

## Research Article

# Metacognition Application: The Use of Chess as a Strategy to Improve the Teaching and Learning of Mathematics

Simon Adjei Tachie <sup>1</sup> and Johnson Motingoe Ramathe<sup>2</sup>

<sup>1</sup>*School of Mathematics, Science and Technology Education, North-West University, South Africa*

<sup>2</sup>*Post Graduate Student, School of Mathematics, Science and Education, North-West University, South Africa*

Correspondence should be addressed to Simon Adjei Tachie; [simon.tachie@gmail.com](mailto:simon.tachie@gmail.com)

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The aim of this study was to explore chess as an alternative strategy to improve the teaching and learning of mathematics. Purposive sampling was used to identify 25 experimental group learners and 26 control group learners. Experimental learners were drawn from two schools that offered chess, while control group learners were drawn from five non-chess-training schools. As such, 51 learners provided the data. The researcher administered a group test in an attempt to triangulate the findings of the study. Using chess as an alternative strategy to improve the teaching and learning of mathematics, it emerged from the main findings that the control group learners made far more contextual errors, compared to the experimental group, on the various problems that were presented. It also emerged that non-experimental learners were poorer in applying required basic steps to arrive at answers to the activities given to them compared to the experimental learners. The study further found that while both groups committed blended contextual and procedural errors, control group learners were dominant as compared to the experimental learners. This implies that the experimental group applied more metacognition in various problems and therefore outperformed the control group in the group test. Based on the above findings, the study concludes that chess training improves both teachers' and learners' application of metacognition in supporting learners' performance in mathematics. The study recommends that since the use of chess serves as an alternative strategy to positively improve the teaching and learning of mathematics, chess training can be introduced to schools to enhance the mathematics performance of learners.

## 1. Introduction

For the past decades in the 21st century, scholars have engaged themselves in related research activities to investigate and suggest effective strategies that may help educators to prepare learners on how to navigate through the increasingly globalized world and inter-connected problems associated with learners' poor performance in mathematics and how these strategies, applied by teachers in teaching and learning situations, can help learners overcome their poor performance in mathematics globally in the 21st century [1]. It is believed that learners need skillful strategies in order to deal with the competitive global market changes and employment through the application of the concept of metacognition; this being abstract in nature and which we focus

on learners' thinking about their own thinking when faced with problem-solving activities in today's sophisticated world so as to further prepare themselves for graduating from colleges of learning [2].

The concept of metacognition as proposed by Flavell [3] in his research work, focused merely on developmental psychology, which particularly targeted children's thinking of their own thinking processes in problem-solving and which was also influenced by Jean Piaget's important work in developmental psychology [4, 5]. Due to the abstract nature of metacognition, different terms such as self-regulation, meta-memory, and executive control are used interchangeably in describing/defining the same basic phenomenon for improving the thinking abilities of learners in problem-solving situations.

The concepts of cognition and metacognition are widely used interchangeably in our daily lives to mean the same thing and it is important to explain the difference between these two concepts for the betterment of our research environment. Flavell [3] explains cognition as “the state of learning and understanding the outside world” whereas metacognition refers to the process of engagement with higher order thinking about how to understand and how to create a better learning experience [4]. Related studies have demonstrated that learners need skills such as problem-solving, metacognition, and critical thinking [6–10] to enable them to think critically when faced with problem-solving situations in order to find appropriate solutions to their problems in teaching and learning. In line with this, Greenstein [11] also states that critical thinking includes the concept of analyzing information, dissecting the problem faced, applying relevant strategies, ideas, logical inquiry, drawing conclusions, evaluating evidence, making accurate judgments, and analyzing assumptions in order to find a solution to a problem which is very important in the game of chess; likewise in the teaching and learning situation [1].

*1.1. Background.* In chess, there are various pieces, namely, Queen, King, Rook, Bishop, Knight, and Pawn, and all have their distinct way of moving across the board. Queens move diagonally, horizontally, and vertically; Kings just move one step in every direction, The Rooks move vertically and horizontally, Bishops move diagonally, the Knights move two squares vertically and then one square horizontally, while the Pawns move on the files and capture diagonally.

Giménez et al. engaged in a test on several games to ascertain the skills correlation that learners had in those games such as Dots and Boxes, Wari, and Traffic Lights in respect to mathematics factors such as numerical and geometric, counting, and rotation. This test was done to investigate exactly how and with what every game correlated in relation to each mathematical factor. The current study’s main focus explores chess as a strategy to improve the teaching and learning of mathematics. Drawing from Todor et al.’s [12] study, resolving the problem-solving of tasks compatibly, can be modelled by children’s practice in chess using Costa and Kallick’s [13] concept of “habit of the mind.” A game of chess is more than a game: it relates to mathematics as it involves calculations, basic arithmetic, and geometry applied in enhancing problem-solving abilities of children. The above can be interpreted as a solid connection between chess and the mathematics domain.

Djakov [14], and Chase and Simon, cited in Trincherro and Sala [15] indicates that chess players can memorize chess positions which arose from previous games because of their good memory. Gobet [16] and Severin et al. [17] further confirm that professional chess players’ knowledge and acquisition of games in the database assist them in evaluating and applying the best combinations, thus ensuring that they play or choose best variations and make the best decisions.

Gobet and Simon [18] concur in that chess players usually have excellent memories, stating that chess players are able to recall positions and random legal positions on a

chessboard. Gobet and Simon [18] further claim that chess players have specific knowledge of structures which are embedded in the long-term memory resulting from prolonged chess practice. This means that if chess players have a good memory of positions and classic games, and they use the latter skill to their advantage in becoming expert chess players, then they are able to apply that skill to mathematics learning, an aspect that this study seeks to explore.

*1.1.1. Knowledge Gained as Learners Learn Chess.* There are different forms of knowledge that learners may acquire and develop as they play chess. Ormrod [19] describes metacognition as one’s “realization and reliance on one’s own subconscious thought processes and vigorous ventures which are utilized to modulate one’s processes to optimize learning and memory”; in other words, comprehension and control of our subconscious thought processes.

Flavell [3] further explains metacognition as thinking about thinking, while Martinez [20] says metacognition is the monitoring and control of the thought algorithm, which Bechtel [21] describes cognition as a dynamic mental mechanism process that can be a chain of information-processing steps.

From Figure 1, for a chess player to make the first move on a chessboard, he or she has to apply his/her metacognitive abilities to think of the possibilities of the first move, of which there are about 20 move possibilities. Bart [23], Jerrim et al. [24], and Kazemi et al. [25] argue that chess is a game that can be helpful to learners’ studies by developing their metacognitive abilities. According to Bart [23], chess is a cognitively demanding activity; subsequent to that, it ameliorates learners’ intelligence, attention, and reasoning abilities, thus developing many skills unrelated to chess.

Jerrim et al. [24] and Garner [26] share the same sentiments: that is, that chess is a cognitive enhancer. A general population that is involved in intellectual activities more often than not present with superior cognitive abilities compared to others [27], and this is found in an intellectual activity such as a game of chess.

