Research Article

The Pressure Buildup Well Test Analysis considering Stress Sensitivity Effect for Deepwater Composite Gas Reservoir with High Temperature and Pressure

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Some deepwater gas reservoirs with high temperature and pressure have obvious stress sensitivity effect resulting in difficulty in well test interpretations. The influence of stress sensitivity effect on the pressure drawdown well test is discussed in many papers. However, the influence on the pressure buildup well test is barely discussed. For practices in oilfields, the quality of pressure data from the drawdown stage of well test is poor due to the influence of production fluctuation. Thus, the pressure data from the buildup stage is used for well test interpretations in most cases. In order to analyze the influence of stress sensitivity effect on the pressure buildup well test, this paper establishes a composite gas reservoir pressure buildup well test model considering the stress sensitivity effect and the hysteresis effect. Numerical solutions to both pressure drawdown and buildup well test models are obtained by the numerical differentiation method. The numerical solutions are verified by comparing with analytical solutions and the homogeneous gas reservoir well test solution. Then, the differences between pressure drawdown and buildup well test curves considering the stress sensitivity effect are compared. The parameter sensitivity analysis is conducted. Compared with the conventional well test curve, the pressure derivative curve of pressure drawdown well test considering the stress sensitivity effect deviates upward from the 0.5 horizontal line at the inner zone radial flow stage, while it deviates upward from the $M/2$ (mobility ratio/2) horizontal line at the outer zone radial flow stage. However, for the pressure buildup well test curve considering the stress sensitivity effect, the pressure derivative curve gradually descends to the 0.5 horizontal line at the inner zone radial flow stage, while it descends to the $M/2$ (mobility ratio/2) horizontal line at the outer zone radial flow stage. The pressure derivative curve of pressure buildup well test considering the hysteresis effect is higher than the curve without considering the hysteresis effect, because the permeability cannot be recovered to its original value in the buildup stage after considering the hysteresis effect. Meanwhile, skin factor and mobility ratio have different effects on pressure drawdown and buildup well test curves. Based on the model, a well test interpretation case from a deepwater gas reservoir with high temperature and pressure is studied. The result indicates that the accuracy of the interpretation is improved after considering the stress sensitivity effect, and the skin factor will be exaggerated without considering the stress sensitivity effect.

1. Introduction

Well test is an important way to obtain physical parameters and evaluate the development performance of gas reservoirs [1–9]. It can help to understand the seepage mechanism and the development law of gas reservoirs with high temper-

ature and pressure. Compared with the conventional gas reservoir, there is more obvious rock deformation during the development process of the gas reservoir with high temperature and pressure resulting in stronger stress sensitivity effect [10–15]. Thus, it is necessary to consider the influence of stress sensitivity effect on well test curves. Many experiments

and production practices show that there is irreversible deformation in some reservoirs during the development process [16–18]. Reservoir permeability decreases with the decrease of formation pressure. However, the permeability cannot be completely recovered to its original value after the formation pressure increases to the initial value if there is irreversible deformation in reservoirs. This phenomenon is called the hysteresis effect [18].

At present, many researchers have studied the influence of stress sensitivity effect on well test curves. However, the influence on the pressure drawdown well test is mainly discussed in these studies [19–30]. For practices in oilfields, the quality of pressure data from the drawdown stage of well test is poor due to the influence of production fluctuation. Thus, the pressure data from the buildup stage is used in most cases of well test interpretations for its good data quality [31–34]. For conventional well test interpretations, the superposition principle is used to obtain solutions to pressure buildup well test models from that of pressure drawdown well tests. However, for well test models considering the stress sensitivity effect, the seepage differential equations are nonlinear equations, and the superposition principle cannot be directly used for nonlinear equations to obtain the solutions to pressure buildup well test models [31]. Therefore, it is necessary to analyze the influence of stress sensitivity effect on the pressure buildup well test to guide well test interpretations. However, the studies about pressure buildup well test models considering the stress sensitivity effect are not sufficient. Zhang et al. established pressure buildup well test models considering the stress sensitivity effect for vertical fractured well and vertical well in a homogeneous reservoir [18, 31]. One of the researches mainly discussed the influence of stress sensitivity effect and hysteresis effect on the variation of fracture conductivity for vertical fractured well in a homogeneous reservoir at the buildup stage [18]. Another research only discussed the influence of stress sensitivity effect on vertical well in a homogeneous reservoir at the buildup stage [31]. There are few researches about the pressure buildup well test of composite gas reservoir considering both the stress sensitivity effect and the hysteresis effect.

Thus, the paper establishes theoretical pressure drawdown and buildup well test models for the composite gas reservoir with high temperature and pressure considering both the stress sensitivity effect and the hysteresis effect. The solutions to these models are obtained by the numerical differentiation method. The numerical solutions to pressure drawdown and buildup well test models are verified by comparing with analytical solutions [23] and the homogeneous gas reservoir well test solution [31]. The differences between pressure buildup and drawdown well test curves considering the stress sensitivity effect are analyzed. The parameter sensitivity analysis is conducted. Then, a field case from a gas reservoir with high temperature and pressure is presented indicating that the accuracy of interpretation is improved after considering the stress sensitivity effect.

