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Research Article

The Effectiveness of Infusion Learning Model in Linear Algebra Course

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This study aims to determine the effectiveness of applying the learning model in linear algebra at a university in the city of Jombang, Indonesia, which is indicated by looking at student learning outcomes, student learning activities, and the ability of lecturers to manage to learn. The quasi-experimental research method was carried out for 3 months involving two classes of students consisting of an experimental class (infusion learning model) and a control class (conventional learning). Data were collected through test sheets (pretest and posttest), student activity observation sheets, and lecturers' ability observation sheets in managing to learn. Data were analyzed using two techniques, namely, inferential statistical analysis and descriptive statistical analysis. Based on inferential statistical analysis, it shows differences in students' argumentation abilities between the control and experimental groups. In addition, based on the results of the descriptive analysis, student learning outcomes in the infusion learning model obtained more than the minimum standard value, students were active in learning activities, and lecturers' abilities were good and very good in managing to learn. Thus, the infusion learning model effectively learns linear algebra with vector subspace topics. These findings indicate that students are enthusiastic about solving problems, building arguments not in dialog and arguments in dialog, and actively discussing with other students in class. We suggest that lecturers apply infusion learning to other math topics so that students can be enthusiastic about learning mathematics in class. Alternatively, lecturers can use the infusion learning model with technology-assisted learning to make learning more interesting for students.

1. Introduction

One of the mathematics materials students consider linear algebra is difficult to learn [1]. In linear algebra, students study vectors and linear transformations [2]. The causes of students having difficulty in learning linear algebra are abstract concepts, the area of application is unusual for students, most students cannot prove, the basic concepts of linear algebra are not displayed geometrically, memorizing concepts without really understanding them, and the inability of students to interpret symbols [3]. In addition, students also experience difficulties in implementing the definitions and concepts they have learned to solve problems [4].

When students solve problems, they need arguments [5–8]. These arguments are used to produce, define, and support a plausible solution. However, some students do not use deductive arguments when delivering arguments [9–12]. In developing argumentation skills, students in solving evidentiary problems must change nondeductive arguments into deductive arguments [13]. Therefore, the learning objectives must gradually improve students' argumentation skills to produce formal evidence [14]. Therefore, research is needed to solve these problems in an enjoyable linear algebra learning process to improve students' argumentation skills.

One of the exciting learning models is infusion learning. Infusion learning is a learning model that aims to assist students in developing argumentation skills [15]. In this model, argumentation skills are taught and identified along with learning content as implemented by Davies [16]. In addition to applying the infusion learning model, Tristanti

Table 1: Differences between a traditional class and an infusion learning class.

Traditional class	Infusion learning class
The lecturer gives students proof questions and asks them to solve the problem individually or in groups and then present it	The lecturer gives proof questions to students and asks students to solve the problem individually in small groups and then present it
Students present their work	Students present their work and must convince the audience and be able to argue the truth of their work
Students solve the proof problems in an inductive way	Students solve proof problems in inductive and deductive ways

and Nusantara [17] have applied the CIRC-typed cooperative learning model and problem-based learning to describe its impact and advantages in students' mathematical arguments.

This infusion learning model refers to the infusion learning approach first introduced by Swartz [18]. The infusion learning approach aims to teach specific critical thinking skills along with different subjects and inculcate necessary thinking skills by teaching defined learning materials. Infusion learning is recommended in the Indonesian curriculum, which is superior to conventional learning models in improving students' argumentation skills and involving students in learning [19].

In recent years, there have been many studies related to infusion learning, including Apache and Rizzo [20] that evaluated the effect of the infusion learning model on the attitudes of students with disabilities in physical education. The results showed a significant positive change in students' attitudes toward disabilities and increased their confidence in learning. In addition, the research of Tristanti and Nusantara [15] described the effect of infusion learning on improving students' argumentation skills. However, the results of this study show that the impact of infusion learning is only based on students' argumentation skills. In contrast, the sign of learning effectiveness is the active involvement of students [21]. In addition to learning outcomes, learning effectiveness is also determined by the level of teaching, time, and appropriate incentives [22]. Based on the abovementioned description, this study aims to test the effectiveness of the infusion learning model in linear algebra learning from three aspects, namely, argumentation skills, student learning activities, and lecturers' ability in managing to learn.

