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Research Article

Source Localization Using RSS Measurements with Sensor Position Uncertainty

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Received signal strength- (RSS-) based localization has attracted considerable attention for its low cost and easy implementation. In plenty of existing work, sensor positions, which play an important role in source localization, are usually assumed perfectly known. Unfortunately, they are often subject to uncertainties, which directly leads to effect on localization result. To tackle this problem, we study the RSS-based source localization with sensor position uncertainty. Sensor position uncertainty will be modeled as two types: Gaussian random variable and unknown nonrandom variable. For either of the models, two semidefinite programming (SDP) methods are proposed, i.e., SDP-1 and SDP-2. The SDP-1 method proceeds from the nonconvex problem with respect to the maximum likelihood (ML) estimation and then obtains an SDP problem using proper approximation and relaxation. The SDP-2 method first transfers the sensor position uncertainties to the source position and then obtains an SDP formulation following a similar idea as in SDP-1 method. Numerical examples demonstrate the performance superiority of the proposed methods, compared to some existing methods assuming perfect sensor position information.

1. Introduction

Over the past decade, localization using wireless sensor network (WSN) has attracted a great deal of research interest for its wide applications such as military, environmental, health, and commercial aspects [1, 2]. It overcomes some limitations of traditional global navigation satellite system- (GNSS-) based methods, e.g., GPS's invalidity in indoor environments and vision-based localization's instability in poor light scenarios. Depending on different physical measurements of radio signal, wireless localization can be divided into several types, such as angle of arrival (AOA), time of arrival (TOA), time difference of arrival (TDOA), and received signal strength (RSS) [3–7]. Among these different schemes, RSS-based localization has the merits of low cost and easy implementation since most wireless devices have built-in receiver modules [8]. For example, a smartphone can be directly utilized to sample the RSS measurements of an emission source.

There has been large amounts of work on wireless localization using RSS measurements [9–13], which includes range-free method and range-based method. Range-free methods do not utilize RSS model directly. They construct RSS database at offline phase and then estimate the location according to the database at online phase. Such methods are generally sensible to environment changes. By comparison, range-based methods estimate the node's location using the RSS measurements at known sensor positions according to specific RSS model. For many applications, they are more attractive and can provide better localization accuracies. The ML and least squares (LS) are traditional range-based methods, which are also nonlinear methods. They suffer from nonconvexities of the formulated estimation problems and need proper initialization. Otherwise, the globally optimal solution can not be guaranteed [14]. Linear methods provide linear estimators through linearizing the RSS model and applying LS estimation, which include linear LS method and weighted linear LS method [15]. Recently, optimization-based methods, e.g., SDP and second-order cone programming (SOCP), have become popular in RSS localization [16–18]. They usually originate from ML estimation but overcome the nonconvexity of ML method and obtain convex problems by using approximation and relaxation. Besides, they generally have better localization accuracy than linear methods.

For RSS-based source localization, sensor positions are essential in addition to RSS measurements. However, in plenty of existing work, a common assumption is that the sensor positions are perfectly known. Unfortunately, the exact information about the sensor positions is not prior known and usually obtained by some self-localization schemes, e.g., GPS, which is inevitably erroneous [19]. As a result, this error (uncertainty) will pass to source localization if it is ignored. Especially, when the sensor positions have large uncertainties, the accuracy of the source localization will be affected significantly.

There has been some work on source localization with sensor position uncertainty, in which the uncertainty is usually assumed Gaussian random variable [19–22]. In [19], a joint ML algorithm is proposed for estimating source position and anchor positions, in which an iterative trust region strategy is used to solve the corresponding nonlinear problem. In [20], ranging measurements are used to formulate a robust localization problem with the ML criterion and an SOCP problem is devised using convex relaxation. In [9], SDP algorithms are devised, which use distance between nodes obtained from TOA measurements. In [21], an RSS difference-based method is presented to localize a source. It uses a constrained adaptive weighted LS technique to obtain an initial estimate and then improves it with a computationally efficient modified Newton method. Overall, most of existing work deals with range measurements (obtained from TOA or TDOA measurements), and only a few is designed for RSS measurements.

In practice, sensor position uncertainty may originate from system error, which is a deterministic parameter. As such, it is reasonable to model the uncertainty as an unknown nonrandom variable. Besides, prior information on its range is probably available. In view of this, in this paper, we study the RSS-based source localization with different assumptions on the sensor position uncertainty. Two types of uncertainties will be considered: Gaussian random case and unknown nonranodm case. For either of the two cases, two SDP methods, i.e., SDP-1 and SDP-2, are presented. The SDP-1 method uses the original model to formulate the ML estimation of the source and sensor positions and transforms its nonconvex problem to an SDP problem with approximation and relaxation techniques. The SDP-2 method transfers the uncertainties of sensor positions to source position and obtains a new model. On basis of this, a new SDP problem is also presented with the ML criterion. In summary, the innovative contribution of this paper is to model the uncertainty of sensor position in RSS-based localization and propose new and efficient localization methods, including modeling the uncertainty as deterministic unknown error and deriving new SDP methods with transformed model.

The rest of the paper is organized as follows. Section 2 presents two different models for the RSS-based source localization with sensor position uncertainty. Section 3 presents the SDP methods for source localization with Gaussian random uncertainty of sensor position. Section 4 presents the SDP methods for source localization with unknown nonrandom uncertainty of sensor position. In Section 5, performance of the proposed methods is demonstrated through numerical examples. Section 6 concludes the paper.

2. Problem Formulation

We consider a WSN consisting of a single source to be localized and n sensors with locations subject to uncertainties. The uncertainties will be modeled as random variables and nonrandom variables, respectively, in this paper. Accordingly, we will provide two formulations of source localization using RSS measurement models with sensor position uncertainties.

2.1. Gaussian Random Uncertainty. For the Gaussian random case, we have the following measurement model:

$$\begin{aligned} P_{i,j} &= P_0 - 10\beta \log_{10} \frac{d_j(\mathbf{x}, \mathbf{x}_j)}{d_0} + n_{i,j}, \\ \mathbf{z}_j &= \mathbf{x}_j + v_j, \; i = 1, 2, \cdots, k, \; j = 1, 2, \cdots, n, \end{aligned} \tag{1}$$

where $P_{i,j}$ denotes the RSS measurement of sensor j at ith sampling time, P_0 is the reference power at a reference distance d_0 ($d_0 = 1m$ is usually assumed), β is the path loss exponent (assumed to be known), $\mathbf{x}$ is the source location, $\mathbf{x}_j$ is the true unknown position of sensor j, $d_j(\mathbf{x}, \mathbf{x}_j)$ denotes the distance between the source and sensor j, and $\mathbf{z}_j$ is the uncertain position of sensor j. $n_{i,j}$ is the noise of RSS measurement, assumed to be independent and identically distributed (i.i.d) zero-mean Gaussian random variable with variances $(\sigma^r_{i,j})^2$, and v_j is the noise of sensor position measurement, also assumed to be Gaussian random variable with zero-mean and covariance matrix $\mathbf{\Phi}_j$. $n_{i,j}$ and v_j are assumed to be mutually independent. It is reasonable to model the position uncertainty as Gaussian random variable, since the sensor positions are inevitably obtained with coupled random errors in some cases. These errors may be generated by a variety of factors. When these factors are independent, the sum of the errors in sensor position approximately has the Gaussian distribution according to the central limit theorem.

