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Research Article

Confidence Levels Measurement of Mobile Phone Selection Using a Multiattribute Decision-Making Approach with Unknown Attribute Weight Information Based on T-Spherical Fuzzy Aggregation Operators

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Advancement in mobile phone (MP) technology has revolutionized the lifestyle. In recent years, we observed that MP technology had been involved in almost all aspects of life, such as communication purposes, e-commerce, mobile baking, and social media connectivity. So, it becomes a hot research topic to select the best MP that fulfills the desired feathers requirement. In this paper, the expert's familiarity with the examined objects is factored into the initial judgments under the T-spherical fuzzy sets (T-SFSs) environment. The T-SFS is the extension of the picture fuzzy (PF) set (PFS), which gives wider scope for finding the most precise options than existing fuzzy frameworks. The multiattribute decision-making (MADM) is a common and valuable method for aggregating information. For MADM, various aggregation operators (AOs) have been created over the years. The article introduces the newly proposed approach T-spherical fuzzy (T-SF) confidence level weighted averaging $T-\mathrm{SFWA}_c$ and T-SF confidence level weighted geometric $T-\mathrm{SFWG}_c$. Also, some desired properties of AOs are discussed, and the T-SF entropy measure is introduced for selecting the weight criteria. A MADM framework is introduced, on the behalf of proposed operators. The proposed MADM framework is applied to solve the real-life example of consumers' preferences to show effectiveness and practicality. Lastly, the developed framework is set side by side with other prevailing approaches to demonstrate the superiority and significance of other existing AOs.

1. Introduction

The MADM approach is a highly effective tool for the aggregation of information. It is used worldwide widely for the selection of the most appropriate option from the list of options on the bases of some specific criteria. However, it is the most difficult problem to select the best option precisely from the complex surrounding. Due to the complex situations of real-world issues Zadeh [1], initially the thought of the fuzzy set (FS) is introduced. Initially, FS, at most, describes the membership degree (MD), but FS is not suitable for complex problems. For the completion of this gap, Atanassov [2] gave thought to the intuitionistic fuzzy set (IFS) which covers both aspects of the problem, acceptance, and rejection, such as MD and nonmembership degree (NMD). Due to this, property idea of IFS got more popularity in the MADM approach for handling complex real-life situations. In few cases, the summation of MD and NMD has exceeded by [0, 1], and conquering these restrictions Yager [3], the thought of the Pythagorean FS (PyFS) framework is introduced which give more compatible results than FS and IFS. Yager [4] introduces the term q-rung orthopair FS (q-ROFS), which can be formed by taking q^{th} power of NMD and MD. Cuong extended the thought of FS and IFS by adding the abstinent degree (AD) and introducing the picture FS (PFS), which provides more freedom for decision-making (DM). However, there is still some limitation in the PFS framework, and to cover this limitation, Mahmood et al. [5] provide the awareness of spherical FS (SFS), which can be obtained by the sum of MD, NMD, and AD. In some situations, the idea of SFS fails to deal with a complex problem. For this, Mahmood et al. [5] generalize the concept of SFS and introduce the new concept of T-SFS which is formed by the "t" power of MD, AD, and NMD. T-SFS environment provides more comfortable zone for DM issues.

AOs are important in the aggregation of confusing and ambiguous information. For this, several AOs have been introduced by several authors; in the IFS environment, Xu and Yager [6] offered the thought of weighted averaging (WA) and weighted geometric (WG) AOs, and Xu [7] examined the resemblance measure issue in the IFS framework. Pei and Zheng [8] studied the novel approach in MADM in the environment of IFS; Joshi [9] studied the idea of generalized PyFS AOs by using the defined parameters of PyFS theory, and Garg [10] studied the concept of confidence Pythagorean fuzzy WA and confidence Pythagorean fuzzy ordered WA operators along with some vital axioms. Hamacher interactive WA operators discussed by Shahzadi et al. [11] and Dombi AOs based on q-ROFS for MADM were presented by [12]. The solution of MADM problem through SVTNH technique was given by Jana et al. [13]. Peng and Yang [14] studied the PyFS Bonferroni AOs in the PyFS framework. Applications of TSFS for clustering analysis were examined by Ozlu and Karaaslan [15]. The thought of Hesitant TSF Dombi operators for group decision was made by Karaaslan et al. [16]. Mahnaz et al. [17] provided the concept of TSFS for Frank AOs, and a novel approach based on TSFS was examined by Khan et al. [18], and TSFS soft set AOs was studied [19] by Guleria and Bajaj.

For complex situations, Mahmood et al. found a new terminology called T-SFS; this is the generality of SFS. On the other hand, the domains of FS, IFS, PyFS, PFS, and q-rung orthopair FS (q-ROFS) are narrow and ineffective to specify the complex situation, while T-SFS has a much larger domain to express the complexity of real-life issues due to their structure. Apart from this significance and superiority under the T-SFS environment, all other existing efforts are unable to describe the familiarity degree (confidence levels (CLs)) in the information fusion stage. The presentation of the alternatives in a MADM issue is determined solely by the listed characteristics; familiarity of CLs with the assessment objects is not taken into account. So, as a result, in the T-SFS context, it is necessary to incorporate the observer's familiarity with the original material. In this article, we highly focused on such types of flaws by including the expert's CL's familiarity and awareness of the analyzed alternatives. To overcome the influence of uncertain and fuzzy information, we derived the a family of mathematical approaches, we proposed the T-SF confidence level weighted averaging $\mathscr{T}-\mathrm{SFWA}_c$, T-SF confidence level ordered weighted averaging $\mathscr{T}-\mathrm{SFOWA}_c$, T-SF confidence level hybrid weighted averaging $\mathscr{T}-\mathrm{SFHWA}_c$, T-SF confidence level weighted geometric $\mathscr{T}-\mathrm{SFWG}_c$, T-SF confidence level ordered weighted geometric $\mathscr{T}-\mathrm{SFOWG}_c$, and T-SF confidence level hybrid weighted geometric $\mathscr{T}-\mathrm{SFHWG}_c$ operators.

With the rapid growth of the wireless telecommunications and cellular phone industries around the world, mobile selection is a trending topic in the research field. MP is utilized in a variety of ways and patterns all over the world. Nowadays, MP selection is going to be a very complex problem due to innovations and advancements in technology. MP consumers are hunting by confusion to selecting the best phone from the variety of phones. To overcome this type of complex situation, many researchers suggested multiple idea in fuzzy frameworks, such as Buyukozjkan [20] studied the MP selection issue by using the thought of intuitionistic fuzzy framework, Rajak and Shaw [21] examined the best mobile health problem under the fuzzy framework, Singh et al. [22] studied the best MP selection problem in fuzzy environment, Arora et al. [23] provided the idea of how MP plays role in behavioral health and time monitoring of sleep, Sama and Kalvakolana [24] examined the effect of MP addiction on human health, Khaw et al. [25] studied the challenging problem and how much we trust on mobile commerce (bank transaction), in fuzzy framework, Saqlain et al. [26] proposed the idea of selection of smart phone, Kuo and Cheng [27] investigated the MP value-added services, Isiklar and Buyukozkan [28] discussed MADM approach to evaluate the MP alternatives, Ling et al. [29] conducted the survey what type of designs people mostly like in MP, Sharma et al. [30] studied deeply benefits of mobile banking system by utilizing fuzzy approach, and Donner and Tellez [31] investigated that facility of online banking utilizing the best MP option.

In addition to this unexpected success in the T-SFSs system, existing WA and WG operators do not include a degree of familiarity with the confusing information. So, without discussing familiarities, such as CLs in WA and WG, operators are unable to provide exact information for ambiguous and awkward data. The experts in the MADM issues give the evaluation on the bases of only mentioned criteria, and familiarity such as CLs of the decision makers with the performance of the object is not included. So, this gap motivates us to introduce the idea of CLs of the decision makers along with the mentioned criteria of the object's performance.

This article is consisting of several parts as succeed. Section 2 contains definitions and the fundamental terminologies that are helpful to understanding this article. Proposed AOs under T-SFS confidence level WA and WG operators are discussed in Section 3. Complete details of the MADM algorithm are provided in Section 4. A numerical illustration is discussed, for highlighting the significance of our proposed AOs in Section 5. To determine the superiority of the projected framework, the comparative analysis is discussed in Section 6. Lastly, a concrete opinion is summarized in Section 7.

2. Preliminaries

The main aim of this section briefly explains the fundamental definitions of T-SFS and their operational laws that will help to understand this article.

2.1. T-Spherical Fuzzy Set and Their Operational Laws

Definition 1 (see [5]). A T-SFS is present with a membership degree (m), abstinence degree (i), and nonmembership degree (n) restricted with the limitation $0 \leq m^t + i^t + n^t \geq 1 \forall t \in Z^+$, and t is always $t \geq 1$. The refusal term can be defined as $r = \sqrt[t]{1-(m^t + i^t + n^t)}$. For appropriateness, the triplet (m, i, n) is stated as T-spherical fuzzy value (T-SFV).

Remark 2

(i) A T-SFS turns into SFS if we take $t = 2$ by Mahmood et al. [5]

(ii) A T-SFS turns into PFS if we take $t = 1$ by Cong [32]

(iii) T-SFS turns into q-ROPFS if we consider $i = 0$ by Yager [4]

(iv) T-SFS turns into PyFS if we consider $i = 0$ and $t = 2$ by Yager [3]

(v) T-SFS turns into IFS if we consider $i = 0$ and $t = 1$ by Atanassov [2]

(vi) T-SFS turns into FS if we consider $i = 0, n = 0$, and $t = 1$ by Zadeh [1]

The above remark shows, distinctly, the benefits of T-SFS and then the other present extended ideas of FS, such as SFS, PFS, q-ROPFS, PyFS, and IFS. A T-SFS explains the following four terms: MD, AD, NMD, and refusal degree. Therefore, it can explain all kinds of human opinions in a competent way rather than other exciting ideas of FS. For example, the voting process is based on four human opinions, such as vote in favor, vote in against, abstain, and refused to vote. In such types of difficulties, the thought of T-SFS gives more accurate results than other fuzzy environments. Furthermore, in T-SFS, the parameter t gives freedom to the DM to choose the value t from the unit interval. Also, there is no limitation for the value t. Therefore, the thought of the T-SFS is preferable to the other existing fuzzy frameworks.

Now, we discussed the CLs of experts under the T-SFS framework. In general, no current efforts in the fusion of T-SFS data incorporate expert CL for their familiarity and awareness of the analyzed choices. As a result, we present T-SFS with weighted averaging and weighted geometric AOs that incorporate expert CL with the assessed possibilities.

