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Research Article

Hypothetical Approach to the Teaching of Trigonometric Concepts Using Cooperative Learning

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In this paper, we propose a teaching strategy to enhance the teaching and learning of trigonometry, called cooperative teaching and learning (CTL). The proposed method stems from the need of developing a pedagogical approach that promotes understanding and interest in the learning of trigonometry based on the cooperative learning (CL). This study therefore sought to explore the features of CTL that promote students' learning of the concepts of trigonometry. The study employed systematic review as the research approach for the collection of the data. The review analysis covered 40 research studies on a meaningful-based pedagogies that allow students to make meaning of the subject matter. The study revealed that: positive interdependence, individual accountability, promotive face-to-face interaction, social skill, and group processing characterize the features of CTL suitable for the teaching of the concept of trigonometry. The study suggests that the teaching and learning of trigonometry through the use of the features of the CTL provide students with the opportunity to actively create ideas and work through high-order trigonometric tasks. The study contends that in order to implement CTL effectively in the mathematics classroom context, the features of CTL must be conceptualized in a way that is consistent with the desired objectives of a given mathematical topic.

1. Introduction

The quest for a meaningful-based teaching methods characterized by features that provoke students' understanding of the teaching and learning of mathematics has been a source of worry in recent times, particularly in the area of trigonometry at the senior high schools (SHSs). Thus, necessitating various research studies into the features of a productive pedagogy that allows students to make meaning of the subject matter. In fact, previous research undertaken by us indicates that teachers' inability to relate to features of a meaningful-based teaching approach have an effect on students in the tertiary institutions [1] and at the SHS in particular [2–4]. There seems to be no unanimity of purpose on a single approach to teaching mathematics [5]. Cooperative learning (CL) has been researched to have a positive effect on students learning. This is because CL affords students an opportunity to make meaning of the subject matter independently and collectively as a group in the classroom context [6]. Literature asserts that CL advances a particular a kind of pedagogy that is superior, motivates and improves the inter and intra relationships among individual group members [7]. Again, by employing a CL approach to teaching, learners are encouraged to work cooperatively, the individuals can appreciate the sense of belongingness to support each one another during the teaching and learning process [8]. Moreover, the classroom context of CL is more relaxed and enjoyable than traditional classes, because there is a positive environment in the classroom [9]. In addition, CL serves as a corrective measure for the elimination of individual differences that has inhibited the progress of the students in the learning environment for decades in literature [10]. In the light of the above considerations, trigonometry as a course of study was prioritized in the current with the view to designing a pedagogy anchored on the CL to improve student's learning of the concept. This is because recent studies have cataloged a number of misunderstandings and misrepresentations largely from the perspective of the students on the concept of trigonometry [11, 12]. In particular, the inability on the part of students to establish the relationships that exist in the three main trigonometric concepts of sine, cosine, and tangent of an angle [13, 14]. The import of this act is mirrored in the abysmal performance of students in trigonometry as highlighted in various examination reports of second cycle institutions [2]. Additionally, the teaching and learning of trigonometrical concepts require an understanding of multiple interrelated mathematical concepts, such as algebraic transformation skills, geometry knowledge, and reasoning of graphical representation of concepts [13, 14]. To this end, a pedagogy purposed to promote students' understanding of the concepts of trigonometry cannot be overemphasized. Nonetheless, teachers choose to employ pedagogical strategies that are not student centered [15]. The lack of student-centered and meaningful pedagogies in the area of mathematics, especially in the field of trigonometry causes students resort to memorization of concepts as a means of learning [16]. It is against this background that, the current study explores features of a productive pedagogy that employs the affordances of CL as a means of teaching trigonometry. This is in view of the multivariate forms of CL being used as a pedagogical tool across different subject matters. In this regard, the absence of clear-cut features of CL that characterize the teaching of a particular subject matter could be attributable to the unwillingness on the part of teachers to implement CL as a teaching method in the classroom context [2, 17]. Thus, in the current study we analyze the components (features) that are inherent to the CL lessons for the teaching of trigonometry. Hence, the need for a CL lesson characterized by distinct features that incorporates these concerns to maximize students' learning of the concepts under trigonometry.

