Research Article

Elastic-Impedance-Based Fluid/Porosity Term and Fracture Weaknesses Inversion in Transversely Isotropic Media with a Tilted Axis of Symmetry

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The rock containing a set of tilted fractures is equivalent to a transversely isotropic (TTI) medium with a tilted axis of symmetry. To implement fluid identification and tilted fracture detection, we propose an inversion approach of utilizing seismic data to simultaneously estimate parameters that are sensitive to fluids and tilted fractures. We first derive a PP-wave reflection coefficient and elastic impedance (EI) in terms of the dip angle, fluid/porosity term, shear modulus, density, and fracture weaknesses, and we present numerical examples to demonstrate how the PP-wave reflection coefficient and EI vary with the dip angle. Based on the information of dip angle of fractures provided by geologic and well data, we propose a two-step inversion approach of utilizing azimuthal seismic data to estimate unknown parameters involving the fluid/porosity term and fracture weaknesses: (1) the constrained sparse spike inversion (CSSI) for azimuthally anisotropic EI data and (2) the estimation of unknown parameters with the low-frequency constrained regularization term. Synthetic and real data demonstrate that fluid and fracture parameters are reasonably estimated, which may help fluid identification and fracture characterization.

1. Introduction

Amplitude variation with offsets (AVO) can be used to obtain the physical properties of a homogeneous isotropic medium [1]. The assumption of isotropic media is not appropriate in the case of being applied to fractured reservoirs [2]. Recently, transversely isotropic media with a horizontal/vertical axis of symmetry (HTI/VTI) are proposed for the description of how horizontal and vertical fractures affect rock properties. Rüger [3] derives a P-wave reflection coefficient in terms of weak anisotropy parameters, which is well used for modeling reflection amplitude variation with offsets and azimuth (AVOAz).

For arbitrarily anisotropic media, derivation of appropriate analytical expressions for reflection coefficient is challenging. Based on the perturbation theory, Vavryčuk [4] derives approximate PP-wave reflection/transmission coefficients for interfaces separating two arbitrarily anisotropic media; Shaw and Sen [5] propose the linearized expression

of PP-wave reflection coefficients for arbitrary weakly anisotropic media. A rock containing tilted fractures is equivalent to a transversely isotropic medium with a tilted symmetry (TTI). Under the assumption of weak anisotropy (WA), Ivanov and Stovas [6] present P-wave reflection coefficient for the boundary between two TTI media. Wang et al. [7] propose the azimuthal AVO formula for TTI media. To study the effect of fracture parameters and the dip angle of symmetry on the reflection amplitudes, we first derive a PP-wave reflection coefficient in terms of fracture parameters for an interface separating two weakly anisotropic TTI media, which will be used for modeling how dip angle affects seismic reflection amplitude.

Fluid identification is an important task in the seismic inversion for fractured reservoirs [8–12]. The normal fracture compliance varies with fluid content; however, the tangential fracture compliance exhibits independence on fluid infills. To estimate fluid type in vertically saturated fractured reservoirs, Shaw and Sen [13] suggest the normal-to-tangential

FIGURE 1: A TTI medium with dip angle ζ of symmetry axis.

compliance ratio as a fluid indicator, which can be estimated using seismic AVOAz data; however, this fluid indicator is affected by both fracture density and shear-to-compressional velocity ratio [13, 14]. Based on the anisotropic poroelasticity theory, Pan et al. [15] use the fluid/porosity term, which is proposed by Russell et al. [16], to discriminate the fluid type; they also provide a potential tool to implement the fluid detection in an oil-bearing fractured reservoir that contains a set of vertical fractures. For a tilted fractured reservoir, we intend to utilize the general linear-slip model to represent a fluid-saturated background medium containing a single set of tilted fractures, and then perform the fluid detection in such a TTI medium. The fluid/porosity is used to identify the fluids in a tilted fractured medium. Unlike the PP-wave reflection coefficient expressed as functions of P- and S-wave velocities, density, and Thomsen-type parameters in a TTI medium, we use a Born formalism to derive a new expression for reflection coefficient in terms of fluid/porosity term, shear modulus, density, and normal and tangential fracture weaknesses. Numerical examples illustrate that the reflectivity is sensitive to the fluid content, and it also exhibits significant dependence on the dip angle of the symmetry axis.

AVOAz inversion approach can be used for the estimation of model parameters, but it is usually subject to the seismic noises [17–19]. Elastic impedance (EI) inversion is now widely used to estimate the physical properties of layers [20–22]. Considering the seismic anisotropy, Martins [23] first derives an anisotropic elastic impedance (EI) equation

based on the PP-wave reflection coefficient in weakly anisotropic media with arbitrary symmetry. Based on a fractured anisotropic rock physics model, Pan et al. [24] propose an azimuthally anisotropic EI inversion approach to estimate a fluid indicator. In this paper, we also derive an azimuthally anisotropic EI equation in terms of fluid/porosity term, shear modulus, density, and normal and tangential fracture weaknesses in a TTI medium. Numerical examples also demonstrate that the azimuthally anisotropic EI is closely related to both fluid content and dip angle of the symmetry axis. Therefore, we propose an EI inversion for the estimates of fluid/porosity term and fracture weaknesses to perform the fluid detection and fracture characterization in a TTI medium. The reliability and stability of the proposed approach are validated by the application of synthetic and real data acquired in the Sichuan Basin.