2. Review of Literature

The infusion learning model is a case-based learning process that presents proof problems so that students are expected to express mathematical arguments in solving and finding solutions to these cases and be able to convince others [19]. The infusion learning model aims to assist students in developing their mathematical argumentation competence. The infusion learning model was developed based on the theory of Swartz [18], Walton [23], and Toulmin [24].

In conventional learning, students are preoccupied with theories, examples, and exercises [25], but the emphasis is not on how students argue. It contrasts with the infusion learning model, where students are allowed to develop argument skills in critical discussions and new situations. The difference between the traditional class and the infusion learning class is described in Table 1.

The following is the syntax of the infusion learning model [19]:

(1) Introduction
   (a) Lecturers prepare students to study. Activities at this stage include conveying learning objectives, teaching materials, and motivation or importance of learning teaching materials.

(2) For the presentation of teaching materials
   (a) Lecturers present teaching materials packaged in student worksheets (LKM). Then the lecturer conveys the components of mathematical arguments, including data, claims, warrants, backing, counterexamples, and qualifiers. The lecturer provides examples of applying mathematical argument components in solving proof problems.

(3) Reasoning
   (a) The lecturer presents a proof problem, asks the students to investigate the truth of a statement, and then is asked to think actively in generating ideas and applying them to solve proof problems accompanied by reasons.

(4) Argument not in dialog
   (a) Students are asked to show and ensure the correct view through an argument addressed to themselves. Students try to convince themselves so that there is something to approach and argue with themselves.

(5) Conveying arguments in small dialogs
   (a) Students are formed into small groups consisting of three students. The formation of small group members is based on heterogeneous ideas used in solving proof problems. Next, students have a critical discussion, where each group member tries to show the correct view by way of an argument addressed to other members. The purpose of the argument in the dialog is to speak that other members can understand and accept, so that other members believe and believe.
   (b) In this phase, the lecturer also provides guidance, guiding the process of solving evidentiary problems in the LKM individually or in groups. This activity aims to assist students who have difficulty solving proof problems correctly and independently.

(6) Presenting arguments in class dialog
   (a) Students present their arguments in class and other students respond to these arguments.

The purpose of the argument in class dialog is to speak, which can be understood and accepted by other students so that other students can believe.

(7) Assessing student arguments
   (a) The lecturer checks the comprehension and provides feedback as an evaluation. Gagne states that feedback (feedback phase) is a phase in learning [26].

(8) In conclusion
   (a) The lecturer concludes and confirms the teaching material studied as a form of strengthening student knowledge.

3. Methods

This research is quasi-experimental research using a pretest–posttest control group design. The experimental class applies the infusion learning model, while the control class applies conventional learning. The experimental and control classes were divided into small groups of students with low, medium, and high math abilities (having heterogeneity). The mathematical ability is based on the grade point average.

The participants of this study were divided into two classes selected using random cluster sampling from three classes in the same class from a university in Jombang, Indonesia. The participants were divided into experimental and control classes [27]. The control class consisted of 26 students, while the experimental group consisted of 28 students. All participants were in the second semester and aged between 18 and 19 years.

The research instrument consisted of students' learning activity, observation sheets, lecturers' ability observation sheets in managing learning, and pretest and posttest. The observation sheet was adapted from the research of Tristanti and Nusantara [19]. Meanwhile, the pretest and posttest were developed by the researcher.

Data on the ability of lecturers to manage learning were analyzed using the average score of the lecturer's ability level adopted from Tristanti and Hidayati [28], as shown in Table 2.

Table 2: Categorization of lecturer ability to manage learning.