2. Cooperative Learning

In recent times, student-centered and effective pedagogical approach have been pivotal in mathematics education, particularly at the SHS. Several studies in connection with student-centered and meaningful-based pedagogical approaches have been examined to have an effect on students' performance in the classroom context [18, 1, 4, 19]. One of such teaching approaches is the CL. Theoretically, CL is grounded in the sociocultural theory [20], social interdependence theory [21], humanist psychology [22], social constructivism [23], and multiple intelligences theory [24]. Since the inception of CL, both past and present scholars have offered multitudes of definitions in literature. For instance, Felder and Brent [25] defined the term *CL* as students working in teams on an assignment or project under conditions in which certain criteria are satisfied, including that the team members be held individually accountable for the complete content of the assignment or project. Again, "the instructional use of small groups so that students work together to maximize their own and each other's learning" (p. 1:14) was coined as the definition of *CL* [26, 27]. Further, *CL* is an instruction that involves students working in teams to accomplish a common goal, under conditions that include some elements [28]. Similarly, Thomas [29] defined *CL* as an interactive process that enables groups or teams of students with different ideas to generate creative solutions to problems. Moreover, Slavin [30] defined CL as "a set of instructional methods in which students work in small, mixed-ability learning groups" (p. 8). Consequently, other scholars also held that, "Cooperative learning is an instructional method in which students work in small, heterogeneous groups to help one another learn" (Strother [31], p. 158). "Cooperation is a structure of interaction designed to facilitate the accomplishment of a specific end product or goal through people working together in groups" (Panitz [32], p. 1). A cursory analysis of these definitions exposes some fundamentals that anchor a learning environment that is CL based. To this end, some scholars have established in literature principles/elements/pillars that inform successful implementation of the CL in a classroom learning environment. In particular, Kagan [33] proposed seven features that work together to create an ideal learning environment conducive to CL. These seven features include structures, teams, management, class building, team building, social skills, and basic principles. According to Slavin [34], CL differs from mere group activities in five specific areas of interest: (a) reward interdependence, (b) task interdependence, (c) individual accountability, (d) teacher-imposed structure, and (e) the use of group competitions. These areas serve as the pivot on which lessons that are cooperative inclined revolve. Educators often operate under the false assumption that placement of students in groups presupposes that they are being "cooperative" and that they are "learning." On the contrary, these assertions are false [35]. According to the Cooperative Learning Centre at the University of Minnesota, CL is a relationship among a group of students that requires five elements: positive interdependence, individual accountability, interpersonal skills, face-to-face promotive interaction, and processing out [35]. These five elements form the "five pillars" of CL which Johnson et al. [26] use as their basis for utilizing such practices in the college classroom. In a more recent study, Kagan [36] advanced that, positive interdependence and individual accountability are two of the four basic principles of CL. To this end, he implied that positive interdependence, individual accountability, equal participation, and simultaneous interaction are the four principles that that anchor the implementation of CL in the learning environment. Again, Jacobs and Seow [37] advocated for eight principles that make the teaching and learning environment CL based. Thus, heterogeneous grouping, teaching collaborative skills, group autonomy, maximum peer interactions, equal opportunity to participate, individual accountability, positive interdependence, and cooperation as a value. Nonetheless, a successful implementation of CL bridges the gap between theory and practice through the design of reflective work [38]. However, in spite of these advantages in the classroom context, some teachers are yet to learn these essential pedagogical skills [39] in the classroom context. In view of the multiplicity of the features of CL in literature as espoused, we reviewed and contextualized features of CL that characterize the pedagogical approach that provokes students' learning of trigonometry in the Ghanaian sociocultural context with particular consideration to the topic of trigonometry in the SHS mathematics curriculum. Thus, in the current study we prioritized on reviewing studies that focus on developing teaching practices that promote student-centered and meaningful-based teaching practices [40] as a means of instructing lessons in trigonometry in the Ghanaian mathematics classroom context. This is because, the absence of student-centered and meaningful-based teaching approaches causes students and teachers claim that the teaching and learning of mathematics is difficult [41].

3. Study Context

In Ghana, teacher-centered pedagogical approach has been endemic in the teaching and learning of mathematics [1, 15]. Thus, teaching practices devoid of lessons that are activity oriented have been observed to be absent in the Ghanaian mathematics classroom learning environment. Further, students' participation in lessons that allow them to make meaning of mathematical concepts individually and collectively as a group are passively employed in the teaching and learning of mathematics. To this end, students' resort to memorization as a means of learning in mathematics [42]. One of the teaching methods recommended by the planners of the Ghanaian mathematics syllabus for the SHSs with the aim of reducing the conceptual difficulty students face in learning mathematics is the CL [43]. However, in the Ghanaian classroom context, teacher's use of CL as a pedagogical tool has been characterized by misconceptions and misrepresentations [2]. The teachers' limited knowledge and skills on the features of CL as an instructional strategy in the classroom context could be a contributory factor for their unwillingness to implement this teaching strategy [26, 27]. In this way, furnishing teachers with the characteristics of what constitutes a CL pedagogical approach will afford them an opportunity to appreciate the dynamics of the implementation of this teaching approach in the classroom context. This is because the National Council for Curriculum and Assessment of Ministry of Education [43] positions the CL-based approach to teaching mathematics, as a vital teaching method aimed at affording mathematics educators' synopsis of what a CL lesson constitutes. Thus, positioning CL as critical to teaching mathematics lessons in the Ghanaian SHS mathematics curriculum. The integration of the CL approach is informed by the provision of the positive effect in the mathematics learning environment [43].

In recent years, the diminishing interest in the teaching and learning of trigonometry has been a source of worry [11, 12]. In particular, the performance of Ghanaian students in the area of trigonometry. This is evidenced in the remarks of the chief examiner that "The questions on trigonometry were poorly answered by most of the candidates. They were unable to visualize the questions and sketch the correct diagram which will help them solve the problem. However, few candidates who made a good sketch were able to solve the question with ease" [44, 45]. These reports were suggestive of the conceptual difficulty students face in answering examination questions that demand the application of concepts in trigonometry at the SHS level. Current studies of which our study is no exception [46–50] contend that the mediums of instruction are fraught with teacher-centered approaches when it comes to the teaching of trigonometry. Again, informed by the complexities of the interrelationships of factors that invalidates the gains in the teaching and learning of trigonometry [13, 14] coupled with multiplicity of the features that characterize CL as a pedagogical tool in literature, we reviewed and conceptualized these features in a way reflective of the Ghanaian mathematics classroom learning environment to teach the concept of trigonometry herein referred to in the study as the cooperative teaching and learning (CTL). Thus, the features of CTL were informed by the sociocultural context of the Ghanaian SHS. In effect, our objective of the current study is to review relevant literature predicated on CL to highlight CTL as instructional method that makes ease the teaching and learning of trigonometry. It is in this regard that, we formulated the following research question to guide the conduct of the study: *What are the features of CTL approach suitable for an effective and meaningful teaching of trigonometry in the SHS classroom?*