2. Theory and Method

2.1. Elastic Stiffness Tensor of a TTI Medium. To describe a transversely isotropic medium with a symmetry axis inclined at an angle ζ in a $x_1 - x_3$ plane (illustrated as in Figure 1), we consider a homogeneous isotropic medium with fluid/porosity term f , shear modulus μ , and density ρ embedded in a single set of parallel tilted fractures. Therefore, the 6×6 elastic stiffness matrix of a long-wavelength equivalent TTI medium with a dip angle ζ of the symmetry axis can be obtained using the stiffness matrix of a VTI medium and the Bond transformation, which is given by

$$\mathbf{C}_{\text{TTI}} = \mathbf{B}_{\zeta} \cdot \mathbf{C}_{\text{VTI}} \cdot \mathbf{B}_{\zeta}^T = \begin{bmatrix} C_{11} & C_{12} & C_{13} & & C_{15} & \\ & C_{22} & C_{23} & & C_{25} & \\ & & C_{33} & & C_{35} & \\ & & & C_{44} & & C_{46} \\ & & & & C_{55} & \\ & & & & & C_{66} \end{bmatrix}, \quad (1)$$

where only the nonzero components in the upper triangle are displayed and the lower triangle is symmetrical; the symbol T denotes the transpose of a matrix; the 6×6 elastic stiffness matrix of a VTI medium in terms of normal and tangential fracture weaknesses δ_N and δ_T can be expressed as [25]

$$\mathbf{C}_{\text{VTI}} = \begin{bmatrix} M_{\text{sat}}(1 - \chi_{\text{sat}}^2 \delta_N) & \lambda_{\text{sat}}(1 - \chi_{\text{sat}} \delta_N) & \lambda_{\text{sat}}(1 - \delta_N) & 0 & 0 & 0 \\ \lambda_{\text{sat}}(1 - \chi_{\text{sat}} \delta_N) & M_{\text{sat}}(1 - \chi_{\text{sat}}^2 \delta_N) & \lambda_{\text{sat}}(1 - \delta_N) & 0 & 0 & 0 \\ \lambda_{\text{sat}}(1 - \delta_N) & \lambda_{\text{sat}}(1 - \delta_N) & M_{\text{sat}}(1 - \delta_N) & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu(1 - \delta_T) & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu(1 - \delta_T) & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix}, \quad (2)$$

and the Bond matrix rotated about the x_3 axis can be written as [6]

$$\mathbf{B}_\zeta = \begin{bmatrix} \cos^2 \zeta & 0 & \sin^2 \zeta & 0 & -\sin 2\zeta & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \sin^2 \zeta & 0 & \cos^2 \zeta & 0 & \sin 2\zeta & 0 \\ 0 & 0 & 0 & \cos \zeta & 0 & \sin \zeta \\ \frac{1}{2} \sin 2\zeta & 0 & -\frac{1}{2} \sin 2\zeta & 0 & \cos 2\zeta & 0 \\ 0 & 0 & 0 & -\sin \zeta & 0 & \cos \zeta \end{bmatrix}, \quad (3)$$

and the elastic stiffness components of a TTI medium are given by

$$\begin{aligned} C_{11} &= (f + \gamma_{\text{sat}}^2 \mu) (\sin^4 \zeta + \cos^4 \zeta) \\ &\quad - M_{\text{sat}} \delta_N (\chi_{\text{sat}}^2 \sin^4 \zeta + \cos^4 \zeta) \\ &\quad + \sin^2 2\zeta \left[\frac{1}{2} M_{\text{sat}} + \frac{1}{2} \lambda_{\text{sat}} \delta_N - \mu \delta_T \right], \\ C_{12} &= f + (\gamma_{\text{sat}}^2 - 2) \mu - \lambda_{\text{sat}} \delta_N (\chi_{\text{sat}} \cos^2 \zeta + \sin^2 \zeta), \\ C_{13} &= f + (\gamma_{\text{sat}}^2 - 2) \mu - \lambda_{\text{sat}} \delta_N \\ &\quad + \frac{1}{4} \sin^2 2\zeta [2\lambda_{\text{sat}} \delta_N - (1 + \chi_{\text{sat}}^2) M_{\text{sat}} \delta_N + 4\mu \delta_T], \\ C_{15} &= \frac{1}{4} \sin 4\zeta (\lambda_{\text{sat}} \delta_N + 2\mu \delta_T) \\ &\quad + \frac{1}{2} \sin^2 2\zeta [\delta_N M_{\text{sat}} (\sin^2 \zeta - \chi_{\text{sat}} \cos^2 \zeta)], \\ C_{22} &= f + \gamma_{\text{sat}}^2 \mu - \chi_{\text{sat}}^2 M_{\text{sat}} \delta_N, \quad C_{23} \\ &= f + (\gamma_{\text{sat}}^2 - 2) \mu - \lambda_{\text{sat}} \delta_N (\chi_{\text{sat}} \sin^2 \zeta + \cos^2 \zeta), \\ C_{25} &= \frac{1}{2} [f + (\gamma_{\text{sat}}^2 - 2) \mu - \chi_{\text{sat}} \lambda_{\text{sat}}] \delta_N \sin 2\zeta, \quad C_{44} \\ &= \mu - \mu \delta_T \cos^2 \zeta, \quad C_{46} = \frac{1}{2} \mu \delta_T \sin 2\zeta, \\ C_{35} &= -\frac{1}{2} (f + \gamma_{\text{sat}}^2 \mu) \delta_N \sin 2\zeta (\chi_{\text{sat}}^2 \sin^2 \zeta + \cos^2 \zeta) \\ &\quad + \frac{1}{4} \sin 4\zeta [2\mu \delta_T - \lambda_{\text{sat}} \delta_N], \\ C_{55} &= \frac{1}{4} \sin^2 2\zeta [2\lambda_{\text{sat}} \delta_N - M_{\text{sat}} \delta_N (\chi_{\text{sat}}^2 + 1) - 4\mu] \\ &\quad + \cos^2 2\zeta [\mu (1 - \delta_T)], \\ C_{66} &= \mu - \mu \delta_T \sin^2 \zeta. \end{aligned} \quad (4)$$

