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Research Article

Study on Statistical Damage Theory Model of Tailings in a Metal Ore under the Action of Moisture Absorption and Dehumidification Circulation

Bing Cao,[1] Hua Wei,[2] Mei-Qi Dou,[3] Lin Hu,[3] and Yao-hui Guo[3]

[1]*Department of Agriculture and Forestry Engineering, Tangshan Vocational and Technical College, Tangshan, 063000 Hebei, China*
[2]*School of Geography and Tourism, Baoding University, Baoding 071000, China*
[3]*School of Resource & Environment and Safety Engineering, University of South China, 421001 Hengyang, China*

Correspondence should be addressed to Yao-hui Guo; 2698068623@qq.com

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In order to explore the variation law of mechanical properties of tailing sand of a metal ore in Hunan Province under the action of moisture absorption and dehumidification circulation, based on the indoor triaxial test results of tailing sand, this paper introduces the statistical damage theory, combined with the physical and mechanical properties of tailing sand, and studies the elastic-plastic mechanical properties of tailing sand on the basis of the Lemaitre strain equivalent theory. Giving full consideration of the change of tailing sand's pore and volume in the deformation process, an improved statistical damage constitutive model is proposed; it can reflect that the residual strength of tailings after the peak value still has bearing capacity under the action of dry wet circulation. Compared with the results of indoor triaxial consolidation undrained test, this constitutive model is more reliable.

1. Introduction

Mineral resources are the material basis for human survival. Many scholars at home and abroad have done a lot of research on mining methods of mineral resources [1, 2]. According to the important premise of China's economic development, the development of mineral resources is gradually increasing, which will produce a large number of tailing waste. Most of the treatment methods of tailing waste are to store them in the open-air tailing pond. In the open-air environment, tailings are often exposed to wind, sunlight, rain, and snow and are in repeated moisture absorption and dehumidification state. When tailings are in this environment for a long time, its internal structure and mechanical properties will change irreversibly, and the degree of mechanical characteristic change gradually increases with the increase of the number of moisture absorption and dehumidification circulations [3].

In 1958, the Soviet plastic mechanics expert Kachanov [4, 5] proposed the concept of "effective stress factor." On this basis, in 1963, the Soviet expert Rabotnov [6, 7] proposed the concept of "damage factor." Later, after unremitting efforts of more and more scholars, in 1977, Lemaitre [8], Chaboche [9], and others used the continuous medium method, according to the irreversible thermodynamic principles, and established the subject of damage mechanics. Based on that, foreign scholars Frantziskonis and Desai [10] proposed that the damaged material can be abstracted into two parts: damaged and undamaged. Xia et al. [11] proposed the expression of microunit strength of soil structure contact surface based on the Mohr-Coulomb yield criterion under plane strain condition and established a statistical damage softening constitutive model of soil structure contact surface on the basis of assuming that the microunit strength obeys Weibull distribution. Based on the Mohr criterion and combined with the theory of damage mechanics, Jiang et al. [12] established a damage constitutive model of rock under triaxial compression; compared with the experimental results, it was found that the simulation of stress-strain relationship curve achieved good effect. Yin et al. [13] adopted the Lade-Duncan and Drucker-Prager criteria to measure

Table 1: Tailing particle group.

Particle composition parameters				
Effective diameter d_{10}/mm	Median size d_{30}/mm	Constrained size d_{60}/mm	Coefficient of nonuniformity C_u	Coefficient of curvature C_c
0.0780	0.1620	0.2000	2.5320	1.6610

the microunit strength of rock and established a statistical damage constitutive model of rock and modified the model. Cao and Zhang [14] discussed the influence of Weibull distribution parameters on the statistical constitutive model of rock damage based on the Mohr-Coulomb criterion and modified the model in combination with the characteristics of triaxial stress-strain curve. Shen [15, 16] optimized the original elastic-plastic damage model, proposed a nonlinear mechanical damage model, and conducted research and analysis for structural clay, which proved that by the new nonlinear damage model the turning point of compression curve and the peak point of shear curve can be automatically calculated; thus, it will be more simple and have more application prospects. Cheng et al. [17] studied the damage problem of rock caused by internal water migration in freeze-thaw environment, and finally, the stress distribution and migration path of pores were concluded. Yuan et al. [18] established a damage constitutive model of rock with correction coefficient by introducing the correction coefficient.

