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Research Article

Novel Complex Intuitionistic Hesitant Fuzzy Distance Measures for Solving Decision-Support Problems

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The key objective of this article is to introduce the innovative idea of a complex intuitionistic hesitant fuzzy set (CIHFS), which blends the intuitionistic hesitant fuzzy set with the complex fuzzy set to address the uncertain information in real-life complex problems. In CIHFS, the range of the membership functions is extended from the subset of the real number to the unit disc under the hesitant environment. To determine how well the CIHFSs can be distinguished from one another, we first propose generalized distance measures and weighted generalized distance measures based on the Hamming, Euclidean, and Hausdorff metrics. Some interesting properties and their relationships are thoroughly discussed. Furthermore, a decision-making framework for selecting the optimal option from the feasible set has been proposed, which is grounded in these distance metrics. For the purpose of proving the method's efficacy, we included examples from pattern recognition and medical diagnostics.

1. Introduction

In the process of making a decision based on many factors, a finite set of options is analyzed, and the alternatives are ranked in accordance with how accurate they seem to the person or people making the decision when all the criteria are looked at together. The rating values for each solution in this method take into account objective information from experts as well as accurate facts. However, it is customarily believed that the information they offer is of a clear kind. The information in many MCDM [1] situations in real life is either ambiguous, imprecise, or unpredictable owing to the system's increasing complexity. When all the criteria are taken into account at once, the multiple attribute decision-making technique analyses and arranges the number of options such that they are reliable and accurate to the decision-maker(s) [2–5]. In this method, projections of each option's rating take both hard data and expert opinions into consideration. However, it is frequently assumed that the information they provide is current. Due to the unpredictability of the framework, the real world contains many MADM issues, such as data that are jumbled, missing, or of questionable form.

A fuzzy set (FS) concept [6] has been investigated as a way to manage it, which is one of the most important ways to deal with ambiguity in multiattribute decision-making. Several extensions of this notion were established after the advent of Zadeh's fuzzy sets. His method makes it possible to manipulate fuzzy sets consistently and logically. It can be applied to a wide range of tasks, such as decision-making [7, 8], pattern recognition [9], and medical diagnosis. The authors in [10, 11] introduced the fuzzy set's elements' similarity measures as a significant means of conveying a person's position as a scale of grades, and the authors in [12] presented the comparative study for similarities using fuzzy number and distance measure. But the disadvantage of FSs is that they only consider membership grades that are favorable. Atanassov in 1983 defined intuitionistic fuzzy sets

(IFSs) as a generalized fuzzy set by addressing the flaw in FSs [13, 14]. He considered both positive and negative membership grades but only to the extent that the sum of the two should be less than or equal to one, and for further study on IFSs, see [15]. The intuitionistic fuzzy set of Atanassov could be the most acceptable fuzzy set (IFS or A-IFS for short). Many processes have been outlined and different applications produced in the last few years for these types of fuzzy sets such as several operations were created by researchers in [16], namely, distance between IFS [17], correlation of IFS [18], pattern recognition [19], decision and game theory in management [20], medical diagnosis [21], and divergence measure [22].

As a result of the advancement of this idea, many researchers are curious about what happens when we transform FS from a complex number to a real number, from a unit disc on a plane. The answer to this problem was offered by the investigations of Ramot et al. [23] on the complex fuzzy set (CFS), which conveys the membership grade as a complex integer that is a component of the unit disc in the complex plane. Two-dimensional data are handled by CFS in a single set. CFS is a useful tool for communicating judgment through grades. In recent years, CFS has gotten a lot of attention. Li and Chiang [24] investigated the complex FS and their approximation of function. Yazdanbakhsh and Dick [25] performed a comprehensive review based on CFS and reasoning. Over and above a typical fuzzy set's benefit, the complex fuzzy set provides a distinctive structure. Many researchers in a variety of fields have employed CFS such as operation, properties and equalities [26], distance and continuity [27], neuro-fuzzy ARIMA forecasting approach [28], arithmetic and aggregation operator [29, 30], Heronian mean operators, and applications on CIFS [31]. Researchers also present the hybrid structures of CFS; for instance, Alkouri and Salleh [32] stated that the innovative idea of complex IFS (CIFS), which is distinguished by complex-valued membership and complex-valued nonmembership. According to the CIFS framework, even the total of the real parts of the complex-valued membership degree and the complex-valued nonmembership degree must be equal to or less than 1, novel decision-making approach under complex Pythagorean FS (CPFS) [33], and distance measures of CPFS and their applications in pattern recognition [34].

Sometimes, people act hesitantly when making decisions regarding things, but the abovementioned theories cannot handle this particular situation. Toora [35] introduced the hesitant fuzzy set (HFS) to address these limitations. The membership function of HFS is restricted with the ranges that are a finite subset of [0, 1]. Many MAGDM difficulties have also been solved by using the HFS [36, 37], neutral networks [38], medical diagnosis [39], market prediction [40], and many more. The scholars combine the hesitant fuzzy set (HFS) with the complex fuzzy set (CFS) to create the complex hesitant fuzzy set (CHFS) while retaining the benefits of similarity measures (SM). Levels of membership in the CHFS theory are complex valued and expressed in a reference frame [41].

On the other hand, the current measures are ineffective when a decision-maker assigns complex-valued membership and no-membership grades in the form of groups. Here, the authors created a complex intuitionistic hesitant fuzzy set (CIHFS) to address these kinds of problems while maintaining the benefits of SM. This integrates the CFS, IFS, and HFS. Degrees of membership and nonmembership are complex valued and expressed in polar coordinates in the CIHFS theory. When a decision-maker was presented with this type of data, which provides two-dimensional information in a single set, for instance,

$$\left\{\left(0.7\ddot{e}^{\widetilde{i}2\pi(0.1)}, 0.1\ddot{e}^{\widetilde{i}2\pi(0.3)}, 0.6\ddot{e}^{\widetilde{i}2\pi(0.3)}\right), \left(0.5\ddot{e}^{\widetilde{i}2\pi(0.7)}, 0.1\ddot{e}^{\widetilde{i}2\pi(0.3)}\right)\right\}. \tag{1}$$

Then, all conceptual frameworks are incapable of dealing with such data. The CIHFS is an effective strategy for addressing practical choice difficulties in the framework of the FS theory when dealing with such type of situations. Compared to other theories such as FS, CFS, HFS, and CHFS, CIHFS is more powerful and all-encompassing when it comes to dealing with difficult and complex data in real-world decisions. The borders of the interpreted CHFS are looked at below because all theories are variations of it.

To learn about the "complex" characteristics and functions related to periodicity and complexity, we may refer to it for additional work on CIFS and its drawbacks [32]. Because all ideas are special cases of the construed CIHFS, the construed CIHFS's edges are investigated as follows:

(1) When we set the CIHFS's imaginary parts to zero, the CIHFS is transformed into CHFS in the shape of

$$\left\{0.5\ddot{e}^{\widetilde{i}2\pi(0.6)}, 0.2\ddot{e}^{\widetilde{i}2\pi(0.8)}, 0.6\ddot{e}^{\widetilde{i}2\pi(0.1)}\right\}. \tag{2}$$

(2) When the CIHFS is allowed to exist as a singleton set, it is transformed into the CIFS, which has the shape of

$$\left\langle\left\{0.5\ddot{e}^{\widetilde{i}2\pi(0.6)}\right\}, \left\{0.6\ddot{e}^{\widetilde{i}2\pi(0.1)}\right\}\right\rangle. \tag{3}$$

(3) When the CIHFS is allowed to exist as a singleton set and nonmembership function is zero, then it transformed into CFS, which has the shape of

$$\left\langle \left\{ 0.5\ddot{e}^{\widetilde{i}2\pi(0.6)} \right\} \right\rangle. \tag{4}$$

(4) The CIHFS is transformed into FS, assuming it is a unique set and the entirely imaginary portions and nonmembership degrees are set to zero.

In addition, the use of generalised distance measures (GDMs) and modified GDMs (MGDMs), which are based on existing methodologies, also exposes the special situations of the established approach. The parameterized distance measures are then created, and their unique applications are explored. To assess the viability and validity of the investigated measures, the established measures are applied in a decision-making framework. Furthermore, the numerical examples for the predetermined metrics are solved to demonstrate the excellence and reliability of the research. The adjusted and parameterized distance measures based on CIHFS are finally validated by comparison with some existing distance measures in order to assess their authenticity.

The following are the article's main factors that contribute:

(1) We first establish the CIHFS, which is the union of the IHFS [42] and the CFS [23], based on existing concepts to control difficult and complicated data in the actual-decision theory.

(2) We made generalized distance measures (GDMs) and distance measures that are changed from standard distance measurements, and we explored their specific cases.

(3) We also constructed the parameterized distance measures and their particular scenarios using current distance and SMs as well as parameterized distance measures obtained from [43].

(4) To determine the feasibility and validity of the investigated measures, the established measures are applied in a decision-making framework that can manage delays in terms of when completion has been developed. Complex intuitionistic hesitant fuzzy sets (CIHFSs) are more advantageous in that they can accurately resolve decision-making problems. Additionally, the numerical examples for the predetermined metrics are solved to demonstrate the excellence and reliability of the research. Finally, the adjusted and parameterized distance measures based on CIHFS are validated by comparison with various existing measures in order to assess their applicability.

1.1. Novelty. Some consequential endowments of the current study are as follows:

(1) A novel idea of complex intuitionistic hesitant fuzzy sets (CIHFSs) has to be presented first.

(2) Also, explore the fundamental operational laws for CIHFSs.

(3) Design a decision-making strategy that employs proposed operators to aggregate uncertain data for decision-making difficulties in the part of a best option for industry development.

The remainder of this study is structured as follows. Section 2 presents some basic concepts of FSs, HFSs, and CHFSs are briefly reviewed. Basic notations and concepts are described. A novel notion of CIHFSs is presented in Sections 3 and 4, respectively. Section 5 presents a decision-making structure. The validity and reliability tests are presented in Section 5 to ensure that the suggested approach is effective. This manuscript comes to a close with Section 6.

2. Preliminaries

This section presented some fundamental concepts of FSs, CFSs, IFSs, HFSs, and CIHFSs and also discussed their core characteristics. κ serves as a fix set throughout this article.

Definition 1 (see [6]). Fuzzy sets $\breve{A}$ is formed such that

$$\breve{A} = \left\{ \left\langle \Xi_{\breve{A}}(\kappa) \right\rangle \mid \kappa \in \chi \right\}, \tag{5}$$

under the condition $0 \leqslant \Xi_{\breve{A}}(\kappa) \leqslant 1$, where $\Xi_{\breve{A}}(\kappa)$ shows the membership of $\breve{A}$ in κ. The collection of all sources of information used in this article for fuzzy numbers on κ is denoted by FS (κ).

Definition 2. Let $\varphi, \psi \in \mathrm{FS}(\kappa)$. Then, the basic operations are defined as follows:

(1) $\varphi^c = \left\{ \left\langle 1 - \Xi_{\varphi}(\kappa) \right\rangle \mid \kappa \in \chi \right\}$
(2) $\varphi \cap \psi = \left\{ \left\langle \min\left\{ \Xi_{\varphi}(\kappa), \Xi_{\psi}(\kappa) \right\} \right\rangle \mid \kappa \in \chi \right\}$
(3) $\varphi \cup \psi = \left\{ \left\langle \max\left\{ \Xi_{\varphi}(\kappa), \Xi_{\psi}(\kappa) \right\} \right\rangle \mid \kappa \in \chi \right\}$

Definition 3 (see [23]). A complex fuzzy set $\acute{A}$ is formed such that

$$\acute{A} = \left\{ \left\langle \Xi_{\acute{A}}(\kappa) \right\rangle \mid \kappa \in \chi \right\}, \tag{6}$$

where $\Xi_{\acute{A}}(\kappa) = ¢_{\acute{A}}(\kappa).\ddot{e}^{\widetilde{i}2\pi(\varphi_{¢\acute{A}}(\kappa))}$ shows the complex's membership in the polar coordinate form, where $¢_{\acute{A}}(\kappa), \varphi_{¢\acute{A}}(\kappa) \in [0, 1]$.

Definition 4. For any two CFNs, $\varphi = \left\langle \kappa, ¢_{\varphi}(\kappa).\ddot{e}^{\widetilde{i}2\pi(\varphi_{¢\varphi}(\kappa))} \right\rangle$ and $\psi = \left\langle \kappa, ¢_{\psi}(\kappa).\ddot{e}^{\widetilde{i}2\pi(\varphi_{¢\psi}(\kappa))} \right\rangle$, then

(1) $\varphi \cup \psi = \left\{ \left\langle \max\left\{ ¢_{\varphi}(\kappa), ¢_{\psi}(\kappa) \right\} \right\rangle \right\} \ddot{e}^{\widetilde{i}2\pi(\max\left\{ \varphi_{\varphi}(\kappa), \varphi_{\psi}(\kappa) \right\}).}$
(2) $\varphi \cap \psi = \left\{ \left\langle \min\left\{ ¢_{\varphi}(\kappa), ¢_{\psi}(\kappa) \right\} \right\rangle \right\} \ddot{e}^{\widetilde{i}2\pi(\min\left\{ \varphi_{\varphi}(\kappa), \varphi_{\psi}(\kappa) \right\}).}$
(3) $\varphi^c = \left\{ \left\langle \left\{ 1 - ¢_{\varphi}(\kappa) \right\} \ddot{e}^{\widetilde{i}2\pi\left\{ 1 - \varphi_{\varphi}(\kappa) \right\}} \right\rangle \right\}$

Definition 5 (see [35]). A hesitant fuzzy number (HFN) $\tilde{A}$ is formed such that

$$\widetilde{A} = \left\{\left\langle \lambda_{\widetilde{A}}(\kappa) \mid \kappa \in \chi \right\rangle\right\}, \tag{7}$$

where $\lambda_{\widetilde{A}}(\kappa)$ is the subset of interval [0, 1] known to be membership grade of $\kappa \in \chi$.

Definition 6 Let $\varphi = \left\langle \lambda_\varphi(\kappa) \right\rangle$ and $\psi = \left\langle \lambda_\psi(\kappa) \right\rangle$ be two HFNs. Then, the basic operations are defined as

(1) $\varphi \cup \psi = \left\{\left\langle P_{¢_1 \in \lambda_\varphi(\kappa), ¢_2 \in \lambda_\psi(\kappa)}\{\max(¢_1, ¢_2)\} \right\rangle\right\}$

(2) $\varphi \cap \psi = \left\{\left\langle P_{¢_1 \in \lambda_\varphi(\kappa), ¢_2 \in \lambda_\psi(\kappa)}\{\min(¢_1, ¢_2)\} \right\rangle\right\}$

(3) $\varphi^c = \left\{\left\langle P_{¢ \in \lambda(\kappa)}\{1 - ¢\} \right\rangle\right\}$

Definition 7 (see [14]). A intuitionistic fuzzy set (IFS) $\overline{A}$ is of the following form:

$$\overline{A} = \left\{\left\langle \Xi_{\overline{A}}(\kappa), \lambda_{\overline{A}}(\kappa) \right\rangle \mid \kappa \in \chi\right\}, \tag{8}$$

where $\Xi_{\overline{A}}(\kappa), \lambda_{\overline{A}}(\kappa)$: $\kappa \longrightarrow [0, 1]$ shows the degree of membership and degree of nonmembership of x in $\breve{A}$, and its value lies between zero and one such that $\Xi_{\overline{A}} + \lambda_{\overline{A}} \leq 1$ and the pair $\left\langle \Xi_{\overline{A}}, \lambda_{\overline{A}} \right\rangle$ is called the intuitionistic fuzzy number.

Definition 8 (see [16]). Let $\varphi = \left\langle \Xi_\varphi(\kappa), \lambda_\varphi(\kappa) \right\rangle$ and $\psi = \left\langle \Xi_\psi(\kappa), \lambda_\psi(\kappa) \right\rangle$ be two IFSs. Then, the basic operation are defined as

(1) $\varphi \cap \psi = \left\{\left\langle \min(\Xi_\varphi(\kappa), \Xi_\psi(\kappa)), \max(\lambda_\varphi(\kappa), \lambda_\psi(\kappa)) \right\rangle \mid \kappa \in \chi\right\}$

(2) $\varphi \cup \psi = \left\{\left\langle \max(\Xi_\varphi(\kappa), \Xi_\psi(\kappa)), \min(\lambda_\varphi(\kappa), \lambda_\psi(\kappa)) \right\rangle \mid \kappa \in \chi\right\}$

(3) $\varphi^c = \left\{\left\langle \lambda_\varphi(\kappa), \Xi_\varphi(\kappa) \right\rangle\right\}$

3. Construction of Complex Intuitionistic Hesitant Fuzzy Sets

We take a look at the concept of CIHFSs and its rudimentary operating principles here. The established work was also confirmed with the aid of various numerical examples.