Definition 3 (see [5]). Let $\eta_i = (m_i^t, i_i^t, n_i^t)$ be the T-SFVs. Then, score functions (SFs) can be defined as follows:

$$\mathfrak{R}(\eta) = m_i^t - i_i^t - n_i^t, \tag{1}$$

with range $[-1, 1]$ and also accuracy function can be defined as follows:

$$\mathcal{S}(\eta_i) = m_i^t + i_i^t + n_i^t, \tag{2}$$

with range [0, 1]. On the bases of the above definition, we define rules for TSFNs η_1 and η_2 as follows:

(i) If $\mathfrak{R}(\eta_1) > \mathfrak{R}(\eta_2)$ which can be signified as $\eta_1 > \eta_2$

(ii) If $\mathfrak{R}(\eta_1) < \mathfrak{R}(\eta_2)$ which can be signified as $\eta_1 < \eta_2$

If $\mathfrak{R}(\eta_1) = \mathfrak{R}(\eta_2)$, then $\eta_1 = \eta_2$ are represented by the same information, the accuracy function, and two numbers η_1 and η_2 in the form of TSFNs which can be defined as follows:

(i) If $\mathcal{S}(\eta_1) > \mathcal{S}(\eta_2)$, which can be represented as $\eta_1 > \eta_2$

(ii) If $\mathcal{S}(\eta_1) < \mathcal{S}(\eta_2)$, which can be denoted as $\eta_1 < \eta_2$

(iii) If $\mathcal{S}(\eta_1) = \mathcal{S}(\eta_2)$, then $\eta_1 = \eta_2$ are represented with the same information

Let any three T-SFSNs $\eta = (m^t, i^t, n^t), \eta_1 = (m_1^t, i_1^t, n_1^t)$ and $\eta_2 = (m_2^t, i_2^t, n_2^t)$, then the following operational laws can be defined as:

(1) $\overline{\eta} = (m^t, i^t, n^t)$

(2) $\eta_1 \vee \eta_2 = [\max\{m_1^t, m_2^t\}, \min\{i_1, i_2\}, \min\{n_1, n_2\}]$

(3) $\eta_1 \wedge \eta_2 = [\min\{m_1^t, m_2^t\}, \max\{i_1, i_2\}, \max\{n_1, n_2\}]$

(4) $\eta_1 \oplus \eta_2 = [(m_1^t + m_2^t - m_1^t m_2^t)^{1/t}, i_1 i_2, n_1 n_2]$

(5) $\eta_1 \otimes \eta_2 = [m_1 m_2, (i_1^t + i_2^t - i_1^t i_2^t)^{1/t}, (n_1^t + n_2^t - n_1^t n_2^t)^{1/t}]$

(6) $\gamma\eta = ((1-(1-m^t)^\gamma)^{1/t}, i^t, n^t). \forall \gamma > 0$

(7) $\eta^\gamma = ((m^t, (1-(1-i^t)^\gamma)^{1/t}, (1-(1-n^t)^\gamma)^{1/t})). \forall \gamma > 0$

These operations are proposed by Liu and Wang [33], and their proves are applicable $\forall \gamma, \gamma_1, \gamma_2 > 0$:

(1) $\eta_1 \oplus \eta_2 = \eta_2 \oplus \eta_1$

(2) $\eta_1 \otimes \eta_2 = \eta_2 \otimes \eta_1$

(3) $\gamma(\eta_1 \oplus \eta_2) = \gamma\eta_1 \oplus \gamma\eta_2$

(4) $(\eta_1 \otimes \eta_2)^\gamma = \eta_1^\gamma \otimes \eta_2^\gamma$

(5) $\gamma_1\eta + \gamma_2\eta = (\gamma_1 + \gamma_2)\eta$

(6) $\eta^{\gamma_1} \otimes \eta^{\gamma_2} = \eta^{\gamma_1+\gamma_2}$

On the bases of the operational laws, we presented the following T-SFS weighted and averaging operators.

2.2. Entropy Measurement for T-SFS.

In this segment, we present the thought of entropy for T-SFS discussed in detail. This entropy measure is a helpful tool for the calculation of the WV.

Definition 4 (see [17]). Let $\eta_1 = (m_1^t, i_1^t, n_1^t)$ and $\eta_2 = (m_2^t, i_2^t, n_2^t)$ be the two sets of T-SFNs on X. Then, a real valued function $\mathcal{E}: T-\text{SFS} \longrightarrow [0, 1]$ can be the entropy for T-SFSs, if it will satisfy the axioms listed as follows:

$\aleph_1$. $\mathcal{E}(\eta_1) = 0 \Longleftrightarrow \eta_1$ is the crisp set

$\aleph_2$. $\mathcal{E}(\eta_1) = 1 \Longleftrightarrow m^t(x) = n^t(x)$ and $i^t(x) = \sqrt[t]{0.25}$ for all $x \in X$

$\aleph_3$. $\mathscr{E}(\eta_1) = \mathscr{E}(\eta_1^c)$

$\aleph_3$. $\mathscr{E}(\eta_1), \leq \mathscr{E}(\eta_2)$ if $i_2^t(x) \leq i_1^t(x)$ and $m_1^t(x) \leq m_2^t(x) \leq n_2^t(x) \leq n_1^t(x)$ or $n_1^t(x) \leq n_2^t(x) \leq m_2^t(x) \leq m_1^t(x)$ for all $x \in X$

Theorem 5. *Let η be the T-SFS on X. The mapping is defined as follows:*

$$\mathscr{E}(\eta) = \sum_{i=1}^{n}\left(1 - \frac{4}{5}\left[\left|m_i^t(x) - n_i^t(x)\right| + \left|i_i^t(x) - 0.25\right|\right]\right), \quad (3)$$

which is called the entropy measure of T-SFS.

3. T-Spherical Fuzzy Averaging Aggregation Operators under Confidence Levels

No existing T-SFS averaging and geometric AOs include the CL of experts for their awareness of the analyzed options. To overcome this situation, we proposed new AOs for the measurement of the CLs of the experts.

3.1. T-Spherical Fuzzy Confidence Level Weighted Averaging Operators. In this section, the AOs and their core axioms are investigated comprehensively.

Definition 6. Let Φ be the set of n T-SFSNs $\eta_i = (m_i^t, i_i^t, n_i^t)$ $\forall\ (i = 1, 2, \ldots, n)$ and CLs are denoted as ℓ_i of η_i with the restriction $0 \leq \ell_i \geq 1$. If the weight vector $\varphi_i = (\varphi_1, \varphi_2, \ldots, \varphi_n)$ with $\varphi_i \leq \in [0, 1]$ and $\sum_{i=1}^{n}\varphi_i = 1$, then the mapping of function $\mathscr{T} - \mathrm{SFWA}_c\colon \Phi^n \longrightarrow \Phi$ is defined as follows:

$$\begin{aligned}&\mathscr{T} - \mathrm{SFWA}_c(\langle\eta_1, \ell_1\rangle, \langle\eta_2, \ell_2\rangle, \ldots, \langle\eta_n, \ell_n\rangle)\\ &\quad = \mathop{\oplus}_{i=1}^{n}\varphi_i(\eta_n, \ell_n) = \varphi_1(\eta_1, \ell_1) \oplus \varphi_2(\eta_2, \ell_2) \oplus \ldots \oplus \varphi_n(\eta_n \ell_n).\end{aligned} \quad (4)$$

It is said to be the CL of the $\mathscr{T}$ − SF weighted averaging operator.

Theorem 7. *Let $\eta_i = (m_i^t, i_i^t, n_i^t)$ be the set of T-SFVs with the condition of CLs $0 \leq \ell_i \geq 1$ and weight vectors (WV) $\varphi_i = (\varphi_1, \varphi_2, \ldots, \varphi_n)$, $\forall\ \varphi_i \in [0, 1]$ and $\sum_{i=1}^{n}\varphi_i = 1$, then their aggregated results by using $\mathscr{T}$ − SFWA$_c$ operator gives value in the form of T-SFV, which is given by the following expression:*

$$\begin{aligned}&\mathscr{T} - \mathrm{SFWA}_c(\langle\eta_1, \ell_1\rangle, \langle\eta_2, \ell_2\rangle, \ldots, \langle\eta_n, \ell_n\rangle)\\ &\quad = \left(\left(1 - \prod_{i=1}^{n}\left(1 - m_i^{\mathscr{T}}\right)^{l_i\varphi_i}\right)^{1/\mathscr{T}}, \prod_{i=1}^{n}(i_i)^{l_i\varphi_i}, \prod_{i=1}^{n}(n_i)^{l_i\varphi_i}\right).\end{aligned} \quad (5)$$

Proof. We can prove the theorem through the technique of the mathematical induction on n.

Taking $n = 2$, we have the following expression:

$$\mathscr{T} - \mathrm{SFWA}_c(\langle\eta_1, \ell_1\rangle, \langle\eta_2, \ell_2\rangle) = \varphi_1(\eta_1, \ell_1) \oplus \varphi_2(\eta_2, \ell_2). \quad (6)$$

On the behalf of the operational laws of T-SFVs, we have the following expression:

$$\begin{aligned}\eta_1, \ell_1 &= \left(\left(1 - \left(1 - m_1^{\mathscr{T}}\right)^{l_1}\right)^{1/\mathscr{T}}, (i_1)^{l_1}, (n_1)^{l_1}\right)\\ &\Longrightarrow \varphi_1(\eta_1, \ell_1) = \left(\left(1 - \left(1 - m_1^{\mathscr{T}}\right)^{\varphi_1}\right)^{1/\mathscr{T}}, (i_1)^{\varphi_1}, (n_1)^{\varphi_1}\right)\\ &= \left(\left(\left(\left(1 - \left(1 - \left(1 - m_1^{\mathscr{T}}\right)^{l_1}\right)^{1/\mathscr{T}}\right)^{\mathscr{T}}\right)^{\varphi_1}\right)^{1/\mathscr{T}}, \left((i_1)^{\ell_1}\right)^{\varphi_1}, \left((n_1)^{\ell_1}\right)^{\varphi_1}\right)\\ &= \left(\left(\left(1 - \left(1 - \left(1 - m_1^{\mathscr{T}}\right)^{l_1}\right)\right)^{\varphi_1}\right)^{1/\mathscr{T}}, \left((i_1)^{\ell_1}\right)^{\varphi_1}, \left((n_1)^{\ell_1}\right)^{\varphi_1}\right)\\ &= \left(\left(\left(1 - m_1^{\mathscr{T}}\right)^{l_1\varphi_1}\right)^{1/\mathscr{T}}, (i_1)^{\ell_1\varphi_1}, (n_1)^{\ell_1\varphi_1}\right).\end{aligned} \quad (7)$$

Similarly, we say that $\varphi_2(\eta_2, \ell_2) = (((1 - m_2^{\mathscr{T}})^{\ell_2\varphi_2})^{1/\mathscr{T}}, (i_1)^{\ell_1\varphi_2}, (n_1)^{\ell_2\varphi_2})$.

Then,

$$\begin{aligned}
\mathcal{T}-\mathrm{SFWA}_c(\langle\eta_1,\ell_1\rangle,\langle\eta_2,\ell_2\rangle) &= \varphi_1(\eta_1,\ell_1)\oplus\varphi_2(\eta_2,\ell_2)\\
&=\left(\left\{\left(\left(1-m_1^{\mathcal{T}}\right)^{l_i\varphi_1}\right)^{1/\mathcal{T}}+\left(\left(1-m_2^{\mathcal{T}}\right)^{\ell_2\varphi_2}\right)^{1/\mathcal{T}}\right\}-\left\{\left(\left(1-m_1^{\mathcal{T}}\right)^{l_i\varphi_1}\right)^{1/\mathcal{T}}.\left(\left(1-m_2^{\mathcal{T}}\right)^{\ell_2\varphi_2}\right)\right\},(i_1)^{\ell_1\varphi_1}.(i_2)^{\ell_2\varphi_2},(n_1)^{\ell_1\varphi_1}.(n_2)^{\ell_2\varphi_2}\right),\\
\mathcal{T}-\mathrm{SFWA}_c(\langle\eta_1,\ell_1\rangle,\langle\eta_2,\ell_2\rangle) &= \left(\left(1-\prod_{i=1}^{2}\left(1-\left(\left(1-m_i^{\mathcal{T}}\right)^{l_i\varphi_i}\right)^{1/\mathcal{T}}\right)\right),\prod_{i=1}^{2}\left((i_i)^{\ell_i\varphi_i}\right),\prod_{i=1}^{2}(n_i)^{\ell_i\varphi_i}\right).
\end{aligned} \tag{8}$$

Consider the result is correct for $n=k$, that is,

$$\begin{aligned}
&\mathcal{T}-\mathrm{SFWA}_c(\langle\eta_1,\ell_1\rangle,\langle\eta_2,\ell_2\rangle,\ldots,\langle\eta_k,\ell_k\rangle)\\
&=\left(\left(1-\prod_{i=1}^{k}\left(1-\left(\left(1-m_i^{\mathcal{T}}\right)^{l_i\varphi_1}\right)^{1/\mathcal{T}}\right)\right),\prod_{i=1}^{k}\left((i_i)^{\ell_i\varphi_i}\right),\prod_{i=1}^{k}(n_i)^{\ell_i\varphi_i}\right).
\end{aligned} \tag{9}$$

