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Research Article

Mathematics and Numeracy as Social and Spatial Practice

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This study of networked classroom activity proposes that a resource-rich point of view is powerful in increasing the engagement of marginalized students in mathematics classes. Our work brings attention to the values, beliefs, and power relations that infuse numeracy practices and adds attention to mathematical dimensions of social spaces. Findings show that the multiple modes available to communicate mathematically, to contribute, and the inquiry-oriented discussions invited students to draw on a variety of expressive modes to engage with complex mathematical concepts. Spatial analyses illuminate the relations among reproduction and production of knowledge, as well as the social space that characterized the networked classroom activity. They also reveal the affordance of emergent, transformed social spaces for youth's use of a variety of social and cultural displays in producing mathematical knowledge. Students extended notions about social space by adding attention to affective features of classroom and school activities.

1. Introduction

We hope with this study to contribute to the literature that illuminates the successful participation of marginalized students in secondary mathematics classes. From this perspective, rather than focusing on the barriers, such youth may face, we examine classroom interactions that foster their engagement in powerful mathematics learning. To do so, we present an exploration of mathematics as social practice that is linked to critical geographies' focus on social space.

We use varied terms throughout the paper to refer to youth and communities that have and continue to be subject to societal inequities as a way to trouble the use of language such as urban and at-risk languages that cover deficit assumptions about people and places. Similar stances are evident in related literature reviewed here. For example, Moschkovich [1] juxtaposed three theoretical stances to examine how each would guide the analysis of two Latina students' construction of understanding and communication about properties of rectangles and about slope. In that article, she illuminated what was learned and what was missed in adopting the theories' principles, arguing that a sociocultural lens provided ways to understand linguistic and interactional resources the participants were drawing upon that were missed in vocabulary acquisition and social construction of meaning perspectives. We approach this analysis similarly in that it is an exercise in examining what we see when we take a social spatial theoretical position and use its central tenets to analyze classroom activity and interaction.

We present findings from a study of mathematical practices of high school youth in the northeast US, building on work in New Literacy Studies or NLS [2, 3]. In this framework, literacy is seen as "literacies" or literacy practices, which are undertaken for specific purposes and bound within social, historical, and cultural contexts [4]. In addition to literacy, this framework changes the ways that we think about other disciplines, including mathematics and science. For many, mathematical and scientific practices can no longer be separated from the larger contexts in which they occur. For example, borrowing from and expanding upon the work of NLS, Calabrese Barton [5] and Ortiz [6] focus on the science practices of urban youth from critical feminist science perspectives to center youths' experiences and perspectives and to uncover science-related resources they develop in everyday life.

Similarly, Baker [7] and Baker and Street's [8] pioneering work has enriched our understanding of what counts as mathematics. Their differentiation between numeracy and mathematics highlights the embedded nature of mathematics in everyday life:

> *Numeracy, then, is taken as the broader term, including both everyday practices and educational aspects, both of which may have a mathematical dimension. Mathematics, then, we take to be a more specialised and abstract set of practices, usually the domain of professional practitioners of both mathematicians in universities and mathematics educators in both Higher Education and schooling. ([9], p. x)*

As such, mathematical activity not only involves content, but also involves values and beliefs, context, and social and institutional relations ([9], p. 17). These dimensions shape what kind of numeracy gets done in particular situations. Their work provides important impetus for our explorations of numeracy as social practice in relation to the creation of classroom social spaces that have mathematical and cultural dimensions as central features.

Our work contributes to the social turn in mathematics education research [7, 9]. Exploring this social turn is informative for deepening the understanding of numeracy that is inevitably constructed as students and teachers make meaning in classrooms. Such work has given rise to views that stress the dynamic, emergent nature of disciplinary content, discourse, and practices that ensue from individuals' and groups' action and interaction (cf., [1, 10]). The focus on social construction in mathematics has included recognition that examining both every day and schooled mathematics practices is critical to understand mathematics learning and teaching and to inform both pedagogy and policy (cf., [7, 11–14]). Attention to both every day and school activity is important to mathematics education research because it broadens the contexts in which mathematics learning is considered to occur. Our contribution to this work is bringing attention to the values, beliefs, and power relations that infuse numeracy practices and adding attention to mathematical dimensions of social spaces. Further, we show the ways that numeracy practices shape and are shaped by the social space of the classroom.

1.1. Spaces as Social Constructions. At first blush, space may seem static (as in classroom space with its requisite desks, tables, chairs, and so forth, staying largely unchanged over decades, particularly in underresourced schools serving nondominant students). However, following theorists including Soja [15], Harvey [16], and de Certeau [17], we propose that, contrary to this rigidity, social space is actually dynamic and volatile. For example, cities are made up of spaces that are constructed and differentiated based on physical, social, and historical dimensions:

> *[they] are marked by socially constructed boundaries that divide areas geographically along racial, ethnic, class, and religious lines. Chicago, New York, Boston, and Toronto, to name a few, all have designations such as "South Side" or "Upper East Side" that mark those spaces and their inhabitants as different from those in others parts of the city. ([18], p. 1)*

Through this kind of sociospatial differentiation, people are located within particular spaces and inscribed with particular social orderings of who they are, what they can do, and how they can be. On another scale, youth experience school spaces as different from neighborhood spaces due to the physical arrangements of people and things, the kinds of actions and talk that are treated as legitimate, and the norms for social relations among children and adults. Finally, in school and classroom spaces, youth are positioned in relation to both the teacher and the discipline of mathematics as, among other things, producers or, more often, receivers of knowledge. This attention to social positioning is important in considering the numeracy learning of traditionally underserved youth, as access to opportunities to engage in rigorous learning depends in important ways on their and their teachers' views of the goals and aims of school as a space learning for particular students (e.g., aligned with their goals and practices or alien to them).

Treating space as a social construction leads us to consider the practices through which spaces are created, how people are positioned in various spaces, and the implications for agency and learning. In this paper, we examine numeracy as a social practice that creates social space, considering in particular its productive nature, or how it is implicated in the construction of space that has social, historical, cultural, and mathematical dimensions, all of which are infused with relations of power [9]. Appropriate questions to consider when looking at practices with an eye toward spatial analysis include the following. How are numeracy and social practices changing classroom social space? Why and into what is this space changing as a result of youths' engagement in numeracy practices?

We find this approach promising in its attention to agency and to the variety of ways in which people engage in numeracy practices, as well as the connections among school, home, and community practices that have mathematical activity as central features (cf., [9, 11, 19, 20]). A social practice view provides a unique perspective on pedagogical issues, given the focus on practices and activities that make up everyday interactions (both in and out of classrooms). Taking on a spatial theory lens extends such work (i.e., funds of knowledge, [12]; culturally relevant pedagogy, [21]; and cultural modeling, [22]) by highlighting the intersections of social, cultural, historical, and physical dimensions of numeracy learning. In this study, we explore work with a networked classroom technology (described below) that requires collaboration among students, fosters generative learning, and transforms both the mathematical content and student-teacher roles [18]. Examination of such practices has the potential to widen the types of practices invited into classroom mathematics activity. As shown in earlier research [23], these activities can be crucial in building on practices of students that are often undervalued or excluded from mathematics teaching and learning in school.