2. Model Description and Solution

2.1. Physical Model and Basic Assumptions. A physical model of a vertical well with fixed gas production rate in a

composite gas reservoir is established shown in Figure 1. The basic assumptions of the model are as follows:

- (1) An isotropic gas reservoir is bounded by impermeable layers at the bottom and top. The gas reservoir has two zones with different permeability and porosity. The radius of inner zone boundary is r_1 , and the outer zone boundary is infinite
- (2) The vertical gas well is on production at fixed rate with the initial pressure p_i followed by a shut-in stage
- (3) The gas in the reservoir is highly compressible fluid with the compressibility C_g , compressibility factor Z , and viscosity μ_g
- (4) Considering the stress sensitivity effect and the hysteresis effect, the gas reservoir permeability changes with the formation pressure resulting in irreversible deformation to the reservoir
- (5) The gravity effect is negligible. The temperature in the reservoir is constant at production and shut-in stages
- (6) The effect of wellbore storage is considered by a constant wellbore storage coefficient. The effective wellbore radius is used to describe the skin damage

2.2. Mathematical Model

2.2.1. Characterization of Stress Sensitivity Effect and Hysteresis Effect. The stress sensitivity coefficient γ is used to describe the change of reservoir permeability with the pressure. Some experimental studies indicate that the relationship between the reservoir permeability and the pressure change conforms to the exponential form [23, 35, 36]. The changes of reservoir permeability with the pressure at pressure drawdown and buildup stages are shown in Figure 2.

At the pressure drawdown stage, the stress sensitivity effect can be described by Equation (1):

$$\gamma_d = \frac{1}{k_{id}} \frac{dk}{dp} \Rightarrow k_d = k_{id} \exp [-\gamma_d(p_i - p_d)]. \quad (1)$$

At the pressure buildup stage, the stress sensitivity effect and the hysteresis effect can be described by Equation (2):

$$\begin{aligned} \gamma_b &= \frac{1}{k_{ib}} \frac{dk}{dp} \Rightarrow k_b = k_{ib} \exp [-\gamma_b(p_i - p_b)] \\ &= k_{id} \exp [-(\gamma_d - \gamma_b)(p_i - p_d)] \exp [-\gamma_b(p_i - p_b)], \end{aligned} \quad (2)$$

where γ_d and γ_b are the stress sensitivity coefficients at pressure drawdown and buildup stage, $1/\text{Pa}$; there will be the hysteresis effect when γ_b is less than γ_d ; k_{id} is the initial permeability of gas reservoir, m^2 ; k is the permeability of gas reservoir, m^2 ; p is the pressure of gas reservoir, Pa ; p_i is the initial pressure of gas reservoir, Pa ; p_d and p_b are the gas reservoir pressure at pressure drawdown and buildup stages,

FIGURE 1: Physical model of vertical well producing in composite gas reservoir.

FIGURE 2: The change of reservoir permeability with pressure.

Pa; k_d and k_b are the gas reservoir permeability at pressure drawdown and buildup stages, m^2 ; k_{ib} is the maximum permeability value that can be restored after the gas reservoir pressure increases to the initial pressure, m^2 .

The gas pseudo-pressure is generally used in gas reservoir well tests, and the definition of the gas pseudo-pressure is as Equation (3):

$$\psi = 2 \int_{p_{sc}}^p \frac{p}{\mu Z} dp, \quad (3)$$

where ψ is the gas pseudo-pressure, Pa/s; μ is the gas viscosity, Pa.s; Z is the gas compressibility factor, decimal; p_{sc} is the pressure at the standard condition, Pa.

Substituting Equation (3) into Equations (1) and (2), stress sensitivity equations in the pseudo-pressure form can be expressed as Equations (4) and (5):

$$\gamma_d = \frac{1}{k_{id}} \frac{dk}{dp} = \frac{1}{k_{id}} \frac{dk}{d\psi_d} \frac{d\psi_d}{dp} = \frac{1}{k_{id}} \frac{dk}{d\psi_d} \frac{p}{\mu Z}, \quad (4)$$

$$\gamma_b = \frac{1}{k_{ib}} \frac{dk}{dp} = \frac{1}{k_{ib}} \frac{dk}{d\psi_b} \frac{d\psi_b}{dp} = \frac{1}{k_{ib}} \frac{dk}{d\psi_b} \frac{p}{\mu Z}. \quad (5)$$

Then, stress sensitivity coefficients in the pseudo-pressure form can be obtained as Equations (6) and (7):

$$\gamma_d^* = \frac{\gamma_d}{(p/(\mu Z))} = \frac{1}{k_{id}} \frac{dk}{d\psi_d} \Rightarrow k_d = k_{id} \exp [-\gamma_d^*(\psi_i - \psi_d)], \quad (6)$$

$$\gamma_b^* = \frac{\gamma_b}{(p/(\mu Z))} = \frac{1}{k_{ib}} \frac{dk}{d\psi_b} \Rightarrow k_b = k_{ib} \exp [-\gamma_b^*(\psi_i - \psi_b)], \quad (7)$$

where γ_d^* and γ_b^* are stress sensitivity coefficients in the pseudo-pressure form at pressure drawdown and buildup stages, Pa/s; ψ_i is the initial pseudo-pressure, Pa/s; ψ_d and ψ_b are the pseudo-pressure at pressure drawdown and buildup stages, Pa/s.

2.2.2. Pressure Drawdown Well Test Model. The differential equation of natural gas seepage considering the stress sensitivity effect and the hysteresis effect is as Equation (8):

$$\frac{1}{r} \frac{\partial}{\partial r} \left[k(p) r \frac{p}{Z \mu} \frac{\partial p}{\partial r} \right] = \phi \frac{\partial}{\partial t} \left(\frac{p}{Z} \right), \quad (8)$$

where r is the radius, m; $k(p)$ is the permeability considering the stress sensitivity effect and the hysteresis effect, m^2 ; t is time, s; ϕ is porosity, decimal.

For the pressure drawdown well test, the comprehensive seepage differential equation considering the stress sensitivity effect in the inner zone of composite gas reservoir is as Equation (9):

$$\frac{1}{r} \frac{\partial}{\partial r} \left[k_{i1} e^{-\gamma_d(p_i - p_1)} r \frac{p_1}{Z \mu_1} \left(\frac{\partial p_1}{\partial r} \right) \right] = \phi_1 \frac{\partial}{\partial t} \left(\frac{p_1}{Z} \right), \quad (9)$$

where k_{i1} is the initial inner zone permeability of composite gas reservoir, m^2 ; p_1 is the inner zone pressure of composite gas reservoir, Pa; μ_1 is the inner zone gas viscosity of composite gas reservoir, Pa.s; ϕ_1 is the inner zone porosity of composite gas reservoir, decimal.