Definition 9. A CIHFS$_{\overline{A}}$ has the form

$$\overline{A} = \left\{\left\langle \Xi_{\overline{A}}(\kappa), \lambda_{\overline{A}}(\kappa) \right\rangle \mid \kappa \in \chi\right\}, \tag{9}$$

where $\Xi_{\overline{A}}(\kappa), \lambda_{\overline{A}} = \left\langle \left\{¢_{\overline{A}_k}(\kappa).\ddot{e}^{\widetilde{\imath}2\pi(\Xi_{¢_{\overline{A}_k}}(\kappa))}, \beta_{\overline{A}_k}(\kappa).\ddot{e}^{\widetilde{\imath}2\pi(\lambda_{\beta_{\overline{A}_k}}(\kappa))}\right\}, k = 1, 2, 3, \ldots, n \right\rangle$; the conditional membership and nonmembership of complex systems is a subset of the disc in the following complex plan: $0 \leq \left\{(\max ¢_{\overline{A}_k}(\kappa)) + (\min \beta_{\overline{A}_k}(\kappa))\right\} \leq 1$ and $0 \leq \left\{\max(\Xi_{¢_{\overline{A}_k}}(\kappa)) + \min(\lambda_{\beta_{\overline{A}_k}}(\kappa))\right\} \leq 1, ¢_{\overline{A}_k}(\kappa), \beta_{\overline{A}_k}(\kappa), \Xi_{¢_{\overline{A}_k}}(\kappa), \lambda_{\beta_{\overline{A}_k}}(\kappa) \in [0, 1]$.

Also, $\overline{A} = \left\langle \kappa, ¢_{\overline{A}_k}(\kappa).\ddot{e}^{\widetilde{\imath}2\pi(\Xi_{¢_{\overline{A}_k}}(\kappa))}, \beta_{\overline{A}_k}(\kappa).\ddot{e}^{\widetilde{\imath}2\pi(\lambda_{\beta_{\overline{A}_k}}(\kappa))} \right\rangle$ is called complex intuitionistic HFN (CIHFN).

Definition 10. Let $\varphi = (¢_{\varphi_k}(\kappa).\ddot{e}^{\widetilde{\imath}2\pi(\varrho_{¢_{\varphi_k}}(\kappa))}, \beta_{\varphi_k}(\kappa).\ddot{e}^{\widetilde{\imath}2\pi(\alpha_{¢_{\varphi_k}}(\kappa))})$ and $\psi = (¢_{\psi_k}(\kappa).\ddot{e}^{\widetilde{\imath}2\pi(\varrho_{¢_{\psi_k}}(\kappa))}, \beta_{\psi_k}(\kappa).\ddot{e}^{\widetilde{\imath}2\pi(\alpha_{¢_{\psi_k}}(\kappa))})$ be two CIHFNs. Then,

(1)

$$\varphi \cup \psi = \left\{\max\left(¢_{\varphi_k}(\kappa), ¢_{\psi_k}(\kappa)\right).\ddot{e}^{\widetilde{\imath}2\pi\left(\max\left(\varrho_{¢_{\varphi_k}}(\kappa), \varrho_{¢_{\psi_K}}(\kappa)\right)\right)}, \min\left(\beta_{\varphi_k}(\kappa), \beta_{\psi_k}(\kappa)\right).\ddot{e}^{\widetilde{\imath}2\pi\left(\min\left(\alpha_{¢_{\varphi_k}}(\kappa), \alpha_{¢_{\psi_K}}(\kappa)\right)\right)}\right\}. \tag{10}$$

(2)

$$\varphi \cap \psi = \left\{\min\left(¢_{\varphi_k}(\kappa), ¢_{\psi_k}(\kappa)\right).\ddot{e}^{\widetilde{\imath}2\pi\left(\min\left(\varrho_{¢_{\varphi_k}}(\kappa), \varrho_{¢_{\psi_K}}(\kappa)\right)\right)}, \max\left(\beta_{\varphi_k}(\kappa), \beta_{\psi_k}(\kappa)\right).\ddot{e}^{\widetilde{\imath}2\pi\left(\max\left(\alpha_{¢_{\varphi_k}}(\kappa), \alpha_{¢_{\psi_K}}(\kappa)\right)\right)}\right\}. \tag{11}$$

(3)

$$c\left\{\alpha_{\overline{A}}(\kappa), \varrho_{\overline{A}}(\kappa)\right\} = \left\{\left\langle \varrho_\varphi(\kappa), \alpha_\varphi(\kappa) \right\rangle\right\}. \tag{12}$$

The CIHFS concept makes it an effective and useful tool for dealing with difficult and sophisticated data in critical decision-making situations. A CIHFS degree is a collection of polar coordinates, which is a subset of the unit disc in the complex plane. In essence, two-dimensional data are saved by the CIHFS as a single set. As will be discussed in more depth in the following section, the CIHFS interpretation is broader in scope than the prevailing theories of FS, CFS, and HFS. To represent the complex-valued membership degree in CHFS [41], polar coordinates are employed. The amplitude term represents the extent to which an item has something, whereas the phase term indicates the supplementary data, which are often related to periodicity. The traditional FS, CFS, and HFS theories are distinguished from

one another by the phase terms, which are brand-new membership degree parameters. The HFS hypothesis only accounts for one dimension at a time, which occasionally causes data leakage.

Definition 11. Let

$$\varphi = \left\{ \begin{array}{c} \left\langle \kappa_1, \left\{0.2\ddot{e}^{\widetilde{i}2\pi(0.5)}, 0.3\ddot{e}^{\widetilde{i}2\pi(0.7)}, 0.6\ddot{e}^{\widetilde{i}2\pi(0.3)}\right\}, \left\{0.3\ddot{e}^{\widetilde{i}2\pi(0.2)}, 0.4\ddot{e}^{\widetilde{i}2\pi(0.5)}\right\}\right\rangle \\ \left\langle \kappa_2, \left\{0.3\ddot{e}^{\widetilde{i}2\pi(0.4)}, 0.6\ddot{e}^{\widetilde{i}2\pi(0.3)}\right\}, \left\{0.2\ddot{e}^{\widetilde{i}2\pi(0.2)}, 0.4\ddot{e}^{\widetilde{i}2\pi(0.5)}\right\}\right\rangle \\ \left\langle \kappa_3, \left\{0.7\ddot{e}^{\widetilde{i}2\pi(0.4)}, 0.3\ddot{e}^{\widetilde{i}2\pi(0.3)}, 0.2\ddot{e}^{\widetilde{i}2\pi(0.2)}\right\}, \left\{0.1\ddot{e}^{\widetilde{i}2\pi(0.2)}, 0.6\ddot{e}^{\widetilde{i}2\pi(0.5)}\right\}\right\rangle \\ \left\langle \kappa_4, \left\{0.8\ddot{e}^{\widetilde{i}2\pi(0.3)}, 0.5\ddot{e}^{\widetilde{i}2\pi(0.4)}, 0.2\ddot{e}^{\widetilde{i}2\pi(0.1)}\right\}, \left\{0.1\ddot{e}^{\widetilde{i}2\pi(0.3)}, 0.2\ddot{e}^{\widetilde{i}2\pi(0.4)}\right\}\right\rangle \end{array} \right\},$$

$$\psi = \left\{ \begin{array}{c} \left\langle \kappa_1, \left\{0.5\ddot{e}^{\widetilde{i}2\pi(0.6)}, 0.2\ddot{e}^{\widetilde{i}2\pi(0.8)}, 0.6\ddot{e}^{\widetilde{i}2\pi(0.1)}\right\}, \left\{0.1\ddot{e}^{\widetilde{i}2\pi(0.5)}, 0.2\ddot{e}^{\widetilde{i}2\pi(0.3)}, 0.3\ddot{e}^{\widetilde{i}2\pi(0.4)}\right\}\right\rangle \\ \left\langle \kappa_2, \left\{0.3\ddot{e}^{\widetilde{i}2\pi(0.4)}, 0.6\ddot{e}^{\widetilde{i}2\pi(0.3)}\right\}, \left\{0.2\ddot{e}^{\widetilde{i}2\pi(0.2)}, 0.4\ddot{e}^{\widetilde{i}2\pi(0.5)}, 0.6\ddot{e}^{\widetilde{i}2\pi(0.5)}\right\}\right\rangle \\ \left\langle \kappa_3, \left\{0.6\ddot{e}^{\widetilde{i}2\pi(0.4)}, 0.3\ddot{e}^{\widetilde{i}2\pi(0.3)}, 0.2\ddot{e}^{\widetilde{i}2\pi(0.2)}, 0.1\ddot{e}^{\widetilde{i}2\pi(0.5)}\right\}, \left\{0.1\ddot{e}^{\widetilde{i}2\pi(0.2)}, 0.6\ddot{e}^{\widetilde{i}2\pi(0.5)}\right\}\right\rangle \\ \left\langle \kappa_4, \left\{0.7\ddot{e}^{\widetilde{i}2\pi(0.3)}, 0.5\ddot{e}^{\widetilde{i}2\pi(0.4)}, 0.2\ddot{e}^{\widetilde{i}2\pi(0.1)}\right\}, \left\{0.2\ddot{e}^{\widetilde{i}2\pi(0.5)}, 0.3\ddot{e}^{\widetilde{i}2\pi(0.5)}\right\}\right\rangle \end{array} \right\}. \tag{13}$$

Example 1. For any two complex intuitionistic hesitant fuzzy numbers, the basic operational laws are described as follows:

(1)

$$\varphi \cup \psi = \left\{ \begin{array}{c} \left\{0.5\ddot{e}^{\widetilde{i}2\pi(0.6)}, 0.3\ddot{e}^{\widetilde{i}2\pi(0.7)}, 0.6\ddot{e}^{\widetilde{i}2\pi(0.3)}\right\}, \left\{0.1\ddot{e}^{\widetilde{i}2\pi(0.2)}, 0.2\ddot{e}^{\widetilde{i}2\pi(0.3)}\right\} \\ \left\{0.3\ddot{e}^{\widetilde{i}2\pi(0.4)}, 0.6\ddot{e}^{\widetilde{i}2\pi(0.3)}\right\}, \left\{0.2\ddot{e}^{\widetilde{i}2\pi(0.2)}, 0.4\ddot{e}^{\widetilde{i}2\pi(0.3)}\right\} \\ \left\{0.7\ddot{e}^{\widetilde{i}2\pi(0.4)}, 0.3\ddot{e}^{\widetilde{i}2\pi(0.3)}, 0.2\ddot{e}^{\widetilde{i}2\pi(0.2)}\right\}, \left\{0.1\ddot{e}^{\widetilde{i}2\pi(0.2)}, 0.6\ddot{e}^{\widetilde{i}2\pi(0.5)}\right\} \\ \left\{0.8\ddot{e}^{\widetilde{i}2\pi(0.3)}, 0.5\ddot{e}^{\widetilde{i}2\pi(0.4)}, 0.2\ddot{e}^{\widetilde{i}2\pi(0.1)}\right\}, \left\{0.1\ddot{e}^{\widetilde{i}2\pi(0.3)}, 0.2\ddot{e}^{\widetilde{i}2\pi(0.4)}\right\} \end{array} \right\}. \tag{14}$$

(2)

$$\varphi \cap \psi = \left\{ \begin{array}{c} \left\{0.2\ddot{e}^{\widetilde{\imath 2\pi}(0.5)}, 0.2\ddot{e}^{\widetilde{\imath 2\pi}(0.7)}, 0.6\ddot{e}^{\widetilde{\imath 2\pi}(0.1)}\right\}, \left\{0.3\ddot{e}^{\widetilde{\imath 2\pi}(0.5)}, 0.4\ddot{e}^{\widetilde{\imath 2\pi}(0.5)}\right\} \\ \left\{0.3\ddot{e}^{\widetilde{\imath 2\pi}(0.4)}, 0.6\ddot{e}^{\widetilde{\imath 2\pi}(0.3)}\right\}, \left\{0.2\ddot{e}^{\widetilde{\imath 2\pi}(0.2)}, 0.4\ddot{e}^{\widetilde{\imath 2\pi}(0.5)}, 0.6\ddot{e}^{\widetilde{\imath 2\pi}(0.5)}\right\} \\ \left\{0.6\ddot{e}^{\widetilde{\imath 2\pi}(0.4)}, 0.3\ddot{e}^{\widetilde{\imath 2\pi}(0.3)}, 0.2\ddot{e}^{\widetilde{\imath 2\pi}(0.2)}, 0.1\ddot{e}^{\widetilde{\imath 2\pi}(0.5)}\right\}, \left\{0.1\ddot{e}^{\widetilde{\imath 2\pi}(0.2)}, 0.6\ddot{e}^{\widetilde{\imath 2\pi}(0.5)}\right\} \\ \left\{0.7\ddot{e}^{\widetilde{\imath 2\pi}(0.3)}, 0.5\ddot{e}^{\widetilde{\imath 2\pi}(0.4)}, 0.2\ddot{e}^{\widetilde{\imath 2\pi}(0.1)}\right\}, \left\{0.2\ddot{e}^{\widetilde{\imath 2\pi}(0.5)}, 0.3\ddot{e}^{\widetilde{\imath 2\pi}(0.5)}\right\} \end{array} \right\}. \tag{15}$$

(3)

$$\varphi^C = \left\{ \begin{array}{c} \left\{0.3\ddot{e}^{\widetilde{\imath 2\pi}(0.2)}, 0.4\ddot{e}^{\widetilde{\imath 2\pi}(0.5)}\right\}, \left\{0.2\ddot{e}^{\widetilde{\imath 2\pi}(0.5)}, 0.3\ddot{e}^{\widetilde{\imath 2\pi}(0.7)}, 0.6\ddot{e}^{\widetilde{\imath 2\pi}(0.3)}\right\} \\ \left\{0.2\ddot{e}^{\widetilde{\imath 2\pi}(0.2)}, 0.4\ddot{e}^{\widetilde{\imath 2\pi}(0.5)}\right\}, \left\{0.3\ddot{e}^{\widetilde{\imath 2\pi}(0.4)}, 0.6\ddot{e}^{\widetilde{\imath 2\pi}(0.3)}\right\} \\ \left\{0.1\ddot{e}^{\widetilde{\imath 2\pi}(0.2)}, 0.6\ddot{e}^{\widetilde{\imath 2\pi}(0.5)}\right\}, \left\{0.7\ddot{e}^{\widetilde{\imath 2\pi}(0.4)}, 0.3\ddot{e}^{\widetilde{\imath 2\pi}(0.3)}, 0.2\ddot{e}^{\widetilde{\imath 2\pi}(0.2)}\right\} \\ \left\{0.1\ddot{e}^{\widetilde{\imath 2\pi}(0.3)}, 0.2\ddot{e}^{\widetilde{\imath 2\pi}(0.4)}\right\}, \left\{0.8\ddot{e}^{\widetilde{\imath 2\pi}(0.3)}, 0.5\ddot{e}^{\widetilde{\imath 2\pi}(0.4)}, 0.2\ddot{e}^{\widetilde{\imath 2\pi}(0.1)}\right\} \end{array} \right\}. \tag{16}$$

4. Proposed Technique

Here, we define a few distance measures for CIHFSs.

Definition 12. Consider φ and ψ as two CIHFSs on set κ. Then, calculate the distance between φ and ψ determined by $d_\varsigma(\varphi, \psi)$, which fulfils the attributes described as follows:

(1) $0 \le d_\varsigma(\varphi, \psi) \le 1$

(2) $d_\varsigma(\varphi, \psi) = 0$ iff $\varphi = \psi$

(3) $d_\varsigma(\varphi, \psi) = d_\varsigma(\psi, \varphi)$

Definition 13. Let φ and ψ be two CIHFSs on set κ. Then, the similarly measurement between φ and ψ is determined by $S_\varsigma(\varphi, \psi)$, which fulfils the attributes described as follows:

(1) $0 \le d_\varsigma(\varphi, \psi) \le 1$

(2) $d_\varsigma(\varphi, \psi) = 1$ iff $\varphi = \psi$

(3) $d_\varsigma(\varphi, \psi) = d_\varsigma(\psi, \varphi)$

Remark 14

(1) If d_ς is the measurement of distance between two CIHFSs φ and ψ, then $d_\varsigma(\varphi, \psi) = 1 - d_\varsigma(\varphi, \psi)$ is the SM between CIHFSs φ and ψ

(2) If d_ς is the similarly measurement of distance between two CIHFSs φ and ψ, then $d_\varsigma(\varphi, \psi) = 1 - d_\varsigma(\varphi, \psi)$ is the distance measure between CIHFSs φ and ψ

Definition 15. Let φ and ψ be two CIHFSs on set X. Then, GCIHND is defined as follows:

$$d_{\varsigma_q}(\varphi, \psi) = \left[\begin{array}{l} \frac{1}{l} \sum_{i=1}^{l} \frac{1}{2} \left(\frac{1}{n} \sum_{h=1}^{n} \left(\left| \mathfrak{C}_{\varphi_h}(\kappa_i) - \mathfrak{C}_{\psi_h}(\kappa_i) \right|^\eta + \left| \varrho_{\mathfrak{C}_{\varphi_h}}(\kappa_i) - \varrho_{\mathfrak{C}_{\psi_h}}(\kappa_i) \right|^\eta \right) \right) \\ + \left(\frac{1}{m} \sum_{h=1}^{m} \left(\left| \beta_{\varphi_h}(\kappa_i) - \beta_{\psi_h}(\kappa_i) \right|^\eta + \left| \alpha_{\mathfrak{C}_{\varphi_h}}(\kappa_i) - \alpha_{\mathfrak{C}_{\psi_h}}(\kappa_i) \right|^\eta \right) \right) \end{array} \right]^{1/\eta}, \tag{17}$$

where $\eta > 0$.