2.2. Unknown Nonrandom Uncertainty. For the nonrandom case, we have the following measurement model:

$$\begin{aligned} P_{i,j} &= P_0 - 10\beta \log_{10} \frac{d_j(\mathbf{x}, \mathbf{x}_j)}{d_0} + n_{i,j}, \\ \mathbf{z}_j &= \mathbf{x}_j + \varepsilon_j, \\ \left|\varepsilon_j\right| &\le a_j, \; j = 1, 2, \cdots, n, \end{aligned} \tag{2}$$

where $\mathbf{z}_j$ is also the uncertain position of sensor j with the unknown deterministic error ε_j. Additionally, some range information on the error is assumed to be available. Other notations have the same meanings as in model (1). $n_{i,j}$ is also assumed i.i.d zero-mean Gaussian random variable with variance $(\sigma^r_{i,j})^2$.

For both models (1) and (2), source localization is to estimate the source location $\mathbf{x}$ using the noisy RSS measurements $P_{i,j}$ and uncertain sensor positions $\mathbf{z}_j$. Two SDP methods will be proposed: SDP-1 and SDP-2, according to either of the two models.

3. Source Localization with Gaussian Random Uncertainty of Sensor Position

First, we consider the source localization with model (1). We first propose an SDP-1 method that originates from the ML estimation using combined RSS and sensor position measurements. Then, we propose an SDP-2 method with a transformed model that transfers the uncertainty of sensor position to source position.

3.1. SDP-1 Method. Let $\mathbf{X} = [\mathbf{x}, \mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_n]$ be the unknown parameter vector comprising true source location and sensor locations. On basis of the assumption on the the noises n_j and v_j as before, the likelihood function of $\mathbf{X}$ given the RSS measurements $\mathbf{P} = [P_{1,1}, \cdots, P_{k,1}, \cdots, P_{1,n} \cdots, P_{k,n}]^T$ and noisy sensor positions $\mathbf{Z} = [\mathbf{z}_1, \mathbf{z}_2, \cdots, \mathbf{z}_n]^T$ can be written as

$$\begin{aligned} p(\mathbf{P}, \mathbf{Z}|\mathbf{X}) &= p(\mathbf{P}|\mathbf{X}) \times p(\mathbf{Z}|\mathbf{X}) \\ &= \prod_{i=1}^{k}\prod_{j=1}^{n} \frac{1}{\sqrt{2\pi}\sigma^r_{i,j}} \exp\left(\frac{-\left(P_{i,j} - P_0 + 10\beta \log_{10}\left(d_j(\mathbf{x}, \mathbf{x}_j)/d_0\right)\right)^2}{2\left(\sigma^r_{i,j}\right)^2}\right) \\ &\quad \times \prod_{j=1}^{n} \frac{1}{\sqrt{2\pi|\mathbf{\Phi}_j|}} \exp\left(\frac{-(\mathbf{z}_j - \mathbf{x}_j)^T \mathbf{\Phi}_j^{-1}(\mathbf{z}_j - \mathbf{x}_j)}{2}\right). \end{aligned} \tag{3}$$

Then, ML estimation of the unknown position vector $\mathbf{X}$ is

$$\widehat{\mathbf{X}} = \arg \min_{\mathbf{X}} \sum_{i=1}^{k}\sum_{j=1}^{n} \frac{1}{\left(\sigma^r_{i,j}\right)^2}\left(P_{i,j} - \tilde{P}_{i,j}\right)^2 + \sum_{j=1}^{n} (\mathbf{z}_j - \mathbf{x}_j)^T \mathbf{\Phi}_j^{-1}(\mathbf{z}_j - \mathbf{x}_j), \tag{4}$$

where $\tilde{P}_{i,j} = P_0 - 10\beta \log_{10}(d_j(\mathbf{x}_j, \mathbf{x}_s)/d_0)$.

Since $P_{i,j} - \tilde{P}_{i,j} = 5\beta(\log_{10} d_j^2 10^{(P_{i,j}-P_0)/5\beta})$ and $\log_{10} d_j^2 10^{(P_{i,j}-P_0)/5\beta} \approx (1/\ln 10)(d_j^2 10^{(P_{i,j}-P_0)/5\beta} - 1)(d_j^2 10^{(P_{i,j}-P_0)/5\beta} \longrightarrow 1$ as $n_{i,j} \longrightarrow 0)$, problem (4) can be approximated as

$$\begin{aligned} \widehat{\mathbf{X}} = \arg \min_{\mathbf{X}} &\sum_{i=1}^{k}\sum_{j=1}^{n} \left(\frac{5\beta}{\ln 10\sigma^r_{i,j}}\right)^2 \left(d_j^2 10^{(P_{i,j}-P_0)/5\beta} - 1\right) \\ &+ \sum_{j=1}^{n} \frac{1}{\left(\sigma^s_j\right)^2} \left\|\mathbf{z}_j - \mathbf{x}_j\right\|^2, \end{aligned} \tag{5}$$

where v_j is assumed to have the same variance $(\sigma^s_j)^2$ in x and y directions ($\mathbf{\Phi}_j$ is diagonal with all elements being $(\sigma^s_j)^2$).

Problem (5) can be further represented as

$$\widehat{\mathbf{X}} = \arg \min_{\mathbf{X}} \sum_{i=1}^{k}\sum_{j=1}^{n} w^r_{i,j}\left(\lambda_{i,j} h_j - 1\right)^2 + \sum_{j=1}^{n} w^s_j \left\|\mathbf{z}_j - \mathbf{x}_j\right\|^2, \tag{6a}$$

$$\text{s.t. } h_j = \left\|\mathbf{x} - \mathbf{x}_j\right\|_2^2, \tag{6b}$$

where $\lambda_{i,j} = 10^{(P_{i,j}-P_0)/(5\beta)}$, $w^r_{i,j} = (5\beta/\ln 10\sigma^r_{i,j})^2$, and $w^s_j = 1/(\sigma^s_j)^2$. It can be seen that the cost function consists of two parts. The first reflects the likelihood of RSS measurements with weighted coefficient $w^r_{i,j}$, and the second reflects the likelihood of sensor positions with weighted coefficient w^s_j.

