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Review Article

Fracture Behavior of Rock with Initial Damage: Theoretical, Experimental, and Numerical Investigations

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Geomaterials such as rock mass often have initial damage under the influence of long-term geological action and hydration corrosion environment. The initial damage affects the integrity and stability of the rock mass, resulting in a difference in the mechanical properties of jointed rock mass and intact one. Therefore, the study of the fracture and failure characteristics of the jointed rock mass is of great significance. Most of the previous researches into the fracture behavior of rock with initial damage are based on model testing, theoretical analysis, and numerical simulation of rock mass with preexisting flaws. This review concentrates on the theoretical, experimental, and numerical efforts that have been devoted to the fracture characteristics of rock or rock-like specimens with preexisting flaws under compression. Some suggestions on the future research work in this field are also given.

1. Introduction

In practical engineering structures, natural rock masses often have initial damage such as joints and fissures under the influence of long-term geological action and hydration corrosion environment. These defects affect the integrity of the rock, resulting in nonlinear, heterogeneous, anisotropy, and other characteristics of the rock mass [1, 2]. The mechanical properties of fissured rock mass are significantly different from that of intact rock. Under the action of crustal stress and external load, stress concentration will appear in defects of rock mass, leading to the initiation, extension, and coalescence of new cracks and finally the destruction of the rock mass [3–7]. Therefore, the study of the fracture and failure characteristics of jointed rock mass is of great significance to engineering.

Most researches on the fracture and mechanism characteristics of jointed rock mass are based on the study of the rock model with preexisting flaws [8–13]. And the present researches mainly focus on the exploration of crack initiation criteria, small model experiment, and numerical simulation. Plenty of scholars have devoted their efforts to this research area and have achieved fruitful results [14–17]. However, with the continuous accumulation of research results in this area, in order to avoid repeated work or meaningless work, it is necessary to analyze and summarize these research results. An effective summary can extract general laws from existing research and use them to guide practice. On the other hand, it can also find vacancies and deficiencies from the existing research, so as to point out the direction for the follow-up research of scholars.

Considering that there are few summary works on the fracture behavior of rock with initial damage at present, this review briefly concludes these achievements in theoretical, experimental, and numerical aspects aiming to keep readers informed of the research progress in this field and give them suggestions on the problems and the future research direction.

2. Theoretical Achievements on Fracture Criteria

Fracture criteria and crack initiation mechanism of flaws under compression are essential problems in rock fracture mechanics. Inclined preexisting flaws in rock models are often under the compression and shear stress condition, and the fracture in these rock models under compression is often a kind of mixed-mode fracture. At present, the most basic and commonly used mixed-mode fracture criteria are the maximum tangential stress criterion [18, 19], maximum energy release rate criterion [20–22], and minimum strain energy density factor criterion [23]. Many other criteria are developed based on them [24].

The maximum tangential stress criterion theory is the most widely used criterion which can be described as the following: (1) crack initiates along the direction of maximum tangential stress, and (2) when the maximum tangential stress at the flaw tip reaches a critical value, the crack initiates. The corresponding formula is as follows:

$$\left.\frac{\partial \sigma_{\theta\theta}}{\partial \theta}\right|_{\theta=\theta_0} = 0, \qquad \left.\frac{\partial^2 \sigma_{\theta\theta}}{\partial \theta^2}\right|_{\theta=\theta_0} < 0, \tag{1}$$

$$\sigma_{\theta\theta}|_{\theta=\theta_0} = \sigma_c, \tag{2}$$

where $\sigma_{\theta\theta}$ is the tangential stress at the flaw tip and σ_c is the critical value of the tangential stress. The analysis results can well reflect the differences of fracture types of cracks under the action of tension and compression, and the results obtained are in good agreement with the experimental results, which makes it widely used in the theoretical research and practical engineering of rock failure [25–28].

Sih [23] proposed the minimum strain energy density factor criterion and described that the crack initiates along the direction of the minimum strain energy density factor when it reaches a critical value:

$$\left.\frac{\partial S}{\partial \theta}\right|_{\theta=\theta_0} = 0, \qquad \left.\frac{\partial^2 S}{\partial \theta^2}\right|_{\theta=\theta_0} > 0, \tag{3}$$

$$S|_{\theta=\theta_0} = S_c, \tag{4}$$

where S is the strain energy density factor and S_c is the critical value of the strain energy density factor. The application of this theory is convenient, and the two-dimensional model is in good agreement with the experimental results [29–31]. However, there are many controversies for the reason that the connection between this theory and the physical nature of material destruction is not clear.