*1.2. Problem Statement.* The problem of poor performance in mathematics and the causes and solutions thereof have been a subject of concern for many years. South Africa currently faces a crisis in mathematics education which has been identified in five cycles of the Trends in International Mathematics and Science Study (TIMSS) conducted since 1995 with Grade 9 learners. Grade 9 South African learners were placed last in the Third International Mathematics and Science Study [28–31] and in 2015, South African learners were some of the lowest performing candidates of the 39 participating countries [32]. A report by Evans [29] indicates that South Africa is ranked second last in the world in terms of mathematics and science proficiency.

For eight years, as a researcher, and as a Head of the Department of Mathematics and Science in high school, I have been working and organizing workshops at my school. During this time, I have come to realize that most mathematics teachers in the circuit use traditional approaches in the teaching and learning of mathematics even though the

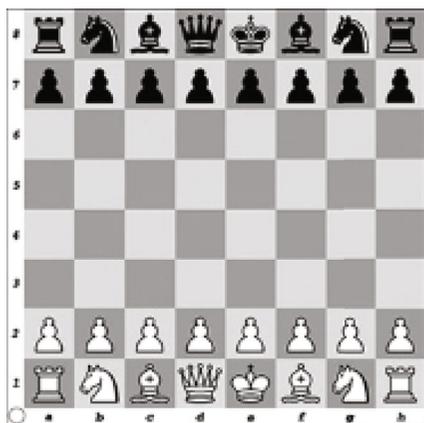


FIGURE 1: The chessboard setup [22].

Department of Basic Education (DBE) has developed a variety of policies aimed at improving teachers' methods of teaching mathematics in the country. Some of the schools in the country have been equipped with computers and software programs to teach mathematics; however, it has not been established whether these have had a positive effect on mathematics performance to date. To the best of my knowledge, little research has been conducted on the use of chess as an alternative strategy to improve the teaching and learning of mathematics, particularly in the Motheo Education District of the Free State Province, hence the current study.

**1.3. The Research Questions.** Based on the above background, the main research question for this study was: *In what ways does chess serve as an alternative strategy to improve the teaching and learning of mathematics in schools?*

The current study further sought to answer the following sub-research questions:

- (i) To what extent do schools provide learners with chess training?
- (ii) To what extent do learners understand chess?
- (iii) How does learners' practice in chess influence their performance in mathematics?

**1.3.1. Theoretical Framework.** Pragmatism as a research paradigm finds its philosophical foundation in the historical contributions of the philosophy of pragmatism [32] and, as such, embraces a plurality of methods. Tashakkori and Teddlie [33] state that as a research paradigm, pragmatism is based on the inclination that researchers should utilize the philosophical and methodological approach that functions well for a certain research problem that is being investigated. It is often related to mixed methods [34–36] where the focus is on the results of the research and on the research questions rather than on the techniques and strategies.

Duffy et al. [35] suggest that pragmatism accepts that there can be single or multiple realities that are open to empirical inquiry. A significant set of ideas of pragmatist philosophy is that knowledge and reality align with beliefs

and habits that are socially constructed [37]. Pragmatists doubt that reality can ever be certain for one and all [38]. They view reality as a standard idea and assert that reality is what functions; thus, they maintain that knowledge claims cannot be completely taken from contingent beliefs, habits, and experiences [39]. For pragmatists, reality is absolute as far as it helps us to get into satisfactory relations with other parts of our experiences [40]. According to Baker and Schaltegger [41] and Ray [42], the truth is whatever proves itself good or what has stood the scrutiny of individual use over time. But it is also of utmost importance to keep in mind that it is not necessarily guaranteed that if pragmatism works, it is now an absolute truth [43]. In pragmatism, an empirical approach takes preference over idealistic or rationalistic approaches [44].

A significant set of ideas of pragmatist epistemology is that knowledge comes from the foundation of experience as in chess training. The understanding of the world is largely impacted by social experiences. Thus, the socialization of individuals or group of people in certain activities were to understand a situation; in this case, the learners practicing of chess to develop themselves mentally to improve their performance in mathematics. Individuals have specific knowledge that emanates from unique experiences. Nonetheless, the bulk of knowledge is socially shared as it comes from socially-shared experiences; therefore, all knowledge is social knowledge [36], hence the use of pragmatic theory in this study. Pragmatist epistemology does not view knowledge as reality [45]. Instead, it is created with the aim of effectively managing people's existence and operations in the world [46]. According to Berkman [47], playing chess requires concentration, visualization, analytical thinking skills, abstract thinking, creativity, critical thinking skills, and bring about cultural enrichment and early intellectual maturity. As a result, learners who train in chess will be able to espouse such skills in their mathematics class and perform better, hence the theory. Many pragmatists in the forefront looked at the work of Dewey and maintained that the epistemology draws from Dewey's concept of inquiry which amalgamates beliefs and actions through a process of inquiry [34, 36, 46].

**1.4. Literature Review.** In our previous article (Tachie and Ramathe in press), we highlighted the game of chess and the way learners learn chess to influence their performance in mathematics. The study revealed that in non-chess-playing schools, only 4% ( $N=2$ ) indicated that they could algebraically notate the game whereas all 20 learners (100%) in chess-playing schools confirmed that they could notate the game. The game of chess comprises a board with 32 chess pieces, 16 of which are white and 16 black.

Figure 2 shows that in chess there are various pieces, namely, a Queen, King, Rook, Bishop, Knight, and Pawn, and all have their distinct way of moving across the board. Queens move diagonally, horizontally, and vertically; Kings just move one step in every direction. The Rooks move vertically and horizontally, the Bishops move diagonally, the Knights move two squares vertically and then one square horizontally, while the Pawns move on the files and capture diagonally.

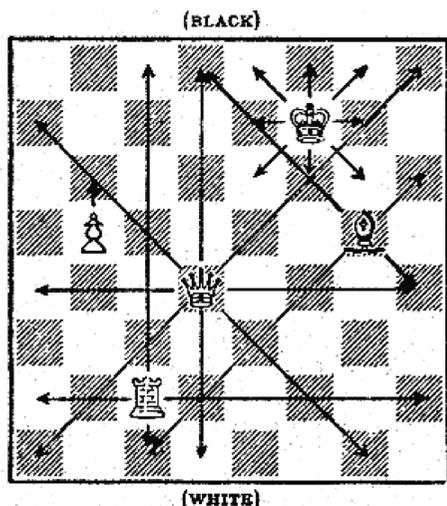


FIGURE 2: Chess piece movement on a chessboard.

*1.4.1. Time Spent Learning Chess.* Time spent on learning and practicing chess is vital in fine-tuning the relevant skills and abilities. According to Gobet and Simon [18], as already stated, mastery of chess needs specialized knowledge, as well as “titanic brain capacity” to learn previous games and current games patterns, ability to recall, fathom, synthesize, apply, evaluate, and problem solve certain move orders so as to win games. This applies to learners from the beginner stage to grand master levels.

Mastery in chess requires deep training and study. It comes only from having more, better, or more efficiently organized knowledge in a domain. Gobet and Campitelli [48] assert that mastery of one’s chess skills requires maximum focus and determination as well as investment in time. They further indicate that the best time to commence chess practice or play in local chess clubs is 12 years old. On the other hand, Blair et al. [49] claim that training in chess and development of chess skills play a significant role in social and academic interactions of individuals. Chess rehearsal extrapolates the abilities which learners can use to comprehend mathematics [50]. Amount of time spent on training in chess is of great significance to mathematics performance of learners, giving an indication of chess being a strategy to improve the teaching of mathematics in schools.

