A New Method Based on Boundary Element Method to Appraise CO₂ Geological Storage Potential in Depleted Shale Gas Reservoirs

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1. Introduction

Greenhouse gases such as CO₂ have increased in the atmosphere leading to climate change concerns [1]. Global efforts have been made to reduce atmospheric CO₂ concentration by carbon sequestration, including geological sequestration, mineral carbonation, and ocean storage [2]. Furthermore, the CO₂ geological sequestration is a widely applied measure. Industrial analogs, including natural gas and acid gas injection projects, have proved that CO₂ can be safely injected and stored [3].

The gas injection into the reservoirs is widely used in petroleum engineering to enhance oil recovery, and it is feasible to store CO₂ in geological formations as a CO₂ reduction option [4]. In the beginning, most research on gas injection for improving oil recovery takes a typical block as an example, which is based on the black oil numerical simulation with less consideration on the mechanism. In 2009, Shoab and Hoffman first conducted mathematical modeling studies on enhanced recovery by CO₂ flooding and CO₂ injection huff-n-puff based on the Elm Coulee field (0.01-0.04 mD) block Bakken unconventional reservoir [5]. Wan et al. studied the influence of the stimulated reservoir volume (SRV) characteristics on the gas injection and huff-n-puff by numerical simulation, including the size of the SRV, fracture space, and stress sensitivity [6].
The research combined with the gas flooding mechanism in the experimental study and the component model was introduced to consider the miscibility based on the actual block data. Alharthy et al. [7] established a numerical model of gas injection stimulation and predicted the production rate of a horizontal well after volume fracturing in the field, which is based on the flow mechanism of CO₂ injection in the tight matrix proposed by Hawthorne et al., including the diffusion effect caused by crude oil expansion, viscosity reduction, and concentration difference [8]. Wan et al. [9] and Pu [10] established the component model about the gas huff-n-puff model considering the miscibility of crude oil and injected gas and carried out a numerical simulation study on enhanced oil recovery in the SRV region. Yu et al. [11] and Zhu et al. [12] used the component model to analyze the sensitivity of each influencing factor of gas flooding. Many numerical methods coupling business software are increasingly used due to advantages considering mechanism in the study of CO₂-EOR simulation [13]. Zuloaga-Molero et al. [14] applied an EDFM (embedded discrete fracture model) to build a compositional reservoir model investigating the effects on CO₂ huff-n-puff and CO₂ continuous injection, including complex fracture geometries, CO₂ diffusion, permeability, and natural fractures. Sun et al. [15] applied the proposed fracture discretization approach to CO₂ huff-n-puff in complex fracture and obtained more accurate simulation results than the dual continuum approach.

Some scholars proposed that saline aquifers can be a good storage site because of their vast volumes. However, oil and gas reservoirs also are suitable for storage, and the geological conditions and storage scale are explicit in the oil and gas exploration. On the other hand, the production well and injection well can be used as carbon dioxide injection and EOR. Liu et al. [32] evaluated the CO₂ sequestration ability of shale under three-dimensional complex geological conditions using a projection-based embedded discrete fracture model. For the semianalytical model method, many scholars study the CO₂ storage process using the semianalytical model. For the numerical simulation method, Ren and Duncan [28] discussed the effects of both injection strategies and reservoir heterogeneity on CO₂ storage by reservoir simulation of carbon storage associated with CO₂ EOR in residual oil zones in San Andres formation of West Texas Evgeny. Their research showed that hydraulically fractured shale formations can be regarded as reservoirs for carbon dioxide sequestration. Andersen and Nilsen [30] evaluated CO₂ storage numerical simulation with thermal effects considering fluid properties and geomechanical stresses and the rate of geochemical reactions caused by temperature change around the injection well. Lashgari et al. [31] used CMG-GEM software to build a field-scale numerical simulation model to analyze the important physical mechanisms (miscibility and CO₂ adsorption, along with gas diffusion) of CO₂ storage and EOR. Liu et al. [32] evaluated the CO₂ sequestration ability of shale under three-dimensional complex geological conditions using a projection-based embedded discrete fracture model. For the semianalytical model method, many scholars study the CO₂ storage process using the semianalytical method by Laplace transformation, source function, and numerical discretion. Chen et al. [33, 34] developed a quick analytical method for estimating the CO₂ storage capacity of depleted shale gas based on pressure transient analysis (PTA). Xiao et al. [24, 35] quantitatively determined the abandonment pressure and analyze related impacts of dynamic parameters on it. Volume-based calculation methods simplified the CO₂ flow behavior and injection rate. Furthermore, the numerical simulation method needs modeling and mesh subdivision to apply to the field scale, which is time consuming. Semianalytical methods are suitable for rectangular or circular boundaries. However, there are few works that discussed the irregular boundary of formation. Therefore, it is vital to find efficient and convenient methods to consider the irregular boundary of formation.