The comprehensive seepage differential equation considering the stress sensitivity effect in the outer zone of composite gas reservoir is as Equation (10):

$$\frac{1}{r} \frac{\partial}{\partial r} \left[k_{i2} e^{-\gamma_d(p_i - p_2)} r \frac{p_2}{Z \mu_2} \left(\frac{\partial p_2}{\partial r} \right) \right] = \phi_2 \frac{\partial}{\partial t} \left(\frac{p_2}{Z} \right), \quad (10)$$

where k_{i2} is the initial outer zone permeability of composite gas reservoir, m^2 ; p_2 is the outer zone pressure of composite gas reservoir, Pa; μ_2 is the outer zone gas viscosity of composite gas reservoir, Pa.s; ϕ_2 is the outer zone porosity of composite gas reservoir, decimal.

After substituting Equation (3) into Equations (9) and (10), gas comprehensive seepage differential equations in the pseudo-pressure form can be expressed as Equations (11) and (12):

$$\frac{1}{r} \frac{\partial \psi_1}{\partial r} + \frac{\partial^2 \psi_1}{\partial r^2} + \gamma_d^* \left(\frac{\partial \psi_1}{\partial r} \right)^2 = \frac{\phi_1 \mu_1 C_g}{k_{i1}} e^{\gamma_d^*(\psi_i - \psi_1)} \frac{\partial \psi_1}{\partial t}, \quad (11)$$

$$\frac{1}{r} \frac{\partial \psi_2}{\partial r} + \frac{\partial^2 \psi_2}{\partial r^2} + \gamma_d^* \left(\frac{\partial \psi_2}{\partial r} \right)^2 = \frac{\phi_2 \mu_2 C_g}{k_{i2}} e^{\gamma_d^*(\psi_i - \psi_2)} \frac{\partial \psi_2}{\partial t}, \quad (12)$$

where ψ_1 is the inner zone pseudo-pressure, Pa/s; ψ_2 is the outer zone pseudo-pressure of composite gas reservoir, Pa/s; C_g is the gas compressibility coefficient, Pa⁻¹.

The initial condition can be expressed as Equation (13):

$$\psi_1(r, t=0) = \psi_2(r, t=0) = \psi_i. \quad (13)$$

The inner boundary considering the skin damage and the well storage effect for the pressure drawdown well test can be expressed as Equation (14):

$$\left[r e^{-\gamma_d^*(\psi_i - \psi_1)} \frac{\partial \psi_1}{\partial r} \right] \bigg|_{r=r_{we}} = \frac{q_{sc} p_{sc} T}{\pi k_{i1} h T_{sc} Z_{sc}} + \frac{C \mu_1}{2 \pi k_{i1} h} \frac{\partial \psi_1}{\partial t}, \quad (14)$$

where $r_{we} = r_w \exp(-S)$ is the effective well radius, m; S is the skin factor, dimensionless; q_{sc} is the production of gas well at the standard condition, m³/s; T_{sc} is the temperature at the standard condition, K; T is the formation temperature, K; C is the wellbore storage coefficient, m³/Pa; h is the effective thickness of reservoir, m.

The pressure and flow rate are equal in the interface between the inner zone and the outer zone as described in Equations (15) and (16):

$$\frac{k_{i1} e^{-\gamma_d^*(\psi_i - \psi_1)} \frac{\partial \psi_1}{\partial r}}{\mu_1} \bigg|_{r=r_1} = \frac{k_{i2} e^{-\gamma_d^*(\psi_i - \psi_2)} \frac{\partial \psi_2}{\partial r}}{\mu_2} \bigg|_{r=r_1}, \quad (15)$$

$$\psi_1(r=r_1, t) = \psi_2(r=r_1, t), \quad (16)$$

where r_1 is the inner zone radius, m.

Considering the infinite outer boundary, Equation (17) is obtained:

$$\lim_{r \rightarrow \infty} \psi_2(r, t) = \psi_i. \quad (17)$$

Thus, the mathematical model of composite gas reservoir pressure drawdown well test considering the stress sensitivity effect can be expressed as Equation (18):

$$\begin{cases} \frac{1}{r} \frac{\partial \psi_1}{\partial r} + \frac{\partial^2 \psi_1}{\partial r^2} + \gamma_d^* \left(\frac{\partial \psi_1}{\partial r} \right)^2 = \frac{\phi_1 \mu_1 C_g}{k_{i1}} e^{\gamma_d^*(\psi_i - \psi_1)} \frac{\partial \psi_1}{\partial t}, \\ \frac{1}{r} \frac{\partial \psi_2}{\partial r} + \frac{\partial^2 \psi_2}{\partial r^2} + \gamma_d^* \left(\frac{\partial \psi_2}{\partial r} \right)^2 = \frac{\phi_2 \mu_2 C_g}{k_{i2}} e^{\gamma_d^*(\psi_i - \psi_2)} \frac{\partial \psi_2}{\partial t}, \\ \left[r e^{-\gamma_d^*(\psi_i - \psi_1)} \frac{\partial \psi_1}{\partial r} \right] \bigg|_{r=r_{we}} = \frac{q_{sc} p_{sc} T}{\pi k_{i1} h T_{sc} Z_{sc}} + \frac{C \mu_1}{2 \pi k_{i1} h} \frac{\partial \psi_1}{\partial t}, \\ \frac{k_{i1} e^{-\gamma_d^*(\psi_i - \psi_1)} \frac{\partial \psi_1}{\partial r}}{\mu_1} \bigg|_{r=r_1} = \frac{k_{i2} e^{-\gamma_d^*(\psi_i - \psi_2)} \frac{\partial \psi_2}{\partial r}}{\mu_2} \bigg|_{r=r_1}, \\ \psi_1(r, t=0) = \psi_2(r, t=0) = \psi_i, \\ \psi_1(r=r_1, t) = \psi_2(r=r_1, t), \\ \lim_{r \rightarrow \infty} \psi_2(r, t) = \psi_i. \end{cases} \quad (18)$$