The maximum energy release rate criterion is another energy-based criterion that is widely applied [32, 33]. It proposed that a crack initiates when the maximum energy release rate reaches a critical value which can be accepted readily at the physical level:

$$\left.\frac{\partial G_\theta}{\partial \theta}\right|_{\theta=\theta_0} = 0, \qquad \left.\frac{\partial^2 G_\theta}{\partial \theta^2}\right|_{\theta=\theta_0} < 0, \tag{5}$$

$$G|_{\theta=\theta_0} = G_c, \tag{6}$$

where G is the energy release rate and G_c is the critical value of the energy release rate. However, the value of the energy release rate G is not easy to obtain when a crack initiates not along the direction of the original flaw, and it is difficult to explain the fracture path observed by experiments.

With the development of rock fracture mechanics, there are many other theories which have been proposed based on the above classical criteria. Matvienko [34] proposed the maximum mean tangential stress (mean tensile stress along the line in front of the incision) theory, which suggested that the crack growth of rock-like materials always extends along the direction of the maximum mean circumferential stress in the area near the flaw. Khan and Khraisheh [35] proposed a modified maximum tangential stress criterion based on maximum tangential stress, taking into account the elastic-plastic boundary of the flaw tip. Shen [36, 37] proposed a modification of the G-criterion for crack propagation subjected to compression named "F-criteria."

These classical and modified criteria are mostly based on mode I fracture (tension fracture) and seldom consider the mode II fracture of compression-shear flaws. As the mode II fracture is nonignorable, many scholars have proposed their theories to predict this mode of fracture [38–40]. Sun [38, 39] analyzed the stress field of the flaw tip under pure shear, developed the criterion of maximum tangential tensile stress, and made it possible to judge the type of shear fracture. Based on the energy release rate criterion, Chang et al. [40] developed a more general composite fracture criterion, which could be used for I-II composite fracture problems, and most of the fracture criteria could be degraded by it.

However, until now, many fracture mechanics theories remain controversial, and there is a lack of a commonly applicable or generally accepted theory. Moreover, theoretical studies on rock fracture mainly focus on 2D crack propagation, and there are few studies on three-dimensional crack because of the complexity of this situation. Therefore, the theoretical research on rock fracture in the future should focus on exploring a more reasonable two-dimensional fracture theory and strengthening the research on three-dimensional crack growth.

3. Experimental Specimens and Results

Experimental research is a primary way to explore the failure mode and fracture mechanism of rock for its function in reflecting the actual situation of rock fracture progress more directly and veritably. In the past decades, scholars have applied many kinds of specimens that vary in materials, numbers, and types of preexisting two-dimensional flaws as shown in Table 1.

Undoubtedly, natural rocks are ideal materials for the experiment and should be the first choice for researchers [41, 42, 44–47, 49, 63, 64]. The most widely used rock materials are granite, sandstone, and marble. In addition, rock-like materials, such as cement mortar, gypsum, glass, and PMMA (polymethylmethacrylate), can also be applied to make specimens to simulate rocks for the reason that they have similar mechanical properties with rock and are easy to produce [50, 51, 54, 56–58].

Many scholars have employed the specimens containing different numbers of preexisting flaws to investigate the fracture characteristics of rock mass with different numbers of joints. Single-flaw specimens are often used to investigate crack initiation and propagation [3, 8, 10, 59], while multiflaw specimens are mainly employed to study the interaction of preexisting flaws and the coalescence after crack initiation and propagation [12, 55, 60]. A large number of experimental studies indicated that the crack initiation from the 2D preexisting flaws could be generally classified into two types [9, 39, 64]: wing (or primary) cracks and secondary cracks as shown in Figure 1. Wing cracks appear first, which are the tensile cracks that initiate from the tips of the flaw and propagate in a stable manner towards the direction of maximum compression. Secondary cracks are generally described as shear cracks or shear zones. Secondary cracks initiate from the tips of the flaw, and two directions are possible: (1) coplanar or quasicoplanar to the flaw and (2) with an inclination similar to the wing cracks but in the opposite direction (antiwing crack).

