Research Article

Is There Complementarity between Teaching and Research? Evidence from Pakistani Higher Education Institutions

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Many activities have been used to impart knowledge and foster the quality of education at higher education institutions: mainly teaching and research. Higher education institutions have typically focused on the adoption of teaching and research independently, but in many instances, both activities coexist. By taking into account the coexistence, this study empirically analyzed why teaching and research activities appear together and how joint adoption of the activities has economic impacts on the performance of the higher education institutions. To do so, this study tested the existence of complementarity between teaching and research using supermodularity through the data envelopment analysis approach. Therefore, the empirical result showed that complementarity between teaching and research confirms that the adoption of one activity strengthens the adoption decision about the other activity. This implies that the institutions that execute both activities simultaneously become more productive rather than adopting a single activity. Moreover, it is important for academic decision-makers to take decisions in order to allow universities to achieve economies of scale.

1. Introduction

Economies of scale at higher education institutions (HEIs) have been gaining attention around the world [1, 2]. Many strategies are being used such as the merger of colleges and campuses to minimize costs and burdens on students without compromising their qualities [3, 4]. Previous studies have focused on measuring the efficiency of campuses affiliated with the departments, employment efficiency, research efficiency, teaching efficiency, etc. [5–8]. Furthermore, the joint adoption of teaching and research in higher education has also been recognized since a long time [9, 10].

Teaching at HEIs differs from teaching in other education institutes because it is correlated with research activities. Commitments to integrate teaching and research in many institutes are observed in order to have excellence in both. Education is a seamless web, and if we hope to have a center of excellence in research, we must have a center of excellence in the classroom [11]. Implementing teaching and research jointly at HEIs helps scholars and students to do their work in accordance with the evidence-based approach, creating new findings as well as transferring the existing knowledge. Moreover, it plays an important role in reflecting allocation of institutional resources, in promotion criteria and in metrics used to measure the performance at individual, institutional, and at system level [12].

When all academic staff involve in research activity besides teaching, they have been playing their own roles in maintaining quality education and creating a good research environment [13]. Therefore, research that adopts along with teaching is a basement for making the institutes to have academic excellence. Based on the evidence, teaching and research are passionate partners rather than shared beneficiaries. However, delivering teaching activity without research leads to failure of the student’s motivation.

Apart from the academic strength, in economic perspectives, if the implementation of one practice raises
marginal return of the other, there exists complementarity between practices. On the contrary, substitutability occurs if the adoption of one of the practices might also decrease marginal return of the other [14]. In the context of supermodularity, two practices are complementary if implementing one practice without excluding another or a total payoff from adopting both practices jointly is greater than their sum [15].

The viewpoint of complementarity is not only a hypothesis but also a meso-level approach. It enables researchers to realize relational phenomena as a guide and is influenced by forces on the lower and higher levels of analysis. Similarly, the notion of supermodularity enables researchers to model the relationship among multiple organizational practices of complex systems as a whole is greater than the sum of its parts in a mathematically rigorous way.

In recent decades, companies, banks, and other organizations have begun to collaborate with each other in order to maximize their profits. On the other hand, other businesses improve their payoff by improving the performance of their activities through the use of a complementarity concept or game theory. However, many HEIs increase their payoff by improving the performance of each of activity independently. Therefore, the importance of complementarity between teaching and research enables to improve the overall performance of HEIs, enhances institutional innovation performance, strengthens the link between activities, and minimizes institutional costs by using certain resources for both activities. Moreover, it is useful for government, educational administrations, and policymakers to monitor institutional performance and save unnecessary funding.

Pakistan is a country that offers a fascinating background in which higher education costs and quality matter a great deal. Pakistan has shown noticeable growth in higher education over more than 15 years. The Higher Education Commission (HEC) has so far affiliated more than 220 higher education institutes—up from around 70 in 2002—and these universities produced more than 20,000 PhDs—up from 3,000 in 2002. These institutes are specialized and general categories. Public institutes are receiving government budgets and generating their own resources, as well. Moreover, the institutions offer degrees at all levels from bachelors to doctorate.

Test for complementarity requires a testing framework that takes into account the whole set of organizational practices [16]. No one has previously discussed the complementarity or substitutability of teaching and research using the supermodularity through the DEA approach. In this study, we have developed a supermodularity through the DEA approach to test the complementarity between teaching and research activities. It tests empirically for the coexistence of teaching and research by assessing the complementary effect that may exist between these activities using available data from the HEIs in Pakistan. Even though complementarity studies have been conducted since the 1990s, the literature on the complementarity of strategies in developing countries is very small. This study addresses a persuasive gap in the prevailing literature. In particular, it is foreseen the application of the supermodularity through the data envelopment analysis (DEA) approach in testing of complementarity. Hence, this will provide important information for an economy where the provision of higher education is growing at such a manifold rate.

The results of this study highlight that the joint efficiency of HEIs is higher than teaching efficiency and research efficiency separately. Institutes being efficient in one of the activities are also efficient in their overall efficiency. And, the institutions that execute both activities simultaneously become more productive rather than adopt a single activity. Hence, this leads to complementarity of teaching and research. In addition, HEIs’ vision and interaction between activities also enhance their complementarity.

The outline of this article is as follows: Section 2 presents a literature review on higher education and an overview of joint adoption of the practices and investigation of the theoretical factors that influence their adoption. Section 3 describes the dataset of the variables and the explanation about the selection of the variables. Section 4 discusses the methodology used to measure efficiency and testing of complementary Section 5 discusses the results for complementarity and the associated conceptual factors. Section 6 concludes the work by outlining contributions for researchers and practitioners.

2. Literature Review

Higher education has long been recognized as a valuable and important tool for promoting economic and social development. Certain countries have reaped the benefits of higher education in terms of the economy, social welfare, and human resources after implementing strategies and initiatives aimed at improving higher education’s competitiveness [17]. Furthermore, higher education has a vital role in encouraging social development and economic growth [18].