Theorem 16. *The GCIHND satisfies the following three properties:*

(1) $0 \le d_{\varsigma}(\varphi, \psi) \le 1$

(2) $d_{\varsigma}(\varphi, \psi) = 0$ *iff* $\varphi = \psi$

(3) $d_{\varsigma}(\varphi, \psi) = d_{\varsigma}(\psi, \varphi)$

Proof

(1) Since $1/n\sum_{h=1}^{n}|¢_{\varphi_h}(\kappa_i) - ¢_{\psi_h}(\kappa_i)|^{\eta} \in [0,1]$, $1/n\sum_{h=1}^{n}|\varrho_{¢_{\varphi_h}}(\kappa_i) - \varrho_{¢_{\psi_h}}(\kappa_i)|^{\eta} \in [0, 1]$, $1/m\sum_{h=1}^{m}|\beta_{\varphi_h}(\kappa_i) - \beta_{\psi_h}(\kappa_i)|^{\eta} \in [0, 1]$, $1/m\sum_{h=1}^{m}|\alpha_{¢_{\varphi_h}}(\kappa_i) - \alpha_{¢_{\psi_h}}(\kappa_i)|^{\eta} \in [0, 1]$, then for $i = 11/2\,(1/n\sum_{h=1}^{n}(|¢_{\varphi_h}(\kappa_1) - ¢_{\psi_h}(\kappa_1)|^{\eta} + |\varrho_{¢_{\varphi_h}}(\kappa_1) - \varrho_{¢_{\psi_h}}(\kappa_1)|^{\eta})$, $(1/m\sum_{h=1}^{m}(|\beta_{\varphi_h}(\kappa_1) - \beta_{\psi_h}(\kappa_1)|^{\eta} + |\alpha_{¢_{\varphi_h}}(\kappa_1) - \alpha_{¢_{\psi_h}}(\kappa_1)|^{\eta})) \in [0, 1]$ for $i = 21/2\,(1/n\sum_{h=1}^{n}(|¢_{\varphi_h}(\kappa_2) - ¢_{\psi_h}(\kappa_1)|^{\eta} + |\varrho_{¢_{\varphi_h}}(\kappa_2) - \varrho_{¢_{\psi_h}}(\kappa_2)|^{\eta}) + (1/m\sum_{h=1}^{m}(|\beta_{\varphi_h}(\kappa_2) - \beta_{\psi_h}(\kappa_2)|^{\eta} + |\alpha_{¢_{\varphi_h}}(\kappa_2) - \alpha_{¢_{\psi_h}}(\kappa_2)|^{\eta})) \in [0,1]$.

By continuing this process, we obtain

$$
\begin{aligned}
&\sum_{i=1}^{l}\frac{1}{2}\left(\frac{1}{n}\sum_{h=1}^{n}\left(\left|¢_{\varphi_h}(\kappa_i) - ¢_{\psi_h}(\kappa_i)\right|^{\eta} + \left|\varrho_{¢_{\varphi_h}}(\kappa_i) - \varrho_{¢_{\psi_h}}(\kappa_i)\right|^{\eta}\right)\right) \\
&\quad + \left(\frac{1}{m}\sum_{h=1}^{m}\left(\left|\beta_{\varphi_h}(\kappa_i) - \beta_{\psi_h}(\kappa_i)\right|^{\eta} + \left|\alpha_{¢_{\varphi_h}}(\kappa_i) - \alpha_{¢_{\psi_h}}(\kappa_i)\right|^{\eta}\right)\right) \in [0, 1] \\
&0 \le \sum_{i=1}^{l}\frac{1}{2}\left(\frac{1}{n}\sum_{h=1}^{n}\left(\left|¢_{\varphi_h}(\kappa_i) - ¢_{\psi_h}(\kappa_i)\right|^{\eta} + \left|\varrho_{¢_{\varphi_h}}(\kappa_i) - \varrho_{¢_{\psi_h}}(\kappa_i)\right|^{\eta}\right)\right) \\
&\quad + \left(\frac{1}{m}\sum_{h=1}^{m}\left(\left|\beta_{\varphi_h}(\kappa_i) - \beta_{\psi_h}(\kappa_i)\right|^{\eta} + \left|\alpha_{¢_{\varphi_h}}(\kappa_i) - \alpha_{¢_{\psi_h}}(\kappa_i)\right|^{\eta}\right)\right) \in [0, 1] \le n \\
&\Longrightarrow 0 \le \frac{1}{l}\sum_{i=1}^{l}\frac{1}{2}\left(\frac{1}{n}\sum_{i=1}^{n}\left(\left|¢_{\varphi_h}(\kappa_i) - ¢_{\psi_h}(\kappa_i)\right|^{\eta} + \left|\varrho_{¢_{\varphi_h}}(\kappa_i) - \varrho_{¢_{\psi_h}}(\kappa_i)\right|^{\eta}\right)\right) \\
&\quad + \left(\frac{1}{m}\sum_{h=1}^{m}\left(\left|\beta_{\varphi_h}(\kappa_i) - \beta_{\psi_h}(\kappa_i)\right|^{\eta} + \left|\alpha_{¢_{\varphi_h}}(\kappa_i) - \alpha_{¢_{\psi_h}}(\kappa_i)\right|^{\eta}\right)\right) \le 1, \\
&\Longrightarrow 0 \le \left[\begin{array}{l}\frac{1}{l}\sum_{i=1}^{l}\frac{1}{2}\left(\frac{1}{n}\sum_{h=1}^{n}\left(\left|¢_{\varphi_h}(\kappa_i) - ¢_{\psi_h}(\kappa_i)\right|^{\eta} + \left|\varrho_{¢_{\varphi_h}}(\kappa_i) - \varrho_{¢_{\psi_h}}(\kappa_i)\right|^{\eta}\right)\right) \\ + \left(\frac{1}{m}\sum_{h=1}^{m}\left(\left|\beta_{\varphi_h}(\kappa_i) - \beta_{\psi_h}(\kappa_i)\right|^{\eta} + \left|\alpha_{¢_{\varphi_h}}(\kappa_i) - \alpha_{¢_{\psi_h}}(\kappa_i)\right|^{\eta}\right)\right)\end{array}\right]^{1/\eta} \le 1, \\
&\Longrightarrow 0 \le d_{\varsigma_g}(\varphi, \psi) \le 1.
\end{aligned}
\tag{18}
$$

(2) By definition, we have

$$
d_{\varsigma_g}(\varphi, \psi) = \left[\begin{array}{l}\frac{1}{l}\sum_{i=1}^{l}\frac{1}{2}\left(\frac{1}{n}\sum_{i=1}^{n}\left(\left|¢_{\varphi_h}(\kappa_i) - ¢_{\psi_h}(\kappa_i)\right|^{\eta} + \left|\varrho_{¢_{\varphi_h}}(\kappa_i) - \varrho_{¢_{\psi_h}}(\kappa_i)\right|^{\eta}\right)\right) \\ + \left(\frac{1}{m}\sum_{h=1}^{m}\left(\left|\beta_{\varphi_h}(\kappa_i) - \beta_{\psi_h}(\kappa_i)\right|^{\eta} + \left|\alpha_{¢_{\varphi_h}}(\kappa_i) - \alpha_{¢_{\psi_h}}(\kappa_i)\right|^{\eta}\right)\right)\end{array}\right]^{1/\eta}. \tag{19}
$$

Suppose that $d_{\varsigma_g}(\varphi,\psi)=0$, then

$$\left[\begin{array}{l}\frac{1}{l}\sum_{i=1}^{l}\frac{1}{2}\left(\frac{1}{n}\sum_{h=1}^{n}\left(\left|\mathfrak{c}_{\varphi_h}(\kappa_i)-\mathfrak{c}_{\psi_h}(\kappa_i)\right|^{\eta}+\left|\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i)-\varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|^{\eta}\right)\right)+\\ \left(\frac{1}{m}\sum_{h=1}^{m}\left(\left|\beta_{\varphi_h}(\kappa_i)-\beta_{\psi_h}(\kappa_i)\right|^{\eta}+\left|\alpha_{\mathfrak{c}_{\varphi_h}}(\kappa_i)-\alpha_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|^{\eta}\right)\right)\end{array}\right]^{1/\eta}=0$$

$$\Longrightarrow\frac{1}{n}\sum_{h=1}^{n}\left|\mathfrak{c}_{\varphi_h}(\kappa_i)-\mathfrak{c}_{\psi_h}(\kappa_i)\right|^{\eta}=0,\frac{1}{n}\sum_{h=1}^{n}\left|\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i)-\varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|^{\eta}=0+\frac{1}{m}\sum_{h=1}^{m}\left|\beta_{\varphi_h}(\kappa_i)-\beta_{\psi_h}(\kappa_i)\right|^{\eta}=0,$$

$$\frac{1}{m}\sum_{h=1}^{m}\left|\alpha_{\mathfrak{c}_{\varphi_h}}(\kappa_i)-\alpha_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|^{\eta}=0\Longrightarrow\left(\left|\mathfrak{c}_{\varphi_h}(\kappa_i)-\mathfrak{c}_{\psi_h}(\kappa_i)\right|=0,\left|\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i)-\varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|=0\right.$$

$$+\left(\left|\beta_{\varphi_h}(\kappa_i)-\beta_{\psi_h}(\kappa_i)\right|=0,\left|\alpha_{\mathfrak{c}_{\varphi_h}}(\kappa_i)-\alpha_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|=0\right)\Longrightarrow\left(\mathfrak{c}_{\varphi_h}(\kappa_i)=\mathfrak{c}_{\psi_h}(\kappa_i),\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i)=\varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right)$$

$$+\left(\beta_{\varphi_h}(\kappa_i)=\beta_{\psi_h}(\kappa_i),\alpha_{\mathfrak{c}_{\varphi_h}}(\kappa_i)=\alpha_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right),\quad\forall\kappa_i\in\kappa\Longrightarrow(\lambda_r(\kappa)=\lambda_s(\kappa))+(\xi_r(\kappa)=\xi_s(\kappa))\Longrightarrow\varphi=\psi.\tag{20}$$

Conversely, suppose $\varphi=\psi$, then

$$\left(\lambda_{\varphi}(\kappa)=\lambda_{\psi}(\kappa)\right)+\left(\xi_{\varphi}(\kappa)=\xi_{\psi}(\kappa)\right)$$

$$\Longrightarrow\left(\mathfrak{c}_{\varphi_h}(\kappa_i)=\mathfrak{c}_{\psi_h}(\kappa_i),\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i)=\varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right)+\left(\beta_{\varphi_h}(\kappa_i)=\beta_{\psi_h}(\kappa_i),\alpha_{\mathfrak{c}_{\varphi_h}}(\kappa_i)=\alpha_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right),\quad\forall\kappa_i\in\chi,$$

$$\Longrightarrow d_{\varsigma_g}(\varphi,\psi)=\left[\begin{array}{l}\frac{1}{l}\sum_{i=1}^{l}\frac{1}{2}\left(\frac{1}{n}\sum_{h=1}^{n}\left(\left|\mathfrak{c}_{\varphi_h}(\kappa_i)-\mathfrak{c}_{\psi_h}(\kappa_i)\right|^{\eta}+\left|\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i)-\varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|^{\eta}\right)\right)\\ +\left(\frac{1}{m}\sum_{h=1}^{m}\left(\left|\beta_{\varphi_h}(\kappa_i)-\beta_{\psi_h}(\kappa_i)\right|^{\eta}+\left|\alpha_{\mathfrak{c}_{\varphi_h}}(\kappa_i)-\alpha_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|^{\eta}\right)\right)\end{array}\right]^{1/\eta}.$$

$$\Longrightarrow d_{\varsigma_g}(\varphi,\psi)=0.\tag{21}$$

□

Remark 17

(1) If $\eta=1$, then GCHIND take up a Hamming complex intuitionistic hesitant normalized distance (HCIHND) between φ and ψ, i.e.,

$$d_{\varsigma_k}(\varphi,\psi)=\left(\begin{array}{l}\frac{1}{l}\sum_{i=1}^{l}\frac{1}{2}\left(\frac{1}{n}\sum_{h=1}^{n}\left(\left|\mathfrak{c}_{\varphi_h}(\kappa_i)-\mathfrak{c}_{\psi_h}(\kappa_i)\right|+\left|\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i)-\varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|\right)\right)\\ +\left(\frac{1}{m}\sum_{h=1}^{m}\left(\left|\beta_{\varphi_h}(\kappa_i)-\beta_{\psi_h}(\kappa_i)\right|+\left|\alpha_{\mathfrak{c}_{\varphi_h}}(\kappa_i)-\alpha_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|\right)\right)\end{array}\right).\tag{22}$$

(2) If $\eta = 2$, then GCIHND takes up an Euclidean complex intuitionistic hesitant normalized distance (ECIHND) between φ and ψ, that is,

$$d_{\varsigma_f}(\varphi,\psi) = \left(\begin{array}{l} \frac{1}{l}\sum_{i=1}^{l}\frac{1}{2}\left(\frac{1}{n}\sum_{i=1}^{n}\left(\left|\mathfrak{c}_{\varphi_h}(\kappa_i) - \mathfrak{c}_{\psi_h}(\kappa_i)\right|^2 + \left|\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i) - \varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|^2\right)\right) \\ + \left(\frac{1}{m}\sum_{h=1}^{m}\left(\left|\beta_{\varphi_h}(\kappa_i) - \beta_{\psi_h}(\kappa_i)\right|^2 + \left|\alpha_{\mathfrak{c}_{\varphi_h}}(\kappa_i) - \alpha_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|^2\right)\right) \end{array} \right)^{1/2}. \tag{23}$$

Definition 18. Let φ and ψ be two CIHFSs on set X and ω_k be a weight for each $\kappa \in \chi$ such that $\sum_{i=1}^{n}\omega_k = 1$. Then, weighted generalized complex intuitionistic hesitant normalized distance (WGCIHND) is defined as follows:

$$d_{\varsigma\omega_g}(\varphi,\psi) = \left[\begin{array}{l} \sum_{i=1}^{l}\omega_k\frac{1}{2}\left(\frac{1}{n}\sum_{h=1}^{n}\left(\left|\mathfrak{c}_{\varphi_h}(\kappa_i) - \mathfrak{c}_{\psi_h}(\kappa_i)\right|^{\eta} + \left|\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i) - \varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|^{\eta}\right)\right) \\ \left(\frac{1}{m}\sum_{h=1}^{m}\left(\left|\beta_{\varphi_h}(\kappa_i) - \beta_{\psi_h}(\kappa_i)\right|^{\eta} + \left|\alpha_{\mathfrak{c}_{\varphi_h}}(\kappa_i) - \alpha_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|^{\eta}\right)\right) \end{array} \right]^{1/\eta}, \tag{24}$$

where $\eta > 0$.

Remark 19

(1) If $\eta = 1$, then WGCIHND takes up the weighted Hamming complex intuitionistic hesitant normalized distance (WHCIHND) between φ and ψ, that is,

$$d_{\varsigma\omega_f}(\varphi,\psi) = \left[\begin{array}{l} \sum_{i=1}^{l}\omega_k\frac{1}{2}\left(\frac{1}{n}\sum_{h=1}^{n}\left(\left|\mathfrak{c}_{\varphi_h}(\kappa_i) - \mathfrak{c}_{\psi_h}(\kappa_i)\right| + \left|\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i) - \varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|\right)\right) \\ + \left(\frac{1}{m}\sum_{h=1}^{m}\left(\left|\beta_{\varphi_h}(\kappa_i) - \beta_{\psi_h}(\kappa_i)\right| + \left|\alpha_{\mathfrak{c}_{\varphi_h}}(\kappa_i) - \alpha_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|\right)\right) \end{array} \right]. \tag{25}$$

(2) If $\eta = 2$, then WGCIHND becomes the weighted Euclidean complex intuitionistic hesitant normalized distance (WECIHND) between φ and ψ, that is,

$$d_{\varsigma\omega_t}(\varphi,\psi) = \left[\begin{array}{l} \sum_{i=1}^{l}\omega_k\frac{1}{2}\left(\frac{1}{n}\sum_{i=1}^{n}\left(\left|\mathfrak{c}_{\varphi_h}(\kappa_i) - \mathfrak{c}_{\psi_h}(\kappa_i)\right|^2 + \left|\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i) - \varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|^2\right)\right) \\ + \left(\frac{1}{m}\sum_{h=1}^{m}\left(\left|\beta_{\varphi_h}(\kappa_i) - \beta_{\psi_h}(\kappa_i)\right|^2 + \left|\alpha_{\mathfrak{c}_{\varphi_h}}(\kappa_i) - \alpha_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|^2\right)\right) \end{array} \right]^{1/2}. \tag{26}$$

Definition 20. Let ψ be a CIHFS on $\kappa = \{\kappa_1, \kappa_2, \kappa_3, \ldots, \kappa_n\}$. Then, for any $\kappa_i \in \chi, l(\gamma_\psi(\kappa_i), \delta_\psi(\kappa_i))$ is the length of $\gamma_\psi(\kappa_i), \delta_\psi(\kappa_i)$. We defined the intuitionistic hesitant degree of $(\gamma_\psi(\kappa_i), \delta_\psi(\kappa_i))$ as $\widetilde{u}_c(\gamma_\psi(\kappa_i), \delta_\psi(\kappa_i)) = 1 - 1/l(\gamma_\psi(\kappa_i), \delta_\psi(\kappa_i))$, and the intuitionistic hesitant degree of ψ is defined as $\widetilde{u}_c(\psi) = 1/m\sum_{h=1}^{m}\psi_c(\gamma_\psi(\kappa_i)\delta_\psi(\kappa_i))$.