Based on the indoor triaxial test results, fully considering the changes of porosity and volume of tailing sand in the deformation process, according to the physical properties of tailings, the damage mechanics principle and Lemaitre strain equivalent theory are introduced, the microunit strength measurement method is established, and the damage model of tailing sample considering the dry wet circulation is proposed. It provides a new idea to explore the mechanical properties of tailing sand under the action of dry wet circulation.

2. Test Overview

The tailings used in this test were taken from a lead-zinc tailing pond in Hunan Province. After several surveys, the sampling point and sampling location are finally determined. The tailings taken from the site are dried. The particle size distribution obtained by screening method is shown in Table 1. It shows that the tailings are poorly graded in Table 1. The specimen used in the triaxial test is a cylinder with a diameter of 39.1 mm and a height of 80 mm. The sample preparation is carried out with the triaxial sample preparation device self-made from acrylic tube. The sample is compacted in four layers, and the intact sample is shown in Figure 1. The moisture absorption process is simulated by adding water manually, and the dehumidification process is realized by drying in an oven. The moisture content changes in the range of 1%-23% [3].

Figure 1: Complete preparation sample.

Figure 2: Fully automatic strain controlled triaxial apparatus.

The fully automatic strain controlled triaxial apparatus developed and designed by Nanjing TKA Technology Co. Ltd. is used in the test, as shown in Figure 2. The equipment loading system is controlled by Windows computer. There are many test types, and the operator can choose according to the test content and then input the corresponding test parameters, such as test rate and confining pressure. The equipment is mainly composed of main engine loading frame, advanced pressure volume controller, and data acquisition instrument. All the data in the test are collected by the computer and stored automatically after the test.

3. Indoor Test Results and Analysis

It can be seen from Figure 3 that the peak strength of tailing sand decreases gradually with the increase of the number of moisture absorption and dehumidification circulations. From the downward trend in the figure, the peak strength decreases greatly from the zero cycle to the third cycle. The peak strength of the sample is 389 kPa when the sample does not undergo the moisture absorption and dehumidification cycle; the peak strength decreases to 340 kPa after three cycles and 322 kPa after six cycles. This may be due to the fact that when the number of moisture absorption and dehumidification cycles is small, some channels are formed in the process of moisture absorption and dehumidification, and the peak strength decreases greatly. When the number of cycles is large, the change of these channels tends to be stable. Therefore, the decrease of peak strength is gentle after three to six cycles.

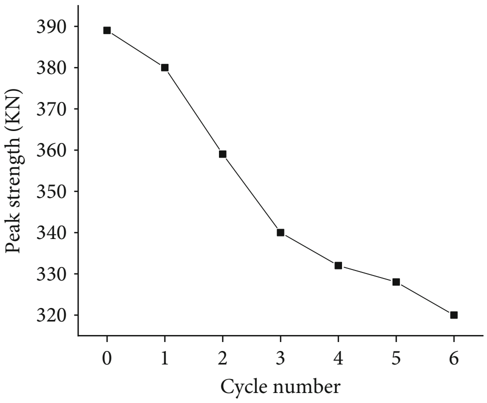

Figure 3: Peak strength of tailing sand under different numbers of moisture absorption and dehumidification cycles.

4. Preliminary Establishment of Tailing Sand Damage Model

According to the principle of damage mechanics, the stress-strain curve of tailings can be divided into five stages, as shown in Figure 4. The first stage is O-A compaction stage. In this stage, fine particles in tailings are gradually pressed into the pores between coarse particles due to external load. The second stage is A-B elastic deformation stage. In this stage, with the increase of axial strain, the stress of tailing sample presents a linear growth trend. The third stage is B-C plastic deformation stage. In this stage, the B-point tailing sample reaches the yield condition, and the stress increment gradually decreases with the increase of strain. The original arrangement and structure of particles in the tailing sample will change, and the pores will gradually derive. The fourth stage is the initial stage of C-D damage. The connection between internal particles of the sample weakens, even gradually breaks, and the particles begin to slide. It can be thought that the tailing sample has been completely damaged at this time. The fifth stage is the D-G post damage stage. In this stage, the tailing pattern has been completely damaged. With the increase of axial strain, some tailing particles are broken and the sliding increases. The stress of the sample gradually decreases with the increase of axial strain, showing the phenomenon of strain softening, finally tending to stable and reaching its residual strength.