Now, consider the results are true for $n=k+1$, that is,

$$\begin{aligned}
&\mathcal{T}-\mathrm{SFWA}_c(\langle\eta_1,\ell_1\rangle,\langle\eta_2,\ell_2\rangle,\ldots,\langle\eta_k,\ell_k\rangle,\langle\eta_{k+1},\ell_{k+1}\rangle)\\
&\quad=\mathcal{T}-\mathrm{SFWA}_c(\langle\eta_1,\ell_1\rangle,\langle\eta_2,\ell_2\rangle,\ldots,\langle\eta_k,\ell_k\rangle)\oplus\varphi_{k+1}\langle\eta_{k+1},\ell_{k+1}\rangle\\
&\quad=\left(\left(1-\prod_{i=1}^{k}\left(1-\left(\left(1-m_i^{\mathcal{T}}\right)^{l_i\varphi_i}\right)^{1/\mathcal{T}}\right)\right),\prod_{i=1}^{k}\left((i_i)^{\ell_i\varphi_i}\right),\prod_{i=1}^{k}(n_i)^{\ell_i\varphi_i}\right)\oplus\left(\left(1-\left(1-m_{(k+1)}^{\mathcal{T}}\right)^{l_i}\right)^{1/\mathcal{T}},\left(i_{(k+1)}\right)^{l_i},\left(n_{(k+1)}\right)^{l_i}\right)\\
&\quad=\left(\begin{array}{c}\left\{\left(1-\prod_{i=1}^{k}\left(1-\left(\left(1-m_i^{\mathcal{T}}\right)^{l_i\varphi_1}\right)^{1/\mathcal{T}}\right)\right)+\left(1-\left(1-m_{(k+1)}^{\mathcal{T}}\right)^{l_i}\right)^{1/\mathcal{T}}\right\}\\ -\left\{\left(1-\prod_{i=1}^{k}\left(1-\left(\left(1-m_i^{\mathcal{T}}\right)^{l_i\varphi_i}\right)^{1/\mathcal{T}}\right)\right).\left(1-\left(1-m_{(k+1)}^{\mathcal{T}}\right)^{l_i}\right)^{1/\mathcal{T}}\right\},\prod_{i=1}^{k}\left((i_i)^{\ell_i\varphi_i}\right).\left(i_{(k+1)}\right)^{l_i},\prod_{i=1}^{k}\left(n_{i_n}^{\mathcal{T}}\right)^{\ell_i\varphi_i}.\left(n_{(k+1)}\right)^{l_i}\end{array}\right)\\
&\quad=\left(1-\prod_{i=1}^{k+1}\left(\left(1-m_i^{\mathcal{T}}\right)^{l_i\varphi_i}\right)^{1/\mathcal{T}},\prod_{i=1}^{k+1}\left((i_i)^{\ell_i\varphi_i}\right),\prod_{i=1}^{k+1}(n_i)^{\ell_i\varphi_i}\right).
\end{aligned} \tag{10}$$

The statement is true for $n=k+1$. Hence, the result satisfies all T-SFVs. □

Property 8 (idempotency). Consider for all $\langle \eta_i, \ell_i \rangle = \langle \eta, \ell \rangle$, $(i = 1, 2, \ldots, n)$, that is, $m_i = m, i_i = i, n_i = n$, and for all $l_i = l$,

$$\mathcal{T} - \mathrm{SFWA}_c(\langle \eta_1, \ell_1 \rangle, \langle \eta_2, \ell_2 \rangle, \ldots, \langle \eta_n, \ell_n \rangle) = \eta\ell. \quad (11)$$

Proof. If $\langle \eta_i, \ell_i \rangle = \langle \eta, \ell \rangle$ for all $(i = 1, 2, \ldots, n)$, then by using Theorem 7, we obtain the following expression:

$$\begin{aligned}
\mathcal{T} - \mathrm{SFWA}_c(\langle \eta_1, \ell_1 \rangle, \langle \eta_2, \ell_2 \rangle, \ldots, \langle \eta_n, \ell_n \rangle) &= \left(\left(1 - \prod_{i=1}^{n} \left(1 - \left(\left(1 - m_i^{\mathcal{T}} \right)^{\ell \varphi_i} \right)^{1/\mathcal{T}} \right) \right)^{1/\mathcal{T}}, \prod_{i=1}^{n} \left((i_i)^{\ell \varphi_i} \right), \prod_{i=1}^{n} (n_i)^{\ell \varphi_i} \right) \\
&= \left(\left(1 - \prod_{i=1}^{n} \left(1 - \left(\left(1 - m_i^{\mathcal{T}} \right)^{\ell \Sigma_{i=1}^{n} \varphi_i} \right)^{1/\mathcal{T}} \right) \right)^{1/\mathcal{T}}, \prod_{i=1}^{n} \left((i_i)^{\ell \Sigma_{i=1}^{n} \varphi_i} \right), \prod_{i=1}^{n} (n_i)^{\ell \Sigma_{i=1}^{n} \varphi_i} \right) \\
&= \left(\left(1 - \prod_{i=1}^{n} \left(1 - \left(\left(1 - m_i^{\mathcal{T}} \right)^{\ell} \right)^{1/\mathcal{T}} \right) \right)^{1/\mathcal{T}}, \prod_{i=1}^{n} \left((i_i)^{\ell} \right), \prod_{i=1}^{n} (n_i)^{\ell} \right) = \eta\ell.
\end{aligned} \quad (12)$$

□

Property 9 (boundedness). If $\eta_i^- = ((m_i^t)_{\min}, (i_i^t)_{\max}, (n_i^t)_{\max})$ and $\eta_i^+ = ((m_i^t)_{\max}, (i_i^t)_{\min}, (n_i^t)_{\min})$, then for every weight φ_i, $\eta_i^- \le \mathcal{T} - \mathrm{SFWA}_c(\langle \eta_1, \ell_1 \rangle, \langle \eta_2, \ell_2 \rangle, \ldots, \langle \eta_n, \ell_n \rangle) \le \eta_i^+$.

Proof. For all i, we obtain $\min(m_i^t) \le m_i^t \le \max(m_i^t)$. Furthermore, this implies that $1 - \min(m_i^t) \le 1 - m_i^t \le 1 - \max(m_i^t)$. Then, ∀ weight φ,

$$\begin{aligned}
\prod_{i=1}^{n} \left(1 - \min\left(m_i^t\right) \right)^{(\max \ell_i)\varphi_i} &\le \prod_{i=1}^{n} \left(1 - m_i^t \right)^{\ell_i \varphi_i} \le \prod_{i=1}^{n} \left(1 - \max\left(m_i^t\right) \right)^{(\min \ell_i)\varphi_i} \\
&\Longrightarrow \left(\left(1 - m_i^t \right)^{\max \ell_i \Sigma_{i=1}^{n} \varphi_i} \right) \\
&\le 1 - \left(1 - \min\left(m_i^t\right) \right)^{\ell_i \varphi_i} \le \left(\left(1 - m_i^t \right)^{\min \ell_i \Sigma_{i=1}^{n} \varphi_i} \right) \\
&\Longrightarrow 1 - \left(\left(\left(1 - \min\left(m_i^t\right) \right)^{\min \ell_i} \right) \right) \\
&\le 1 - \prod_{i=1}^{n} \left(1 - m_i^t \right)^{\ell_i \varphi_i} \le 1 - \left(1 - \max\left(m_i^t\right) \right)^{\ell_i} \\
&\Longrightarrow \left(1 - \left(1 - \min\left(m_i^t\right) \right)^{\min \ell_i} \right) \le \left(1 - \prod_{i=1}^{n} \left(1 - m_i^t \right)^{\ell_i \varphi_i} \right)^{1/\mathcal{T}} \\
&\le \left(1 - \left(1 - \max\left(m_i^t\right) \right)^{\max \ell_i} \right) \Longrightarrow \left(m_i^t\right)^{\min} \le \left(1 - \prod_{i=1}^{n} \left(1 - m_i^t \right)^{\ell_i \varphi_i} \right)^{1/\mathcal{T}} \le \left(m_i^t\right)^{\max}.
\end{aligned} \quad (13)$$

Now,

$$\min\left(i_i^t\right) \le i_i^t \le \max\left(i_i^t\right) \Longleftrightarrow \left(\min\left(i_i^t\right)\right)^{\min \ell_i} \le \prod_{i=1}^{n} \left(i_i^t\right)^{\ell_i \varphi_i} \le \left(\max\left(i_i^t\right)\right)^{\max \ell_i} \Longrightarrow \left(i_i^t\right)^{\ell_i \varphi_i} \le \left(i_i^t\right)^{\max}_{\ell_i \varphi_i}. \tag{14}$$

Furthermore,

$$\min\left(n_i^t\right) \le i_i^t \le \max\left(n_i^t\right) \Longleftrightarrow \left(\min\left(n_i^t\right)\right)^{\min \ell_i} \le \prod_{i=1}^{n} \left(n_i^t\right)^{\ell_i \varphi_i} \le \left(\max\left(n_i^t\right)\right)^{\max \ell_i} \Longrightarrow \left(n_i^t\right)^{\ell_i \varphi_i} \le \left(n_i^t\right)^{\max}_{\ell_i \varphi_i}. \tag{15}$$

From the above investigation, if $\mathcal{T} - \text{SFWA}_c\ (\langle \alpha_1, \ell_1 \rangle, \langle \alpha_2, \ell_2 \rangle, \ldots, \langle \alpha_n, \ell_n \rangle) = \alpha = ((m_i^t)_\alpha, (i_i^t)_\alpha, (n_i^t)_\alpha)$, then we have $m^{\min}_{\alpha_i, \ell_i} \le m_\alpha \le m^{\max}_{\alpha_i, \ell_i}$, $i^{\min}_{\alpha_i, \ell_i} \le i_\alpha \le i^{\max}_{\alpha_i, \ell_i}$, and $n^{\min}_{\alpha_i, \ell_i} \le n_\alpha \le n^{\max}_{\alpha_i, \ell_i}$. So, we conclude that from the definition of the SF,

$$\eta^- \le \mathcal{T} - \text{SFWA}_c\left(\langle \eta_1, \ell_1 \rangle, \langle \eta_2, \ell_2 \rangle, \ldots, \langle \eta_n, \ell_n \rangle\right) \le \eta^+. \tag{16}$$

□

Property 10 (monotonicity). Consider $\eta_i^* = (m_{i*}^t, i_{i*}^t, n_{i*}^t)$ be the set of n T-SFVs with the condition $m_i^t \le m_{i*}^t$, $i_i^t \ge i_{i*}^t, n_i^t \ge n_{i*}^t$ of and for every weight vector φ

$$\mathcal{T} - \text{SFWA}_c\left(\langle \eta_1, \ell_1 \rangle, \langle \eta_2, \ell_2 \rangle, \ldots, \langle \eta_n, \ell_n \rangle\right) \le \mathcal{T} - \text{SFWA}_c\left(\langle \eta_1^*, \ell_1 \rangle, \langle \eta_2^*, \ell_2 \rangle, \ldots, \langle \eta_n^*, \ell_n \rangle\right). \tag{17}$$

Proof. Since, for every i, we have $m_{\eta_i}^t \le m_{\eta_i^*}^t, i_{\eta_i}^t \ge i_{\eta_i^*}^t, n_{\eta_i}^t \ge n_{\eta_i^*}^t$, then

$$1 - m_{\eta_i^*}^t \le 1 - m_{\eta_i}^t \Longrightarrow \prod_{i=1}^{n} \left(1 - m_{\eta_i^*}^t\right)^{\ell_i \varphi_i} \le \prod_{i=1}^{n} \left(1 - m_{\eta_i}^t\right)^{\ell_i \varphi_i} \Longrightarrow \left(1 - \prod_{i=1}^{n} \left(1 - m_{\eta_i}^t\right)^{\ell_i \varphi_i}\right)^{1/\mathcal{T}} \le \left(1 - \prod_{i=1}^{n} \left(1 - m_{\eta_i^*}^t\right)^{\ell_i \varphi_i}\right)^{1/\mathcal{T}}. \tag{18}$$

Also, $\prod_{i=1}^n (i_{\eta_i}^t)^{\ell_i \varphi_i} \ge \prod_{i=1}^n (i_{\eta_i^*}^t)^{\ell_i \varphi_i}$ and $\prod_{i=1}^n (n_{\eta_i}^t)^{\ell_i \varphi_i} \ge \prod_{i=1}^n (n_{\eta_i^*}^t)^{\ell_i \varphi_i}$.