By introducing an auxiliary variable $\mathbf{Z} = \mathbf{X}^T\mathbf{X}$, problem (6a) and (6b) can be converted into the following SDP problem:

$$\widehat{\mathbf{X}} = \arg \min_{\substack{t_{i,j}, s_j, h_j, \\ \mathbf{X}, \mathbf{Z}}} \sum_{i=1}^{k}\sum_{j=1}^{n} t_{i,j} + \sum_{j=1}^{n} s_j, \tag{7a}$$

$$\text{s.t. } w^r_{i,j}\left(\lambda_{i,j} h_j - 1\right)^2 \le t_{i,j}, \tag{7b}$$

$$w^s_j\left(\mathbf{z}_j - \mathbf{x}_j\right)^T\left(\mathbf{z}_j - \mathbf{x}_j\right) \le s_j, \tag{7c}$$

$$\mathbf{x}_j = \mathbf{X}(:, j+1), \tag{7d}$$

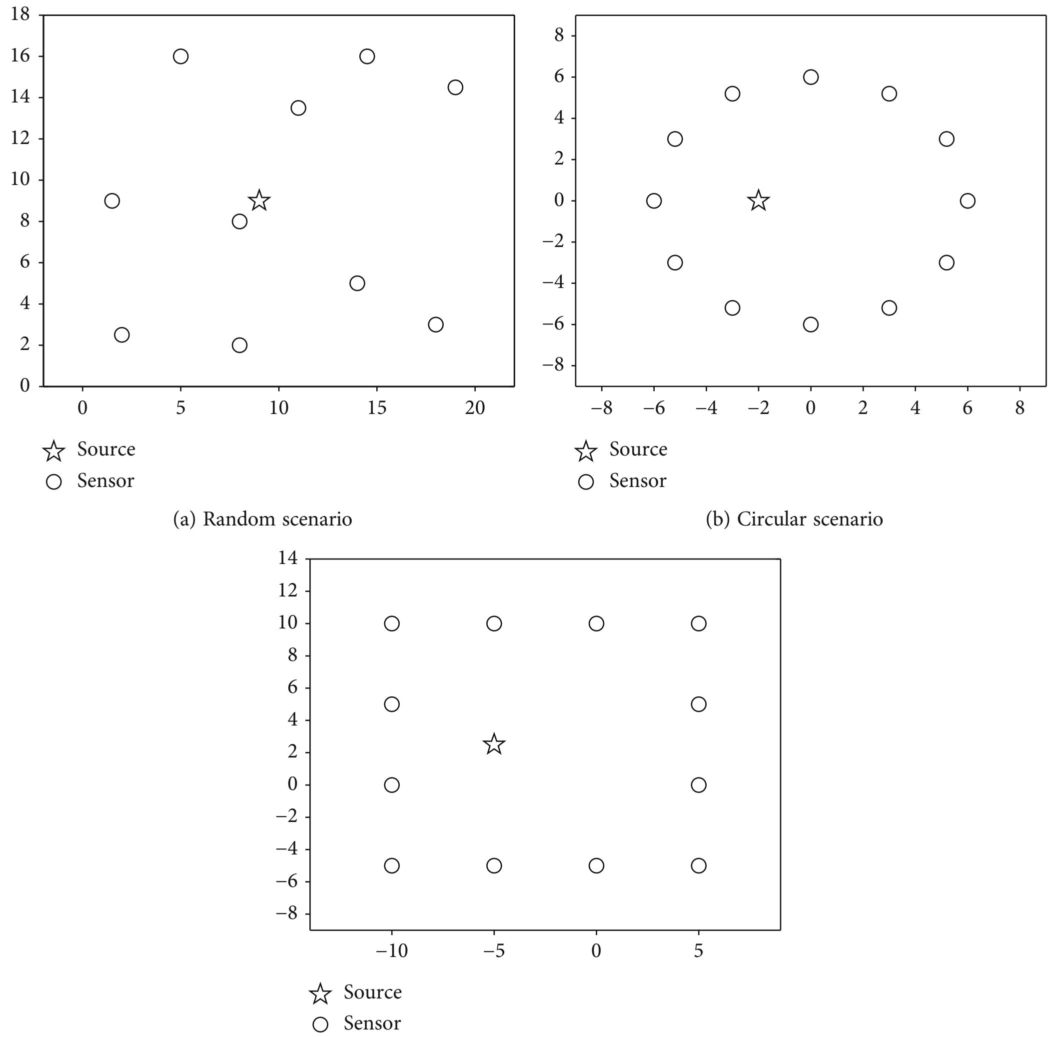
$$h_j = \begin{bmatrix} \mathbf{0}_2 \\ \mathbf{e}_1 - \mathbf{e}_j \end{bmatrix}^T \begin{bmatrix} \mathbf{I}_2 & \mathbf{X} \\ \mathbf{X}^T & \mathbf{Z} \end{bmatrix} \begin{bmatrix} \mathbf{0}_2 \\ \mathbf{e}_1 - \mathbf{e}_j \end{bmatrix}, \tag{7e}$$

$$\begin{bmatrix} \mathbf{I}_2 & \mathbf{X} \\ \mathbf{X}^T & \mathbf{Z} \end{bmatrix} \ge \mathbf{0}_{n+3}, \tag{7f}$$

where $\mathbf{e}_j$ is a $(n+1) \times 1$ unit vector with jth element being 1. Note that $\mathbf{Z} = \mathbf{X}^T\mathbf{X}$ is relaxed to $\begin{bmatrix} \mathbf{I}_2 & \mathbf{X} \\ \mathbf{X}^T & \mathbf{Z} \end{bmatrix} \ge \mathbf{0}_{n+3}$ such that it becomes a linear matrix inequality and satisfies the constraint of an SDP [23]. Besides, h_j in (7e) is actually

$$h_j = d_j^2 = \left\|\mathbf{x}_s - \mathbf{x}_j\right\|_2^2 = \mathbf{a}_j^T \mathbf{A} \mathbf{a}_j, \tag{8}$$

where $\mathbf{a}_j = [0 \quad 0 \quad 1 \quad 0 \quad \cdots \quad -1 \quad \cdots \quad 0]^T$ with the dimension of $(n+3) \times 1$ and the $(j+3)$th element being -1, $\mathbf{A} = \begin{bmatrix} 1 & 0 & x & x_1 & \cdots & x_n \\ 0 & 1 & y & y_1 & \cdots & y_n \\ x & y & \mathbf{x}^T\mathbf{x} & \mathbf{x}^T\mathbf{x}_1 & \cdots & \mathbf{x}^T\mathbf{x}_n \\ x_1 & y_1 & \mathbf{x}_1^T\mathbf{x} & \mathbf{x}_1^T\mathbf{x}_1 & \cdots & \mathbf{x}_1^T\mathbf{x}_n \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ x_n & y_n & \mathbf{x}_n^T\mathbf{x} & \mathbf{x}_n^T\mathbf{x}_1 & \cdots & \mathbf{x}_n^T\mathbf{x}_n \end{bmatrix}$, $\mathbf{x} = [x, y]^T$, and $\mathbf{x}_j = [x_j, y_j]^T$.



(a) Random scenario

(b) Circular scenario

(c) Rectangular scenario

Figure 1: Three scenarios.

