mathfrak{Y}_4)\left(-\text{Ln}\left(\varphi_{\alpha_4}\right)\right)^3\right)^{(1/3)}} \\ 1-e^{-\left((\mathfrak{Y}_1)\left(-\text{Ln}\left(1-\xi_{\alpha_1}\right)\right)^3+(\mathfrak{Y}_2)\left(-\text{Ln}\left(1-\xi_{\alpha_2}\right)\right)^3+(\mathfrak{Y}_3)\left(-\text{Ln}\left(1-\xi_{\alpha_3}\right)\right)^3+(\mathfrak{Y}_4)\left(-\text{Ln}\left(1-\xi_{\alpha_4}\right)\right)^3\right)^{(1/3)}} \\ 1-e^{-\left((\mathfrak{Y}_1)\left(-\text{Ln}\left(1-\psi_{\alpha_1}\right)\right)^3+(\mathfrak{Y}_2)\left(-\text{Ln}\left(1-\psi_{\alpha_2}\right)\right)^3+(\mathfrak{Y}_3)\left(-\text{Ln}\left(01-\psi_{\alpha_3}\right)^3+(\mathfrak{Y}_4)\left(-\text{Ln}\left(1-\psi_{\alpha_4}\right)\right)^3\right)^{(1/3)}} \end{pmatrix} \\ &=\begin{pmatrix} e^{-\left((0.1613)(-\text{Ln}(0.78))^3+(0.3332)(-\text{Ln}(0.56))^3+(0.2938)(-\text{Ln}(0.69))^3+(0.2118)(-\text{Ln}(0.49))^3\right)^{(1/3)}} \\ 1-e^{-\left((0.1613)(-\text{Ln}(1-0.89))^3+(0.3332)(-\text{Ln}(1-0.78))^3+(0.2938)(-\text{Ln}(1-0.43))^3+(0.2118)(-\text{Ln}(1-0.77))^3\right)^{(1/3)}} \\ 1-e^{-\left((0.1613)(-\text{Ln}(1-0.56))^3+(0.3332)(-\text{Ln}(1-0.98))^3+(0.2938)(-\text{Ln}((1|-0.51)^3+(0.2118)(-\text{Ln}(1-0.45))^3\right)^{(1/3)}} \end{pmatrix} \\ &=(0.5815,\ 0.7845,\ 0.9343).\end{aligned} \tag{45}$$

We can state the following characteristics of the SVNAAPWG operator by using the Aczel–Alsina operations on SVNNs.

Theorem 32 (idempotency). *Let $\alpha_{\jmath}=(\varphi_{\alpha_{\jmath}},\xi_{\alpha_{\jmath}},\psi_{\alpha_{\jmath}})$, $\jmath=1,2,3,\ldots,n$, be the set of identical SVNNs, $\alpha_n=\alpha$ for all α. Then, we have the following equation:*

$$\text{SVNAAPWG}(\alpha_1,\alpha_2,\ldots,\alpha_n)=\alpha. \tag{46}$$

Theorem 33 (boundedness). *Let $\alpha_{\jmath}=(\varphi_{\alpha_{\jmath}},\xi_{\alpha_{\jmath}},\psi_{\alpha_{\jmath}})$, $\jmath=1,2,3,\ldots,n$, be the set of SVNNs. Let $\alpha^-=\min(\alpha_1,\alpha_2,\ldots,\alpha_n)$, and $\alpha^+=\max(\alpha_1,\alpha_2,\ldots,\alpha_n)$. Then, we have the following equation:*

$$\alpha^-\leq\text{SVNAAPWG}(\alpha_1,\alpha_2,\ldots,\alpha_n)\leq\alpha^+. \tag{47}$$

Theorem 34 (Monotonicity). *Let $\alpha_{\jmath}$ and $\alpha'_{\jmath}$, $\jmath = 1, 2, 3, \ldots, n$, be any two sets of SVNNs, if $\alpha_{\jmath} \leq \alpha'_{\jmath}$ for all α. Then, we have the following equation:*

$$\mathrm{SVNAAPWG}(\alpha_1, \alpha_2, \ldots, \alpha_n) \leq \mathrm{SVNAAPWG}(\alpha'_1, \alpha'_2, \ldots, \alpha'_n). \tag{48}$$

We can prove all these theorems in the same way as explained in Section 4.

6. Model of MADM Techniques with SVNN Information

In this section, we solved a MADM technique by using SVNAAPWA and SVNAAPWG operators under the system of SVN information. Consider $(Б_1, Б_2, \ldots, Б_\kappa)$ to be the set of alternatives and $Ɇ = (Ɇ_1, Ɇ_2, \ldots, Ɇ_n)$ to be the set of attributes with corresponding weight vectors $\omega = (\omega_1, \omega_2, \ldots \omega_n)^T$, $(\jmath = 1,2, \ldots n)$, $\omega_{\jmath} > 0$, and $\sum_{\jmath=1}^{n} \omega_{\jmath} = 1$. A decision matrix $R = (Y_{\eta\jmath})_{\kappa\times n}$ containing the information under SVNNs given by decision-making is as follows:

$$R = \left(Y_{\eta\jmath}\right)_{\kappa\times n} = \begin{pmatrix} \left(\varphi_{\alpha_{11}}, \xi_{\alpha_{11}}, \psi_{\alpha_{11}}\right) & \left(\varphi_{\alpha_{12}}, \xi_{\alpha_{12}}, \psi_{\alpha_{12}}\right) & \ldots & \left(\varphi_{\alpha_{1n}}, \xi_{\alpha_{1n}}, \psi_{\alpha_{1n}}\right) \\ \left(\varphi_{\alpha_{21}}, \xi_{\alpha_{21}}, \psi_{\alpha_{21}}\right) & \left(\varphi_{\alpha_{22}}, \xi_{\alpha_{22}}, \psi_{\alpha_{22}}\right) & \ldots & \left(\varphi_{\alpha_{2n}}, \xi_{\alpha_{2n}}, \psi_{\alpha_{2n}}\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(\varphi_{\alpha_{\kappa 1}}, \xi_{\alpha_{\kappa n}}, \psi_{\alpha_{\kappa 1}}\right) & \left(\varphi_{\alpha_{\kappa 2}}, \xi_{\alpha_{\kappa 2}}, \psi_{\alpha_{\kappa 2}}\right) & \vdots & \left(\varphi_{\alpha_{\kappa n}}, \xi_{\alpha_{\kappa n}}, \psi_{\alpha_{\kappa n}}\right) \end{pmatrix}. \tag{49}$$

In the decision matrix, each 3-tuple $(\varphi_{\alpha_{\eta\jmath}}, \xi_{\alpha_{\eta\jmath}}, \psi_{\alpha_{\eta\jmath}})$ represents the value of SVNN and $\varphi_{\alpha_{\eta\jmath}} \in [0, 1]$, $\xi_{\alpha_{\eta\jmath}} \in [0, 1]$, and $\psi_{\alpha_{\eta\jmath}} \in [0, 1]$ such that $0 \leq \varphi_{\alpha_{\eta\jmath}} + \xi_{\alpha_{\eta\jmath}} + \psi_{\alpha_{\eta\jmath}} \leq 3$. There are two types of attributes: cost factor and beneficial factor. If a cost factor is involved, then the decision matrix is transformed into the following normalized matrix:

$$R = \left(\overline{Y_{\eta\jmath}}\right)_{\kappa\times n} = \begin{cases} \left(\varphi_{\eta\jmath}, \xi_{\eta\jmath}, \psi_{\eta\jmath}\right), & \text{if benefit factor,} \\ \left(\psi_{\eta\jmath}, \xi_{\eta\jmath}, \varphi_{\eta\jmath}\right), & \text{if cost factor.} \end{cases} \tag{50}$$

We will follow the following steps of the algorithm to solve a given MADM technique by the decision maker.

Step 1: the decision maker collects the information and arranged it in a decision matrix based on SVNNs.

Step 2: if the cost factor is involved in the set of attributes, then we need to transform the decision matrix into a normalized matrix.

Step 3: calculate support value as follows:

$$Supp\left(Y_{\eta\jmath}, Y_{\eta\tilde{ȇ}}\right) = 1 - d\left(Y_{\eta\jmath}, Y_{\eta\tilde{ȇ}}\right),$$
$$\text{where} \tag{51}$$
$$\eta = 1,2, \ldots, \kappa, \jmath, \tilde{ȇ} = 1,2, \ldots, n,$$

where $d(Y_{\eta\jmath}, Y_{\eta\tilde{ȇ}}) = (1/3)(|\varphi_{\eta\jmath} - \varphi_{\eta\tilde{ȇ}}| + |\xi_{\eta\jmath} - \xi_{\eta\tilde{ȇ}}| + |\psi_{\eta\jmath} - \psi_{\eta\tilde{ȇ}}|)$ is the expression for distance.

Step 4: compute weighted support of $\alpha_{\eta\jmath}$ by using the weights of the characteristics $\omega = (\omega_1, \omega_2, \ldots \omega_n)^T$, $(\jmath = 1, 2, \ldots n)$, $\omega_{\jmath} > 0$, and $\sum_{\jmath=1}^{n} \omega_{\jmath} = 1$:

$$\mathfrak{A}\left(Y_{\eta\jmath}\right) = \sum_{\substack{\jmath=1, \tilde{ȇ}=1, \\ \jmath \neq \tilde{ȇ}.}}^{n} \omega_{\jmath} Supp\left(Y_{\eta\jmath}, Y_{\eta\tilde{ȇ}}\right),$$
$$\eta = 1, 2, \ldots, \kappa, \jmath, \tilde{ȇ} = 1, 2, \ldots, n, \tag{52}$$

and investigate weights corresponding to the SVNNs of $Y_{\eta\jmath}$, $\eta = 1, 2, \ldots, \kappa$, $\jmath = 1, 2, \ldots, n$.

$$\mathfrak{Y}_{\eta\jmath} = \frac{\omega_{\jmath}\left(1 + \mathfrak{A}\left(Y_{\eta\jmath}\right)\right)}{\sum_{\jmath=1}^{n} \omega_{\jmath}\left(1 + \mathfrak{A}\left(Y_{\eta\jmath}\right)\right)}, \quad \eta = 1, 2, \ldots, \kappa, \jmath = 1, 2, \ldots, n, \tag{53}$$

where the associated weight vector of $Y_{\eta\jmath}$ is $\mathfrak{Y}_{\eta\jmath} > 0$, $\sum_{\jmath=1}^{n} \mathfrak{Y}_{\eta\jmath} = 1$, $\eta = 1,2, \ldots, \kappa$, and $\jmath = 1,2, \ldots, n$.

