Hindawi
Education Research International
Volume 2022, Article ID 5200153, 5 pages
https://doi.org/10.1155/2022/5200153

Research Article

Some Examples of Materialist Dialectics in the Concept of Higher Mathematics

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Received 5 February 2022; Accepted 6 May 2022; Published 23 May 2022

Academic Editor: Ayoub Bahnasse

Some examples of dialectics philosophy in higher mathematics are illustrated in this paper. Firstly, the principle of interconversion between quality and quantity in dialectics philosophy is quantified by the mathematical definition of the limit theory. Secondly, some natural and social phenomena imply the definition of continuous function in incremental form and it is a new explanation for the Zeno paradox. Finally, the dialectics relationship between the local change and the whole change of some variables is discussed in the differential median theorems.

1. Introduction

Higher mathematics is a basic mathematical course for undergraduates, including limit theory for a function, derivative and differential, indefinite integral, multiple integral, and so on. At present, many textbooks are based on the limit theory to discuss differential and integral. Differential is a method to discuss the rate and variation of variable change, including the calculation of derivative and differential. Integral, including the concepts of indefinite integral and definite integral, is the inverse operation of derivative and differential. The calculus is widely used in many fields, such as seeking instantaneous velocity in physics, doing work with variable force, etc.; in economics, marginal analysis, elasticity analysis, etc.; in geometry, the slope of a curve, the area of a trapezoid with curved edges, etc. It not only provides scientific principles and calculation methods for natural phenomena, but also expounds dialectical ideology and philosophy of life from a new perspective. As the mathematician Demollins pointed out [1], "without mathematics, we cannot see through the depth of philosophy; without philosophy, we cannot see through the depth of mathematics; and without both, people cannot see through anything." In ancient Greece, Heraclitus, Socrates, Zeno, Plato, Aristotle, and so on discovered and used "Dialectics" firstly. After thousands of years of repeated honing, the mathematical thinking method and "Dialectics" are skillfully combined to form a unique philosophical thinking method, that is mathematical dialectics [2, 3]. However, to learn dialectics in calculus well is to understand materialist dialectics more accurately from a new viewpoint. Therefore, the integration of the elements of philosophical education into the teaching of professional knowledge of calculus not only reflects the course characteristics and philosophical background of calculus but also is the teaching goal of the fundamental task of establishing morality and cultivating talents of the general secretary of the internship. However, "it is not easy to teach the philosophical course well, because it is very demanding." "The key to running the philosophical theory course well lies in the teachers, and the key lies in giving full play to the teachers' enthusiasm, initiative, and creativity." [4] Therefore, it is a high demand and high-tech teaching work to integrate the elements of philosophical education into calculus teaching. According to the research work of above literature and the teaching experience of calculus in recent years, several examples of calculus in ideological and philosophical education are discussed in this paper, mainly including the principle of interconversion between quality and quantity by the mathematical definition of limit theory; some explanations for natural, social phenomena, and the Zeno paradox by the definition of continuous function in incremental form; and the dialectics relationship between the local change and the whole change by the differential median theorems.