In order to reflect the whole process of stress-strain deformation of tailings, based on the above five stages of stress-strain deformation of tailings, the constitutive model of stress-strain deformation process of tailings is studied and discussed.

Tailing is different from natural soil. As the broken geotechnical material after beneficiation, its particle shape is irregular and its particle hardness is relatively higher than other soil mass. After the circulation action of moisture absorption and dehumidification, the mechanical properties of tailings will change greatly. Therefore, referring to many research results, a damage model more suitable for tailings is established. Finally, the following assumptions are put forward:

(1) A large number of pores and microcracks will be generated in the tailings after the circulation action of moisture absorption and dehumidification

(2) It is assumed that the tailing sample after the circulation action of moisture absorption and dehumidification is composed of three parts: the damaged part, the undamaged part, and the pore. The total area is A, and the corresponding areas of each part are A_1, A_2, and A_3

(3) Because the damage location in the tailing sample is random, there is no damage crack in an isolated area, so the strain in each part of tailings can be regarded as equal

(4) Assuming that the stress of the tailings sample is σ_i, the stress of the damaged part is σ_i^s, then the undamaged part is σ_i^r, and the pore is unable to bear the function, so its stress is zero

According to the above, the model was established as

$$\sigma_i A = \sigma_i^s A_1 + \sigma_i^{\prime} A_2, \tag{1}$$

$$A = A_1 + A_2 + A_3, \tag{2}$$

$$A_3 = nA, \tag{3}$$

where n represents porosity.

For the selection of damage variable, according to the definition, $D = A_2/(A_1 + A_2)$, and then, according to equations (1), (2) and (3), the following results can be obtained:

$$\sigma_i = (1 - n)[\sigma_i{}^r(1 - D) + \sigma_i{}^s D]. \tag{4}$$

Equation (4) is the tailing damage model established in this paper. A large number of research results show that the plastic deformation of geotechnical materials is accompanied by damage, and the greater the damage variable D, the greater the plastic deformation of geotechnical materials. The analysis shows that when the tailings are completely damaged, i.e., $D = 1$, equation (4) can be written as $\sigma = \sigma_i^s(1 - n)$, and the residual strength after complete damage of tailings is σ_i^s. According to the stress-strain curve of tailing samples, the value will not change when the tailing stress reaches the residual stress.

5. Establishment of Damage Constitutive Relation of Tailing Sand

Hereby, $\sigma_i^{\prime}$ and σ_i^s are the stress values on the micro level, and 'n' is the physical quantity gradually changing with the deformation of tailing sample, which cannot be directly measured by test. Therefore, it needs to be expressed by the quantity that can be measured under macro conditions.

A large number of studies have proved that the stress-strain relationship obeys the generalized Hooke's law when there is no damage and failure of geotechnical materials. So there are

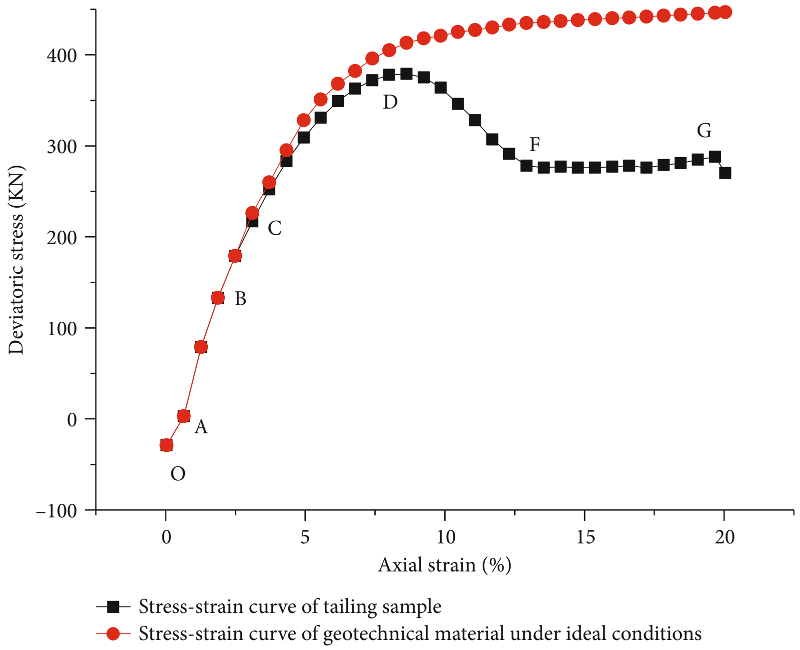

FIGURE 4: Stress-strain curve of tailings under confining pressure of 100 kPa.