Water in the shale will reduce the storage capacity due to gas-liquid two-phase flow and the occupied water volume. Moreover, the CO₂-water-shale reaction has implications for CO₂ flow [36]. Chen et al. [33, 34] evaluate the depleted shale CO₂ storage capacity by a single-phase flow for more convenient and quick evaluation. Similarly, Myshakin et al.
[37] only consider CO₂ flow, single-phase flow, to evaluate flow regimes and storage efficiency. So the model is simplified to derive a semianalytical single-phase flow solution with the complex boundary in this paper. The semianalytical solution evaluation is carried out under relatively ideal conditions.

The boundary element method (BEM) has been used widely in fluid mechanics, fracture mechanics, and contact mechanics to solve linear partial differential equations formulated as integral equations. Many scholars have applied and modified the BEM into petroleum and environmental engineering [38–40]. The production and pressure calculation in forming the irregular boundary can be solved by BEM [41, 42].

A quick and reasonable boundary element method for estimating CO₂ storage capacity of depleted shale gas formation considering arbitrarily shaped boundaries is introduced in this paper. This physical model is built based on the microseismic data, and BEM solves the mathematical model. The star-delta transformation deals with interconnected fracture segments, and the results have a high agreement with commercial software for the regular boundary. Furthermore, a real case with a horizontal well after large-scale hydraulic fracturing in depleted shale gas reservoirs is discussed. The influence of injection pressure, stress sensitivity coefficient, fracture permeability, and shale matrix permeability on the storage capacity was studied.

2. Methodology for Estimating CO₂ Storage Potential

2.1. Physical Model. Microseismic detection is used widely to analyze the underground situation, which is shown in Figure 1(a). There is a microseismic map, which uses acoustic signals to determine the underground fractures. The same color points represent the same fracture, and the dense colored dots indicate fractures. Therefore, fracture networks consist of hydraulic and secondary fractures, explained and derived by microseismograms shown as Figure 1(b). The black straight line represents the horizontal well, and the area within the black curve represents the stimulated reservoir volume (SRV). It can be seen that the SRV with irregular boundaries has hydrofractures to easily inject gas, which can be used as a CO₂ storage place.

2.2. Mathematics Model and Solution. Figure 1(b) provides a schematic diagram of the horizontal well after large-scale hydraulic fracturing. The stimulated reservoir volume region shape is irregular. The following assumptions are provided:

(1) Only isothermal single-phase flow exists
The SRV is a dual porosity continuum medium. Gas flows only from the matrix system into the fracture system.

The gas flowing into the well from the formation is only caused by hydraulic fractures.

CO₂ adsorbs in the reservoirs following the Langmuir isothermal adsorption law. The adsorption only exists in the matrix system [33, 34].

Hydraulic fractures and secondary fractures are infinite conductivity fractures.

Gravity and capillary forces were ignored in the study.

The stress sensibility exists in the fracture system leading to permeability changing with pressure [43].

The velocity equation and flow equation based on the law of conservation of mass can be written as follows:

Matrix system:

\[ \nu_m = -D_K \text{ grad } V, \]

and

\[ \text{ div } (\rho_m \nu_m) + \rho_m \frac{\partial V}{\partial t} = 0. \]  \hspace{1cm} (1)

Fracture system:

\[ \nu_f = -\frac{3.6K_f}{\mu} \text{ grad } \rho, \]

and

\[ \text{ div } (\rho_v \nu_f) - \rho_v \frac{\partial V}{\partial t} = \frac{\partial (\rho_f \phi)}{\partial t}. \]  \hspace{1cm} (2)

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**Table 1:** Reservoir parameters and production parameters (data from the literature [24]).