Step 5: apply our derived approaches of the SVNAAPWA and SVNAAPWG operators on information depicted in the decision matrix.

$$R_\eta = \mathrm{SVNAAPWA}\left(Y_{11}, Y_{22}, \ldots, Y_{\kappa n}\right),$$
$$R_\eta = \mathrm{SVNAAPWG}\left(Y_{11}, Y_{22}, \ldots, Y_{\kappa n}\right). \tag{54}$$

Step 6: compute the score value of R_η, $\eta = 1,2, \ldots, \kappa$, under consideration of SVNs and rearrange all acquired score values of alternative to choose a desirable optimal option.

If the score values of optimal options are the same, the decision maker faces some difficulties in the selection process and can categorize or classify desirable options from a group of options. For this purpose, the accuracy function is

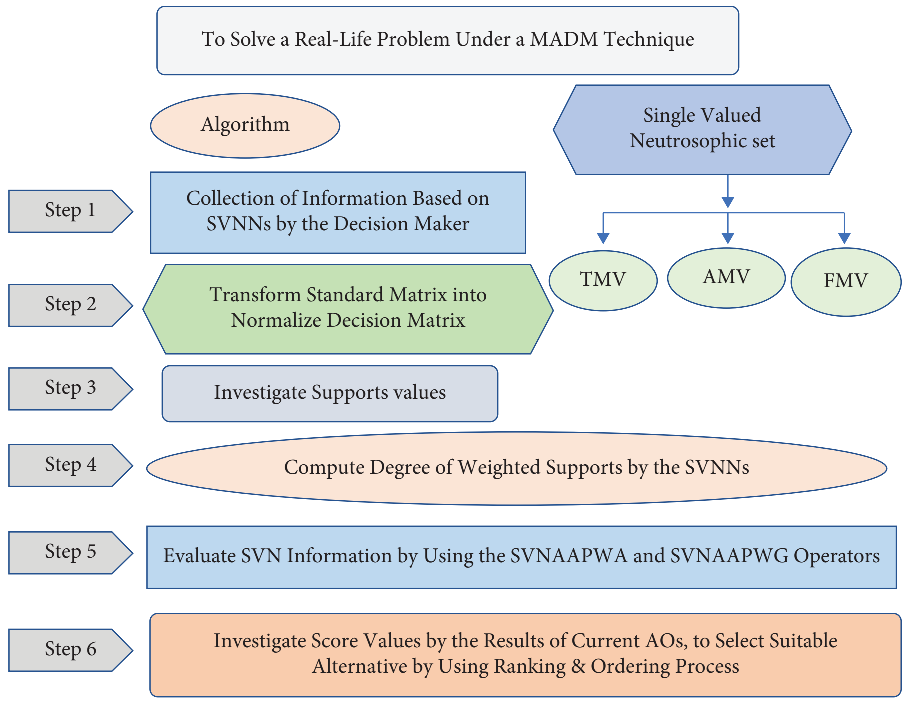

Figure 1: Flowchart of an algorithm.

Table 2: SVN information given by the decision maker.

	$Ɇ_1$	$Ɇ_2$	$Ɇ_3$	$Ɇ_4$	$Ɇ_5$
$Б_1$	(0.67, 0.85, 0.95)	(0.52, 0.48, 0.76)	(0.75, 0.59, 0.83)	(0.28, 0.69, 0.84)	(0.96, 0.81, 0.99)
$Б_2$	(0.97, 0.49, 0.83)	(0.57, 0.83, 0.79)	(0.81, 0.92, 0.98)	(0.47, 0.68, 0.89)	(0.95, 0.86, 0.93)
$Б_3$	(0.59, 0.64, 0.78)	(0.76, 0.84, 0.86)	(0.79, 0.94, 0.28)	(0.56, 0.73, 0.85)	(0.84, 0.76, 0.49)
$Б_4$	(0.58, 0.76, 0.56)	(0.65, 0.76, 0.92)	(0.47, 0.82, 0.38)	(0.58, 0.75, 0.91)	(0.86, 0.75, 0.72)

defined in equation (14). Figure 1 also covers the steps of an algorithm of the MADM problem.

6.1. Application. The construction sector is one of the modern agencies and organizations that has a big impact on the economy of any nation. All of the real estate and infrastructure surrounding us were constructed by various parts of the construction economy. The building sector is an essential kind that significantly affects a nation's economic growth. The construction industry, which is driven by investment, is very important to the government. The government enters into contracts with the building trade to construct infrastructure for the health, transportation, and educational sectors. The prosperity of each nation depends on the construction industry.

6.2. Numerical Example. In this numerical example, we investigate suitable construction materials for the construction process of a building. There are several construction materials but we have selected suitable cement for the construction process of our building. Consider five

Table 3: The consequences of our proposed methodologies.