Definition 21. Let φ and ψ be two CIHFSs on set κ. Then, we define GCIHND including the intuitionistic degree between φ and ψ as

$$d_{\varsigma_{f_g}}(\varphi,\psi) = \left[\begin{array}{l} \dfrac{1}{2l}\displaystyle\sum_{i=1}^{l}\left|\psi_c\left(\gamma_\varphi(\kappa_i)\right) - \psi_c\left(\gamma_\psi(\kappa_i)\right), \psi_c\left(\delta_\varphi(\kappa_i)\right) - \psi_c\left(\delta_\psi(\kappa_i)\right)\right|^\eta \\ +\dfrac{1}{2}\left(\begin{array}{l} \dfrac{1}{n}\displaystyle\sum_{h=1}^{n}\left(\left|\mathfrak{c}_{\varphi_h}(\kappa_i) - \mathfrak{c}_{\psi_h}(\kappa_i)\right|^\eta + \left|\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i) - \varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|^\eta\right) \\ +\dfrac{1}{m}\displaystyle\sum_{h=1}^{m}\left(\left|\beta_{\varphi_h}(\kappa_i) - \beta_{\psi_h}(\kappa_i)\right|^\eta + \left|\alpha_{\mathfrak{c}_{\varphi_h}}(\kappa_i) - \alpha_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|^\eta\right) \end{array}\right) \end{array}\right]^{1/\eta}, \tag{27}$$

where $\eta = 0$.

Remark 22

(1) If $\eta = 1$, then GCIHND including the intuitionistic hesitant degree becomes HCIHND including the intuitionistic hesitant degree between φ and ψ, that is,

$$d_{\varsigma_{f_f}}(\varphi,\psi) = \left[\begin{array}{l} \dfrac{1}{2l}\displaystyle\sum_{i=1}^{l}\left|\psi_c\left(\gamma_\varphi(\kappa_i)\right) - \psi_c\left(\gamma_\psi(\kappa_i)\right), \psi_c\left(\delta_\varphi(\kappa_i)\right) - \psi_c\left(\delta_\psi(\kappa_i)\right)\right| \\ +\dfrac{1}{2}\left(\begin{array}{l} \dfrac{1}{n}\displaystyle\sum_{h=1}^{n}\left(\left|\mathfrak{c}_{\varphi_h}(\kappa_i) - \mathfrak{c}_{\psi_h}(\kappa_i)\right| + \left|\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i) - \varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|\right) \\ +\dfrac{1}{m}\displaystyle\sum_{h=1}^{m}\left(\left|\beta_{\varphi_h}(\kappa_i) - \beta_{\psi_h}(\kappa_i)\right| + \left|\alpha_{\mathfrak{c}_{\varphi_h}}(\kappa_i) - \alpha_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|\right) \end{array}\right) \end{array}\right]. \tag{28}$$

(2) If $\eta = 2$, then GCIHND including the intuitionistic hesitant degree becomes ECIHND including the intuitionistic hesitant the difference in φ and ψ, that is,

$$d_{\varsigma_{f_g}}(\varphi,\psi) = \left[\begin{array}{l} \dfrac{1}{2l}\displaystyle\sum_{i=1}^{l}\left|\psi_c\left(\gamma_\varphi(\kappa_i)\right) - \psi_c\left(\gamma_\psi(\kappa_i)\right), \psi_c\left(\delta_\varphi(\kappa_i)\right) - \psi_c\left(\delta_\psi(\kappa_i)\right)\right|^2 \\ +\dfrac{1}{2}\left(\begin{array}{l} \dfrac{1}{n}\displaystyle\sum_{h=1}^{n}\left(\left|\mathfrak{c}_{\varphi_h}(\kappa_i) - \mathfrak{c}_{\psi_h}(\kappa_i)\right|^2 + \left|\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i) - \varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|^2\right) \\ +\dfrac{1}{m}\displaystyle\sum_{h=1}^{m}\left(\left|\beta_{\varphi_h}(\kappa_i) - \beta_{\psi_h}(\kappa_i)\right|^2 + \left|\alpha_{\mathfrak{c}_{\varphi_h}}(\kappa_i) - \alpha_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|^2\right) \end{array}\right) \end{array}\right]^{1/2}. \tag{29}$$

Definition 23. Let φ and ψ be two CIHFS on κ and ω_k be a weight for each $\kappa_i \in \chi$ such that $\sum_{i=1}^{l}\omega_k = 1$. Then, we defined WGCIHND including the intuitionistic hesitant the difference in φ and ψ as

$$d_{\varsigma_{\omega f_g}}(\varphi,\psi) = \left[\frac{1}{2}\sum_{i=1}^{l}\omega_k\left(\begin{array}{c}\left|\psi_c\left(\gamma_\varphi(\kappa_i)\right)-\psi_c\left(\gamma_\psi(\kappa_i)\right),\psi_c\left(\delta_\varphi(\kappa_i)\right)-\psi_c\left(\delta_\psi(\kappa_i)\right)\right|^\eta \\ +\frac{1}{2}\left(\begin{array}{c}\frac{1}{n}\sum_{h=1}^{n}\left(\left|\mathfrak{c}_{\varphi_h}(\kappa_i)-\mathfrak{c}_{\psi_h}(\kappa_i)\right|^\eta+\left|\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i)-\varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|^\eta\right) \\ +\frac{1}{m}\sum_{h=1}^{m}\left(\left|\beta_{\varphi_h}(\kappa_i)-\beta_{\psi_h}(\kappa_i)\right|^\eta+\left|\alpha_{\mathfrak{c}_{\varphi_h}}(\kappa_i)-\alpha_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|^\eta\right)\end{array}\right)\end{array}\right)\right]^{1/\eta}. \tag{30}$$

Remark 24

(1) If $\eta = 1$, then WGCIHND including the intuitionistic hesitant degree becomes WHCIHND including the intuitionistic hesitant the difference in φ and ψ, that is,

$$d_{\varsigma_{\omega f_f}}(\varphi,\psi) = \left[\frac{1}{2}\sum_{i=1}^{l}\omega_k\left(\begin{array}{c}\left|\psi_c\left(\gamma_\varphi(\kappa_i)\right)-\psi_c\left(\gamma_\psi(\kappa_i)\right),\psi_c\left(\delta_\varphi(\kappa_i)\right)-\psi_c\left(\delta_\psi(\kappa_i)\right)\right| \\ +\frac{1}{2}\left(\begin{array}{c}\frac{1}{n}\sum_{h=1}^{n}\left(\left|\mathfrak{c}_{\varphi_h}(\kappa_i)-\mathfrak{c}_{\psi_h}(\kappa_i)\right|+\left|\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa i)-\varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|\right) \\ +\frac{1}{m}\sum_{h=1}^{m}\left(\left|\beta_{\varphi_h}(\kappa_i)-\beta_{\psi_h}(\kappa_i)\right|+\left|\alpha_{\mathfrak{c}_{\varphi_h}}(\kappa_i)-\alpha_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|\right)\end{array}\right)\end{array}\right]. \tag{31}$$

(2) If $\eta = 2$, we define GCIHND including the intuitionistic hesitant degree between φ and ψ as

$$d_{\varsigma_{\omega g\ddot{e}}}(\varphi,\psi) = \left[\frac{1}{2}\sum_{i=1}^{l}\omega_k\left(\begin{array}{c}\left|\psi_c\left(\gamma_\varphi(\kappa_i)\right)-\psi_c\left(\gamma_\psi(\kappa_i)\right),\psi_c\left(\delta_\varphi(\kappa_i)\right)-\psi_c\left(\delta_\psi(\kappa_i)\right)\right|^2 \\ +\frac{1}{2}\left(\begin{array}{c}\frac{1}{n}\sum_{h=1}^{n}\left(\left|\mathfrak{c}_{\varphi_h}(\kappa_i)-\mathfrak{c}_{\psi_h}(\kappa_i)\right|^2+\left|\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i)-\varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|^2\right) \\ +\frac{1}{m}\sum_{h=1}^{m}\left(\left|\beta_{\varphi_h}(\kappa_i)-\beta_{\psi_h}(\kappa_i)\right|^2+\left|\alpha_{\mathfrak{c}_{\varphi_h}}(\kappa_i)-\alpha_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|^2\right)\end{array}\right)\end{array}\right]^{1/2}. \tag{32}$$

If we take account into different choices between the intuitionistic hesitant degrees, membership values, and nonmember values, then the measurements of distance are described as follows.

Definition 25. Let φ and ψ be two CIHFSs on κ. Then, we defined GCIHND including the intuitionistic hesitant degree with preference between φ and ψ as

$$d_{\varsigma_{p_{f_g}}}(\varphi,\psi)=\left[\frac{1}{2n}\sum_{i=1}^{l}\left(\begin{array}{c}\zeta_c\left(\left|\psi_c\left(\gamma_\varphi(\kappa_i)\right)-\psi_c\left(\gamma_\psi(\kappa_i)\right),\psi_c\left(\delta_\varphi(\kappa_i)\right)-\psi_c\left(\delta_\psi(\kappa_i)\right)\right|^\eta\right)\\ +\frac{\eta_c}{2}\left(\begin{array}{c}\frac{1}{n}\sum_{h=1}^{n}\left(\left|¢_{\varphi_h}(\kappa_i)-¢_{\psi_h}(\kappa_i)\right|^\eta+\left|\varrho_{¢_{\varphi_h}}(\kappa_i)-\varrho_{¢_{\psi_h}}(\kappa_i)\right|^\eta\right)\\ +\frac{1}{m}\sum_{h=1}^{m}\left(\left|\beta_{\varphi_h}(\kappa_i)-\beta_{\psi_h}(\kappa_i)\right|^\eta+\left|\alpha_{¢_{\varphi_h}}(\kappa_i)-\alpha_{¢_{\psi_h}}(\kappa_i)\right|^\eta\right)\end{array}\right)\end{array}\right)\right]^{1/\eta}, \tag{33}$$

where $\eta>0$, $0\le\zeta_c,\eta_c\le 1$ and $\zeta_c+\eta_c=1$.

Remark 26

(1) If $\eta=1$, then GCIHND as well as the intuitionistic hesitant degree with the preference becomes HCIHND as well as hesitant degree with the preference between φ and ψ, that is,

$$d_{\varsigma_{p_{f_f}}}(\varphi,\psi)=\left[\frac{1}{2n}\sum_{i=1}^{l}\left(\begin{array}{c}\zeta_c\left(\left|\psi_c\left(\gamma_\varphi(\kappa_i)\right)-\psi_c\left(\gamma_\psi(\kappa_i)\right),\psi_c\left(\delta_\varphi(\kappa_i)\right)-\psi_c\left(\delta_\psi(\kappa_i)\right)\right|\right)\\ +\frac{\eta_c}{2}\left(\begin{array}{c}\frac{1}{n}\sum_{h=1}^{n}\left(\left|¢_{\varphi_h}(\kappa_i)-¢_{\psi_h}(\kappa_i)\right|+\left|\varrho_{¢_{\varphi_h}}(\kappa_i)-\varrho_{¢_{\psi_h}}(\kappa_i)\right|\right)\\ +\frac{1}{m}\sum_{h=1}^{m}\left(\left|\beta_{\varphi_h}(\kappa_i)-\beta_{\psi_h}(\kappa_i)\right|+\left|\alpha_{¢_{\varphi_h}}(\kappa_i)-\alpha_{¢_{\psi_h}}(\kappa_i)\right|\right)\end{array}\right)\end{array}\right)\right]. \tag{34}$$

(2) If $\eta=2$, then GCIHND as well as the intuitionistic hesitant degree with the preference becomes HCIHND as well as the hesitant degree with the preference between φ and ψ, that is,

$$d_{\varsigma_{f_{g_{\ddot{e}}}}}(\varphi,\psi)=\left[\frac{1}{2n}\sum_{i=1}^{l}\left(\begin{array}{c}\zeta_c\left(\left|\psi_c\left(\gamma_\varphi(\kappa_i)\right)-\psi_c\left(\gamma_\psi(\kappa_i)\right),\psi_c\left(\delta_\varphi(\kappa_i)\right)-\psi_c\left(\delta_\psi(\kappa_i)\right)\right|^2\right)\\ +\frac{\eta_c}{2}\left(\begin{array}{c}\frac{1}{n}\sum_{h=1}^{n}\left(\left|¢_{\varphi_h}(\kappa_i)-¢_{\psi_h}(\kappa_i)\right|^2+\left|\varrho_{¢_{\varphi_h}}(\kappa_i)-\varrho_{¢_{\psi_h}}(\kappa_i)\right|^2\right)\\ +\frac{1}{m}\sum_{h=1}^{m}\left(\left|\beta_{\varphi_h}(\kappa_i)-\beta_{\psi_h}(\kappa_i)\right|^2+\left|\alpha_{¢_{\varphi_h}}(\kappa_i)-\alpha_{¢_{\psi_h}}(\kappa_i)\right|^2\right)\end{array}\right)\end{array}\right)\right]^{1/2}, \tag{35}$$

where $\zeta_c=0$; it means that we are not considering the intuitionistic hesitant degree and $d_{\varsigma_{p_{f_g}}},d_{\varsigma_{p_{f_f}}},d_{\varsigma_{f_{g_{\ddot{e}}}}}$ become measures $d_{\varsigma_g},d_{\varsigma_k},d_{\varsigma_f}$.

Definition 27. Let φ and ψ be two CIHFSs on κ and ω_k be a weight for each $\kappa_i\in\chi$ such that $\sum_{i=1}^{l}\omega_k=1$. Then, we defined WGCIHND as well as the intuitionistic hesitant degree as a preference between φ and ψ as

$$d_{\varsigma_{\omega_{p_{f_{\ddot{e}}}}}}(\varphi,\psi)=\left[\frac{1}{2}\sum_{j=1}^{l}\omega_k\left(\begin{array}{c}\zeta_c\left(\left|\psi_c\left(\gamma_\varphi(\kappa_i)\right)-\psi_c\left(\gamma_\psi(\kappa_i)\right),\psi_c\left(\delta_\varphi(\kappa_i)\right)-\psi_c\left(\delta_\psi(\kappa_i)\right)\right|^\eta\right)\\ +\frac{\eta_c}{2}\left(\begin{array}{c}\frac{1}{l}\sum_{h=1}^{l}\left(\left|¢_{\varphi_h}(\kappa_i)-¢_{\psi_h}(\kappa_i)\right|^\eta+\left|\varrho_{¢_{\varphi_h}}(\kappa_i)-\varrho_{¢_{\psi_h}}(\kappa_i)\right|^\eta\right)\\ +\frac{1}{m}\sum_{h=1}^{m}\left(\left|\beta_{\varphi_h}(\kappa_i)-\beta_{\psi_h}(\kappa_i)\right|^\eta+\left|\alpha_{¢_{\varphi_h}}(\kappa_i)-\alpha_{¢_{\psi_h}}(\kappa_i)\right|^\eta\right)\end{array}\right)\end{array}\right)\right]^{1/\eta}. \tag{36}$$

Remark 28

(1) If $\eta = 1$, then WGCIHND including the intuitionistic hesitant degree as a preference becomes WHCIHND including the intuitionistic hesitant degree as a preference between φ and ψ, that is,

$$d_{\varsigma_{\omega p_{g\ddot{e}}}}(\varphi,\psi)=\left[\frac{1}{2}\sum_{i=1}^{l}\omega_k\left(\begin{array}{c}\zeta_c\left(\left|\psi_c\left(\gamma_\varphi(\kappa_i)\right)-\psi_c\left(\gamma_\psi(\kappa_i)\right),\psi_c\left(\delta_\varphi(\kappa_i)\right)-\psi_c\left(\delta_\psi(\kappa_i)\right)\right|\right)\\ +\frac{\eta_c}{2}\left(\begin{array}{c}\frac{1}{n}\sum_{h=1}^{n}\left(\left|\mathfrak{c}_{\varphi_h}(\kappa_i)-\mathfrak{c}_{\psi_h}(\kappa_i)\right|+\left|\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i)-\varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|\right)\\ +\frac{1}{m}\sum_{h=1}^{m}\left(\left|\beta_{\varphi_h}(\kappa_i)-\beta_{\psi_h}(\kappa_i)\right|+\left|\alpha_{\mathfrak{c}_{\varphi_h}}(\kappa_i)-\alpha_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|\right)\end{array}\right)\end{array}\right)\right]. \tag{37}$$