It is also a well-known issue around the world that students equate teaching and research. As a result, reforms must be undertaken on both sides, in both education and research institutes [19]. The tensions that arise as students strive to interpret the roles of research and teaching in their future careers, and how these tensions influence their developing academic identities, are little understood [20].

Teaching is a research-based profession, and the involvement of scholars in both activities is important for coordinating higher education processes. As a result, teaching and research are inextricably connected. According to [21, 22], a good scholar in higher education should be active in research. Therefore, teaching effectiveness metrics are associated with research proficiency since they are inputs for each other.

Since there are many activities performed in the HEI, it is critical to understand how to enhance the institution’s performance as a system. Several studies have been conducted in order to improve the performance of HEIs and have come to different conclusions. For instance, Bessent and Ruggiero concluded that educational improvement happens when the organizations in charge of the production and implementation of activities use all available resources.
to achieve optimal results [21, 22]. On the other hand, according to [23], an inefficient institution is one in which there are opportunities to boost production with a fixed number of resources.

Many studies have been undertaken to clarify the significance of the implementation of teaching and research activities separately and jointly, and this idea has been evolved in HEIs since recent years. According to the survey of institutional strategies and teaching and learning plans of the 39 publicly Australian-funded universities, about 33 institutes inspired to adopt teaching and research jointly at some level. On the contrary, the Australian Quality Agency reported that many universities aspired to adopt these activities simultaneously but did not properly translate into practice, because the institutes did not grasp the significance of combination well before adoption.

Researchers have explained the benefits of adopting teaching and research jointly in higher education to professionals, administrators, and academic staff in shaping higher education decision-making policy and disseminating ideas on higher education policy setting. Some of the works on this literature are measurement and achievement of academic excellence due to joint adoption [24], knowledge transfer [25], allocation of institutional resources [26], economic scale in universities, [27] and competitive pressure [28].

2.1. Complementarity and Supermodularity. Complementarity perspective is not a theory of organizational structure but rather an approach that helps a researcher to understand relational processes, and the joint adoption of the activities produces greater benefit than the activities separately [29]. It also helps to develop and consider how different activities are merged and recombined [30]. Additionally, it arises from the cooperation of heterogeneous or homogeneous practices when the nature of the relationships between practices brings great economic benefit to the organization rather than the practices functioning separately [31, 32].

There have been many studies conducted on complementarity between types of innovations since Schumpeter’s era. For instance, studies have focused on innovations that include improvements in production processes and products, and improvements in distribution, promotion, and business operations. Hence, these innovation patterns guide the studies to concentrate on one type of innovation.

The concept of complementarity has indeed been discussed in economic perspective on the work of Roberts and Milgrom [33]. Many studies have used the mathematical analysis of supermodularity on lattices as a method to formulating complementarities [34, 35]. Supermodularity is a mathematical expression equivalent to the argument of “the gain from increasing every component, is more than the sum of gains from the separate individual increases” [14, 36].

Presently, researchers have used interaction concepts and cluster processes to examine the complementarity-in-performance between practices. For instance, the relationship between technical advancement and profit margins was investigated by Schmidt and Rammer. There is a strong trend to incorporate technical and nontechnological developments, and both types are mutually related. The result showed that the impact of nontechnical innovation on the firm’s income margins is much lower than the impact of technological innovation. In the opposite, the combination of technical and nontechnological innovation has a positive impact on the performance of a firm; Sapprasert and Clausen examined the complementarity between technological and organizational innovation on the performance of firms. The empirical result showed that the joint effect of both types of innovation on organizational performance is positive and significant.

Based on the empirical evidence, the concept of complementarity has been applied in several disciplines. Some of them are certified labels and brands, process and product innovation, labor skill and innovation strategies [37], different government innovation policies, information technology, workplace organization and new product and service innovation, adoption of different information technologies in emergency healthcare, different types of labor in the determination of trade patterns, and use of external knowledge across different stages of new product development.

2.2. Theoretical Framework and Hypothesis Development

2.2.1. Interaction between Teaching and Research. To build the institutional framework, the HEIs have a complex structure of interconnected characteristics of resources, activities, and processes. Many HEIs are committed to combining teaching and research in order to be competitive in both. It is essential for HEIs to provide quality teaching and conduct good research in order to provide skilled human power to a country. Since higher education makes such a significant contribution to the country’s economic growth, the institutes must increase their performance. On the other hand, institutes face difficulties in improving their efficiency through the implementation of integrated and coherent practices [38, 39].

Hypothesis 1. The interaction between teaching and research has a positive effect on the performance of HEIs and supports their complementarity.

2.2.2. HEIs’ Vision. A vision could offer a picture of a potential future along with motivation and guidelines for the activities via common expectations and alternate sets of laws. In fact, a vision may establish synergy between an organization’s different divisions. On the basis of various levels of HEIs’ vision, the institute can employ teaching and research separately. However, the mission of the higher education provides a clear catalyst for operations incorporation, which can build a constructive outlook towards the institutes when the institution could improve the possibility of employee participation in teaching and research initiatives.
Hypothesis 2. HEIs’ vision can be projected to have beneficial effects on efficiency of HEIs and supports their complementarity.

2.2.3. Complementarities-in-Performance. Complementarities-in-performance analyzes the impact on the performance of the organization that adopts its practices jointly and shows the effect of economic value of these practices rather than utilizing the practice individually. Lattice theory and supermodularity suggested that certain practices become mutually complementary if they are implemented together with each strengthening the involvement of the other [37, 40]. Therefore, the complementarity of the activities could produce greater economic advantages over a single activity.

Hypothesis 3. The joint implementation of teaching and research results in greater economic efficiency of HEIs relative to a single activity and supports their complementarity.