$$\sigma_1' = E'\varepsilon_1' + \mu'\left(\sigma_2' + \sigma_3'\right)\sigma_1' = E'\varepsilon_1' + \mu'\left(\sigma_2' + \sigma_3'\right), \tag{5}$$

where E' is the elastic modulus when there is no damage and μ' is Poisson's ratio when there is no damage.

For the tailing sand, the relationship between the elastic modulus, Poisson's ratio, and the value of tailing samples without damage is as follows:

$$\begin{aligned} E &= (1-n)E'E = (1-n)E', \\ \mu &= \mu'. \end{aligned} \tag{6}$$

According to reference [19], the following results can be obtained:

$$n = 1 - \frac{(1-n_0)(1-D) + P_{23}\sigma_3}{P_{24}\varepsilon_1 + 1 + (-P_{24}\varepsilon_1 + (P_{21}/\varepsilon_1) - P_{22} - 1)D}, \tag{7}$$

where

$$P_{21} = P_{23}P_{25}, \tag{8}$$

$$P_{22} = P_{23}P_{26}, \tag{9}$$

$$P_{23} = \frac{2\left(1-\mu-2\mu^2\right)}{E'}, \tag{10}$$

$$P_{24} = 2\mu - 1, \tag{11}$$

$$P_{25} = \frac{(1+\mu)\sigma_c^2 \cot\varphi}{\left[3(1+\tan\alpha)E'\right]}, \tag{12}$$

$$P_{26} = \frac{c(\cot\varphi + 2\sin\alpha)}{[\sin\alpha(1+\tan\alpha)]}. \tag{13}$$

The values of c and φ here are the cohesion and internal friction angle of tailing samples at yield.

Considering that the damaged part and the undamaged part are a whole and the damaged part is completely random, so

$$\varepsilon_i = \varepsilon_i' = \varepsilon_i', \tag{14}$$

where ε_i, ε_i', ε_i'' $(i = 1, 2, 3)$ refer to the macroscopic strain of tailing sample and the undamaged part, as well as the micro strain value of the damaged part.

The purpose of this paper is to investigate the damage process of tailings under triaxial loading, so $\sigma_2 = \sigma_3$. Substituting equation (13) into (5), the following results can be obtained:

$$\sigma_1' = E'\varepsilon_1 + 2\mu'\sigma_3', \tag{15}$$

$$\sigma_3' = E'\varepsilon_3 + \mu'\left(\sigma_1' + \sigma_2'\right). \tag{16}$$

For the volume strain of tailings after loading, there is

$$V_1 = (1+\varepsilon_1)(1+\varepsilon_2)(1+\varepsilon_3)dxdydz. \tag{17}$$

By simplifying the above formula, the following results can be obtained:

$$\varepsilon_V = \varepsilon_1 + \varepsilon_2 + \varepsilon_3. \tag{18}$$

According to the relation equation of porosity in reference [15], the following results can be obtained:

$$n = \frac{(n_0 - \varepsilon_V)}{(1 - \varepsilon_V)}. \tag{19}$$

Substituting equation (17) into (18), together with $\varepsilon_2 = \varepsilon_3$, the relational expression of ε_1 and ε_3 can be obtained:

$$\frac{n - n_0}{n - 1} = \varepsilon_1 + 2\varepsilon_3. \tag{20}$$

By simplifying the above formula, the following results can be obtained:

$$\varepsilon_1 = \frac{n - n_0}{n - 1} - 2\varepsilon_3. \tag{21}$$

By equations (14) and (15), there is

$$\sigma_3' = \frac{E'}{1 - 2\mu'^2 - \mu'}\varepsilon_3 + \frac{E\mu'}{1 - 2\mu'^2 - \mu'}\varepsilon_1, \tag{22}$$

$$\sigma_1' = \frac{E'\left(1 - \mu'\right)}{1 - 2\mu'^2 - \mu'}\varepsilon_1 + \frac{2\mu' E'}{1 - 2\mu'^2 - \mu'}\varepsilon_3. \tag{23}$$

For the determination of the yield surface of tailings, firstly, the yield condition reaches the stress condition when the tailing sample initially reaches the plastic state. According to a large number of experiments, this paper selects M-C strength criterion as the yield criterion of tailings.

$$\sigma_1^s = \frac{(1 + \sin\varphi_s)\sigma_3^s + 2c_s\cos\varphi_s}{1 - \sin\varphi_s}. \tag{24}$$

The values of c_s and φ_s here are the cohesion and internal friction angle of residual stress after tailing samples are damaged.