In the above equations, f is the fluid/porosity term, which can be expressed as [16]

$$f = (\rho V_P^2)_{\text{sat}} - \gamma_{\text{dry}}^2 (\rho V_S^2)_{\text{sat}} = M_{\text{sat}} - \gamma_{\text{dry}}^2 \mu, \quad (5)$$

where $\mu = \mu_{\text{sat}} = (\rho V_S^2)_{\text{sat}} = \mu_{\text{dry}}$ is the shear (S-wave) modulus, $M_{\text{sat}} = (\rho V_P^2)_{\text{sat}} = \lambda_{\text{sat}} + 2\mu$ is the compressional (P-wave) modulus, and $\chi_{\text{sat}} = \lambda_{\text{sat}}/M_{\text{sat}} = 1 - 2\mu/M_{\text{sat}}$, $f/\mu = \gamma_{\text{sat}}^2 - \gamma_{\text{dry}}^2$, and $\gamma_{\text{dry}}^2 = [V_P^2/V_S^2]_{\text{dry}} = \lambda_{\text{dry}}/\mu + 2$.

2.2. Linearized PP-Wave Reflection Coefficient of a TTI Medium. To derive a linearized PP-wave reflection coefficient of a TTI medium, we represent a weakly anisotropic TTI medium as a volume of scatters embedded in a homogeneous isotropic background [5]. Following Shaw and Sen [13], we use the seismic scattering theory and stationary phase method to derive the linearized PP-wave reflection coefficient, which is related to a scattering function $\mathbf{S}(\mathbf{x})$ as

$$R_{\text{PP}} = \frac{1}{4\rho \cos^2 \theta} \mathbf{S}(\mathbf{x}), \quad (6)$$

where

$$\mathbf{S}(\mathbf{x}) = \Delta\rho \cos 2\theta + \Delta\mathbf{C} \cdot \boldsymbol{\eta}, \quad (7)$$

and in which $\mathbf{x}$ is a position vector of horizontal interface, $\Delta\rho$ is the perturbation in density, $\Delta\mathbf{C}$ is the first-order perturbation in stiffness tensor, and $\boldsymbol{\eta}$ is a 6×6 matrix related to the angle of incidence and azimuth (θ and ϕ , respectively), which is demonstrated in Shaw and Sen [13].

The final expression for linearized PP-wave reflection coefficient of a TTI medium is given by

$$R_{\text{PP}}^{\text{TTI}}(\theta, \phi; \zeta) = R_{\text{PP}}^{\text{iso}}(\theta) + R_{\text{PP}}^{\text{ani}}(\theta, \phi; \boldsymbol{\delta}), \quad (8)$$

where the isotropic background part of reflection coefficient of a TTI medium is written as

$$R_{\text{PP}}^{\text{iso}}(\theta) = a_f(\theta) \frac{\Delta f}{f} + a_\mu(\theta) \frac{\Delta \mu}{\mu} + a_\rho(\theta) \frac{\Delta \rho}{\rho}, \quad (9)$$

in which

$$\begin{aligned} a_f(\theta) &= \left(1 - \frac{\gamma_{\text{dry}}^2}{\gamma_{\text{sat}}^2} \right) \frac{\sec^2 \theta}{4}, \quad a_\mu(\theta) \\ &= \frac{\gamma_{\text{dry}}^2 \sec^2 \theta}{\gamma_{\text{sat}}^2 4} - \frac{2}{\gamma_{\text{sat}}^2} \sin^2 \theta, \quad \text{and } a_\rho(\theta) = \frac{1}{2} - \frac{\sec^2 \theta}{4}, \end{aligned} \quad (10)$$

which is consistent with the result of Russell et al. [16], and the bar represents an average of the physical properties at upper and lower interfaces, and the symbol Δ represents the contrasts in physical properties across the interface.

The anisotropic part of the reflection coefficient of a TTI medium in Equation (8) can be written as

$$R_{\text{PP}}^{\text{ani}}(\theta, \phi; \boldsymbol{\delta}) = a_{\delta_N}(\theta, \phi; \zeta) \Delta \delta_N + a_{\delta_T}(\theta, \phi; \zeta) \Delta \delta_T, \quad (11)$$

where

$$a_{\delta_N}(\theta, \phi; \zeta) = -\frac{\sec^2\theta}{4} \left[\begin{aligned} &(\chi_{\text{sat}} \cos^2\zeta + \sin^2\zeta)^2 \sin^4\theta \cos^4\phi + (\cos^2\zeta + \chi_{\text{sat}} \sin^2\zeta)^2 \cos^4\theta \\ &+ 2\chi_{\text{sat}}(\chi_{\text{sat}} \cos^2\zeta + \sin^2\zeta) \sin^4\theta \sin^2\phi \cos^2\phi \\ &+ 2\chi_{\text{sat}}(\cos^2\zeta + \chi_{\text{sat}} \sin^2\zeta) \sin^2\theta \cos^2\theta \sin^2\phi \\ &+ \chi_{\text{sat}}^2 \sin^4\theta \sin^4\phi + \left(2\chi_{\text{sat}} - \frac{(1 - \chi_{\text{sat}})^2 \sin^2 2\zeta}{2}\right) \sin^2\theta \cos^2\theta \cos^2\phi \end{aligned} \right], \quad (12)$$