In order to obtain the dimensionless mathematical model, some dimensionless variables are introduced as follows:

$$\begin{aligned} \psi_{1D} &= \frac{\pi k_{i1} h T_{sc} (\psi_i - \psi_1)}{q_{sc} p_{sc} T}, \\ \psi_{2D} &= \frac{\pi k_{i1} h T_{sc} (\psi_i - \psi_2)}{q_{sc} p_{sc} T}, \\ t_D &= \frac{k_{i1} t}{\phi_1 \mu_1 C_g r_w^2}, \\ \omega &= \frac{\phi_1 C_{g1}}{\phi_2 C_{g2}}, \\ r_D &= \frac{r}{r_{we}} = \frac{r}{r_w \exp(-S)}, \\ r_{1D} &= \frac{r_1}{r_{we}} = \frac{r_1}{r_w \exp(-S)}, \\ \gamma_{dD}^* &= \gamma_d^* \frac{\mu_i Z_i}{2 p_i} \frac{q_{sc} p_{sc} T}{\pi k_{i1} h T_{sc}}, \\ \gamma_{bD}^* &= \gamma_b^* \frac{\mu_i Z_i}{2 p_i} \frac{q_{sc} p_{sc} T}{\pi k_{i1} h T_{sc}}, \\ M &= \frac{k_{i1} / \mu_1}{k_{i2} / \mu_2}, \\ C_D &= \frac{C}{2 \pi h \phi_1 C_g r_w^2}, \end{aligned} \quad (19)$$

where ψ_{1D} is the dimensionless pseudo-pressure in the inner zone; ψ_{2D} is the dimensionless pseudo-pressure in the outer zone; t_D is the dimensionless time; ω is the storage ratio between the inner zone and the outer zone; r_D is the dimensionless radius; r_{1D} is the dimensionless inner zone radius; γ_{dD}^* and γ_{bD}^* are dimensionless stress sensitivity coefficients at pressure drawdown and buildup stages, respectively; M is the mobility ratio between the inner zone and the outer zone; C_D is the dimensionless wellbore storage coefficient.

Substituting dimensionless variables into Equation (18), the dimensionless mathematical model of the pressure drawdown well test considering the stress sensitivity effect can be expressed as Equation (20):

$$\begin{cases} \frac{1}{r_D} \frac{\partial \psi_{1D}}{\partial r_D} + \frac{\partial^2 \psi_{1D}}{\partial r_D^2} - \gamma_{dD}^* \left(\frac{\partial \psi_{1D}}{\partial r_D} \right)^2 = \frac{e^{\gamma_{dD}^* \psi_{1D}}}{e^{2S}} \frac{\partial \psi_{1D}}{\partial t_D}, \\ \frac{1}{r_D} \frac{\partial \psi_{2D}}{\partial r_D} + \frac{\partial^2 \psi_{2D}}{\partial r_D^2} - \gamma_{dD}^* \left(\frac{\partial \psi_{2D}}{\partial r_D} \right)^2 = \frac{\omega}{M} \frac{e^{\gamma_{dD}^* \psi_{2D}}}{e^{2S}} \frac{\partial \psi_{2D}}{\partial t_D}, \\ \left(C_D \frac{\partial \psi_{1D}}{\partial t_D} - e^{-\gamma_{dD}^* \psi_{1D}} \frac{\partial \psi_{1D}}{\partial r_D} \right) \bigg|_{r_D=1} = 1, \\ \psi_{1D}(r_D, t_D=0) = \psi_{2D}(r_D, t_D=0) = 0, \\ e^{-\gamma_{dD}^* \psi_{1D}} \frac{\partial \psi_{1D}}{\partial r_D} \bigg|_{r_D=r_{1D}} = e^{-\gamma_{dD}^* \psi_{2D}} \frac{1}{M} \frac{\partial \psi_{2D}}{\partial r_D} \bigg|_{r_D=r_{1D}}, \\ \psi_{1D}(r_D=r_{1D}, t_D) = \psi_{2D}(r_D=r_{1D}, t_D), \\ \lim_{r_D \rightarrow \infty} \psi_{2D}(r_D, t_D) = 0. \end{cases} \quad (20)$$

2.2.3. Pressure Buildup Well Test Model. For the pressure buildup well test, the gas production q_{sc} in the inner boundary condition Equation (14) is zero. The stress sensitivity coefficient of the pressure buildup stage is introduced into

the mathematical model. Similarly, the dimensionless mathematical model of the pressure buildup well test considering the stress sensitivity effect and the hysteresis effect can be expressed as Equation (21):