3. Data

This section presents inputs and outputs used to measure teaching efficiency, research efficiency, and overall efficiency of HEIs. The identification of appropriate variables is a crucial step in the model development process. As a result, in order to estimate the efficiency of a HEI, the required variables must be chosen.

To measure the HEI’s efficiency, it is difficult to use market-oriented outputs such as profit and economic value of inputs [40]. In addition, no precise definition is stated in the selection of appropriate inputs and outputs at the higher education setup. However, a standard technique for selecting appropriate inputs and outputs used to calculate the efficiency of HEIs is to align the assessment goal, and the chosen inputs and outputs must have a positive relationship among themselves. For instance, in the work of [41], the benchmarks to measure the performance of research activity of higher education were the number of publications and the number of supervised PhD theses.

Because outputs have intangible and tangible properties, it is difficult to measure outputs empirically [42]. Based on some studies, the researchers disagreed on what is the best way to measure the “education” output. However, they proposed that a perfect predictor of the performance of higher education correlates with the number of students it educates [43].

Input criteria for higher education are usually faced with the same consistency and commitment problems seen on the output side, but there is less controversy on how to quantify them. Some input variables substitute one another, and we have chosen only those that represent the maximum of the specific input. Technical efficiency measure naturally uses physical input units, whereas cost efficiency uses expenditure-based units.

Increasing enrollment to HEIs and conducting academic research studies with limited funding does not imply that the HEIs operate at the highest level of efficiency. Hence, there are certainly other factors that contribute to increasing the efficiency of HEIs. According to several studies, the variables in measuring the efficiency of HEIs are faculty members and research grant [44], laboratories, libraries, and graduates [45], enrollment and graduate rate [46], PhD theses supervised and publication [41], and average graduate results [47].

The data have been collected from the annual reports of 40 Pakistan HEIs for three years (2017–2019). For teaching activity, inputs are the number of hours a day with respect to a faculty member carrying out teaching activity ($x_1$), the number of hours a day with respect to teaching activity is performed in a laboratory ($x_2$), the number of libraries ($x_3$), and the number of enrollment courses students are taught ($x_4$), while outputs are the number of graduated students from the courses taught ($y_1$), the average graduates’ results ($y_2$), and the graduate rate ($y_3$). For research activity, inputs are the number of hours a day with respect to a faculty member carrying out research activities ($x_1$), the number of hours a day with respect to research activity is performed in a laboratory ($x_2$), the number of PhD theses supervised ($x_3$), and the amount of research grant in millions ($x_4$), while the output is the number of published research papers and books in the impact factor journal ($y_1$). Figure 1 illustrates the composite of six inputs and four outputs to estimate the HEI’s efficiency during the given time interval.

From the list of teaching inputs and research inputs, faculty member and laboratory are shared inputs. Teaching and research activities at HEI carry out an average of eight hours a day. Since shared inputs may or may not use the same hours to perform each activity, it requires an allocation of the share inputs into each activity to measure its efficiency. Moreover, disaggregation of shared resources is very important for the calculation of the efficiency of each activity because summing up of the efficiency of each activity does not provide the overall efficiency of the institute.

4. Methodology

This section discusses the model used to determine whether teaching and research are complementarity. As a result, two steps are performed: first, estimate teaching efficiency, research efficiency, and overall (joint) efficiency using the DEA model; and second, examine the existence of complementarity between teaching and research using supermodularity through DEA approach, that is, supermodularity concept on the basis of the activity’s efficiency computed by the DEA model.

4.1. Measuring Efficiency. DEA is a nonparametric mathematical programming approach used to measure the relative efficiency of decision-making units (DMUs) involving multiple inputs and multiple outputs. The DEA was first introduced by Farrell using only single input and single output [48]. Later, this model was developed using multiple inputs and multiple outputs and works under the assumption of Constant Returns to Scale (CRS), that is, the...
CCR model [49]. The CCR model was revised and developed into a BCC model that works under the assumption of Variable Returns to Scale (VRS) [50].

The advantage in computing the efficiency of HEIs using the DEA approach is to include multiple inputs and multiple outputs without any prior information and require only the quantities of inputs and outputs. This makes the analysis suitable, even if it is difficult to observe the price of inputs and outputs. Moreover, it helps to identify efficient and inefficient institutions and gives direction on how the inefficient institutions can be efficient. However, stochastic frontier analysis estimates the efficiency of the institutions with a specification

**Figure 1:** The composite six inputs and four outputs.
bias since it requires to define a priori functional form of the production frontier [51].

The DEA model can be categorized into input orientation and output orientation. An input orientation model is a certain number of outputs produced using a minimum level of inputs to compare the efficiency of DMUs, while the output orientation model is a certain number of inputs to maximize outputs to compare the efficiency of DMUs. Since HEIs produce invisible products like skills and knowledge and also the variable costs are relatively low, the output orientation model is a convenient approach to measure the HEIs’ efficiency.

For this case, there are ‘n’ DMUs where every DMU \( j \) (\( j = 1, 2, \ldots, n \)) employs ‘m’ inputs \( x_{ij} \) (\( i = 1, 2, \ldots, m \)) and also produces ‘s’ outputs \( y_{hj} \) (\( h = 1, 2, \ldots, s \)). Suppose that the input weights \( u_i \) (\( i = 1, 2, \ldots, m \)) and also output weights \( v_h \) (\( h = 1, 2, \ldots, s \)) are variables. Assume that DMU \( j \) to be computed to design DMU \( k \) (\( k = 1, 2, \ldots, n \)), then the efficiency of each DMU \( \theta_k \) can be computed as a linear programming problem and is given by

\[
\text{max} \, \theta_k = \sum_{h} v_h y_{hk},
\]

Subject to
\[
\sum_{i} u_i x_{ik} = 1, \quad \sum_{h} v_h y_{hj} - \sum_{i} u_i x_{ij} \leq 0, \quad v_h, u_i \geq 0. \quad \forall h, \forall i.
\] 

Note that \( u_i \) and \( v_h \) must be larger than a small positive number \( \varepsilon \) to ignore any undesirable variables by assigning zero to \( u_i \) and \( v_h \) [52]. This tiny number of \( \varepsilon \) is known as a non-Archimedean number [50].