Various studies have suggested a range of times for chess training. Sala et al. [51] indicate that some studies suggest that learners participate in a chess lesson first and then allocate 30 minutes for learners to play against each other, while some studies give learners two-hour lessons daily spread over 12 days resulting in a total of 24 hours to introduce new skills. Sala and Gobet [52] report having given learners 30 hours of chess instruction. Trincherro [53] blended 10 to 15 hours for a chess course, supported by computer-assisted training (CAT) to enhance learners’ mathematical-solving ability. In the current study, I took note of the research and worked with learners that were introduced to chess training over a whole year while they were studying

mathematics at the same time. In the subsequent year, the study was conducted with the hope of seeing the influence of chess training on mathematics performance.

Studies conducted by Trincherro [54] and Kazemi et al. [25], looked at the effects of chess courses on Grades 3, 5, 8, and 9 learners’ mathematical problem-solving ability. The current study focused on Grade 9 learners who were participating in chess training as well as those who did not participate in chess training but all took mathematics as a subject.

*1.4.2. Experimental Groups vs Control Groups.* Various studies have been conducted to compare a normal group’s performance with a dyscalculia group with a significant difference being found [54]. In another study, Trincherro [53] reported that three types of groups were compared: the group that was actively involved in chess, the group actively involved in checkers and was treated as a control group, the third group that was passive. The group that played chess together with the passive group indicated a slightly better performance in comparison to the active control group (playing checkers) in mathematics problem-solving. The three groups showed no statistically significant difference in mathematical problem-solving. However, the results showed that in counting skills, problem-solving, and computational tasks, the students who received chess training had higher mathematical scores [55].

In the evaluation of the role of chess heuristics in promoting children’s mathematical problem-solving skills, Trincherro [53] compared the effectiveness of two different types of chess training in enhancing mathematical problem-solving abilities. A meta-analysis reported that chess players outperformed non-chess players in several cognitive skills [56]. In a study conducted by Aciego et al. [57] in Northern Italy, the comparison group, which played soccer or basketball in comparison to the chess group, had better cognitive abilities, better coping and problem-solving capacities, and better socio-affective development. In the study conducted by Hong and Bart [54] in Korea, the experimental group (students at risk for academic failure) received chess lessons for three months (12 lessons) and the last six lessons took place in a computer lab with chess software in pursuit of establishing if there was an impact of chess on cognitive abilities and minimal impact was found. The study presents the paradigm adopted. Regarding pragmatism, this study explains the history of pragmatism and pragmatism as a research paradigm.

*1.5. Research Approach.* In this study, a qualitative research approach was used. The research design is the case study design used for the structural framework that guided the researchers in the planning and implementation of the research study. It also helped the researchers to achieve optimal control over the factors that could influence the study [58]. Polit and Beck [59] contend that a research design maps out a detailed plan which speaks to the research questions and helps attain the research objectives, hence the use of a case study in this research.

*1.6. Population of the Participants.* The population of the participants consisted of Grade 9 learners from seven high schools in the Thaba Nchu region of the Free State in South Africa, the researchers having established which schools offered chess training. Grade 9 learners normally have a solid mathematical foundation hence being selected. Grade 9 learners who were doing mathematics at the seven (7) chess-playing schools were subsequently sampled for the data collection.

*1.7. Sample of the Participants.* Various studies have yielded different results and the researcher has found loopholes. This study, conducted in a rural area of South Africa, focuses on secondary school learners who were aged 15-16. All participants were normal chess-playing and normal non-chess-playing participants and the training was done over a year by chess instructors who are mathematics educators as well. Purposive sampling was used to identify an experimental group and a control group at Schools X and Y. Purposive sampling is a non-probability sampling or purposeful sampling procedure, which involves selecting certain participants based on a specific purpose rather than randomly [60]. Purposive sampling was used in this study to select participants for the second phase of data collection. The researchers made use of the control group and experimental group in chess training from the seven (7) schools; five (5) schools were found to have no chess training while two (2) schools provided chess training. The schools were given pseudonyms, they being Schools A, B, C, D, and E and were the ones which did not offer chess training which Schools X and Y offered chess training to their learners.

From Schools A, B, C, D, E, X, and Y, 26 learners indicated that they did not have chess training and were hand-picked in accordance with Table 1 as the control group.

Twenty-six learners of the non-experimental group were given a test totalling 50 marks (Annexure H) and the procedural errors and the contextual errors were examined. Learner performance results during the year recorded on SA-SAMs as well as classwork and homework were used for document analysis. In addition, lessons were observed.

In Table 2, from School X and Y, 25 learners formed the experimental group of 15 and 10, respectively, as indicated by Table 1 and the same scope of work that was administered to the non-experimental group was also administered to them.

*1.8. Instruments for Data Collection.* Table 3 summarizes the alignment of the research questions to the instrument utilized.

The test was administered to the groups and these results were compared to learner performance recorded in SA-SAMS. Prior to the commencement of the main study, a pilot study with three (3) learners who were not part of the final study was conducted using the test. The test was scrutinized by the supervisor and revisions made according to initial findings. In the control group, the group test was administered with the assistance of educators of the seven various schools. The learners concerned wrote the test dur-

TABLE 1: Distribution of control group learners.

Schools	Number of control group learners	Control group learners' pseudonyms
School A	4	A, B, C, D
School B	4	E, F, G, H
School C	4	I, J, K, L
School D	4	M, N, O, P
School E	4	Q, R, S, T
School X	3	U, V, W
School Y	3	X, Y, Z

ing their study periods and the test was out of 50. The test was written on the same day across the seven schools.

*1.9. Group Test and SA-SAMS as Additional Research Instruments.* The researcher also administered a group test in an attempt to triangulate the findings of the study. This instrument was used in order to improve validity of the findings as it was used to check learners' performance under a controlled environment during which they were closely supervised on common items. These results were an independent source of evidence to gauge learners' general mathematics performance which helped to validate the findings.

## 2. Data Analysis, Presentation, and Discussion

Document analysis was conducted in an attempt to establish the influence of chess training on mathematics performance on Grade 9. Qualitative data gathered from two of the seven schools is presented, interpreted, and compared with the information obtained from the literature study.

*2.1. Influence of Learners' Practice in Chess on Their Learning of Mathematics.* In this section, comparison of test results is done and errors are identified.

*2.2. Comparison of Test Result Errors.* All learners wrote a test and after scoring the test, the researchers were able to identify the procedural and contextual errors. Procedural errors are those mistakes that learners make but have some understanding of how to solve the problem. Contextual errors are those mistakes which learners make when they have no understanding of the problem. We sampled several procedural errors and several contextual errors from the test to identify how learners performed in general. At this stage, the idea was to establish whether there were any significant differences in learners' performance in the selected tasks.

The test was marked out of 50. Marks were recorded and the type of errors which learners made were classified into three themes, namely, contextual errors, procedural errors, and a combination of both contextual and procedural errors, namely, blended errors. Analysis of the qualitative data in this section involved comparing similarities and differences in findings and this was executed in the format of themes as follows:

- (i) Theme 1: Contextual errors.

TABLE 2: Distribution of experimental group learners.

