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Research Article

Comparative Analyses Concerning Triaxial Compressive Yield Criteria of Coal with the Presence of Pore Water

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The least absolute deviation is used as a metric to analyze the applicability of five yield criteria, to describe the yield characteristics of coal based on triaxial compressive strength tests on natural, water-saturated, and seepage coal samples with the presence of pore water. The results show that the strength of coal exhibits nonlinear characteristics with the increase of confining pressure, which the linear Coulomb criterion fails to authentically describe. Although the parabolic Mohr criterion can describe the nonlinearity feature more decently than the linear yield criterion, the fitting error is significant, and the uniaxial compressive strength of coal is overestimated. The Hoek-Brown criterion, quadratic polynomial criterion, and exponential criterion yield decent fitting quality for the coal rock. In particular, the exponential strength criterion can accurately reflect the actual uniaxial compressive strength of the rock. However, the differential principle yield stress for an infinite confining pressure calculated from the exponential strength criterion is lower than the measured value. Furthermore, by employing effective stress principle to analyze the yield criteria for the saturated and seepage coal samples, one can find that the quadratic polynomial criterion and the exponential criterion can also reflect the changes of yield characteristics during the fluid-solid coupling triaxial compression test.

1. Introduction

In the subsurface mining engineering, hydroelectric power generation, petroleum, and geological engineering applications [1, 2], how to calculate and predict the rock strength is of great significance to the researchers, e.g., engineering designers. The accuracy of the calculation here directly dictates the quality and safety of engineering construction, and then affects the safety of workers. However, the rock strength is closely related to the geological environment. The mechanical properties of the rock also differ in response to the change of rock stress and geological environment. The test of rock strength under various stress states has been a hot research topic for scholars. With the advancement of experimental techniques for testing rock mechanics, the corresponding rock strength evaluation criteria have also been subject to continuous corrections and improvements.

Since its advent in 1776, the Coulomb yield criterion has become one of the most widely used yield criteria in geotechnical engineering [3]. In 1900, Mohr extended the Coulomb shear strength criterion to account for the three-dimensional stress state, leading to the M-C criterion [4, 5]. According to the behaviors of fracturing and rock failure and by taking into consideration the triaxial testing data and the rock mass in situ measurement results, Hoek and Brown put forward in 1980 the Hoek-Brown empirical yield criterion [6], which, based on the experimental validation and practical improvements, has been widely accepted by the rock mechanics and

engineering community. In addition, many scholars have extended the yield criteria based on experimental measurement and theoretical analyses, giving rise to numerous new criteria, e.g., Drucker-Prager failure criterion [7], Griffith criterion and its associated Murrell three-dimensional extension [8], Mogi yield criterion [9], power function-based yield criterion [10–12], exponential yield criterion [13, 14], and unified strength theory [15].

With the continuous application and development of yield criterion in engineering practice, scholars have carried out experiments and analyses on the yield criteria under various stress paths based on different rock types, with a plethora of results published. Among them, Zhao [16] establishes a dynamic Mohr-Coulomb criterion and a dynamic Hoek-Brown criterion for a low loading rate through a series of experiments, e.g., uniaxial compression and triaxial compression. Based on the linear Mogi criterion, Al-Ajmi and Zimmerman [17, 18] established the Mogi-Coulomb criterion by analyzing the relationship between the fitting parameters and the antishearing strength parameters based on linear Mogi criterion. Costamagna and Bruhns [19] establish a nonlinear strength criterion with four complex parameters, which can be transformed into multiple variants based on different selections of parameters. You [20, 21] thoroughly analyze the stress-strain curves of defective rock samples, based on which the impact of confining pressure on the triaxial strength of rock samples is determined, and the yield criteria associated with various rock types are systematically analyzed. Although significant achievements have been made concerning the yield criteria, these criteria are all subject to certain practical limitations with a lack of universal applicability. Therefore, systematic studies are still required for developing yield criteria under different conditions.

Many engineering practices have indicated that the rock is not only affected by the stress field in the Earth's crust but also affected by the presence of water to certain degrees. In some cases, even high osmotic pressure exists in the rock, leading to a complicated water environment that affects the rock. In order to explore the influence of groundwater on rock strength, Ojo and Brook [22] summarize the relevant research results and conclude that the existence of water tends to weaken rock strength, with the increase of humidity compromising the compressive and tensile strengths of rock. Vásárhelyi and Ván [23, 24] test the uniaxial compressive strength and the modulus of elasticity of dry and saturated limestone and find that the strength and modulus of elasticity are reduced by about 34% after the saturation of rock with water. Baud et al. [25] systematically analyze the water's rock-weakening effect based on mechanical and chemical mechanisms and find that the water's effect for reducing triaxial compressive strength is attributed to a decrease in the surface energy and the internal friction coefficient of the rock. For the subsurface mining application, Peng et al. [26] study the mechanical properties of deep coal rock samples under five different confining pressure conditions using a triaxial compression test system. Medhurst and Brown [27] conduct a conventional triaxial compression test on a large coal sample using a rock servo tester. The results show that with an increase of confining pressure, the peak strength of the coal

sample increases and the expansion deformation decreases. Bell and Jermy [28] analyze the strength and deformation characteristics of coal samples under different stress conditions by performing compressive tensile and permeability tests on coal samples from the East Transvaal coal field. In addition, most of the scholars consider the impact of groundwater environment on the mechanical properties of rocks by conducting related experiments for various rock types. However, the measured results were primarily used to analyze the rock strength and deformation under various stress conditions based on the classical Coulomb yield criterion. In the case of underground mining engineering, due to the joint effects of in situ stress field and seepage field, the coal yield characteristics are more complicated, calling the effectiveness of the classical yield criteria in reflecting the actual rock strength under various stress states into question. Therefore, in-depth investigation in this regard is needed to ensure the reliable safety design of coal mines. To this end, the present paper obtains the conventional triaxial compressive strength of natural and saturated rock as well as the correlation between stress and strain with the presence of pore water seepage under various confining pressures by employing a MTS815 servo controlled rock mechanic testing system. Linear, parabolic and exponential yield criteria are used to characterize the strength of the rock sample subject to various stresses and water environments as well as the applicability of various yield criteria. The results obtained here can serve as a reference for the safety design concerning the operation of underground coal mines.