Model (1) runs ‘n’ times to measure the relative efficiency of all DMUs, and each DMU chooses input weights and output weights to optimize its efficiency. DMU’s relative efficiency score is between 0 and 1 (inclusive), and a DMU’s relative efficiency score is one that can be regarded as efficient; otherwise, it is inefficient.

While performing teaching and research, there are some inputs associated with teaching only, some are associated only with research, whereas some inputs associated with both activities are known as “shared inputs.”

Let \( \mu_i \) be the proportion of the number of hours per day carrying out teaching and then \( 1 - \mu_i \) be the proportion of the number of hours per day carrying out research with the range for \( \mu_i \) being

\[
0 < \mu_i < 1,
\]

where \( i \) be the number of shared inputs for research and teaching of the given DMU.

In the following DEA models, inputs and outputs associated with teaching and research are denoted by the superscript \( T \) and \( R \), respectively. Similarly, shared inputs associated with teaching and research are denoted by the superscript \( S \). Weights associated with shared teaching inputs and shared research inputs are denoted by the superscript \( TS \) and \( RS \), respectively.

According to available data, there are 40 DMUs or HEIs: DMU \( j \) (\( j = 1, 2, \ldots, 40 \)). Each DMU \( j \) uses 6 inputs \( x_{ij} \) (\( i = 1, 2, \ldots, 6 \)) and produces 4 outputs \( y_{hj} \) (\( h = 1, 2, \ldots, 4 \)), assuming the input and output weights \( u_i \) (\( i = 1, 2, \ldots, 6 \)) and \( v_h \) (\( h = 1, 2, \ldots, 4 \)), respectively. Let \( 1 - \mu_i \) be evaluated to design DMU \( k \) (\( k = 1, 2, 3, \ldots, 40 \)). Then, teaching efficiency \( (T_k) \) of each DMU \( k \) can be evaluated using linear programming:

\[
\text{max} \sum_{h=1}^{s} v_h^T y_{hk}^T,
\]

Subject to
\[
\sum_{i=1}^{m} u_i^T x_{ik} + \sum_{i=1}^{6} u_i^T x_{ik} = 1,
\]

\[
\sum_{h=1}^{s} v_h^T y_{hj} - \sum_{i=1}^{6} u_i^T x_{ij} - \sum_{i=1}^{6} u_i^T x_{ij} \leq 0, \quad \forall j, \forall i,
\]

\[
0 < \mu_i \leq 1, \quad i = 1, 2,
\]

\[
v_h, u_i, \varepsilon, \forall h, \forall i.
\]

Research efficiency \( (R_k) \) of each DMU \( k \) can be evaluated using linear programming:

\[
\text{max} \sum_{h=1}^{s} v_h^R y_{hk}^R,
\]

Subject to
\[
\sum_{i=1}^{6} u_i^R x_{ik} + \sum_{i=1}^{6} u_i^R x_{ik} = 1,
\]

\[
v_h^R y_{hj}^R - \sum_{i=1}^{6} u_i^R x_{ij}^R - \sum_{i=1}^{6} u_i^R x_{ij}^R \leq 0, \quad \forall j, \forall i,
\]

\[
0 < \mu_i \leq 1, \quad i = 1, 2,
\]

\[
v_h, u_i, \varepsilon, \forall h, \forall i.
\]

It is known that the overall efficiency of HEIs consists of the efficiency of teaching and research. However, it is not obtained by summing up teaching efficiency and research efficiency because these activities have shared inputs. Hence, to measure the overall efficiency of HEIs, use a joint DEA model that takes into account shared inputs between the activities.

Using the concept of goal programming, the objective function of the overall efficiency of DMU \( k \) is defined as

\[
e_k = \lambda_k T_k + (1 - \lambda_k) R_k,
\]
where \( \lambda_k \) is the weight of teaching efficiency and \( 1 - \lambda_k \) is the weight of research efficiency for the DMU\(_k\).

Let \( \lambda_k \) be the weight of \( T_k \) for the DMU\(_k\) and

\[
\lambda_k = \frac{\sum_{i=1}^{r} u_{ik}^T \left( \mu_i x_{ik}^T \right) + \sum_{i=r+1}^{w} u_{ik}^T x_{ik}^T + \sum_{i=1}^{r} u_{ik}^{RS} \left( 1 - \mu_i \right) x_{ik}^S + \sum_{i=w+1}^{m} u_{ik}^R x_{ik}^R}{\sum_{i=1}^{r} u_{ik}^T \left( \mu_i x_{ik}^T \right) + \sum_{i=r+1}^{w} u_{ik}^T x_{ik}^T + \sum_{i=1}^{r} u_{ik}^{RS} \left( 1 - \mu_i \right) x_{ik}^S + \sum_{i=w+1}^{m} u_{ik}^R x_{ik}^R}
\]  

(6)

then, \( 1 - \lambda_k \) is the weight of \( R_k \) for the DMU\(_k\) and

\[
1 - \lambda_k = \frac{\sum_{i=1}^{r} u_{ik}^{RS} \left( 1 - \mu_i \right) x_{ik}^S + \sum_{i=1}^{r} u_{ik}^{RS} \left( 1 - \mu_i \right) x_{ik}^S + \sum_{i=w+1}^{m} u_{ik}^R x_{ik}^R}{\sum_{i=1}^{r} u_{ik}^T \left( \mu_i x_{ik}^T \right) + \sum_{i=r+1}^{w} u_{ik}^T x_{ik}^T + \sum_{i=1}^{r} u_{ik}^{RS} \left( 1 - \mu_i \right) x_{ik}^S + \sum_{i=w+1}^{m} u_{ik}^R x_{ik}^R}
\]  