Schools	Number of experimental group learners	Experimental group learners' pseudonyms
School X	15	AA, AB, AC, AD, AE, AF, AG, AH, AI, AJ, AK, AL, AM, AN, AO
School Y	10	AP, AQ, AR, AS, AT, AU, AV, AW, AX, AY

TABLE 3: Alignment of data collection instruments to the research questions.

Questions	Data collection
Main question	
How does training in chess influence the performance of learners in mathematics?	Questionnaire, document analysis, and observation
Sub-questions	
To what extent do schools provide learners with chess training?	Interviews
To what extent do learners understand chess?	Interviews
To what extent does learners' practice in chess influence their performance in mathematics?	Interviews, document analysis, and observation

(ii) Theme 2: Procedural errors.

(iii) Theme 3: Blended contextual and procedural errors.

2.3. *Theme 1: Contextual Errors.* Participants were assessed on the manner in which they simplified expressions. The expected first step was to expand the expression by removing the brackets, and the second step was to apply the FOIL method which is to multiply the FIRST, OUTER, INNER, and LAST terms to simplify the expression.

In Figure 3, a control group participant from School A brought down the first term as it was, while the second term was simplified correctly. However, in the second step, the learner did not apply the FOIL method. This was supposed to expand the first term by multiplying the brackets but, instead, the learner squared what was inside the bracket. This was wrong. The learner had applied metacognition in solving the problem but fell short of giving him/her an opportunity to expand the first term by multiplying the bracket in order to arrive at the correct answer even though we were not really interested in the correct answer but rather how the procedure was applied so as to arrive at the right (or wrong) answer. Clearly, the learner could not apply skillful strategies through the application of the concept of metacognition in solving the problem, which of course focuses on the learner's thinking about his/her own thinking and that could have helped him/her to arrive at the correct answer using all the necessary steps involved [4, 5].

In Figure 4, the experimental learner AX from School X made a number of incorrect algebraic manipulations to render the whole exercise futile. The first mistake was the failure to apply the FOIL method to correctly expand the first bracket. This was followed by the incorrect interpretation of the negative sign which was converted into a multiplication sign by simply multiplying the terms in the second bracket next to the negative sign. Failure to cautiously treat the subtraction sign as an algebraic operation that separates terms was one of the sources of the problem for this exper-

imental learner in comparison to the control group learners. Though the learner's final answer was correct as per the previous step, the contextual mistakes that were committed earlier had an effect on the final answer as a result of the incorrect application of metacognitive abilities in solving the problem [4, 5]. This was similar to the previous learner's errors. According to Brown [61] and Flavell [3], metacognitive knowledge consists of awareness and understanding that can help a learner learn effectively [5]. The above learners simply failed to engage in higher order thinking about how to understand and present the problem step-by-step, in order to create a better learning experience that could lead to a permanent change in learning in life [4]. In this regard, the teachers should take their time to guide learners in their effort to teach mathematics for effective understanding of their learners. Teachers should also take their time to carefully explain the mathematical operations or signs, where they should be used or not used, and what happens if they use in the wrong place when attempting to build or improve their metacognitive skills of thinking.

To arrive at the correct answer, the following steps were to be followed:

$$(x - 3)^2 - x(x + 4) = x^2 - 6x + 9 - x^2 - 4x = -10x + 9 \checkmark \checkmark \quad (1)$$

As such, it is clear that despite the fact that learners were drawn from two different groups, it emerged that they both made the same error but merely differed in the ways used and steps taken when making the mistakes.

In another question, a control group learner X drawn from chess-training school Y was also observed to have committed the same error (Figure 5).

The control group learner was supposed to apply the distribution law which was to multiply each term inside the bracket by the outside term. However, learner X instead applied factorizing by grouping which did not apply in this

$$\begin{aligned} \text{iii. } & (x-3)^2 - x(x+4) \\ & (x-3)^2 - 22x - 4x \\ & x^2 - 39 - 22x - 4x \\ & x^2 - 22x - 4x - 9 \\ & -4x - 9 \end{aligned}$$

FIGURE 3: Control group learner B, School A.

$$\begin{aligned} \text{iii. } & (x-3)^2 - x(x+4) \\ & = (x^2+9)(-x^2-4x) \\ & = x^2(-x^2-4x) + 9(-x^2-4x) \\ & = -x^4 - 4x^3 - 9x^2 - 36x \end{aligned}$$

FIGURE 4: Experimental learner AX, School X.

$$\begin{aligned} \text{i. } & 3a^2bc^2(3a^2-4b-c) \\ & = 3a^2(3a^2-4b-c)bc^2 \\ & = 9a^4 - 12a^2b - 3a^2c + 3a^2c^2 - 4b^2c^2 \end{aligned}$$

FIGURE 5: Control group learner X, School Y.

instance. This is because there must be two common brackets and outside each bracket a plus or minus sign; only then can factorizing by grouping apply. This means that the learner got the first step wrong and, in addition, failed to note that  $3a^2bc^2$  is a single term which thus cannot be split as manipulated. The learner even proceeded to poorly simplify the incorrect previous step.

While the final answer was not entirely correct, an experimental learner in Figure 4 generally performed better in the sense that  $3a^2bc^2$  was correctly treated as a single term (see Figure 6).

In addition, while the learner failed to uphold all exponential laws correctly on variables, there were correct algebraic manipulations. As such, there were not entirely the same challenges that the control group learner encountered but were experienced by the experimental group learner. In fact, the control group learners also failed to note that  $3a^2bc^2$  is a single term which thus cannot be split as was manipulated by one learner. The learner even proceeded to poorly simplify the incorrect previous step. The question one may ask is: Do teachers take their time to explain what is meant by a term, expression, etc. to learners when they are executing their duties as mathematics educators? In most cases, educators minimize or ignore this simple information learners require and the steps learners should take when confronting any problem which leads them to make simple mistakes in their attempts to solve mathematical problems thus resulting in poor performance in the subject and which further derails their metacognitive thinking abilities. In short, learners often fail to apply certain skills such as prob-

$$\begin{aligned} \text{i. } & 3a^2bc^2(3a^2-4b-c) \\ & = 9a^4bc - 12a^2b^2c - 3a^2bc^2 \end{aligned}$$

FIGURE 6: Experimental learner AR, School Y.

$$\begin{aligned} \text{v. } & \frac{4x^{-2}}{(4x)^{-2}} \\ & = \frac{4x^{-2}}{4^{-2}} \\ & = \frac{-4x^2}{-4x^2} \\ & = 1 \end{aligned}$$

FIGURE 7: Learner K, School C.

lem-solving, metacognition, and critical thinking [6–10] that enable them to think critically when faced with problem-solving situations.

The question on simplifying fractions using exponential laws was generally poorly done especially by the control group learners. The ability to follow the correct steps was critical.

In the case of Figure 7, the first step taken by the control group learner K from School C was correct in distributing the exponent to each term inside the bracket in the denominator. In the second step, the learner was supposed to apply the Laws of Exponents and specifically the second law of exponents. This says when you divide, the base is the same as the one that should subtract the exponents. The learner should have identified the fact that there were two bases, 4 and x, then applied the Laws of Exponents correctly. However, the control group learner was completely incorrect as he/she dropped the negative exponents and introduced a negative sign on the terms.

Figure 8 shows the attempt made by the experimental group learner and the mistakes made.