2. The Context

We examine numeracy practices in an urban high school's mathematics classes using a networked technology designed to leverage the power of groups in rigorous, generative learning. The networked technology is Hubnet and Participatory Simulations (PartSims; [24] (extensive information about this networked system is available at http://ccl.northwestern.edu/ps/ and about SimCalc at http://www.simcalc.umassd.edu/)), a member of a class of technologies that focus on shared construction of mathematics learning (e.g., SimCalc MathWorlds [25]; Texas Instruments Navigator system). The system involves graphing calculators that are connected to hubs that have a wireless connection to a computer which functions as a central server. In PartSims, students

> *act out the roles of individual system elements and then see how the behavior of the system as a whole can emerge from these individual behaviors. The emergent behavior of the system and its relation to individual participant actions and strategies can then become the object of collective discussion and analysis. ([26], p. 2)*

Traffic flow in a traffic grid, spread of disease, and the motion of elevators are examples of phenomena explored in PartSims, while the content involved includes but is not limited to linear, trigonometric, and exponential functions; regression; equivalence; rates of change; graph analysis; and modelling. PartSims provide opportunities for youth to be central actors and producers of mathematical discourse and practice. They are important to our examination of social practices and spaces that emerge from youths' activities and interactions, highlighting that youth are active agents in creating practices and social spaces. This view of students is in stark contrast to the conventional classrooms that position them as recipients of norms, practices, and discourses, at the mercy of adult and institutional exercises of power. As we show below, it is also a productive arena for examining the creation of social space because youth have ample opportunity to interact and to exercise agency as to what mathematics is explored as well as how the exploration proceeds, transforming classroom practices to those that invite participation by more students and less control by teachers.

The particular PartSim that is the focus of this paper is Gridlock, which involves each student individually controlling a traffic light at a specific intersection in a traffic grid and working collectively to optimize traffic flow (more extensive description of the activity is included below). The numeracy involved includes the mathematics of variation and change, working with positive and negative numbers, graph analysis, and connections to traffic flow in the city in which the classroom is situated. Gridlock is appropriate for conducting a multidimensional exploration of the construction of social spaces. For example, students work with multiple, linked representations (i.e., graphs of number of stopped cars, average wait time, and average speed), as well as tasks that require whole-class coordination and collaboration to be successful. Gridlock is also rich with mathematical discourse and practice, including representations, visualization, language, and gesture serving to mediate learning and interaction. In addition, Gridlock involves rich mathematical content and reasoning in ways that are similar to SimCalc [25, 27]: "underlying ideas of calculus (variable rates of changing quantities, the accumulation of those quantities, connections between rates and accumulations, and approximations) are taught . . . and are rooted . . . in children's everyday experiences" ([28], p. 289).

Important for this study are the following: connections among (1) discourse and practice and (2) mathematical content and reasoning are central to the recently adopted Common Core Curriculum Standards for mathematics [29]:

> *The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. (http://www.p12.nysed.gov/ciai/cores.html)*

The emphasis on understanding as a critical starting point for introducing students to more formalized content in mathematics is exactly what Gridlock and other PartSims are designed to do. As seen in the findings reported below, foundational understandings of concepts and mathematical representations were built across all the events and classrooms we studied, preparing students for rigorous mathematics learning.

Beyond mathematics classrooms, understanding the mathematics of change and variation is important for many topics in the curriculum beyond algebra and calculus. As discussed earlier, social sciences, economics, and history increasingly rely on dynamic models to understand complex phenomena. Interactive media that simultaneously present qualitative and quantitative representations are potentially useful in many fields for helping learners bridge between experience and abstraction ([28], p. 291).

Both the widely applicable mathematics involved and the grounding in personal experience production can alter classroom social spaces that are ripe with opportunities to exercise agency and learn powerful mathematics. As Soja [15] notes, "It is precisely this possibility of meaningful spatial [and content] transformation that gives to the production of space a significant practical and political dimension" (p. x), a dimension that we view as critically important in research involving underserved youth. The transformed social space and access to rigorous mathematics are particularly salient for urban schools serving students of color, where expectations of students and provision of resources are often low [21, 30, 31].

The paper is organized as follows: the theoretical framework is explained in terms of what we can gain by looking at numeracy practices as creating social spaces, particularly in understanding youth engagement in numeracy events, or "occasions when numeracy activity is integral to the nature of the participants' interactions and their interpretive processes" ([9]; p. 20). Spatialized and spatializing practices [32] are proposed as useful tools in understanding and explaining sociocultural processes involved in youths' engagement in numeracy activity in their mathematics classes at school. We present findings from our study in high school mathematics classrooms in Rochester, NY, and conclude with a discussion of implications for research and pedagogy.

3. Theoretical Framework

We work in this study to particularize examinations of social space to numeracy activity in classroom contexts involving youth from nondominant groups. We draw on particularly on de Certeau and Harvey [16, 17] in conceptualizing social space involving knowledge/discourse, technologies (e.g., symbol systems, calculators, and curricula), and practice (Foucault (1986) and Bhabha (1994) are also informative along these lines.). Across all these conceptualizations, space is a social sphere that is constructed through peoples' material and ideal activity.

Following Street, Baker, and colleagues [7, 9, 33], our starting point is the social and cultural practices involved in numeracy events; we view those practices as activities that create space that has social, political, historical, cultural, and mathematical dimensions. From this view, such activity shapes and is shaped by discipline-specific ways of speaking, interacting, acting, and using analytical and physical tools. Numeracy practices, then, emerge from interactions involving both numeracy—language, representations, symbolic and notational systems, forms of argumentation—and the contexts in which numeracy activity is taking place (e.g., classroom, grocery store, and engineering team). Integral to those contexts are the power relations that permeate them, given that participants are accorded differing status in relation to content and to each other in different contexts.

We add attention to the creation of social space through practices that shape the kinds of activities and modes of participation treated as appropriate or legitimate in particular places involving particular people [15, 19, 34]. Such practices also shape the kinds of interactions, social relations, and knowledge that result. In addition, the ways in which everyday activity inserts dynamism, mutability, and challenge into dominant norms are made explicit. This complements Street et al.'s [9] work on the content, context, values and beliefs, and social and institutional relations involved in numeracy events and practices not only by looking at relations among, for example, school and home numeracies, but also by uncovering the ways that those spaces are both produced and potentially transformed. Street and colleagues' works are instrumental in making explicit that there are power relations and differential valuing of particular practices, that is, school numeracy over home or community numeracy practices. However, social spatial theory gives us a way to examine in some detail how those power relations and values come to be.