2. Quantifying the Principle of Interconversion between Quality and Quantity in Dialectics from Limit Theory

The idea of the limit theory has been born in the sprouting period of calculus. For example, the problem of stick cutting in "Tianxia chapter" of Mei and Zhang-Hua [5–7] (Zhuangzi (About BC369–BC286: His name is Zhuang Zhou. During the Warring States period, he was born in Song Guo. He is the representative of the Taoist school, the successor and developer of Lao Tzu's philosophical thought, and the founder of the pre-Qin Zhuangzi school. Their philosophy was "Lao Zhuang philosophy." His main works include "Free and Unfettered Travel," "Qi Wu Lun," "Tianxia Pian," and so on, which are included in Zhuangzi): "the hammer of one foot cuts half every day, and it will last forever." That is to say, the wooden stick of one foot cuts half every day, and its length decreases continuously, and tends to zero. Another example is Liu Hui's cyclotomic method [8–11] (Liu Hui (About 225–295): He is one of the great mathematicians during the Wei and Jin Dynasties of China. His masterpieces are "nine chapters arithmetic note" and "island arithmetic Sutra." He presents the methods of mouhe square cover, weight difference, cyclotomic method, and so on) in the period of the Three Kingdoms, which used the inscribed regular polygon to calculate the area of a circle. In Figure 1, it can be seen that with the increasing number of sides of inscribed regular polygon, the area of regular polygon trends to the area of circle. According to the above examples, if something changes, such as the length of the stick, the area of the positive polygon, etc., it can be recognized as a variable. The above two examples show the trend of their change, that is, with the increase of independent variables n such as number of edges of a positive polygon and number of days, dependent variable an (such as area and stick length) trends to a constant. This is called the limit of series. It can be understood that when the independent variable n is large enough, the distance between the dependent variable (denoted as a_n) and the constant (denoted as a) is much smaller. So, the mathematical definition of a number sequence limit is given in the following Definition 1.

Definition 1. (see [12]) If a number sequence a_n is limited by a constant a, it is equivalent to that for an arbitrary given positive and sufficient small number $\varepsilon > 0$, there exists a sufficient large natural number N, when $n > N$, $|a_n - a| < \varepsilon$ holds.

The dialectics philosophy in Definition 1 can be interpreted distinctly by Figure 2. Firstly, a point a is given on the number axis and the distance from point $a - \varepsilon$ and $a + \varepsilon$ to a is ε. If something's change can be recognized as a number

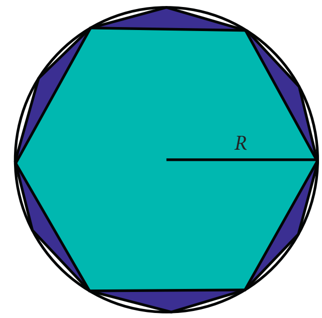

FIGURE 1: Liu Hui's cyclotomic method.

sequence a_n and it is limited by a constant a, it is equivalent to that the variable a_n changes with the increase of n and a_n trends to a constant a. According to the description of Definition 1, for an arbitrary given positive number $\varepsilon > 0$, there exists a natural number N, when $n > N$, $|a_n - a| < \varepsilon$. That means, if $n > N$, $a - \varepsilon < a_n < a + \varepsilon$. Its implications can be intuitively understood from Figure 2. If $n > N$, a_n is located between $a - \varepsilon$ and $a + \varepsilon$. That is the quality which a_n satisfies ($|a_n - a| < \varepsilon$). For an given positive number $\varepsilon > 0$, there exists a natural number N, when $n > N$, a_n satisfies the quality ($|a_n - a| < \varepsilon$). On the other words, a_n changes with the increase of n. a_1 does not necessarily satisfies the quality ($|a_1 - a| < \varepsilon$). $a_2, a_3, \ldots, a_N$ do not necessarily satisfies the quality ($|a_n - a| < \varepsilon$). That is, they are not located between $a - \varepsilon$ and $a + \varepsilon$, as shown in Figure 2. With the increase of n, if $n > N$, all a_n satisfy the quality ($|a_n - a| < \varepsilon$). That is, if $n > N$, all a_n are located between $a - \varepsilon$ and $a + \varepsilon$, also as shown in Figure 2.