2. Triaxial Strength Test and Yield Criterion

2.1. Triaxial Strength Test of Coal. An MTS815 electrohydraulic servo rock test system was applied to three types of coal samples from the Yanzhou Coal Mining Co. Dongtan coal mine [29, 30]. Triaxial compression tests of natural and saturated coal and a triaxial seepage compression test of saturated coal were carried out. The MTS815 electrohydraulic servo rock test system is equipped with three independent servo systems to control the axial pressure, confining pressure, and pore pressure. Test applies a certain axial compression σ_1, confining pressure σ_3, pore pressure P_1, and always maintain $P_1 < \sigma_3$; osmotic pressure is produced at the two ends of the test pieces ($\Delta P = P_1 - P_2$) by reducing the pore pressure at the end of the specimen to P_2, so that pore water seepage under the effects of osmotic pressure is through the pore and fracture. When a triaxial seepage compression test of DT coal is carried out, the pore water pressures P_1 and P_2 are 3.5 MPa and 2 MPa, respectively. The coal sample is fully saturated with water before the seepage test. When a triaxial seepage compression test of BD and YH coal is carried out, the pore water pressures P_1 and P_2 are 3.8 MPa and 2.3 MPa. Prior to testing, we selected coal samples with no obvious cracks. We then used the ultrasonic velocity measurement method to screen the selected coal samples and used those samples which exhibited similar sonic velocities and waveforms for testing. The results of the triaxial strength test and the data utilized for assessing the yield criteria are shown in Table 1. The principle of triaxial tests is shown in Figure 1.

TABLE 1: Test results of ultimate strength of coal samples.

DT[1] natural coal		DT water-saturated nonseeping coal		DT seepage coal		BD[2] natural coal [33]		XH[3] natural coal [34]		BD seepage coal [34]		YC[4] natural coal [32]	
σ_3 (MPa)	σ_{1max} (MPa)	σ_3 (MPa)	σ_{1max} (MPa)	σ_3 (MPa)	σ_{1max} (MPa)	σ_3 (MPa)	σ_{1max} (MPa)	σ_3 (MPa)	σ_{1max} (MPa)	σ_3 (MPa)	σ_{1max} (MPa)	σ_3 (MPa)	σ_{1max} (MPa)
0	16.078	0	11.094	5	41.267	0	30.16	0	21.07	4	33.78	0	16.5
0	15.68	0	12.181	7	55.242	2	44.38	2	39.13	6	40.21	0	18.13
5	51.659	5	47.617	8	59.113	5	63.04	5	59.04	8	52.24	0	17.87
5	48.682	5	43.675	12	69.864	8	76.89	8	67.99	XH seepage coal [34]		0	18.22
15	82.552	15	69.619	—	—	12	92.51	11	76.77	σ_3 (MPa)	σ_{1max} (MPa)	5	47.74
15	81.811	15	65.818	—	—	16	106.07	14	89.91	4	24.32	10	59.13
25	104.548	25	101.212	—	—	20	112.99	17	103.77	6	29.77	20	76.62
25	106.615	25	95.659	—	—	24	123.16	20	113.32	8	33.89	25	97.24

[1]DT denotes Dongtan coal mine. [2]BD denotes Baodian coal mine. [3]XH denotes Xinhe coal mine. [4]C denotes Yangcun coal mine.
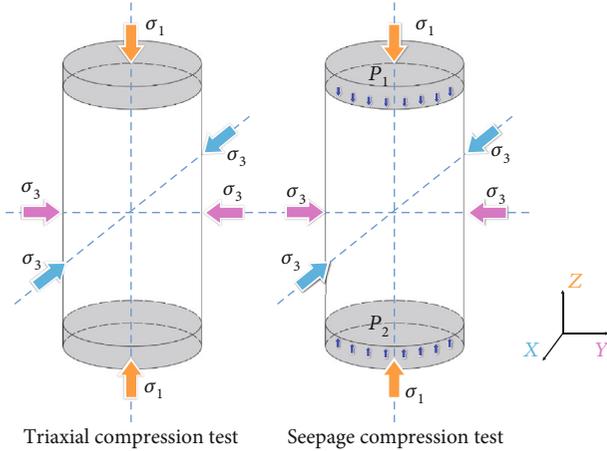
FIGURE 1: Diagram of the tests' principle.

2.2. Mathematical Expression of Yield Criteria

2.2.1. Linear Yield Criterion.
Coulomb yield criterion is established by assuming that the rock failure is induced by sheer stress, and the maximum shear stress the rock can withstand τ_{max} is jointly dictated by the cohesion c and the internal friction [31], i.e.,

$$\tau_{max} = c + \mu\sigma, \tag{1}$$

where μ is the internal friction coefficient; σ denotes the normal stress. The equation above can be expressed based on the principle stress as follows:

$$\sigma_{1\ max} = \frac{2c \cos \varphi}{1 - \sin \varphi} + \frac{1 + \sin \varphi}{1 - \sin \varphi}\sigma_3, \tag{2}$$

where σ_{1max} denotes the maximum principal stress; σ_3 denotes the minimum principal stress; φ denotes the internal friction angle.

2.2.2. Parabolic Yield Criterion.
Considering the symmetry, the parabolic Mohr criterion (You 2010) can be written as follows:

$$\sigma = \frac{\tau_{max}^2}{aT_0} - T_0, \tag{3}$$

where T_0 denotes the uniaxial tensile strength of rock, and a denotes the a parameter related to the compressive strength of the rock R:

$$a = \frac{R^2}{\left(1 + \sqrt{1 + R}\right)^2}. \tag{4}$$

The corresponding principal stress relationship can be expressed as follows:

$$(\sigma_{1\ max} - \sigma_3)^2 = 2aT_0(\sigma_{1\ max} + \sigma_3) + 4aT_0^2 - a^2T_0^2. \tag{5}$$

The equation above can also be expressed as follows:

$$(\sigma_{1\ max} - \sigma_3)^2 = A(\sigma_{1\ max} + \sigma_3) + C, \tag{6}$$

where A and C are unknown parameters, which can be determined based on linear regression to finalize the yield criterion expression. Equation (6) can be equivalently written as follows:

$$\sigma_{1\ max} = \sigma_3 + \sigma_c - \sigma_D + \sqrt{4(\sigma_c - \sigma_D)\sigma_3 + \sigma_D^2}, \tag{7}$$

where σ_c denotes the uniaxial compressive strength of a complete rock, and σ_D denotes an unknown parameter to be determined here.

2.2.3. Hoek-Brown Yield Criterion.
Based on the experimental testing of rock strength, Hoek and Brown put forward an empirical yield criterion, whose general form [6] is shown below:

$$\sigma_{1\ max} = \sigma_3 + \sigma_c \left(\frac{m\sigma_3}{\sigma_c} + s\right)^n, \tag{8}$$

where σ_c denotes the uniaxial compressive strength of a complete rock; m, s, and n are constants dictated by the characteristics of the rock. For a typical complete rock, $s = 1$ and $n = 0.5$.

For a complete rock, Equation (8) can be simplified as follows:

$$\sigma_{1\ max} = \sigma_3 + \sigma_c \left(\frac{m\sigma_3}{\sigma_c} + 1\right)^{0.5}. \tag{9}$$

Equation (9) is also referred to as the special Hoek-Brown criterion, which can also be rewritten as follows:

$$(\sigma_{1\ max} - \sigma_3)^2 = m\sigma_c\sigma_3 + \sigma_c^2. \tag{10}$$

The square of the principal stress deviator is linearly correlated with the minimum principal stress, based on which the parameters m and σ_c can be determined by applying linear regression to the experimental data.