4. Data and Methods

The study considered in the research design advanced by Khan et al. [51] as a guide to the systematic review of related articles purposed to address the present study's research question: *What are the features of CTL lessons suitable for an effective and meaningful teaching of trigonometry in the SHS classroom?* The research question informed the kinds of studies to contemplate for the analysis. The analysis covered pieces of evidence from empirical studies published in the area of CL as well as student-centered and meaningful-based pedagogical inclinations aimed at improving student's performance in mathematics. This is to identify features of a teaching approach that facilitate the teaching and learning process in the classroom context. The method involves identifying, analyzing, and interpreting the available evidence of the current research questions in an unbiased and replicable manner [52]. The gradual implementation of this process was achieved by rigorously applying preestablished steps in identifying from the multiplicity of features of CL that characterize a pedagogy that provokes students understanding of trigonometry. In particular, several references in the area of CL together with student-centered and meaningful-based teaching methods were searched to identify relevant articles to gain a comprehensive, detailed overview of the study. Electronic databases: Web of Science, Scopus, Google, Google Scholar, Research Gates, Springer, Eric, and SAGE were examined for vital literature in English language, for instance "features of CL in teaching mathematics," "CL use as pedagogical approach in education," "student-centered and meaningful teaching methods in teaching trigonometry," etc. Originally, 55 studies were generally selected grounded on their connotation to literature in mathematics education in Ghana, CL use in the teaching and learning of mathematics. For the purposes of avoiding repetitions, the results of the information were further reviewed to only studies (total of 40) that prioritized the potentials of the features of CL application in mathematics and its implication for teachers and students, student-centered and meaningful-based teaching strategies with CL, CL-oriented knowledge and pedagogical skills as well as problems in relation to the teaching and learning of trigonometry. The 40 designated revisions were subjected to further scrutiny after which a summary was provided taking into consideration the respective backgrounds,

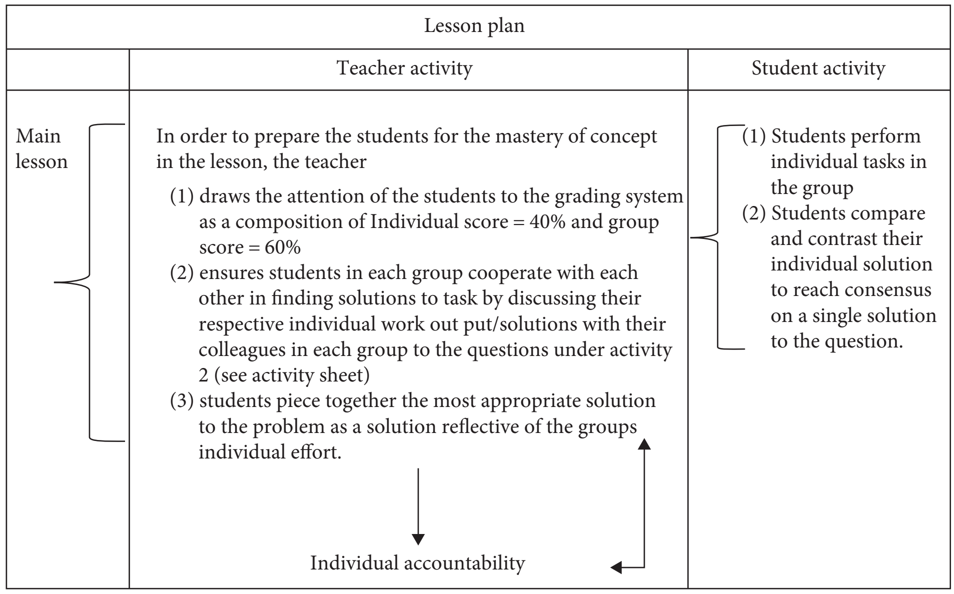

Figure 1: Aspect of lesson plan on trigonometric ratios of sine, cosine, and tangent of angles showing individual accountability (Authors' Own Construction, 2023).

results, findings, and conclusions of such studies. The findings in relation to the review process were then employed as a lens in identifying features suitable for an effective and meaningful teaching of trigonometry in the Ghanaian SHS mathematics classroom.

Further, in order to contextualize these features as identified in the literature in the Ghanaian mathematics classroom context, the lessons designed were subjected to the experts' appraisal. Thus, the lessons were given to experts with at least 10 years' teaching experience in mathematics education, particularly in the Ghanaian mathematics curriculum context. In view of the results from the review process, two lessons: *CTL-Lesson 1* (CTL-L1) and *CTL-Lesson 2* (CTL-L2), were designed based on the features identified.

5. Results

Based on the features identified, lessons informed by the mathematics curriculum for SHS in Ghana [43] were designed. In particular, some concepts in the area of trigonometry: *trigonometric ratios of sine*, *cosine*, *and tangent of an angle* and *angles of elevation and depression* served as the teaching topics that anchored the construction of the lessons CTL-L1 and CTL-L2, respectively. In order to ensure the content and facial validity of the lessons developed, CTL-L1 and CTL-L2 were given to experts to appraise. This was purposed to improve the quality and practicability of the lessons. Hence, the inputs (in the form of feedback) received from the experts were incorporated in the development of the final versions of the lessons. Informed by the literature review, the study established positive interdependence, individual accountability, social skills, promotive face-to-face interaction, and group processing as the features that characterize CTL that is purposed to facilitate the teaching of trigonometry. This section discusses the features of CTL that informed the construction of aspects of CTL-L1 and CTL-L2. For effective and meaningful pedagogy within the context of teaching trigonometry in the classroom context at the SHS level.