Substituting equation (23) into (4), the following results can be obtained:

$$\sigma_3 = (1 - n)\left[\sigma_3'(1 - D) + \sigma_3^s D\right]. \tag{25}$$

By simplifying the above formula, the following results can be obtained:

$$\sigma_3^s = \frac{1}{(1 - n)D}\sigma_3 - \frac{1 - D}{D}\sigma_3'. \tag{26}$$

Substituting equation (21) into (25) and then into (23), the following results can be obtained:

$$\sigma_1^s = \frac{M}{(1 - n)D}\sigma_3 - MNP\varepsilon_3 - \mu' MNP\varepsilon_1 + Q, \tag{27}$$

where

$$M = \frac{1 + \sin\varphi_s}{1 - \sin\varphi_s}, \tag{28}$$

$$N = \frac{E'}{1 - 2\mu'^2 - \mu'}, \tag{29}$$

$$P = \frac{1 - D}{D}, \tag{30}$$

$$Q = \frac{2c_s\cos\varphi_s}{1 - \sin\varphi_s}. \tag{31}$$

Substituting equations (22) and (26) into (4), the following results can be obtained:

$$\sigma_1 = (1 - n)(1 - D)N\left[\left(1 - \frac{2\mu'}{1 - \sin\varphi_s}\right)\varepsilon_1 + \left(2\mu' - M\right)\varepsilon_3\right] + M\sigma_3 + (1 - n)QD. \tag{32}$$

Equation (32) is the damage model of tailings established in this paper.

The key to establish the evolution model of tailings by using statistical damage theory is to determine the microunit strength of tailings. Cao et al. [20] established a measurement method of rock microunit strength on the basis of the Drucker-Prager material yield criterion. This method can comprehensively reflect the influence of stress state on microunit strength, but the disadvantage is that it cannot reflect the influence of damage threshold on damage measurement. Therefore, this paper uses the relevant methods proposed in reference [21]; thus, the measurement method of microunit strength of tailings can be obtained:

$$F = \sigma_1' - \frac{1 + \sin\varphi}{1 - \sin\varphi}\sigma_3' - \frac{2c_s\cos\varphi}{1 - \sin\varphi}. \tag{33}$$

By simplifying the above formula, the following results can be obtained:

$$F = \sigma_1' - M\sigma_3' - Q. \tag{34}$$

Substituting equations (21) and (22) into (34), the following results can be obtained:

$$F = N\left[\left(1 - \mu' - M\mu'\right)\varepsilon_1 + \left(2\mu' - M\right)\varepsilon_3\right] - Q. \tag{35}$$

Equation (35) is the microunit strength measurement method established in this chapter. Based on the idea of references [12, 21], assuming that the microunit strength of tailings conforms to Weibull distribution, the statistical evolution model of tailing damage can be obtained as

$$D = \begin{cases} 1 - \exp\left[-\left(\dfrac{F}{F_0}\right)^m\right], F \geq 0, \\ 0, F < 0, \end{cases} \tag{36}$$

where m and F_0 are Weibull distribution parameters of tailing microunit strength. The model also reflects the influence of damage threshold on the evolution of the model. Only when the threshold is exceeded, the tailings will be damaged.

Substituting equation (35) into (32), the damage constitutive model of tailings in triaxial test can be obtained.