2.2.4. Quadratic Polynomial Criterion.
In order to facilitate the regression of experimental data, one can introduce additional unknown parameters to the criterion formula to modify the existing yield criterion. Also, a multitude of options exist for correcting the yield criterion. The general form of the parabolic yield criterion can be rewritten in a quadratic polynomial form [13]:

$$(\sigma_{1\ max} - \sigma_3)^2 = A\sigma_{1\ max} + B\sigma_3 + C, \tag{11}$$

where $A, B,$ and C are unknown parameters. In the case where $A = B$, the equation above becomes Equation (6), while

Table 2: Test results concerning ultimate strength of coal samples.

| Category | σ_c (MPa) | Coulomb criterion | | | Parabolic Mohr criterion | | | Hoek-Brown criterion | | | Quadratic polynomial criterion | | | |
		c (MPa)	φ (°)	δ_{2m} (MPa)	σ_c (MPa)	σ_D (MPa)	δ_{2m} (MPa)	m	σ_c (MPa)	δ_{2m} (MPa)	m	σ_c (MPa)	σ_D (MPa)	δ_{2m} (MPa)
DT natural	15.879	10.05	29.13	6.35	26.7	0.0001688	4.53	17.44	16.08	2.42	10.29	16.39	$2.17e-06$	0.88
BD natural	30.16	12.71	32.55	5.56	37.87	$2.457e-14$	4.47	14.93	30.16	2.55	8.338	30.16	8.61	1.66
XH natural	21.07	7.454	38.15	2.98	29.75	0.006519	6.22	19.61	21.07	1.86	17.51	21.07	14.95	1.80
YC natural	17.68	5.096	31.31	3.74	20.32	$2.651e-08$	3.00	10.79	18.13	2.30	5.484	18.01	0.0002742	1.90
DT water-saturated	11.638	3.382	34.02	5.25	20.37	$1.291e-07$	5.31	15.91	12.18	3.27	10.89	12.45	$6.427e-08$	2.64
DT seepage		10.17	29.36	2.27	23.23	$8.244e-05$	2.72	9.546	24.44	2.16	35.34	6.406	0.05028	2.16
BD seepage		3.348	40.08	1.24	15.15	0.00016	2.67	1859	0.1197	1.48	1608	26.19	$1.248e+04$	3.27
XH seepage		4.84	24.23	0.29	8.959	$9.518e-07$	0.25	5.197	12.56	0.22	6.578	6.873	$1.673e-12$	0.14

TABLE 3: Fitted results of exponential yield criterion.

Category	σ_c (MPa)	Exponential yield criterion			
		K_0	Q_0 (MPa)	Q_∞ (MPa)	δ_{2m} (MPa)
DT natural	15.879	8.059	16.08	84.83	1.41
BD natural	30.16	7.62	30.16	110.1	0.49
XH natural	21.07	5.876	26.72	145.9	2.40
YC natural	17.68	5.075	18.13	89.1	2.36
DT water-saturated	11.638	7.892	12.18	74.61	3.82
DT seepage		14.01	$1.052e - 08$	63.22	1.11
BD seepage		1.279	26.96	26.28	0.05
XH seepage		6.771	5.569	29.27	0.00097

a special form of Hoek-Brown criterion is obtained by setting $A = 0$ in the equation above. Equation (11) can be explicitly expressed as follows:

$$\sigma_{1\,max} = \sigma_3 + \sigma_c - \sigma_D + \sqrt{m\sigma_c\sigma_3 + \sigma_D^2}. \qquad (12)$$

2.2.5. Exponential Yield Criterion. By recognizing that the heterogeneity, cohesive force, and friction cannot coexist in a rock material, You [14, 31] postulates that with an increase of minimum principal stress, the maximum shear stress or the principal stress deviator in rock tends to approach a constant. Therefore, an exponential yield criterion with three unknown parameters is obtained as follows:

$$\sigma_{1\,max} - \sigma_3 = Q_\infty - (Q_\infty - Q_0) \exp\left[-\frac{(K_0 - 1)\sigma_3}{Q_\infty - Q_0}\right], \quad (13)$$

where Q_0 denotes the uniaxial compressive strength, Q_∞ denotes the yield differential principle stress, and K_0 denotes the impact factor on strength for a zero confining pressure.

2.3. Comparative Analysis on Yield Criteria. In this paper, the triaxial compressive test data under different coal conditions are used to compare and analyze the applicability of the following five yield criteria.

(1) Coulomb yield criterion, Equation (2)

(2) Parabolic Mohr criterion, Equations (6) and (7)

(3) Hoek-Brown yield criterion, Equations (9) and (10)

(4) Quadratic polynomial criterion, Equations (11) and (12)

(5) Exponential yield criterion, Equation (13)

3. Comparative Analyses of Yield Criteria

3.1. Metrics for Assessing the Regression Quality of Yield Criteria. The least square method is used to determine the unknown parameters in the formula, which can minimize the summation over the square of error between the model prediction and measurement data.

$$\delta_1 = \sum[\sigma_s - f(\sigma_3)]^2. \qquad (14)$$

You [13] conducts regression on the experimental data by using two different methods, i.e., minimizing the sum of square of error and minimizing the sum of absolute error. The study indicates that the latter approach can highlight the outliers in the measurement data and ensure that the measured points are distributed over both sides of the fitted curve in a balanced manner while staying as close as possible to the fitted curve.

$$\delta_2 = \sum \text{abs}[\sigma_s - f(\sigma_3)]. \qquad (15)$$

For this reason, the present study is aimed at minimizing the sum of absolute error so as to determine the unknown parameters in the formula and uses δ_{2m} to indicate the average fitting error:

$$\delta_{2m} = \frac{\sum \text{abs}[\sigma_s - f(\sigma_3)]}{N} = \frac{\delta_2}{N}, \qquad (16)$$

where N denotes the number of data points.