Lecturer ability level (LAL)	Criteria
LAL = 4.00	Very good
3.00 ≤ LAL < 4.00	Good
2.00 ≤ LAL < 3.00	Medium
1.00 ≤ LAL < 2.00	Not good enough
0.00 ≤ LAL < 1.00	Not good

The ability of lecturers to manage learning is said to be good if the average score of each aspect assessed for each meeting is in the good or very good category.

Student learning activity data were analyzed using percentages. The percentage of student activity is determined based on the time allocation planned in the lesson plan. The suitability of the ideal activity determines the effectiveness of student learning activities, which is indicated by the ideal time of 5% with a tolerance of 0.1%.

The data on argumentation ability were collected by giving pretest and posttest. The pretest was given before learning, while the posttest was given after learning in the experimental and control classes. The research used a pretest and posttests consisting similar two essay questions. Each item was tested for validity and reliability in different classes from the control and experimental classes. Each question were assessed using a score of 0–50. The score details convey the data: 0–5; apply warrants: 0–30; using backing: 0–10; conclusion: 0–5. Score. Therefore, each subject's total pretest and posttest scores ranged from 0 to 100. The following is a description of the score description:

(1) Convey the data
   (a) If do not convey the data, then given a score of 0
   (b) If convey the incorrect data, then given a score of 2
   (c) If convey the incomplete but correct data, then given a score of 3
   (d) If convey the complete and correct data, then given a score of 5

(2) Apply warrant
   (a) If do not apply a warrant, then given a score of 0
   (b) If apply a warrant but it is wrong, then given a score of 2
   (c) If apply a warrant that is not complete but correct, then given a score of 15
   (d) If apply a warrant that is complete and correct, then given a score of 30

(3) Using backing
   (a) If do not using backing, then given a score of 0
   (b) If using backing but wrong, then given a score of 2
   (c) If using backing is not complete but correct, then given a score of 5
   (d) If using backing is complete and correct, then given a score of 10

(4) Convey conclusion
   (a) If do not convey a conclusion, then given a score of 0
   (b) If the conclusion is wrong, then given a score of 2
   (c) If the conclusion is not complete but correct, then given a score of 3
   (d) If the conclusion is complete and correct, then given a score of 5

The pretest and posttest validity tests used product–moment correlation, while the reliability test used alpha coefficient [29]. Pretest and posttest are considered valid if the R_{xy} value is >0.40. At the same time, the pretest and posttest meet the reliability criteria if the r_{11} value is 0.71–0.85 [30].

Data were analyzed using two techniques, namely, inferential and descriptive statistics. Inferential statistical data analysis was used to see the difference in the argumentation

Pretest	Posttest
Define $W = \{(a, 0) \mid a \in \mathbb{R}\}$. Check if W is a subspace of $\mathbb{R}^2$!	Define $W = \{(a, b, c) \in \mathbb{R}^3 \mid b = a + c\}$. Check if W is a subspace of $\mathbb{R}^3$!

Figure 1: Test instruments.

Table 3: Reliability and validity of pretest instruments.

	Correlations	Sig. (2-tailed)	Reliability
Problem_1	0.705	0.000	0.645
Problem_2	0.554	0.003	

Table 4: Reliability and validity of posttest instruments.

	Correlations	Sig. (2-tailed)	Reliability
Problem_1	0.727	0.000	0.652
Problem_2	0.517	0.007	

Table 5: Normality test of experimental class and control class data using Kolmogorov–Smirnov.

Class	Data	Kolmogorov–Smirnov[a]			Shapiro–Wilk		
		Statistic	*df*	Sig.	Statistic	*df*	Sig.
Control	Pretest_1	0.131	26	0.200*	0.940	26	0.133
	Posttest_2	0.164	26	0.070*	0.936	26	0.107
Experiment	Pretest_1	0.139	28	0.180*	0.960	28	0.356
	Posttest_2	0.160	28	0.065*	0.936	28	0.087

*This is a lower bound of the true significance. [a]Lilliefors significance correction.

ability of students who took lessons on infusion learning models and conventional learning with linear algebra material. Before the pretest–posttest data were analyzed using inferential statistics, the first data were tested for normality and homogeneity.