$$a_{\delta_T}(\theta, \phi; \zeta) = \frac{1}{\gamma_{\text{sat}}^2} \left[\begin{aligned} &\cos^2 2\zeta (\sin^2\theta \cos^2\phi) - \frac{1}{4} \sin^2 2\zeta \left(\frac{\sin^2\theta \tan^2\theta \cos^4\phi}{+\cos^2\theta - 2 \sin^2\theta \cos^2\phi} \right) \\ &+ \cos^2\zeta (\sin^2\theta \sin^2\phi) - \sin^2\zeta (\sin^2\theta \tan^2\theta \sin^2\phi \cos^2\phi) \end{aligned} \right],$$

and $\delta^T = (\delta_N, \delta_T)$ represents the vector of fracture weaknesses of rotationally invariant fractures.

To demonstrate the sensitivity of the PP-wave reflection coefficient to the fluid content, three physical properties of an interface separating an isotropic overburden from a tilted fractured medium are given in Table 1. We first assume that the dip angle of the symmetry axis is 60° , and three kinds of titled fractured reservoirs are dry (or gas-saturated), partially saturated, and fully saturated, respectively. Figures 2(a)–2(c) demonstrate that the reflection amplitudes are sensitive to the fluid content, especially for the gas-filled reservoir. Then, we assume that the tilted fracture reservoir is dry, and test the influence of dip angle on the reflection coefficient. Figures 3(a)–3(f) show the effect of the dip angle of the symmetry axis in a TTI medium on the reflection amplitude, and Figures 4(a) and 4(b) show the relative error in PP-wave reflection coefficient if a TTI medium with the dip angle of the symmetry axis ($\zeta = 60^\circ$) is modelled as an HTI ($\zeta = 90^\circ$) or a VTI ($\zeta = 0^\circ$) model. We find that the PP-wave reflection amplitudes exhibit significant effect on the dip angle of the symmetry axis, and the estimate of AVOAz in a TTI medium may be wrong if we neglect the dip angle of the symmetry axis.

In the case of $\zeta = 0^\circ$, Equation (8) reduces to the linearized PP-wave reflection coefficient of a VTI medium with a symmetry axis along the x_3 direction, which can be expressed as

$$R_{\text{PP}}^{\text{VTI}}(\theta, \phi; \zeta = 0^\circ) = a_f(\theta) \frac{\Delta f}{f} + a_\mu(\theta) \frac{\Delta \mu}{\mu} + a_\rho(\theta) \frac{\Delta \rho}{\rho} + a_{\delta_N}(\theta; \zeta = 0^\circ) \Delta \delta_N + a_{\delta_T}(\theta; \zeta = 0^\circ) \Delta \delta_T, \quad (13)$$

where

$$a_{\delta_N}(\theta; \zeta = 0^\circ) = -\frac{\sec^2\theta}{4} \left[\frac{2}{\gamma_{\text{sat}}^2} \sin^2\theta - 1 \right]^2, \quad a_{\delta_T}(\theta; \zeta = 0^\circ) = \frac{1}{\gamma_{\text{sat}}^2} \sin^2\theta. \quad (14)$$

For $\zeta = 90^\circ$, Equation (8) reduces to the linearized PP-wave reflection coefficient of an HTI medium with a symmetry axis along the x_1 direction, which can be expressed as

$$R_{\text{PP}}^{\text{HTI}}(\theta, \phi; \zeta = 90^\circ) = a_f(\theta) \frac{\Delta f}{f} + a_\mu(\theta) \frac{\Delta \mu}{\mu} + a_\rho(\theta) \frac{\Delta \rho}{\rho} + a_{\delta_N}(\theta, \phi; \zeta = 90^\circ) \Delta \delta_N + a_{\delta_T}(\theta, \phi; \zeta = 90^\circ) \Delta \delta_T, \quad (15)$$

where

$$a_{\delta_N}(\theta, \phi; \zeta = 90^\circ) = -\frac{\sec^2\theta}{4} \left[\frac{2}{\gamma_{\text{sat}}^2} (\sin^2\theta \sin^2\phi + \cos^2\theta) - 1 \right]^2, \\ a_{\delta_T}(\theta, \phi; \zeta = 90^\circ) = \frac{1}{\gamma_{\text{sat}}^2} \sin^2\theta \cos^2\phi (1 - \tan^2\theta \sin^2\phi). \quad (16)$$

Equation (13) is exactly the same as that of an HTI medium published in Pan et al. [15].

2.3. Azimuthally Anisotropic EI Parameterization of a TTI Medium. Following Connolly [20] and Martins [23], the PP-wave reflection coefficient of a horizontal interface separating two weakly TTI media can be expressed using the EI terms, which is given by

$$2R_{\text{PP}}^{\text{TTI}}(\theta, \phi; \zeta) \approx \frac{\Delta \text{EI}_{\text{PP}}(\theta, \phi; \zeta)}{\overline{\text{EI}}_{\text{PP}}(\theta, \phi; \zeta)} \approx 2a_f(\theta) \frac{\Delta f}{f} + 2a_\mu(\theta) \frac{\Delta \mu}{\mu} + 2a_\rho(\theta) \frac{\Delta \rho}{\rho} + 2a_{\delta_N}(\theta, \phi; \zeta) \Delta \delta_N + 2a_{\delta_T}(\theta, \phi; \zeta) \Delta \delta_T. \quad (17)$$

where the symbol $\text{EI}_{\text{PP}}(\cdot)$ represents the azimuthally anisotropic EI of a TTI medium, and $\Delta \text{EI}_{\text{PP}}(\cdot)$ represents the perturbation in azimuthally anisotropic EI; the bar of EI term denotes the average values of the upper and lower interfaces.