3.2. SDP-2 Method. Consider model (1). The true sensor position can be written as

$$\mathbf{x}_j = \mathbf{z}_j - \mathbf{v}_j. \tag{9}$$

Then, the RSS measurement can be represented as

$$P_{i,j} = P_0 - 10\beta \log_{10} \frac{d_j(\mathbf{x}, \mathbf{z}_j - \mathbf{v}_j)}{d_0} + n_{i,j}. \tag{10}$$

Since $d_j(\mathbf{x}, \mathbf{z}_j - \mathbf{v}_j) = d_j(\mathbf{x} + \mathbf{v}_j, \mathbf{z}_j)$, (10) is equivalent to

$$P_{i,j} = P_0 - 10\beta \log_{10} \frac{d_j(\mathbf{x} + \mathbf{v}_j, \mathbf{z}_j)}{d_0} + n_{i,j}. \tag{11}$$

As such, a new model for RSS localization with sensor position uncertainty can be obtained as

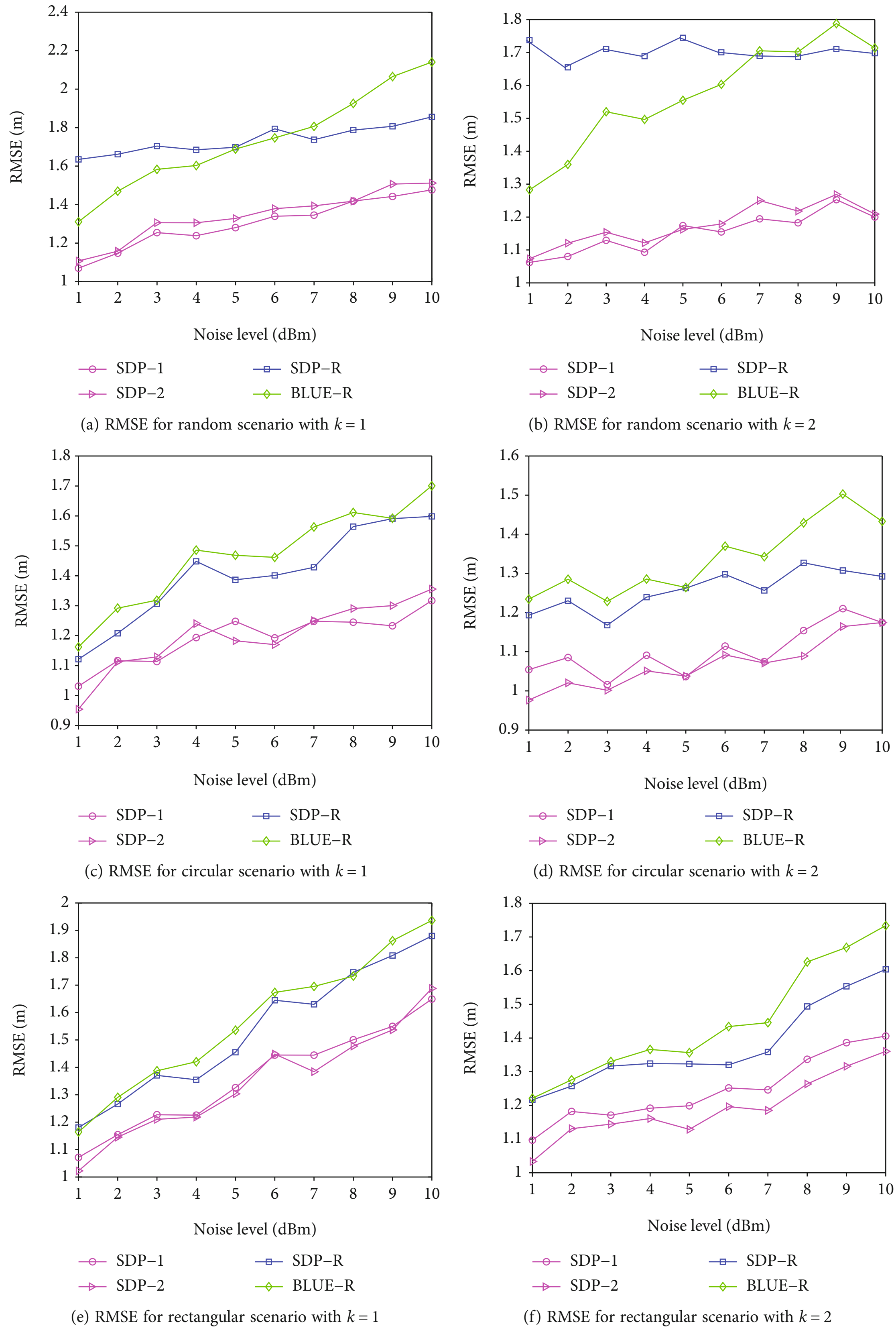
$$P_{i,j} = P_0 - 10\beta \log_{10} \frac{d_j\left(\mathbf{x}_j^v, \mathbf{z}_j\right)}{d_0} + n_{i,j}, \tag{12a}$$

$$\mathbf{x}_j^v = \mathbf{x} + \mathbf{v}_j,\ i = 1, 2, \cdots, k,\ j = 1, 2, \cdots, n, \tag{12b}$$

where $\mathbf{x}_j^v$ is a new unknown variable, and other notations are defined as before.

3.2.1. Remark. Compared to model (1), the new model (12a) and (12b) transfers the uncertainties in sensor positions to source position and introduces new variable x_j^v, which can be seen as a measurement of source position. By this, it is easier for optimization since the unknown variables become fewer in the RSS function of the new model. That is, only x_j^v is unknown for $d_j(\mathrm{x}_j^v, \mathrm{z}_j)$ while both x and x_j are unknown for $d_j(\mathrm{x}, \mathrm{x}_j)$.

Let $\mathbf{X}^v = [\mathbf{x}_1^v, \mathbf{x}_2^v, \cdots, \mathbf{x}_n^v]$ be the noisy source locations and $\mathbf{Y} = [\mathbf{x}, \mathbf{X}^v]$. Assume that $\mathbf{x}_j^v$ is available in (12b), we can formulate the likelihood functions of $\mathbf{X}^v$ and $\mathbf{x}$, respectively.