5.1. Positive Interdependence. Positive interdependence as conceptualized in the study meant putting students into groups with some assigned responsibilities on the subject matter during the teaching and learning process. The study revealed that, in employing this feature students in each group elect themselves into positions of group leaders, time checkers, and recorders. This was to ensure a shared responsibility (division of the content [task] of the subject matter among the group members) in each group. Students are assigned into groups in order to explore and interact with one another. This positions them to discover and learn the content-driven affordances for the representation of the subject matter. Again, it is evident (see Figure 1) that, the success of the group is collectively dependant on the individual members in the groups. This is because (a) each group member's efforts are required and as such indispensable for the success of the entire group members and (b) each group member has a unique contribution to make to the joint effort because of his or her resources and/or role and task responsibilities as assigned. This is intended to depict how the use of positive interdependence informed the design of the two lessons CTL-L1 and CTL-L2. For instance, in designing CTL-L2 on the topic of "*angles of elevation and depression*" positive interdependence was purposed to draw on the strengths of the good students in the group to the advantage of the weak ones. Thus, students provide complementary ideas on the assigned task in the various groups during the teaching and process. This was to afford the individual group members to make meaning of the "*angles of elevation and depression*." This feature also positions teachers to provide a facilitatory role in the learning environment. In this way, teachers attend to students with peculiar problems and support them to understand the concept under instruction. Figure 2

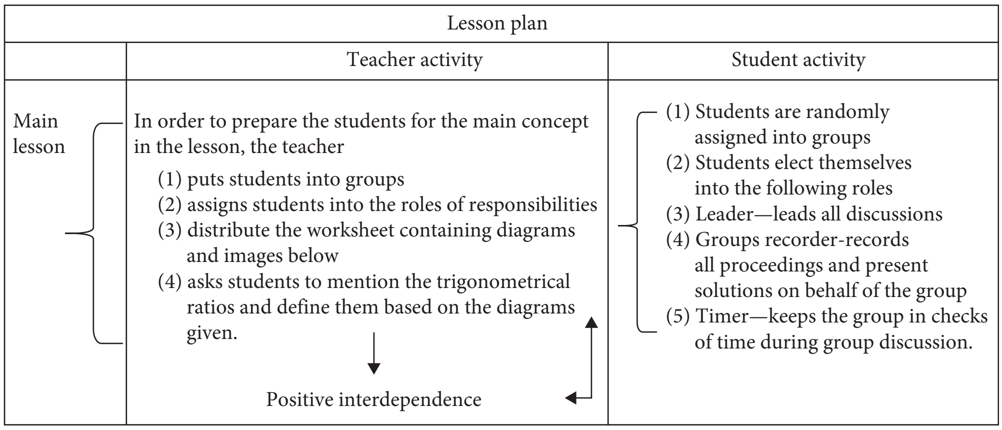

Figure 2: Aspect of lesson plan on angles of elevation showing positive interdependence (Authors' Own Construction, 2023).

depicts aspect of lesson plan that shows positive interdependence on the *angles of elevation and depression*.

As evidenced from Figure 1, the use of positive interdependence in the teaching of mathematics is likely to reduce competition among individual students and create a shared responsibility where the success of the group is contingent on the collective effort of the individual group members in the learning environment. As a result, its purpose fits the realities in the SHS mathematics classroom context.

5.2. Individual Accountability. The feature of individual accountability was purposed to afford students in the group master the concept taught in the classroom independently as an individual and collectively as a group. The lessons were designed to elicit responses/solutions to questions from two perspectives; (i) individual level-where students on their own finds a solution to the questions posed and (ii) group level-where the individual group members reconcile their individual solutions together to reach consensus on one true solution as the group's final answer to the question. Further, the study showed that in applying individual accountability, it positions the learners as active participants in the group activities. In this way, the likelihood of "free riders" or "social loafing" during group activities are minimsed. As a result, they learn to do the assigned task collectively so that they can do it more easily when they are alone (independently) (see Figure 1). In CTL-L1, individual accountability was designed on the topic of the trigonometric ratios of sine, cosine, and tangent of angles. Figure 1 presents a snapshot of how the individual accountability was designed to facilitate the teaching of the trigonometric ratios of sine, cosine, and tangent of angles.

The review posits that, the design of individual accountability in the teaching of mathematics creates a conducive and cooperative-based learning environment that allow students to experience at first hand the practical and realistic effect of the subject matter which hitherto had been taught in the abstract. As a result, its purpose fits the realities in the SHS mathematics classroom context.

5.3. Promotive Face-to-Face Interaction. Central to the enactment of the CTL in this study was the design of promotive face-to-face interaction feature in the CTL-L1 and CTL-L2. This is because, promotive face-to-face interaction involves the development of activities that allow the learner to be at the center of the lesson. Thus, making the lesson student centered. Its incorporation in the lesson ensures that students interact, reason, and reach compromise on the subject matter with one another in the group. The use of the promotive face-to-face interaction feature in the design of the CTL lessons was meant to engage students in the various groups to share ideas on the subject matter in the lesson. This is realized as a result of involving students in activity-oriented lessons that are tailored towards the conceptualization of the subject matter under instruction (as in the design of aspects of CTL-L1 and CTL-L2 on the topics of *the three trigonometric ratios of sine*, *cosine*, *and tangent of angles and the angles of elevation*, *respectively*). In particular, the use of promotive face-to-face interaction in the design of lessons on the trigonometric ratios of sine, cosine, and tangent of angles created an environment where students' exchange of ideas was prioritized to establish the concept. Figure 3 details the collective group activities students shared ideas on to deduce their own definition of the trigonometric ratios of sine, cosine, and tangent of an angle.

From Figure 4, it could be envisaged that, the design of promotive face-to-face interaction creates an enabling environment where prior knowledge of individual students is brought to bear on a concept, shared, and modified in the learning environment through the exchange of respective individual ideas on the subject matter. As a result, its purpose fits the realities in the SHS mathematics classroom context.