Therefore,

$$\Longrightarrow \left(\left(1 - \prod_{i=1}^{n} \left(1 - m_{\eta_i}^t\right)^{\ell_i \varphi_i}\right)^{1/\mathcal{T}}\right)^{\mathcal{T}} - \left(\prod_{i=1}^{n} \left(i_{\eta_i}^t\right)^{\ell_i \varphi_i}\right)^{\mathcal{T}} - \left(\prod_{i=1}^{n} \left(n_{\eta_i}^t\right)^{\ell_i \varphi_i}\right)^{\mathcal{T}} \le \left(\left(1 - \prod_{i=1}^{n} \left(1 - m_{\eta_i^*}^t\right)^{\ell_i \varphi_i}\right)^{1/\mathcal{T}}\right)^{\mathcal{T}} - \left(\prod_{i=1}^{n} \left(i_{\eta_i^*}^t\right)^{\ell_i \varphi_i}\right)^{\mathcal{T}} - \left(\prod_{i=1}^{n} \left(n_{\eta_i^*}^t\right)^{\ell_i \varphi_i}\right)^{\mathcal{T}}. \tag{19}$$

If $\mathcal{T} - \text{SFWA}_c\left(\langle \eta_1, \ell_1 \rangle, \langle \eta_2, \ell_2 \rangle, \ldots, \langle \eta_n, \ell_n \rangle\right) = (m_i^t, i_i^t, n_i^t) = \eta$ and $\mathcal{T} - \text{SFWA}_c\left(\langle \eta_1^*, \ell_1 \rangle, \langle \eta_2^*, \ell_2 \rangle, \ldots, \langle \eta_n^*, \ell_n \rangle\right) = \eta_i^* = (m_{i*}^t, i_{i*}^t, n_{i*}^t) = \eta^*$, then, for $\mathfrak{R}(\eta) \le \mathfrak{R}(\eta^*)$, two following conditions exist,

(1) If $\mathfrak{R}(\eta) < \mathfrak{R}(\eta^*)$, then by using the SF, we obtain the following expression:

$$\mathcal{T} - \text{SFWA}_c\left(\langle \eta_1, \ell_1 \rangle, \langle \eta_2, \ell_2 \rangle, \ldots, \langle \eta_n, \ell_n \rangle\right) < \mathcal{T} - \text{SFWA}_c\left(\langle \eta_1^*, \ell_1 \rangle, \langle \eta_2^*, \ell_2 \rangle, \ldots, \langle \eta_n^*, \ell_n \rangle\right). \tag{20}$$

(2) If $\mathfrak{R}(\eta) = \mathfrak{R}(\eta^*)$, then we obtain the following expression:

$$\left(\left(1 - \prod_{i=1}^{n} \left(1 - m_{\eta_i}^t\right)^{\ell_i \varphi_i}\right)^{1/\mathcal{T}}\right)^{\mathcal{T}} - \left(\prod_{i=1}^{n} \left(i_{\eta_i}^t\right)^{\ell_i \varphi_i}\right)^{\mathcal{T}} - \left(\prod_{i=1}^{n} \left(n_{\eta_i}^t\right)^{\ell_i \varphi_i}\right)^{\mathcal{T}} = \left(\left(1 - \prod_{i=1}^{n} \left(1 - m_{\eta_i^*}^t\right)^{\ell_i \varphi_i}\right)^{1/\mathcal{T}}\right)^{\mathcal{T}} - \left(\prod_{i=1}^{n} \left(i_{\eta_i^*}^t\right)^{\ell_i \varphi_i}\right)^{\mathcal{T}} - \left(\prod_{i=1}^{n} \left(n_{\eta_i^*}^t\right)^{\ell_i \varphi_i}\right)^{\mathcal{T}}. \tag{21}$$

Thus, we have for every $(m^t_{\eta_i})^{\ell_i\varphi_i} \le (m^t_{\eta_i^*})^{\ell_i\varphi_i}$, $(i^t_{\eta_i})^{\ell_i\varphi_i} \ge (i^t_{\eta_i^*})^{\ell_i\varphi_i}$, and $(n^t_{\eta_i})^{\ell_i\varphi_i} \ge (n^t_{\eta_i^*})^{\ell_i\varphi_i}$,

$$\Longrightarrow\left(\left(1-\prod_{i=1}^{n}\left(1-m^t_{\eta_i}\right)^{\ell_i\varphi_i}\right)^{1/\mathcal{T}}\right)^{\mathcal{T}} = \left(\left(1-\prod_{i=1}^{n}\left(1-m^t_{\eta_i^*}\right)^{\ell_i\varphi_i}\right)^{1/\mathcal{T}}\right)^{\mathcal{T}}, \tag{22}$$

and $(\prod_{i=1}^n (i^t_{\eta_i})^{\ell_i\varphi_i})^{\mathcal{T}} = (\prod_{i=1}^n (i^t_{\eta_i^*})^{\ell_i\varphi_i})^{\mathcal{T}}$, $(\prod_{i=1}^n (n^t_{\eta_i})^{\ell_i\varphi_i})^{\mathcal{T}} = (\prod_{i=1}^n (n^t_{\eta_i^*})^{\ell_i\varphi_i})^{\mathcal{T}}$

Now, by using the accuracy function, we have the following expression:

$$\begin{aligned}\aleph(\eta) &= \left(\left(1-\prod_{i=1}^{n}\left(1-m^t_{\eta_i}\right)^{\ell_i\varphi_i}\right)^{1/\mathcal{T}}\right)^{\mathcal{T}} + \left(\prod_{i=1}^{n}\left(i^t_{\eta_i}\right)^{\ell_i\varphi_i}\right)^{\mathcal{T}} + \left(\prod_{i=1}^{n}\left(n^t_{\eta_i}\right)^{\ell_i\varphi_i}\right)^{\mathcal{T}}\\ &= \left(\left(1-\prod_{i=1}^{n}\left(1-m^t_{\eta_i^*}\right)^{\ell_i\varphi_i}\right)^{1/\mathcal{T}}\right)^{\mathcal{T}} + \left(\prod_{i=1}^{n}\left(i^t_{\eta_i^*}\right)^{\ell_i\varphi_i}\right)^{\mathcal{T}} + \left(\prod_{i=1}^{n}\left(n^t_{\eta_i^*}\right)^{\ell_i\varphi_i}\right)^{\mathcal{T}}\\ &= \aleph(\eta^*).\end{aligned} \tag{23}$$

Hence,

$$\begin{aligned}&\mathcal{T}-\text{SFWA}_c(\langle\eta_1,\ell_1\rangle,\langle\eta_2,\ell_2\rangle,\ldots,\langle\eta_n,\ell_n\rangle)\\ &\quad<\mathcal{T}-\text{SFWA}_c(\langle\eta_1^*,\ell_1\rangle,\langle\eta_2^*,\ell_2\rangle,\ldots,\langle\eta_n^*,\ell_n\rangle).\end{aligned} \tag{24}$$

□

Definition 11. Let Φ be the set of n T-SFVs $\eta_i = (m^t_i, i^t_i, n^t_i)$ $\forall\,(i = 1, 2, \ldots, n)$ and CLs are denoted as ℓ_i of η_i with restriction $0 \le \ell_i \ge 1$. If the weight vector $\varphi_i = (\varphi_1, \varphi_2, \ldots, \varphi_n)$ with $\varphi_i \le \in [0,1]$ and $\sum_{i=1}^n \varphi_i = 1$, then the mapping of function $\mathcal{T}-\text{SFOWA}_c\colon \Phi^n \longrightarrow \Phi$ is defined as follows:

$$\begin{aligned}&\mathcal{T}-\text{SFWOA}_c(\langle\eta_1,\ell_1\rangle,\langle\eta_2,\ell_2\rangle,\ldots,\langle\eta_n,\ell_n\rangle)\\ &\quad=\bigoplus_{i=1}^{n}\varphi_i\left(\eta_{n(\sigma)},\ell_n\right).\end{aligned} \tag{25}$$

It is said to be the CLs of the $\mathcal{T}-$ SF ordered weighted averaging operator. Here, for every i, $(\sigma_{(1)}, \sigma_{(2)}, \ldots, \sigma_{(n)})$ is a permutation of $(1, 2, \ldots, n)$ with condition $\eta_{\sigma_{(i-1)}} \ge \eta_{\sigma_{(i)}}$.

Theorem 12. *Let $\eta_i = (m^t_i, i^t_i, n^t_i)$ be the set of T-SFVs with the condition of CLs $0 \le \ell_i \ge 1$ and WV $\varphi_i = (\varphi_1, \varphi_2, \ldots, \varphi_n)$, $\forall\,\varphi_i \in [0,1]$, and $\sum_{i=1}^n \varphi_i = 1$, then their aggregated outcomes by applying $\mathcal{T}-\text{SFOWA}_c$ the AO is a T-SFV, which is given by the following expression:*

$$\mathcal{T}-\text{SFOWA}_c(\langle\eta_1,\ell_1\rangle,\langle\eta_2,\ell_2\rangle,\ldots,\langle\eta_n,\ell_n\rangle) = \left(\left(1-\prod_{i=1}^{n}\left(1-m^{\mathcal{T}}_{\sigma_{(i)}}\right)^{l_i\varphi_i}\right)^{1/\mathcal{T}}, \prod_{i=1}^{n}\left(i_{\sigma_{(i)}}\right)^{l_i\varphi_i}, \prod_{i=1}^{n}\left(n_{\sigma_{(i)}}\right)^{l_i\varphi_i}\right). \tag{26}$$

Proof. It is easy to prove as Theorem 7. □

Definition 13. Let Φ be the set of n T-SFVs $\eta_i = (\dot{m}^t_i, \dot{i}^t_i, \dot{n}^t_i)$ $\forall\,(i = 1, 2, \ldots, n)$ and CLs are denoted as ℓ_i of η_i with the restriction $0 \le \ell_i \ge 1$. If the weight vector $\varphi_i = (\varphi_1, \varphi_2, \ldots, \varphi_n)$ with $\varphi_i \le \in [0,1]$ and $\sum_{i=1}^n \varphi_i = 1$, for the maintenance of equilibrium, where n is the balancing coefficient, then the mapping of function $\mathcal{T}-\text{SFHWA}_c\colon \Phi^n \longrightarrow \Phi$ defined as follows:

$$\begin{aligned}&\mathcal{T}-\text{SFHWA}_c(\langle\dot{\eta}_1,\ell_1\rangle,\langle\dot{\eta}_2,\ell_2\rangle,\ldots,\langle\dot{\eta}_n,\ell_n\rangle)\\ &\quad=\bigoplus_{i=1}^{n}\varphi_i\left((\dot{\eta})_{(\dot{\sigma})n},\ell_n\right).\end{aligned} \tag{27}$$

These are said to be the CLs of the $\mathcal{T}-$ SF hybrid weighted averaging operator. Here, for every i, $(\dot{\sigma}_{(1)}, \dot{\sigma}_{(2)}, \ldots, \dot{\sigma}_{(n)})$ is a permutation of $(1, 2, \ldots, n)$ with condition $\dot{\eta}_{\sigma_{(i-1)}} \ge \dot{\eta}_{\sigma_{(i)}}$.