Notions of space involving a relationship among physical, mental, and social spheres of activity are useful in studies of numeracy practices across contexts because they highlight the dynamic and multidimensional aspects of space-creating activity, that is, physical, historical, sociocultural, political, and mathematical dimensions. These spheres exist in a mutually constitutive relationship, where each influences and is influenced by the activity in the other, while retaining distinct features that make them unique arenas for action. In particular, for our work, the networked activity we examined has design principles that guide both the architecture of the system of networked calculators and the orchestration of activities that involve physical, mental, and social space. Students are physically connected via the calculator-hub-server system, transforming the classroom from a collection of individuals to a collective body (physical, social) that constructs and analyzes mathematical objects and complex systems (both social and mathematical). Students build conceptual understandings to scaffold work with the more abstract and formal content involved (mathematical). The resources students brought to bear in our study include informal and formal language (i.e., African American English, Spanish, "urban" language, and European American English) (Student coresearchers in an earlier study [23] coined this term when asked about language practices they saw in the videotapes of classroom activities that we analyzed together. For them, urban language is a hybrid derived from a variety of sources.), gesture, overlapping talk, and collective construction of meaning (sociocultural). As we show, transformation of dominated spaces that have developed over time is a key feature (political). It is the explicit recognition that social space involves multiple, overlapping spheres of activity that does important conceptual work for our research.

3.1. Spatialized and Spatializing Practices

3.1. Spatialized and Spatializing Practices. Building on earlier work on spatialized and spatializing practices ([15, 32]), we bring together theories of social practice and social space in that discourse/knowledge, practice, technologies, and space are treated as operating in a mutually constitutive relation with each other. Spatialized practices are those that are often viewed as "natural" or appropriate, based on historical, political, and social notions; these practices largely reproduce social space (e.g., teachers' and students' compliance with desks arranged in rows and the resulting reproduction of traditional teacher/student relations). Engagement in them is largely unreflective. Certain paths and actions are presupposed for particular places and people (e.g., teachers move mostly at the front of the classroom and determine who speaks and about what; students remain seated and respond when asked to speak); their practices are spatialized [15].

Spatializing practices, on the other hand, are those involved in appropriation and production of space. de Certeau [17] gives an example of such modification of the existing practices and built environments, treating an act of walking in a city as

> *. . . a process of appropriation of the topographical system on the part of the pedestrian (just as the speaker appropriates and takes on language); it is a spatial acting-out of the place (just as the speech act is an acoustic acting-out of language); and it implies relations among differentiated positions. (p. 97-98)*

Walkers are channeled in their paths by the built environment, but they also have agency to modify those paths by taking/making shortcuts or meandering rather than taking the "efficient" route as laid out by a transportation planner. The relations among differentiated positions thus involve both the planner as one who "determines" where streets and walls may go to efficiently direct activity and the walker who appropriates the built environment for particular purposes (a relaxing stroll, a quick way home). Spatializing practices can involve both production and reproduction, depending on the purposes people have for engaging in them. It is this productive quality that entails possibilities for transformation that we see as important for our work aiming at understanding numeracy practices that involve nondominant youths' creation of social spaces in classrooms and schools.

4. Methods

Again, the guiding questions for our study are as follows. How are numeracy practices changing classroom social space? Why and into what is this space changing as a result of youths' engagement in numeracy practices? Given our theoretical framework, ethnographic style methods are appropriate. LeCompte and Schensul [35] note that ethnography uses everyday practices as a lens for interpretation as well as exploring the sociopolitical and historical nature of phenomena. Rather than testing hypotheses, we are seeking to understand numeracy practices involving urban youth and the meanings numeracy has for them. To do so, we need not only to observe numeracy practices, but also "to start talking to people, listening to them and linking their immediate experience out to other things that they do as well" ([9]; p. 19).

5. Setting

The school Biddy Mason Academy (a pseudonym) is a high school in Rochester, NY, located in a large urban school district that has been designated as the most needy in the state, surpassing districts in New York City, Buffalo, and Albany in the proportion of students served who are living in poverty. Our study was conducted during the 2005–2008 academic years. The State Department of Education reported that, in 2007-08, the school served 1891 students from grades seven to 12. Of these students, 68% were non-Hispanic Black, 20% were Hispanic, 10% were non-Hispanic White, and 1% was American Indian, Alaskan, Asian, or Pacific Islander. Additionally, 78% received free or reduced lunch and 5% were designated as English language learners. Over the prior three years, only ~51% of students had received regents diplomas that lead to postsecondary education, and ~13% were designated as "noncompleters." In 2005-2006, only 67% of students tested met or exceeded the minimum passing score (65%) on the state-mandated Mathematics A test (algebra and geometry). In 2004-2005, no ethnic/racial group but Whites met the NCLB-required performance index in mathematics. The graduation rate for that year for Hispanics was 30%; for Blacks was 53%; for Whites was 60%; and for students living in poverty was 53%. As with most schools in the US and especially urban schools, the culture of the school is shaped by an intense focus on standardized test scores and accountability, creating an atmosphere of tight control of curriculum and pedagogy. It is also a very large, overcrowded school that focuses on strict discipline (As of 2009, a new principal was hired who has undertaken substantial changes that have, according to news reports and statements of students, begun to change some aspects of the climate. The state-mandated mathematics curriculum had not changed as that of 2010.). Teachers are working to support student learning in this atmosphere and students are working to negotiate the context, too. Given these data, we view this as a perfect setting for examining the ways that numeracy practices are implicated in the construction of social spaces that are transformative, that open the spaces of possibility for students who are often viewed from deficit-rather than resource-rich perspectives.

5.1. Positioning Ourselves as Researchers. Our being Caucasian while the majority of students being African American or Puerto Rican required our attention to inevitable issues of status and power in analyzing, interpreting, and reporting our participants' experiences. To address these issues, we positioned ourselves as learners in relation to our participants (students in particular) and we came from the point of view that youth have developed and draw on powerful cultural and social practices they have appropriated from their communities. Thus, an important aspect of this methodology entailed incorporating students themselves into the research project as coresearchers (or participant) researchers. We involved students from a variety of cultural/ethnic/racial backgrounds (African American, Puerto Rican, Haitian, Caucasian, Nigerian, and Asian American) in analyzing and interpreting data together. They collaborated with us in study groups, following González and colleagues' work in funds of knowledge (e.g., [36]), during which we analyzed data to identify the culturally valued and mathematically rich practices (e.g., use of signs and symbols, artifacts, and social relations) in school and their youth peer communities. Four students' work with us supported the analyses reported here, three who are African American and one who is a recent immigrant from Nigeria. Two are female and two are male. Our work together increased the trustworthiness of our interpretations and helped maintain ethical, respectful, and productive relationships with participants.