Descriptive statistical analysis was used to test the effectiveness of the infusion learning model on linear algebra material. Student activities are effective if the percentage of each aspect observed at each meeting is in the ideal timeframe. The ability of lecturers to manage learning is said to be effective if the average score of each aspect assessed for each meeting is in the good or very good category. Data on students' argumentation skills were analyzed descriptively to describe students' mathematical argumentation skills based on tests.

4. Results and Discussion

The research instrument was pretest and posttest for the experimental and control groups. The test is designed concerning the student's argumentative ability. This test contains proving problems. An example of the problem used in this study is shown in Figure 1.

Before the pretest and posttest on, the control and experimental groups must be distributed to specific classes for validation and reliability tests. The test is considered valid if the R_{xy} value >0.40 and the test is said to be reliable if $r_{11} > 0.60$. The results of the validation test through SPSS are presented in Tables 3 and 4.

Table 3 shows that the correlation values of problems 1 and 2 are 0.705 and 0.554, respectively. It shows that both pretest questions are valid because the R_{xy} value is >0.40. Cronbach's α value r_{11} is 0.645, indicating that the pretest is reliable because the r_{11} value is >0.60. Meanwhile, Table 4 shows that the correlation values for problems 1 and 2 are 0.727 and 0.517, respectively. It indicates that both posttest questions are valid because the R_{xy} value is >0.40. Cronbach's α value r_{11} is 0.652, indicating that the posttest is reliable because $r_{11} > 0.60$.

This study found the results of inferential statistical data analysis and descriptive data analysis obtained during the research process.

4.1. Inferential Statistical Data Analysis. Before carrying out statistical tests, normality and homogeneity tests were first carried out. The normality test is to determine whether the sample data taken are normally distributed. Normality is essential for inferential statistics, which aims to generalize the results of sample data analysis.

Based on Table 5, the Kolmogorov–Smirnov test shows that the control class's Sig. values of the pretest and posttest data are 0.200 and 0.070 > 0.05, respectively. Thus, Ho is accepted, meaning that the pretest and posttest data for the control class came from a normally distributed population. Meanwhile, the pretest and posttest data in the experimental

Table 6: Test of homogeneity of variances.

Levene statistic	*df*1	*df*2	Sig.
0.105	1	52	0.747

Table 7: Group statistics.

	Kelas	*N*	Mean	Std. deviation	Std. error mean
Score	Control class	26	62.31	6.361	1.247
	Experiment class	28	89.64	6.372	1.204

Table 8: Independent samples test.

t	*df*	Sig. (2-tailed)	Mean difference	Std. error difference	95% confidence interval of the difference	
					Lower	Upper
−15.764	52	0.000	−27.335	1.734	−30.815	−23.856

Table 9: Comparison of student learning outcomes in the experimental class and the control class.

Information	Experiment class	Control class
Average learning outcomes	89.64	62.31
Percentage of students who finished studying	100%	42.3%

class were 0.180 and $0.065 > 0.05$, respectively, so the pretest and posttest data for the experimental class came from a normally distributed population.

Table 6 shows the significant value of Levene's test results of 0.747, which indicates that the data variance in the experimental and control classes is homogeneous. Because students' argumentation ability data are normally distributed and have homogeneous variants, an independent test can be performed to describe the effect of student's initial ability (x) on students' argumentation ability in each experimental and control group.

Table 7 shows the mean of each class; namely, in the control class, the value is 62.31, which is lower than the experimental class, which is 89.64. Meanwhile, Table 8 shows that the two-way (t-tailed) significance value is $0.000 < 0.05$. Therefore, there is a difference in mathematical argumentation ability between the control and experimental groups.

4.2. Descriptive Statistical Data Analysis Results. Analysis of the data obtained regarding the implementation of the infusion learning and conventional learning models are as follows.

4.3. Student Learning Outcomes. Table 9 shows the comparison of student learning outcomes in the experimental class and the control class.