(a) RMSE for random scenario with $k = 1$

(b) RMSE for random scenario with $k = 2$

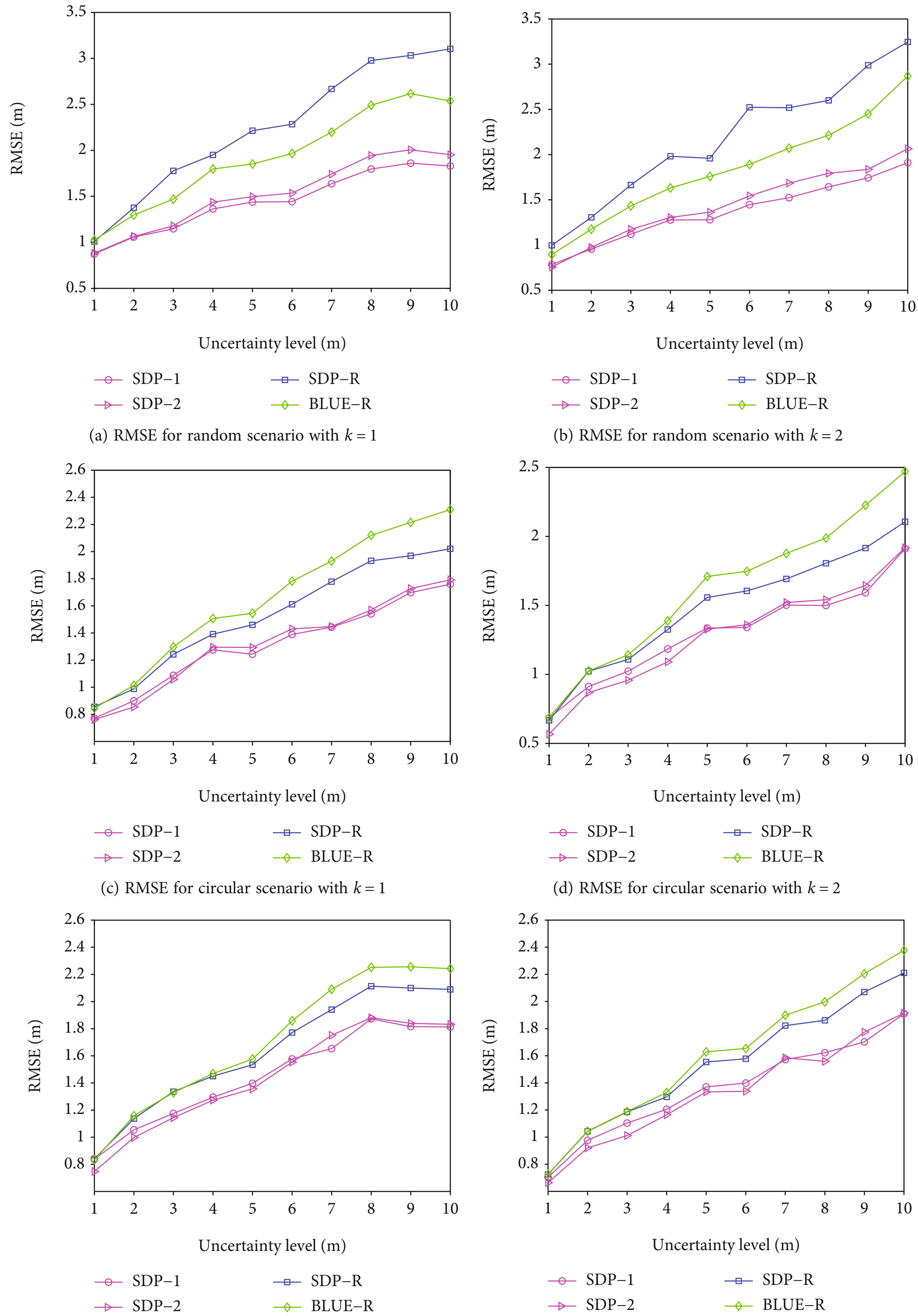
(c) RMSE for circular scenario with $k = 1$

(d) RMSE for circular scenario with $k = 2$

(e) RMSE for rectangular scenario with $k = 1$

(f) RMSE for rectangular scenario with $k = 2$

Figure 2: RMSE versus the noise level of RSS measurement for random case.



(a) RMSE for random scenario with $k = 1$

(b) RMSE for random scenario with $k = 2$

(c) RMSE for circular scenario with $k = 1$

(d) RMSE for circular scenario with $k = 2$

(e) RMSE for rectangular scenario with $k = 1$

(f) RMSE for rectangular scenario with $k = 2$

Figure 3: RMSE versus the uncertainty level of sensor position for random case.

$$p(\mathbf{P}|\mathbf{X}^v) = \prod_{i=1}^{k}\prod_{j=1}^{n}\frac{1}{\sqrt{2\pi}\sigma_{i,j}^r}\exp\left(\frac{-\left(P_{i,j}-P_0+10\beta\log_{10}\left(d_j\left(\mathbf{x}_j^v,\mathbf{z}_j\right)/d_0\right)\right)^2}{2\left(\sigma_{i,j}^r\right)^2}\right), \quad (13)$$

$$p(\mathbf{X}^v|\mathbf{x}) = \prod_{j=1}^{n}\frac{1}{\sqrt{2\pi|\mathbf{\Phi}_j|}}\exp\left(\frac{-\left(\mathbf{x}_j^v-\mathbf{x}\right)^T\mathbf{\Phi}_j^{-1}\left(\mathbf{x}_j^v-\mathbf{x}\right)}{2}\right). \quad (14)$$

Then, a combined ML estimation problem can be formulated through the common $\mathbf{X}^v$ as following:

$$\widehat{\mathbf{Y}} = \arg\min_{\mathbf{Y}}\sum_{i=1}^{k}\sum_{j=1}^{n}\frac{1}{\left(\sigma_{i,j}^r\right)^2}\left(P_{i,j}-P_{i,j}^v\right)^2+\sum_{j=1}^{n}\left(\mathbf{x}_j^v-\mathbf{x}\right)^T\mathbf{\Phi}_j^{-1}\left(\mathbf{x}_j^v-\mathbf{x}\right), \quad (15)$$

where $P_{i,j}^v = P_0 - 10\beta\log_{10}(d_j(\mathbf{x}_j^v,\mathbf{z}_j)/d_0)$.

By following the similar procedures from (4) to (7a), (7b), (7c), (7d), (7e) and (7f), an SDP problem with respect to (15) can be obtained as

$$\arg\min_{t_{i,j},s_j,h_j,\mathbf{X},\mathbf{Z}}\sum_{j=1}^{n}\sum_{i=1}^{k}w_{i,j}^r t_{i,j}+\sum_{j=1}^{n}w_j^s s_j, \quad (16a)$$

$$\text{s.t.}\left(\lambda_{i,j}h_j-1\right)^2\le t_{i,j}, \quad (16b)$$

$$h_j = \mathbf{Z}(i+1,i+1)-2\left(\mathbf{z}_j\right)^T\mathbf{Y}(:,i+1)+2\left(\mathbf{z}_j\right)^T\mathbf{z}_j, \quad (16c)$$

$$s_j = \begin{bmatrix}\mathbf{0}_2\\ \mathbf{e}_1-\mathbf{e}_j\end{bmatrix}^T\begin{bmatrix}\mathbf{I}_2 & \mathbf{Y}\\ \mathbf{Y}^T & \mathbf{Z}\end{bmatrix}\begin{bmatrix}\mathbf{0}_2\\ \mathbf{e}_1-\mathbf{e}_j\end{bmatrix}, \quad (16d)$$

$$\begin{bmatrix}\mathbf{I}_2 & \mathbf{Y}\\ \mathbf{Y}^T & \mathbf{Z}\end{bmatrix}\ge\mathbf{0}_{n+3}, \quad (16e)$$