5.2. Networked Classrooms. The chosen classrooms were important for this study because youth had more control of the content and processes of learning and activity than we saw in their "regular" classroom sessions (We collected video data one day in each of the participating teachers' classes we were involved in on days that the networked system was not in use. Also, the coresearchers and students participating in focus groups corroborated our interpretation of "regular" classroom activities.). Also, the nature of the mathematics involved engaged students (and teachers) in the production of rich mathematical discourse and numeracy practice, giving us insights into the ways that youth can play a part in spatializing practices that transform mathematics classrooms as social spaces.

We worked with three mathematics teachers, two in their inclusive classrooms, over four years. For the two years of work reported here, we spent two full days per month working with PartSims in five classrooms (16 days/90 class sessions total in year one; 12 days/60 sessions in year two). Gridlock sessions numbered 32 sessions in year one and 24 sessions in year two. Class titles of the courses we covered included prealgebra, algebra, integrated mathematics, and mathematics competency. Classes lasted for 42 minutes, and class sizes ranged from seven to 33 students.

5.3. Data Sources and Analytic Techniques. Videotapes of two sessions of Gridlock PartSim classroom activity and interviews with 93 students comprise the data corpus. We chose to focus on two sessions because of (1) the exploratory nature of our study, (2) the aim of ethnography to provide thick descriptions [37], and (3) the high quality of the video and audio. Two cameras were used so that we were able to analyze simultaneous front-and back-of-the-classroom views. There were twenty-eight students in one class and ten in the other. The interviews included but were not limited to specific questions about Gridlock; other questions addressed other PartSims and more general impressions and advice students would share.

Two university researchers analyzed the video data by producing written descriptions of the Gridlock activity, resulting in an analysis partitioned into three episodes according to the flow of activity across the sessions: before Gridlock (before the simulation was run), Gridlock (during the simulation), and after Gridlock (after the simulation was run). The two authors analyzed the written descriptions of numeracy activity within and across episodes independently to identify the mathematics involved as well as spatialized and spatializing practices, following Buendía and Ares [32]. Frequent forays back into the videos were conducted as a way to maintain closeness to the data. Consensus was reached through discussion.

We identified actions, interactions, and utterances that lead to maintenance or transformation of conventional roles of students and teachers (e.g., physical arrangements of people, who was speaking, and what they were saying). For example, spatialized practice found in a pregridlock introduction involved the teacher at the front of the room, calling on individual students to describe good or bad traffic and resisting overlapping talk. Conversely, spatializing practices found in a Gridlock phase included teachers and researchers moving to the back and sides of the room, multiparty talk among students, and students collectively controlling the dialogue. Finally, we worked with our student coresearchers, first viewing the videotaped sessions together without any discussion of our prior analysis and then discussing both collaborative and university researcher findings to corroborate, challenge, and extend our analyses. Through these varied analyses, we were able to partition the classroom sessions into major episodes and characterize the social and mathematical dimensions within each episode.

Interview data were analyzed by university researchers as supplementary to the above analyses and helped heighten our understanding of participants' perspectives of numeracy practices. Three researchers analyzed the same interview transcript independently, discussed our coding, analyzed another transcript independently, and reached consensus through further discussion.

6. Findings

We organize our findings in two sections. The first presents our analyses of the numeracy practices found in the networked classroom activity, including the mathematics and the control of symbols, signs, and knowledge. The second focuses on our analyses of spatialized and spatializing practices involved in the creation of social spaces, along with students' experiences and perceptions of numeracy practices as personal, political, and value-laden. The latter helped illuminate issues of power and ideology in numeracy practices associated with Gridlock.

6.1. Gridlock as a Site of Numeracy Practice. In this PartSim, students' collective activity formed traffic flow in a grid, along with real-time emerging, linked graphs of the number of stopped cars, average wait time, and average speed over time. All of these representations were displayed visually at the front of the class (see Figure 1). Students logged in using a numbered or three-letter username and were assigned to an intersection, designated by that same "name" in the projected traffic grid. When there were more intersections than students, the computer controlled those intersections not under human control by changing the light at a fixed interval. It was possible to alter certain aspects of the simulation such as the number of cars in the traffic grid, the speed at which the cars travel, the number of computer-controlled traffic lights (e.g., size of the grid), and the time on the simulation's clock that elapses before the computer changes traffic lights under its control. When the simulation was being run, students called out to each other to change lights, implemented the strategy they developed to optimize traffic flow (e.g., coordinate their light changes with the computer), laughed, and engaged in overlapping talk.

As shown below, numeracy practices engaged students in the creation and analysis of a complex dynamic system (e.g., traffic flow in the system, graphs of number of stopped cars, average speed, and average wait time). Mathematical discourse and practice involved strategizing at both