$$\left\{ \begin{array}{l} \frac{1}{r_D} \frac{\partial \psi_{1D}}{\partial r_D} + \frac{\partial^2 \psi_{1D}}{\partial r_D^2} - \gamma_{bD}^* \left(\frac{\partial \psi_{1D}}{\partial r_D} \right)^2 = R_1 \frac{e^{\gamma_{bD}^* \psi_{1D}}}{e^{2S}} \frac{\partial \psi_{1D}}{\partial t_D}, \\ \frac{1}{r_D} \frac{\partial \psi_{2D}}{\partial r_D} + \frac{\partial^2 \psi_{2D}}{\partial r_D^2} - \gamma_{bD}^* \left(\frac{\partial \psi_{2D}}{\partial r_D} \right)^2 = R_2 \frac{\omega}{M} \frac{e^{\gamma_{bD}^* \psi_{2D}}}{e^{2S}} \frac{\partial \psi_{2D}}{\partial t_D}, \\ \left(R_1 C_D \frac{\partial \psi_{1D}}{\partial t_D} - e^{-\gamma_{bD}^* \psi_{1D}} \frac{\partial \psi_{1D}}{\partial r_D} \right) \Big|_{r_D=1} = 0, \\ \psi_{1D}(r_D, t_D=0) = \psi_{2D}(r_D, t_D=0) = 0, \\ e^{-\gamma_{bD}^* \psi_{1D}} \frac{\partial \psi_{1D}}{\partial r_D} \Big|_{r_D=r_{1D}} = e^{-\gamma_{bD}^* \psi_{2D}} \frac{1}{M} \frac{\partial \psi_{2D}}{\partial r_D} \Big|_{r_D=r_{1D}}, \\ \psi_{1D}(r_D=r_{1D}, t_D) = \psi_{2D}(r_D=r_{1D}, t_D), \\ \lim_{r_D \rightarrow \infty} \psi_{2D}(r_D, t_D) = 0, \end{array} \right. \quad (21)$$

where R_1 is the ratio of the initial permeability and the maximum permeability that can be restored after the gas reservoir pressure increases to the initial value in the inner zone, dimensionless; R_2 is the ratio of the initial permeability and the maximum permeability that can be restored after the gas reservoir pressure increases to the initial value in the outer zone, dimensionless.

2.3. Solutions to Mathematical Models

2.3.1. Solution to Pressure Drawdown Well Test Model by Perturbation Method. Kikani and Pedrosa proposed the perturbation method to solve the mathematical model for the pressure drawdown well test considering the stress sensitivity effect [23]. First, the dimensionless pseudo-pressure is transformed as Equation (22):

$$\psi_{1D} = \frac{-\ln(1 - \gamma_{dD}^* \xi_{1D})}{\gamma_{dD}^*}, \quad \psi_{2D} = \frac{-\ln(1 - \gamma_{dD}^* \xi_{2D})}{\gamma_{dD}^*}. \quad (22)$$

Substituting Equation (22) into Equation (20), the dimensionless pressure drawdown well test mathematical model can be transformed as Equation (23):

$$\left\{ \begin{array}{l} \frac{1}{r_D} \frac{\partial \xi_{1D}}{\partial r_D} + \frac{\partial^2 \xi_{1D}}{\partial r_D^2} = \frac{e^{-2S}}{(1 - \gamma_{dD}^* \xi_{1D})} \frac{\partial \xi_{1D}}{\partial t_D}, \\ \frac{1}{r_D} \frac{\partial \xi_{2D}}{\partial r_D} + \frac{\partial^2 \xi_{2D}}{\partial r_D^2} = \frac{\omega}{M} \frac{e^{-2S}}{(1 - \gamma_{dD}^* \xi_{2D})} \frac{\partial \xi_{2D}}{\partial t_D}, \\ \left(C_D \frac{\partial \xi_{1D}}{\partial t_D} - r_D \frac{\partial \xi_{1D}}{\partial r_D} \right) \Big|_{r_D=1} = \frac{1}{(1 - \gamma_{dD}^* \xi_{1D})}, \\ \xi_{1D}(r_D=r_{1D}, t_D) = \xi_{2D}(r_D=r_{1D}, t_D), \\ \frac{\partial \xi_{1D}}{\partial r_D} \Big|_{r_D=r_{1D}} = \frac{1}{M} \frac{\partial \xi_{2D}}{\partial r_D} \Big|_{r_D=r_{1D}}, \\ \xi_{1D}(r_D, t_D=0) = \xi_{2D}(r_D, t_D=0) = 0, \\ \lim_{r_D \rightarrow \infty} \xi_{2D}(r_D, t_D) = 0. \end{array} \right. \quad (23)$$

After applying zero-order approximation to ξ_{1D} and ξ_{2D} in Equation (23) [23], the new mathematical model can be expressed as Equation (24):

$$\left\{ \begin{array}{l} \frac{1}{r_D} \frac{\partial \xi_{1D}}{\partial r_D} + \frac{\partial^2 \xi_{1D}}{\partial r_D^2} = e^{-2S} \frac{\partial \xi_{1D}}{\partial t_D}, \\ \frac{1}{r_D} \frac{\partial \xi_{2D}}{\partial r_D} + \frac{\partial^2 \xi_{2D}}{\partial r_D^2} = \frac{\omega}{M} e^{-2S} \frac{\partial \xi_{2D}}{\partial t_D}, \\ \left(C_D \frac{\partial \xi_{1D}}{\partial t_D} - r_D \frac{\partial \xi_{1D}}{\partial r_D} \right) \Big|_{r_D=1} = 1, \\ \xi_{1D}(r_D=r_{1D}, t_D) = \xi_{2D}(r_D=r_{1D}, t_D), \\ \frac{\partial \xi_{1D}}{\partial r_D} \Big|_{r_D=r_{1D}} = \frac{1}{M} \frac{\partial \xi_{2D}}{\partial r_D} \Big|_{r_D=r_{1D}}, \\ \xi_{1D}(r_D, t_D=0) = \xi_{2D}(r_D, t_D=0) = 0, \\ \lim_{r_D \rightarrow \infty} \xi_{2D}(r_D, t_D) = 0. \end{array} \right. \quad (24)$$