3.2. Comparative Analyses of Coal Strength Criteria. The present study applies curve fitting to the data of triaxial compression test on the coal rock samples as well as the triaxial compression test data published by Su et al. [32] and Yang et al. [33, 34] (as shown in Table 1), based on Coulomb criterion, parabolic Mohr criterion, Hoek-Brown criterion, quadratic polynomial criterion, and exponential strength criterion, respectively, with the fitted parameters shown in Tables 2 and 3, and the fitted curves associated with various yield criteria under the same coal rock condition shown in Figure 2. To facilitate the comparison, the fitting errors corresponding to the five yield criteria studied here are shown with a column diagram in Figure 3. It can be seen that for the natural water-bearing coal samples, the linear Coulomb yield criterion exhibits a large fitting error, while the parabolic Mohr criterion yields a lower fitting error compared with the linear Coulomb yield criterion for all tested cases
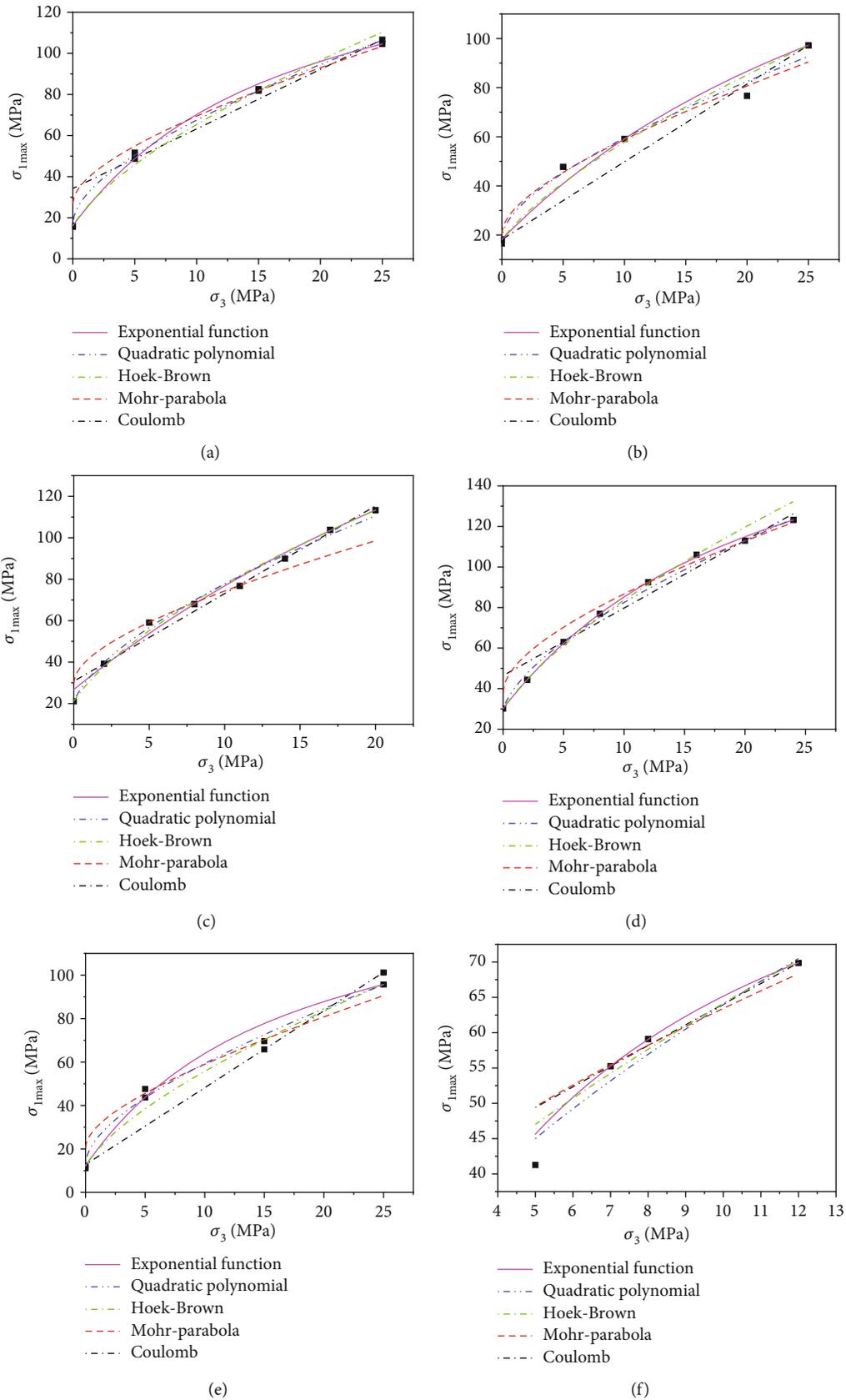
(a)

(b)

(c)

(d)

(e)

(f)
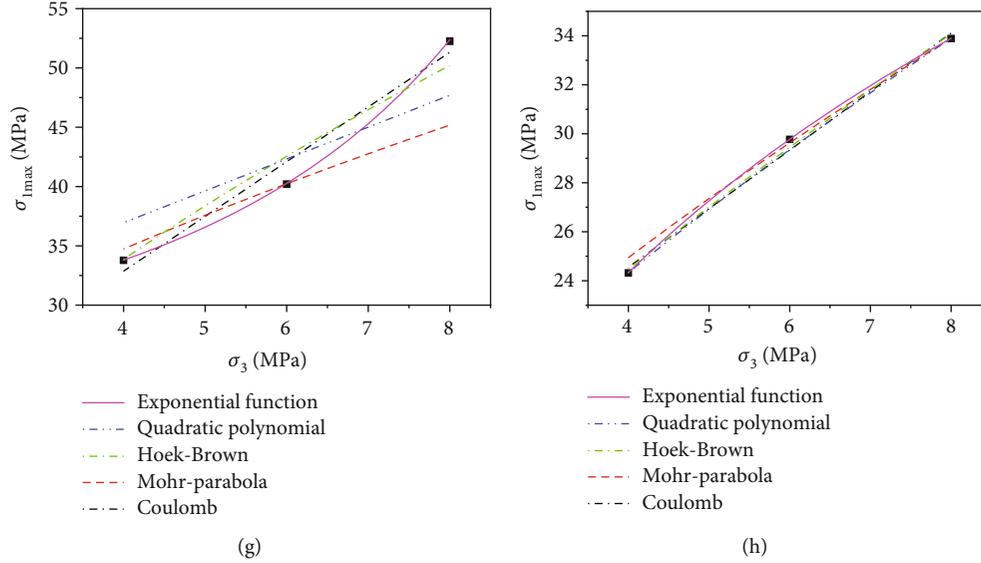
FIGURE 2: Continued.

(g)

(h)

FIGURE 2: Triaxial compressive strength of coal samples and fitted curves of five yield criteria. (a) DT Natural coal. (b) BD Natural coal. (c) XH Natural coal. (d) YC Natural coal. (e) DT Water-saturated coal. (f) DT Seepage coal. (g) BD Seepage coal. (h) XH Seepage coal.

except the XH natural coal sample. However, the uniaxial compressive strength of a complete rock σ_c obtained based on Equation (7) is significantly higher than that of the measured data. In other words, the calibrated parabolic Mohr criterion tends to overestimate the uniaxial compressive strength of the rock. For the Hoek-Brown criterion, quadratic polynomial criterion, and exponential strength criterion, the corresponding fitting errors are all less than those associated with the linear yield criterion and the parabolic Mohr criterion. Except that the quadratic polynomial criterion fails to decently fit the BD and XH natural coal samples, the fitted uniaxial strength of coal rock derived from the three criteria above all agree well with experimental data, and the performances of Hoek-Brown and exponential yield criteria are particularly satisfactory.