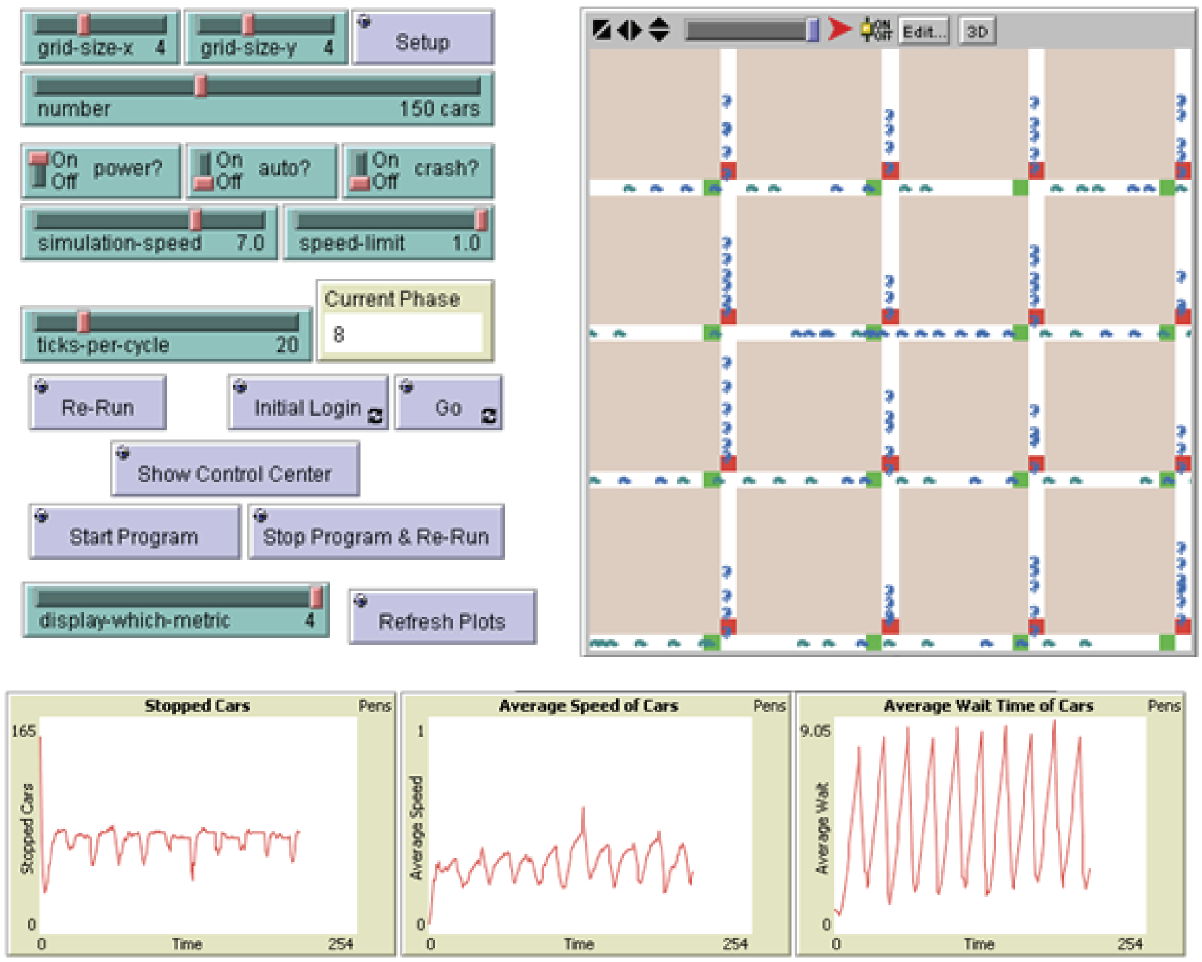

Figure 1: Projected display of the Gridlock participatory simulation.

individual and collective levels, linking representations to each other and to the traffic flow and using informal and formal language as well as physical gestures to communicate mathematically. The excerpt below from field notes and video transcriptions shows students' collective, multimodal construction of interpretations of and relationships among graphs during one of the postgridlock activities. Though not captured in this excerpt and while the individual students were talking, many other students were chiming in as well, engaging in overlapping talk as described above.

There are seven students in the classroom. Their student desks are placed in rows; several of the young men are squished in their seats because they are big for them. Though there are plenty of desks near the front, all but one young woman sit at the back of the room or on the side nearest the door. Three researchers (Nancy, Dawn, and Al) and Mrs. H., the regular teacher, are placed around the sides of the room, watching the activity and/or staffing video cameras. The upfront space is displayed and students' user names for individual intersections are visible; the grid has eight intersections, meaning one is computer controlled. The group is running its second simulation, working hard on this one to beat the freshman class' time to reach Gridlock. One young man exclaims several times that, "We gotta beat the freshmen, beat the freshmen." All the students' eyes are focused on the screen, their fingers pressing calculator buttons that change the traffic light at their intersection. The atmosphere in the room is intense as they operate their lights, keeping track of the flow using the emerging graphs (they had worked through the graphs' meaning after the first Gridlock). Two students are calling out to the class that the number of stopped cars is going up. Another chimes in that the wait time is "crazy." Al calls their attention to the front after N. stops the simulation, explaining that they have reached Gridlock. Students keep talking about their progress until Al remarks that all the cars stopped and how much time it took them (they did not beat the freshmen this round). He then draws their attention to the three graphs and asks what they see in them (see Figure 2). Overlapping talk ceases as they switch to individual turn taking.

Big L: It's a lot of movement. It's a lot of movement (pointing to the graph of stopped cars).

Al: In the graph? What does that movement in the graph say to you?

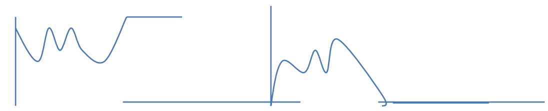

Figure 2: Gridlock graphs projected in real time in upfront space.

Big L: Lots of cars, stop and go. (moves hand up and down as he speaks)

Al: Okay, so there's a lot of cars. . .the number of cars that are stopped is changing, going up and down. And then, finally, what's happening here at the end? (pointing to graph again, where the line is horizontal and at the top of the y axis)

Ronald: It goes straight.

Instructor: What's that mean?

Big L: Constant speed maybe? (motioning with hand to illustrate a horizontal line)

Al: What's that graph graphing?

Lydia: Stopped cars.

Patricia: Oh! And and, all the cars stopped at the same time.

Al: At this point in time, all the cars are stopped. What's this graph telling you? (pointing to graph of average speed)

Big L: Everybody's crashed (laughter among students)

Concepción: Average speed.

Big L: A whole bunch of cars have a different pace.

Maria: It's how fast the cars go within the same period of time?

Big L: No. Oh, yeah, that's right.

Al: So what's this saying then?

Concepción: They started going fast, then they were goin' slow for a long time, and then stopped (gesturing with hand and arm, starting with hand up high and swooping down in an arc).

In this episode, the students and instructor coconstructed the understanding of how the graphs are related to the motion of the cars in the grid. (In terms of individual learning within the collective, Concepción understood that average speed varied and that average represented the speed of the cars as a group. The use of the word "they" indicates that she had an implicit understanding of the nature of "average," and "average speed." Hers is the most obvious individual understanding being exhibited. Importantly, we do not claim to have analyzed individual learning, as the focus in the study is on group-level construction of knowledge and practice.) The mathematics of change, an important strand of school mathematics [38, 39], grounds their analysis. In particular, accumulation and rate of change were two central constructs that were the focus of their interpretations. First, the varying shape of the graph of the stopped cars (the "lot of movement" and the "straight" portions) was used to make sense of the varied and increasing number of stopped cars as the system moved to gridlock. As the class moved to investigate the graph of average speed, the analysis shifted to the rate of change (going fast and then they were going slow for a long time and then stopped). In both cases, important work in linking the abstract representations to the more concrete simulated phenomenon was central. Though not shown in the above excerpt, later in this episode the class examined the link between the number of stopped cars and the average speed, an important analysis of the relationship between rate and amount, a fundamental concept in calculus.

The mathematics of change seen in the above excerpt and found across Gridlock sessions is vital to students' learning, "not only because of its critical role historically and the present day in mathematics, the sciences, and the social sciences, but also because the concepts of the mathematics of change are rooted in everyday experiences" ([40], p. 90; see also [39, 41]). Also, exploring dynamic systems is increasingly seen to be critical to understanding such things as adaptive systems, chaos, and self-organization [26]. Finally, numerous practices included in New York State's Common Core Curriculum Standards for mathematics practices [29] are evident in our data, including the following.

(1) Make sense of problems and persevere in solving them: explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends.

(2) Reason abstractly and quantitatively: make sense of quantities and their relationships in problem situations.

(3) Construct viable arguments and critique the reasoning of others: make conjectures and build a logical progression of statements to explore the truth of their conjectures and reason inductively about data, making plausible arguments that take into account the context from which the data arose.

(4) Model with mathematics: apply the mathematics they know to solve problems arising in everyday life, society, and the workplace and analyze those relationships mathematically to draw conclusions. (http://www.p12.nysed.gov/ciai/cores.html).