5.4. Social Skills. The social skill feature of the CTL lesson was designed to provide a conducive atmosphere during the group activities of the students in the classroom context. Thus, an atmosphere where the learning of mathematics was not intimidatory, students' expressions were devoid of mocking and individual respective views were respected by the group members. Especially, during the intra- and inter-group discussion stages in the lesson. Moreover, it sought to foster discussions on the guideline prerequisite for a successful discussion/activity in each group. This is aimed at achieving and implementing good classroom management practices for learning to thrive. For instance, on the topic: "angles of elevation and

Lesson plan		
	Teacher activity	Student activity
Main lesson Activity 1 Definition of sine, tangent, and cosine of an acute angle in a right-angle triangle (20 min)	In order for students to acquire their own practical understanding of the concept of sine, tangent, and cosine of an angle in a right-angle triangle, the teacher (1) students piece together the most appropriate solution to the problem as a solution reflective of the groups individual effort. (i) presents an activity of the processes leading to the exploration of the comparison of the ratios of length in a right-angle triangle and guides students to (ii) measure and record the lengths of adjacent a (cm) to the angle (iii) measure and record the lengths of hypotenuse b (cm) to the angle (iv) measure and record the lengths of opposite c (cm) to the angle of measurement and finally, (v) with the help of the calculator find the ratios of the measured lengths as indicated in the respective tables in the activity sheet 1. Promotive face to face interaction	(1) Students explore the activities as guided by the teacher in groups and respond to the question on the activity sheet (under activity 1) based on their observations and newly constructed knowledge. Some discussion points expected from students: Sin$A$ (length of opposite side to angle A)/(length of hypotenuse side to angle A) Cos$A$ (length of adjacent side to angle A)/(length of hypotenuse length to angle A) Tan$A$ (length of opposite side to angle A)/(length of adjacent side to angle A) $f(x) = \text{Sin}x$ = opposite/hypotenuse $f(x) = \text{Cos}x$ = adjacent/hypotenuse $f(x) = \text{Tan}x$ = opposite/adjacent

Figure 3: Aspect of the plan on the trigonometric ratios of sine, cosine, and tangent of angle showing promotive face-to-face interaction (Authors' Own Construction, 2023).

Lesson plan		
	Teacher activity	Student activity
Main lesson Activity 1 angles of elevation and depression (20min).	In order for students to apply their own understanding of activity 1 to find solution to practical questions in trigonometry, the teacher (1) guides students to establish rules needed for a small group interaction (2) fosters whole group discussions on the construction of the list of items as a common standard practice expected of each member in the groups. Social skills	(1) Students reminiscent of think pair share model codevelop guidelines for group discussions to ensure interactions within each group (2) Students assist in harmonizing the guidelines collectively developed in class.

Figure 4: Aspect of lesson plan showing social skills on the angles of elevation and depression (Authors' Own Construction, 2023).

depression." A social skills feature of the CTL was designed in the lesson to establish the ground rules the students could associate with so as to have a fruitful deliberation in the group. Thus, students reminiscent of think pair share model could codevelop guidelines with the view to: (1) respecting the views of all group members, whoever gets the solution correct owes the rest of the group members an explanation, (2) cooperating with one another on all the task assigned to the group, and (3) arriving at solutions which is reflective of the collective effort of the individual group members. Figure 4 shows an aspect of lesson plan showing social skills on the *angles of elevation and depression*.

Thus, based on the findings of the current study, the use of social skill positions the teacher to create an environment where various behavioral tendencies that disturb the conduct of group activities are put to check. In this way, good classroom management practices are adhered to and observed during the lessons. As a result, the social skills feature of the CTL was found to be appropriate for the teaching of mathematics in the Ghanian SHS classroom.

5.5. Group Processing. In order to promote an evaluation in respect of the students own understanding of the subject matter, the group processing, as a feature of CTL was

Lesson plan		
	Teacher activity	Student activity
Conclusion stage (15 min)	In order to provide clarity and correct errors with the aim of helping students to grasp concept taught, the teacher (1) calls to the board, the group's recorder to present solutions on behalf of the group (2) fosters whole group discussions on the various presentations on the solutions to the problem (3) solicits alternate methods/inputs of finding solution to the questions. Finally, sums up the whole lesson by highlighting the key concepts discussed as follows: Group processing	(1) Group leaders lead discussions on the solutions to the problems after the group's recorder presentation of the solution on the board (2) Students provide alternate solutions to the solutions presented on the board.

Figure 5: Aspect of lesson plan showing group processing on the angles of elevation and depression (Authors' Own Construction, 2023).

conceptualized in the current study as an activity that enables students in the groups (after piecing their individual solutions to a task together) project their solution (after reaching consensus on the methods used in getting that solution) on the board. This is aimed at generating alternate solutions and procedures from the other group members in the classroom. In addition, this feature was designed in the CTL lesson to afford teachers the opportunity to provide clarity to students' questions. Again, the teachers summarize the main highlights (core points) in the lesson to the understanding of students in class. In this way, students have unfettered access to a variety of techniques in solving a particular question. Further, the teacher recognizes and capitalizes on the identified weaknesses of the students to offer responses that are problem specific and appropriate to the needs of the students in the classroom. The group's presentation on the board enables the students to express their views and ideas and ultimately help in conscientising the teacher as to how the groups understood the subject matter in all the activities that were undertaken by the students in the various groups. For instance, in designing CTL-L2 on the topic: "angles of elevation and depression." Group processing as an activity in the teaching and learning process was designed as a means of evaluating the concept. As such, it positions the groups to project their solution on the board for in-class discussions after which the teacher sums up the lesson by providing clarity to any misconceptions (if any) from the students. Figure 5 shows an aspect of lesson plan showing the feature on group processing for the topic on: *angles of elevation and depression*.