Theorem 14. *Let $\eta_i = (\dot{m}^t_i, \dot{i}^t_i, \dot{n}^t_i)$ be the set of T-SFVs with the condition of CLs $0 \le \ell_i \ge 1$ and WV $\varphi_i = (\varphi_1, \varphi_2, \ldots, \varphi_n)$, $\forall\,\varphi_i \in [0,1]$, and $\sum_{i=1}^n \varphi_i = 1$, for the maintenance of equilibrium, where n is the balancing coefficient, then their aggregated outcomes by applying $\mathcal{T}-\text{SFHWA}_c$, we obtain a T-SFV, which is given by the following expression:*

$$\mathcal{T}-\text{SFHWA}_c(\langle\dot{\eta}_1,\ell_1\rangle,\langle\dot{\eta}_2,\ell_2\rangle,\ldots,\langle\dot{\eta}_n,\ell_n\rangle)=\left(\left(1-\prod_{i=1}^{n}\left(1-\dot{m}_{\sigma(i)}^{\mathcal{T}}\right)^{l_i\varphi_i}\right)^{1/\mathcal{T}},\prod_{i=1}^{n}\left((\dot{i})_{\sigma(i)}\right)^{l_i\varphi_i},\prod_{i=1}^{n}\left((\dot{n})_{\sigma(i)}\right)^{l_i\varphi_i}\right). \tag{28}$$

Proof. It is easy to verify as Theorem 7. □

3.2. T-Spherical Fuzzy Confidence Level Weighted Geometric Operators

Definition 15. Let Φ be the set of n T-SFSNs $\eta_i=(m_i^t,i_i^t,n_i^t)$ $\forall\,(i=1,2,\ldots,n)$ and CLs are denoted as ℓ_i of η_i with the restriction $0\le\ell_i\ge 1$. If the weight vector $\varphi_i=(\varphi_1,\varphi_2,\ldots,\varphi_n)$ with $\varphi_i\le\in[0,1]$ and $\sum_{i=1}^{n}\varphi_i=1$, then the mapping of function $\mathcal{T}-\text{SFWG}_c\colon\Phi^n\longrightarrow\Phi$ is defined as follows:

$$\begin{aligned}&\mathcal{T}-\text{SFWG}_c(\langle\eta_1,\ell_1\rangle,\langle\eta_2,\ell_2\rangle,\ldots,\langle\eta_n,\ell_n\rangle)\\&\quad=\bigotimes_{i=1}^{n}\varphi_i(\eta_n,\ell_n)=\varphi_1(\eta_1,\ell_1)\otimes\varphi_2(\eta_2,\ell_2)\otimes\ldots\otimes\varphi_n(\eta_n\ell_n).\end{aligned} \tag{29}$$

This is said to be the CLs of the $\mathcal{T}-$SF weighted geometric operator.

Theorem 16. *Let $\eta_i=(m_i^t,i_i^t,n_i^t)$ be the set of T-SFVs with the condition of CLs $0\le\ell_i\ge1$ and WV $\varphi_i=(\varphi_1,\varphi_2,\ldots,\varphi_n)$, $\forall\,\varphi_i\in[0,1]$, and $\sum_{i=1}^{n}\varphi_i=1$, then their aggregated outcomes by applying $\mathcal{T}-\text{SFWG}_c$ we obtain a T-SFV, which is given by the following expression:*

$$\mathcal{T}-\text{SFWG}_c(\langle\eta_1,\ell_1\rangle,\langle\eta_2,\ell_2\rangle,\ldots,\langle\eta_n,\ell_n\rangle)=\left(\prod_{i=1}^{n}(m_i)^{l_i\varphi_i},\left(1-\prod_{i=1}^{n}\left(1-i_i^{\mathcal{T}}\right)^{l_i\varphi_i}\right)^{1/\mathcal{T}},\left(1-\prod_{i=1}^{n}\left(1-n_i^{\mathcal{T}}\right)^{l_i\varphi_i}\right)^{1/\mathcal{T}}\right). \tag{30}$$

Proof. We can easily verify this theorem through the technique of mathematical induction method .

Taking $n=2$, we have the following expression:

$$\mathcal{T}-\text{SFWG}_c(\langle\eta_1,\ell_1\rangle,\langle\eta_2,\ell_2\rangle)=\varphi_1(\eta_1,\ell_1)\otimes\varphi_2(\eta_2,\ell_2). \tag{31}$$

On the behalf of the operational laws of T-SFVs, we have the following expression:

$$\begin{aligned}\eta_1,\ell_1&=\left((m_1)^{l_1},\left(1-\left(1-i_1^{\mathcal{T}}\right)^{l_1}\right)^{1/\mathcal{T}},\left(1-\left(1-n_1^{\mathcal{T}}\right)^{l_1}\right)^{1/\mathcal{T}}\right)\\&\Longrightarrow\varphi_1(\eta_1,\ell_1)=\left((m_i)^{\varphi_1},\left(1-\left(1-i_1^{\mathcal{T}}\right)^{\varphi_1}\right)^{1/\mathcal{T}},\left(1-\left(1-n_1^{\mathcal{T}}\right)^{\varphi_1}\right)^{1/\mathcal{T}}\right)\\&=\left(\left((m_1)^{\ell_1}\right)^{\varphi_1},\left(\left(\left(1-\left(1-\left(1-i_1^{\mathcal{T}}\right)^{l_1}\right)^{1/\mathcal{T}}\right)^{\mathcal{T}}\right)^{\varphi_1}\right)^{1/\mathcal{T}},\left(\left(\left(1-\left(1-\left(1-n_1^{\mathcal{T}}\right)^{l_1}\right)^{1/\mathcal{T}}\right)^{\mathcal{T}}\right)^{\varphi_1}\right)^{1/\mathcal{T}}\right)\\&=\left(\left((m_1)^{\ell_1}\right)^{\varphi_1},\left(\left(1-\left(1-\left(1-i_1^{\mathcal{T}}\right)^{l_1}\right)\right)^{\varphi_1}\right)^{1/\mathcal{T}},\left(\left(1-\left(1-\left(1-n_1^{\mathcal{T}}\right)^{l_1}\right)\right)^{\varphi_1}\right)^{1/\mathcal{T}}\right)\\&=\left((m_1)^{\ell_1\varphi_1},\left(\left(1-i_1^{\mathcal{T}}\right)^{l_1\varphi_1}\right)^{1/\mathcal{T}},\left(\left(1-n_1^{\mathcal{T}}\right)^{l_1\varphi_1}\right)^{1/\mathcal{T}}\right).\end{aligned} \tag{32}$$

Similarly, we say that $\varphi_2(\eta_2, \ell_2) = ((m_2)^{\ell_1 \varphi_2}, ((1 - i_2^{\mathcal{T}})^{\ell_2 \varphi_2})^{1/\mathcal{T}}, ((1 - n_2^{\mathcal{T}})^{\ell_2 \varphi_2})^{1/\mathcal{T}})$ Then,

$$
\begin{aligned}
\mathcal{T} - \mathrm{SFWG}_c(\langle \eta_1, \ell_1 \rangle, \langle \eta_2, \ell_2 \rangle) &= \varphi_1(\eta_1, \ell_1) \otimes \varphi_2(\eta_2, \ell_2) \\
&= \Bigg((m_1)^{\ell_1 \varphi_1} . (m_2)^{\ell_2 \varphi_2}, \left\{ \left(\left(1 - i_1^{\mathcal{T}}\right)^{l_i \varphi_1} \right)^{1/\mathcal{T}} + \left(\left(1 - i_2^{\mathcal{T}}\right)^{\ell_2 \varphi_2} \right)^{1/\mathcal{T}} \right\} \\
&\quad - \left\{ \left(\left(1 - i_1^{\mathcal{T}}\right)^{l_i \varphi_1} \right)^{1/\mathcal{T}} . \left(\left(1 - i_2^{\mathcal{T}}\right)^{\ell_2 \varphi_2} \right) \right\}, \left\{ \left(\left(1 - n_1^{\mathcal{T}}\right)^{l_i \varphi_1} \right)^{1/\mathcal{T}} + \left(\left(1 - n_2^{\mathcal{T}}\right)^{\ell_2 \varphi_2} \right)^{1/\mathcal{T}} \right\} \\
&\quad - \left\{ \left(\left(1 - n_1^{\mathcal{T}}\right)^{l_i \varphi_1} \right)^{1/\mathcal{T}} . \left(\left(1 - n_2^{\mathcal{T}}\right)^{\ell_2 \varphi_2} \right) \right\} \Bigg) \\
\mathcal{T} - SFWG_c(\langle \eta_1, \ell_1 \rangle, \langle \eta_2, \ell_2 \rangle) &= \left(\prod_{i=1}^{2} \left((m_i)^{\ell_i \varphi_i} \right), \left(1 - \prod_{i=1}^{2} \left(1 - \left(\left(1 - i_i^{\mathcal{T}}\right)^{l_i \varphi_i} \right)^{1/\mathcal{T}} \right) \right), \left(1 - \prod_{i=1}^{2} \left(1 - \left(\left(1 - n_i^{\mathcal{T}}\right)^{l_i \varphi_i} \right)^{1/\mathcal{T}} \right) \right) \right).
\end{aligned}
\tag{33}
$$

Consider the result is satisfy for $n = k$, that is,

$$
\mathcal{T} - \mathrm{SFWG}_c(\langle \eta_1, \ell_1 \rangle, \langle \eta_2, \ell_2 \rangle, \ldots, \langle \eta_k, \ell_k \rangle) = \begin{pmatrix} \prod_{i=1}^{k} \left((m_i)^{\ell_i \varphi_i} \right), \left(1 - \prod_{i=1}^{k} \left(1 - \left(\left(1 - i_i^{\mathcal{T}}\right)^{l_i \varphi_1} \right)^{1/\mathcal{T}} \right) \right) \\ \left(1 - \prod_{i=1}^{k} \left(1 - \left(\left(1 - n_i^{\mathcal{T}}\right)^{l_i \varphi_1} \right)^{1/\mathcal{T}} \right) \right) \end{pmatrix}. \tag{34}
$$

Now, consider the results are true for $n = k + 1$, that is,

$$
\begin{aligned}
&\mathcal{T} - \mathrm{SFWG}_c(\langle \eta_1, \ell_1 \rangle, \langle \eta_2, \ell_2 \rangle, \ldots, \langle \eta_k, \ell_k \rangle, \langle \eta_{k+1}, \ell_{k+1} \rangle) \\
&\quad = \mathcal{T} - SFWG_c(\langle \eta_1, \ell_1 \rangle, \langle \eta_2, \ell_2 \rangle, \ldots, \langle \eta_k, \ell_k \rangle) \otimes \varphi_{k+1} \langle \eta_{k+1}, \ell_{k+1} \rangle \\
&\quad = \left(\prod_{i=1}^{k} \left((m_i)^{\ell_i \varphi_i} \right), \left(1 - \prod_{i=1}^{k} \left(1 - \left(\left(1 - i_i^{\mathcal{T}}\right)^{l_i \varphi_i} \right)^{1/\mathcal{T}} \right) \right), \left(1 - \prod_{i=1}^{k} \left(1 - \left(\left(1 - n_i^{\mathcal{T}}\right)^{l_i \varphi_i} \right)^{1/\mathcal{T}} \right) \right) \right) \\
&\quad \otimes \left(\left(m_{(k+1)} \right)^{l_i}, \left(1 - \left(1 - i_{(k+1)}^{\mathcal{T}}\right)^{l_i} \right)^{1/\mathcal{T}}, \left(1 - \left(1 - n_{(k+1)}^{\mathcal{T}}\right)^{l_i} \right)^{1/\mathcal{T}} \right) \\
&\quad = \begin{pmatrix} \prod_{i=1}^{k} \left((m_i)^{\ell_i \varphi_i} \right) . \left(i_{(k+1)}^{\mathcal{T}} \right)^{l_i} \\ \left\{ \left(1 - \prod_{i=1}^{k} \left(1 - \left(\left(1 - i_i^{\mathcal{T}}\right)^{l_i \varphi_1} \right)^{1/\mathcal{T}} \right) \right) + \left(1 - \left(1 - i_{(k+1)}^{\mathcal{T}}\right)^{l_i} \right)^{1/\mathcal{T}} \right\} \\ - \left\{ \left(1 - \prod_{i=1}^{k} \left(1 - \left(\left(1 - i_i^{\mathcal{T}}\right)^{l_i \varphi_i} \right)^{1/\mathcal{T}} \right) \right) . \left(1 - \left(1 - i_{(k+1)}^{\mathcal{T}}\right)^{l_i} \right)^{1/\mathcal{T}} \right\} \\ \left\{ \left(1 - \prod_{i=1}^{k} \left(1 - \left(\left(1 - n_i^{\mathcal{T}}\right)^{l_i \varphi_1} \right)^{1/\mathcal{T}} \right) \right) + \left(1 - \left(1 - n_{(k+1)}^{\mathcal{T}}\right)^{l_i} \right)^{1/\mathcal{T}} \right\} \\ - \left\{ \left(1 - \prod_{i=1}^{k} \left(1 - \left(\left(1 - n_i^{\mathcal{T}}\right)^{l_i \varphi_i} \right)^{1/\mathcal{T}} \right) \right) . \left(1 - \left(1 - n_{(k+1)}^{\mathcal{T}}\right)^{l_i} \right)^{1/\mathcal{T}} \right\} \end{pmatrix} \\
&\quad = \left(\prod_{i=1}^{k+1} \left((m_i)^{\ell_i \varphi_i} \right), \left(1 - \prod_{i=1}^{k+1} \left(\left(1 - i_i^{\mathcal{T}}\right)^{l_i \varphi_i} \right)^{1/\mathcal{T}} \right), \left(1 - \prod_{i=1}^{k+1} \left(\left(1 - n_i^{\mathcal{T}}\right)^{l_i \varphi_i} \right)^{1/\mathcal{T}} \right) \right).
\end{aligned}
\tag{35}
$$