SVNAAPWA	SVNAAPWG
(0.8348, 0.6384, 0.7870)	(0.7125, 0.6841, 0.8342)
(0.6422, 0.7349, 0.8238)	(0.6223, 0.7786, 0.8363)
(0.7566, 0.8293, 0.5623)	(0.7265, 0.8829, 0.8450)
(0.4855, 0.7076, 0.8707)	(0.4604, 0.7095, 0.8739)
(0.9209, 0.8013, 0.7503)	(0.9039, 0.8082, 0.8962)

different types of cement $Ɇ = (Ɇ_1, Ɇ_2, Ɇ_3, Ɇ_4, Ɇ_5)$; panel select suitable cement based on given characteristics (attributes) $Б = (Б_1, Б_2, Б_3, Б_4)$. Here, $Б_1$ represents the color of cement, $Б_2$ represents the materials, $Б_3$ represents the quality of the cement, and $Б_4$ represents the availability of cement in the market.

To evaluate and investigate the suitable cement decision maker provided a set of weight vectors $\mathfrak{Y} = (0.20,0.35, 0.30,0.15)$ for the selection of the best cement, we follow the following steps of the Algorithm 1.

After ranking and ordering of score values, we observed that $Ɇ_1 \succ Ɇ_5 \succ Ɇ_3 \succ Ɇ_4 \succ Ɇ_2$ and $Ɇ_5 \succ Ɇ_1 \succ Ɇ_2 \succ Ɇ_3 \succ Ɇ_4$ for SVNAAPWA and SVNAAPWG operators. We have seen $Ɇ_1$ is the best objective (cement), by both AOs of the

TABLE 4: Score values by the SVNAAPWA and SVNAAPWG operators.

AOs	$\mathbf{R}(Ɇ_1)$	$\mathbf{R}(Ɇ_2)$	$\mathbf{R}(Ɇ_3)$	$\mathbf{R}(Ɇ_4)$	$\mathbf{R}(Ɇ_5)$	Ranking and ordering
SVNAAPWA	0.4698	0.3612	0.4550	0.3024	0.4564	$Ɇ_1 \succ Ɇ_5 \succ Ɇ_3 \succ Ɇ_4 \succ Ɇ_2$
SVNAAPWG	0.3981	0.3358	0.3328	0.2923	0.3998	$Ɇ_5 \succ Ɇ_1 \succ Ɇ_2 \succ Ɇ_3 \succ Ɇ_4$

Step 1: the decision maker collects the information under the system of SVNNs as depicted in Table 2.
Step 2: there is no need to perform the transformation of the decision matrix into a normalized matrix because there is no involved cost factor in the set of attributes.
Step 3: investigate support by given information using $Supp(Y_{\eta\jmath}, Y_{\eta\tilde{\tilde{e}}}) = 1 - d(Y_{\eta\jmath}, Y_{\eta\tilde{\tilde{e}}})$ where $\eta = 1,2,\ldots,\kappa$, $\jmath, \tilde{\tilde{e}} = 1,2,\ldots,n$.
Step 4: compute weighted supports $\mathfrak{A}(Y_{\eta\jmath})$, $\eta = 1,2,\ldots,\kappa$, $\jmath, \tilde{\tilde{e}} = 1,2,\ldots,n$ based on weight vectors $(0.20, 0.35, 0.30, 0.15)$ of criteria associated with the alternatives.
And also investigate weights $\mathfrak{Y}_{\eta\jmath}$ of $Y_{\eta\jmath}$, $\eta = 1,2,\ldots,\kappa$, $\jmath, \tilde{\tilde{e}} = 1,2,\ldots,n$ associated with the SVNNs, acquired results shown in the following matrix.

$$\mathfrak{Y} = \begin{pmatrix} 0.2075 & 0.3319 & 0.3017 & 0.1589 \\ 0.2027 & 0.3395 & 0.2963 & 0.1615 \\ 0.2073 & 0.3385 & 0.2972 & 0.1570 \\ 0.2057 & 0.3367 & 0.2970 & 0.1607 \\ 0.2100 & 0.3370 & 0.2907 & 0.1622 \end{pmatrix}$$

Step 5: We apply our proposed methodologies of SVNAAPWA and SVNAAPWG operators for the parametric value of $\mathbb{N} = 1$. Table 3 covers the results of the SVNAAPWA and SVNAAPWG operators.

$$\text{SVNAAPWA}(Y_{11}, Y_{22}, \ldots, Y_{\kappa n}) = \begin{pmatrix} 1 - e^{-\left(\sum_{\jmath=1}^{n} (\mathfrak{Y}_{\jmath})(-\mathrm{Ln}(1-\varphi_{\alpha_{\eta\tilde{\tilde{e}}}}))^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ e^{-\left(\sum_{\jmath=1}^{n} (\mathfrak{Y}_{\jmath})(-\mathrm{Ln}(\xi_{\alpha_{\eta\tilde{\tilde{e}}}}))^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ e^{-\left(\sum_{\jmath=1}^{n} (\mathfrak{Y}_{\jmath})(-\mathrm{Ln}(\psi_{\alpha_{\eta\tilde{\tilde{e}}}}))^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \end{pmatrix}$$