The study suggests that, the design of group processing in the lesson affords students access to a variety of related solutions to the mathematical problem. This is as a result of the multiplicity of presentations on a particular mathematical problem offered by the student groups during the teaching and learning processes. As a such, group processing feature of the CTL was found apt for the teaching of mathematics in the Ghanian SHS classroom.

Further, the review of the literature showed that an activity worksheet could be employed as a medium to project the five features of the CTL. This is because it provides the medium to effectively engage learners in the teaching and learning process. The activity sheets also provide the opportunity for students to become active participants in the learning environment. In this way, practical problems are collectively solved as a group as well as the sharing of ideas and experiences during group activities. The involvement of activity-oriented sheet in the design the of lessons positions teachers to plan lessons in such a way that he/she does not dominate during the teaching and learning process. It is characterized by sections: introductory activity section (instructions) and main activities section (see aspects of CTL-L1 and CTL-L2). The introductory activity section was designed in the form of preamble to introduce students to the main activities. This was purposed to prepare the students in the processes involved in carrying out the activities on the worksheet. Also, the main activities for both lessons (see aspects of CTL-L1 and CTL-L2) were designed based on the features of the CTL. In addition, they were designed to align with the instructional learning objectives in the SHS mathematics curriculum. The main activities were designed to be self-explanatory, in that the teacher needs not make direct input but serve as a guide during the teaching and learning process. The essence of which was to put the students at the center of the learning in order to construct their own understanding of the subject matter. The activities in the lessons were purposefully designed to promote the *positive interdependence*, *social skills*, *promotive face-to-face interaction*, *individual accountability*, *and group processing* features of CTL. As employed (see Figures 1–5), the features of CTL were designed in the form of activities to: engage students to share ideas on a subject matter (*promotive face-to-face interaction*); ensure students learn together but performs independently (*individual group accountability*); put students into groups of specifically assigned responsibilities on a learning task during group activities (*positive interdependence*); enable students brainstorm on the do's and don'ts (guidelines)

Core mathematics (SHS 2)
The angles of elevation and depression

Student's activity worksheet sheet

Activity 1: Determining the appropriate trigonometric ratio in a right-angle triangle.
Instruction

In groups of three, you are presented with following diagrams to enable you share ideas by cooperating with your group members for the purposes of presenting your group's solution to the class, the appropriate trigonometric ratio needed in finding the missing parameters in the angles below:

Positive interdependence
Promotive face to face interaction
Group processing
Individual accountability
Social skills

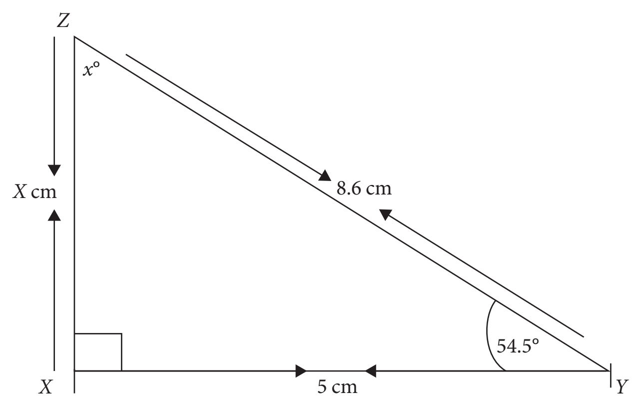

Figure 6: Illustration of the five color coded features as projected in activity 1 of CTL-L2 (Authors' Own Construction, 2023).

prerequisite for successful group discussions/activities (*social skill*); and promote evaluation in respect of the subject matter taught in the classroom (*group processing*). Hence, it found expression in the study since it embodied and projected the features of the CTL as a pedagogical tool in the classroom context. An example of the projection of these features of CTLA-L2 as detailed in Figure 6 is shown using activity 1 for illustrations.

Thus, the activity sheet was intended to enhance the student-centered approach to learning. It aids students in the conceptualization of the subject matter through the affordances of the features of the CTL. As a result of which it motivates learners to be preoccupied in CTL lessons.

Finally, the student-centered approach to learning was the foundation on which all classroom preparations imagined for encouraging differing information acquisition and expression with the CTL-L1 and CTL-L2 revolved. Thus, creating an atmosphere where students share ideas, interact with the readily available learning materials, rely on one another in finding solutions to the task, respect the contrary opinions of their colleagues in the group, and most importantly see their peers as a resource in the classroom rather than a competitor. Also, by accepting the student-centered strategy as pedagogical approach to the acquisition of knowledge, the features of the CTL were put into action to encourage students to be active in the teaching and learning process with the view to constructing their own understanding of the concept under instruction. In reference to the features as discussed, it is the considered opinion of the current study that, CTL-L1 and CTL-L2 as designed by the researchers fit the realities in the classroom context at the SHS level in a manner that makes the teaching of trigonometry meaningful.

6. Discussions and Limitations of the Study

The main research question was to determine the features of CTL lesson that best suit the classroom context at the SHS level. Informed by the literature reviewed [15, 17] (Roseth et al., 2008), the two lessons were designed. The CTL-L1 and CTL-L2 designed had the following: (1) student-centered approach and (2) an activity-based worksheet as compliments that projected the features of CTL-based lessons.

The CTL lessons provoke students understanding of the subject through the provision of hands-on and practical-based activities in the classroom [2, 53, 12].