The statement is true for $n = k + 1$. Hence, the result satisfies all T-SFVs. □

Property 17 (Idempotency). Consider for all $\langle \eta_i, \ell_i \rangle = \langle \eta, \ell \rangle$, $(i = 1,2,\ldots,n)$, that is, $m_i = m, i_i = i, n_i = n$, and for all $l_i = l$, then,

$$\mathcal{T} - \mathrm{SFWA}_c(\langle \eta_1, \ell_1 \rangle, \langle \eta_2, \ell_2 \rangle, \ldots, \langle \eta_n, \ell_n \rangle) = \eta\ell. \quad (36)$$

Proof. If $\langle \eta_i, \ell_i \rangle = \langle \eta, \ell \rangle$ for all $(i = 1, 2, \ldots, n)$, then by utilizing Theorem 16, we obtain the following expression:

$$\begin{aligned}
\mathcal{T} - \mathrm{SFWG}_c(\langle \eta_1, \ell_1 \rangle, \langle \eta_2, \ell_2 \rangle, \ldots, \langle \eta_n, \ell_n \rangle) &= \begin{pmatrix} \prod_{i=1}^{n}\left((m_i)^{\ell\varphi_i}\right), \left(1 - \prod_{i=1}^{n}\left(1 - \left(\left(1 - i_i^{\mathcal{T}}\right)^{\ell\varphi_i}\right)^{1/\mathcal{T}}\right)\right) \\ \left(1 - \prod_{i=1}^{n}\left(1 - \left(\left(1 - n_i^{\mathcal{T}}\right)^{\ell\varphi_i}\right)^{1/\mathcal{T}}\right)\right) \end{pmatrix} \\
&= \begin{pmatrix} \prod_{i=1}^{n}\left((m_i)^{\ell\Sigma_{i=1}^{n}\varphi_i}\right), \left(1 - \prod_{i=1}^{n}\left(1 - \left(\left(1 - i_i^{\mathcal{T}}\right)^{\ell\Sigma_{i=1}^{n}\varphi_i}\right)^{1/\mathcal{T}}\right)\right)^{1/\mathcal{T}} \\ \left(1 - \prod_{i=1}^{n}\left(1 - \left(\left(1 - n_i^{\mathcal{T}}\right)^{\ell\Sigma_{i=1}^{n}\varphi_i}\right)^{1/\mathcal{T}}\right)\right)^{1/\mathcal{T}} \end{pmatrix} \\
&= \begin{pmatrix} \prod_{i=1}^{n}\left((m_i)^{\ell}\right), \left(1 - \prod_{i=1}^{n}\left(1 - \left(\left(1 - i_i^{\mathcal{T}}\right)^{\ell}\right)^{1/\mathcal{T}}\right)\right)^{1/\mathcal{T}} \\ \left(1 - \prod_{i=1}^{n}\left(1 - \left(\left(1 - n_i^{\mathcal{T}}\right)^{\ell}\right)^{1/\mathcal{T}}\right)\right)^{1/\mathcal{T}} \end{pmatrix} \\
&= \eta\ell.
\end{aligned} \quad (37)$$

□

Property 18 (Boundedness). If $\eta_i^- = ((m_i^t)_{\min}, (i_i^t)_{\max}, (n_i^t)_{\max})$ and $\eta_i^+ = ((m_i^t)_{\max}, (i_i^t)_{\min}, (n_i^t)_{\min})$, then for every weight φ_i, $\eta_i^- \leq \mathcal{T} - \mathrm{SFWG}_c(\langle \eta_1, \ell_1 \rangle, \langle \eta_2, \ell_2 \rangle, \ldots, \langle \eta_n, \ell_n \rangle) \leq \eta_i^+$.

Property 19 (Monotonicity). Consider $\eta^* = (m_{i*}^t, i_{i*}^t, n_{i*}^t)$ be the set of n T-SFVs with the condition $m_i^t \leq m_{i*}^t, i_i^t \geq i_{i*}^t, n_i^t \geq n_{i*}^t$ of and for every weight vector φ

$$\begin{aligned}
&\mathcal{T} - \mathrm{SFWG}_c(\langle \eta_1, \ell_1 \rangle, \langle \eta_2, \ell_2 \rangle, \ldots, \langle \eta_n, \ell_n \rangle) \\
&\quad \leq \mathcal{T} - \mathrm{SFWG}_c(\langle \eta_1^*, \ell_1 \rangle, \langle \eta_2^*, \ell_2 \rangle, \ldots, \langle \eta_n^*, \ell_n \rangle).
\end{aligned} \quad (38)$$

Definition 20. Let Φ be the set of n T-SFVs $\eta_i = (m_i^t, i_i^t, n_i^t)$ and CLs are denoted as ℓ_i of η_i with restriction $0 \leq \ell_i \geq 1$. If the weight vector $\varphi_i = (\varphi_1, \varphi_2, \ldots, \varphi_n)$ with $\varphi_i \leq \in [0, 1]$ and $\sum_{i=1}^{n} \varphi_i = 1$, then the mapping of function $\mathcal{T} - \mathrm{SFOWG}_c$: $\Phi^n \longrightarrow \Phi$ defined as follows:

$$\begin{aligned}
&\mathcal{T} - \mathrm{SFWOG}_c(\langle \eta_1, \ell_1 \rangle, \langle \eta_2, \ell_2 \rangle, \ldots, \langle \eta_n, \ell_n \rangle) \\
&\quad = \bigotimes_{i=1}^{n} \varphi_i\left(\eta_{(\sigma)n}, \ell_n\right).
\end{aligned} \quad (39)$$

This is said to be the CLs of the $\mathcal{T}$ − SF ordered weighted geometric operator. Here, for every i, $(\sigma_{(1)}, \sigma_{(2)}, \ldots, \sigma_{(n)})$ is a permutation of $(1, 2, \ldots, n)$ with condition $\eta_{\sigma_{(i-1)}} \geq \eta_{\sigma_{(i)}}$.

Theorem 21. *Let $\eta_i = (m_i^t, i_i^t, n_i^t)$ be the set of T-SFVs with the condition of CLs $0 \le \ell_i \ge 1$ and WV $\varphi_i = (\varphi_1, \varphi_2, \ldots, \varphi_n)$, $\forall \varphi_i \in [0,1]$ and $\sum_{i=1}^{n} \varphi_i = 1$, then their aggregated outcomes by applying $\mathcal{T}$ − SFOWG$_c$ we obtain a T-SFV, which is given by the following expression:*

$$\mathcal{T} - \mathrm{SFOWG}_c(\langle \eta_1, \ell_1 \rangle, \langle \eta_2, \ell_2 \rangle, \ldots, \langle \eta_n, \ell_n \rangle) = \left(\prod_{i=1}^{n} \left(m_{\sigma_{(i)}} \right)^{l_i \varphi_i}, \left(1 - \prod_{i=1}^{n} \left(1 - i_{\sigma_{(i)}} \right)^{l_i \varphi_i} \right)^{1/\mathcal{T}}, \left(1 - \prod_{i=1}^{n} \left(1 - n_{\sigma_{(i)}} \right)^{l_i \varphi_i} \right)^{1/\mathcal{T}} \right). \tag{40}$$

Proof. It can be proved a similarly as Theorem 16. □

Definition 22. Let Φ be the set of n T-SFVs $\eta_i = (\dot{m}_i^t, \dot{i}_i^t, \dot{n}_i^t)$ $\forall (i = 1, 2, \ldots, n)$ and CLs are denoted as ℓ_i of η_i with the restriction $0 \le \ell_i \ge 1$. If the weight vector $\varphi_i = (\varphi_1, \varphi_2, \ldots, \varphi_n)$ with $\varphi_i \le \in [0,1]$ and $\sum_{i=1}^{n} \varphi_i = 1$, for the maintenance of equilibrium, where n is the balancing coefficient, then the mapping of function $\mathcal{T}$ − SFHWG$_c$: $\Phi^n \longrightarrow \Phi$ defined as follows:

$$\begin{aligned} &\mathcal{T} - \mathrm{SFHWG}_c(\langle \dot{\eta}_1, \ell_1 \rangle, \langle \dot{\eta}_2, \ell_2 \rangle, \ldots, \langle \dot{\eta}_n, \ell_n \rangle) \\ &\quad = \bigotimes_{i=1}^{n} \varphi_i \left((\dot{\eta})_{(\dot{\sigma})n}, \ell_n \right). \end{aligned} \tag{41}$$

These are said to be the CLs of the $\mathcal{T}$ − SF hybrid weighted geometric operator. Here, for every i, $(\dot{\sigma}_{(1)}, \dot{\sigma}_{(2)}, \ldots, \dot{\sigma}_{(n)})$ is a permutation of $(1, 2, \ldots, n)$ with condition $\dot{\eta}_{\sigma_{(i-1)}} \ge \dot{\eta}_{\sigma_{(i)}}$.

Theorem 23. *Let $\eta_i = (\dot{m}_i^t, \dot{i}_i^t, \dot{n}_i^t)$ be the set of T-SFVs with the condition of CLs $0 \le \ell_i \ge 1$ and WV $\varphi_i = (\varphi_1, \varphi_2, \ldots, \varphi_n)$, $\forall \varphi_i \in [0,1]$ and $\sum_{i=1}^{n} \varphi_i = 1$, for the maintenance of equilibrium, where n is the balancing coefficient, then their aggregated outcomes by applying the $\mathcal{T}$ − SFHWG$_c$, we obtain a T-SFV, which is given by the following expression:*

$$\mathcal{T} - \mathrm{SFHWG}_c(\langle \dot{\eta}_1, \ell_1 \rangle, \langle \dot{\eta}_2, \ell_2 \rangle, \ldots, \langle \dot{\eta}_n, \ell_n \rangle) = \left(\prod_{i=1}^{n} \left(\dot{m}_{\sigma_{(i)}}^{\mathcal{T}} \right)^{l_i \varphi_i}, \left(1 - \prod_{i=1}^{n} \left(1 - (\dot{i})_{\sigma_{(i)}} \right)^{l_i \varphi_i} \right)^{1/\mathcal{T}}, \left(1 - \prod_{i=1}^{n} \left(1 - (\dot{n})_{\sigma_{(i)}} \right)^{l_i \varphi_i} \right)^{1/\mathcal{T}} \right). \tag{42}$$

Proof. It is easy to prove as Theorem 16. □

4. MADM Algorithm under Confidence Levels

In this part, a DM technique created on developed operators is projected by solving the MCDM problems. A daily-life consumer selection problem is also considered to effectively demonstrate the DM method.