The very first step (identifying the Lowest Common Denominator (LCD)) was correct. However, the learner failed to correctly apply the required steps to arrive at the correct answer. For instance, the learner was supposed to say 4 on the first term goes so many times into the LCD and then multiply by the numerator on the first term; in the second term, the learner should say 2 goes so many times into the LCD and then multiply by the numerator of the second term. Lastly, he/she should say 4 goes so many times into the LCD and then multiply by the numerator of the third term. The learner did not apply the correct steps and thus arrived at the incorrect expression.

iv.  $\frac{2x+1}{4} - \frac{x+3}{2} - \frac{1}{4}$

~~$2x - 2x - 6$~~

~~$= -6$~~

FIGURE 8: Learner AN, School X.

ii.  $\frac{-2(3-2) - (3+2)}{5 - (2-4)}$

~~$= -7$~~

~~$6$~~

FIGURE 9: Learner N, School D.

i. Calculate the value of  $2x^4 + 3x^3 + 7x + 12$  if  $x = -2$

~~$= 2(16) + 3(-8) + 7(-2) + 12$~~

~~$= 38 - 24 - 14 + 12$~~

~~$= -10 - 18$~~

FIGURE 10: Learner AS, School Y.

2.4. *Theme 2: Procedural Errors.* In this example, learners were required to identify and apply the correct concept. In this case, the learner was expected to apply the BODMAS Rule, namely, Bracket of Division, Multiplication, Addition, and Subtraction.

Learner N from Figure 9 in a non-chess-training school was a control group learner who omitted the required steps; this gives an indication that he/she used the calculator. Learners were instructed not to use a calculator. The lack of a clear step-by-step approach which does not show all workings does not guarantee a learner getting full marks, even if the learner is correct.

In this question, learners were simply required to use substitution to arrive at an answer.

The experimental group learner AS of School Y in Figure 10 missed marks when he/she failed to do the actual substitution before any simplification. In order to calculate by substitution, the learner should have substituted -2 where he/she saw "x" in the whole expression. The learner failed to show the full correct substitution in the first step except on the third term. The learner correctly presented the second step but did not add and subtract correctly in the last step; hence, he/she arrived at the incorrect answer.

In comparison to the control group, learner L from School C in Figure 11 scored lower marks than the experimental group learner due to a complete failure to do the actual substitution and subsequently arriving at 36 instead of 32. Though there was consistency in getting the final answer wrong, on average the control group Learner L scored less than the experimental Learner, AS. As such, the experimental and the control group learners made procedural errors but they were not entirely similar.

2.5. *Theme 3: Blended Contextual and Procedural Errors.* We also established that learners from both groups committed a blend of contextual and procedural errors.

In Figure 12, the control group learner W was supposed to solve the expression within three steps. On the first term, 2 on the denominator of the mixed fraction inside the bracket should have been multiplied by 2 whole plus 1 on the numerator to get  $(5/2)2$  and then he/she solved the second term by converting the decimal fraction (0,5) to fraction  $(1/2)2$  since it was not possible to simplify the whole expression with fractions and decimals mixed without the use of a

calculator; thus, the use of a calculator contributed to him/her making a contextual error. The expected procedure was:

$$\left(2\frac{1}{2}\right)^2 + (0,5)^2 = \left(\frac{5}{2}\right)^2 + \frac{1}{4} = \frac{25}{4} + \frac{1}{4} = \frac{26}{4} = \frac{13}{2} \cdot \checkmark \checkmark \quad (2)$$

Figure 13 shows how a learner from the control group committed a procedural error by doing the first step incorrectly.

It is unclear how the participant arrived at the second step without using a calculator. Furthermore, in the second step, the learner missed the addition and was also unclear about how -2 in the previous step became 2; thus, all the workings were incorrect.

In the same question, experimental group learner AQ made a contextual error.

The learner AQ from Figure 14 got the first step incorrect because he/she calculated the square root of 144 and 25 in the same root instead of adding 144 and 25 first in the first term thus everything is incorrect. The learner was also unclear about how became 5. This meant that the learner failed to apply the correct manipulation in step 1 which affected the rest of the workings.

Learner D in Figure 15 was a control group learner who committed a procedural error.

The learner correctly changed the division sign to a multiplication sign by reciprocal, which means that the numerator becomes the denominator and the denominator becomes the numerator on the second term. The learner was then to take out the common factor on the first term which was  $2x$ . He/she should then have simplified the expression to the simplest form. The learner completed the first term correctly by changing the division sign and bringing down the first term as it was; however, when coming to the second step, the learner did not take out the common factor. The learner rather decided to add unlike terms, and as a result, the whole solution was incorrect.

i. Calculate the value of  $2x^4 + 3x^3 + 7x + 12$  if  $x = -2$

$$2x^4 + 3x^3 + 7x + 12 = 2(-2)^4 + 3(-2)^3 + 7(-2) + 12$$

$$= 2(16) + 3(-8) - 14 + 12$$

$$= 32 - 24 - 14 + 12$$

$$= 10$$

FIGURE 11: Learner L, School C.

ii.  $(2\frac{1}{2})^2 + (0,5)^2$

$$(4\frac{1}{4}) + 0,25$$

$$= 4\frac{1}{2}$$

FIGURE 12: Learner W, School X.

iii.  $(\sqrt{144 + 25} + 3\sqrt{25}) \div (\sqrt{2})^2$

$$(13 + 15) \div 2$$

$$= 28 \div 2$$

$$= 14$$

FIGURE 13: Learner H, School.

iii.  $(\sqrt{144 + 25} + 3\sqrt{25}) \div (\sqrt{2})^2$

$$= (12 + 15) \div 2$$

$$= 27 \div 2$$

$$= 13,5$$

FIGURE 14: Learner AQ, School Y.

iv.  $\frac{2x^2 + 2x}{x} \div \frac{2}{x}$

$$\frac{2x^2 + 2x}{x} \times \frac{x}{2} = \frac{2x^3 + 2x^2}{2} = x^3 + x^2$$

FIGURE 15: Learner D, School A.

2.6. *Quantitative Representations.* Figure 16 shows results for control group test results emerging from the written exercises and tests.

The test results from the control group from seven non-chess-playing schools indicated that out of 26 learners who wrote the test, 10 made both contextual and procedural errors, while 11 made contextual errors and 5 made procedural errors.

Figure 17 shows the results of the experimental group selected from the two chess-playing schools who wrote the same test as the control group.

The results from the 25 learners in the two schools, Schools X and Y, were as follows: One learner was found to have both contextual errors and procedural errors, four learners were found to have made only contextual errors, while 10 learners were found to have procedural errors, and 10 of the learners were found to have made none of the errors.

For a chess player to make the first move on a chess-board, he or she has to think of the possibilities of the first move, there being about 20 move possibilities. Bart [23], Jerrim et al. [24], and Kazemi et al. [25] argue that chess is a sport that can be helpful to learners' studies. According to Bart [23], chess is a cognitively demanding activity which subsequently ameliorates learners' intelligence, attention, and reasoning abilities thus benefiting many skills unrelated to chess. Jerrim et al. [24] and Garner [26] share the sentiments that chess is a cognitive enhancer. It is argued that learners involved in intellectual activities such as chess, more often than not, demonstrate glimpses of superior cognitive abilities compared to others who are not involved in such activities [27].