<table>
<thead>
<tr>
<th>Reservoir parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abandonment pressure (MPa)</td>
<td>1.5</td>
</tr>
<tr>
<td>Formation temperature (K)</td>
<td>333</td>
</tr>
<tr>
<td>Horizontal length (m)</td>
<td>1000</td>
</tr>
<tr>
<td>Formation thickness (m)</td>
<td>15</td>
</tr>
<tr>
<td>Total compressibility of natural fracture (MPa⁻¹)</td>
<td>5.0 × 10⁻⁴</td>
</tr>
<tr>
<td>Total compressibility of the matrix (MPa⁻¹)</td>
<td>4.5 × 10⁻⁴</td>
</tr>
<tr>
<td>Total compressibility of hydraulic fracture (MPa⁻¹)</td>
<td>5.0 × 10⁻⁴</td>
</tr>
<tr>
<td>Porosity of natural fracture (-)</td>
<td>0.001</td>
</tr>
<tr>
<td>Porosity of matrix (-)</td>
<td>0.08</td>
</tr>
<tr>
<td>Porosity of hydraulic fracture (-)</td>
<td>0.3</td>
</tr>
<tr>
<td>Initial permeability of natural fracture (mD)</td>
<td>0.1</td>
</tr>
<tr>
<td>Permeability of matrix (mD)</td>
<td>5 × 10⁻³</td>
</tr>
<tr>
<td>Permeability of hydraulic fracture (mD)</td>
<td>5000</td>
</tr>
<tr>
<td>Hydraulic fracture width (m)</td>
<td>0.003</td>
</tr>
<tr>
<td>Hydraulic fracture number (-)</td>
<td>4</td>
</tr>
<tr>
<td>CO₂ Langmuir pressure (MPa)</td>
<td>8</td>
</tr>
<tr>
<td>CO₂ Langmuir volume (sm³/m³)</td>
<td>8</td>
</tr>
<tr>
<td>Gas diffusion coefficient D (m²/s)</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

**Table 2:** Reservoir parameters and production parameters.

<table>
<thead>
<tr>
<th>Reservoir parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abandonment pressure (MPa)</td>
<td>1.5</td>
</tr>
<tr>
<td>Matrix porosity (-)</td>
<td>0.092</td>
</tr>
<tr>
<td>Matrix permeability (mD)</td>
<td>5 × 10⁻³</td>
</tr>
<tr>
<td>Hydraulic fracture porosity (-)</td>
<td>0.4</td>
</tr>
<tr>
<td>Hydraulic fracture permeability (mD)</td>
<td>10000</td>
</tr>
<tr>
<td>Matrix porosity (-)</td>
<td>0.092</td>
</tr>
<tr>
<td>Hydraulic fracture opening (mm)</td>
<td>10</td>
</tr>
<tr>
<td>Formation thickness (m)</td>
<td>15</td>
</tr>
<tr>
<td>Matrix compression coefficient (MPa⁻¹)</td>
<td>1.07 × 10⁻⁴</td>
</tr>
<tr>
<td>Fracture compression factor (MPa⁻¹)</td>
<td>1.07 × 10⁻⁴</td>
</tr>
<tr>
<td>CO₂ Langmuir pressure (MPa)</td>
<td>8</td>
</tr>
<tr>
<td>CO₂ Langmuir volume (sm³/m³)</td>
<td>8</td>
</tr>
<tr>
<td>Stress sensitivity coefficient (1/MPa)</td>
<td>0.2</td>
</tr>
</tbody>
</table>
Basic equations for CO₂ can be obtained from the following assumptions:

**CO₂ adsorption equation:**
\[ V = V_L \frac{p}{p + p_L} \]  

**Permeability-pressure equation**
\[ K_f = K_{fi} e^{-\alpha (p - p_f)} \]  

The flow model, which consists of the matrix system and fracture system after dimensionless transformation, can be written as follows:

**Matrix system:**
\[ \frac{1}{r_{mD}^2} \frac{\partial}{\partial r_{mD}} \left( r_{mD}^2 \frac{\partial V_D}{\partial r_{mD}} \right) = \frac{1}{D_{KD}} \frac{\partial V_D}{\partial t_D} \]  

**Fracture system:**
\[ \frac{\partial^2 m_{1D}}{\partial r_{ID}^2} + 2 \frac{\partial m_{1D}}{\partial r_{ID}} - \gamma_D \left( \frac{\partial m_{1D}}{\partial r_{ID}} \right)^2 = e^{\omega} \left[ \omega \frac{\partial m_{1D}}{\partial t_D} + (1 - \omega) \frac{\partial V_D}{\partial t_D} \right] \]  