Our data show that students engaged in a majority of the items listed. Thus, rigorous mathematical skills and practices were important to the classes' practices using the PartSims. They performed these skills and practices as numeracy rather than mathematics, given numeracy's definition: practices involving everyday and educational aspects, both of which have mathematical dimensions.

Regarding numeracy tied to students' everyday experiences, we can see the development of formalized descriptions of good and bad traffic across the three excerpts presented above. In the pregridlock session, students defined good traffic as continuous movement of cars and efficient use of time. Bad traffic involved no movement and too many cars. Less formal language was used "Good traffic is when I can get to where I need to go in the amount of time I need to get there". After the Gridlock simulation, students' descriptions of good traffic involved coordination of lights nearby, traffic not getting backed up, and the traffic grid having the right number of cars going at the right speed. Finally, students used the graphs as metrics of traffic flow, showing that students were using them to evaluate or gauge their progress. The collective construction of optimal traffic flow involves continuous movement of cars rather than lots of cars stopping on crowded streets. Also, reducing the number of cars later in the simulation (after Gridlock) is related to the valuing of less crowded streets, connecting back to everyday experiences of driving in Rochester by analyzing the graphs of three metrics of movement.

Importantly for this study, analyses show that the multiple modes available to communicate mathematically, to contribute, and the inquiry-oriented discussions invited students to draw on a variety of resources to engage with concepts of dynamic systems and the mathematics of variation. The animated, lively discussion among participants built on a variety of individual contributions and communicative forms (informal and formal language, gesture) as they developed a collective story of the graphs. We explore these multiple dimensions of the construction of the networked space next. The excerpts presented in this section, representative of activity across sessions and classrooms, give a sense of how social space was created in the Gridlock activity.

6.2. Resources and Control of Content and Process. As stated above, we analyzed pregridlock, Gridlock, and postgridlock episodes to examine the social spaces that emerged from students' engagement with the PartSim. Focus on interactions, utterances, gestures, and use of artifacts (e.g., graphs in the upfront space) highlights how each of the three episodes contained particular modes of production, including who controlled the mode, and were characterized by differing social relations, including the physical arrangements/connections among participants where the locus of control of content and process lay.

6.2.1. Before Gridlock. In both of the classroom sessions, routine classroom practices found in most mathematics classrooms were prominent at the outset—students sitting in desks arranged in rows facing the front of the classroom where the teacher was teaching. The researcher/instructor (As this was early in our work with teachers, they chose to have us teach while they watched, participated in the simulations, and chimed in about content and strategies as they saw fit.) stood in "teacher space," using a yardstick to point to the various elements of the upfront display (traffic grid, movement of traffic, graphs, etc.). He also introduced the task. The Mayor of the town was unhappy with traffic patterns and had asked the students to come up with strategies for optimizing flow (for more extensive description of this simulation, see http://ccl.northwestern.edu/ps/ps.shtml). Students, for the most part, appeared to be focused on the explanation being given. A heavy adult presence in the classrooms (teacher, student teacher, student aide, and three researchers) was highly suggestive of power relations in a typical classroom.

Prior to discussing Gridlock in the class with 28 students, the instructor/researcher (This particular researcher, Al, was involved in teaching and analyzing data, though not in the writing of the final version of this manuscript.) introduced himself and the other members of the research team using first names (regular teachers were called Mrs. H. or Mr. B.), relaxing some of the conventional practices seen above.

The use of first names and then asking the students to describe what they deemed good and bad traffic without emphasizing the need for speaking in turn or raising hands lead to a more personal, relaxed space where social relations shifted to a less hierarchical, student-centered one. The following excerpt captures these exchanges.

> *Al: What is good traffic?*
>
> *Terrance: The cars is moving - everybody movin'*
>
> *Sylvia: At the same pace*
>
> *Terrance: What do you mean the same pace, driving like old ladies, I hate people like that!*
>
> *Shawna: Yeah, I like it when everybody be goin' slow so I can just pass right by them. Yeah, I'll have my Mustang and it'll just be...*
>
> *Sheldon: Good traffic is when I can get to where I need to go in the amount of time I need to get there. [laughter, "aw," lots of heads nodding]*
>
> *Al: How about bad traffic, what's that?*
>
> *Multiple students, overlapping talk: Bad traffic is New York City! It sure is. Ain't nobody ain't movin'. People on bikes. Like in the hood, Jack, on like Genesee Street [a local street].*
>
> *Al: What are some of the causes of bad traffic?*
>
> *Multiple students: Too many cars, we need to reduce population, people need to stop having kids, need to take the bus, people need to use roller blades... (This excerpt is also seen in [42]. The analysis reported here is based on a different theoretical frame.)*

Students dominated the discussion in this session, though the researcher/instructor asked the prompting questions. Good and bad traffic were defined based on students' contributions and on their out-of-school experiences. The references to a particular local street and to New York City grounded the exchange in familiar physical spaces, while the informal, “urban” language used situated it in a social, linguistic context that reflected the youth’s peer and community memberships. Thus, these practices transformed the social space of the classroom into one that centered youth’s judgments, their experiences, and their modes of communication. This more relaxed atmosphere was noted and appreciated by students, as evidenced in our analyses of interview data (below).

6.2.2. Gridlock. As calculators were distributed among students, the social space among the students changed as they were quite literally connected to each other physically by the calculators and hubs. They were also charged with working together to optimize traffic flow. This movement from the typical individualized classroom setup to a more collective or group setup was a transformative move that leveraged the power of the students as a group, centering agency in creating numeracy practice and understanding in their interactions. With the beginning of the Gridlock simulation, the typical physical space and roles occupied by the adults diminished as the students negotiated the use of calculators and engaged with each other and the upfront space. This marked student appropriation of much more of the classroom space than they typically had. Control of the mathematical symbols and signs was moved to the students as a collective as the activity unfolded. In both classes, as the simulation proceeded the researchers, teacher, and student teacher moved to the back and sides of the classroom—a physical representation of giving up the teacher space or allowing/inviting the students to appropriate that space. Students became very vocal, calling out names of intersections as they attempted to keep traffic flowing through the grid.

> *Who’s 8?*
>
> *Who’s 22?*
>
> *Who’s number 8? Number 8?*
>
> *Who’s 4?*
>
> *Who’s 9?*
>
> *22, change!*
>
> *[students continue with the simulation until gridlock occurs and cars can no longer move. N. asks what were students’ strategies?]*
>
> *Talk to each other.*
>
> *Turn your light when too many are there.*
>
> *Look at lights around you.*
>
> *[One student suggest a coordination of effort. Another student suggests leaving a gap so the cars do not back up at intersections. Students suggest speeds for cars and want all three graphs showing. They decide to reduce the number of cars and the speed. When simulation is started again, the same pattern of calling out to each other to change lights ensues.] (video data, 10/2005)*

The contributions above were spread across numerous students, so that control of the group’s efforts was distributed among participants. There was considerable overlapping speech aimed at coordinating the group’s efforts. Additionally, calculators became a means of interacting with the simulation as the students used them to change the traffic lights. Although the researchers remained at the back of the room in a clear attempt to turn over classroom space to the students, both the teachers and student teacher moved about and their voices could be heard as well, but not to the same extent as the students. These kinds of practices typified groups’ efforts across both classes, with students as a collective controlling the modes of production via speech and electronic gesture. Again, this shift in control to students and to their collective efforts marks a significant change for them, as their classrooms were normally teacher-centered and focused on individual efforts. The resources available to students (each other, everyday experiences, multiple mathematical representations) and the agency they could exercise in the transformed space developed what Yackel and Cobb [10] discuss as sociomathematical norms: “classroom social norms that sustain inquiry based discussion and argumentation. . .[and] regulate mathematical argumentation and influence learning opportunities” (p. 458).