Guided by the review of the literature, Prawatt and Floden [54], Johnson and Johnson [28], Rittle-Johnson et al. [50], and Alfieri et al. [46] the five features of CTL as defined in CTL lessons (see Figures 1–6) were introduced in the design of the lessons for the solitary determination of putting into practice how meaningful teaching with CTL could be achieved. This pedagogy positions teachers to be versatile in their use of the features of CTL. The findings are in line with a study conducted by Darling-Hammond [38] who proposed that such pedagogical approach attempts to bridge the gap between theory and practice through the design of reflective work. Also, motivated by the teacher's knowledge of the features of CTL to design and enact successful learning outcomes with CTL lessons further affirms Johnson and Johnson's [17] proposition that, the features of CTL are the fulcrum around which the design of the lessons revolves. With reference to these features, the CTL-L1 and CTL-L2 lessons appear to position the teaching of SHS trigonometry with a student-centered approach to learning that were activity oriented. The specific features of the CTL-L1 and CTL-L2 are discussed in the following paragraphs:

Positive interdependence as a feature in the lesson aligned with the views expressed as "group members sink or swim together" in respect of its performance [19]. Moreover, "We need contribution from each of my team members if we are doing to succeed". These activities inexplicably set the tone to demonstrate the power of positive interdependence which was a prerequisite for successful design of CTL activities throughout the lessons. As a result, it was grounded in literature and considered as apt in the design and development of CTL lessons.

The inclusion of the individual accountability feature in the CLT-lessons was essential for guaranteeing that the teaching and learning process occurs in an environment that is highly motivated by effective participation of the group members at individual and collective levels during group activities. Further, it provides result-oriented discussions to aid the conceptualization of the concept under instruction. This is reflective of the studies conducted by Johnson et al. [55]. Thus, the individual accountability seems to suggest an active individual participation and mastery of the concept of the subject matter in the mathematics learning environment.

The promotive face-to-face interaction feature in the CTL lesson was critical in ensuring that, the methodology of teaching occur in an environment that accommodates exchanges in ideas, reflections, and learners' individual experiences with the aid of the teaching and learning materials, teacher learner activities, and the student worksheet of activities. This aligns with the model of CL in the studies of Slavin [6]. Hence, the promotive face-to-face interaction feature of the exemplary material seems to suggest inclusivity and participation in the mathematics classroom. This design agrees with studies of Fritz et al. [41] who averred that, the absence of student-centered and activity-based teaching approach causes students and teachers claim that mathematics is difficult.

The social skill was a necessity in ensuring a whole class participation in the activities of the classroom. As such, learners' involvement in the task and respect for the views expressed by group members via the inter- and intra-group interactions in the classroom learning environment, served as the anchor on which this feature revolves in the teaching processes. This however, contrasted with the study view of Johnson et al. [56] and Slavin [6] who ascribed social skill to the characteristics of the development of leadership trait of the students. It was however, synonymous with the studies of Opitz [57]. Thus, the use of this feature in the CTL-based pedagogical lessons to brainstorm on the do's and don'ts (guidelines) prerequisite for successful group discussions/activities in the classroom. As a consequence, this feature ultimately provided a congenial atmosphere for CTL lessons to thrive in the mathematics learning environment which was crucial in this study. Further, the activities under this feature are synonymous to the studies of Shoaib and Ullah [4] who found classroom as a key academic and learning site in which the nature of interaction between teachers and students and among peers has a strong impact on students' learning skills.

The feature of group processing informs the instructor's methodology as well as the group learning strategies in finding alternative understandings of mathematics problems. This is because, it brings clarity to the concept. Further, it affords teachers the opportunity to attend to the weak students in the class through the feedback received from the students. This feature although, anew to the traditional mode of instruction seems to agree with the studies of Dalinger et al. [39] who argue for teachers to position themselves to learn these essential pedagogical skills aimed at improving students' learning of the subject matter. Hence, its employment in the CTL lessons as the last effort to bridge the gap of understanding between the weak and good students in the class.

The inclusion of the activity-based sheet served as the medium to highlight and project the five features of the CTL lessons. Thus, it gives practical meaning to the features of the CTL during instructions. To this end, it reduces boredom and actively engages students in the construction of their own understanding of the concept. This is in line with "Students must be active during learning" assertions of Agyei [15]. Thus, providing a sharp deviation from the traditional method of teaching where teachers are unlikely to select and interpret instructional practices that support motivation and learning [58].

The current study was limited in scope, although extensive review of relevant literature was conducted. In view of the limitations of the search strategy exclusively to studies published in English to the detriment of other languages, may have denied us access to relevant published works that could have been of tremendous value to the study. Moreover, the review may have omitted relevant publications before or after this period. To this end, employing only electronic databases limited the reviewed scientific evidence. This