For water-saturated coal and compressive seepage coal, the linear yield criterion and the parabolic Mohr criterion still yield large errors in fitting the measured data. The cohesive force of the natural coal derived from the Coulomb criterion is smaller than those of all the other coal samples, e.g., water-saturated and seepage coal samples, except for the DT coal sample. The internal friction angle of the natural coal sample is smaller than those of all the water-saturated and seepage coal samples, except for the XH water-seeping coal with well-developed fractures and pore space [34], whose internal friction angle is smaller than that of the DT water-saturated coal and slightly larger than that of the DT natural coal. Due to the complex distribution of fine joints, fractures, and pore space in the coal, the linear yield criterion cannot authentically reflect the actual variation of coal strength, and thus, the applicability of the internal friction angle computed from the linear criterion to describing the mechanical characteristics of the rock needs further experimental validation.

In addition to the BD seepage coal test, the Hoek-Brown criterion, the quadratic polynomial criterion, and the expo-

nential yield criterion can also decently fit the experimental strength data of the water-saturated and water-seeping rock samples, with the exponential yield criterion giving the best result, as shown in Figure 3. However, considering the mechanical interpretation underlying the exponential yield criterion, the DT seepage uniaxial compressive strength Q_0 derived from the exponential yield criterion shows a local minimum value, which is inconsistent with the reality. In addition, the fitting parameters of the exponential yield criterion, Q_0, all give low fitting errors, implying its ability to truly reflect the uniaxial strength of the coal rock. It is noteworthy that, for both the natural coal rock and the water-saturated water-seeping coal rock, the yield differential principle stress, Q_∞, derived from the exponential yield criterion is lower than the measured value. Intuitively speaking, however, the exponential yield criterion exhibits a major advantage in fitting the strength data of the water-seeping coal. Therefore, it is necessary to conduct further experimental validation concerning the applicability of the exponential yield criterion to analyzing the coal rock strength.

4. Comparative Analyses of Yield Criteria with the Presence of Pore Water in Coal

In 1936, Terzaghi proposed the principle of effective stress [35]. According to the principle of effective stress, the total stress in the soil is equal to the sum of the effective stress and the pore water pressure, and only the change of effective stress will cause the change of strength. In the field of geotechnical engineering, the complex groundwater environment and the huge engineering risks associated with it have prompted engineering professionals to introduce the principle of effective stress into rock mechanics for rock strength analysis. To further analyze the effect of pore pressure on the mechanical properties of coal, the effective stress principle and the aforementioned test results are
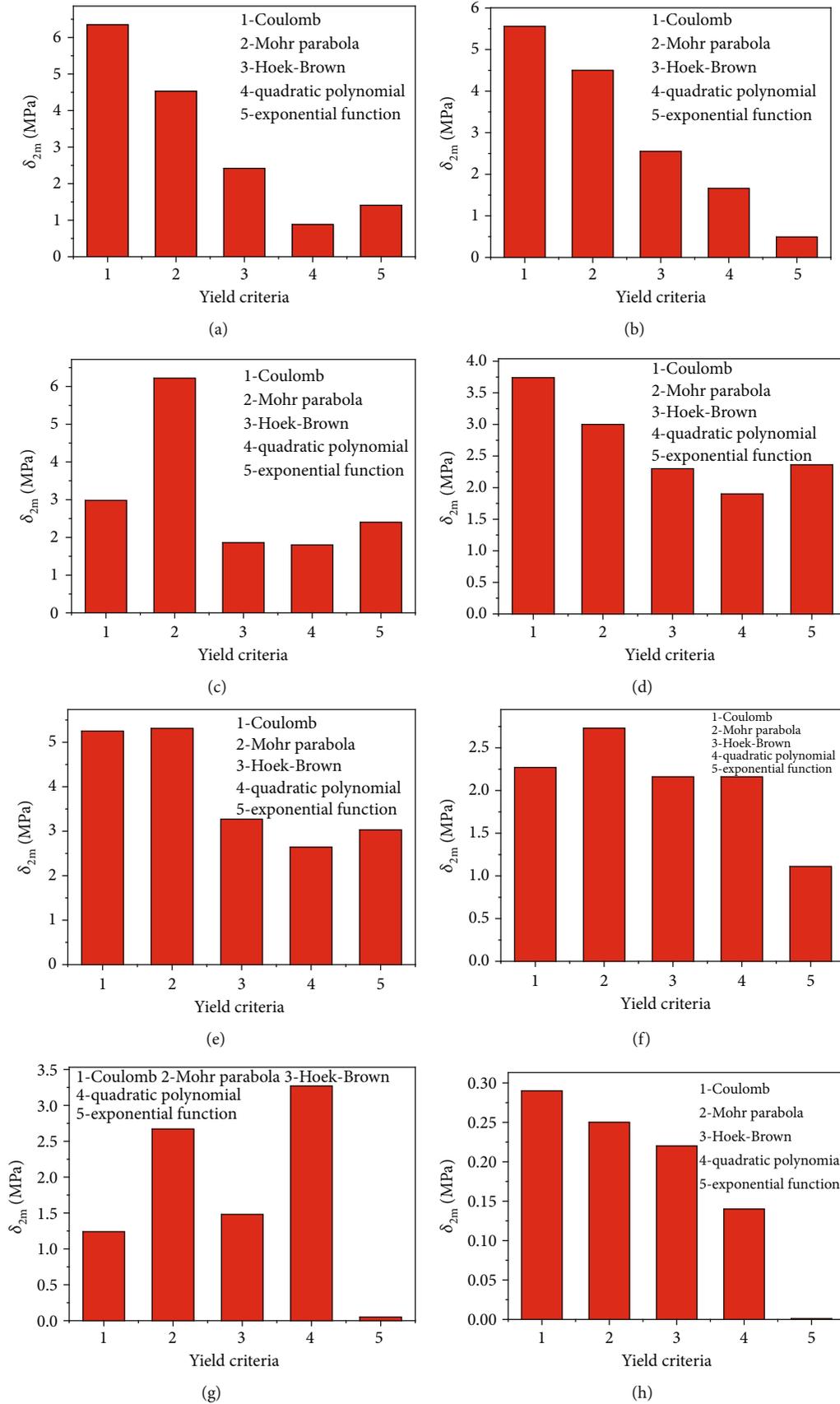
FIGURE 3: Mean fitting deviations of five yield criteria. (a) DT Natural coal. (b) BD Natural coal. (c) XH Natural coal. (d) YC Natural coal. (e) DT Water-saturated coal. (f) DT Seepage coal. (g) BD Seepage coal. (h) XH Seepage coal.

Table 4: Triaxial strength fitting results of five effective yield criteria for saturated coal based on the least absolute deviation.