(2) If $\eta = 2$, then GCIHND including the intuitionistic hesitant degree with the preference becomes HCIHND including the hesitant degree with the preference between φ and ψ, that is,

$$d_{\varsigma_{\omega p_{h\ddot{e}}}}(\varphi,\psi)=\left[\frac{1}{2}\sum_{i=1}^{l}\omega_k\left(\begin{array}{c}\zeta_c\left(\left|\psi_c\left(\gamma_\varphi(\kappa_i)\right)-\psi_c\left(\gamma_\psi(\kappa_i)\right),\psi_c\left(\delta_\varphi(\kappa_i)\right)-\psi_c\left(\delta_\psi(\kappa_i)\right)\right|^2\right)\\ +\frac{\eta_c}{2}\left(\begin{array}{c}\frac{1}{n}\sum_{h=1}^{n}\left(\left|\mathfrak{c}_{\varphi_h}(\kappa_i)-\mathfrak{c}_{\psi_h}(\kappa_i)\right|^2+\left|\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i)-\varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|^2\right)\\ +\frac{1}{m}\sum_{h=1}^{m}\left(\left|\beta_{\varphi_h}(\kappa_i)-\beta_{\psi_h}(\kappa_i)\right|^2+\left|\alpha_{\mathfrak{c}_{\varphi_h}}(\kappa_i)-\alpha_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|^2\right)\end{array}\right)\end{array}\right)\right]^{1/2}. \tag{38}$$

Definition 29. Let φ and ψ be two CIHFSs on κ. Then, we defined modified generalized complex intuitionistic hesitant normalized distance (MGCIHND) between φ and ψ as follows:

$$d_{\varsigma_{n_h}}(P,Q)=\left[\frac{1}{l}\sum_{i=1}^{l}\frac{1}{2}\left(\begin{array}{c}\frac{1}{n}\sum_{h=1}^{n}\left(\left|\frac{\mathfrak{c}_{\varphi_h}(\kappa_i)-\mathfrak{c}_{\psi_h}(\kappa_i)}{\mathfrak{c}_{\varphi_h}(\kappa_i)+\mathfrak{c}_{\psi_h}(\kappa_i)}\right|^{\eta}+\left|\frac{\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i)-\varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)}{\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i)+\varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)}\right|^{\eta}\right)\\ +\frac{1}{m}\sum_{h=1}^{m}\left(\left|\frac{\beta_{\varphi_h}(\kappa_i)-\beta_{\psi_h}(\kappa_i)}{\beta_{\varphi_h}(\kappa_i)+\beta_{\psi_h}(\kappa_i)}\right|^{\eta}+\left|\frac{\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i)-\varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)}{\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i)+\varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)}\right|^{\eta}\right)\end{array}\right)\right]^{1/\eta}, \tag{39}$$

where $\eta = 0$.

Remark 30

(1) If $\eta = 1$, then MGCIHND becomes the modified Hamming complex intuitionistic hesitant normalized distance (MHCIHND) between φ and ψ, that is,

$$d_{\varsigma_{m_h}}(P,Q) = \left[\frac{1}{l}\sum_{i=1}^{l}\frac{1}{2}\left(\begin{array}{l}\frac{1}{n}\sum_{h=1}^{n}\left(\left|\frac{\mathfrak{c}_{\varphi_h}(\kappa_i)-\mathfrak{c}_{\psi_h}(\kappa_i)}{\mathfrak{c}_{\varphi_h}(\kappa_i)+\mathfrak{c}_{\psi_h}(\kappa_i)}\right|+\left|\frac{\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i)-\varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)}{\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa i)+\varrho_{\mathfrak{c}_{\psi_h}}(\kappa i)}\right|\right)\\ +\frac{1}{m}\sum_{h=1}^{m}\left(\left|\frac{\beta_{\varphi_h}(\kappa_i)-\beta_{\psi_h}(\kappa_i)}{\beta_{\varphi_h}(\kappa_i)+\beta_{\psi_h}(\kappa_i)}\right|+\left|\frac{\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i)-\varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)}{\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i)+\varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)}\right|\right)\end{array}\right)\right]. \tag{40}$$

(2) If $\eta = 2$, then MGCIHND becomes the modified Euclidean complex intuitionistic hesitant normalized distance (MECIHND) between φ and ψ, i.e.,

$$d_{\varsigma_{m_r}}(P,Q) = \left[\frac{1}{l}\sum_{i=1}^{l}\frac{1}{2}\left(\begin{array}{l}\frac{1}{n}\sum_{h=1}^{n}\left(\left|\frac{\mathfrak{c}_{\varphi_h}(\kappa_i)-\mathfrak{c}_{\psi_h}(\kappa_i)}{\mathfrak{c}_{\varphi_h}(\kappa_i)+\mathfrak{c}_{\psi_h}(\kappa_i)}\right|^2+\left|\frac{\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i)-\varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)}{\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i)+\varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)}\right|^2\right)\\ +\frac{1}{m}\sum_{h=1}^{m}\left(\left|\frac{\beta_{\varphi_h}(\kappa_i)-\beta_{\psi_h}(\kappa_i)}{\beta_{\varphi_h}(\kappa_i)+\beta_{\psi_h}(\kappa_i)}\right|^2+\left|\frac{\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i)-\varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)}{\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i)+\varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)}\right|^2\right)\end{array}\right)\right]^{1/2}. \tag{41}$$

Definition 31. Let φ and ψ be two CIHFSs on κ and ω_k be a weight for each $\kappa_i \in \chi$ such that $\sum_{i=1}^{l}\omega_k = 1$. Then, we defined modified generalized complex intuitionistic hesitant normalized distance (MWGCIHND) between φ and ψ as follows:

$$d_{\varsigma_{m_f}}(P,Q) = \left[\frac{1}{l}\sum_{h=1}^{l}\omega_k\frac{1}{2}\left(\begin{array}{l}\frac{1}{n}\sum_{h=1}^{n}\left(\left|\frac{\mathfrak{c}_{\varphi_h}(\kappa_i)-\mathfrak{c}_{\psi_h}(\kappa_i)}{\mathfrak{c}_{\varphi_h}(\kappa_i)+\mathfrak{c}_{\psi_h}(\kappa_i)}\right|^{\eta}+\left|\frac{\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i)-\varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)}{\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i)+\varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)}\right|^{\eta}\right)+\\ \frac{1}{m}\sum_{h=1}^{m}\left(\left|\frac{\beta_{\varphi_h}(\kappa_i)-\beta_{\psi_h}(\kappa_i)}{\beta_{\varphi_h}(\kappa_i)+\beta_{\psi_h}(\kappa_i)}\right|^{\eta}+\left|\frac{\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i)-\varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)}{\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i)+\varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)}\right|^{\eta}\right)\end{array}\right)\right]^{1/\eta}, \tag{42}$$

where $\eta > 0$.

Remark 32

(1) If $\eta = 1$, then MWGCIHND becomes modified weight Hamming complex intuitionistic hesitant normalized distance (MWHCIHND) between φ and ψ, that is,

$$d_{\varsigma_{m_t}}(P,Q) = \left[\frac{1}{l}\sum_{i=1}^{l}\omega_k\frac{1}{2}\left(\begin{array}{l}\frac{1}{n}\sum_{h=1}^{n}\left(\left|\frac{\mathfrak{c}_{\varphi_h}(\kappa_i)-\mathfrak{c}_{\psi_h}(\kappa_i)}{\mathfrak{c}_{\varphi_h}(\kappa_i)+\mathfrak{c}_{\psi_h}(\kappa_i)}\right|+\left|\frac{\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i)-\varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)}{\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i)+\varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)}\right|\right)\\ +\frac{1}{m}\sum_{h=1}^{m}\left(\left|\frac{\beta_{\varphi_h}(\kappa_i)-\beta_{\psi_h}(\kappa_i)}{\beta_{\varphi_h}(\kappa_i)+\beta_{\psi_h}(\kappa_i)}\right|+\left|\frac{\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i)-\varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)}{\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i)+\varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)}\right|\right)\end{array}\right)\right]. \tag{43}$$

(2) If $\eta = 2$, then MWGCIHND becomes modified weight Euclidean complex intuitionistic hesitant normalized distance (MWHCIHND) between φ and ψ, that is,

$$d_{\varsigma_{m_y}}(P,Q) = \left[\frac{1}{l}\sum_{i=1}^{l}\omega_k\frac{1}{2}\left(\begin{array}{c}\frac{1}{n}\sum_{h=1}^{n}\left(\left|\frac{¢_{\varphi_h}(\kappa_i)-¢_{\psi_h}(\kappa_i)}{¢_{\varphi_h}(\kappa_i)+¢_{\psi_h}(\kappa_i)}\right|^2+\left|\frac{\varrho_{¢_{\varphi_h}}(\kappa_i)-\varrho_{¢_{\psi_h}}(\kappa_i)}{\varrho_{¢_{\varphi_h}}(\kappa_i)+\varrho_{¢_{\psi_h}}(\kappa_i)}\right|^2\right)\\ +\frac{1}{m}\sum_{h=1}^{m}\left(\left|\frac{\beta_{\varphi_h}(\kappa_i)-\beta_{\psi_h}(\kappa_i)}{\beta_{\varphi_h}(\kappa_i)+\beta_{\psi_h}(\kappa_i)}\right|^2+\left|\frac{\varrho_{¢_{\varphi_h}}(\kappa_i)-\varrho_{¢_{\psi_h}}(\kappa_i)}{\varrho_{¢_{\varphi_h}}(\kappa_i)+\varrho_{¢_{\psi_h}}(\kappa_i)}\right|^2\right)\end{array}\right)\right]^{1/2}. \tag{44}$$

Definition 33. Let φ be a CIHFS on $\kappa = \{\kappa_1, \kappa_2, \kappa_3, \ldots, \kappa_n\}$. Then, for each $\kappa_i = \kappa, m(\gamma_\psi(\kappa_i), \delta_\psi(\kappa_i))$ is the cardinal number of $\gamma_\psi(\kappa_i), \delta_\psi(\kappa_i)$. The credibility factor of $\gamma_\psi(\kappa_i), \delta_\psi(\kappa_i)$ is given as $C_c(\gamma_\psi(\kappa_i), \delta_\psi(\kappa_i)) = (m(\gamma_\psi(\kappa_i), \delta_\psi(\kappa_i)))^{-1}$. Now, let φ and ψ be two CIHFS. Then, the credibility element between φ and ψ is defined as

$$C_c\big((\gamma_\varphi(\kappa_i), \gamma_\psi(\kappa_i)), (\delta_\varphi(\kappa_i), (\delta_\psi(\kappa_i))\big) = \left[C_c\big(\gamma_\varphi(\kappa_i)\big)C_c\big(\gamma_\psi(\kappa_i)\big)\right]^{1/2} + \left[C_c\big(\delta_\varphi(\kappa_i)\big)C_c\big(\delta_\psi(\kappa_i)\big)\right]^{1/2}, \tag{45}$$

and the normalized credibility element is defined as

$$C_c^*\big((\gamma_\varphi(\kappa_i), \gamma_\psi(\kappa_i)), (\delta_\varphi(\kappa_i), (\delta_\psi(\kappa_i))\big) = \frac{\left[C_c\big(\gamma_\varphi(\kappa_i)\big)C_c\big(\gamma_\psi(\kappa_i)\big)\right]^{1/2}}{\sum_{i=1}^{m}\left[C_c\big(\gamma_\varphi(\kappa_i)\big)C_c\big(\gamma_\psi(\kappa_i)\big)\right]^{1/2}} + \frac{\left[C_c\big(\delta_\varphi(\kappa_i)\big)C_c\big(\delta_\psi(\kappa_i)\big)\right]^{1/2}}{\sum_{i=1}^{m}\left[C_c\big(\delta_\varphi(\kappa_i)\big)C_c\big(\delta_\psi(\kappa_i)\big)\right]^{1/2}}. \tag{46}$$

A novel distance metric based on the credibility factor has been established.

Definition 34. Let φ and ψ be two CIHFSs on κ. Then, we defined the novel GCIHND between φ and ψ as

$$d_{\varsigma_{c_h}}(\varphi,\psi) = \left[\sum_{i=1}^{l}\left[\begin{array}{c}C_c^*\big((\gamma_\varphi(\kappa_i), \gamma_\psi(\kappa_i)), (\delta_\varphi(\kappa_i), \delta_\psi(\kappa_i))\big)\\ \frac{1}{2}\left(\begin{array}{c}\frac{1}{n}\sum_{h=1}^{n}\left(\left|¢_{\varphi_h}(\kappa_i)-¢_{\psi_h}(\kappa_i)\right|^\eta+\left|\varrho_{¢_{\varphi_h}}(\kappa_i)-\varrho_{¢_{\psi_h}}(\kappa_i)\right|^\eta\right)\\ +\frac{1}{m}\sum_{h=1}^{m}\left(\left|\beta_{\varphi_h}(\kappa_i)-\beta_{\psi_h}(\kappa_i)\right|^\eta+\left|\alpha_{¢_{\varphi_h}}(\kappa_i)-\alpha_{¢_{\psi_h}}(\kappa_i)\right|^\eta\right)\end{array}\right)\end{array}\right]^\eta\right]^{1/\eta}, \tag{47}$$

where $\eta = 0$.

Remark 35

(1) If $\eta = 1$, then the novel GCIHND becomes novel HCIHND between φ and ψ as follows:

$$\left(C_c^*\left(\gamma_\varphi(\kappa_i), \gamma_\psi(\kappa_i)\right), \left(\delta_\varphi(\kappa_i), \left(\delta_\psi(\kappa_i)\right)\right)\right),$$

$$d_{\varsigma_{c_k}}(\varphi, \psi) = \sum_{i=1}^{l} \frac{1}{2}\left(\begin{array}{c} \frac{1}{n}\sum_{h=1}^{n}\left(\left|\mathfrak{c}_{\varphi_h}(\kappa_i) - \mathfrak{c}_{\psi_h}(\kappa_i)\right| + \left|\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i) - \varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|\right) \\ + \frac{1}{m}\sum_{h=1}^{m}\left(\left|\beta_{\varphi_h}(\kappa_i) - \beta_{\psi_h}(\kappa_i)\right| + \left|\alpha_{\mathfrak{c}_{\varphi_h}}(\kappa_i) - \alpha_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|\right) \end{array}\right). \tag{48}$$

(2) If $\eta = 2$, then the novel GCIHND becomes novel HCIHND between φ and ψ as follows:

$$d_{\varsigma_{c_q}}(\varphi, \psi) = \left[\sum_{i=1}^{l}\left[\frac{1}{2}\left(\begin{array}{c} C_c^*\left(\left(\gamma_\varphi(\kappa_i), \gamma_\psi(\kappa_i)\right), \left(\delta_\varphi(\kappa_i), \left(\delta_\psi(\kappa_i)\right)\right)\right) \\ \frac{1}{n}\sum_{h=1}^{n}\left(\left|\mathfrak{c}_{\varphi_h}(\kappa_i) - \mathfrak{c}_{\psi_h}(\kappa_i)\right|^2 + \left|\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i) - \varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|^2\right) \\ + \frac{1}{m}\sum_{h=1}^{m}\left(\left|\beta_{\varphi_h}(\kappa_i) - \beta_{\psi_h}(\kappa_i)\right|^2 + \left|\alpha_{\mathfrak{c}_{\varphi_h}}(\kappa_i) - \alpha_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|^2\right) \end{array}\right)\right]\right]^{1/2}. \tag{49}$$

Definition 36. Let φ and ψ be two CIHFSs on κ and ω_k be a weight for each $\kappa_i \in \chi$ such that $\sum_{i=1}^{l}\omega_k = 1$ and $0 \le \omega_k \le 1$. Then, we defined the novel WGCIHND as follows:

$$d_{\varsigma_{w_{c_h}}}(\varphi, \psi) = \left[\sum_{i=1}^{l}\omega_k\left[\frac{1}{2}\left(\begin{array}{c} C_c^*\left(\left(\gamma_\varphi(\kappa_i), \gamma_\psi(\kappa_i)\right), \left(\delta_\varphi(\kappa_i), \delta_\psi(\kappa_i)\right)\right) \\ \frac{1}{n}\sum_{h=1}^{n}\left(\left|\mathfrak{c}_{\varphi_h}(\kappa_i) - \mathfrak{c}_{\psi_h}(\kappa_i)\right|^\eta + \left|\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i) - \varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|^\eta\right) \\ + \frac{1}{m}\sum_{h=1}^{m}\left(\left|\beta_{\varphi_h}(\kappa_i) - \beta_{\psi_h}(\kappa_i)\right|^\eta + \left|\alpha_{\mathfrak{c}_{\varphi_h}}(\kappa_i) - \alpha_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|^\eta\right) \end{array}\right)\right]\right]^{1/\eta}, \tag{50}$$

where $\eta = 0$.