The collection of alternatives is considered, such as $\{\mathcal{E}_1, \mathcal{E}_2, \ldots, \mathcal{E}_n\}$, that is projected on the bases of the criteria $\{C_1, C_2, \ldots, C_n\}$. Considering WV $\varphi_i = (\varphi_1, \varphi_2, \ldots, \varphi_n)^{\mathcal{T}}$, $\forall \varphi_i \ge 0, \varphi_i \in [0,1]$, and $\sum_{i=1}^{n} \varphi_i = 1$, the WVs are calculated through the entropy measure for TSFS defined in section 7. Also, consider the DS such as $\{\mathcal{D}_1, \mathcal{D}_2, \ldots, \mathcal{D}_n\}$ having WV $\varphi = (\varphi_1, \varphi_2, \ldots, \varphi_n)^{\mathcal{T}}$ which is satisfy the condition $\varphi_k > 0$, $(k = 1, 2, \ldots, n)$ and $\sum_{k=1}^{n} \varphi_i = 1$, assign the valuation of each option against each criterion in the term of $\mathcal{T}$ − SFNs and it is represented as $(\eta_{ij}^k)_{m \times n} = ((m^t)_{ij}^k, (i^t)_{ij}^k, (n^t)_{ij}^k)_{m \times n}$, where in the view of the expert $\mathcal{D}_k$, $(m^t)_{ij}^k$, $(i^t)_{ij}^k$, and $(n^t)_{ij}^k$ represent the degree of that alternative, with restriction $0 \le (m^t)_{ij}^k, (i^t)_{ij}^k, (n^t)_{ij}^k \ge 1 \,\forall\, t \in \mathrm{Z}^+$, $(i = 1, 2, \ldots, n)$, and $\forall (k = 1, 2, \ldots, n)$. To include the concept of CLs, ℓ_{ij}^k with condition $0 \le \ell_{ij}^k \ge 1$, the following steps are present effectively to assist the proposed operators in DM problems:

Individual expert judgments should be established for each alternative against the specified set of criteria in the term of $\mathcal{T}$ − SFNs with CLs and then built the matrix on the bases of matching the expert's judgment such that $(\aleph_{ij}^k)_{m \times n} = (\langle (m^t)_{ij}^k, (i^t)_{ij}^k, (n^t)_{ij}^k \rangle, \ell_{ij}^k)$.

Step 1. First of all, transform our T-SFVs matrix into the standardized decision matrix.

Step 2. The calculation of the WV entropy measure of T-SFS is used from equation (3) for each i^{th} criteria.

$$\mathcal{E}(\eta) = \sum_{i=1}^{n} \left(1 - \frac{4}{5} \left[\left| m_i^t(x) - n_i^t(x) \right| + \left| i_i^t(x) - 0.25 \right| \right] \right). \tag{43}$$

The intrinsic data of i^{th} criteria divergence are denoted by div$_i$. = 1 − $\mathcal{E}_i$. The calculation for the weight criteria is as follows [34]:

$$\varphi = \frac{\mathrm{div}_i}{\sum_{i=1}^{n} \mathrm{div}_i}. \tag{44}$$

Step 3. By using the proposed $\mathcal{T}-\mathrm{SFWA}_c$, $\mathcal{T}-\mathrm{SFWG}_c$ AOs, we aggregate the information which is provided in the form of matrix $(\aleph_{ij}^{k})_{m\times n}$.

$$\mathcal{T}-\mathrm{SFWA}_c(\langle\eta_1,\ell_1\rangle,\langle\eta_2,\ell_2\rangle,\ldots,\langle\eta_n,\ell_n\rangle)=\left(\left(1-\prod_{i=1}^{n}\left(1-m_i^{\mathcal{T}}\right)^{l_i\varphi_i}\right)^{1/\mathcal{T}},\prod_{i=1}^{n}(i_i)^{l_i\varphi_i},\prod_{i=1}^{n}(n_i)^{l_i\varphi_i}\right),$$
$$\mathcal{T}-\mathrm{SFWG}_c(\langle\eta_1,\ell_1\rangle,\langle\eta_2,\ell_2\rangle,\ldots,\langle\eta_n,\ell_n\rangle)=\left(\prod_{i=1}^{n}(m_i)^{l_i\varphi_i},\left(1-\prod_{i=1}^{n}\left(1-i_i^{\mathcal{T}}\right)^{l_i\varphi_i}\right)^{1/\mathcal{T}},\left(1-\prod_{i=1}^{n}\left(1-n_i^{\mathcal{T}}\right)^{l_i\varphi_i}\right)^{1/\mathcal{T}}\right). \tag{45}$$

Step 4. Utilizing the SF as described in Definition 3, we rank the SF in descending order.

$$\mathfrak{R}(\eta)=m_\eta^t-n_\eta^t. \tag{46}$$

Step 5. While keeping in our mind φ_i, rank the data and then select the best option.

Step 6. End.

For clarity, the flowchart in Figure 1 shows the MADM algorithm based on the T-SFS environment.

5. Numerical Example

Our proposed technique in MP selection and evaluation is applied in Pakistan. The Pakistani retail market has played a significant part in the Pakistani economical structure. MP is the most competent way of electronic communication. According to the "Global System for Mobile Communication" survey in 2022, there are more than 186.9 million MP users in Pakistan. The huge number of MP users and gadgets in Pakistan, as well as recent publications, has sparked our interest in developing a DM model for choosing the best MP. From this perspective, the objective of this paper is to identify the finest MP option on the behalf of customer preferences.

Example 1. This case study discusses the investigation of the best MP selection problem, in which customers choose a suitable MP from a list of MPs based on CLs T-SFS standards. In this problematic situation, the customer must select MP which will allow for more efficient features. In this scenario, information is collected from multiple consumers and the decision matrix is constructed having attributes $\aleph_i=(1,2,3,4)$ as the best option, "operating system," "memory storage," "camera resolution," and "processor speed." These four attributes have a weight vector $\varphi=(0.24,0.22,0.25,0.26)^{\mathcal{T}}$, and these weights are calculated by the help of the formula discussed in equation (15). Also, considerable importance is given to the expert's opinions in the selection of the best MP. It is considered that the opinions of four experts having WV (0.31,0.20,0.30,0.19) give their assessment in the form of TSFNs. Table 1 presents the DMs analytical monitoring of four possibilities based on four attributes and the CLs of the experts. The MADM method illuminates this difficult situation, and a detailed procedure is given as follows:

Step 1. From anonymous DMs, we collect data for the four alternatives, and also, we mentioned the CLs on expert and then create the decision matrix as given in Table 1.

Step 2. Aggregate the data in Table 1 by applying the proposed $\mathcal{T}-\mathrm{SFWA}_c$ and $\mathcal{T}-\mathrm{SFWG}_c$ AOs to derive the overall T-SFVs. Table 2 shows the aggregation results.

Step 3. Calculate the SF based on their aggregated results by using Definition 4 as shown in Table 3.

Step 4. Table 4 shows the ranking of technology-developing enterprises based on the scoring function presented in Table 3. It is significant to observe that " > " means "preferred to." As can be seen, regardless of the AOs utilized, the technology-developing enterprises are ordered similarly, with $\aleph_1$ being the finest emerging technology enterprise.

Figure 2 shows the graphical depiction of score values by utilizing aggregation results of the proposed $\mathcal{T}-\mathrm{SFWA}_c$ and $\mathcal{T}-\mathrm{SFWG}_c$ operators. The vertical line demonstrates the range of the SF form $[-1,1]$. The arrangement of the score values in descending order by using aggregated data from Table 3 can be shown in Table 4.

By utilizing the $\mathcal{T}-\mathrm{SFWA}_c$ and $\mathcal{T}-\mathrm{SFWG}_c$ AOs, we obtained the ordering of score values $\aleph_2>\aleph_4>\aleph_3>\aleph_1$ and $\aleph_2>\aleph_4>\aleph_3>\aleph_1$, respectively, which shows clearly that $\aleph_2$ is the best option from the list of options. We also observed here, by using $\mathcal{T}-\mathrm{SFWA}_c$ and $\mathcal{T}-\mathrm{SFWG}_c$, AO's ordering of alternatives will not remain the same for both AOs. So, it would be depending upon the experts whether they select the $\mathcal{T}-\mathrm{SFWA}_c$ or $\mathcal{T}-\mathrm{SFWG}_c$ AO for the aggregation of data.

6. Comparative Studies

The main purpose of this segment is to create a comparison of our purposed work with present literature and illustrate the superiority of our projected AOs over others. We consider the following existing AOs and compare them with our aggregated outcomes of $\mathcal{T}-\mathrm{SFWA}_c$ and $\mathcal{T}-\mathrm{SFWG}_c$ AOs. For it, we compare TSFWA and TSFWG AOs which are proposed

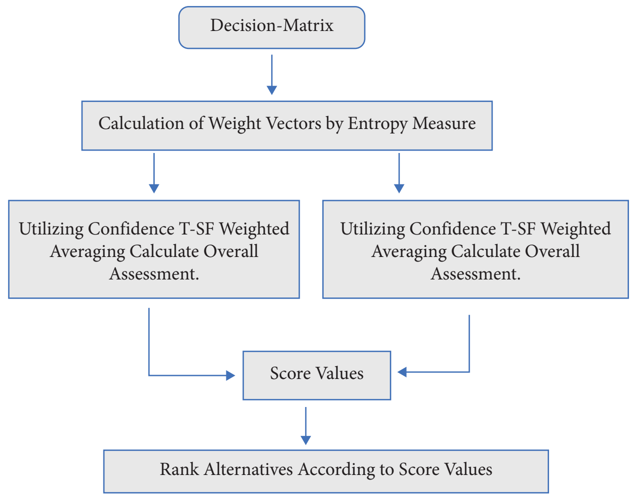


Figure 1: Flowchart of the MADM algorithm.

Table 1: Matrix for selection the best option.

	C_1	C_2	C_3	C_4
$\aleph_1$	$\left[\begin{pmatrix} 0.17 \\ 0.61,0.69 \end{pmatrix}, 0.31\right]$	$\left[\begin{pmatrix} 0.15 \\ 0.34,0.81 \end{pmatrix}, 0.20\right]$	$\left[\begin{pmatrix} 0.11 \\ 0.51,0.49 \end{pmatrix}, 0.30\right]$	$\left[\begin{pmatrix} 0.39 \\ 0.66,0.41 \end{pmatrix}, 0.19\right]$
$\aleph_2$	$\left[\begin{pmatrix} 0.40 \\ 0.12,0.33 \end{pmatrix}, 0.31\right]$	$\left[\begin{pmatrix} 0.44 \\ 0.37,0.21 \end{pmatrix}, 0.20\right]$	$\left[\begin{pmatrix} 0.38 \\ 0.41,0.19 \end{pmatrix}, 0.30\right]$	$\left[\begin{pmatrix} 0.25 \\ 0.72,0.67 \end{pmatrix}, 0.19\right]$
$\aleph_3$	$\left[\begin{pmatrix} 0.30 \\ 0.29,0.51 \end{pmatrix}, 0, 31\right]$	$\left[\begin{pmatrix} 0.25 \\ 0.45,0.31 \end{pmatrix}, 0.20\right]$	$\left[\begin{pmatrix} 0.19 \\ 0.38,0.63 \end{pmatrix}, 0.30\right]$	$\left[\begin{pmatrix} 0.41 \\ 0.11,0.17 \end{pmatrix}, 0.19\right]$
$\aleph_4$	$\left[\begin{pmatrix} 0.25 \\ 0.29,0.45 \end{pmatrix}, 0.31\right]$	$\left[\begin{pmatrix} 0.61 \\ 0.23,0.67 \end{pmatrix}, 0.20\right]$	$\left[\begin{pmatrix} 0.16 \\ 0.77,0.31 \end{pmatrix}, 0.30\right]$	$\left[\begin{pmatrix} 0.35 \\ 0.24,0.23 \end{pmatrix}, 0.19\right]$

Table 2: The aggregation results.