For the two model parameters m and F_0, the determination of their values is the key to the establishment of the tailing constitutive model. Therefore, in this chapter, the stress and strain values corresponding to the peak points of the stress-strain curve of tailings under different confining pressures are set as σ_{ss} and ε_{ss}; then, equation (31) can be derived as

$$\left.\frac{d\sigma_1}{d\varepsilon_1}\right|_{\sigma_1=\sigma_{ss},\varepsilon_1=\varepsilon_{ss}} = 0. \tag{37}$$

Equations (36) and (26) are combined and solved to get the following results:

$$m = -\frac{K_1^{ss} F_{ss}}{L_1 K_2^{ss}(1-D_{ss}) \ln (1-D_{ss})}, \tag{38}$$

$$F_0 = \frac{F_{ss}}{[-\ln (1-D_{ss})]^{1/m}}, \tag{39}$$

where

$$\begin{aligned} K_1^{ss} &= -\mu' N(1-D_{ss}), \\ F_{ss} &= L_1\varepsilon_{ss} - k_0, \\ K_2^{ss} &= Q + 3\left(\mu' - \frac{1}{2}\right)N\varepsilon_{ss}, \\ D_{ss} &= \frac{\sigma_{ss} - \left(1-2\mu' N\right)\varepsilon_{ss} - M\sigma_3}{Q + \left(1-2\mu' N\right)\varepsilon_{ss}}. \end{aligned} \tag{40}$$

The parameters m and F_0 of the statistical damage constitutive model of tailings can be determined by equations (37) and (38). However, σ_{ss} and ε_{ss} represent only the stress and strain values corresponding to the peak value under a certain confining pressure, so in order to establish the damage constitutive model of tailings under different confining pressures, it is necessary to determine the calculation method of σ_{ss} and ε_{ss}. Thus, the method of reference [20] is used hereby to calculate σ_{ss} and ε_{ss} by the following formula:

$$\begin{aligned} \sigma_{ss} &= \frac{2c_f \cos \varphi_f + \left(1+\sin \varphi_f\right)\sigma_3}{1-\sin \varphi_f}, \\ \varepsilon_{ss} &= b + a\sigma_3, \end{aligned} \tag{41}$$

where a, b are constants and φ_f and c_f are the internal friction angle and cohesion corresponding to the peak stress of tailings.

6. Model Validation

After the indoor consolidated undrained triaxial test of tailing samples was done, according to the stress-strain curve, the relevant mechanical parameters are $E = 78.46$ and $\mu = 0.258$ and the internal friction angle and cohesion at the peak point are 39.1° and 25.5 kPa. The internal friction angle and cohesion at yield are 24.72° and 25.5 kPa. The cohesion and internal friction angle of residual stress are 36.67 kPa and 15.95°, respectively. By substituting these parameters into the damage constitutive model of tailings established before, the stress-strain theoretical curve under this condition can be obtained, which is compared with the previous test curve, as shown in Figure 5. The fitting curve of damage constitutive model is in good agreement with the stress-strain curve of indoor tailing samples.

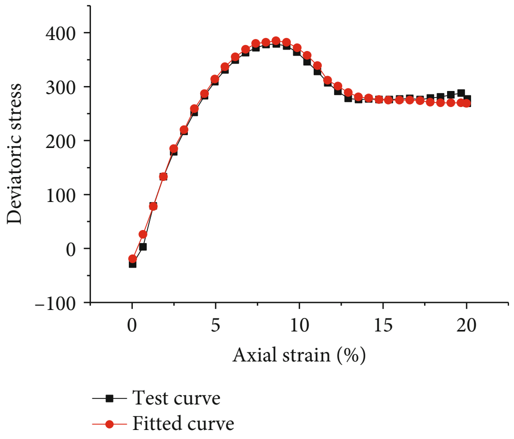

Figure 5: Comparison of theoretical and experimental curves.

7. Conclusion

Based on the indoor triaxial consolidation undrained test results of tailing sand, the internal structure changes of tailing sand after repeated moisture absorption and dehumidification cycles are studied, and the statistical damage model is established in accordance with the statistical damage theory.

On the basis of the Lemaitre strain equivalent theory and considering the influence of pore and volume change on tailings in shear test, a statistical damage constitutive model which can reflect the stress-strain process of tailings is established. The model can reflect the characteristics of bearing capacity of tailings after damage.

In this paper, the principle of damage mechanics is introduced. According to the stress-strain curve obtained from the triaxial shear test of tailings, the loading deformation process of tailings can be divided into (1) elastic deformation stage, (2) plastic deformation stage, (3) gradual damage stage, and (4) post damage stage.

The model established in this paper can reflect the volume deformation process of tailings after triaxial test, with good fit. Fewer parameters are required, which is conducive to engineering application.

Data Availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare no conflicts of interest.

Acknowledgments

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