Apart from a single flaw, specimens containing two or more flaws were always employed to investigate the coalescence pattern of the cracks. Shen et al. [65] observed that the coalescence modes of the cracks are determined by the fracture-bridge inclination and indicated 5 types of coalescence. Bobet and Einstein [48] carried out uniaxial and biaxial compression tests on gypsum specimens with preexisting flaws and identified five different types of coalescence with a combination of tensile and shearing processes. Park and Bobet [60] found that in specimens with more than two flaws, coalescence could be produced by the linkage through wing cracks, secondary cracks, or their combination. Figure S1 presents some experimental results of rock specimens containing more than one flaw. It can be seen clearly that the preexisting flaws in the specimen are interrelated, and different numbers and locations of flaws will lead to different types of crack coalescence.

Due to the limitation of test conditions, most of the experimental studies mentioned above are about the initiation and propagation of two-dimensional flaws. However, most of the flaws in the actual rock mass are three-dimensional flaws, which are usually in the inner part of the rock mass. Moreover, most of the simulated materials used by scholars are opaque materials, making it difficult to directly observe the initiation and propagation process of rock fractures and understand the propagation state of cracks in the rock mass under different stress states in real time. Therefore, the researches of three-dimensional flaws have made slow progress in a long period of time. In the 1990s, Dyskin et al. [66–68] used resin materials to make specimens with three-dimensional surface flaws. Uniaxial and triaxial compression tests were then carried out on these specimens after treatment at low temperature. The experimental results show that the growth of the wing crack stops when it grows to a certain extent because the enveloped wing crack is generated at the edge of the prefabricated flaw. That is completely different from 2D situations. In 2004, Wong et al. [58] employed marble and PMMA specimens containing three-dimensional surface flaws to study the expansion mechanism of this kind of flaw. During the test, it was found that not only a wing crack but also a petal-shaped crack (Figure S2) which has not appeared in 2D situations appeared in the tip area of the flaw. This study shows that the propagation process of a surface crack is influenced by the material properties, specimen thickness, flaw depth, and flaw inclination. In 2016, Zhu et al. [69] developed a kind of transparent rock-like material with similar properties to rock to observe and study the propagation and connection mechanism of the internal three-dimensional flaw. The experimental results show that the secondary cracks have different propagation and transfixion modes under different bridge angles and flaw spacing. The secondary cracks observed in the test included wing cracks, antiwing cracks, and petal-like cracks under the action of tension and shear.

No matter in two-dimensional or three-dimensional cases, the numerous above-mentioned experimental results show that the initial damage in rock has a huge impact on its mechanical properties and cracking process, which is reflected in the following: (1) The strength of rock with initial damage is far less than that of intact rock. (2) The initial damage is the inducement of crack initiation. Under the action of load, almost all of the cracks initiate from the preexisting flaws. And (3) the number, location, and type of the initial flaws will determine the initiation, propagation, and coalescence of cracks that are induced by external stress, resulting in different failure modes of specimens.

Almost all of the experimental study on three-dimensional flaws found that there are many differences between three-dimensional flaws and two-dimensional flaws in rock mass, which indicates that an experimental test of three-dimensional flaws is extremely necessary for the study of actual rock mass flaws. In addition, most of the present three-dimensional experiments focus on three-dimensional surface flaws using completely homogeneous transparent materials. Considering that most flaws in the rock mass are located in the inside of the rock mass and the rock material

Table 1: Different kinds of specimens for rock fracture test.

Materials		Flaw type	Flaw number
Natural rocks	Granite	Open	1 [3, 8, 41], 2 [8, 12, 42], 3 [43]
	Sandstone	Closed	2 [44]
		Open	1 [3], 2 [45], 3 [46]
	Marble	Open	1 [13, 47, 48], 2 [49]
Rock-like materials	Cement mortar	Closed	Multiple [50, 51]
		Open	1 [52, 53], 2 [11, 54, 55], 3 [11], multiple [56, 57]
	PMMA	Closed	1 [8], 2 [8]
		Open	1 [8], 2 [8, 58, 59]
	Gypsum	Closed	1 [10, 13, 47], 2 [48, 60], 3 [60], multiple [60]
		Open	1 [10, 18, 47, 61], 2 [47, 60, 61], 3 [58, 60–62], multiple [60, 62]