According to the above analysis, the variable a_n changes with the increase of natural number n. Before the N term, a_n changes in quantity. After the N term, a_n satisfies the quality ($|a_n - a| < \varepsilon$). So, it can be seen that the mathematical symbol language of a number sequence limit implicates the principle of qualitative change caused by quantitative change in dialectics. Moreover, when quantitative change of a_n leads to qualitative change, it also point out the degree of quantitative change. That means there are qualitative changes in a_n after reaching the N term. After qualitative change in a_n, it shows a stable state, that is, $n > N$, all a_n are located between $a - \varepsilon$ and $a + \varepsilon$, as shown in Figure 2. This shows that the variables have better controllability. Mathematically, it is called convergence. The given positive number ε is relatively fixed. But it is arbitrarily small, and it reveals the measure of approximation between a_n and a. If the given ε changes, the quality which a_n satisfies ($|a_n - a| < \varepsilon$) also changes. Therefore, ε can be understood as a measure of the quality which a_n satisfies ($|a_n - a| < \varepsilon$). That is, there exists quantitative change in the quality. Based on the above analysis, Definition 1 implicates the principle of interconversion between quality and quantity in dialectics. It provides the measure of the principle, including the degree of quantitative change (number N) and measure of the quality which a_n satisfies ($|a_n - a| < \varepsilon$). It also implicates another philosophical element, that is, dialectical relationship between the absoluteness and relativity, including relative fixation and arbitrariness of ε. Some work implicates this principle, such as oil exploration, mining, and so on. Nothing can be obtained before N days. After N days, there is harvest every day, more or less. Just as solving a difficult problem, there may be no progress for a long time. The solution may be found after N days. It is called "no a single step, no even a thousand miles."

3. The Model of Continuous Phenomena and Its Explanation for Zeno Paradox

In the natural environment, two or three peach blossoms can be seen in spring. The duck may know it firstly when the river becomes warm in spring. Because the temperature of water in a short time changes very little, and although each stream is very small, there is no river or sea without a small stream. In our journey of life, although each a step is very small, you cannot complete a thousand mile journey without taking every a small step. These are all common phenomena in nature and our lives. They share a common feature. Every unit, such as the change of water temperature in an instant, a step or a stream, is very small. If these (such as water temperature, air temperature, mileage, and so on) are recognized as some variables, they have the same feature, that is, when the independent variable changes very small, the change of dependent variable is also very small. Mathematically, this phenomena is called to be continuous. How to quantify the phenomena? An independent variable is usually denoted by x and the dependent variable is denoted by y. In this way, y is a continuous function about x. In order to establish a model for the continuous phenomena, Δx is introduced for the variation of the independent variable x and Δy denotes the variation of the dependent variable y. The continuous change is that the variation of dependent variable Δy is very small when the independent variable Δx changes very little. That means when Δx is close to 0, Δy is also close to 0. Thus, the mathematical model of the continuous change is the continuous function in mathematics.

Definition 2. (see [12]) Assume that a function $y = f(x)$ is defined in a neighborhood at x_0. Define Δx as $\Delta x = x - x_0$, where x is near the point x_0 and $\Delta y = f(x) - f(x_0)$. If $\Delta x \longrightarrow 0$, $\Delta y \longrightarrow 0$, that is, $\lim_{\Delta x \longrightarrow 0} \Delta y = 0$, then, the function $y = f(x)$ is continuous at the point x_0.

It is easy to be seen that Definition 2 implicates the mathematical model of the continuous phenomena. If Definition 2 is discussed by dialectical philosophy, there are some interesting findings. There are two sides for everything in dialectical philosophy. And so are the continuous phenomena. Its influence cannot be ignored. For everyone, in a short period of time, both positive progress caused by hard work and negative progress caused by slack and carelessness are all very small. These are not easy to attract our attention. For example, the learning process of college students is continuous. If the learning tasks are completed on time every day, although every progress is small, not only the examination can be passed easily after a semester, but also it can contribute to the further study. If the learning tasks are dealt with perfunctorily or finished a little less every day, although every tiny negative effects are not easy to be found, not only the examination may be failed or the class may be restudy after a semester, but also it will affect the learning of follow-up courses. The people's reaction to the continuous phenomena is obtuse due to continuous principle. But small changes accumulated day by day will eventually become a significant qualitative change, such as "the bank of thousands of miles was destroyed by the ant nest." As a mathematical model of continuous principle, continuous function reveals that calculus is derived from nature, from our daily life and it also implicates materialist dialectics, that is quantitative change leads to qualitative change.

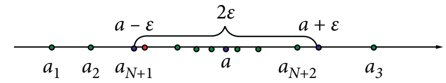

Figure 2: "ε-N" definition of the limit of an series of numbers.