The boundary condition can be written as
\[ \left. \frac{\partial V_D}{\partial r_{ID}} \right|_{r_{ID} = 0} = 0, \quad V_D|_{r_{ID} = 1} = V_{ED}, \quad V_D|_{t_D = 0} = 0. \]  

**Fracture system:**
\[ \lim_{\varepsilon_D \to 0} 2 e^{-\gamma_D} \frac{\partial m_{1D}}{\partial r_{ID}} \bigg|_{r_{ID} = \varepsilon_D} = -\tilde{q}_{mD} m_{1D}|_{r_{ID} = -\varepsilon_D} = 0, \quad m_{1D}|_{t_D = 0} = 0. \]  

A point source function is obtained by using Laplace transform, Pedrosa's substitution [38], and solution of the CO₂ flow model [33, 34]. Moreover, the dimensionless definitions of this article are given in the appendix.

\[ \alpha \left( \frac{p}{p_f} \right) \]  

\[ \frac{\bar{D}_D}{2} = \frac{D_{KD} e^{-u \sqrt{s/D_{KD}}}}{r_{sD}}, \]  

where
\[ r_{sD} = \sqrt{(x_D - x_wD)^2 + (y_D - y_wD)^2}, \]
\[ u = \omega s + (1 - \omega)3 \alpha D_{KD} \left[ \sqrt{s/D_{KD}} \coth \left( \sqrt{s/D_{KD}} \right) - 1 \right]. \]
The integral equation of the dimensionless pressure using Green’s secondary identity can be written as

$$\theta_D(x_D, y_D) = \frac{1}{\pi} \int_{x_D} \left( \bar{G}_{D} \frac{\partial \bar{P}_D}{\partial x_D} - \bar{P}_D \frac{\partial \bar{G}_{D}}{\partial x_D} \right) d\Gamma + \pi \int_{x_D} \bar{q}_{wD} \bar{G}_{D} d\Omega. $$

(11)

The fracture segments can be considered as point sources written as Equation (9). And then, the second term on the right side of Equation (11) can be simplified as

$$\pi \int_{x_D} \bar{q}_{wD} \bar{G}_{D} d\Omega = \pi \sum_{j=1}^{N_F} \bar{\eta}_{Fj} \bar{S}_{FDj}(x_{Dj}, y_{Dj}, x_{wDj}, y_{wDj}, \theta_{Fj}; s). $$

(12)

where the source function for the jth segment is given by

$$\bar{S}_{FDj}(x_{Dj}, y_{Dj}, x_{wDj}, y_{wDj}, \theta_{Fj}; s) = \frac{1}{2} \Delta L_{FDj} \int_{-1}^{1} \bar{G}_{D} d\xi. $$

(13)

The boundary is divided into many boundary elements by BEM, and the number of the element on a boundary is $N_b$, as shown in Figure 2(b). For the irregular zone, the
discretized form of Equation (11) is Equation (14).

$$\theta_D(x_D, y_D) = \frac{1}{\pi} \int_{x_D} \left( \bar{G}_{D} \frac{\partial \bar{P}_D}{\partial x_D} - \bar{P}_D \frac{\partial \bar{G}_{D}}{\partial x_D} \right) d\Gamma + \pi \sum_{j=1}^{N_F} \bar{q}_{FDj} \bar{S}_{FDj} + \pi \sum_{j=1}^{N_b} \bar{q}_{wDj} \bar{G}_{Dj} d\Omega. $$

(14)

where $\Delta L_D$ is the jth element on the boundary $\Gamma_D$.

The finite difference method (FDM) is used to discretize the fracture flow formula, as shown in Figure 2(a). The boundary can be discretized into $N_b$ segments, and the fracture can be discretized into $N_F$ segments. So $2 \times N_b + 2 \times N_F$ variables, including the pressure and saturation of every segment, can be solved by $2 \times N_b + 2 \times N_F$ equations, which consists of $N_b + N_F$ equations of the boundary condition. And $N_F$ equations from BEM are shown as Equation (15), and $N_b$ equations from FDM are shown as Equation (16).

$$T \bar{P}_F + R \bar{q}_F + \bar{p}_w b = 0, $$

(15)

where $T, R, and b$ are corresponding coefficient matrixes and vector, respectively.

$$A \bar{p}_D + B(\partial \bar{p}_D/\partial n_D) + C \bar{q}_F = 0 $$

(16)

where $A, B, and C$ are the corresponding coefficient matrixes.