2.7. *Performance in Classwork and Homework.* Frank and Hondt [62] and Scholz et al. [55] state that many studies reported aligned factors in chess instruction with mathematical problem-solving; however, only a few of those studies utilized sound methodological methods; therefore, the conclusions that were reached are questionable. Interestingly, Isabella as cited in McDonald [22] reviewed various studies which indicated aligning the effects of chess on mathematics problem-solving in classrooms. She alludes to the fact that benefits of chess in mathematics might arise from the fact that chess uses notations which might assist learners in understanding mathematics. Isabella further indicates that notation is associated with visio-spatial patterns on a chess-board, while mathematics is associated with pure symbolic manipulation.

2.8. *Comparison of Overall Group Test and SA-SAMS Results.* The overall performance of the two groups in comparison was also analyzed and is presented in Figure 18.

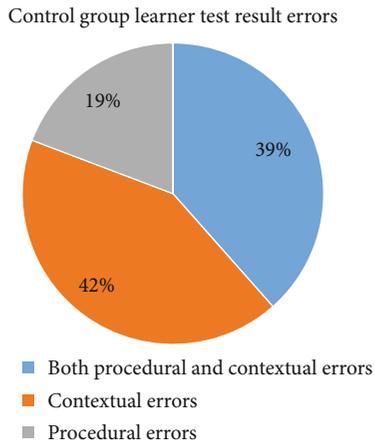


FIGURE 16: Control group test result errors.

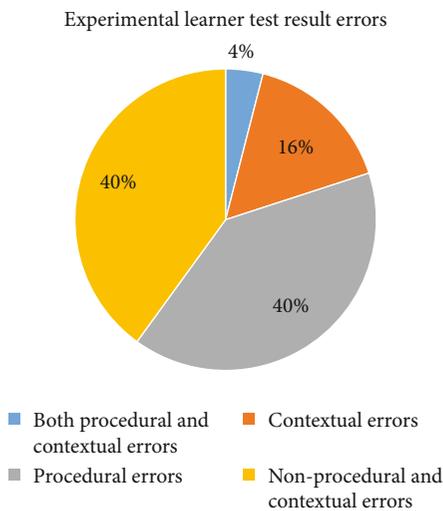


FIGURE 17: Experimental group test result errors.

This figure shows interesting results pertaining to the overall achievement of participants in the two groups on the different tasks. It emerged that in the group test administered by the researchers, experimental learners had an average performance of 78%, which was double the mean mark obtained by the control group. It further emerged that in Term 1, the experimental group had a 30% mean mark compared to 17% of the control group, while in Term 3, the control group obtained a mean of 28% compared to 39% which was obtained by the experimental group. Overall, it was clear that the experimental group outperformed the control group in the group test and the SA-SAMS results from Terms 1 and 3.

**2.9. Five-Number Summary Comparison of Group Test and SA-SAMS Marks.** The overall performance of the two groups is reflected using a five-number summary, which is a set of descriptive statistics that provides information about a data set. It consists of the five most important sample percentiles—the sample minimum, the lower quartile or first quar-

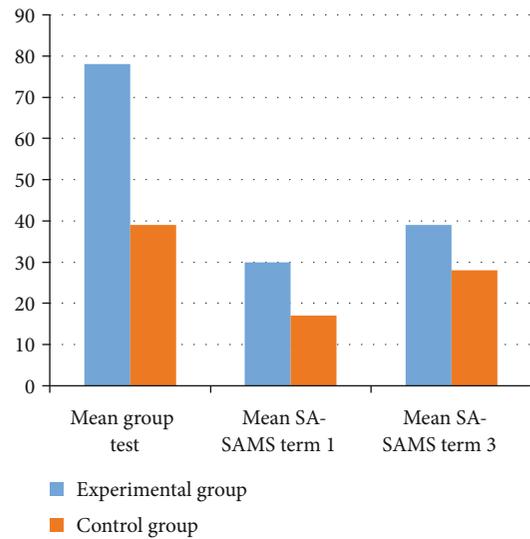


FIGURE 18: Comparison of group test and SA-SAMS results.

tile, the median, the second quartile, and the maximum. The five-number summary is presented in Table 4.

Table 4 indicates that the learners from the experimental group outperformed the control group in all aspects. For instance, in the test, there was a difference of 7% between the minimum marks, with the best learner in the control group attaining 60% compared to the experimental learner who scored 100%. The SA-SAMS marks also showed a similar trend despite variations in the range of marks for each component. There were also reduced differences in SA-SAMS Term 3 performance compared to the other two assessments. Despite that, experimental learners’ performance was far better than the control group learners’ performance throughout.

In various studies done, with some comparing a normal group’s performance with a dyscalculia group, a significant difference was found [54]. In one study, the group playing chess was compared to an active control group (playing checkers) and a passive control group. While the chess-treated group and the passive control group slightly outperformed the active chess-playing control group in mathematical problem-solving, the differences were not statistically significant. The three other groups showed no statistically significant difference in mathematical problem-solving. The results showed that in the counting skills, problem-solving, and computational tasks, the learners who received chess training had higher mathematics scores [55].

### 3. Summary of the Findings

**3.1. Influence of Learners’ Practice in Chess on Their Learning of Mathematics.** The following were observed by the researchers during the study.

#### 3.1.1. Contextual Errors

- (i) The study found that control group learners made far more contextual errors compared to the experimental group on various problems that were presented. For

TABLE 4: Five-number summary of group test and SA-SAMS results.

Assessment	Group	Five-number summary				
		Min	Q <sub>1</sub>	Median	Q <sub>2</sub>	Max
Group test	Control	20	30	40	47	60
	Experimental	27	67	83	97	100
SA-SAMS Term 1	Control	3	10	15	28	28
	Experimental	14	24	32	34	40
SA-SAMS Term 3	Control	16	21	26	32	51
	Experimental	27	26	41	42	53

instance, in one of the problems in the second step, the learner did not apply the FOIL method which was supposed to expand the first term by multiplying the brackets; the learner squared what was inside the bracket thus going totally wrong

- (ii) Failure to cautiously treat the subtraction sign as an algebraic operation that separates terms was one of the sources of the problem for one experimental learner in comparison to the control group learners
- (iii) Control group learners also failed to note that  $3a^2bc^2$  is a single term which thus cannot be split as was manipulated by one learner. The learner even proceeded to poorly simplify the incorrect previous step

### 3.1.2. Procedural Errors

- (i) The study also found out that control group learners were poorer in applying the BODMAS Rule compared to the experimental learners. As such, some ended up omitting the required steps and used the calculator, even though the instruction was for learners not to use calculators
- (ii) The use of calculators was also possibly evident in the experimental group when learners missed marks for failing to do the actual substitution before any simplification

### 3.1.3. Blended Contextual and Procedural Errors

- (i) A control group learner failed to properly first convert a mixed fraction into an improper fraction before first-stage simplification. Furthermore, the learner did not convert the decimal number to a fraction since it was not possible to simplify the whole expression with fractions and decimals mixed without the use of a calculator, thus the use of a calculator contributed to committing a contextual error
- (ii) Addition of unlike terms was prevalent among learners from the non-experimental group and more so than learners from the experimental group

## 4. Contribution of the Study to the Field

The present study is significant to the extent that it will offer concrete evidence to link chess training and good performance in mathematics, thereby adding to the existing body of knowledge regarding tools that can be used to facilitate the teaching and learning of mathematics. If chess training is found to have an influence on mathematics performance, it means that learners would do well in mathematics and could result in the production of a generation of engineers, architects, and many other mathematics-related professions. Chess can then be incorporated into the teaching of mathematics, and teachers and the Department of Basic Education would benefit immensely as teaching and learning of mathematics would be more effective and fun. Furthermore, the ability to develop a more focused approach through increased concentration and developing emotional intelligence are characteristics which indirectly are associated with success in life yet they are exhibited by most chess players as well as mathematicians. The study is based in the rural areas, with communities previously disadvantaged, so learners performing in mathematics would bring economic emancipation to the society as whole. Binev et al. [63] indicated that the European parliament and Spanish parliament have agreed to the implementation of a chess course in schools where it functions as an educational tool which takes place during school hours.