Moreover, Definition 2 of continuous functions also has an important role in solving the Zeno paradox "Flying Arrow is Not Moving" [13, 14]. Zeno pointed out that since the arrow has a temporary position at any moment in its flight, it is no different from immobility in this position [15, 16]. Time is a point on the timeline. The moment represents a very short period of time. No matter how short is a period of time, the time is continuous. The flying arrow is a fusion of time and space. It occupies a space or a position in the space at every time of the flight. Mathematically, each time can be denotes by a symbol t, and a position in the corresponding space can be represented by $y(t)$. In materialist dialectics, movement refers to the changes and processes of all things and phenomena. Quiescence is the relative state of the motion. In physics, motion and quiescence are relative. An object is called moved if its position changes relative to an object of reference. Also, an object is called static if its position is unchanged. For the flying arrow from time t_0 to time t, its corresponding position is from position $y(t_0)$ to $y(t)$. Assume that Δt is introduced for the variation at the time t_0, that is $\Delta t = t - t_0$, and Δy denotes the variation of the location variable y, that is $\Delta y = y(t) - y(t_0)$. So, the variation Δt is a period of time. The moment means that the variation Δt is small enough. Moreover, the variation Δy can implicate the motion and quiescence of the objects. If the variation Δy is equal to 0 for any moment, no matter how short is a period of time, this means the position of an object is unchanged at time t_0. On the other words, the object is static. If the variation Δy is always not equal to 0 for any moment, no matter how short is a period of time, this means the position of an object is changed at time t_0. On the other words, the object is motional. According to the continuity of flying arrow, if $\Delta t \longrightarrow 0$ and $\Delta y \longrightarrow 0$. However, Δy is always not equal to 0 for any moment, no matter how small is the Δt. In mathematics, Zeno's paradox is obviously wrong. So, the mathematical model of continuous phenomena also has an important role in explaining Zeno's paradox.

4. Establishing the Relationship between the Local and the Whole from the Mean Value Theorem

Another content with philosophical elements in calculus is the mean value theorem. The differential mean value theorems are important theorems which implicate the relationship between functions and derivatives, and it is also the theoretical basis of calculus. It includes Lagrangian mean value theorem, Cauchy mean value theorem, and so on.

Lagrangian mean value theorem [12]: if a function $f(x)$ is continuous on a closed interval $[a, b]$, and it is differentiable in the open interval (a, b), then there exists more than one point ξ in the open interval (a, b) $(a < \xi < b)$ so that

$$\frac{f(b) - f(a)}{b - a} = f'(\xi). \quad (1)$$

Cauchy mean value theorem [12]: if functions $f(x)$ and $g(x)$ are continuous on a closed interval $[a, b]$, and they are derivative in the open interval (a, b), and $g(x) \neq 0$ for every point $x \in (a, b)$; then, there exists more than one point ξ in the open interval (a, b) $(a < \xi < b)$ so that

$$\frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f'(\xi)}{g'(\xi)}. \quad (2)$$

Roll mean value theorem [12]: if a function $f(x)$ is continuous on a closed interval $[a, b]$, and it is differentiable in the open interval (a, b), $f(a) = f(b)$ then, there exists more than one point ξ in the open interval (a, b) $(a < \xi < b)$ so that