Category	ζ	Coulomb criterion			Parabolic Mohr criterion			Hoek-Brown criterion		
		c (MPa)	φ (°)	δ_{2m} (MPa)	σ_c (MPa)	σ_D (MPa)	δ_{2m} (MPa)	m	σ_c (MPa)	δ_{2m} (MPa)
	0.01	3.403	34.99	5.47	21.53	0.0005366	5.59	16.89	12.18	3.44
DT water-saturated	0.05	3.522	39.58	6.68	23.83	$2.09e-05$	6.33	23.23	12.18	4.63
	0.08	4.458	42.97	8.23	27.13	$1.573e-07$	8.07	33.1	12.18	6.97

Category	ζ	Quadratic polynomial criteria				ζ	Exponential yield criterion			
		m	σ_c (MPa)	σ_D (MPa)	δ_{2m} (MPa)		K_0	Q_0 (MPa)	Q_∞ (MPa)	δ_{2m} (MPa)
	0.01	10.5	13.31	$1.073e-06$	2.94	0.01	8.758	12.18	73.85	3.99
DT water-saturated	0.05	8.424	18.34	0.0001455	4.66	0.05	16.17	12.18	71.28	5.08
	0.08	21.03	12.18	$3.368e-08$	5.73	0.08	59.29	12.18	55.24	6.91

used to evaluate the applicability of the Coulomb criterion, the parabolic Mohr criterion, the Hoek-Brown criterion, the quadratic polynomial criterion, and the exponential yield criterion to computing the strength of water-saturated and water-seeping rock samples. Herein, the maximum axial stress σ_{1max} and the confining pressure σ_3 corresponding to the coal rock solid skeleton under the pore pressure are expressed as the effective maximum axial stress σ'_{1max} and the effective confining pressure σ'_3.

4.1. Comparative Analyses of Yield Criteria in Water-Saturated Nonseeping Coal. During triaxial compression tests on the water-saturated rock sample, the water present within the fractures and pore spaces between solid particles in the solid skeleton of the coal rock is subject to a combination of axial, confining, and pore pressure [36, 37]. Existing research has shown that under the physical processes of water lubrication, softening, argillization, drying and wetting, freezing and thawing, the chemical processes of dissolution, hydration, hydrolysis, acidification, and oxidation, and the mechanical effects of pore water pressure, the mechanical property parameters of rock, such as strength, all decrease in a groundwater environment [38–40]. Although the structure and composition of coal rock is different from other rocks, studies have shown that the strength limit of coal rock is also related to its degree of water saturation and pore pressure [41, 42]. During the saturated nonseeping compression test, the pore pressure enclosed in the solid skeleton also increased in response to the compression of the solid skeleton, and the resulting pore pressure is related to the volumetric stress of the coal rock skeleton. Therefore, the pore pressure inside the coal rock should be $\zeta(\sigma_1 + 2\sigma_3)$, where ζ denotes the coefficient related to the porosity and its distribution within the coal rock, with the value ranging between 0 and 1. As a result, the effective strength associated with the saturated nonpermeating coal rocks can be calculated as follows:

$$\left.\begin{array}{l} \sigma'_{1\ max} = \sigma_{1\ max} - \zeta(\sigma_{1\ max} + 2\sigma_3) \\ \sigma'_3 = \sigma_3 - \zeta(\sigma_{1\ max} + 2\sigma_3) \end{array}\right\}. \tag{17}$$

Using the five yield criteria above, the experimental data tied to the effective strength of DT saturated coal under different conditions are fitted, and the average fitting errors are analyzed, as shown in Table 4. For the sake of conciseness, this paper only presents the fitted parameters and the corresponding fitting errors of the effective coal strength for ζ being 0.01, 0.05, and 0.08. In order to intuitively illustrate the performance of each criterion in fitting the measurement data of saturated coal rock, the fitting errors of various yield criteria are shown in a column diagram (Figure 4), with Figure 5 showing the corresponding fitted curves of the five yield criteria for $\zeta = 0.05$.

It can be seen that the Coulomb criterion and the parabolic Mohr criterion yield large fitting errors, and the parabolic Mohr criterion tends to overestimate the uniaxial compressive strength of saturated coal rock. In addition, it is found from the Coulomb criterion-based fitting data under different pore pressure conditions that the internal friction angle and cohesion of saturated coal increase with the pore pressure. By contrast, the Hoek-Brown criterion, the quadratic polynomial criterion, and the exponential yield criterion are superior to the Coulomb criterion and the parabolic Mohr criterion in terms of the fitting quality of effective saturated rock strength. For a variety of pore pressures, the fitting errors of the three methods are comparable. All three methods can decently fit the uniaxial strength data of saturated coal rock.

Among the three yield criteria with small fitting errors, the Hoek-Brown criterion is regarded as a special case of the second-degree polynomial rule [14] (with $A = 0$ in Equation (11)). Figure 6 shows the fitted curves of quadratic polynomial criterion and exponential yield criterion under different pore pressures. It can be seen in the figure that the fitting parameter m, as a measure of the rock hardness, increases with the pore pressure, that is, the rock skeleton gets reinforced under the influence of the pore pressure. Besides, from the fitting quality of the exponential yield criterion, one can conclude that with an increase of pore pressure, the yield differential principal stress Q_∞ of the saturated coal rock gradually decreases, i.e., the increase of pore pressure compromises the yield stress of
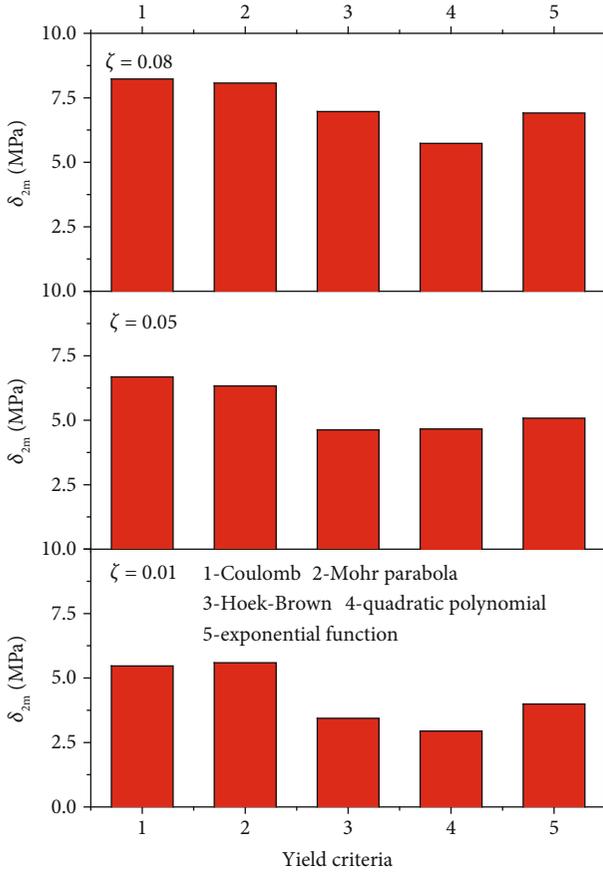
Figure 4: Mean fitting deviations of yield criteria for saturated DT coal samples.
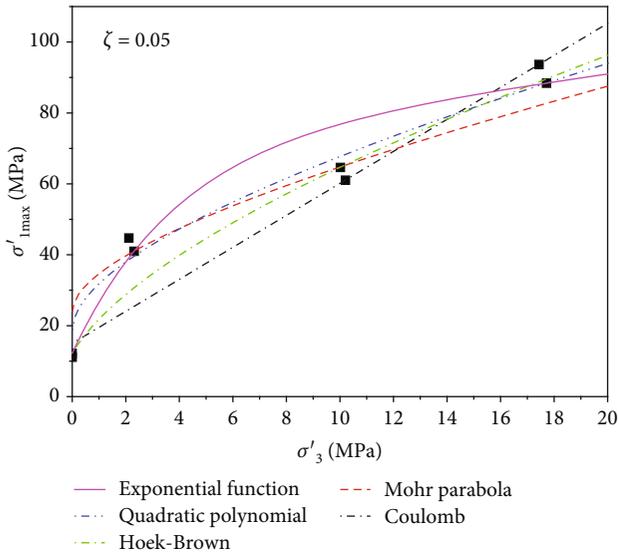