$$\text{And, SVNAAPWG}(Y_{11}, Y_{22}, \ldots, Y_{\kappa n}) = \begin{pmatrix} e^{-\left(\sum_{\jmath=1}^{n} (\mathfrak{Y}_{\jmath})(-\mathrm{Ln}(\varphi_{\alpha_{\eta\tilde{\tilde{e}}}}))^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ 1 - e^{-\left(\sum_{\jmath=1}^{n} (\mathfrak{Y}_{\jmath})(-\mathrm{Ln}(1-\xi_{\alpha_{\eta\tilde{\tilde{e}}}}))^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \\ 1 - e^{-\left(\sum_{\jmath=1}^{n} (\mathfrak{Y}_{\jmath})(-\mathrm{Ln}(1-\psi_{\alpha_{\eta\tilde{\tilde{e}}}}))^{\mathbb{N}}\right)^{(1/\mathbb{N})}} \end{pmatrix}$$

It is worth noticing that the results obtained in Table 3 are well in accordance with the Definition #.
Step 6: investigate score values by using the consequences of the SVNAAPWA and SVNAAPWG operators which are shown in Table 3. Obtained results of the score values are shown in Table 4.

ALGORITHM 1: Procedure for decision-making process.

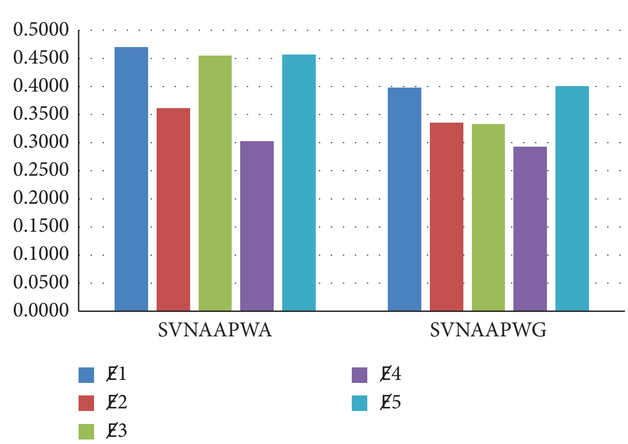

FIGURE 2: Graphical representation of the score values depicted in Table 4.

Table 5: The consequences of the SVNAAPWA operator for the variation of ℕ.

	$\mathbf{R}(Ɇ_1)$	$\mathbf{R}(Ɇ_2)$	$\mathbf{R}(Ɇ_3)$	$\mathbf{R}(Ɇ_4)$	$\mathbf{R}(Ɇ_5)$	Ranking and ordering
$\mathbb{N} = \mathbf{1}$	0.4698	0.3612	0.4550	0.3024	0.4564	$Ɇ_1 \succ Ɇ_5 \succ Ɇ_3 \succ Ɇ_4 \succ Ɇ_2$
$\mathbb{N} = \mathbf{10}$	0.6039	0.4660	0.6100	0.3302	0.5474	$Ɇ_3 \succ Ɇ_1 \succ Ɇ_5 \succ Ɇ_2 \succ Ɇ_4$
$\mathbb{N} = \mathbf{25}$	0.6254	0.4894	0.6306	0.3407	0.5613	$Ɇ_3 \succ Ɇ_2 \succ Ɇ_5 \succ Ɇ_2 \succ Ɇ_4$
$\mathbb{N} = \mathbf{40}$	0.6309	0.4958	0.6363	0.3445	0.5653	$Ɇ_3 \succ Ɇ_1 \succ Ɇ_5 \succ Ɇ_2 \succ Ɇ_4$
$\mathbb{N} = \mathbf{65}$	0.6344	0.5000	0.6402	0.3475	0.5682	$Ɇ_3 \succ Ɇ_1 \succ Ɇ_5 \succ Ɇ_2 \succ Ɇ_4$
$\mathbb{N} = \mathbf{80}$	0.6354	0.5012	0.6414	0.3485	0.5691	$Ɇ_3 \succ Ɇ_1 \succ Ɇ_5 \succ Ɇ_2 \succ Ɇ_4$
$\mathbb{N} = \mathbf{100}$	0.6363	0.5023	0.6435	0.3494	0.5699	$Ɇ_3 \succ Ɇ_1 \succ Ɇ_5 \succ Ɇ_2 \succ Ɇ_4$
$\mathbb{N} = \mathbf{135}$	0.6373	0.5034	0.6439	0.3504	0.5708	$Ɇ_3 \succ Ɇ_1 \succ Ɇ_5 \succ Ɇ_2 \succ Ɇ_4$
$\mathbb{N} = \mathbf{150}$	0.6376	0.5038	0.6439	0.3507	0.5711	$Ɇ_3 \succ Ɇ_1 \succ Ɇ_5 \succ Ɇ_2 \succ Ɇ_4$
$\mathbb{N} = \mathbf{200}$	0.6382	0.5045	0.6446	0.3514	0.5716	$Ɇ_3 \succ Ɇ_1 \succ Ɇ_5 \succ Ɇ_2 \succ Ɇ_4$

Table 6: The consequences of the SVNAAPWG operator for the variation of ℕ.