Remark 37

(1) If $\eta = 1$, then the novel GCIHND becomes novel HCIHND between φ and ψ as follows:

$$\left(C_c^*\left(\gamma_\varphi(\kappa_i), \gamma_\psi(\kappa_i)\right), \left(\delta_\varphi(\kappa_i), \delta_\psi(\kappa_i)\right)\right),$$

$$d_{\varsigma_{w_{c_k}}}(\varphi, \psi) = \sum_{i=1}^{l} \omega_k \frac{1}{2} \left(\begin{array}{c} \frac{1}{n} \sum_{h=1}^{n} \left(\left| \mathfrak{c}_{\varphi_h}(\kappa_i) - \mathfrak{c}_{\psi_h}(\kappa_i) \right| + \left| \varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i) - \varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i) \right| \right) \\ + \frac{1}{m} \sum_{h=1}^{m} \left(\left| \beta_{\varphi_h}(\kappa_i) - \beta_{\psi_h}(\kappa_i) \right| + \left| \alpha_{\mathfrak{c}_{\varphi_h}}(\kappa_i) - \alpha_{\mathfrak{c}_{\psi_h}}(\kappa_i) \right| \right) \end{array} \right). \tag{51}$$

(2) If $\eta = 2$, then the novel GCIHND becomes novel HCIHND between φ and ψ as follows:

$$d_{\varsigma\omega_{c_q}}(\varphi, \psi) = \left[\sum_{i=1}^{l} \omega_k \left[\frac{1}{2} \left(\begin{array}{c} C_c^*\left(\left(\gamma_\varphi(\kappa_i), \gamma_\psi(\kappa_i)\right), \left(\delta_\varphi(\kappa_i), \delta_\psi(\kappa_i)\right)\right) \\ \frac{1}{n} \sum_{h=1}^{n} \left(\left| \mathfrak{c}_{\varphi_h}(\kappa_i) - \mathfrak{c}_{\psi_h}(\kappa_i) \right|^2 + \left| \varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i) - \varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i) \right|^2 \right) \\ + \frac{1}{m} \sum_{h=1}^{m} \left(\left| \beta_{\varphi_h}(\kappa_i) - \beta_{\psi_h}(\kappa_i) \right|^2 + \left| \alpha_{\mathfrak{c}_{\varphi_h}}(\kappa_i) - \alpha_{\mathfrak{c}_{\psi_h}}(\kappa_i) \right|^2 \right) \end{array} \right) \right]^2 \right]^{1/2}. \tag{52}$$

We take into account the conservative element α_{cc} and the risk element β_{cc} for a thorough comprehension of the connection between cardinalities and the worth of CIHFSs. Incorporating these factors, the above described novel distance measures become weighted.

$$d_{\varsigma_{wc_h}}(\varphi, \psi) = \left[\sum_{i=1}^{l} \omega_k \left[\left(\frac{1}{2} \left(\begin{array}{c} \left(\frac{1}{n} \sum_{h=1}^{n} \left| \mathfrak{c}_{\varphi_h}(\kappa_i) - \mathfrak{c}_{\psi_h}(\kappa_i) \right|^\eta + \left| \varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i) - \varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i) \right|^\eta \right) \\ + \left(\frac{1}{m} \sum_{h=1}^{m} \left| \beta_{\varphi_h}(\kappa_i) - \beta_{\psi_h}(\kappa_i) \right|^\eta + \left| \alpha_{\mathfrak{c}_{\varphi_h}}(\kappa_i) - \alpha_{\mathfrak{c}_{\psi_h}}(\kappa_i) \right|^\eta \right) \end{array} \right) \right)^{\beta_{cc}} \right]^\eta \right]^{1/\eta},$$

$$d_{\varsigma_{\omega_{c_k}}}(\varphi,\psi)=\left[\sum_{i=1}^{l}\omega_k\left[\begin{array}{c} C_c^*\left(\left(\gamma_\varphi(\kappa_i),\gamma_\psi(\kappa_i)\right),\left(\delta_\varphi(\kappa_i),\delta_\psi(\kappa_i)\right)\right)^{\alpha_{cc}} \\ \left(\frac{1}{2}\left(\begin{array}{c}\frac{1}{n}\sum_{h=1}^{n}\left(\left|\mathfrak{c}_{\varphi_h}(\kappa_i)-\mathfrak{c}_{\psi_h}(\kappa_i)\right|+\left|\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i)-\varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|\right) \\ +\frac{1}{m}\sum_{h=1}^{m}\left(\left|\beta_{\varphi_h}(\kappa_i)-\beta_{\psi_h}(\kappa_i)\right|+\left|\alpha_{\mathfrak{c}_{\varphi_h}}(\kappa_i)-\alpha_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|\right)\end{array}\right)\right)^{\beta_{cc}}\end{array}\right]\right],$$

$$d_{\varsigma_{\omega_{c_q}}}(\varphi,\psi)=\left[\sum_{h=1}^{I}\omega_k\left[\begin{array}{c} C_c^*\left(\left(\gamma_\varphi(\kappa_i),\gamma_\psi(\kappa_i)\right),\left(\delta_\varphi(\kappa_i),\delta_\psi(\kappa_i)\right)\right)^{\alpha_{cc}} \\ \left(\frac{1}{2}\left(\begin{array}{c}\frac{1}{l}\sum_{h=1}^{l}\left(\left|\mathfrak{c}_{\varphi_h}(\kappa_i)-\mathfrak{c}_{\psi_h}(\kappa_i)\right|+\left|\varrho_{\mathfrak{c}_{\varphi_h}}(\kappa_i)-\varrho_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|\right) \\ +\frac{1}{m}\sum_{h=1}^{m}\left(\left|\beta_{\varphi_h}(\kappa_i)-\beta_{\psi_h}(\kappa_i)\right|+\left|\alpha_{\mathfrak{c}_{\varphi_h}}(\kappa_i)-\alpha_{\mathfrak{c}_{\psi_h}}(\kappa_i)\right|\right)\end{array}\right)\right)^{\beta_{cc}}\end{array}\right]^2\right]^{1/2}, \tag{53}$$

where $\alpha_{cc},\beta_{cc}\in[0,1]$ and $\alpha_{cc}+\beta_{cc}=1$.

5. An Approach to Multicriteria Decision-Making Using CIHF Information

Consider an MCDM with complex intuitionistic hesitant fuzzy information and assume that p is different alternatives $\Delta=(\Delta_1,\Delta_2,\Delta_3,\Delta_4,\Delta_5,\ldots,\Delta_j)$; $P=(1,2,3\ldots,j)$ and q criteria $K=\{\kappa_1,\kappa_2,\kappa_3,\kappa_4,\ldots,\kappa_d\}$; $q=(1,2,3\ldots,d)$ and the weight vectors for the criteria are $=\{W_1,W_2,W_3,\ldots,W_d\}$, where $W_q\geq 0$ and $\sum_{q=1}^{d}W_q=1$.

The procedure of the presented MCDM technique by using CIHFS is provided below for helping in the selection of the best alternative.

Step 1: We will gather the information in complex intuitionistic hesitant fuzzy form by the experts as follows:

$$\widehat{S}=\left[\left\langle\Xi_{\widehat{S}}(\kappa),\lambda_{\widehat{S}}(\kappa)\right\rangle_{pq}\right]_{jd},\quad \kappa\in\chi. \tag{54}$$

Step 2: Assume the alternative as follows:

$$\Delta^*=\left\{\begin{array}{c}\left\langle\kappa_1,\left(1\ddot{e}^{\widetilde{i}2\pi(1)}\right),\left(1\ddot{e}^{\widetilde{i}2\pi(1)}\right)\right\rangle \\ \left\langle\kappa_2,\left(1\ddot{e}^{\widetilde{i}2\pi(1)}\right),\left(1\ddot{e}^{\widetilde{i}2\pi(1)}\right)\right\rangle \\ \left\langle\kappa_3,\left(1\ddot{e}^{\widetilde{i}2\pi(1)}\right),\left(1\ddot{e}^{\widetilde{i}2\pi(1)}\right)\right\rangle \\ \left\langle\kappa_4,\left(1\ddot{e}^{\widetilde{i}2\pi(1)}\right),\left(1\ddot{e}^{\widetilde{i}2\pi(1)}\right)\right\rangle\end{array}\right\}. \tag{55}$$

Step 3: If there is an imbalance in the length of CIHFE, the shorter CIHFE added the value $\left\{(0.0\ddot{e}^{\widetilde{i}2\pi(0.0)}),(0.0\ddot{e}^{\widetilde{i}2\pi(0.0)})\right\}$ until the lengths are equal.

Step 4: We will measure the deviations between each alternative and ideal alternative by taking (α_{cc},β_{cc}) and η.

Step 5: We will rank the alternative in the descending

Note: The alternative ranking order varies when the values of the parameters are modified.

The flowchart of the proposed algorithm is shown in Figure 1.

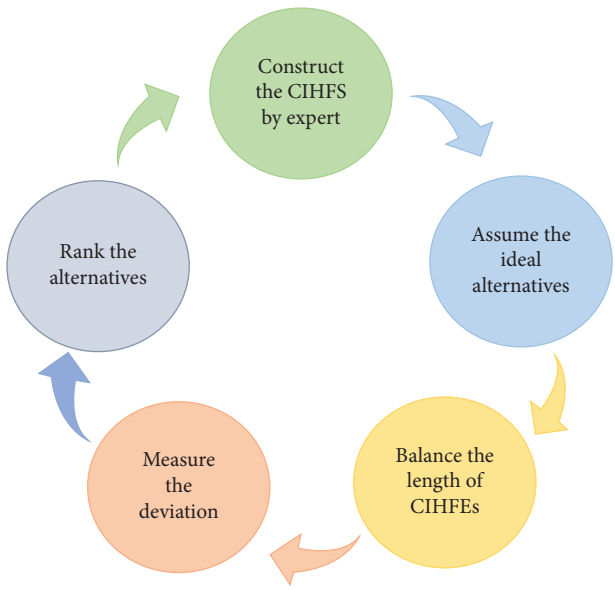

Figure 1: Flowchart of the algorithm.

6. Numerical Illustration

Based on four criteria that integrate under complex intuitionistic hesitant fuzzy information, we offered a multiattribute strategy for evaluating the factor for industry development. We take into account a numerical example of selecting the best course of action that supports the technological advancement of the manufacturing industry.

6.1. Case Study. The business model has traditionally been thought of by management scholars as an innovation hub, organizing schematic, conceptual model reasoning, or marketing strategy. The model has been used but not much is known about it. This research aims to address this void by exploring the repercussions and limitations of using the business model as a strategic instrument and by providing a strategy-as-practice approach. A single-case study approach was used to examine the benefits and drawbacks of using the business model as a strategic tool in a high-tech company. A useful structural template for outlining a company's existing business model is provided by the business model. However, rather than being an analytical instrument with a clear procedure of steps, it is more of a symbolic artefact meant to spark brainstorming and discussion of potential strategies rather than a practical tool. The business model concept's original technical and linguistic legitimacy is severely constrained when used in practice. A collective lock-in to the existing strategic identity, however, may evolve in the course of legitimization. Managers must be aware of these constraints and strike the right balance inside the company. The research contributes a social practices perspective to the business model topic by focusing on the repercussions and limitations of using the notion of the business model as a planning tool in a real-world setting. Making judgments about business operations and expansion is the management's primary responsibility at work. We see many examples of decision-making abilities in daily operations, including managing the staff, providing customer service, increasing productivity, and hiring new employees.

To demonstrate the use and efficacy of the suggested distance measure, we applied it to the setting of CIHFSs. In this part, we provide an example of how the proposed fuzzy decision-making techniques might be put to use in the real world. The success of the suggested ways will be confirmed, and a comparison study will also be done.

We want to select the best CEO for industry that will help grow the business. The five experts take the interviews and collect the information based on criteria given and find the result that which CEO can grow the business well and give the investor more and more benefits. The criteria for evaluations are given below.

6.2. Human Resources. Visualize yourself as the chief executive officer of a brand-new Internet store. As your business expands, it becomes more important than ever to recruit top talent to help you achieve your ultimate aim of creating the finest online shopping experience possible. You will need to staff your business with highly competent professionals that specialise in areas such as software engineering, advertising, operations, material management, and supply chain management. Because it is an online venture, you will not need a physical location to house employees. Qualified remote employees may provide the essential technical support and services online as well. Having the right number of local employees combined with remote workers will help you complete the duties at a reasonable cost. Video chats, electronic letters, and online chitchat are all viable options for keeping the team spirit

alive. You will also have the option of hiring individuals who are geographically distant but can work together over the Internet to create innovative solutions.

6.3. Production. One of the most prevalent situations requiring managerial decision-making is deciding where to put a manufacturing plant. As your business expands and consumer demand increases, you will have to increase the output. The next step would be to calculate how much more capacity installation is required to meet demand effectively. The manufacturing processes cannot be carried out without figuring out what tools and workers would be needed. Keep in mind that the end aim is to grow production sustainably so that you may scale up or down without considerable extra expense.

6.4. Marketing. Throughout their lifetimes, the vast majority of companies will alter their logo and overall visual identity. Initially, a company's branding and target audience are limited to its immediate vicinity, but when the company expands, it often finds itself in need of a rebrand. Changes to a company's logo, official mascot, or even name are commonplace when the leadership feels the need to signal a shift in the company's identity, competence, or vision. Decision-making procedures are shown by rebranding projects, which take into account the company's core beliefs, products, target markets, cultural relevance, and commercial objectives.

6.5. Client Servicing. When first launching a company, focus on winning as many clients and contracts as possible. The general belief is that working longer hours results in higher earnings. Numerous real-world decision-making instances, however, show that this approach is not the most effective long-term tactic. For instance, you may discover that a long-standing client consumes an inordinate amount of time and effort without justifying it monetarily. Maybe they were one of your first, and they were essential in your company's early stages. It is essential, however, to change with the times. It is possible that the customer or project that helped you get started will not work after you have matured a little more. Therefore, there are instances when tough choices need to be made.

Here,

(1) κ_1 = Human resources,

(2) κ_2 = Production,

(3) κ_3 = Marketing,

(4) κ_4 = Client servicing. The weight vector $W = (0.15, 0.3, 0.2, 0.35)$.

The expert data under CIHFSs are given in Table 1.

Ideal information for $(\alpha_{cc}, \beta_{cc}) = (0.9, 0.1)$ is given in Table 2.

The ranking results are shown in Figure 2.

Ideal information for $(\alpha_{cc}, \beta_{cc}) = (0.7, 0.3)$ is given in Table 3.

The ranking results under $(\alpha_{cc}, \beta_{cc}) = (0.7, 0.3)$ are shown in Figure 3.

Ideal information for $(\alpha_{cc}, \beta_{cc}) = (0.5, 0.5)$ is given in Table 4.

The ranking results under $(\alpha_{cc}, \beta_{cc}) = (0.5, 0.5)$ are shown in Figure 4.

Ideal information for $(\alpha_{cc}, \beta_{cc}) = (0.3, 0.7)$ is given in Table 5.

The ranking results under $(\alpha_{cc}, \beta_{cc}) = (0.3, 0.7)$ are shown in Figure 5.

7. Comparison Study

This section demonstrates the integrity and superiority of the proposed algorithm by contrasting the work under investigation with some other existing works in the literature.

7.1. Comparison with the Study by Garg et al. [41]. To determine the integrity and superiority of the proposed algorithm, we compare our results with the existing work proposed by Garg et al. [41]. We utilized the data from [41] given in Table 6 to compare the similarity measures of the proposed algorithm as follows:

Ideal information CIHFSs for $(\alpha_{cc}, \beta_{cc}) = (0.9, 0.1)$ is given in Table 7.

The ranking results under $(\alpha_{cc}, \beta_{cc}) = (0.9, 0.1)$ are shown in Figure 6.

Ideal information CIHFSs for $(\alpha_{cc}, \beta_{cc}) = (0.7, 0.3)$ is given in Table 8.

The ranking results under $(\alpha_{cc}, \beta_{cc}) = (0.7, 0.3)$ are shown in Figure 7.

Ideal information CIHFSs for $(\alpha_{cc}, \beta_{cc}) = (0.5, 0.5)$ is given in Table 9.

The ranking results under $(\alpha_{cc}, \beta_{cc}) = (0.5, 0.5)$ are shown in Figure 8.

Ideal information CIHFSs for $(\alpha_{cc}, \beta_{cc}) = (0.3, 0.7)$ is given in Table 10.

The ranking results under $(\alpha_{cc}, \beta_{cc}) = (0.3, 0.7)$ are shown in Figure 9.

We compared the SMs we looked at to several already known SMs defined by Wang et al. [44]. From the foregoing discussion, it is clear that our suggested SMs can articulate a higher degree of vagueness in information and place it in the context of real-world problems. We investigated the SMs from the perspective of CIHFS and found that they are both more general and more amenable to dealing with real problems than the currently available alternatives. Results of a comparison between existing and proposed distance measures for Example 2 are given in Tables 7–10.