Regarding the mastery of learning outcomes, 28 students in the experimental class passed the lesson (who scored more than the minimum standard >75). Thus, the percentage of students learning completeness is 100%. While in the control class, 15 of the 26 students failed the learning process; thus, it can be concluded that students in the control class did not pass the study.

4.4. Student Activity Observation Results. The results of observations on student activities in learning that apply the infusion learning model for three meetings are expressed in percentages. The conclusions of the observations are presented in Table 10.

Table 10 shows the time of students learning activities in learning (no. 1–16) >5% and the treatment of students who are not relevant to learning activities (no. 17) <0.1%. Therefore, student learning activities that apply infusion learning can be concluded to be effective. However, some students do irrelevant activities in conventional classrooms, such as studying other materials and playing games. Therefore, the control class does not meet the adequate category.

4.5. Observation Results of Lecturer Ability to Manage Learning. The results of observing the ability of lecturers to manage the infusion learning model can be seen in Table 11.

From Table 11, it can be seen that the average value of each aspect assessed in managing the infusion learning model learning from three meetings is in the good or very good category. This shows that educators who use the infusion learning model are effective in managing learning. Meanwhile, in conventional learning, the lecturer's ability to manage learning is good or good enough.

The achievement of the learning effectiveness of the infusion learning model is determined based on the argumentation ability of students, the ability of lecturers to manage learning, and student learning activities. Based on the learning effectiveness criteria, it can be concluded that the infusion learning model is effective for teaching linear algebra.

The difference in argumentation ability occurs in the control and experimental classes because the infusion

Table 10: Results of observation of student learning activities.

No.	Aspects observed	Activity percentage at meeting I	II	III	Average
1.	Reading teaching materials and learning indicators through student worksheets (LKM)	6.75	6.74	6.78	6.76
2.	Linking teaching materials with material that has been studied through LKM	6.50	6.60	6.90	6.67
3.	Work on an individual LKM	6.65	6.60	6.50	6.58
4.	Describing argument components (data, claim, warrant, backing, qualifier)	6.60	6.30	5.60	6.17
5.	Generating ideas that are used to solve problems	6.80	6.90	6.80	6.83
6.	Giving reasons for each idea used, combine, and insert ideas	5.50	5.70	5.60	5.60
7.	Describing the ideas used to solve the problem	6.50	6.30	6.60	6.47
8.	Making schematic of mathematical argument	6.43	6.23	6.30	6.32
9.	Asking himself directly about his thoughts, reflecting on what thoughts were done, how to do them, and how effective the thinking was	6.23	6.31	6.28	6.27
10.	Presenting ideas and arguments that have been compiled	5.50	5.70	5.60	5.60
11.	Exchanging ideas in small groups	6.30	6.00	6.50	6.27
12.	Convincing group members that the idea is correct by using logical arguments	5.52	6.05	6.14	5.90
13.	Expressing the discussion results with logical arguments in front of the class	6.4	6.32	6.20	6.31
14.	Responding to or dealing with presentations made by friends	6.10	6.30	6.14	6.18
15.	Listening to the emphasis/reinforcement on the discussion results in the truth of the arguments presented	6.50	6.67	6.72	6.63
16.	Inferring or emphasizing the core material that has been studied	5.67	5.24	5.31	5.41
17.	Treatment that is not relevant to learning activities	0.05	0.04	0.03	0.04
	Total percentage of learning time	100	100	100	100

Table 11: Observation result in managing the infusion learning.

Rated aspects	Meeting 1	2	3	Average
Phase 1: Introduction				
Informing the topic to be studied and learning indicators through student worksheets (LKM)	3	4	4	3.67
Assigning students to link teaching materials with material that has been studied through LKM	4	4	4	4.00
Allowing students to ask questions that are not clear	3	3	4	3.33
Explain or provide direction to students about questions asked by students (things that are not understood)	3	4	4	3.67
Phase 2: Presentation of teaching materials				
Presenting information on teaching materials through LKM	4	4	4	4.00
Allowing students to do an introduction to initial knowledge at the LKM	3	3	4	3.33
Discussing the introduction to initial knowledge with students	4	4	4	4.00
Inviting students to ask questions about material that has not been understood	3	3	4	3.33
Giving direction to students about questions asked by students (things that have not been understood)	3	4	4	3.67
Phase 3: Reasoning				
Asking students to solve problems in the LKM by revealing the components of mathematical arguments, namely, data, claims, warrants, backing, and qualifiers and making mathematical argument schemes	3	3	4	3.33
Allowing students to ask questions related to problem solving, if something is not understood	4	4	4	4.00
Provide direction related to student questions by providing stimulation so that they can come up with ideas to solve problems	4	4	4	4.00