$$f'(\xi) = 0. \quad (3)$$

These implicate the relationship between the function value $f(x)$ and its derivative value $f\prime(\xi)$. The derivative value $f\prime(\xi)$ is the instantaneous change rate of the function $f(x)$ at a point x, which denotes the local feature of the function $f(x)$. These differential mean value theorems play an important role in dialectic relationship between the local and the whole. This association is quantitative. The mean variation ratio $(f(b) - f(a))/(b - a)$ of the function $f(x)$ on a whole closed interval $[a, b]$ can be inferred by its transient change rate at a local point ξ in the open interval (a, b), that is, the derivative $f\prime(\xi)$, as described in the Lagrangian mean value theorem. For example, there are about 200 kilometers from Shanghai to Hangzhou. If you choose to drive at 1 : 00, you will reach your destination at about 4 : 00. In whole journey, assume that the distance function is $f(x)$, the mean speed is $(f(4) - f(1))/(4 - 1) = 200/3 \approx 67$ km/h, that is, $(200 \times 1000)/(3 \times 3600) \approx 18.5$ m/s, which is the mean variation ratio of the distance function $f(x)$ with time. During the drive, it will be found that the instantaneous speed is 67 km per hour at some time ξ between 1 : 00 and 4 : 00. This shows that for continuous differentiable variables, its global variation can be obtained at local points. Moreover, the ratio $(f(b) - f(a))/(g(b) - g(a))$ between the different continuous differentiable function $f(x)$ and $g(x)$ on a whole closed interval $[a, b]$ can be inferred by ratio of their transient change rate at a local point ξ in the open interval (a, b), that is, the derivatives' ratio $f\prime(\xi)/g\prime(\xi)$, as described in Cauchy mean value theorem. In the above example, consider another continuous differentiable variable $g(x)$, that is the fuel in the car. Assume that 20 L of fuel is consumed between 1 : 00 and 4 : 00, it can be denoted by $g(4) - g(1)$. During the period from 1 : 00 to 4 : 00, the average fuel consumption is $(g(4) - g(1))/(4 - 1) \approx 6.7$ L/h. However, average fuel consumption with per kilometer is the ratio $(g(4) - g(1))/(f(4) - f(1)) = 0.1$ L/km, that is 0.1 ml/m. During the drive, it will be found that the instantaneous fuel consumption is 0.1 ml/m at some time ξ between 1 : 00 and 4 : 00.

Based on the above example analysis, the differential mean value theorem reveals the quantitative relationship between the whole variation of the variables and the local derivatives. This also illustrates the dialectical relationship between whole and local in dialectics. Observing equations (1) and (2) from left to right, the whole is composed of local, properties of the whole can be obtained at the local points. Observing equations (1) and (2) from right to left, the local points are in the whole, and the whole variation will also determine the local points.

The relationship among these differential mean value theorems is derived from special to general. Roll mean value theorem is a special case of the Lagrangian mean value theorem, and the Lagrangian mean value theorem is a special case of Cauchy mean value theorem. This also implicates the dialectical relationship between local and whole. On the one hand, Roll theorem is a part case of the Lagrange and Cauchy theorem, which can be derived from the Lagrange and Cauchy theorem. On the other hand, the conclusions of Lagrange theorem and Cauchy theorem can also be proved by Roll theorem. This implies the properties of the whole can also be derived by the part. So, these differential mean value theorems are the dialectical relationship of unified coexistence. According to the above discussion, a profound dialectical relationship between local and the whole can be shown from these differential mean value theorems.

5. Discussion and Conclusion

In this paper, some examples of dialectics philosophy in higher mathematics are discussed, based on the definition of series limit, differential mean value theorems, and continuous function. Firstly, the Definition 1 of series limit implicates the principle of interconversion between quality and quantity in dialectics. It provides the measure of the principle, including the degree of quantitative change (number N) and measure of the quality which a_n satisfies ($|a_n - a| < \varepsilon$). It also implicates another philosophical element, that is dialectical relationship between the absoluteness and relativity, including relative fixation and arbitrariness of ε. Secondly, the differential mean value theorem reveals the quantitative relationship between the whole variation of the variables and the local derivatives. This also illustrates the dialectical relationship between whole and local in dialectics. Finally, as a mathematical model of continuous phenomena, the continuous function reveals that high mathematics is derived from nature and our daily life, and it also implicates materialist dialectics, that is, quantitative change leads to qualitative change. Explained by the definition of continuous function, Zeno paradox is obviously wrong. However, there are many concepts and principle in high mathematics which are rich in ideological and philosophical elements and profound in thought. The work should be continued and more new discoveries will be explored in our future research.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work is supported by the Planning of Philosophy and Social Sciences in Zhejiang Province of China (19NDQN340YB).

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