Equation (24) is the same as the well test model of composite gas reservoir without considering the stress sensitivity effect [37]. Based on the Laplace transform and the property of Bessel function, the solution to Equation (24) in the Laplace space can be expressed as Equation (25):

$$\bar{\xi}_{wD} = \frac{1}{u} \frac{R_{TP}}{u C_D R_{TP} + \beta M_{TP}}, \quad (25)$$

where u is the variable in the Laplace space,

$$\begin{aligned} R_{TP} &= 1 + \frac{I_0(\beta)}{K_0(\beta)} \left[\frac{\beta K_0(r_{1D} \sqrt{N} \beta) K_1(r_{1D} \beta) - (\sqrt{N}/M) \beta K_0(r_{1D} \beta) K_1(r_{1D} \sqrt{N} \beta)}{[(\sqrt{N}/M) \beta K_1(r_{1D} \sqrt{N} \beta) I_0(r_{1D} \beta) + \beta K_0(r_{1D} \sqrt{N} \beta) I_1(r_{1D} \beta)]} \right], \\ M_{TP} &= \frac{K_1(\beta)}{K_0(\beta)} - \frac{I_1(\beta)}{K_0(\beta)} \left[\frac{\beta K_0(r_{1D} \sqrt{N} \beta) K_1(r_{1D} \beta) - (\sqrt{N}/M) \beta K_0(r_{1D} \beta) K_1(r_{1D} \sqrt{N} \beta)}{[(\sqrt{N}/M) \beta K_1(r_{1D} \sqrt{N} \beta) I_0(r_{1D} \beta) + \beta K_0(r_{1D} \sqrt{N} \beta) I_1(r_{1D} \beta)]} \right], \\ \beta &= \sqrt{u} \exp(-S), \quad N = \frac{\omega}{M}. \end{aligned} \quad (26)$$

Based on the numerical inversion method and the perturbation method [23], the solution to the pressure drawdown well test model considering the stress sensitivity effect for the composite gas reservoir can be expressed as Equation (27):

$$\psi_{wD} = \frac{-\ln(1 - \gamma_{dD}^* \xi_{wD})}{\gamma_{dD}^*}. \quad (27)$$

For the conventional well test without considering the stress sensitivity effect, the solution to the pressure buildup well test mathematical model, which is a linear equation, can be obtained by the superposition principle based on the solution to the pressure drawdown well test model. However, the superposition principle cannot be applied to the well test model considering the stress sensitivity effect, which is a nonlinear equation. Thus, it is difficult to get the

solution to the pressure buildup well test model considering the stress sensitivity effect by the superposition principle.

2.3.2. Solutions to Pressure Drawdown and Buildup Well Test Models by Numerical Method. The well test models considering the stress sensitivity effect and the hysteresis effect as Equations (20) and (21) are second-order nonlinear equations. The analytical solution to the pressure drawdown well test model obtained by the perturbation method is an approximate result. Thus, the numerical differentiation method is used to solve these equations.

Considering dramatic pressure changes near the well-bore, the unequal-space grid division is used. Taking $x_D = \ln r_D$, Equation (28) is obtained:

$$\frac{\partial \psi_D}{\partial r_D} = e^{-x_D} \frac{\partial \psi_D}{\partial x_D}, \quad \frac{\partial^2 \psi_D}{\partial r_D^2} = e^{-2x_D} \frac{\partial^2 \psi_D}{\partial x_D^2} - e^{-2x_D} \frac{\partial \psi_D}{\partial x_D}. \quad (28)$$

Substituting Equation (28) into the dimensionless pressure drawdown well test model Equation (20), the model can be written as Equation (29):

$$\left\{ \begin{array}{l} \frac{\partial^2 \psi_{1D}}{\partial x_D^2} - \gamma_{dD}^* \left(\frac{\partial \psi_{1D}}{\partial x_D} \right)^2 = \frac{e^{2x_D + \gamma_{dD}^* \psi_{1D}}}{e^{2S}} \frac{\partial \psi_{1D}}{\partial t_D}, \\ \frac{\partial^2 \psi_{2D}}{\partial x_D^2} - \gamma_{dD}^* \left(\frac{\partial \psi_{2D}}{\partial x_D} \right)^2 = \frac{\omega}{M} \frac{e^{2x_D + \gamma_{dD}^* \psi_{2D}}}{e^{2S}} \frac{\partial \psi_{2D}}{\partial t_D}, \\ \left(C_D \frac{\partial \psi_{1D}}{\partial t_D} - e^{-\gamma_{dD}^* \psi_{1D}} \frac{\partial \psi_{1D}}{\partial x_D} \right) \Big|_{x_D=0} = 1, \\ \psi_{1D}(x_D = \ln r_{1D}, t_D) = \psi_{2D}(x_D = \ln r_{1D}, t_D), \\ e^{-\gamma_{dD}^* \psi_{1D}} \frac{\partial \psi_{1D}}{\partial x_D} \Big|_{x_D = \ln r_{1D}} = e^{-\gamma_{dD}^* \psi_{2D}} \frac{1}{M} \frac{\partial \psi_{2D}}{\partial x_D} \Big|_{x_D = \ln r_{1D}}, \\ \psi_{1D}(x_D, t_D = 0) = \psi_{2D}(x_D, t_D = 0) = 0, \\ \lim_{x_D \rightarrow \infty} \psi_{2D}(x_D, t_D) = 0. \end{array} \right. \quad (29)$$

The difference in space is as Equation (30):

$$x_D(i) = \begin{cases} (i-1)\Delta x_{1D} & i = 1, 2, 3, \dots, N, \\ \ln r_{1D} + (i-N)\Delta x_{2D} & i = N+1, N+2, N+3, \dots, NN, \end{cases} \quad (30)$$

where $\Delta x_{1D} = \ln r_{1D}/(N-1)$; $\Delta x_{2D} = \ln(R_D/r_{1D})/(NN-N)$; N is the discrete number in space of the inner zone, NN is the total discrete number in space of inner and outer zones, and R_D is the dimensionless outer zone boundary radius.

The logarithmic interval is taken for time as Equation (31):

$$t_D(j) = 10^{-6+(j-1)\Delta t}, \quad \Delta t = \frac{\log(T_D) + 6}{O-1}, \quad (31)$$

where $j = 1, 2, 3, \dots, O$, O is the discrete number of time and T_D is the dimensionless time at the end of the simulation.