where $w_{i,j}^r$, w_j^s, $\lambda_{i,j}$, and $\mathbf{e}_j$ are defined as before. Also, s_j in (16d) is actually

$$s_j = \mathbf{a}_j^T\mathbf{B}\mathbf{a}_j, \quad (17)$$

where

$$\mathbf{B} = \begin{bmatrix} 1 & 0 & x & x_1^v & \cdots & x_n^v\\ 0 & 1 & y & y_1^v & \cdots & y_n^v\\ x & y & \mathbf{x}^T\mathbf{x} & \mathbf{x}^T\mathbf{x}_1^v & \cdots & \mathbf{x}^T\mathbf{x}_n^v\\ x_1^v & y_1^v & (\mathbf{x}_1^v)^T\mathbf{x} & (\mathbf{x}_1^v)^T\mathbf{x}_1^v & \cdots & (\mathbf{x}_1^v)^T\mathbf{x}_n^v\\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots\\ x_n^v & y_n^v & (\mathbf{x}_n^v)^T\mathbf{x} & (\mathbf{x}_n^v)^T\mathbf{x}_1^v & \cdots & (\mathbf{x}_n^v)^T\mathbf{x}_n^v\end{bmatrix}, \quad (18)$$

$\mathbf{a}_j$ is defined as before, and $\mathbf{x}_j^v = [x_j^v, y_j^v]^T$.

Table 1: Running time (s) for random case.

Method	$k = 1$	$k = 2$
SDP-1	1.78	2.29
SDP-2	1	1.49

4. Source Localization with Unknown Nonrandom Uncertainty of Sensor Position

Next, we consider the source localization with model (2). Prior information on the range of the sensor position error is assumed available. Taking this as a constraint in localization, two SDP methods will be also presented: SDP-1 and SDP-2.

4.1. SDP-1 Method. According to model (2), given the RSS measurements of all sensors, an ML estimation for the source location can be obtained using the constraints on the the sensor positions as

$$\begin{aligned}\widehat{\mathbf{X}} = \arg\ &\min_{\mathbf{X}}\sum_{i=1}^{k}\sum_{j=1}^{n}\left(P_{i,j}-\tilde{P}_{i,j}\right)^2,\\ \text{s.t.}\ &\left\|\mathbf{z}_j-\mathbf{x}_j\right\|\le a_j,\end{aligned} \quad (19)$$

where $\mathbf{X}$, $\tilde{P}_{i,j}$ are defined as before.

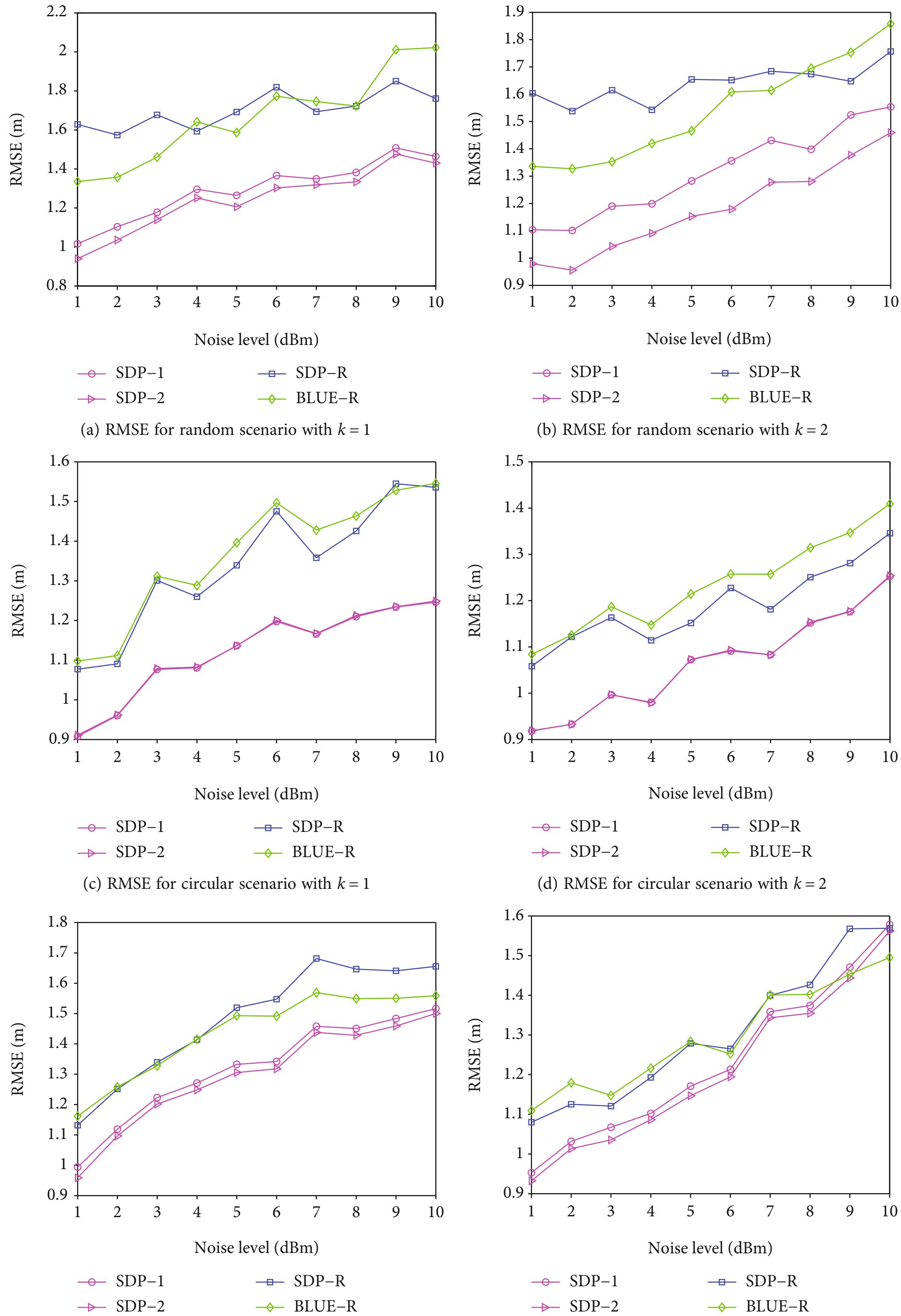
That is

$$\begin{aligned}\widehat{\mathbf{X}} = \arg\ &\min_{\mathbf{X}}\sum_{i=1}^{k}\sum_{j=1}^{n}\left(\lambda_{i,j}h_j-1\right)^2,\\ \text{s.t.}\ &h_j = \left\|\mathbf{x}-\mathbf{x}_j\right\|_2^2,\\ &\left\|\mathbf{z}_j-\mathbf{x}_j\right\|\le a_j,\end{aligned} \quad (20)$$