6.2.3. After Gridlock. During a study group session with the four student coresearchers, our attention was drawn to one particular instance when the class was focused on using the resulting graphs to develop a class-wide strategy for optimizing traffic flow. One student, Arturo, came to the front of the class to use the upfront space to describe a strategy using the graphs of traffic flow to explain his idea of curves representing optimal flow. As such, he appropriated the teacher space both physically and symbolically. One of the student coresearchers noticed that, “the class boosted up and had more feedback to Arturo” than to the instructor who had been leading the discussion (video data, 03.25.2006). Another noted that Arturo made a connection to driving in Rochester, “so kids paid attention to him” (video data, 03.25.2006). When asked whose knowledge was important, our student colleagues instantly recognized Arturo as being the locus of both knowledge and control in the segment. This privileging of Arturo’s explanation and authority transformed the social space in that student knowledge rather than teacher knowledge was important. Additionally, the connection Arturo made between driving in Rochester and the graphical representations of measure of traffic flow privileged students knowledge, tying the meaning of the more abstract representational representations with their concrete experiences as drivers and/or passengers.

Across the classes and episodes and using the affordances of the networked activity, students not only collectively created multiple mathematical representations of good (or bad) traffic flow, they also transformed the social space of the classroom from a teacher-centered to a student-centered one connected to students' out-of-school experiences. In this altered classroom space, the content was enriched by the transformed social and institutional roles taken on by the participants. Regarding control over signs, symbols, and activity, there was a mixture of both traditional, adult-controlled activity, and moves to a space changed to one in which students were in charge of the direction of the activity and the kinds of numeracy practices involved. As seen above, this transformed space was prominent across the before gridlock, gridlock, and after gridlock episodes.

6.3. Production and Reproduction of Social Spaces. Works by de Certeau [17] and Street et al. [9] are helpful in drawing together multiple dimensions of networked classroom activity (e.g., social, historical, cultural, ideological, and political). We add to that analysis attention to mathematical dimensions of social space. The exploration of the practices characteristic of the two classrooms' Gridlock simulations and students' interview responses highlight their experiences and perceptions of numeracy practices as personal, political, and value-laden and illuminate issues of power and ideology in numeracy practice. Importantly, though we discuss them separately, there is a blurring of spatialized and spatializing practices as the activities unfold.

6.3.1. Spatialized Practices, Reproduced Spaces. Spatialized practices lead to the construction of space that can be described as hegemonic, designed in advance, where knowledge is both controlled and imparted in a particular manner, such that everything and everyone has its place. In many "typical" classrooms, the spatial arrangement of subjects and objects reflects both the unknowing (usually students) and the knowing (usually teachers), with teachers acting on students as they impart disciplinary knowledge. There were a number of factors serving to position students this way in the Gridlock classes, one being the race/ethnicity of the adults in the class. The teacher, aide, student teacher and two of the researchers were White females, while the third (and teaching) researcher was a White male. These were the people in charge of classes made up of primarily African American and Latina/o students, mirroring inequitable social and power relations in the larger US society. A related issue is the power associated with the technology brought into the class by the researchers who operated the equipment and taught when in the classroom, as well as with their knowledge and control of this technology. The researchers, with their technology and command of it, served to position students and teachers/aide in conventional social and institutional roles (school versus university, researcher versus teacher). At times in the Gridlock activity, the instructor/researcher controlled the signs, symbols, and knowledge (requiring turn-taking rather than overlapping talk, directing attention to the graphs and walking the class through their analysis). In addition, as a design principle is determined by the technology developers and the researcher/instructors, the technology, curriculum, and pedagogy themselves represent an utopian view of what mathematics classrooms should look like (student-centered, inquiry oriented) and what mathematical concepts and topics are important in school (dynamic systems, mathematics of variation and change). In other words, such "space is the interpretive locale of the creative artist and the artful architect, visually or literally re-presenting the work in the image of their subjective imaginaries; the utopian urbanist seeking social and spatial justice through the application of better ideas, good intentions, and improved social learning" ([15], p. 79). However, this activity did not last long in Gridlock as we were committed to engaging students in transformed social spaces by inviting participation so that they had control of the signs, symbols, and knowledge ("opening" up the classroom by asking them to connect their ideas of what comprises good/bad traffic to the graphical representations, as one example).

6.3.2. Spatializing Practices, Produced Spaces. Spatial practice theories consider the social space of numeracy practices—not just the description of the layout, but what Lefebvre ([43], as cited in [15]) describes as people being both users and producers. It is a space where the traditional roles can be deconstructed and then reconstructed once again "with new possibilities heretofore unthought of" inside the traditional space of the classroom or school ([15], p. 81). In this view, we consider how people are positioned and position themselves in relation to others and institutions. In the Gridlock activity, students acted as a collective body, rather than a collection of bodies. With the use of calculators, each were traffic lights, controlling the flow of traffic through the grid. By talking with each other, calling out names of intersections to be changed to keep traffic flowing, they controlled traffic flow through the grid. In this way, the typical activity of the mathematics classroom was disrupted as students collectively produced mathematical objects, such as the graphs and the motion of the cars in the system. Because students were working collectively as opposed to individually, classroom space became a "we" space, not the usual "I" space that one finds in a high school mathematics classroom. Based on our observations of the classes when the network was not being implemented and our interviews with students, this was an unusual transformation. In this case, the social space of the classroom, historically a space of individual students separated in their individual desks, became transformed into a space of a collective of students, interconnected via their calculators and their coordination of efforts. Additionally, this interconnectedness of the students flies in the face of the historical construction of the school classroom, which places the locus of control in the hands of the teacher and focuses on individual student effort and learning. Engagement with Gridlock, by its connected and dynamic nature, alters that traditional, teacher-controlled classroom space into a more student controlled, collaborative one. Hence, we see the activity of Gridlock inserting dynamism and mutability into numeracy practice and challenging the dominant norms of the mathematics classroom.