(7)

Hence, \( \lambda_k T_k + (1 - \lambda_k) R_k \) is the ratio of virtual outputs to virtual inputs and the objective function of the model and also this would seem appropriate to be maximized. Then, the overall efficiency (\( e_k \)) of DMU\(_k\) can be evaluated using linear programming:

\[
\text{max} \ e_k = \left[ \frac{\lambda_k}{\sum_{h=1}^{q} v_h^T y_{hk}} \right] + \left[ \frac{1 - \lambda_k}{\sum_{h=1}^{q} v_h^R y_{hk}} \right],
\]

Subject to \( \sum_{h=1}^{q} v_h^T y_{hk} - \sum_{i=1}^{r} \left( u_{i}^{RS} \right) x_{ij}^S + \sum_{i=r+1}^{w} u_{i}^T x_{ij}^T \leq 0, \forall j, \)

\[
\sum_{h=1}^{q} v_h^T y_{hk} - \sum_{i=1}^{r} \left( u_{i}^{RS} \right) x_{ij}^S + \sum_{i=r+1}^{w} u_{i}^T x_{ij}^T \leq 0, \forall j, \]

\[0 < \mu_i < 1, \quad i = 1, \ldots, r,\]

\[v_h, u_i \geq 0, \forall h, \forall i.\]

By substituting the value of \( \lambda_k \) in (8) and using the transformation into linear form suggested by the Charnes, model (8) can be transformed into a linear programming form [53]:

\[
\text{max} \ e_k = \sum_{h=1}^{q} v_h^T y_{hk} + \sum_{h=1+q}^{s} v_h^R y_{hk},
\]

Subject to \( \sum_{i=1}^{r} u_{i}^{TS} \left( \mu_i x_{ik}^T \right) + \sum_{i=r+1}^{w} u_{i}^T x_{ik}^T + \sum_{i=1}^{r} u_{i}^{RS} \left( 1 - \mu_i \right) x_{ik}^S + \sum_{i=w+1}^{m} u_{i}^R x_{ik}^R = 1, \)

\[
\sum_{h=1+q}^{s} v_h^R y_{hk} - \sum_{i=1}^{r} \left( u_{i}^{RS} \right) x_{ij}^S + \sum_{i=r+1}^{w} u_{i}^T x_{ij}^T \leq 0, \forall j, \]

\[
\sum_{h=1}^{q} v_h^T y_{hk} - \sum_{i=1}^{r} \left( u_{i}^{TS} \right) \left( \mu_i x_{ik}^T \right) - \sum_{i=r+1}^{w} u_{i}^T x_{ij}^T \leq 0, \forall j, \]

\[0 < \mu_i < 1, \quad i = 1, \ldots, r,\]

\[v_h, u_i \geq 0, \forall h, \forall i.\]
Consequently, the overall efficiency of \((ek)\) can be calculated using the linear programming problem and is given by

\[
\max \sum_{h=1}^{3} v_h^T y_{hk} + v_i^R y_{ik} ,
\]

Subject to \( \sum_{i=1}^{2} u_i^{TS} (\mu_i x_{ik}^S) + \sum_{i=1}^{4} u_i^T x_{ik}^T + \sum_{i=1}^{3} u_i^{RS} (1 - \mu_i x_{ik}^S) + \sum_{i=5}^{6} u_i^R x_{ik}^R = 1, \)

\( \sum_{h=1}^{1} v_h^T y_{hj} + v_i^R y_{ij} - \sum_{i=1}^{2} u_i^{TS} (\mu_i x_{ij}^S) - \sum_{i=1}^{4} u_i^T x_{ij}^T - \sum_{i=1}^{3} u_i^{RS} (1 - \mu_i x_{ij}^S) - \sum_{i=5}^{6} u_i^R x_{ij}^R \leq 0, \forall j, \)

\( v_i^R y_{ij} - \sum_{i=1}^{2} u_i^{RS} (1 - \mu_i x_{ij}^S) - \sum_{i=5}^{6} u_i^R x_{ij}^R \leq 0, \forall j, \)

\( 0 < \mu_i < 1, \quad i = 1, 2, \)

\( v_h, u_i \geq \varepsilon, \forall h, \forall i. \)

Each model runs 40 times to calculate the efficiency of all DMUs, and each DMU chooses input weights and output weights that maximize its efficiency and \( \varepsilon \) is taken as the value of 0.01. In this study, the chosen value of \( \mu_i \) is higher than 1—\( \mu_i \) because teaching has a high proportion of allocation of the shared input than the research.

Despite all features of using the DEA model, there are certain restrictions. Some of these are as follows: (i) the problem of serial correlation occurs, and it does not give any interpretation of the data generating process (DGP), and (ii) it raises uncertainty about the results in the form of inconsistency and invalidity. To deal with all these limitations, we have used a bootstrap technique that gives reliable and statistically significant results and is known as “bias-free.”

Therefore, the bootstrap DEA approach proposed by Simar is used to eliminate the sensitivity of efficiency scores resulting from the (in) efficiency distribution in the sample [54]. It is a resampling technique with a replacement for the given observations and imitates the underlying true model’s data generation process. In addition, it generates several estimates that can be used for statistical inference.