The star-delta transformation is an effective method to deal with the flow of multiple adjacent fracture units. The transmissibility between the two adjacent segments with four fracture segments is given by Karimi-Fard et al. [44] and Slough et al. [45].

$$T_{D_j}^{*} = \frac{Y_{D_1} Y_{D_2}}{\sum_{k=1}^{4} Y_{D_k}}. $$

(17)

Assuming that another segment connected to segment 4 is labeled 5. Thus, the flow equation of segment 4 can be expressed as

$$\bar{T}_{D_4}^{*} \bar{P}_{FD} + \bar{T}_{D_4}^{*} \bar{P}_{FD} + \bar{T}_{D_4}^{*} \bar{P}_{FD} - (\bar{T}_{D_1}^{*} \bar{T}_{D_2}^{*} + \bar{T}_{D_3}^{*} + \bar{T}_{D_4}^{*} + \bar{T}_{D_5}^{*} + \bar{\alpha}_{D_4}) \bar{P}_{FD} = 0 $$

(18)
where the transmissibility $T^*_{D1,A}$, $T^*_{D2,A}$, and $T^*_{D3,A}$ can be evaluated by use of Equation (17).

2.3. CO₂ Storage Potential Calculation. Based on the injection well transient pressure solution, the downhole pressure and the injection gas rate can be obtained. When downhole pressure reaches the target pressure, we can get the corresponding injection time $t_i$. Therefore, the CO₂ storage capacity equals the total injected CO₂ volume, which can be written as:

$$V_{in} = \int_0^{t_i} Q dt.$$  \hspace{1cm} (19)

3. Verification

In this section, the developed model is validated by commercial software CMG (2012) and semianalytical models from Chen et al. [34] and Xiao et al. [24]. The parameters are shown in Table 1 and Figure 3. In total, the targeted model has a grid system of $55 \times 55 \times 1$, among which each grid is 50 m in the $x$ and $y$ directions, while it is 15 m in the $z$ direction. The selected region covers an area of $2750 \times 2750$ m.

The model validation study is conducted by comparing the cumulative injection volume under different constrained pressures, based on the same condition with regular boundary. For constant injection rate and constrained injection pressure, we compare calculated results. The new model has a high agreement with commercial software so that the new model can be regarded as an effective method, shown in Figure 4. In this case, the conventional rectangle-boundary-formation storage capacity is evaluated to verify the new method’s accuracy. However, commercial software evaluation needs to use local grid refinement or irregular grid subdivision for irregular boundary reservoirs. The advantages of the new method are mainly in dealing with reservoirs with complex boundaries, without subdivision grid. The semianalytical method is adopted to evaluate the buried storage capacity according to reservoir parameters. Therefore, the semianalytical method has higher computational efficiency than the numerical simulation method.

4. Case Study

Based on an actual case from the shale gas formation, capacity evaluation is studied in this section. Furthermore, the effects of critical parameters are investigated.

4.1. Background. A depleted shale gas formation has been selected as the targeted formation, which is located in China. China has enormous development potential for shale gas, and it can be a prospective location for CO₂ storage projects to reduce greenhouse gases. The case’s payzone depth ranges from 1730 m to 1745 m with a shale gas formation, and the parameters are shown in Table 2. The microseismic detection and the scale of the reservoir are shown in Figure 1.

4.2. Simulation Results. Based on this method, the CO₂ geological storage capacity can be evaluated in the actual case for the irregular boundary. After 60, 90, 180, and 360 days, the system pressure distribution is shown as Figure 5.

4.3. Effects of Boundary Shape. The innovation of the method is that it can effectively deal with the actual shale reservoir’s irregular geological boundary and have faster calculation speed than the numerical solution. The boundary shape of the reservoir is usually simplified to a circular or rectangular shape. We compared the cumulative CO₂ injection under different boundary shapes. We simulated the model with regular boundary; one is a rectangular boundary with a length of 810 m and a width of 470 m, and the other is a circular boundary with a radius of 410 m. And the cumulative CO₂ injection volume after 720 days is shown in Figure 6. Pressure distribution after 720 days with different...
boundary shapes under the same injection condition is shown in Figure 7. It can be seen that the shape of the reservoir boundary affects the calculation of the CO₂ storage capacity.