TABLE 1: Physical properties of an interface separating an isotropic overburden from a tilted fractured medium (modified from Shaw and Sen, 2005).

Medium		ρ (g/cc)	μ (GPa)	f (GPa)	δ_N (\)	δ_T (\)
Isotropic overburden		2.21	3.18	2.99	—	—
Tilted fractured reservoir	Dry	2.00	2.00	3.33	0.15	0.10
	Partially filled				0.03	0.10
	Fluid filled				0.00	0.15

FIGURE 2: The sensitivity of PP-wave reflection coefficient to the fluid content of titled fractured reservoirs with fixed dip angle, where (a) is dry or gas-saturated, (b) is partially saturated, and (c) is fully saturated.

Equation (17) demonstrates that the relative contrast in $EI_{PP}(\cdot)$ can be expressed as a sum of the relative contrasts in the physical properties of a TTI medium.

Following Martins [23] and Pan et al. [24], Equation (17) can lead to

$$EI_{PP}^{TTI}(\theta, \phi; \zeta) = EI_{PP}^{iso}(\theta) EI_{PP}^{ani}(\theta, \phi; \delta), \quad (18)$$

where the isotropic background part of azimuthally anisotropic EI (Equation (18)) of a TTI medium is written as

$$EI_{PP}^{iso}(\theta) = EI_0 \left(\frac{f}{f_0} \right)^{2a_f(\theta)} \left(\frac{\mu}{\mu_0} \right)^{2a_\mu(\theta)} \left(\frac{\rho}{\rho_0} \right)^{2a_\rho(\theta)}, \quad (19)$$

and in which the normalizing constant $EI_0 = [\gamma_{sat}^2 / \gamma_{dry}^2 / \gamma_{sat}^2 / \gamma_{dry}^2 - 1]^{1/4} (\gamma_{sat}^2)^{1/4} f_0^{1/2} \mu_0^{1/4} \rho_0^{1/4}$ [26].

The anisotropic part of azimuthally anisotropic EI (Equation (18)) of a TTI medium is written as

$$EI_{PP}^{ani}(\theta, \phi; \delta) = \exp [2a_{\delta_N}(\theta, \phi; \zeta) \delta_N + 2a_{\delta_T}(\theta, \phi; \zeta) \delta_T], \quad (20)$$

where $\exp(\cdot)$ represents an exponential function.

Similarly, Figures 5(a)–(c) show the sensitivity of a PP-wave azimuthally anisotropic EI Equation (18) for dry, partially saturated, and fully saturated fractured reservoirs with the dip angle of symmetry axis being 60° , and Figures 6(a)–

FIGURE 3: The effect of dip angle of symmetry axis in a TTI medium on the PP-wave reflection coefficient. (a) 0° (VTI), (b) 30° , (c) 45° , (d) 60° , (e) 75° , and (f) 90° (HTI).

6(f) show the effect of the dip angle of the symmetry axis on the azimuthally anisotropic EI. Figures 7(a) and 7(b) show the relative error in azimuthally anisotropic EI if a TTI medium ($\zeta = 60^\circ$) is modelled as an HTI ($\zeta = 90^\circ$) or a VTI ($\zeta = 0^\circ$) model. The numerical results about the effects of the

dip angle of the symmetry axis on these azimuthally anisotropic EIs demonstrate that the dip angle of the symmetry axis of a TTI medium should be concerned when we estimate the fracture weaknesses based on the analysis of EI variations with angles of incidence and azimuth.

FIGURE 4: The relative error in PP-wave reflection coefficient if a TTI medium with dip of symmetry axis is modelled as an (a) HTI medium or a (b) VTI medium.

FIGURE 5: The sensitivity of PP-wave azimuthally anisotropic EI to the fluid content of titled fractured reservoirs with fixed dip angle, where (a) is dry or gas-saturated, (b) is partially saturated, and (c) is fully saturated.

FIGURE 6: The effect of dip angle of symmetry axis in a TTI medium on the PP-wave azimuthally anisotropic EI, where (a) 0° (VTI), (b) 30°, (c) 45°, (d) 60°, (e) 75°, and (f) 90° (HTI).

FIGURE 7: The relative error in azimuthally anisotropic EI if a TTI medium with dip of symmetry axis is modelled as an (a) HTI medium or a (b) VTI medium.

For $\zeta = 0^\circ$, Equation (18) reduces to the EI of a VTI medium with a symmetry axis along the x_3 direction, which can be expressed as

$$\begin{aligned} EI_{PP}^{VTI}(\theta; \zeta = 0^\circ) = & EI_0 \left(\frac{f}{f_0} \right)^{2a_f(\theta)} \left(\frac{\mu}{\mu_0} \right)^{2a_\mu(\theta)} \left(\frac{\rho}{\rho_0} \right)^{2a_\rho(\theta)} \\ & \cdot \exp [2a_{\delta_N}(\theta; \zeta = 0^\circ)\delta_N \\ & + 2a_{\delta_T}(\theta; \zeta = 0^\circ)\delta_T]. \end{aligned} \quad (21)$$