Table 1: Basic information of interview experts in CIHFSs.

Alternative	κ_1	κ_2	κ_3	κ_4
Δ_1	$\left\{\left(\begin{matrix}0.4e^{\widetilde{i2\pi}(0.24)}\\0.5e^{\widetilde{i2\pi}(0.25)}\end{matrix}\right),\left(\begin{matrix}0.9e^{\widetilde{i2\pi}(0.6)}\\0.4e^{\widetilde{i2\pi}(0.5)}\end{matrix}\right)\right\}$	$\left\{\left(\begin{matrix}0.7e^{\widetilde{i2\pi}(0.1)}\\0.1e^{\widetilde{i2\pi}(0.3)}\\0.6e^{\widetilde{i2\pi}(0.3)}\end{matrix}\right),\left(\begin{matrix}0.5e^{\widetilde{i2\pi}(0.7)}\\0.1e^{\widetilde{i2\pi}(0.3)}\end{matrix}\right)\right\}$	$\left\{\left(\begin{matrix}0.5e^{\widetilde{i2\pi}(0.4)}\\0.8e^{\widetilde{i2\pi}(0.6)}\\0.4e^{\widetilde{i2\pi}(0.3)}\end{matrix}\right),\left(\begin{matrix}0.1e^{\widetilde{i2\pi}(0.1)}\\0.4e^{\widetilde{i2\pi}(0.25)}\\0.7e^{\widetilde{i2\pi}(0.3)}\end{matrix}\right)\right\}$	$\left\{\left(\begin{matrix}0.3e^{\widetilde{i2\pi}(0.5)}\\0.5e^{\widetilde{i2\pi}(0.1)}\end{matrix}\right),\left(\begin{matrix}0.3e^{\widetilde{i2\pi}(0.33)}\\0.6e^{\widetilde{i2\pi}(0.25)}\end{matrix}\right)\right\}$
Δ_2	$\left\{\left(\begin{matrix}0.5e^{\widetilde{i2\pi}(0.3)}\\0.3e^{\widetilde{i2\pi}(0.5)}\end{matrix}\right),\left(\begin{matrix}0.6e^{\widetilde{i2\pi}(0.4)}\\0.4e^{\widetilde{i2\pi}(0.6)}\end{matrix}\right)\right\}$	$\left\{\left(\begin{matrix}0.2e^{\widetilde{i2\pi}(0.3)}\\0.3e^{\widetilde{i2\pi}(0.5)}\end{matrix}\right),\left(\begin{matrix}0.5e^{\widetilde{i2\pi}(0.2)}\\0.6e^{\widetilde{i2\pi}(0.4)}\end{matrix}\right)\right\}$	$\left\{\left(\begin{matrix}0.5e^{\widetilde{i2\pi}(0.2)}\\0.6e^{\widetilde{i2\pi}(0.3)}\end{matrix}\right),\left(\begin{matrix}0.1e^{\widetilde{i2\pi}(0.2)}\\0.8e^{\widetilde{i2\pi}(0.5)}\end{matrix}\right)\right\}$	$\left\{\left(\begin{matrix}0.7e^{\widetilde{i2\pi}(0.3)}\\0.4e^{\widetilde{i2\pi}(0.5)}\\0.2e^{\widetilde{i2\pi}(0.3)}\end{matrix}\right),\left(\begin{matrix}0.1e^{\widetilde{i2\pi}(0.33)}\\0.6e^{\widetilde{i2\pi}(0.25)}\end{matrix}\right)\right\}$
Δ_3	$\left\{\left(\begin{matrix}0.7e^{\widetilde{i2\pi}(0.5)}\\0.6e^{\widetilde{i2\pi}(0.4)}\\0.3e^{\widetilde{i2\pi}(0.3)}\end{matrix}\right),\left(\begin{matrix}0.5e^{\widetilde{i2\pi}(0.5)}\\0.3e^{\widetilde{i2\pi}(0.3)}\end{matrix}\right)\right\}$	$\left\{\left(\begin{matrix}0.2e^{\widetilde{i2\pi}(0.22)}\\0.4e^{\widetilde{i2\pi}(0.5)}\end{matrix}\right),\left(\begin{matrix}0.6e^{\widetilde{i2\pi}(0.5)}\\0.3e^{\widetilde{i2\pi}(0.13)}\end{matrix}\right)\right\}$	$\left\{\left(\begin{matrix}0.5e^{\widetilde{i2\pi}(0.25)}\\0.6e^{\widetilde{i2\pi}(0.5)}\end{matrix}\right),\left(\begin{matrix}0.7e^{\widetilde{i2\pi}(0.1)}\\0.3e^{\widetilde{i2\pi}(0.5)}\end{matrix}\right)\right\}$	$\left\{\left(\begin{matrix}0.4e^{\widetilde{i2\pi}(0.2)}\\0.6e^{\widetilde{i2\pi}(0.5)}\end{matrix}\right),\left(\begin{matrix}0.3e^{\widetilde{i2\pi}(0.2)}\\0.4e^{\widetilde{i2\pi}(0.4)}\\0.6e^{\widetilde{i2\pi}(0.4)}\end{matrix}\right)\right\}$
Δ_4	$\left\{\left(\begin{matrix}0.7e^{\widetilde{i2\pi}(0.5)}\\0.3e^{\widetilde{i2\pi}(0.3)}\end{matrix}\right),\left(\begin{matrix}0.6e^{\widetilde{i2\pi}(0.5)}\\0.2e^{\widetilde{i2\pi}(0.25)}\end{matrix}\right)\right\}$	$\left\{\left(\begin{matrix}0.6e^{\widetilde{i2\pi}(0.50)}\\0.5e^{\widetilde{i2\pi}(0.25)}\end{matrix}\right),\left(\begin{matrix}0.4e^{\widetilde{i2\pi}(0.45)}\\0.3e^{\widetilde{i2\pi}(0.30)}\end{matrix}\right)\right\}$	$\left\{\left(\begin{matrix}0.1e^{\widetilde{i2\pi}(0.3)}\\0.3e^{\widetilde{i2\pi}(0.35)}\end{matrix}\right),\left(\begin{matrix}0.8e^{\widetilde{i2\pi}(0.1)}\\0.2e^{\widetilde{i2\pi}(0.2)}\end{matrix}\right)\right\}$	$\left\{\left(\begin{matrix}0.2e^{\widetilde{i2\pi}(0.25)}\\0.5e^{\widetilde{i2\pi}(0.16667)}\\0.6e^{\widetilde{i2\pi}(0.5)}\end{matrix}\right),\left(\begin{matrix}0.3e^{\widetilde{i2\pi}(0.1)}\\0.4e^{\widetilde{i2\pi}(0.25)}\\0.5e^{\widetilde{i2\pi}(0.45)}\end{matrix}\right)\right\}$
Δ_5	$\left\{\left(\begin{matrix}0.6e^{\widetilde{i2\pi}(0.2)}\\0.5e^{\widetilde{i2\pi}(0.5)}\\0.4e^{\widetilde{i2\pi}(0.3)}\end{matrix}\right),\left(\begin{matrix}0.2e^{\widetilde{i2\pi}(0.3)}\\0.5e^{\widetilde{i2\pi}(0.7)}\end{matrix}\right)\right\}$	$\left\{\left(\begin{matrix}0.7e^{\widetilde{i2\pi}(0.5)}\\0.6e^{\widetilde{i2\pi}(0.4)}\\0.4e^{\widetilde{i2\pi}(0.5)}\end{matrix}\right),\left(\begin{matrix}0.3e^{\widetilde{i2\pi}(0.4)}\\0.6e^{\widetilde{i2\pi}(0.2)}\end{matrix}\right)\right\}$	$\left\{\left(\begin{matrix}0.8e^{\widetilde{i2\pi}(0.4)}\\0.7e^{\widetilde{i2\pi}(0.15557)}\end{matrix}\right),\left(\begin{matrix}0.6e^{\widetilde{i2\pi}(0.33)}\\0.1e^{\widetilde{i2\pi}(0.4)}\end{matrix}\right)\right\}$	$\left\{\left(\begin{matrix}0.1e^{\widetilde{i2\pi}(0.5)}\\0.4e^{\widetilde{i2\pi}(0.25)}\end{matrix}\right),\left(\begin{matrix}0.4e^{\widetilde{i2\pi}(0.65)}\\0.3e^{\widetilde{i2\pi}(0.35)}\end{matrix}\right)\right\}$

Table 2: Information of interview experts in CIHFSs to each alternative and the ideal alternative $(\alpha_{cc}, \beta_{cc}) = (0.9, 0.1)$.

η	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5
$\eta = 1$	0.2941	0.2806	0.2924	0.2886	0.2931
$\eta = 2$	0.2948	0.2809	0.2927	0.2896	0.2935
$\eta = 6$	0.2974	0.2821	0.2940	0.2931	0.2948
$\eta = 10$	0.2996	0.2833	0.2951	0.2959	0.2961
Ranking					
$\Delta_2 > \Delta_4 > \Delta_3 > \Delta_5 > \Delta_1$					
$\Delta_2 > \Delta_4 > \Delta_3 > \Delta_5 > \Delta_1$					
$\Delta_2 > \Delta_4 > \Delta_3 > \Delta_5 > \Delta_1$					
$\Delta_2 > \Delta_3 > \Delta_4 > \Delta_5 > \Delta_1$					

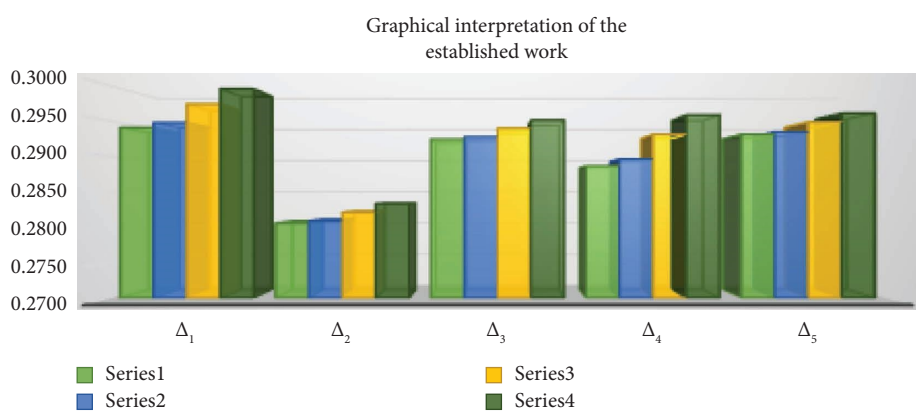

Figure 2: Graphical interpretation of a standard for $(\alpha_\{cc\}, \beta_\{cc\} = 0.9, 0.1)$ parameter values.

Table 3: Deviation basic information of interview experts in CIHFSs to each alternative and the ideal alternative $(\alpha_{cc}, \beta_{cc}) = (0.7, 0.3)$.

η	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5
$\eta = 1$	0.4031	0.3634	0.4002	0.4002	0.3989
$\eta = 2$	0.4037	0.3666	0.4006	0.4010	0.3994
$\eta = 6$	0.4063	0.3765	0.4020	0.4042	0.4012
$\eta = 10$	0.4086	0.3825	0.4035	0.4070	0.4029
Ranking					
$\Delta_2 > \Delta_5 > \Delta_3 > \Delta_4 > \Delta_1$					
$\Delta_2 > \Delta_5 > \Delta_3 > \Delta_4 > \Delta_1$					
$\Delta_2 > \Delta_5 > \Delta_3 > \Delta_4 > \Delta_1$					
$\Delta_2 > \Delta_5 > \Delta_3 > \Delta_4 > \Delta_1$					

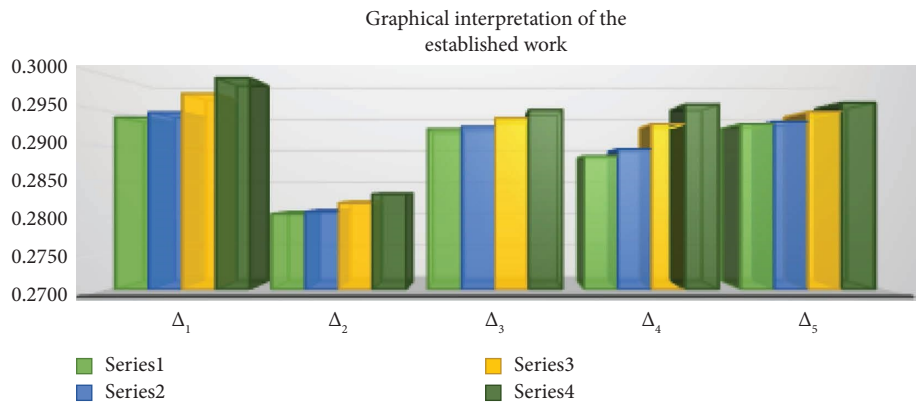

Figure 3: Graphical interpretation of a standard for $(\alpha_\{cc\}, \beta_\{cc\} = 0.7, 0.3)$ parameter values.

Table 4: Deviation basic information of interview experts in CIHFSs to each alternative and the ideal alternative $(\alpha_{cc}, \beta_{cc}) = (0.5, 0.5)$.

η	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5
$\eta = 1$	0.5527	0.4757	0.5477	0.5553	0.5430
$\eta = 2$	0.5534	0.4873	0.5482	0.5561	0.5436
$\eta = 6$	0.5563	0.5148	0.5502	0.5593	0.5462
$\eta = 10$	0.5590	0.5261	0.5522	0.5628	0.5487
Ranking					
$\Delta_2 > \Delta_5 > \Delta_3 > \Delta_1 > \Delta_4$					
$\Delta_2 > \Delta_5 > \Delta_3 > \Delta_1 > \Delta_4$					
$\Delta_2 > \Delta_5 > \Delta_3 > \Delta_1 > \Delta_4$					
$\Delta_2 > \Delta_5 > \Delta_3 > \Delta_1 > \Delta_4$					

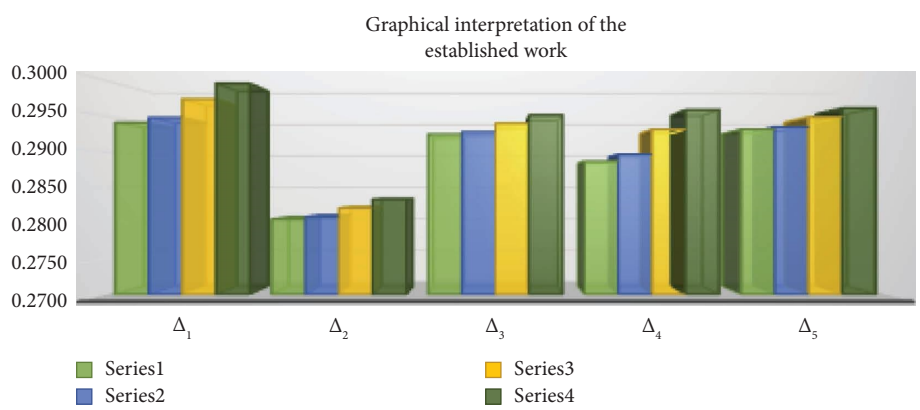

Figure 4: Graphical interpretation of a standard for $(\alpha_\{cc\}, \beta_\{cc\} = 0.5, 0.5)$ parameter values.

Table 5: Deviation basic information of interview experts in CIHFSs to each alternative and the ideal alternative $(\alpha_{cc}, \beta_{cc}) = (0.3, 0.7)$.

η	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5
$\eta = 1$	0.7582	0.6287	0.7497	0.7711	0.7391
$\eta = 2$	0.7592	0.6562	0.7504	0.7721	0.7400
$\eta = 6$	0.7631	0.7098	0.7535	0.7769	0.7439
$\eta = 10$	0.7667	0.7275	0.7566	0.7824	0.7478
Ranking					
$\Delta_2 > \Delta_5 > \Delta_3 > \Delta_1 > \Delta_4$					
$\Delta_2 > \Delta_5 > \Delta_3 > \Delta_1 > \Delta_4$					
$\Delta_2 > \Delta_5 > \Delta_3 > \Delta_1 > \Delta_4$					
$\Delta_2 > \Delta_5 > \Delta_3 > \Delta_1 > \Delta_4$					

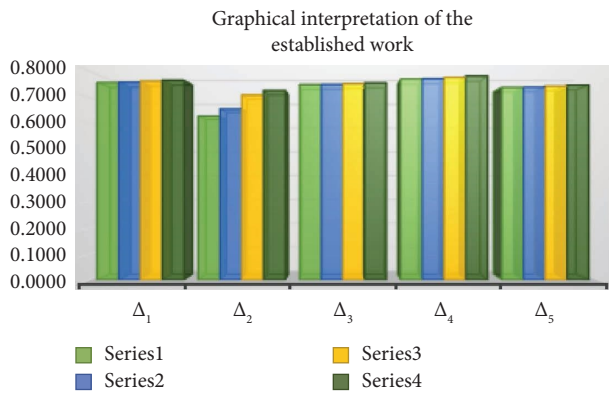

Figure 5: Graphical interpretation of a standard for $(\alpha_\{cc\}, \beta_\{cc\} = 0.3, 0.7)$ parameter values.