For the dimensionless well test mathematical model Equation (29), the finite difference discretization is carried out in space and time as Equations (30) and (31). Then,

the seepage equation discretization result for the composite gas reservoir can be expressed as Equation (32):

$$\begin{aligned} & a(i)\psi_{Di-1}^j + b(i)\psi_{Di}^j + c(i)\psi_{Di+1}^j + d(i)\left(\psi_{Di}^j\right)^2 + e(i)\psi_{Di}^j\psi_{Di+1}^j \\ & + f(i)\left(\psi_{Di+1}^j\right)^2 + g(i)e^{r_{dD}^*\psi_{Di}^j} + h(i)\psi_{Di}^j e^{r_{dD}^*\psi_{Di}^j} = 0. \end{aligned} \quad (32)$$

When $i = 1, 2, \dots, N$, then

$$\begin{aligned} a(i) &= \frac{1}{\Delta x_{1D}^2}, \\ b(i) &= -\frac{2}{\Delta x_{1D}^2}, \\ c(i) &= \frac{1}{\Delta x_{1D}^2}, \\ d(i) &= -\frac{\gamma_{dD}^*}{\Delta x_{1D}^2}, \\ e(i) &= \frac{2\gamma_{dD}^*}{\Delta x_{1D}^2}, \\ f(i) &= -\frac{\gamma_{dD}^*}{\Delta x_{1D}^2}, \\ g(i) &= \frac{e^{2(i-1)\Delta x_{1D}}}{e^{2S}(t_D^j - t_D^{j-1})} \psi_{Di}^{j-1}, \\ h(i) &= -\frac{e^{2(i-1)\Delta x_{1D}}}{e^{2S}(t_D^j - t_D^{j-1})}. \end{aligned} \quad (33)$$

When $i = N+1, N+2, \dots, NN$, then

$$\begin{aligned} a(i) &= \frac{1}{\Delta x_{2D}^2}, \\ b(i) &= -\frac{2}{\Delta x_{2D}^2}, \\ c(i) &= \frac{1}{\Delta x_{2D}^2}, \\ d(i) &= -\frac{\gamma_{dD}^*}{\Delta x_{2D}^2}, \\ e(i) &= \frac{2\gamma_{dD}^*}{\Delta x_{2D}^2}, \\ f(i) &= -\frac{\gamma_{dD}^*}{\Delta x_{2D}^2}, \\ g(i) &= \frac{\omega}{M} \frac{e^{2(i-1)\Delta x_{2D}}}{e^{2S}(t_D^j - t_D^{j-1})} \psi_{Di}^{j-1}, \\ h(i) &= -\frac{\omega}{M} \frac{e^{2(i-1)\Delta x_{2D}}}{e^{2S}(t_D^j - t_D^{j-1})}. \end{aligned} \quad (34)$$

The discretization result of the initial condition can be expressed as Equation (35):

$$\psi_{Di}^1 = 0, i = 1, 2, \dots, NN. \quad (35)$$

The discretization result of the inner boundary condition can be expressed as Equation (36):

$$b(1)\psi_{D1}^j + c(1)\psi_{D2}^j + g(1)e^{\gamma_{ad}^* \psi_{Di}^j} + h(1)\psi_{D1}^j e^{\gamma_{ad}^* \psi_{Di}^j} = 0, \quad (36)$$

where

$$\begin{aligned} b(1) &= -\frac{1}{\Delta x_{1D}}, \\ c(1) &= \frac{1}{\Delta x_{1D}}, \\ g(1) &= 1 + \frac{C_D \psi_{D1}^{j-1}}{t_D^j - t_D^{j-1}}, \\ h(1) &= -\frac{C_D}{t_D^j - t_D^{j-1}}. \end{aligned} \quad (37)$$

The discretization result of the interface condition can be expressed as Equation (38):

$$\frac{\psi_{DN} - \psi_{DN-1}}{\Delta x_1} = \frac{1}{M} \frac{\psi_{DN+1} - \psi_{DN}}{\Delta x_2}. \quad (38)$$

The discretization result of the outer boundary condition can be expressed as Equation (39):

$$\psi_{DNN}^j = 0. \quad (39)$$

As for the pressure buildup test, the model is obtained after modifying the internal boundary condition and introducing the stress sensitivity coefficient of the buildup stage. Then, the seepage equation discretization result for the composite gas reservoir at the buildup stage can be expressed as Equation (40):

$$\begin{aligned} &a(i)\psi_{Di-1}^j + b(i)\psi_{Di}^j + c(i)\psi_{Di+1}^j + d(i)\left(\psi_{Di}^j\right)^2 \\ &+ e(i)\psi_{Di}^j \psi_{Di+1}^j + f(i)\left(\psi_{Di+1}^j\right)^2 + g(i)e^{\gamma_{bd}^* \psi_{Di}^j} \\ &+ h(i)\psi_{Di}^j e^{\gamma_{bd}^* \psi_{Di}^j} = 0. \end{aligned} \quad (40)$$

When $i = 1, 2, \dots, N$, then

$$\begin{aligned} a(i) &= \frac{1}{\Delta x_{1D}^2}, \\ b(i) &= -\frac{2}{\Delta x_{1D}^2}, \\ c(i) &= \frac{1}{\Delta x_{1D}^2}, \\ d(i) &= -\frac{\gamma_{bd}^*}{\Delta x_{1D}^2}, \\ e(i) &= \frac{2\gamma_{bd}^*}{\Delta x_{1D}^2}, \\ f(i) &= -\frac{\gamma_{bd}^*}{\Delta x_{1D}^2}, \\ g(i) &= R_1 \frac{e^{2(i-1)\Delta x_{1D}}}{e^{2S}\left(t_D^j - t_D^{j-1}\right)} \psi_{Di}^{j-1}, \\ h(i) &= -R_1 \frac{e^{2(i-1)\Delta x_{1D}}}{e^{2S}\left(t_D^j - t_D^{j-1}\right)}. \end{aligned} \quad (41)$$