For $\zeta = 90^\circ$, Equation (18) reduces to the EI of an HTI medium with a symmetry axis along the x_1 direction, which can be expressed as

$$\begin{aligned} EI_{PP}^{HTI}(\theta, \phi; \zeta = 90^\circ) = & EI_0 \left(\frac{f}{f_0} \right)^{2a_f(\theta)} \left(\frac{\mu}{\mu_0} \right)^{2a_\mu(\theta)} \left(\frac{\rho}{\rho_0} \right)^{2a_\rho(\theta)} \\ & \cdot \exp [2a_{\delta_N}(\theta, \phi; \zeta = 90^\circ)\delta_N \\ & + 2a_{\delta_T}(\theta, \phi; \zeta = 90^\circ)\delta_T]. \end{aligned} \quad (22)$$

2.4. Azimuthally Anisotropic EI Inversion for Fluid/Porosity Term and Fracture Weaknesses in a TTI Medium. To linearly invert the vector of model parameters $\mathbf{m}^T = (\ln f, \ln \mu, \ln \rho, \delta_N, \delta_T)$, Equation (18) can be expressed in the matrix form,

$$[\mathbf{L}_{EI}]_{MNL \times 1} = [\mathbf{X}]_{MNL \times 5L} [\mathbf{m}]_{5L \times 1}, \quad (23)$$

where M , N , and L are the numbers of incidence angles, azimuths, and samples, respectively; $\mathbf{L}_{EI}$ denotes the vector of logarithmic EI data, which can be estimated using the method of the constrained sparse spike inversion (CSSI); $\mathbf{X}$ denotes the forward operator matrix related to the coefficient weighting matrix; and $\mathbf{m}$ denotes the vector of model parameters, which are given by

$$\begin{aligned} [\mathbf{L}_{EI}]_{MNL \times 1} &= [\mathbf{LEI}(\theta_1, \phi_1; \zeta), \dots, \mathbf{LEI}(\theta_i, \phi_j; \zeta), \dots, \\ &\quad \mathbf{LEI}(\theta_M, \phi_N; \zeta)]^T, \\ [\mathbf{X}]_{MNL \times 5L} &= [\mathbf{A}_f \quad \mathbf{A}_\mu \quad \mathbf{A}_\rho \quad \mathbf{A}_{\delta_N} \quad \mathbf{A}_{\delta_T}], \\ [\mathbf{m}]_{5L \times 1} &= [\mathbf{L}_f \quad \mathbf{L}_\mu \quad \mathbf{L}_\rho \quad \delta_N \quad \delta_T]^T; \\ [\mathbf{LEI}(\theta_i, \phi_j; \zeta)]_{L \times 1} &= \left[\ln \frac{\text{EI}(t_1, \theta_i, \phi_j; \zeta)}{\text{EI}_0}, \dots, \right. \\ &\quad \left. \ln \frac{\text{EI}(t_L, \theta_i, \phi_j; \zeta)}{\text{EI}_0} \right]^T, \\ [\mathbf{A}_f]_{MNL \times L} &= [\mathbf{a}_f(\theta_1), \dots, \mathbf{a}_f(\theta_i), \dots, \mathbf{a}_f(\theta_M)]^T, \\ [\mathbf{A}_\mu]_{MNL \times L} &= [\mathbf{a}_\mu(\theta_1), \dots, \mathbf{a}_\mu(\theta_i), \dots, \mathbf{a}_\mu(\theta_M)]^T, \\ [\mathbf{A}_\rho]_{MNL \times L} &= [\mathbf{a}_\rho(\theta_1), \dots, \mathbf{a}_\rho(\theta_i), \dots, \mathbf{a}_\rho(\theta_M)]^T, \\ [\mathbf{A}_{\delta_N}]_{MNL \times L} &= [\mathbf{a}_{\delta_N}(\theta_1, \phi_1; \zeta), \dots, \mathbf{a}_{\delta_N}(\theta_i, \phi_j; \zeta), \dots, \\ &\quad \mathbf{a}_{\delta_N}(\theta_M, \phi_N; \zeta)]^T, \\ [\mathbf{A}_{\delta_T}]_{MNL \times L} &= [\mathbf{a}_{\delta_T}(\theta_1, \phi_1; \zeta), \dots, \mathbf{a}_{\delta_T}(\theta_i, \phi_j; \zeta), \dots, \\ &\quad \mathbf{a}_{\delta_T}(\theta_M, \phi_N; \zeta)]^T; \\ [\mathbf{L}_f]_{L \times 1} &= \left[\ln \frac{f(t_1)}{f_0}, \dots, \ln \frac{f(t_L)}{f_0} \right]^T, \\ [\mathbf{L}_\mu]_{L \times 1} &= \left[\ln \frac{\mu(t_1)}{\mu_0}, \dots, \ln \frac{\mu(t_L)}{\mu_0} \right]^T, \\ [\mathbf{L}_\rho]_{L \times 1} &= \left[\ln \frac{\rho(t_1)}{\rho_0}, \dots, \ln \frac{\rho(t_L)}{\rho_0} \right]^T, \\ [\delta_N]_{L \times 1} &= [\delta_N(t_1), \dots, \delta_N(t_L)]^T, \\ [\delta_T]_{L \times 1} &= [\delta_T(t_1), \dots, \delta_T(t_L)]^T; \\ [\mathbf{a}_f(\theta_i)]_{L \times L} &= \text{diag} [2a_f(t_1, \theta_i), \dots, 2a_f(t_L, \theta_i)], \end{aligned}$$

$$[\mathbf{a}_\mu(\theta_i)]_{L \times L} = \text{diag} [2a_\mu(t_1, \theta_i), \dots, 2a_\mu(t_L, \theta_i)],$$

$$[\mathbf{a}_\rho(\theta_i)]_{L \times L} = \text{diag} [2a_\rho(t_1, \theta_i), \dots, 2a_\rho(t_L, \theta_i)],$$

$$\begin{aligned} [\mathbf{a}_{\delta_N}(\theta_i, \phi_j)]_{L \times L} &= \text{diag} [2a_{\delta_N}(t_1, \theta_i, \phi_j), \dots, \\ &\quad 2a_{\delta_N}(t_L, \theta_i, \phi_j)], \end{aligned}$$

$$\begin{aligned} [\mathbf{a}_{\delta_T}(\theta_i, \phi_j)]_{L \times L} &= \text{diag} [2a_{\delta_T}(t_1, \theta_i, \phi_j), \dots, \\ &\quad 2a_{\delta_T}(t_L, \theta_i, \phi_j)]. \end{aligned}$$