The accuracy of the bootstrapped estimation depends on residual model variance and the inherent bias of the bootstrap process, and both factors vary with sample size. The residual variance is the source of bootstrapping variability. Bias in bootstrap is any variance value due to the random resampling process in the bootstrap. Especially if the sample is not large and the findings are scattered, the impact of bias can be spread. Consequently, amending bias to bootstrap adjusts the distribution of the estimator to its predicted value. Thus, the steps for using the bootstrapping approach to eliminate the sensitivity of efficiency scores are the following:

1. Utilizing the DEA model for the estimation of efficiency scores
2. Draw efficiency scores with a replacement for the empirical distribution (ED)
3. Divide the original efficiency scores by the pseudo-efficiency scores taken from (smoothed) empirical distribution to get a bootstrap set of pseudo-inputs
4. Use the new set of pseudo-inputs and apply the DEA model to the same set of outputs and compute the bootstrapped efficiency scores
5. Repeat steps (ii)–(iv) “B” times and take bootstrapped scores for testing statistical inferences and hypotheses

4.2. Measuring Complementarity. To test complementarity between practices of a system, there are two broad approaches: complementarities-in-use and complementarities-in-performance. Complementarities-in-use arises from the connection of the practices such that the employment of one practice also requires the inclusion of another practice. In this context, the practices fit well, and they have a jointly beneficial and positive relationship. As researchers analyze complementarities-in-use, they have attempted to recognize relatedness in the use of multiple practices and also demonstrate that these practices appear to be correlated. Complementarities-in-performance deals with the effect of the practices on the performance of the organization by adopting the practices jointly. In particular, it reveals that joint adoption of the practices could produce greater economic advantages to organization rather than adopt a single activity [55].
4.2.1. Lattice Theory. Lattice theory is a division of mathematics based on the partially ordered sets and implemented to optimize the benefit of an organization. The structure of the lattice theory is based on discrete variables in the problems of optimization. In this study, the nodes of the lattice represent activities performed by HEIs. These are teaching only, research only, and both of them. Hence, the possibility combination represents by “C” and C \((x, y) = \{(1, 0), (0, 1), (1, 1): x = \text{teaching}, y = \text{research}\} \). The lattice for HEI is displayed in Figure 2.

4.2.2. Supermodular Function. Supermodularity is a concept that helps to determine the optimal solution for a given problem. It consists of a function defined on a lattice and does not need any specific functional form, continuity, or differentiability [34]. According to [14], the supermodular function is defined as “the sum of changes in the payoff function when several arguments are increased separately is less than a change resulting in increasing all arguments together.”

4.2.3. Testing of Complementarity. To test the existence of complementarity between teaching and research using complementarity-in-performance approach, the complementarity-in-performance approach explores the combined impact of teaching and research on the overall performance of HEI. According to [30], the main problem with empirical complementarity analysis is related to the divisibility criterion of the variables of choice. However, these activities are discrete in nature and are resolved by applying the lattice or order theory, which does not require continuity. More specifically, let \( f \) be a function of two variables. In higher education context, there are three possible combinations \((x, y) = \{(1, 0), (0, 1), (1, 1): x = \text{teaching}, y = \text{research}\} \). The function \( f \) is supermodular only if the implementation of two activities jointly gives better performance rather than the adoption of activity separately. By estimating the teaching efficiency, research efficiency and overall efficiency of HEIs using the DEA model, the supermodular function of \((1, 0), (0, 1) \) and \((1, 1) \) is defined as

\[
\begin{align*}
A \text{ function } f(1, 0) &= T_k, \\
A \text{ function } f(0, 1) &= R_k, \\
A \text{ function } f(1, 1) &= e_k,
\end{align*}
\]

where \( T_k, R_k, \) and \( e_k \) are teaching efficiency score, research efficiency score, and overall (joint) efficiency score of DMU \( k \), respectively. Then, by using supermodularity through the DEA model approach,

\[
f(1, 1) \geq f(1, 0) \text{ and } f(1, 1) \geq f(0, 1) \implies \text{Hypothesis 3.}
\]

\[
(12)
\]

5. Discussion

This section discusses the results of teaching efficiency, research efficiency, and their joint efficiency and the relationship between teaching and research on the basis of complementarities in-performance approach along with HEIs’ vision and interaction between the activities.

5.1. Descriptive Statistics and Analysis. Table 1 illustrates the descriptive statistics of inputs and outputs of 40 HEIs in Pakistan. During three years (2017–2019), 40 HEIs awarded degrees to 62168 students and published 9737 academic papers and books. Such outputs were achieved through the use of 19011 faculty members, 129 libraries, 1664 laboratories, 376 PhD theses supervised, and about 926.2 million Pakistani rupees financed.

The data were collected from public and private sector universities of Pakistan. Private sector universities have a smaller number of faculty, laboratories, PhD thesis supervised, etc., whereas public sector universities show higher numbers in all the input and output variables. However, a large number of laboratories depict the significance of research activities in Pakistani HEIs. As an output, 1500 research publications also show the commitment of faculty towards research activities. This also suggests that faculty burden has a significant share towards both teaching and research activities. It is important to note that there is no distinction between teaching and research faculty jobs in Pakistan. A single faculty has to perform teaching as well as research activities. In view of this, the adoption of teaching and research activities could also be an indication of economies of scale.

5.2. The Relative Efficiency Scores of HEIs. Efficiency measurement of HEIs computed using the DEA model and the teaching efficiency scores, research efficiency scores, and overall (joint) efficiency scores for 40 HEIs under observations are given in Table 2. The relative efficiency score for each HEI lies between zero and one (inclusive). An institution that has the efficiency score of one is an efficient institution; otherwise, it is an inefficient institution, and one is the threshold value for an efficient institution [50].