	$\mathbf{R}(Ɇ_1)$	$\mathbf{R}(Ɇ_2)$	$\mathbf{R}(Ɇ_3)$	$\mathbf{R}(Ɇ_4)$	$\mathbf{R}(Ɇ_5)$	Ranking and ordering
$\mathbb{N} = \mathbf{1}$	0.3981	0.3358	0.3328	0.2923	0.3998	$Ɇ_5 \succ Ɇ_1 \succ Ɇ_2 \succ Ɇ_3 \succ Ɇ_4$
$\mathbb{N} = \mathbf{10}$	0.2942	0.2860	0.2133	0.2443	0.3473	$Ɇ_5 \succ Ɇ_1 \succ Ɇ_2 \succ Ɇ_4 \succ Ɇ_3$
$\mathbb{N} = \mathbf{25}$	0.2746	0.2687	0.1959	0.2258	0.3374	$Ɇ_5 \succ Ɇ_1 \succ Ɇ_2 \succ Ɇ_4 \succ Ɇ_3$
$\mathbb{N} = \mathbf{40}$	0.2695	0.2633	0.1912	0.2195	0.3347	$Ɇ_5 \succ Ɇ_1 \succ Ɇ_2 \succ Ɇ_4 \succ Ɇ_3$
$\mathbb{N} = \mathbf{65}$	0.2662	0.2597	0.1882	0.2148	0.3329	$Ɇ_5 \succ Ɇ_1 \succ Ɇ_2 \succ Ɇ_4 \succ Ɇ_3$
$\mathbb{N} = \mathbf{80}$	0.2651	0.2586	0.1873	0.2133	0.3323	$Ɇ_5 \succ Ɇ_1 \succ Ɇ_2 \succ Ɇ_4 \succ Ɇ_3$
$\mathbb{N} = \mathbf{100}$	0.2641	0.2575	0.1865	0.2120	0.3318	$Ɇ_5 \succ Ɇ_1 \succ Ɇ_2 \succ Ɇ_4 \succ Ɇ_3$
$\mathbb{N} = \mathbf{135}$	0.2631	0.2565	0.1857	0.2106	0.3314	$Ɇ_5 \succ Ɇ_1 \succ Ɇ_2 \succ Ɇ_4 \succ Ɇ_3$
$\mathbb{N} = \mathbf{150}$	0.2628	0.2562	0.1854	0.2102	0.3312	$Ɇ_5 \succ Ɇ_1 \succ Ɇ_2 \succ Ɇ_4 \succ Ɇ_3$
$\mathbb{N} = \mathbf{200}$	0.2621	0.2555	0.1849	0.2093	0.3309	$Ɇ_5 \succ Ɇ_1 \succ Ɇ_2 \succ Ɇ_4 \succ Ɇ_3$

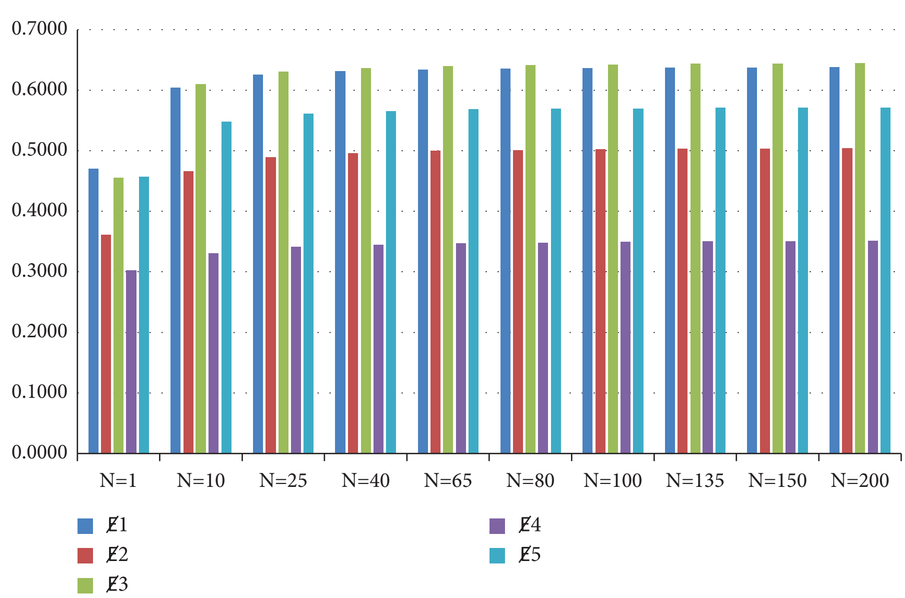

Figure 3: Graphical representation of the score values depicted in Table 5.