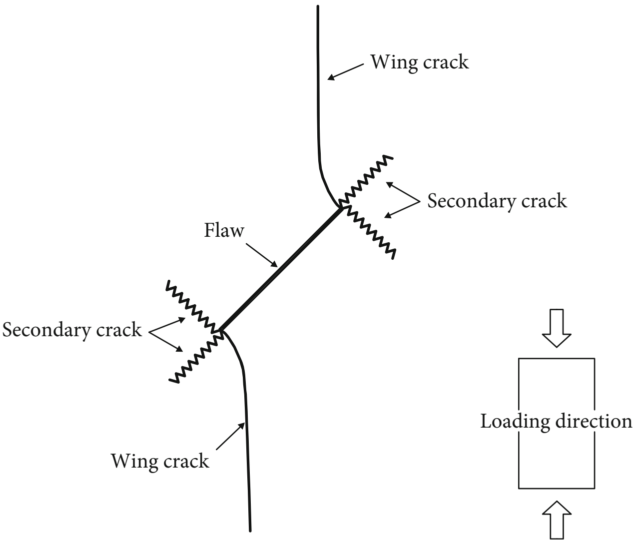

Figure 1: Simplified crack pattern in precracked specimens under uniaxial compression [10].

is heterogeneous, the future research in this field should focus on three-dimensional internal flaws and find more suitable materials to simulate the influence of heterogeneity on the fracture of the rock mass.

4. Numerical Simulation Methods

In recent years, with the deepening of mathematical and mechanical theories and the rapid development of computer technology, numerical simulation methods have been widely used in the theoretical research and engineering problem processing of geotechnical engineering [70–77]. The main numerical simulation methods for the jointed rock mass-related problem are the Finite Element Method (FEM), Finite Difference Method (FDM), Element-Free Method (EFM) and Boundary Element Method (BEM), Discrete Element Method (DEM), Numerical Manifold Method (NMM), and Discontinuous Deformation Analysis (DDA).

FEM is the widely used simulation method in the study of rock fracture mechanics [78, 79]. The solution of the stress field at the crack tip, the calculation of the stress intensity factor, and the J-integral can all be solved by FEM. Bittencourt et al. [80] used a local mesh adjustment technique to simulate crack propagation of linear elastic materials in the two-dimensional finite element program. Li and Wong [81] analyzed the influence of a preexisting flaw inclination angle on the initiation position and angle of the potential cracks by FEM analysis on the stress field distribution. However, FEM has many inconveniences in simulating the rock fracture process. For example, when analyzing crack growth and some extremely large deformation problems, it needs to constantly redraw the grid, thus increasing the workload. To avoid such problems, in 1999, Belytschko [82, 83] proposed a new method to deal with the discontinuity problem—Extended Finite Element Method (XFEM). In XFEM, the finite element mesh and crack are independent of each other, which makes it convenient to analyze discontinuities of cracked bodies, leading to its widespread application [84–86]. Xie et al. [87] used XFEM to investigate the crack initiation and propagation in rock-like material with closed fissure under uniaxial compression. Zhuang et al. [88] compared the fracture behavior of unfilled and filled preexisting flaws by XFEM and found that they were different in crack initiation stress and angle.

FLAC^{3D} is the most widely used FDM software which can solve many complex engineering problems that are difficult to be simulated by the finite element program because of its fast Lagrangian explicit finite-difference method. Fu et al. [89] imported a new modified elastic-brittle theory into FLAC^{3D} to simulate the fracture development of jointed rock mass, and the results are in good agreement with the experiment. By employing FLAC^{3D}, Guo et al. [90] perfectly captured the general features of brittle materials under compression including the fracturing process and AE events as well as stress-strain curves and found that peak stress and crack initiation stress are both heterogeneity dependent.