(*continued*)

Table 11: Continued.

Rated aspects	Meeting 1	Meeting 2	Meeting 3	Average
Phase 4: Arguments not in-dialog				
Asking students to directly ask themselves about their thoughts, what to do, how to do it, and how effective their thinking is	3	4	4	3.67
Asking students to reflect on the truth of their answers to convince themselves that there is something to approach and argue with themselves	3	3	4	3.33
Phase 5: Aarguments in small dialogue				
Forming small groups (2–3 students) based on student answers	4	4	4	4.00
Asking students to havea discussion	4	4	4	4.00
Asking students to submit ideas and arguments that have been compiled	3	3	4	3.33
Asking students to exchange ideas	3	4	4	3.67
Asking students to convince their group members that their ideas are correct with logical arguments	3	3	4	3.33
Asking students to give conclusions from the results of their group discussions	4	4	4	4.00
Phase 6: Arguments in large/class dialog				
Asking group representatives to present the results of the discussion with logical arguments	4	4	4	4.00
Asking students to respond to presentations made by their friends	3	3	4	3.33
Phase 7: Assessing student arguments				
Paying attention to and checking/correcting student arguments and student work	3	4	4	3.67
Providing emphasis/reinforcement on the discussion results in the truth of the arguments presented	3	4	4	3.67
Phase 8: Conclusion				
Asking students to conclude or emphasize the core of the material that has been studied	3	3	4	3.33
Delivering the material that will be studied at the next meeting	4	4	4	4.00

learning model is a means of practising argumentation skills. Argumentation skills are determined by the amount of practice facilitated in the infusion learning model. Therefore, the more practice, the more skilled at arguing. Osborne [31] stated that arguing is a long process that requires repeated practice and experience. In addition, the difference in argumentation ability occurs because the infusion learning model provides opportunities for students in learning activities with a strong understanding of basic, factual, and application knowledge, students demonstrate accurate and effective communication skills in writing and verbally, working independently and cooperatively in small groups [19].

Lecturer's ability to manage the infusion learning model affects the development of students' thinking processes and arguments. Conner et al. [32] stated that teacher actions could develop students' argumentation skills because teachers can encourage students to explain, write, and justify their reasons during class discussions. Student arguments depend on the culture in the classroom, the nature of the assignments, and the type of reasoning emphasized by the teacher [19]. Therefore, the purpose of infusion learning instruction is to gradually improve students' argumentation skills, which impacts students' proving abilities. It is in line with the expectation [14] that instructional learning should be clear and gradually improve the students' evidence scheme moving towards a formal (deductive) proof scheme.

Based on Toulmin's argument scheme, the answers of students in the experimental class to question number 2 have brought up all components of the argument. In addition, students can also understand the concept of subvector space. Students' answers have shown the correct line of thinking and arguments that contain data, claims, warrants, backing, and rebuttals. The data, warrants, and backings put forward have led to claims. The following (Figures 2 and 3) are the students' written and oral answers.