Figure 5: Triaxial compressive strength of water-saturated coal samples and fitted curves of five yield criteria.
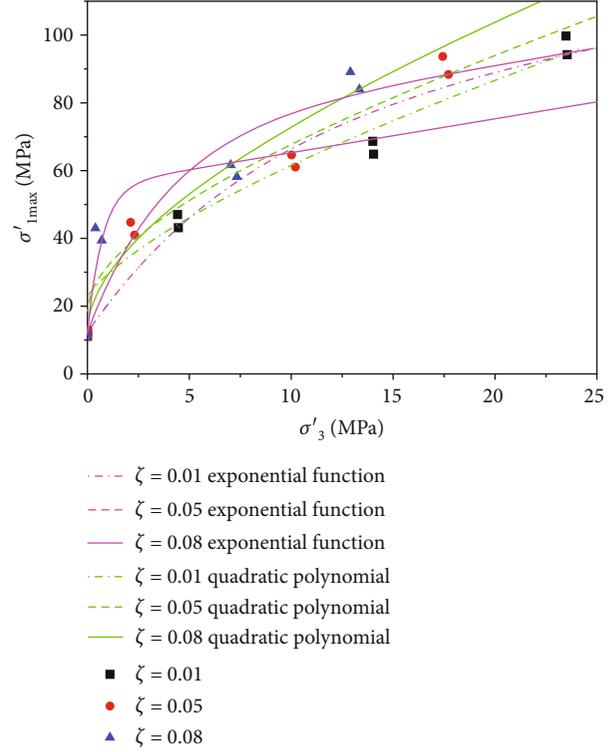

Figure 6: Triaxial compressive strength of water-saturated coal samples and the fitted curves of two yield criteria under different pore pressures.

coal rock. Nonetheless, the yield differential principal stress Q_∞ fitted from the exponential yield criterion is still lower than the experimentally measured value. This deviation, however, is acceptable from the perspective of engineering construction safety.

4.2. Comparative Analyses of Yield Criteria in Water-Saturated Seeping Coal. The internal pores of coal samples are not always connected, so the internal pore water pressure is not necessarily equal to the water pressure exerted by the end face. Therefore, the pore water pressure in coal samples should be δP, where δ is the coefficient related to the porosity of coal and its connected state, and its value is related to the porosity of coal and its connected state. In addition, due to the existence of osmotic pressure differences in coal, pore water pressure is a function of pore distribution coordinate, but it is difficult to accurately determine its distribution state during the test. Therefore, two extreme assumptions were adopted to simplify estimation of the range of effective stress values on coal under conditions of saturated drainage.

(1) Assuming that the coal sample is absolutely dense, i.e., the porosity ϕ is 0 (completely impermeable), the water pressure applied to the specimen section is equivalent to the axial stress superimposed on the specimen section. Therefore, the effective stress on the coal is as follows:

TABLE 5: Triaxial strength fitting results of five types of effective strength criteria for coal during a seepage process based on the least absolute.

Category	ϕ	Coulomb criterion			Parabolic Mohr criterion			Hoek-Brown criterion		
		c (MPa)	φ (°)	δ_{2m} (MPa)	σ_c (MPa)	σ_D (MPa)	δ_{2m} (MPa)	m	σ_c (MPa)	δ_{2m} (MPa)
DT seepage	0	12.14	29.36	2.27	27.42	3.412e − 05	1.69	5.085	40.16	2.09
	$\phi\pm$	11.29	29.36	2.28	24.72	0.001393	2.68	5.649	36.14	2.09
BD seepage	0	6.546	40.08	1.24	27.85	5.867	1.87	8.983	27.97	1.54
	$\phi\pm$	5.284	40.08	1.24	18.55	6.171e − 06	2.01	12.48	20.14	1.54
XH seepage	0	6.55	24.23	0.29	20.05	10.46	0.17	3.24	20.14	0.22
	$\phi\pm$	5.875	24.23	0.29	16.79	7.194	1.58	3.72	17.55	0.22