Then, an SDP problem for the estimation of the source location $\widehat{\mathbf{X}}(:,1)$ can be formulated as

$$\begin{aligned}\arg\ &\min_{t_{i,j},h_j,\mathbf{X},\mathbf{Z}}\sum_{i=1}^{k}\sum_{j=1}^{n}t_{i,j},\\ \text{s.t.}\ &\left(\lambda_{i,j}h_j-1\right)^2\le t_{i,j},\\ &\left\|\mathbf{z}_j-\mathbf{x}_j\right\|\le a_j,\\ &\mathbf{x}_j = \mathbf{X}(:,j+1),\\ &h_j = \begin{bmatrix}\mathbf{0}_2\\ \mathbf{e}_1-\mathbf{e}_j\end{bmatrix}^T\begin{bmatrix}\mathbf{I}_2 & \mathbf{X}\\ \mathbf{X}^T & \mathbf{Z}\end{bmatrix}\begin{bmatrix}\mathbf{0}_2\\ \mathbf{e}_1-\mathbf{e}_j\end{bmatrix}\\ &\begin{bmatrix}\mathbf{I}_2 & \mathbf{X}\\ \mathbf{X}^T & \mathbf{Z}\end{bmatrix}\ge\mathbf{0}_{n+3},\end{aligned} \quad (21)$$

By solving this SDP problem, an estimate of the source location is given by $\widehat{\mathbf{x}} = \widehat{\mathbf{X}}(:,1)$.



(a) RMSE for random scenario with $k = 1$

(b) RMSE for random scenario with $k = 2$

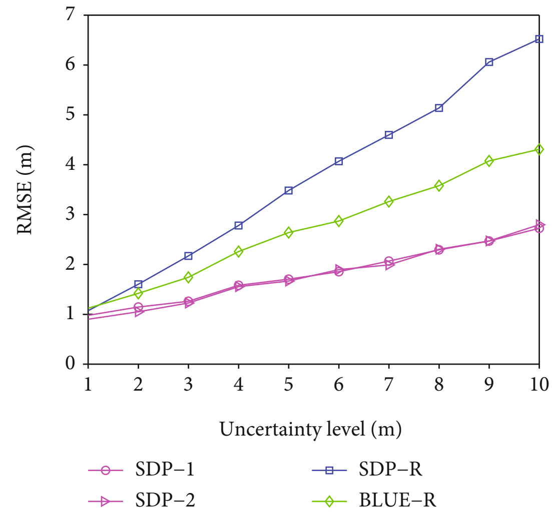
(c) RMSE for circular scenario with $k = 1$

(d) RMSE for circular scenario with $k = 2$

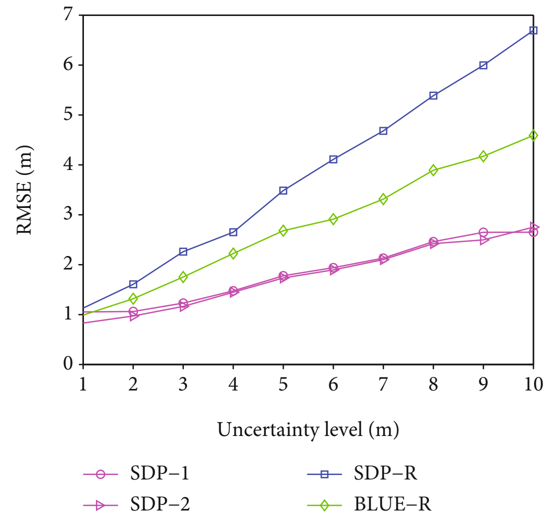
(e) RMSE for rectangular scenario with $k = 1$

(f) RMSE for rectangular scenario with $k = 2$

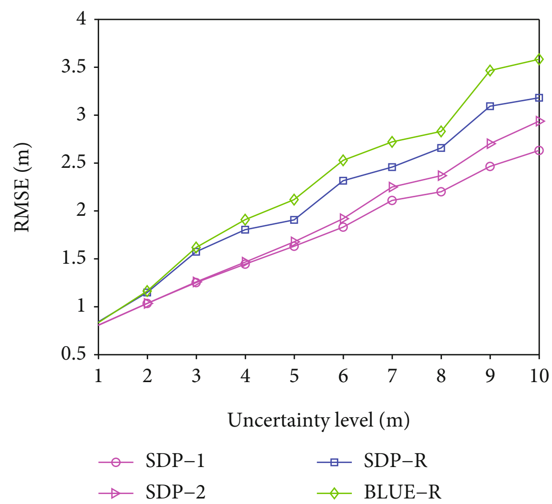
Figure 4: RMSE versus the noise level of RSS measurement for nonrandom case.



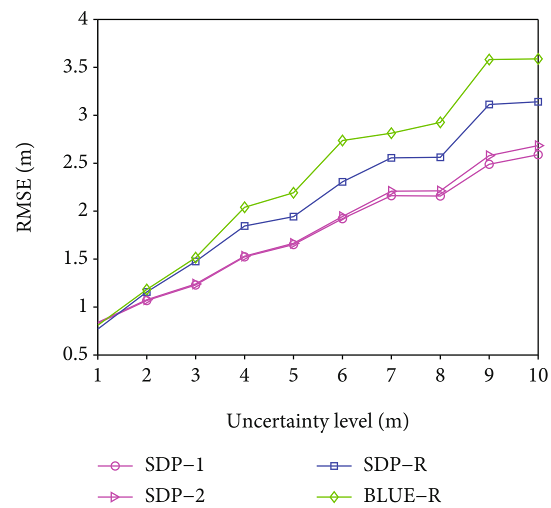
(a) RMSE for random scenario with $k = 1$



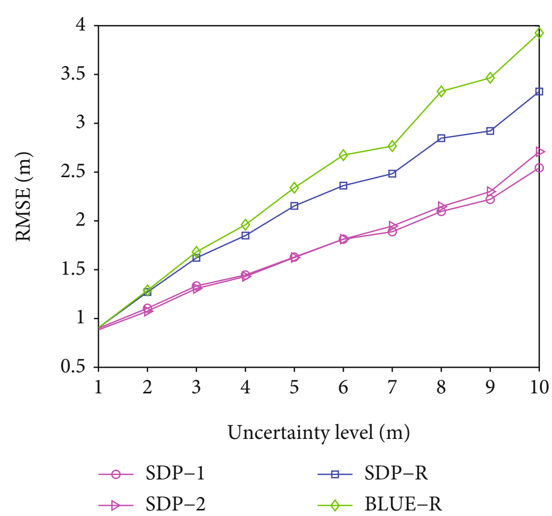
(b) RMSE for random scenario with $k = 2$



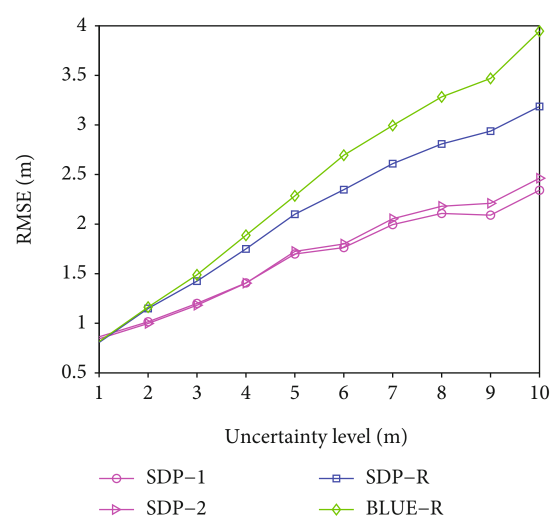
(c) RMSE for circular scenario with $k = 1$



(d) RMSE for circular scenario with $k = 2$



(e) RMSE for rectangular scenario with $k = 1$



(f) RMSE for rectangular scenario with $k = 2$

Figure 5: RMSE versus the uncertainty level of sensor position for nonrandom case.