When teaching and research were taken separately, 7 out of 40 DMUs were efficient in teaching with a mean of 0.6586, a standard deviation of 0.2845, and the lowest score of 0.2829. On the other hand, 7 out of 40 DMUs were efficient in research with a mean of 0.5325, a standard deviation of 0.2779, and the lowest score of 0.0734. Moreover, 12 out of 40 DMUs were efficient in overall performance with a mean of 0.8216, a standard deviation of 0.1982, and the lowest score is 0.3745. The interesting result is that the mean and minimum score of jointly adopted
Table 1: Descriptive statistics of HEIs for three years (n = 40).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description of variables</th>
<th>Sum</th>
<th>Mean</th>
<th>SD</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>Number of faculty members</td>
<td>19011</td>
<td>475.28</td>
<td>586.63</td>
<td>35</td>
<td>3193</td>
</tr>
<tr>
<td>$x_2$</td>
<td>Number of laboratories</td>
<td>1664</td>
<td>41.6</td>
<td>66.7</td>
<td>3</td>
<td>297</td>
</tr>
<tr>
<td>$x_3$</td>
<td>Number of libraries</td>
<td>129</td>
<td>3.23</td>
<td>1.98</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>$x_4$</td>
<td>Number of enrollment course taught students</td>
<td>278637</td>
<td>6965.93</td>
<td>8279.15</td>
<td>620</td>
<td>38058</td>
</tr>
<tr>
<td>$x_5$</td>
<td>Number of PhD thesis supervised</td>
<td>376</td>
<td>9.4</td>
<td>8.05</td>
<td>4</td>
<td>35</td>
</tr>
<tr>
<td>$x_6$</td>
<td>Amount of research grant (in millions Pakistani rupees)</td>
<td>926.2</td>
<td>23.15</td>
<td>33.9</td>
<td>0.5</td>
<td>170.04</td>
</tr>
<tr>
<td>$y_1$</td>
<td>Number of graduated students from course taught</td>
<td>62168</td>
<td>1554.2</td>
<td>1531.2</td>
<td>203</td>
<td>6707</td>
</tr>
<tr>
<td>$y_2$</td>
<td>Average graduates’ result (CGPA)</td>
<td>—</td>
<td>3.19</td>
<td>0.12</td>
<td>2.9</td>
<td>3.43</td>
</tr>
<tr>
<td>$y_3$</td>
<td>Graduate rate</td>
<td>—</td>
<td>25.42</td>
<td>13.53</td>
<td>7</td>
<td>62.02</td>
</tr>
<tr>
<td>$y_4$</td>
<td>Number of published research papers, books, and conferences</td>
<td>9737</td>
<td>243.43</td>
<td>275.63</td>
<td>3</td>
<td>1500</td>
</tr>
</tbody>
</table>

1 Number of HEIs was restricted to 40 because of accessibility issues. However, there are studies published with even lesser number of universities, for example, [56, 57].

Table 2: Relative efficiency scores of HEIs.

<table>
<thead>
<tr>
<th>DMU</th>
<th>Teaching efficiency scores</th>
<th>Research efficiency scores</th>
<th>Overall efficiency scores</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.3656</td>
<td>0.0734</td>
<td>0.3745</td>
</tr>
<tr>
<td>2</td>
<td>0.5559</td>
<td>0.2755</td>
<td>0.5615</td>
</tr>
<tr>
<td>3</td>
<td>0.3854</td>
<td>0.7816</td>
<td>0.7922</td>
</tr>
<tr>
<td>4</td>
<td>1.0000</td>
<td>0.2416</td>
<td>1.0000</td>
</tr>
<tr>
<td>5</td>
<td>1.0000</td>
<td>0.5083</td>
<td>1.0000</td>
</tr>
<tr>
<td>6</td>
<td>0.9474</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>7</td>
<td>0.3919</td>
<td>0.8317</td>
<td>0.8411</td>
</tr>
<tr>
<td>8</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>9</td>
<td>0.3586</td>
<td>0.5274</td>
<td>0.5364</td>
</tr>
<tr>
<td>10</td>
<td>0.7787</td>
<td>0.4824</td>
<td>0.7792</td>
</tr>
<tr>
<td>11</td>
<td>0.8109</td>
<td>0.4691</td>
<td>0.8148</td>
</tr>
<tr>
<td>12</td>
<td>0.3798</td>
<td>0.2181</td>
<td>0.5365</td>
</tr>
<tr>
<td>13</td>
<td>0.3481</td>
<td>0.4735</td>
<td>0.4760</td>
</tr>
<tr>
<td>14</td>
<td>0.4029</td>
<td>0.4643</td>
<td>0.4750</td>
</tr>
<tr>
<td>15</td>
<td>0.9728</td>
<td>0.3605</td>
<td>0.9741</td>
</tr>
<tr>
<td>16</td>
<td>0.9798</td>
<td>0.3695</td>
<td>0.9798</td>
</tr>
<tr>
<td>17</td>
<td>1.0000</td>
<td>0.3855</td>
<td>1.0000</td>
</tr>
<tr>
<td>18</td>
<td>0.6097</td>
<td>0.3670</td>
<td>0.8108</td>
</tr>
<tr>
<td>19</td>
<td>0.4180</td>
<td>0.2410</td>
<td>0.5912</td>
</tr>
<tr>
<td>20</td>
<td>0.8257</td>
<td>0.2785</td>
<td>0.9756</td>
</tr>
<tr>
<td>21</td>
<td>0.3993</td>
<td>0.3070</td>
<td>0.5847</td>
</tr>
<tr>
<td>22</td>
<td>0.2829</td>
<td>0.4776</td>
<td>0.6607</td>
</tr>
<tr>
<td>23</td>
<td>0.3974</td>
<td>0.4255</td>
<td>0.5095</td>
</tr>
<tr>
<td>24</td>
<td>0.9230</td>
<td>0.3695</td>
<td>0.9312</td>
</tr>
<tr>
<td>25</td>
<td>0.6082</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>26</td>
<td>0.4252</td>
<td>0.5427</td>
<td>0.8050</td>
</tr>
<tr>
<td>27</td>
<td>0.3100</td>
<td>0.4420</td>
<td>0.6012</td>
</tr>
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<td>28</td>
<td>1.0000</td>
<td>0.5614</td>
<td>1.0000</td>
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<tr>
<td>29</td>
<td>0.4893</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>30</td>
<td>0.9510</td>
<td>0.6020</td>
<td>0.9912</td>
</tr>
<tr>
<td>31</td>
<td>0.5854</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>32</td>
<td>0.7660</td>
<td>0.5286</td>
<td>0.7755</td>
</tr>
<tr>
<td>33</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>34</td>
<td>0.6123</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>35</td>
<td>1.0000</td>
<td>0.9902</td>
<td>1.0000</td>
</tr>
<tr>
<td>36</td>
<td>0.9864</td>
<td>0.4555</td>
<td>0.9872</td>
</tr>
<tr>
<td>37</td>
<td>0.9587</td>
<td>0.1283</td>
<td>0.9594</td>
</tr>
<tr>
<td>38</td>
<td>0.9831</td>
<td>0.3131</td>
<td>0.9901</td>
</tr>
<tr>
<td>39</td>
<td>0.3978</td>
<td>0.5328</td>
<td>0.8068</td>
</tr>
<tr>
<td>40</td>
<td>0.7031</td>
<td>0.2740</td>
<td>0.7427</td>
</tr>
</tbody>
</table>
activities is higher than teaching and research efficiency scores, separately. This indicates that the joint adoption of both activities follows economies of scale and hence complement each other.