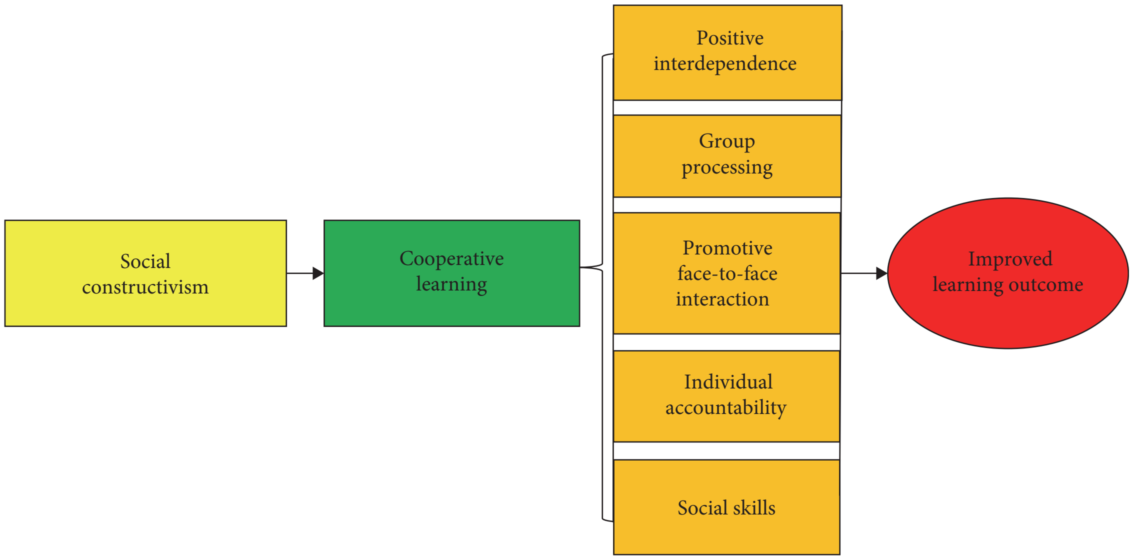

Figure 7: Proposed features of CL for teaching trigonometry [2].

notwithstanding, the study is current with recent publications in the area of research. Additionally, in order to reduce the likelihood of losing vital information which was beyond the control of the researchers, efforts were made to conduct the current study with strict adherence to the review of literature using the guidelines on systematic review processes. Thus, reports which were deemed technical and proceeding emanating from the conferences were examined for information relevant to the current study as advanced by Kitchenham and Charters [52]. In a nutshell, it is critical to note that, as stipulated in the data and methods section, the data on the student-centered and meaningful-based teaching approach and in particular, the CTL correspond to the descriptions in each of the articles, which in most cases, do not focus solely on those aspects.

7. Conclusions and Implications of the Study

In conclusion, grounded in the literature, the features of the CTL lessons as discussed above seemed to project student-centered and meaningful-based teaching of trigonometry. The features of the CTL-based lessons are pivotal in motivating teachers to get a better insight of the CTL as a pedagogical strategy in the classroom context. This could equip teachers to develop competencies in the integration of CTL in their methodology of teaching in a meaningful and effective manner. However, it is significant to note that, even though these features (as discussed above) could be effective in promoting cooperative-based teaching of SHS mathematics especially in the area of trigonometry in the Ghanaian context, this may not necessarily be the case in other countries as a result of differences in the contextual settings of the study. That notwithstanding, it is worthy to note that in meeting the specificities as would be expected in the other contextual settings, the features of CTL could be adapted. Based on our findings, *we hypothesize that an improvement in learning outcomes would be predicated on lessons which teachers design based on the features of CTL in a manner that allows students to make meaning of the materials independently and collectively as a group*. Consequently, we propose a framework (Figure 7) contingent on the five main features of CTL as a means of achieving a meaningful and effective pedagogy to improve the teaching of trigonometry. Figure 7 depicts proposed features of CL for teaching trigonometry.

The proposed framework highlights the social constructivism as the study's overarching theory of which CTL as a pedagogical strategy could be employed to improve the teaching and learning of trigonometry.

The study unveiled important implications in relation to the discussions on the features of the CTL. The findings imply that, to achieve a CL-based environment, the features of CTL must be conceptualized to meet the intended goals of a particular lesson. Further, the study suggests that in order to reduce the abstract nature of the trigonometrical concepts, lessons under the concepts of trigonometry should be designed to reflect on the features of CTL with worksheet of activities as a complement. To this end, the study proposes guidelines for the teacher's mastery of CTL techniques with the focus on the design of lessons and the training of the class to work within this technique. Thus, a CTL lesson comprises the use of:

(1) Features of CTL (positive interdependence, individual accountability, promotive face-to-face interaction, social skills, and group processing) [17, 28].

(2) Framework(s) to: (1) ground a particular type of knowledge required for the incorporation of CTL in a way that makes the student an active participant in the teaching and learning process 19, 28, 46, 50].

(3) Activity-oriented worksheets that employ the affordances of the features of CTL to facilitate the learner's conceptual understanding of the subject matter [15].

(4) Student-centered approach to learning that promote classroom learning environment practices aimed at improving students' conceptual understanding of the subject matter [19].

Furthermore, in detailing the practical implementation of the CTL approach in the classroom context, it is suggested that, to solve a particular trigonometric problem, the teacher puts students in groups with assigned roles in the trigonometric problem (positive interdependence). He/she encourages individual participation of the students (individual accountability) in the quest to find a solution to the problem by allowing students to agree to disagree, conjecture, and negotiate their own solution to the problem individually and collectively as a group (promotive face-to-face interaction). In order for a congenial atmosphere to thrive for inter- and intra-group deliberations in the learning environment, the teacher fosters whole class discussions on the do's and don'ts (guidelines) prerequisite for the successful conduct of the lessons (social skills). Finally, an opportunity is afforded the groups to present their work to the class with the view to soliciting alternate methods of solving the problem (group processing). As a result of which the teacher provides feedback to the students in relation to the weaknesses identified in solving the problem.

In a nutshell, we recommend that Colleges of Education in Ghana and other professional training institutions consider using the CTL lessons as exemplary lessons in a future initiative for their trainees. Such an initiative will afford pre-service and in-service teachers with reliable samples on the effective deployment of CTL as a pedagogical strategy in the classroom context.

Data Availability

The research data used to support the findings of this study may be released upon application to the (Institutional Review Board, Ghana—Ethical Clearance Form ID-UCCIRB/CES/2021/75), who can be contacted at (University of Cape Coast, Ghana).

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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