4.4. Sensitivity Analysis

4.4.1. Injection Pressure. Figure 8 shows the cumulative injection volume (storage volume) of CO₂ under different injection pressures. Obviously, the greater the injection pressure, the more cumulative injection amount of CO₂ (Figure 8). Figure 9 can optimize constrained injection pressure of CO₂ storage.

4.4.2. Stress Sensitivity Coefficient. Figure 10 shows the cumulative injection volume of CO₂ after 360 days under different stress sensitivity coefficients. It can be seen that the increase of fracture permeability caused by stress sensitivity has a significant influence on shale storage potential, especially when \( a \) is greater than 0.3. Therefore, the premise of accurately evaluating the CO₂ storage potential of shale reservoirs correctly considers and characterizes stress sensitivity.

4.4.3. Fracture and Shale Matrix Permeability. The cumulative injection volume of CO₂ after 360 days with different fracture permeability conditions without considering stress sensitivity is shown in Figure 11(a). It can be seen from Figure 11(a) that when the fracture conductivity is minimal, the CO₂ storage potential of the reservoir is meager. With the increase of fracture conductivity, the CO₂ reservoir potential increases sharply. It then tends to be stable, indicating that we need to find abandoned wells with better fracturing effects on the one hand. On the other hand, it is not necessary to blindly seek the high conductivity of fractures. The same law is reflected in the cumulative injection volume of CO₂ injected after 360 days with different matrix permeabilities (Figure 11(b)).

CO₂ sequestration is a long-term and complex process considering not only the storage capacity of the reservoir but also the possibility of leakage after storage. It is necessary to comprehensively consider the effectiveness and safety to finding suitable CO₂ reservoirs. It should be noted that the purpose of this method is only to provide quick and effective reference for researchers.

5. Conclusions

This study appraises CO₂ geological storage potential based on the boundary element method, evaluating the storage capacity considering arbitrarily shaped boundaries. Furthermore, this paper discusses the influence of relevant parameters on the storage capacity of a horizontal well undergoing volumetric fracturing in depleted shale gas reservoirs.

(1) The evaluation results based on the boundary element method have good agreement with commercial software so that the method can be regarded as an effective method

(2) The boundary shape of the reservoir is critical to evaluate the CO₂ geological storage capacity of depleted reservoirs accurately. The method proposed in this paper can deal with any complex boundary and simulate the CO₂ storage process effectively and efficiently

(3) The CO₂ geological storage capacity under different injection pressures, stress sensitivity coefficients, fracture permeabilities, and shale matrix permeabilities were analyzed. CO₂ geological storage design optimization should be carried out to achieve maximum benefits

The main advantage of the method proposed in this paper is that it offers quick and efficient access to CO₂
sequestration-potential estimation considering the irregular boundary, which helps select wells and monitor facilities for storing CO₂ in residual depleted shale reservoirs. Of course, some complex mechanisms were ignored, which is our subsequent work.

Appendix

Dimensionless definitions for fracture flow and flow models [30, 31]:

\[ t_D = \frac{3.6K_{fi}l}{\mu(\phi_g + (K_{fi}h/(1.842 * 10^{-3})))}h^2, \]
\[ V_D = V - V_i, \quad q_D = \frac{q(t)}{q_w}, \]
\[ P_{FD} = \frac{2\pi h(p_i - p_F)}{q_i B_i^2}, \]
\[ y_D = \frac{3.684 * 10^{-3} p_w q_w T}{K_{fi}T_e h (p_L + p_i)(p_L + p_i)}, \]
\[ \alpha = \frac{3.684 * 10^{-3} \mu Z_p a_k T}{K_{fi}T_e h}, \]
\[ \omega = \frac{\phi C_g}{\phi c_g + (K_{fi}h/(1.842 * 10^{-3})))}, \]
\[ D_{KD} = \frac{D_{fi} \mu(\phi c_g + (K_{fi}h/(1.842 * 10^{-3})))h^2}{3.6K_{fi}R_m^2}, \]
\[ q_{FD} = \frac{2\kappa a_F}{q_i}, \]
\[ c_{FD} = \frac{k_F a_F}{\kappa}, \]
\[ \eta_{FD} = \frac{k_F (\phi c_i)}{k (\phi c_i)}. \]  

(A.1)