There are some HEIs that are efficient in either teaching or research and also efficient in their overall performance. This implies that higher education adopting both activities is more beneficiary rather than adopting a single activity. Moreover, each activity has a positive impact on the overall efficiency. Hence, HEIs executing both activities simultaneously have better efficiency scores than those who exercise any one activity. This study addressed the relationship between teaching and research on the basis of the proposed hypotheses. First, using the supermodularity concept on the basis of the activity’s efficiency computed by the DEA model (i.e., supermodularity through DEA approach), the complementarity between teaching and research was tested. That is $f(1, 1) \geq f(1, 0)$ and $f(1, 1) \geq f(0, 1)$. This suggests that implementing both activities at the same time is more efficient for the HEI than implementing only one. Based on this finding, it is deduced that there exists complementarity between teaching and research, and thus, the joint implementation of teaching and research results in greater economic efficiency of HEIs relative to a single activity and supports their complementarity.

HEIs improve their productivity even further by focusing on teaching and research because adopting both activities at the same time helps them to develop their core competencies and gain more value. Furthermore, institutes that implemented both activities concurrently are significantly more likely to produce better performance in those activities.

Second, the interaction between teaching and research at HEIs helps academics and students do their work in an evidence-based approach and generate new ideas. Moreover, it could improve the efficiency of each one. As a result, as teaching and research efficiency increases, the institute’s overall performance improves as well. Therefore, the interaction between teaching and research has a positive effect on the performance of HEIs and supports their complementarity.

Finally, educational administrators may be advised to establish the institute’s creative vision by demonstrating its goals for the promotion of sustainable teaching and research growth. A good organizational culture that values sustainability would enhance the integration of these activities and promote staff involvement in teaching and research. This is the input for joint practices to be implemented and their performance increases. When the performance of these activities improves, the performance of higher education as a whole improves. Therefore, HEIs’ vision can be projected to have beneficial effects on the efficiency of HEIs and supports their complementarity.

5.3. Graphical Description of Complementary Activities of HEIs. From the graphical description of efficiency scores of HEIs as in Figure 3, it helps to identify easily the positive correlation between the activities to form of overall efficiency and thus support complementarity relation between teaching and research.

The above figure also points out the location of HEIs according to the efficiency in the respective fields. HEIs with efficiency scores of more than the 50th percentile in teaching and research show significantly higher overall efficiency scores than those with less than the 50th percentile. This also indicates that if an HEI is highly efficient in teaching and research, it is more likely to gain economies of scope proportionately higher than those that are least efficient. Hence, the assumption of economies of scope in higher education has been proved using the theory of complementarity.

6. Conclusion

In this study, the authors have used supermodularity through the DEA model to test the existence of complementarity between teaching and research using the data from 40 Pakistani HEI over three years. The empirical result showed that an HEI that is efficient in either teaching or research is also efficient in its overall performance. This
implies that it is more advantageous for the institutions to adopt both activities instead of adopting a single activity. Moreover, the joint implementation of teaching and research provides an opportunity for institutions to work effectively on both activities and then enhance their performance. Moreover, the joint adoption of teaching and research leads to better HEIs’ performance rather than the adoption of a single activity. This result strongly supports the argument that there is joint interdependence between teaching and research, that is, the complementarity relation between teaching and research. In addition, the interaction of the activities and HEIs’ vision enhances the existence of complementarity.

Improving the HEIs’ performance demands a top management concern. For this reason, higher education is experimenting with combining teaching and research. The results supported the existence of complementarity between teaching and research. Moreover, educational administrators require tight integration of teaching and research to capture the positive effects. Therefore, joint implementation of these activities in higher education leads to increased capacity in introducing process innovations like the faculty members update their knowledge with current information.

6.1. Contribution and Implication for Researchers. The proposed methodology will help researchers in measuring the efficiency of DMUs having shared inputs. A conventional DEA model evaluates aggregate measures of DMUs’ performance, accompanying the components measure that makes up the aggregate value. The difficulty here is that the development of an appropriate model has to do with the presence of a shared resource on the side of inputs and mechanisms for allocating such resources to each component. In this context, this study is the pioneering work in systematic analysis. The empirical result validates the theoretical basis of those strategic groups.

The existence of complementarity between teaching and research at HEI acknowledges the arguments for relationship between teaching and research themselves as the researchers explained in several studies.

6.2. Contributions and Implications for Practitioners. In continuation to the study results, it is proposed that managers in the higher education go for simultaneous adoption of teaching and research. The administration should therefore concentrate on a consistent and specific line of products to compete on the basis of quality and operational excellence. In order to generate higher performance, a combination of creativity and core product function can be fruitful for higher education. This can be achieved by sticking the quality product line while evaluating the actions of the activities with care.

Data Availability

The data are included in the manuscript.

Disclosure

This is a part of Ph.D. research work in COMSATS University, Islamabad.

Conflicts of Interest

The authors declare that they do not have conflicts of interest.

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