## 5. Conclusion

In conclusion, based on overall performances in the assessments analyzed, experimental group learners outperformed control group learners in the group test as well as the SA-SAMS marks from Terms 1 and 3. The experimental group's teachers applied metacognitive abilities in their executed activities more than the controlled group's teachers.

## 6. Recommendations for Practice

From the conclusion in line with the findings of the study, the following recommendations are made:

- (i) Since the influence of chess training has been proven in the literature, it is recommended that chess training be introduced to schools in order to enhance the mathematics performance of learners

- (ii) Chess training should be encouraged by administrators and embraced by teachers in order to ameliorate mathematics performance
- (iii) While marking mathematics activities, it is advisable to use positive marking in procedural errors if all steps are shown as this assist in the building of confidence

6.1. *Recommendations for Further Study.* Based on the findings of this study, the following recommendations are suggested for further study:

- (i) A large-scale study on the influence of chess among mathematics learners in high schools in South Africa could be conducted
- (ii) A compilation of a well-structured program on how to go about introducing chess as a learning area/subject
- (iii) Chess as an extra-mural activity by way of clubs and competitors/championships be formed or encouraged in schools to support the development of learners' metacognitive abilities in mathematics learning

## Data Availability

The data is captured in the article.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

## References

- [1] M. Haviz and I. M. Maris, "Measuring mathematics and science teachers' perception on thinking and acting in 21st-century learning," *Journal for the Education of Gifted Young Scientists*, vol. 8, no. 4, pp. 1319–1328, 2020.
- [2] S. B. Kaufman, "Opening up openness to experience: a four-factor model and relations to creative achievement in the arts and sciences," *The Journal of Creative Behavior*, vol. 47, no. 4, pp. 233–255, 2013.
- [3] J. H. Flavell, "Metacognition and cognitive monitoring: a new area of cognitive-developmental inquiry," *American Psychologist*, vol. 34, no. 10, pp. 906–911, 1979.
- [4] B. B. Akaydin, A. Yorulmaz, and H. Cokcaliskan, "Investigation of primary school students' metacognitive awareness and decision-making skill," *International Journal of Progressive Education*, vol. 16, no. 4, pp. 157–171, 2020.
- [5] J. H. van Velzen, "Are students intentionally using self-reflection to improve how they learn? Conceptualising self-induced self-reflective thinking," *Reflective Practice*, vol. 16, no. 4, pp. 522–533, 2015.
- [6] M. M. Al-Gaseem, B. S. Bakkar, and A. Z. Suhail, "Metacognitive thinking skills among talented science education students," *Journal for the Education of Gifted Young Scientists*, vol. 8, no. 2, pp. 897–904, 2020.
- [7] A. M. Amin, A. D. Corebima, S. Zubaidah, and S. Mahanal, "The correlation between metacognitive skills and critical thinking skills at the implementation of four different learning strategies in animal physiology lectures," *European Journal of Educational Research*, vol. 9, no. 1, pp. 143–163, 2020.
- [8] İ. İdawati, P. Setyosari, D. Kuswandi, and U. L. F. A. Saida, "The effects of problem solving method and cognitive flexibility in improving university students' metacognitive skills," *Journal for the Education of Gifted Young Scientists*, vol. 8, no. 2, pp. 657–680, 2020.
- [9] J. W. Son and M. Y. Lee, "Exploring the relationship between preservice teachers' conceptions of problem solving and their problem-solving performances," *International Journal of Science and Mathematics Education*, vol. 19, no. 1, pp. 129–150, 2021.
- [10] D. T. Tiruneh, A. Verburgh, and J. Elen, "Effectiveness of critical thinking instruction in higher education: a systematic review of intervention studies," *Higher Education Studies*, vol. 4, no. 1, pp. 1–17, 2014.
- [11] L. M. Greenstein, *Assessing 21st Century Skills: A Guide to Evaluating Mastery and Authentic Learning*, Corwin Press, 2012.
- [12] O. A. Todor, "Feuerstein instrumental enrichment program," in *International Conference of Scientific Paper (hal. 10-13)*, Spiru Haret University Press, Brasov, 2013.
- [13] A. L. Costa and B. Kallick, *Learning and leading with habits of mind: 16 essential characteristics for success*, ASCD, 2008.
- [14] J. N. Djakov, *Psychologie des Schachspiels: auf der Grundlage psychotechnischer Experimente an den Teilnehmern des Internationalen Schachturniers zu Moskau 1925*, Gruyter, 1927.
- [15] R. Trincherro and G. Sala, "Chess training and mathematical problem-solving: the role of teaching heuristics in transfer of learning," *Eurasia Journal of Mathematics, Science and Technology Education*, vol. 12, no. 3, pp. 655–668, 2016.
- [16] F. Gobet, "A pattern-recognition theory of search in expert problem solving," *Thinking & Reasoning*, vol. 3, no. 4, pp. 291–313, 1997.
- [17] F. T. Severin, M. W. Eysenck, and M. T. Keane, *Cognitive Psychology: A student's Handbook*, Psychology Press, 5th edition, 2005.
- [18] F. Gobet and H. A. Simon, "The roles of recognition processes and look-ahead search in time-constrained expert problem solving: evidence from grand-master-level chess," *Psychological Science*, vol. 7, no. 1, pp. 52–55, 1996.
- [19] J. Ormrod, *Educational psychology. Developing learners*, Pearson, 2006.
- [20] M. E. Martinez, "What is metacognition?," *Phi Delta Kappan*, vol. 87, no. 9, pp. 696–699, 2006.
- [21] W. Bechtel, *Mental mechanisms: philosophical perspectives on cognitive neuroscience*, Psychology Press, 2007.
- [22] ChessSteps.com, "Chess board setup," <http://www.chesssteps.com/chess-board-setup/>.
- [23] W. M. Bart, "On the effect of chess training on scholastic achievement," *Frontiers in Psychology*, vol. 5, p. 762, 2014.
- [24] J. Jerrim, L. Macmillan, J. Micklewright, M. Sawtell, and M. Wiggins, *Chess in schools: evaluation report and executive summary*, Education Endowment Foundation, 2016.
- [25] F. Kazemi, M. Yektayar, and A. M. B. Abad, "Investigation the impact of chess play on developing meta-cognitive ability and math problem-solving power of students at different levels of education," *Procedia-Social and Behavioral Sciences*, vol. 32, pp. 372–379, 2012.