Table 6: Comparison of basic information of interview experts in CIHFSs.

Alternative	κ_1	κ_2	κ_3	κ_4
Δ_1	$\left\{\begin{pmatrix} 0.5e^{\sqrt{2}\pi(0.25)} \\ 0.4e^{\sqrt{2}\pi(0.5)} \\ 0.3e^{\sqrt{2}\pi(0.16667)} \end{pmatrix}, \begin{pmatrix} 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \end{pmatrix}\right\}$	$\left\{\begin{pmatrix} 0.9e^{\sqrt{2}\pi(0.125)} \\ 0.8e^{\sqrt{2}\pi(0.25)} \\ 0.7e^{\sqrt{2}\pi(0.5)} \\ 0.1e^{\sqrt{2}\pi(1)} \end{pmatrix}, \begin{pmatrix} 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \end{pmatrix}\right\}$	$\left\{\begin{pmatrix} 0.5e^{\sqrt{2}\pi(0.1667)} \\ 0.4e^{\sqrt{2}\pi(0.5)} \\ 0.2e^{\sqrt{2}\pi(0.08333)} \end{pmatrix}, \begin{pmatrix} 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \end{pmatrix}\right\}$	$\left\{\begin{pmatrix} 0.9e^{\sqrt{2}\pi(0.5)} \\ 0.6e^{\sqrt{2}\pi(1)} \\ 0.5e^{\sqrt{2}\pi(0.1)} \\ 0.3e^{\sqrt{2}\pi(0.5)} \end{pmatrix}, \begin{pmatrix} 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \end{pmatrix}\right\}$
Δ_2	$\left\{\begin{pmatrix} 0.5e^{\sqrt{2}\pi(0.125)} \\ 0.3e^{\sqrt{2}\pi(0.5)} \end{pmatrix}, \begin{pmatrix} 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \end{pmatrix}\right\}$	$\left\{\begin{pmatrix} 0.9e^{\sqrt{2}\pi(0.08333)} \\ 0.7e^{\sqrt{2}\pi(0.0.16667)} \\ 0.6e^{\sqrt{2}\pi(0.5)} \\ 0.5e^{\sqrt{2}\pi(0.5)} \\ 0.2e^{\sqrt{2}\pi(1)} \end{pmatrix}, \begin{pmatrix} 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \end{pmatrix}\right\}$	$\left\{\begin{pmatrix} 0.8e^{\sqrt{2}\pi(0.5)} \\ 0.6e^{\sqrt{2}\pi(0.25)} \\ 0.5e^{\sqrt{2}\pi(1)} \\ 0.1e^{\sqrt{2}\pi(0.5)} \end{pmatrix}, \begin{pmatrix} 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \end{pmatrix}\right\}$	$\left\{\begin{pmatrix} 0.7e^{\sqrt{2}\pi(1)} \\ 0.4e^{\sqrt{2}\pi(0.5)} \\ 0.3e^{\sqrt{2}\pi(0.25)} \end{pmatrix}, \begin{pmatrix} 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \end{pmatrix}\right\}$
Δ_3	$\left\{\begin{pmatrix} 0.7e^{\sqrt{2}\pi(0.5)} \\ 0.6e^{\sqrt{2}\pi(0.4)} \end{pmatrix}, \begin{pmatrix} 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \end{pmatrix}\right\}$	$\left\{\begin{pmatrix} 0.9e^{\sqrt{2}\pi(0.1)} \\ 0.6e^{\sqrt{2}\pi(0.5)} \end{pmatrix}, \begin{pmatrix} 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \end{pmatrix}\right\}$	$\left\{\begin{pmatrix} 0.7e^{\sqrt{2}\pi(0.125)} \\ 0.5e^{\sqrt{2}\pi(0.25)} \\ 0.3e^{\sqrt{2}\pi(0.08333)} \end{pmatrix}, \begin{pmatrix} 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \end{pmatrix}\right\}$	$\left\{\begin{pmatrix} 0.6e^{\sqrt{2}\pi(0.5)} \\ 0.4e^{\sqrt{2}\pi(1)} \end{pmatrix}, \begin{pmatrix} 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \end{pmatrix}\right\}$
Δ_4	$\left\{\begin{pmatrix} 0.8e^{\sqrt{2}\pi(0.5)} \\ 0.7e^{\sqrt{2}\pi(1)} \\ 0.4e^{\sqrt{2}\pi(0.25)} \\ 0.3e^{\sqrt{2}\pi(0.5)} \end{pmatrix}, \begin{pmatrix} 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \end{pmatrix}\right\}$	$\left\{\begin{pmatrix} 0.7e^{\sqrt{2}\pi(0.5)} \\ 0.4e^{\sqrt{2}\pi(0.125)} \\ 0.2e^{\sqrt{2}\pi(1)} \end{pmatrix}, \begin{pmatrix} 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \end{pmatrix}\right\}$	$\left\{\begin{pmatrix} 0.8e^{\sqrt{2}\pi(0.1)} \\ 0.1e^{\sqrt{2}\pi(1)} \end{pmatrix}, \begin{pmatrix} 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)}, \end{pmatrix}\right\}$	$\left\{\begin{pmatrix} 0.9e^{\sqrt{2}\pi(0.25)} \\ 0.8e^{\sqrt{2}\pi(0.5)} \\ 0.6e^{\sqrt{2}\pi(0.25)} \end{pmatrix}, \begin{pmatrix} 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \end{pmatrix}\right\}$
Δ_5	$\left\{\begin{pmatrix} 0.9e^{\sqrt{2}\pi(1)} \\ 0.7e^{\sqrt{2}\pi(0.25)} \\ 0.6e^{\sqrt{2}\pi(0.1),} \\ 0.3e^{\sqrt{2}\pi(0.08333)} \\ 0.1e^{\sqrt{2}\pi(0.5)} \end{pmatrix}, \begin{pmatrix} 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \end{pmatrix}\right\}$	$\left\{\begin{pmatrix} 0.8e^{\sqrt{2}\pi(1)} \\ 0.7e^{\sqrt{2}\pi(0.5)} \\ 0.6e^{\sqrt{2}\pi(0.25)} \\ 0.4e^{\sqrt{2}\pi(0.5)} \end{pmatrix}, \begin{pmatrix} 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \end{pmatrix}\right\}$	$\left\{\begin{pmatrix} 0.9e^{\sqrt{2}\pi(0.25)} \\ 0.8e^{\sqrt{2}\pi(1)} \\ 0.7e^{\sqrt{2}\pi(0.16667),} \end{pmatrix}, \begin{pmatrix} 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \end{pmatrix}\right\}$	$\left\{\begin{pmatrix} 0.9e^{\sqrt{2}\pi(0.25)} \\ 0.7e^{\sqrt{2}\pi(0.5)} \\ 0.6e^{\sqrt{2}\pi(0.16667)} \\ 0.3e^{\sqrt{2}\pi(1)} \end{pmatrix}, \begin{pmatrix} 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \\ 0e^{\sqrt{2}\pi(0)} \end{pmatrix}\right\}$

Table 7: Comparison of deviation basic information of interview experts in CIHFSs to each alternative and the ideal alternative $(\alpha_{cc}, \beta_{cc}) = (0.9, 0.1)$.

η	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5
$\eta = 1$	0.2938	0.2917	0.2996	0.2981	0.3010
$\eta = 2$	0.2945	0.2948	0.3001	0.2996	0.4050
$\eta = 6$	0.2975	0.3078	0.3021	0.3058	0.3065
$\eta = 10$	0.3005	0.3201	0.3035	0.3124	0.3114
Ranking					
$\Delta_2 > \Delta_1 > \Delta_4 > \Delta_3 > \Delta_5$					
$\Delta_1 > \Delta_2 > \Delta_4 > \Delta_3 > \Delta_5$					
$\Delta_1 > \Delta_3 > \Delta_4 > \Delta_5 > \Delta_2$					
$\Delta_1 > \Delta_3 > \Delta_5 > \Delta_4 > \Delta_2$					

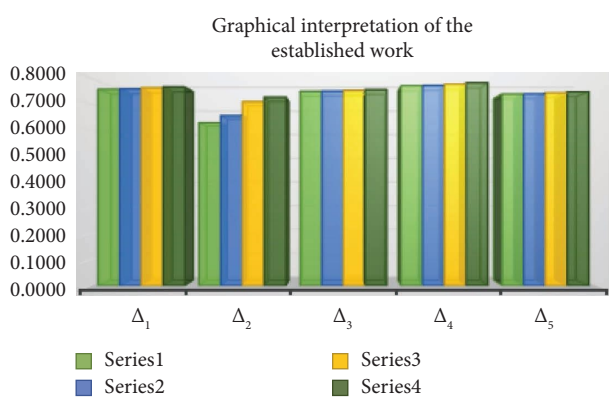

Figure 6: Graphical interpretation of a standard for $(\alpha_\{cc\}, \beta_\{cc\} = 0.9, 0.1)$ parameter values.

Table 8: Comparison of deviation basic information of interview experts in CIHFSs to each alternative and the ideal alternative $(\alpha_{cc}, \beta_{cc}) = (0.7, 0.3)$.

η	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5
$\eta = 1$	0.4239	0.4207	0.4224	0.4223	0.4342
$\eta = 2$	0.4249	0.4235	0.4227	0.4238	0.5249
$\eta = 6$	0.4292	0.4361	0.4237	0.4299	0.4452
$\eta = 10$	0.4335	0.4491	0.4248	0.4364	0.4553
Ranking					
$\Delta_2 > \Delta_4 > \Delta_3 > \Delta_1 > \Delta_5$					
$\Delta_3 > \Delta_2 > \Delta_4 > \Delta_1 > \Delta_5$					
$\Delta_3 > \Delta_1 > \Delta_4 > \Delta_2 > \Delta_5$					
$\Delta_3 > \Delta_1 > \Delta_4 > \Delta_2 > \Delta_4$					

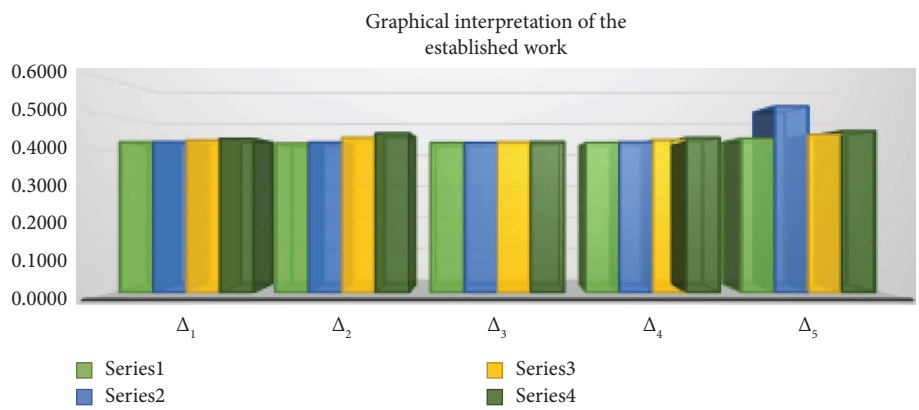

Figure 7: Graphical interpretation of a standard for $(\alpha_\{cc\}, \beta_\{cc\} = 0.7, 0.3)$ parameter values.

Table 9: Comparison of deviation basic information of interview experts in CIHFSs to each alternative and the ideal alternative $(\alpha_{cc}, \beta_{cc}) = (0.5, 0.5)$.

η	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5
$\eta = 1$	0.6115	0.6072	0.5963	0.5986	0.6265
$\eta = 2$	0.6130	0.6096	0.5966	0.6000	0.6938
$\eta = 6$	0.6192	0.6204	0.5978	0.6059	0.6483
$\eta = 10$	0.6254	0.6326	0.5991	0.6120	0.6683
Ranking					
$\Delta_3 > \Delta_4 > \Delta_2 > \Delta_1 > \Delta_5$					
$\Delta_3 > \Delta_4 > \Delta_2 > \Delta_1 > \Delta_5$					
$\Delta_3 > \Delta_4 > \Delta_1 > \Delta_2 > \Delta_5$					
$\Delta_3 > \Delta_4 > \Delta_1 > \Delta_3 > \Delta_5$					

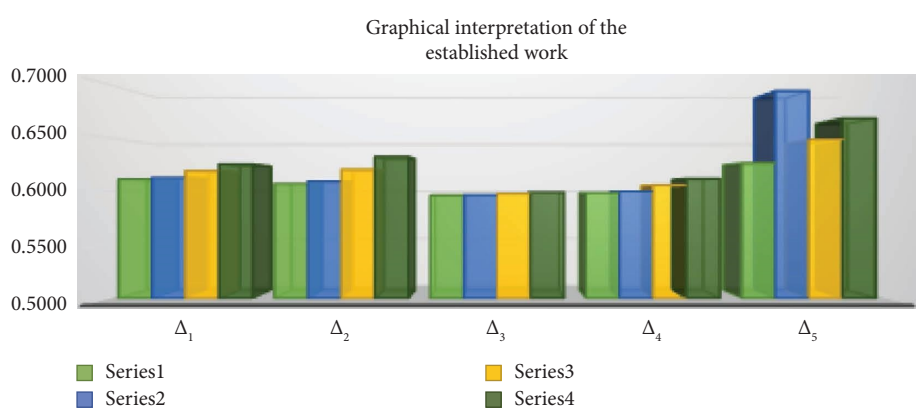

Figure 8: Graphical interpretation of a standard for $(\alpha_\{cc\}, \beta_\{cc\} = 0.5, 0.5)$ parameter values.

Table 10: Comparison of deviation basic information of interview experts in CIHFSs to each alternative and the ideal alternative $(\alpha_{cc}, \beta_{cc}) = (0.3, 0.7)$.

η	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5
$\eta = 1$	0.8823	0.8772	0.8428	0.8487	0.9044
$\eta = 2$	0.8844	0.8790	0.8442	0.8502	0.9356
$\eta = 6$	0.8933	0.8870	0.8496	0.8561	0.9466
$\eta = 10$	0.9023	0.8965	0.8547	0.8619	0.9838
Ranking					
$\Delta_3 > \Delta_4 > \Delta_2 > \Delta_1 > \Delta_5$					
$\Delta_3 > \Delta_4 > \Delta_2 > \Delta_1 > \Delta_5$					
$\Delta_3 > \Delta_4 > \Delta_2 > \Delta_1 > \Delta_5$					
$\Delta_3 > \Delta_4 > \Delta_2 > \Delta_1 > \Delta_5$					

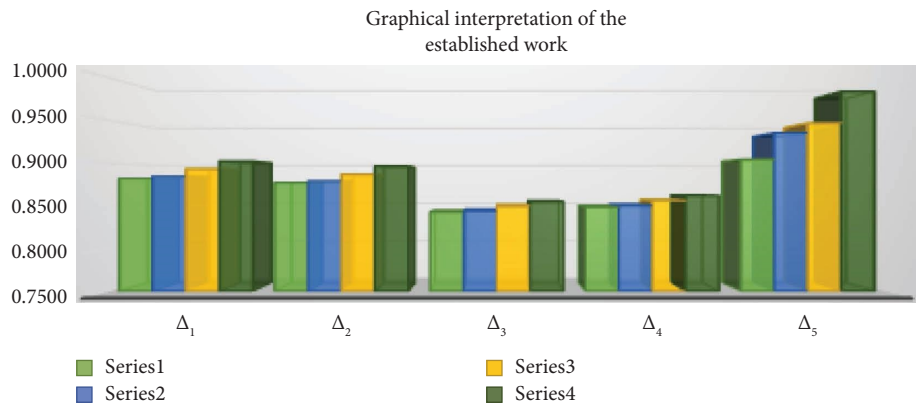

Figure 9: Graphical interpretation of a standard for $(\alpha_\{cc\}, \beta_\{cc\} = 0.3, 0.7)$ parameter values.

8. Conclusion

In order to give decision-makers in MAGDM challenges more freedom, the technique of CIHFSs was developed in the context of fuzzy sets, hesitant fuzzy sets, and complex sets. This was done by qualitatively specifying the evaluation values. In contrast to conventional fuzzy models, a CIHFS model may address issues with consistency, precision, and imperfection in real-world situations. We described the CIHFSs operators, which are effective and flexible operators for coping with uncertain MCGDM circumstances. The authenticity and effectiveness of the technique are also demonstrated by using an example to maintain the business for the industry. Because it can handle partial, uncertain, inconsistent, and other types of data that are common in real-world situations, the suggested CIHFS multicriteria decision-making method is better suited to real-world scientific and engineering applications. The method proposed in this research complements current decision-making strategies and gives decision-makers a useful tool. In terms of potential future works that are good enough to warrant, several essential subjects are still present. We will continue to work on expanding and applying the established operators to more domains (such as EDAS and TODIM) in the future.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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