Additionally, students were constructing representations and understanding of the mathematics of change through their actions and interactions, engaging with concepts and relationships normally reserved for calculus courses. While it is through the curriculum and technology of Gridlock that this engagement occurs, it is important to acknowledge that, “mathematical concepts reside not in physical materials, computer software, or prescribed classroom activities but in what students do and experience” ([40]; p. 87). Thus, it is through interaction with the dynamic medium of the PartSim that transformations of student understanding of dynamic systems, rates of change, accumulation, and the relationships among them transpire.

It should be noted here that the students who engaged with the Gridlock activity were not students for whom the mathematics of change was part of their mathematical “diet.” Earlier it was noted that the students in the study reported here were found in prealgebra, algebra, integrated mathematics, and mathematics competency classes in one of the lowest performing schools in the state’s mathematical measures. Utilizing a more student-friendly approach to calculus enabled the students in the classes we studied to enrich their understanding of the mathematics of change. This understanding enabled the students to bring to bear their knowledge of traffic in Rochester to help them explain what was happening in the graphical representations that are artifacts of the Gridlock simulation as well as analyze their own actions to maximize traffic flow through the grid, a far more mathematically rich activity than their regular classroom work.

6.4. Constructing Safe Spaces. For students interviewed in focus groups, these transformed spaces involved feelings of comfort and communalism important to their engagement. A prominent theme that we found in analyzing focus group interview data about networked classroom activity had to do with safety. This development caught our attention as important in considering the social practices that supported numeracy learning among traditionally marginalized students. The networked classroom as “safe social space” was important to students’ negotiating their ways through this institution of schooling, a space that has been oppressive for many of the youth in our study. Our observations outside classrooms corroborated the notion of tension. Uniformed sentries were numerous, and assistant principals with walkie-talkies circulated, giving a sense of being under surveillance and strict control. The atmosphere was similar to that in many urban schools that serve marginalized populations, with overly strict discipline (cf., [44]) and a lack of trust between adults and students (cf., [45, 46]). As one interviewee put it, working with PartSims was “more like having fun—it’s better to have your mind busy—so much tension in the classes here—if it’s so strict because then you won’t want to do nothing cause you’re all mad and they [teachers] got an attitude—you do not want to learn like that—you want to have fun.”

Students extended our notions about social space by adding attention to affective features of classroom and school activities. Focus group data analyses revealed sentiments about being able to work together and to help each other in networked classroom activity. Freedom to interact and act (this included hands-on learning, opportunity to help each other) and a relaxed and joyful learning atmosphere were the primary themes that spoke to affective elements of network-mediated activity. Many of their responses pointed to the changed classroom climate, for example, as follows.

> *Boy: “cause your level of comfort, you become with each other—it’s like when you in the classroom and you work it out one way or another but like that [networked] activity it was just like everybody was having fun so it’s like—kind of like forget where you at but you do not—it just means everybody having fun.”*

Students’ discussions of interactions during the networked activities were characterized by comments such as, “because really you know, when you’re doing the technology you’re not really worried about it [surveillance] because it’s like your time to do the technology piece…and we’ll be talking like we’re going home.” The invitation offered in the more relaxed atmosphere the students described in networked activity is important in terms of allowing a focus on learning, rather than on contestation, tension, and conflict. As one student remarked, “yeah it makes you want to save math for friends.” These spaces were constructed by youth in ways that worked against the surveillance and control they experienced as central to school and classroom spaces.

7. Discussion

The view of space as socially constructed through interaction and engagement in practices that have social, political, historical, and mathematical dimensions and of numeracy practices as both spatialized and spatializing seems to be very productive in looking at how students take the floor in networked classroom activity. The space of the classroom is changed to one where collaboration, collective effort, informal and formal language, and gesture are all embraced, where the “control of social relations and knowledge” ([9]; p. 21) is more centrally in the hands of the students. As spaces of resistance to dominant norms and relations of power, as well as to hegemonic notions of the kinds of numeracy practices and definitions of what is “mathematics” (e.g., “schooled” math is legitimate and other math is not), Gridlock as a site of numeracy practice seems to involve possibilities for transformation, for creation of new social spaces. These new social spaces are student-controlled, collaborative, and safe.

8. Conclusion

Issues of power and pedagogy make clear that Gridlock provides opportunity for students to engage in numeracy practices that create “the terrain for the generation of “counterspaces,” spaces of resistance to the dominant order arising precisely from their subordinate, peripheral or marginalized positioning” ([15], p. 68). Viewing numeracy as a social practice provided us a way to develop a more nuanced understanding of practices of youth that situates numeracy practices within institutional settings. Our studies of Gridlock examine spaces where there is opportunity to act, to engage in spatializing practices in a transformative way. Gridlock is a space where invitation to participate in a broad array of ways is operating. It is a pedagogical intervention that invites the use of social and cultural practices that may or may not be evident in the traditional math classroom. Once the door is open to widening both the content and conduct of numeracy, the domain of mathematics becomes intertwined in a productive rather than reproductive dialectic relationship with social practices. As a result, the numeracy practices involved in Gridlock create a social space that can involve transformation of students, of content, and of "appropriate" mathematical and social activity.

Our claim is that it is youths' social practices around numeracy that are central to the creation of transformed social spaces. In Gridlock, formal math is not only designed into the activities, but also includes a broader notion of mathematics—not what is usually found in curriculum—because it attends to the dynamics of complex systems and the mathematics of change, fosters students' acting on as well as interpreting mathematical representations of phenomena, and emphasizes the connection of mathematical representations and concepts to students' everyday lives and varied ways of doing math. As Roschelle et al. [41] note, "the mathematics of change and variation (MCV), despite its importance. . .is packed away in a course, Calculus, that sits at the end of a long series of prerequisites that filter out 90% of the population. This is especially true for students from economically poorer neighborhoods and families" (p. 47).

Our student coresearchers have helped us make sense of what is happening in the classroom when the networked technology has been in place. Additionally, they have helped us see some of the everyday mathematical practices they engage in among themselves. As we challenged their notions of traditional mathematics, they in turn located such challenges in the work we do in their classrooms as we analyzed classroom data together. Our understanding of some of the mathematically rich, culturally valued practices they bring to classroom learning in has become much more tangible.

8.1. What Is Gained in Looking at Social Space and Numeracy Practices? Street et al.'s [9] work focuses attention on content, context, values and beliefs, and social and institutional relations. This highlights ideoplogy, power, and situated learning. Attending as well to the spatial relations operating in the different sites in which numeracy practices are operating adds an examination of how social spaces define and are defined through activity and interaction in historically derived, politically and ideologically charged, and power infused arenas. As Soja [15] notes, "We must be insistently aware of how space can be made to hide consequences from us, how relations of power and discipline are inscribed into the apparently innocent spatiality of social life, how human geographies become filled with politics and ideology" (p. 6). There is a connection to be made between the notions of social practice involving ideology, power, content, values and beliefs, and social and institutional relations and social space involving physical, historical, ideological, cultural, and political dimensions, a connection that made us look at how people create spaces where particular activities happen.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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