- [26] R. Garner, "Chess makes a dramatic comeback in primary schools," 2012, <http://www.independent.co.uk/news/education/education-news/chess-makes-a-dramatic-comeback-in-primary-schools-8301313.html>.
- [27] J. Ruthsatz, D. Detterman, W. S. Griscom, and B. A. Cirullo, "Becoming an expert in the musical domain: it takes more than just practice," *Intelligence*, vol. 36, no. 4, pp. 330–338, 2008.
- [28] K. S. S. B. Brende, *The Global Competitiveness Report*, 2014.
- [29] S. Evans, "SA ranks its maths and science second last in the world," 2013, <http://mg.co.za/article/2013-04-17-sas-maths-science-education-ranked-second-last-in-world>.
- [30] S. J. Howie, T. A. Marsh, J. Allummoottil, M. Glencross, C. Deliwe, and C. Hughes, "Middle school students' performance in mathematics in the Third International Mathematics and Science Study: South African realities," *Studies in Educational Evaluation*, vol. 26, no. 1, pp. 61–77, 2000.
- [31] S. J. Plomp and T. Howie, "English Language Proficiency and Other Factors Influencing Mathematics Achievement at Junior Secondary Level in South Africa," 2001, <https://files.eric.ed.gov/fulltext/ED455250.pdf>.
- [32] V. Reddy, M. Visser, L. Winnaar et al., *TIMSS 2015: Highlights of Mathematics and Science Achievement of Grade 9 South African Learners*, Human Sciences Research Council, 2015.
- [33] A. Tashakkori and C. Teddlie, *Mixed methodology: combining qualitative and quantitative approaches*, SAGE Publications, 1998.
- [34] G. Biesta, "Pragmatism and the philosophical foundations of mixed methods research," *Sage Handbook of Mixed Methods in Social and Behavioral Research*, pp. 95–118, 2010.
- [35] R. D. Duffy, K. L. Autin, and E. M. Bott, "Work volition and job satisfaction: examining the role of work meaning and person–environment fit," *The Career Development Quarterly*, vol. 63, no. 2, pp. 126–140, 2015.
- [36] D. L. Morgan, "Pragmatism as a paradigm for social research," *Qualitative Inquiry*, vol. 20, no. 8, pp. 1045–1053, 2014.
- [37] V. Yefimov, *On pragmatist institutional economics*, vol. 49016, MPRA Paper, 2004.
- [38] J. Pansiri, "Pragmatism: a methodological approach to researching strategic alliances in tourism," *Tourism and Hospitality Planning & Development*, vol. 2, no. 3, pp. 191–206, 2005.
- [39] K. R. Howe, "Against the quantitative-qualitative incompatibility thesis or dogmas die hard," *Educational Researcher*, vol. 17, no. 8, pp. 10–16, 1988.
- [40] W. James, "What pragmatism means," in *Pragmatism Class. Am. Philos. Essent. Readings Interpret. Essay*, J. Stuhr, Ed., Oxford University Press, 2nd edition, 2000.
- [41] M. Baker and S. Schaltegger, "Pragmatism and new directions in social and environmental accountability research," *Accounting, Auditing & Accountability Journal*, vol. 28, no. 2, pp. 263–294, 2015.
- [42] L. Ray, "Pragmatism and critical theory," *European Journal of Social Theory*, vol. 7, no. 3, pp. 307–321, 2004.
- [43] J. R. Tiles and R. D. Boisvert, *John Dewey: Rethinking our Time*, vol. 48, no. 4, 1998 SUNY Press, 1998.
- [44] R. Frega, "Pragmatist Epistemologies," *Lexington Books*, 2011.
- [45] R. Rorty, "Pragmatism, relativism, and irrationalism," *Proceedings and Addresses of the American Philosophical Association*, vol. 53, no. 6, pp. 717–738, 1980.
- [46] G. Goldkuhl, "Pragmatism vs interpretivism in qualitative information systems research," *European Journal of Information Systems*, vol. 21, no. 2, pp. 135–146, 2012.
- [47] R. M. Berkman, "The chess and mathematics connection: more than just a game," *Mathematics Teaching in the Middle School*, vol. 9, no. 5, pp. 246–250, 2004.
- [48] F. Gobet and G. Campitelli, "The role of domain-specific practice, handedness, and starting age in chess," *Developmental Psychology*, vol. 43, no. 1, pp. 159–172, 2007.
- [49] C. Blair, P. D. Zelazo, and M. T. Greenberg, "Measurement of executive function in early childhood: a special issue of developmental neuropsychology," vol. 28 Tech. Rep. 2, Psychology Press, 1st edition, 2005.
- [50] A. W. Root, *Science, math, checkmate: 32 chess activities for inquiry and problem solving*, Teacher Ideas Press, 2008.
- [51] G. Sala, A. Gorini, and G. Pravettoni, "Mathematical problem-solving abilities and chess: an experimental study on young pupils," *SAGE Open*, vol. 5, no. 3, article 2158244015596050, 2015.
- [52] G. Sala and F. Gobet, "Do the benefits of chess instruction transfer to academic and cognitive skills? A meta-analysis," *Educational Research Review*, vol. 18, pp. 46–57, 2016.
- [53] R. Trinchero, *Can chess training improve Pisa scores in mathematics. An experiment in Italian primary school*, Kasparov Chess Foundation Europe, Paris, 2013, [https://aperto.unito.it/retrieve/handle/2318/142194/23444/Trinchero\\_KCFE.pdf](https://aperto.unito.it/retrieve/handle/2318/142194/23444/Trinchero_KCFE.pdf).
- [54] S. Hong and W. M. Bart, *Cognitive Effects of Chess Instruction on Students at Risk for Academic Failure*, University of Minnesota, 2007.
- [55] M. Scholz, H. Niesch, O. Steffen et al., "Impact of chess training on mathematics performance and concentration ability of children with learning disabilities," *International Journal of Special Education*, vol. 23, no. 3, pp. 138–148, 2008.
- [56] G. Sala and F. Gobet, "Does far transfer exist? Negative evidence from chess, music, and working memory training," *Current Directions in Psychological Science*, vol. 26, no. 6, pp. 515–520, 2017.
- [57] R. Aciego, L. García, and M. Betancort, "The benefits of chess for the intellectual and social-emotional enrichment in school-children," *The Spanish Journal of Psychology*, vol. 15, no. 2, pp. 551–559, 2012.
- [58] S. K. Grove, N. Burns, and J. R. Gray, *Designs for Quantitative Nursing Research: Quick-Access Chart Descriptive Study Designs Correlational Study Designs Quasi-experimental Study Designs*, Elsevier, 2013.
- [59] D. F. Polit and C. T. Beck, *Nursing Research: Principles and Methods*, Lippincott Williams & Wilkins, 2003.
- [60] C. Teddlie and F. Yu, "Mixed methods sampling," *Journal of Mixed Methods Research*, vol. 1, no. 1, pp. 77–100, 2007.
- [61] A. Brown, *Metacognition, motivation, and understanding*, Lawrence Erlbaum Associates Publishers, 1987.
- [62] A. Frank and W. Hondt, "Aptitudes and learning chess in Zaire," *Psychopathologie Africaine*, vol. 15, no. 1, pp. 81–98, 1979.
- [63] S. Binev, J. Attard-Montalto, N. Deva, M. Mauro, and H. Takkula, "Declaration of the European parliament," 2011, Retrieved from, <http://www.europarl.europa.eu/sides/getDoc.do>.