Other dimensionless definitions are \( x_D = x/L_D, \) \( y_D = y/L_D, \) \( \varepsilon_D = \varepsilon/L_D, \) \( x_{wD} = x_{w}/L_D, \) \( y_{wD} = y_{w}/L_D, \) \( \Delta L_{DFi} = \Delta L_{fi}/L_D, \) \( r_D = r/L_D, \) \( r_{wD} = r_{w}/h, \) \( L_{FD} = L_f/h, \) and .

\[ p_D = \frac{2\pi k_h(p_i - p)}{q_i B_i^2}, \]
\[ q_{wD} = \frac{2\kappa a_w}{q_i}, \]
\[ p_{DF} = \frac{2\pi k_h(p_i - p_F)}{q_i B_i^2}, \]
\[ q_{DF} = \frac{2\kappa a_F}{q_i}. \]  

(A.2)

Nomenclature

\( D_{fi} \): Knudsen diffusion coefficient, \( m^2 \cdot h^{-1} \)
\( V \): CO₂ concentration, \( sm^3/m^3 \)
\( v \): CO₂ flow velocity, \( m/h \)
\( a \): Stress sensitivity coefficient, \( 1/MPa \)
\( r_m \): Radial distance in matrix system, m
\( B \): Formation liquid volume factor, \( m^3/m^3 \)
\( c_{Df} \): Discrete fracture compressibility, \( Pa^{-1} \)
\( c_{Dg} \): Inner zone matrix compressibility, \( Pa^{-1} \)
\( c_{Do} \): Outer zone matrix compressibility, \( Pa^{-1} \)
\( C_D \): Dimensionless wellbore-storage coefficient
\( F_i \): Storability ratio between the inner and outer zones
\( G \): Green function for modified Helmholtz equation
\( h \): Formation thickness, m
\( k_F \): Discrete fracture permeability, \( m^2 \)
\( k \): Matrix permeability, \( m^2 \)
\( M \): Mobility ratio between the inner and outer zones
\( N_i \): Number of node point of boundary \( \Gamma \)
\( p_I \): Initial formation pressure, Pa
\( p \): Matrix pressure, Pa
\( p_w \): Wellbore pressure, Pa
\( q_\omega \): Boundary pressure, m²/s
\( q_F \): Flux per unit length entering the fracture, m²/s
\( p_c \): Reference rate, m²/s
\( q_w \): Flow density to fracture, s⁻¹
\( r_w \): Wellbore radius, m
\( s \): Laplace transformation variable
\( S_i \): Skin factor for flow choking
\( S_F \): Source function for fracture segment
\( t \): Time, s
\( \alpha_D \): Coefficient of dimensionless pressure of fracture segment
\( y \): \( y \)-coordinate, m
\( y_{wD} \): \( y \)-coordinate of midpoint of fracture segment \( i, m \)
\( \omega_F \): Discrete fracture width, m
\( \Delta L_{DFi} \): Length of discrete fracture segment \( i, m \)
\( r_{i} \): Spherical distance in natural fracture system, m
\( \beta_D \): Coefficient of dimensionless flow rate of fracture segment
\( \gamma_D \): Dimensionless transmissibility between the fracture segment and interface
\( e \): Direction of discrete fracture, m
\( x \): Reference length in the system, m
\( \eta \): Local coordinate for boundary element
\( \theta \): Angle
\( \mu \): Fluid viscosity, Pa·s
\( \phi \): Matrix porosity, fraction
\( \lambda \): Interporosity flow parameter
\( \Omega \): SRV zone
\( \Gamma \): Boundary
\( p \): Density, kg/m³
\( \iota_{DFi} \): Dimensionless transmissibility between fracture segments \( i \) and \( j \) after star-delta transformation
\( x \): \( x \)-coordinate, m
\( x_{wD} \): \( x \)-coordinate of midpoint of fracture segment \( i, m \)
\( t_{FD} \): Dimensionless time
Dimensionless transmissibility between fracture segments $i$ and $j$.

**Superscripts**

$\mathbf{-}$: Laplace transform

**Subscripts**

$D$: Dimensionless

$F$: Discrete fracture

$i$: Discrete fracture segment index

$K$: Boundary nodal point index

$L$: Boundary element index.

**Data Availability**

Previously reported data were used to support this study and are available. These prior studies (and datasets) are cited at relevant places within the text as references [24, 34]. And the other data used to support the findings of this study are included within the article.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

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**References**