| Category	ϕ	Quadratic polynomial criterion				ϕ	Exponential yield criterion			
		m	σ_c (MPa)	σ_D (MPa)	δ_{2m} (MPa)		K_0	Q_0 (MPa)	Q_∞ (MPa)	δ_{2m} (MPa)
DT seepage	0	2.801	30.84	0.001214	1.65	0	14.28	20.68	60.05	0.17
	$\phi\pm$	4.342	24.96	4.045e − 05	1.85	$\phi\pm$	19.71	2.559e − 09	60.64	0.23
BD seepage	0	1.026e + 04	29	4.099e + 04	0.94	0	4.617	28.12	1.377e + 04	1.25
	$\phi\pm$	1.316e + 04	22.93	4.133e + 04	1.13	$\phi\pm$	1.715	28.03	26.28	0.0054
XH seepage	0	0.7782	19.68	2.075	0.00065	0	3.288	19.87	29.27	0.0026
	$\phi\pm$	1.94	13.62	1.056e − 13	0.089	$\phi\pm$	4.296	15.73	29.27	0.0012
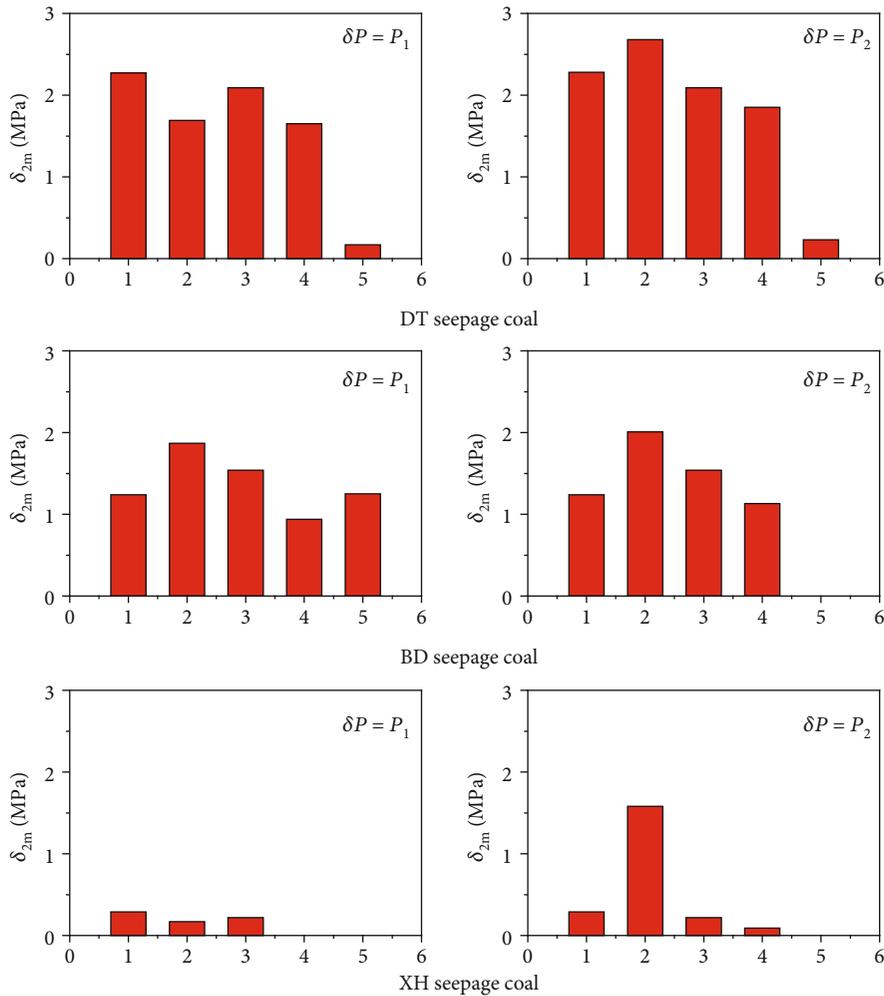
FIGURE 7: Mean fitting deviations of strength criteria for seepage coal samples.

$$\left. \begin{array}{l} \sigma_1'{}_{\max} = \sigma_{1\ \max} + P \\ \sigma_3' = \sigma_3 \end{array} \right\}. \tag{18}$$

(2) Assuming that the porosity of the coal sample is large enough to make its structure similar to that of soil, the effective stress on the coal is as follows:

$$\left. \begin{array}{l} \sigma_1'{}_{\max} = \sigma_{1\ \max} \\ \sigma_3' = \sigma_3 - P \end{array} \right\}. \tag{19}$$

Due to the complex structure of pores and fractures in coal, it is impossible to specifically calculate the effective stress on the internal skeleton of coal. However, from formula (18) and formula (19), it can be seen that the effective stress on seepage coal is somewhere between the two. Therefore, based on the above two extreme hypotheses, the maximum value of effective stress is selected for analysis by fitting data of the effective strength of seepage coal. Table 5 shows the fitting parameters and average fitting deviation of different strength criteria of seepage coal.

It can be seen from Figure 7 that the Hoek-Brown criterion, quadratic polynomial criterion, and exponential yield criterion are better than the Coulomb criterion and parabolic Mohr criterion, and especially the exponential yield criterion. Figure 8 shows the fitting curve of the DT seepage coal quadratic polynomial criterion and the exponential yield criterion, from which we can see that the exponential yield criterion has a good fitting deviation for the effective strength of seepage coal. However, due to the limited amount of coal seepage test data, the exponential yield criterion for the partial seepage test coal data, the fitting parameters of uniaxial compression strength Q_0, and the ultimate principal stress difference Q_∞ are not accurate. Therefore, to apply the exponential yield criterion to the effective strength of seepage coal, further seepage tests are needed.

5. Conclusion

(1) With an increase of confining pressure, the coal rock strength shows a nonlinear characteristic. The linear Coulomb yield criterion cannot describe this behavior well, showing a large fitting error. It is therefore necessary to conduct experimental validation when it comes to using its internal friction angle and cohesion to describe the variation of the rock's mechanical properties

(2) Although the parabolic Mohr yield criterion can reflect to certain extent the nonlinear characteristics of coal rock strength, its fitting error is too large, and the fitted parameters are inconsistent with the reality

(3) The Hoek-Brown criterion, quadratic polynomial criterion, and exponential yield criterion all exhibit decent fitting quality, i.e., they can all reflect the characteristics of coal strength under natural and fluid-structure interactions. The exponential yield criterion, in particular, can truly reflect the uniaxial com-
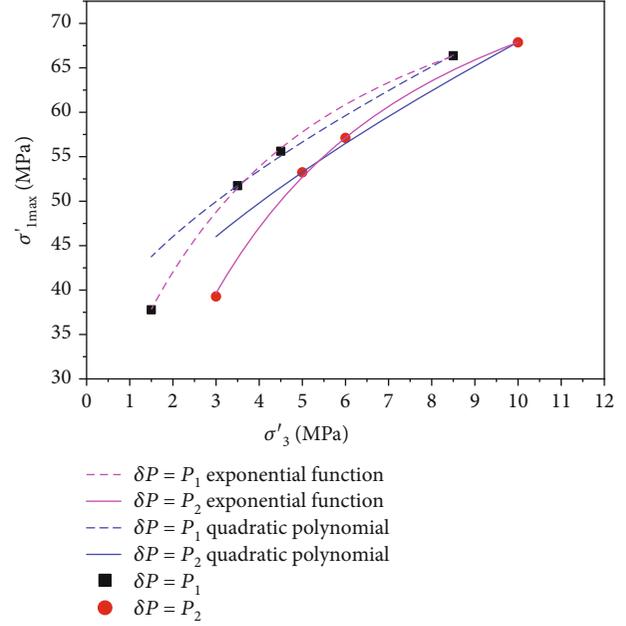

FIGURE 8: Triaxial compression strengths of DT coal samples and fitting curves of two strength criteria under seepage pressures.

pressive strength of coal. Although the yield strength of the rock derived from the exponential yield criterion is less than the measured value, such conservative approach is favored for ensuring engineering safety

Although the test and analysis results presented in this paper provide some insights into the applicability of various yield criteria to characterize the yield variation (especially with the presence of fluid-solid interaction), the size and self-consistency of experimental dataset is limited; thereby, the conclusions presented in this paper are preliminary. As such, the applicability of various yield criteria still requires further experimental analyses.

Data Availability

The data used to support the findings of this study have not been made available.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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