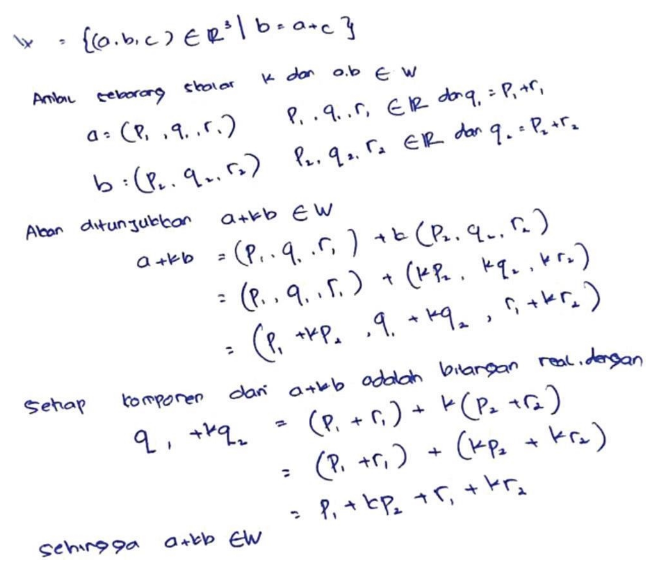

Figure 2: The students' written answers.

On the statement defined $W = \{(a, b, c) \in \mathbb{R}^3 \mid b = a + c\}$. Check if W is a subspace of $\mathbb{R}^3$ the data is $W = \{(a, b, c) \in \mathbb{R}^3 \mid b = a + c\}$. Based on these data, it can be seen that W is a subset of $\mathbb{R}^3$ and there is $(0,0,0) \in W$ so W is not an empty set.
Backing includes the conditions for the subvector space, namely W is a nonempty subset of the vector space V. The set W is a subspace V of if and only if for every scalar k and $a, b \in W$ apply $a + kb \in W$.
Meanwhile, warrants are steps to prove that W is a subspace of $\mathbb{R}^3$. Take any scalar k and $a, b \in W$, can be written

$a = (p_1, q_1, r_1)$, $p_1, q_1, r_1 \in \mathbb{R}$ and $q_1 = p_1 + r_1$
$b = (p_2, q_2, r_2)$, $p_2, q_2, r_2 \in \mathbb{R}$ and $q_2 = p_2 + r_2$

Will be shown $a + kb \in W$. Note that $+ kb = (p_1, q_1, r_1) + k(p_2, q_2, r_2) = (p_1, q_1, r_1) + (kp_2, kq_2, kr_2) = (p_1 + kp_2, q_1 + kq_2, r_1 + kr_2)$. Each component of $a + kb$ is a real number, with $q_1 + kq_2 = (p_1 + r_1) + k(p_2 + r_2) = (p_1 + r_1) + (kp_2 + kr_2) = p_1 + kp_2 + r_1 + kr_2$. So that $a + kb \in W$, W is a subspace of $\mathbb{R}^3$.
The qualifier is $W = \{(a, b, c) \in \mathbb{R}^3 \mid b = a + c\}$ W is a subspace of $\mathbb{R}^3$.
While the rebuttal is the definition of the element W added a number, for example, $W = \{(a, b, c) \in \mathbb{R}^3 \mid b = a + c + 5\}$, so W is not a subspace of $\mathbb{R}^3$, because $k = 2$ and $a = (1, 3, -1)$ obtained $ka = (2, 6, -2)$ In which $6 \neq 2 + (-2) + 5$.

Figure 3: The students' oral answers.