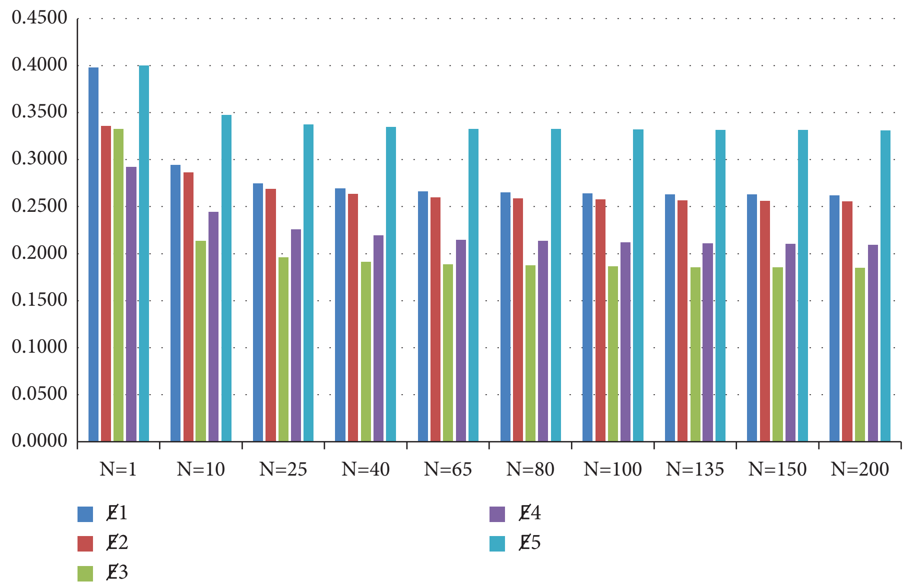

Figure 4: Graphical representation of the score values depicted in Table 6.

Table 7: The results of the comparative analysis.

AOs	Environment	Ranking
SVNAAPWA (current work)	SVNNs	$Ɇ_1 \succ Ɇ_5 \succ Ɇ_3 \succ Ɇ_4 \succ Ɇ_2$
SVNAAPWG (current work)	SVNNs	$Ɇ_5 \succ Ɇ_1 \succ Ɇ_2 \succ Ɇ_3 \succ Ɇ_4$
SVNWA [62]	SVNNs	$Ɇ_1 \succ Ɇ_5 \succ Ɇ_3 \succ Ɇ_2 \succ Ɇ_4$
SVNWG [62]	SVNNs	$Ɇ_1 \succ Ɇ_5 \succ Ɇ_2 \succ Ɇ_3 \succ Ɇ_4$
SVNDWA [61]	SVNNs	$Ɇ_1 \succ Ɇ_3 \succ Ɇ_5 \succ Ɇ_2 \succ Ɇ_4$
SVNDWG [61]	SVNNs	$Ɇ_1 \succ Ɇ_5 \succ Ɇ_2 \succ Ɇ_3 \succ Ɇ_4$
SVNEWA [63]	SVNNs	$Ɇ_1 \succ Ɇ_5 \succ Ɇ_3 \succ Ɇ_2 \succ Ɇ_4$
SVNEWG [63]	SVNNs	$Ɇ_1 \succ Ɇ_5 \succ Ɇ_3 \succ Ɇ_2 \succ Ɇ_4$
SVNAAWA [59]	SVNNs	$Ɇ_1 \succ Ɇ_5 \succ Ɇ_3 \succ Ɇ_2 \succ Ɇ_4$
SVNAAWA [59]	SVNNs	$Ɇ_1 \succ Ɇ_5 \succ Ɇ_2 \succ Ɇ_3 \succ Ɇ_4$
Aiwu et al. [64]	IVSVNNs	Failed
Zhou et al. [65]	IVSVNNs	Failed
Mahmood and Ali [66]	CSVNNs	Failed

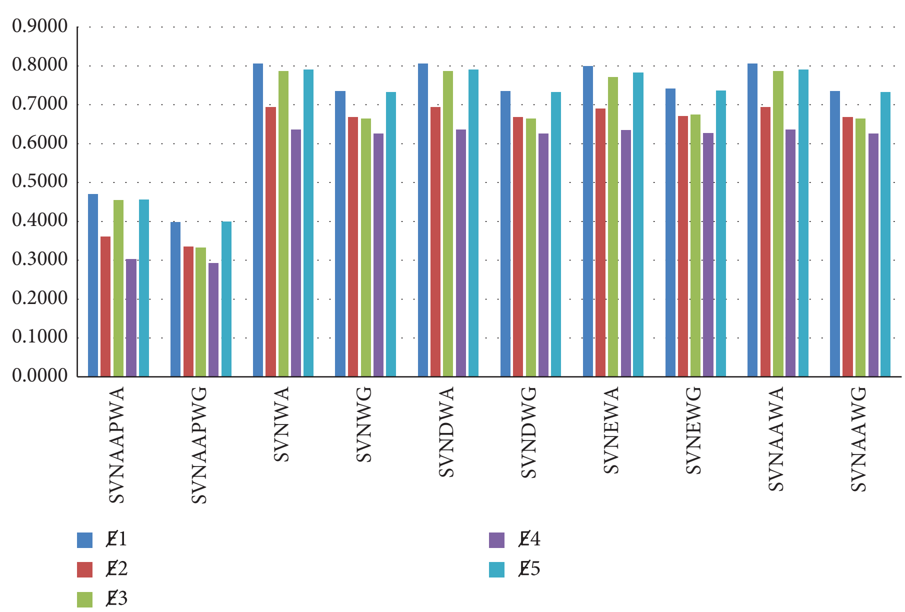

Figure 5: The results of the comparative study.

Table 8: Abbreviations and their meanings.