	$\mathrm{T-SFWA_c}$	$\mathrm{T-SFWG_c}$
$\aleph_1$	$\begin{pmatrix} 0.1880, \\ 0.8561, \\ 0.8742 \end{pmatrix}$	$\begin{pmatrix} 0.6522, \\ 0.0269, \\ 0.0485 \end{pmatrix}$
$\aleph_2$	$\begin{pmatrix} 0.2707, \\ 0.7523, \\ 0.7441 \end{pmatrix}$	$\begin{pmatrix} 0.7824, \\ 0.0182, \\ 0.0121 \end{pmatrix}$
$\aleph_3$	$\begin{pmatrix} 0.2187, \\ 0.7343, \\ 0.7004 \end{pmatrix}$	$\begin{pmatrix} 0.7267, \\ 0.0039, \\ 0.0183 \end{pmatrix}$
$\aleph_4$	$\begin{pmatrix} 0.2954, \\ 0.7812, \\ 0.7886 \end{pmatrix}$	$\begin{pmatrix} 0.7303, \\ 0.0327, \\ 0.0137 \end{pmatrix}$

Table 3: The score function of aggregated results.

	$\mathrm{T-SFWA_c}$	$\mathrm{T-SFWG_c}$
$\aleph_1$	−0.5828	0.1810
$\aleph_2$	−0.3013	0.3748
$\aleph_3$	−0.4062	0.2790
$\aleph_4$	−0.3792	0.2846

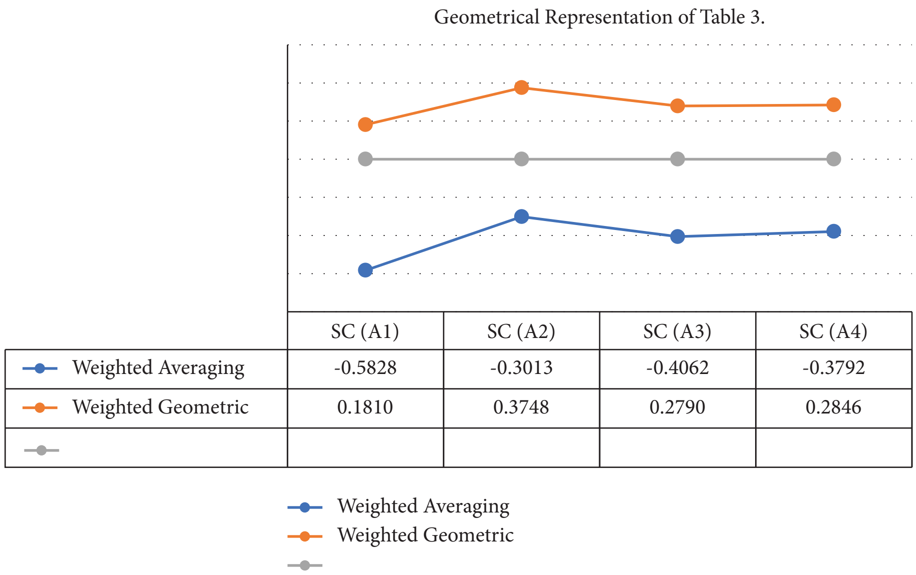


Figure 2: Geometrical representation of the score function.

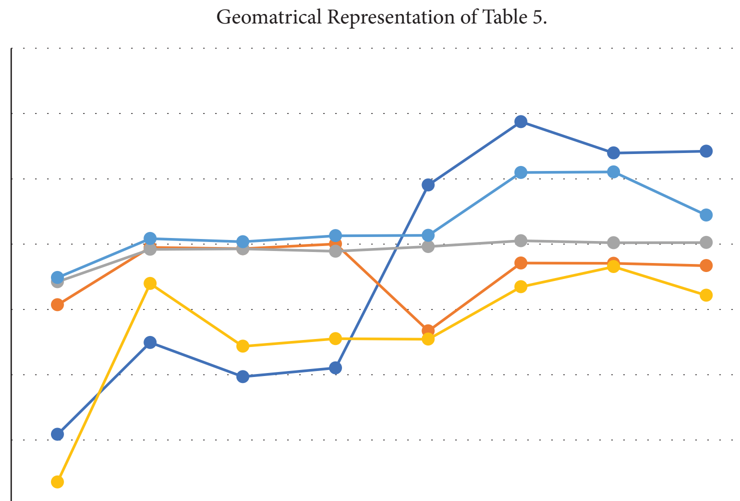
Table 4: The ranking of score function.

	Ordering
$T-SFWA_c$	$\aleph_2 > \aleph_4 > \aleph_3 > \aleph_1$
$T-SFWG_c$	$\aleph_2 > \aleph_4 > \aleph_3 > \aleph_1$

Table 5: A comparative study of proposed work with present work (Table 1).

Methods	Operators	Score values	Ranking outcomes
Proposed operators	$\mathcal{T}-SFWA_c$ $\mathcal{T}-SFWG_c$	$\Re(c_1) = -0.5828, \Re(c_2) = -0.3013, \Re(c_3) = -0.4062, \Re(c_4) = -0.3792$ $\Re(c_1) = 0.1810, \Re(c_2) = 0.3748, \Re(c_3) = 0.2790, \Re(c_4) = 0.2846$	$\aleph_2 > \aleph_5 > \aleph_3 > \aleph_1$ $\aleph_2 > \aleph_4 > \aleph_3 > \aleph_1$
Ullah et al. [35]	TSFWA TSFWG	$\Re(c_1) = -0.1861, \Re(c_2) = -0.0100, \Re(c_3) = -0.0141, \Re(c_4) = -0.0004$ $\Re(c_1) = -0.2659, \Re(c_2) = -0.0580, \Re(c_3) = -0.0588, \Re(c_4) = -0.0661$	$\aleph_4 > \aleph_2 > \aleph_3 > \aleph_1$ $\aleph_2 > \aleph_3 > \aleph_4 > \aleph_1$
Ullah et al. [36]	TSFHWA TSFHWG	$\Re(c_1) = -0.1155, \Re(c_2) = -0.0160, \Re(c_3) = -0.0145, \Re(c_4) = -0.0219$ $\Re(c_1) = -0.0075, \Re(c_2) = 0.0103, \Re(c_3) = 0.0041, \Re(c_4) = 0.0048$	$\aleph_3 > \aleph_2 > \aleph_4 > \aleph_1$ $\aleph_2 > \aleph_4 > \aleph_3 > \aleph_1$
Hussain et al. [37]	TSFAAWA TSFAAWG	$\Re(c_1) = -0.7286, \Re(c_2) = -0.1209, \Re(c_3) = -0.3128, \Re(c_4) = -0.2896$ $\Re(c_1) = -0.2913, \Re(c_2) = -0.1307, \Re(c_3) = -0.0691, \Re(c_4) = -0.1570$	$\aleph_2 > \aleph_4 > \aleph_3 > \aleph_1$ $\aleph_3 > \aleph_2 > \aleph_4 > \aleph_1$
Mahmood et al. [38]	TSFDWA TSFDWG	$\Re(c_1) = -0.1025, \Re(c_2) = 0.0169, \Re(c_3) = 0.0071, \Re(c_4) = 0.0258$ $\Re(c_1) = 0.0267, \Re(c_2) = 0.2192, \Re(c_3) = 0.2211, \Re(c_4) = 0.0886$	$\aleph_4 > \aleph_2 > \aleph_3 > \aleph_1$ $\aleph_3 > \aleph_2 > \aleph_4 > \aleph_1$
Ye et al. [43]	IVIFPWA IVIFPWG	Unable to specify	Failed
Jiang et al. [40]	IFS WA IFS WG	Unable to specify	Failed
Rani and Garg [41]	CIFS WA CIFS WG	Unable to specify	Failed
Wei [42]	PF WA PF WG	Unable to specify	Failed
Zhang [44]	PyFP WA PyFP WA	Unable to specify	Failed

by Ullah et al. [35], T-SF Hamacher WA (TSFHWA) and T-SF Hamacher WG (TSFHWG) AOs presented by Ullah et al. [36], T-SF Aczel Alsina WA (TSFAAWA) and T-SF Aczel Alsina WG (TSFAAWG) AOs proposed by Hussain et al. [37], T-SF Dombi WA (TSFDWA) and T-SF Dombi WG (TSFDWG) AOs proposed by Mahmood et al. [38], and also we observed that WA, WG operators proposed by Ullah et al. [39], Jiang [40], studied IF set WA (IFSWA) and IF set WG (IFSWG), Rani and Garg [41], proposed complex IF set WA (CIFSWA) and complex IF set WG (CIFSWG) operators, Wei [42], examined Picture fuzzy WA (PFWA) and Picture fuzzy WG (PFWG) operators, Pythagorean fuzzy set WA



	SC (A1)	SC (A2)	SC (A3)	SC (A4)	SC (A1)	SC (A2)	SC (A3)	SC (A4)
	Weighted Averaging				Weighted Geometric			
Proposed Operator	-0.5828	-0.3013	-0.4062	-0.3792	0.1810	0.3748	0.2790	0.2846
Ullah et al. [35]	-0.1861	-0.0100	-0.0141	0.0004	-0.2659	-0.0580	-0.0588	-0.0661
Ullah et al. [36]	-0.1155	-0.0160	-0.0145	-0.0219	-0.0075	0.0103	0.0041	0.0048
Hussain et al. [37]	-0.7286	-0.1209	-0.3128	-0.2896	-0.2913	-0.1307	-0.0691	-0.1570
Mahmood et al. [38]	-0.1025	0.0169	0.0071	0.0258	0.0267	0.2192	0.2211	0.0886

Figure 3: Geometrical interpretation of comparative analysis.

(PyFPWA) and Pythagorean fuzzy power WG (PyFPWG) are unable to handle TSFVs-based information. Table 5 shows the comparative study of our proposed work with present AOs to for Table 1.

The geometric depiction of the comparative analysis in Table 5 is shown in Figure 3. It can be easily observed from the graphical representation that $\mathcal{T}-\mathrm{SFWA}_c$ and $\mathcal{T}-\mathrm{SFWG}_c$ AOs provided more precise outcomes than other prevailing AOs in the fuzzy system. We also noticed that the following WA, WG, IFSWA, IFSWG, CIFSWA, CIFSWG, PFWA, PFWG, PFWA, PFWG, PyFPWA, and PyFPWG AOs do not aggregate the data due to limitations in their structure.

The diagnosed $\mathcal{T}-\mathrm{SFWA}_c$ and $\mathcal{T}-\mathrm{SFWG}_c$ AOs are more valuable and are dominant tools for the aggregation of unreliable and awkward information than other existing WA/WG AOs. The existing AOs aggregate the information only on the bases of the attribute's weight. On the other hand, taking advantage of $\mathcal{T}-\mathrm{SFWA}_c$ and $\mathcal{T}-\mathrm{SFWG}_c$ AOs also considers the familiarity like CLs of the DM with the performance of the object.

7. Conclusion

The major aim of this research is to evaluate the best MP alternative by considering numerous attributes and consumer preferences. As a result, MADM approaches are required to accurately identify the most appropriate MP option in the assessment problem. The IFS, PFS, PyFS, and q-ROFS theories are very helpful tools for expressing information under an uncertain environment in MADM issues. It is also notable that how the drawbacks and limited nature of these existing AOs are improved under the TSFS environment. Thus, experts have more freedom for expressing their views on the most suitable option in T-SFS theory. However, present T-SFS AOs are based on the assumption that experts are well-versed in the evaluation of objects, whenever all experts gave the same level of confidence to their assessments of the various alternatives. For this, the current research proposes a T-SF confidence averaging and confidence geometric AOs that take into account expert CLs during the evaluation under the T-SFS framework. During the evaluation of expert CLs, these defined AOs are accurately explaining real-life circumstances more precisely under a T-SF environment. Ultimately, a detailed discussion was conducted to elaborate on the superiority and applicability of the proposed AOs over existing AOs.

In the near future, we will expand the range the T-SF confidence averaging and averaging AOs on cubic set Schweizer–Sklar Heronian Mean AOs by Khan [45], an algorithm for fuzzy soft set by Peng and Garg [46], interval-valued Hamacher AOs by Liu. [47], and bipolar PyFS discussed by Mandal [48].

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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