(24)

Following Pan et al. [24], we use a decorrelation matrix $\mathbf{E}$ to implement the process of decorrelation among model parameters, and then $\tilde{\mathbf{X}} = \mathbf{X}\mathbf{E}^{-1}$ and $\tilde{\mathbf{m}} = \mathbf{E}\mathbf{m}$, in which the matrix $\mathbf{E}^{-1}$ denotes the inverse of a matrix. And after decorrelation, Equation (23) is given by

$$[\mathbf{L}_{EI}]_{MNL \times 1} = [\tilde{\mathbf{X}}]_{MNL \times 5L} [\tilde{\mathbf{m}}]_{5L \times 1}. \quad (25)$$

Using the Bayesian framework [24], the posterior probability density function (PDF) $p(\tilde{\mathbf{m}}|\mathbf{L}_{EI})$ is jointly resolved by integrating a Cauchy prior PDF $p(\tilde{\mathbf{m}})$ and a Gaussian likelihood function $p(\mathbf{L}_{EI}|\tilde{\mathbf{m}})$,

$$\begin{aligned} p(\tilde{\mathbf{m}}|\mathbf{L}_{EI}) &\propto p(\tilde{\mathbf{m}}) \cdot p(\mathbf{L}_{EI}|\tilde{\mathbf{m}}) \\ &\approx \prod_{i=1}^{5L} \left[\frac{1}{1 + \tilde{\mathbf{m}}_i^2 / \sigma_{\tilde{\mathbf{m}}}^2} \right] \cdot \exp \left[-\frac{(\tilde{\mathbf{X}}\tilde{\mathbf{m}} - \mathbf{L}_{EI})^T (\tilde{\mathbf{X}}\tilde{\mathbf{m}} - \mathbf{L}_{EI})}{2\sigma_{EI}^2} \right], \end{aligned} \quad (26)$$

where $p(\cdot)$ represents a PDF, σ_{EI}^2 is the variance of EI noises, and $\sigma_{\tilde{\mathbf{m}}}^2$ is the variance of model parameters. Maximizing Equation (26) and combining the initial-model-based low-frequency constrained regularization gives

$$\begin{aligned} \Psi(\tilde{\mathbf{m}}) &= (\tilde{\mathbf{X}}\tilde{\mathbf{m}} - \mathbf{L}_{EI})^T (\tilde{\mathbf{X}}\tilde{\mathbf{m}} - \mathbf{L}_{EI}) \\ &\quad + 2\sigma_{EI}^2 \sum_{j=1}^{5L} \ln(1 + \tilde{\mathbf{m}}_j / \sigma_{\tilde{\mathbf{m}}}^2) \\ &\quad + \sum_{i=1}^5 \tau_{\tilde{\mathbf{m}}_i} (\Sigma_{\tilde{\mathbf{m}}_i} - \Gamma_{\tilde{\mathbf{m}}_i} \tilde{\mathbf{m}}_i)^T (\Sigma_{\tilde{\mathbf{m}}_i} - \Gamma_{\tilde{\mathbf{m}}_i} \tilde{\mathbf{m}}_i), \end{aligned} \quad (27)$$

where $\Psi(\cdot)$ is the final objective function; $\tau_{\tilde{\mathbf{m}}_i}$ is the low-frequency constrained regularization coefficient of model parameters; $\Gamma_{\tilde{\mathbf{m}}_i} = \int_{t_1}^{t_i} \cdot dt$; and $\Sigma_{\tilde{\mathbf{m}}_i} = 1/2 * \ln(\tilde{\mathbf{m}}_i / \tilde{\mathbf{m}}_{i0})$, in which $\tilde{\mathbf{m}}_{i0}$ represents the initial values of model parameters. We then solve the objective function Equation (27),

$$[\mathbf{\Lambda}]_{5L \times 1} = [\mathbf{Z}]_{5L \times 5L} [\tilde{\mathbf{m}}]_{5L \times 1}, \quad (28)$$

FIGURE 8: Synthetic azimuthal seismic angles with different SNRs, where (a) is noise free, (b) is 5, and (c) is 2. Note that the four azimuths are 0° , 45° , 90° , and 135° , respectively.

FIGURE 9: Comparison between the original (blue), initial (green), and inverted (red) fluid/porosity term, shear modulus, density, and fracture weaknesses using the synthetic data with different SNRs, where (a) is noise free, (b) is 5, and (c) is 2.

FIGURE 10: Continued.