Abbreviations	Meanings
AOs	Aggregation operators
PyFS	Pythagorean fuzzy set
NS	neutrosophic set
A-TNM	Aczel–Alsina t-norm
A-TCNM	Aczel–Alsina t-conorm
SVNWA	single-valued neutrosophic weighted average
SVNWG	single-valued neutrosophic weighted geometric
SVNDWA	single-valued neutrosophic Dombi weighted average
SVNDWG	single-valued neutrosophic Dombi weighted geometric
SVNEWA	single-valued neutrosophic Einstein weighted average
SVNEWG	single-valued neutrosophic Einstein weighted geometric
SVNAAWA	single-valued neutrosophic Aczel–Alsina weighted average
SVNAAWG	single-valued neutrosophic Aczel–Alsina weighted geometric
SVNAAPWA	single-valued neutrosophic Aczel–Alsina power-weighted averaging
MADM	Multi-attribute decision making
SVNAAPWG	single-valued neutrosophic Aczel–Alsina power weighted geometric
TV	Truth value
AV	Abstinence value
FV	Falsity value
SVNS	single-valued neutrosophic set
SVN	single-valued neutrosophic
SVNN	single-valued neutrosophic number
IVIFS	Interval-valued intuitionistic fuzzy set
IVPyFS	Interval-valued pythagorean fuzzy set
FS	Fuzzy set
IFS	Intuitionistic fuzzy set
PA	Power average
TNM	t-norm
TCNM	t-conorm

SVNAAPWA and SVNAAPWG operators. We also present the score values of the SVNAAPWA and SVNAAPWG operators in the graphical representation of Figure 2.

6.3. Influence of Different Parametric Values of $\mathbb{N}$ *on Our Proposed Methodologies.* To determine the effect of different parametric values $\mathbb{N}$ on the results of our proposed methodologies, we evaluated our discussed AOs for various parametric values of $\mathbb{N}$, and the results of SVNAAPWA and SVNAAPWG operators are shown in Tables 5 and 6, respectively. We observed that when parametric values of $\mathbb{N}$ increase, the results of our proposed methodologies gradually increase and the ranking of score values remain unchanged. This obtained technique of the ranking of score values provides isotonicity property, enabling decision-makers to select the ideal value following their preferences.

We also studied the results of score values of SVNAAPWA and SVNAAPWG operators in a graphical representation in Figures 3 and 4, respectively.

7. Comparative Study

In this section, we compared the results of invented works with the results of existing AOs. We applied some existing AOs on a decision matrix given by the decision maker. To find stability and feasibility of our invented works, we applied different existing AOs such as SVN Dombi weighted AOs given by Chen and Ye [61], SVN weighted average and geometric AOs given by Peng et al. [62], AOs of SVNNs given by [59], SVN Einstein weighted AOs given by Ye et al. [63], AOs of interval-valued SVNSs (IVSVNNs) given by Aiwu et al. [64], AOs of IVSVNNs based on frank operations developed by the Zhou et al. [65], and the theory of SVNNs in the framework of complex SVNNs (CSVNNs) by utilizing the idea of prioritization methods explored by Mahmood and Ali [66]. The results of applied existing AOs [59, 61–66] are shown in Table 7.

We observe that existing AOs discussed in [49–51] failed to aggregate this information. The graphical representation of the results of the comparative study is shown in Figure 5, which are depicted in Table 7.

8. Conclusion

The MADM technique is a convenient mathematical process that is used in every field of life, such as computational and environmental science, green supplier enterprises, construction development, game theory, computer programming, and social decision-making. In our daily life, a MADM technique plays a vital role in performing a selection process for suitable objects. AOs are the useful and effective components of a decision-making or MADM technique. In this article, we expressed the theoretic concepts of SVNNs, having a large capacity to handle vague and imprecision information under different fuzzy environments. Another useful and well-known theoretic concept provides support for different input arguments. There are several types of algebraic sum and algebraic products generated by the numerous triangular norms, but Aczel–Alsina aggregation expressions provide an effective and flexible smooth approximation. Recently, numerous research scholars utilized theoretic concepts of Aczel–Alsina aggregation expressions for new creation and evaluated some appropriate real-life applications. By inspiring the effectiveness and reliability of Aczel–Alsina aggregation tools, we derived some new approaches, including SVNAAPWA and SVNAAPWG operators. Some appropriate properties of our derived approaches are also characterized. To solve a MADM technique, we presented an application based on the selection process under consideration of construction enterprises. To check the reliability and validity of our derived approaches, we gave a numerical example to evaluate a suitable construction material based on SVN information. We made a comprehensive comparative study to compare the results of the existing AOs with our proposed methodologies.

Future studies can further expand on the existing model by breaking down the true value into smaller "subtruths," abstinence into smaller "subabstinences," and falsity values into smaller "subfalsehoods." Moreover, we will extend our proposed methodologies in the framework of PF Maclaurin symmetric mean [67] and bipolar soft sets [68]. Moreover, we also explored our invented approaches in the form of a complex bipolar fuzzy soft set [69] and a T-spherical fuzzy set [70]. Furthermore, Table 8 covers all abbreviations used in this article.

Data Availability

No underlying data were collected or produced in this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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