With the rapid development of computing technology, more and more scholars pay attention to DEM to simulate the crack initiation and propagation of rock materials [91]. DEM treats rock materials as a collective composed of a large number of tiny particles. For crack initiation and propagation, the PFC has been widely accepted by scholars, and the numerical results show great agreement with the experimental results [92–103]. Potyondy [92] introduced the basic principle of particle flow in detail and adopted the bonded particle model (BPM) to simulate the mechanical properties of rock material. Manouchehrian et al. [93] applied BPM in PFC2D to study the effect of the flaw orientation on the crack propagation mechanism in brittle materials such as rocks under various compressive loads. The numerical results show that the flaw inclination angle and confinement pressure have a strong influence on crack initiation and propagation behavior. Based on the parallel bond model, Zhang and Wong [94] investigated the crack initiation and propagation under uniaxial compressive loading and got similar results with Manouchehrian et al. [93].

Figure S3 shows a comparison of the simulation results of the three most commonly used methods for simulating rock fracture processes. The crack path can be seen clearly in XFEM, but restricted by the single selection of the cracking criteria, only wing cracks appear in the simulation results, and no other secondary cracks appear. In FLAC^{3D}, because of the indivisibility of the mesh, there is no real crack in the simulation results, only the plastic elements can be used to represent the rock failure, and the crack shown in the figure is composed of the plastic elements. Compared with the above two methods, PFC has a great advantage in simulating rock cracking. The fracture of the bond between particles is applied to simulate the rock fracture, which is more in line with the actual situation. In addition, it can simulate the initiation and propagation of tensile cracks and shear cracks simultaneously. This makes its simulation results closest to the experimental results in many simulation methods. Therefore, DEM that is represented by PFC gradually becomes the most widely employed method to simulate rock fracture behavior.

It is worth noting that although many researchers have employed numerical methods to simulate the compression of rock specimens with initial flaws, the simulation results often differ from the real situation. The main reason is that the present simulation methods often assume that the material is completely homogeneous in the simulation and do not consider the weakening of the mechanical properties of the material caused by the expansion of cracks in the compression process, which is quite different from the actual situation. Therefore, future research on numerical simulation should focus on solving the above two problems to make the simulation results more practical.

5. Discussion and Conclusions

The study of the fracture and failure characteristics of jointed rock mass is of considerable significance to engineering. This review summarizes the recent achievements of investigation into the fracture behaviors of rock with initial damage in theoretical, experimental, and numerical aspects, respectively, and gives some suggestions on future researches.

Many fracture criteria have been proposed that can describe the fracture behavior of precracked specimens under compression. However, until now, there is a lack of a commonly applicable or generally accepted theory. And there are few studies on three-dimensional flaws. Therefore, the theoretical research on rock fracture in the future should focus on exploring a more reasonable two-dimensional fracture theory and strengthening the research on three-dimensional crack growth.

Plenty of experimental results on rock specimens with preexisting flaws show that there are often two types of cracks in a 2D situation: wing crack and secondary crack. And the coalescence types of cracks in the specimen with multiple flaws are dependent on geometry of flaws. Moreover, many differences between three-dimensional flaws and two-dimensional flaws in rock mass have been found, which indicates that an experimental test of three-dimensional flaws is extremely necessary for the study of actual rock mass flaws. Therefore, future research in this field should focus on three-dimensional internal flaws and find more suitable materials to simulate the influence of heterogeneity on the fracture of the rock mass.

Numerical simulation is a beneficial supplement and verification of the experimental analysis and theoretical research. Different kinds of methods have been indicated that could substantially simulate the fracture behavior of specimens with preexisting flaws. DEM, which shows a substantial advantage of crack initiation and propagation, has been gradually accepted by more scholars. Future research on numerical simulation should focus on considering the heterogeneity of the material and the weakening of the mechanical properties of the material in the process of crack growth to make the simulation results more practical.

Conflicts of Interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Authors' Contributions

Hui Zhang provides the idea of the article and writes the text. Panpan Guo offers detailed guidance on the method of the article. Yixian Wang mainly collects the theoretical achievements of this research area. Yanlin Zhao, Lin Hang, Yan Liu, and Yahui Shao revise the final version of the paper.

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Supplementary Materials

Figure S1: some experimental results of rock specimens containing more than one flaw: (a) two flaws [8]; (b) three flaws [43]; (c) multiple flaws [51]. Figure S2: petal cracks in a specimen containing a 3D surface flaw observed by Wong et al. [58]. Figure S3: different simulation results of cracks in rock specimens: (a) XFEM [86]; (b) FLAC^{3D} [89]; (c) PFC [51]. *(Supplementary Materials)*

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