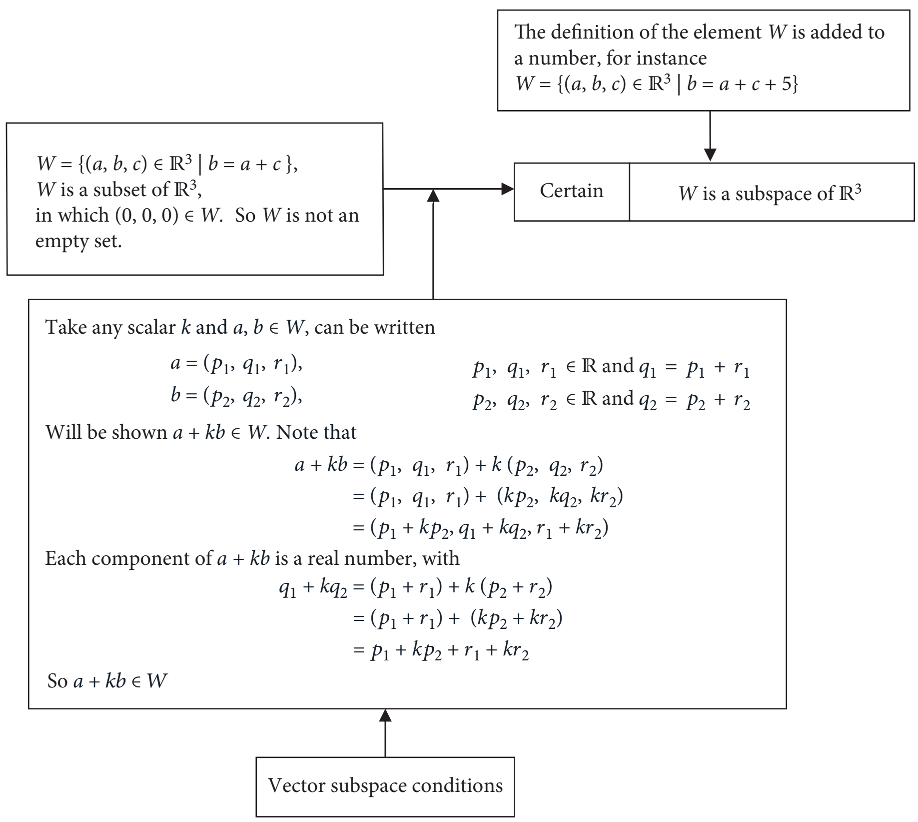

Figure 4: Student mathematical argumentation scheme.

The visualization that can be given for the student's answer is as Figure 4.

The student argument is formal because it uses deductive warrants [9, 33]. Students use formal mathematical truths to guarantee the conclusion of the argument [14] where students use chunks of axioms to establish the truth. For example, in operating $k(p_2, q_2, r_2) = kp_2, kq_2, kr_2$ where students use the rule of multiplication of scalars with vectors.

In addition to data, claims, backing, and warrants, students have conveyed the right qualifier, namely, a pair of deductive warrants with absolute qualifications. It means that students understand that nonformal arguments are used to reduce uncertainty while formal arguments are used to eliminate uncertainty. Students also think about a rebuttal, where an exception condition rebuttal the conclusion. In this case, the students convey no rebuttal in his argument. Therefore, the infusion learning model also emphasizes the use of qualifiers and rebuttals in argumentation. It answers the concerns of Inglis et al. [9] that qualifiers and rebuttals are not used in mathematics education; the role of qualifiers and rebuttals is underestimated, marginalized, or eliminated, even though they help to analyze the arguments presented by students.

5. Conclusions

Based on the study results, it can be concluded that the infusion learning model in linear algebra learning on subspace topics meets the effective criteria. Student learning outcomes are more significant than the standard values, active student activities, and the ability of lecturers to manage learning in good or very good criteria. In addition, the experimental class that applied the infusion model was more effective than the control class that used conventional learning. Thus, the infusion learning model showed higher learning outcomes than the conventional group.

Further research should be conducted to determine the importance of the infusion learning model and its relevance in mathematics. This research can be repeated on different topics, either in mathematics or other disciplines. Furthermore, experimental studies can be carried out to find the dynamics of the infusion learning model to compare arguments, not in dialog with arguments in dialog. Or the use of specific technologies to describe the effect of the process of applying the infusion learning model.

Data Availability

The data that support the findings of this study are available from the corresponding author upon reasonable request. Data (student learning outcomes) are used to support the findings of this study, as shown in Table 9. The data (results of the pretest and posttest instrument trials) are used to support the findings of this study, as shown in Table 5. Data (student learning activities) used to support the findings of this study are included in the article, as shown in Table 10. Data (the ability of lecturers in managing learning) used to support the findings of this study are included in the article, as shown in Table 11. The data (of one subject's argument) used to support the findings of this study are included in the article, as shown in Figure 2.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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