FIGURE 10: Partially incidence-angle-stacked seismic data with five azimuths, where (a) is the near-angle seismic data, (b) is the middle-angle seismic data, and (c) is the far-angle seismic data. Note that the five azimuths are 20° (ranged from 0° to 40°), 55° (ranged from 35° to 75°), 90° (ranged from 70° to 110°), 125° (ranged from 105° to 145°), and 160° (ranged from 140° to 180°), respectively.

where

$$\mathbf{Z} = \tilde{\mathbf{X}}^T \tilde{\mathbf{X}} + 2\sigma_{\text{EI}}^2 / \sigma_{\tilde{\mathbf{m}}}^2 \mathbf{Q}_{\text{Cauchy}} + \sum_{i=1}^5 \tau_{\tilde{\mathbf{m}}_i} \mathbf{\Gamma}_{\tilde{\mathbf{m}}_i}^T \mathbf{\Gamma}_{\tilde{\mathbf{m}}_i} \quad (29)$$

$$\mathbf{\Lambda} = \tilde{\mathbf{X}}^T \mathbf{L}_{\text{EI}} + \sum_{i=1}^5 \tau_{\tilde{\mathbf{m}}_i} \mathbf{\Gamma}_{\tilde{\mathbf{m}}_i}^T \mathbf{\Sigma}_{\tilde{\mathbf{m}}_i}, \quad (30)$$

and in which $\mathbf{Q}_{\text{Cauchy}}$ denotes the Cauchy sparse matrix.

We finally use the iteratively reweighted least-squares (IRLS) algorithm to solve Equation (28) [24].

3. Examples

Synthetic and real data acquired from a work area in the Sichuan Basin are used to verify the reliability and stability of our proposed inversion approach. The tectonic location is located at the western end of the large uplift belt in the middle section of West Sichuan depression. The porosity of the target reservoir mainly distributes in the range of 2–4% with an average of 3.75%, and the permeability is generally lower than $0.1 \times 10^{-3} \mu\text{m}^2$. It is a tight reservoir of ultra-low-porosity and ultra-low-permeability, in which the fractures play an important role in improving the production of the gas reservoir. Due to massive faults, the subsurface fractures in this area is mainly north-south direction. From the distribution diagram of core fracture occurrence, the low-angle fractures are dominant, which account for about 80% of the

total. So, we suppose that the tilted angle ζ of fractured rocks in this area is about 45° based on the known microresistivity image logging (FMI). In general, the fractured rocks in this area can be equivalent to a TTI model. Then, we use the proposed azimuthally anisotropic EI inversion to perform the fluid detection in an equivalent TTI medium.

A well log model is first used to synthesize the azimuthal angle gathers, and Figure 8(a) shows the noise-free azimuthal angle gathers. We add random noise to the synthetic seismic gathers, and Figures 8(b) and 8(c) show the generated seismic gathers. Figures 9(a)–9(c) show the inversion results of fluid/porosity term, shear modulus, density, and fracture weaknesses using the synthetic data with different SNRs (signal-to-noise ratios). From the inversion results, we observe that the model parameters are estimated well in the case that the synthetic data contains no noises or moderate noises. In the case that the data contains a great deal of random noise (SNR is relatively lower), the fluid parameter is estimated well; however, the fracture parameters cannot be inverted reasonably.

Next, we use the real data in this area to further validate the feasibility of the proposed azimuthally anisotropic EI inversion approach. Figures 10(a)–10(c) show the partially near, middle, and far incidence-angle-stacked seismic data at five azimuths (five azimuths are 20° , 55° , 90° , 125° , and 160°). In Figure 10, the red line indicates the location of well drilling. To estimate the fluid and fracture parameters, we first use the partially incidence-angle-stacked seismic data to invert the azimuthally anisotropic EI data with five

FIGURE 11: Continued.

FIGURE 11: Inverted azimuthally anisotropic EI data with five azimuths, where (a) is the near-angle EI data, (b) is the middle-angle EI data, and (c) is the far-angle EI data. Note that the five azimuths are 20° (ranged from 0° to 40°), 55° (ranged from 35° to 75°), 90° (ranged from 70° to 110°), 125° (ranged from 105° to 145°), and 160° (ranged from 140° to 180°), respectively.

FIGURE 12: Inverted background fluid and elastic parameters of a TTI medium, where (a) shows the fluid/porosity term, (b) shows the shear modulus, and (c) shows the density.

FIGURE 13: Inverted fracture weaknesses of a TTI medium, where (a) shows the normal fracture weakness and (b) shows the tangential fracture weakness.

azimuths, and Figures 11(a)–11(c) show the inverted EI data. We observe that the inverted EI data are geologically reasonable and satisfying. Finally, we use the inverted EI data to estimate the fluid and fracture parameters. Figures 12(a)–12(c) show the inverted fluid/porosity term, shear modulus, and density, respectively. Figures 13(a) and 13(b) show the inverted normal and tangential fracture weaknesses. From the inversion results, we observe that the fluid/porosity term can be used to demonstrate the gas-bearing reservoir; however, the vertical resolution in this area is not fair enough to perform good fluid identification.

4. Conclusion

To perform the fluid detection in a TTI medium, we derive the PP-wave azimuthal reflection coefficient and EI equations in terms of fluid/porosity term and fracture weaknesses. Before the inversion process, we first get the prior information of fracture normal and tilt angle. We then implement a two-step inversion approach to estimate the fluid/porosity term and fracture weaknesses in a TTI medium, which involves (1) the estimation of azimuthally anisotropic EI data from the azimuthal seismic data and (2) the estimation of fluid/porosity term and fracture weaknesses from the inverted EI data. Synthetic data is used to validate the feasibility of the proposed two-step azimuthal anisotropic EI inversion approach. Real data demonstrates that we can perform fluid detection and fracture characterization in a gas-saturated TTI medium with the estimated fluid/porosity term and fracture weaknesses. Of course, this approach can be extended to an oil-bearing fractured reservoir with TTI symmetry.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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