4.2. SDP-2 Method. By transferring the uncertainties of sensor positions to source position as in the random case, a new model with respect to model (2) can be also obtained as

$$P_{i,j} = P_0 - 10\beta \log_{10} \frac{d_j\left(\mathbf{x}_j^v, \mathbf{z}_j\right)}{d_0} + n_{i,j}, \quad \left\|\mathbf{x}_j^v - \mathbf{x}\right\| \le a_j, \tag{22}$$

where $\mathbf{x}_j^v$ denotes the source location with error.

On basis of the ML estimation with the new model, an estimate of the source location can be also obtained by solving the following SDP problem

$$\begin{aligned} &\arg \min_{t_{i,j}, h_j, \mathbf{X}, \mathbf{Z}} \sum_{i=1}^{k} \sum_{j=1}^{n} t_{i,j}, \\ &\text{s.t.} \quad \left(\lambda_{i,j} h_j - 1\right)^2 \le t_{i,j}, \\ &\left\|\mathbf{x}_j^v - \mathbf{x}\right\| \le a, \\ &\mathbf{x} = \mathbf{Y}(:,1), \\ &\mathbf{x}_j^v = \mathbf{Y}(:,j+1), \\ &h_j = \mathbf{Z}(i+1, i+1) - 2\left(\mathbf{z}_j\right)^T \mathbf{Y}(:,i+1) + 2\left(\mathbf{z}_j\right)^T \mathbf{z}_j, \\ &\begin{bmatrix} \mathbf{I}_2 & \mathbf{Y} \\ \mathbf{Y}^T & \mathbf{Z} \end{bmatrix} \ge \mathbf{0}_{n+3}, \end{aligned} \tag{23}$$

5. Numerical Examples

The proposed methods were compared with the best linear unbiased estimator in [15] (denoted by BLUE-R) and the SDP estimator in [16] (denoted by SDP-R), both of which assume perfect sensor position information. Different scenarios of sensor placement were used as in Figure 1, including random, circular, and rectangular cases. The effects of the levels of both RSS noise and sensor position uncertainty were illustrated. The path-loss exponent and transmit power in the RSS model were set to $\beta = 3$ and $P_0 = -40$ dBm, respectively. All SDP methods were implemented through a standard CVX toolbox using SeDuMi [24]. The root-mean-square error (RMSE) was used averaged over 200 Monte Carlo runs in each example to evaluate the localization accuracy.

5.1. Gaussian Random Uncertainty. Figure 2 shows the RMSE versus the noise standard deviation of RSS measurement, in which σ_j^s is fixed as $3m$ and $\sigma_{i,j}^r$ varies from 1 to 10 dBm. As the magenta lines show, the proposed methods clearly help improve the localization accuracy compared with the methods in the literature for all cases, especially for random scenario. The SDP-1 method performs better than SDP-2 method in random scenario but worse than SDP-2 method in circular and rectangular scenarios for most cases. Besides, the RMSEs of all methods grow more slowly with sampling for twice than once.

Table 2: Running time (s) for nonrandom case.

Method	$k = 1$	$k = 2$
SDP-1	1	1.25
SDP-2	1.28	1.38

Figure 3 shows the RMSE versus the uncertainty level of sensor position, in which $\sigma_{i,j}^r$ is fixed as 2 dBm and σ_j^s varies from 1 to 10 m. As with the results in Figure 2, the proposed methods provide lower RMSE than the methods in the literature for all cases. Also, the SDP-1 method performs better than the SDP-2 method in random scenario. However, neither of them shows performance priority than the other one in other scenarios considered. The BLUE-R method performs better than the SDP-R method in random scenario, but an opposite result is shown in other two scenarios. Table 1 shows the relative running time of the two proposed methods (all relative to SDP-2 with $k = 1$). Obviously, their computational cost increases with more sampling times, due to the increasing RSS measurements. The SDP-2 method is more computationally efficient than the SDP-1 method for both one and two sampling times.

5.2. Unknown Nonrandom Uncertainty. Figure 4 shows the RMSE versus the noise level of RSS measurement, in which $\varepsilon_j = 3m$ is used and $\sigma_{i,j}^r$ varies from 1 to 10 dBm. As is shown, the proposed methods provide better performance than the BLUE-R and SDP-R methods as expected for most cases. Besides, the SDP-2 method performs better than the SDP-1 method for random and rectangular scenarios but close to the SDP-1 method for circular scenario.

Figure 5 shows the RMSE versus the uncertainty level of sensor position, in which $\sigma_{i,j}^r$ is fixed as 2 dBm and the maximum threshold a_j varies from 1 to 10 m while the true error ε_j is given randomly during $[1, a_j]$ for each Monte Carlo run. Also, the proposed methods provide better performance than the methods in the literature. The SDP-1 and SDP-2 methods have close performance except in the case of circular and rectangular scenarios with large uncertainty level.

Table 2 shows the relative running time of the SDP-1 and SDP-2 methods for the nonrandom uncertainty (all relative to SDP-1 with $k = 1$). Obviously, the SDP-1 method is more computationally efficient than SDP-2 method, which is opposite from the result in Table 1.

6. Conclusion

In this paper, we have studied the source localization using RSS measurements with sensor position uncertainty. By modeling the uncertainty as Gaussian random and unknown nonrandom variables, two SDP methods, i.e., SDP-1 and SDP-2, are proposed, respectively. For random case, the SDP-1 method proceeds from the ML estimation with combined RSS and sensor position measurements and then transforms the corresponding nonconvex problem to an

SDP problem with proper approximation and relaxation techniques. The SDP-2 method transfers sensor position uncertainty to the source position and obtains a new model. Then, an SDP problem is also obtained on basis of the ML estimation, which uses the similar techniques as in SDP-1 method. For nonrandom case, sensor position uncertainties are used as some constraints in formulating the ML estimation. Also, two SDP methods are obtained by following the similar idea as in the random case. Numerical examples verify the good performance of the proposed methods. Compared with some existing methods that assume perfect sensor position information, the proposed methods clearly show better localization accuracies. As for the two proposed methods, they have their own superiorities in terms of both localization accuracy and computational efficiency in some cases.

Data Availability

No underlying data was collected or produced in this study.

Conflicts of Interest

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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