When $i = N + 1, N + 2, \dots, NN$, then

$$\begin{aligned} a(i) &= \frac{1}{\Delta x_{2D}^2}, \\ b(i) &= -\frac{2}{\Delta x_{2D}^2}, \\ c(i) &= \frac{1}{\Delta x_{2D}^2}, \\ d(i) &= -\frac{\gamma_{bd}^*}{\Delta x_{2D}^2}, \\ e(i) &= \frac{2\gamma_{bd}^*}{\Delta x_{2D}^2}, \\ f(i) &= -\frac{\gamma_{bd}^*}{\Delta x_{2D}^2}, \\ g(i) &= R_2 \frac{\omega}{M} \frac{e^{2(i-1)\Delta x_{2D}}}{e^{2S}\left(t_D^j - t_D^{j-1}\right)} \psi_{Di}^{j-1}, \\ h(i) &= -R_2 \frac{\omega}{M} \frac{e^{2(i-1)\Delta x_{2D}}}{e^{2S}\left(t_D^j - t_D^{j-1}\right)}. \end{aligned} \quad (42)$$

The discretization result of the inner boundary condition at the buildup stage can be expressed as Equation (43):

$$b(1)\psi_{D1}^j + c(1)\psi_{D2}^j + g(1)e^{\gamma_{bd}^* \psi_{Di}^j} + h(1)\psi_{D1}^j e^{\gamma_{bd}^* \psi_{Di}^j} = 0, \quad (43)$$

FIGURE 3: Comparison between the numerical solution and the analytical solution to the pressure drawdown well test model without considering the stress sensitivity effect.

FIGURE 4: Comparison between the numerical solution and the perturbation solution to the pressure drawdown well test considering the stress sensitivity effect.

FIGURE 5: Comparison of the homogeneous gas reservoir pressure buildup well test curve obtained from the paper and the research of Zhang and He.

FIGURE 6: Well test curves for the pressure drawdown well test in composite gas reservoir.

FIGURE 7: Well test curves for the pressure buildup well test in composite gas reservoir.

FIGURE 8: Well test curves of the pressure drawdown well test under different stress sensitivity coefficients.

FIGURE 9: Well test curves of the pressure buildup well test under different stress sensitivity coefficients.

FIGURE 12: Well test curves of the pressure drawdown well test under different skin factors.

FIGURE 10: Well test curves of the pressure drawdown well test considering the hysteresis effect.

FIGURE 13: Well test curves of the pressure buildup well test under different skin factors.

where

$$\begin{aligned}
 b(1) &= -\frac{1}{\Delta x_{1D}}, \\
 c(1) &= \frac{1}{\Delta x_{1D}}, \\
 g(1) &= R_1 \frac{C_D \psi_{D1}^{j-1}}{t_D^j - t_D^{j-1}}, \\
 h(1) &= -R_1 \frac{C_D}{t_D^j - t_D^{j-1}}.
 \end{aligned} \tag{44}$$

The discretization results of the interface condition and the outer zone boundary condition for the pressure buildup well test are the same as Equations (38) and (39).

Numerical well test models of drawdown and buildup stages for the composite gas reservoir considering the stress sensitivity effect and the hysteresis effect are established by Equations (32)–(39) and Equations (40)–(43). The Newton-Simpson method is used to solve the nonlinear equation to obtain pressure drawdown and buildup well test solutions.

FIGURE 11: Well test curves of the pressure buildup well test considering the hysteresis effect.

3. Type Curve of Composite Gas Reservoir Well Test

3.1. Model Verification

3.1.1. Verification of Pressure Drawdown Well Test Model. When dimensionless stress sensitivity coefficients γ_{dD}^* and γ_{bD}^* are taken as zero, the model is the same as the conventional pressure drawdown well test model of composite gas reservoir [37]. In order to verify the model in the paper, the type curve obtained by the numerical method without considering the stress sensitivity effect is compared with the conventional composite gas reservoir well test curve obtained by the analytical solution shown in Figure 3.

As shown in Figure 3, the type curves of pressure drawdown well test obtained by the numerical solution and the analytical solution are the same when dimensionless stress sensitivity coefficients are set to zero. The curves show common characteristics. The pressure derivative curve is a 0.5 horizontal line at the inner zone radial flow stage, while it is a $M/2$ (mobility ratio/2) horizontal line at the outer zone radial flow stage. Thus, the numerical solution in the paper is reliable.

In order to further verify the model, the type curves of pressure drawdown well test considering the stress sensitivity effect obtained by the numerical method and the perturbation method [23] are compared shown in Figure 4.

It can be seen from Figure 4 that the numerical solution is consistent with the solution obtained by the perturbation method, which verifies the reliability of the numerical solution.

3.1.2. Verification of Pressure Buildup Well Test Model. In order to verify the pressure buildup well test model of composite gas reservoir in the paper, the model is simplified into a homogeneous gas reservoir model considering the stress sensitivity effect by setting $M = 1$, $\omega = 1$, and $\gamma_{bD}^* = \gamma_{dD}^*$. With those parameter settings, the model does not reflect the influence of the hysteresis effect and the differences of permeability and porosity between the inner zone and the outer zone. Then, the result of the model is compared with the research of Zhang and He [31] shown in Figure 5.

It can be seen from Figure 5 that the well test curve obtained from the simplified model in the paper is consistent with the curve from the research of Zhang and He [31]. Thus, the numerical solution to the pressure buildup well test model in the paper is reliable.

3.2. Type Curve Analysis. Figures 6 and 7 are type curves for pressure drawdown and buildup well tests in a composite gas reservoir. The corresponding parameter values are $C_D = 1$, $S = 1$, $M = 0.5$, $\omega = 1$, $r_{1D} = 500$, $\gamma_{dD}^* = 0$ or 0.04 , and $\gamma_{bD}^* = 0$ or 0.04 . And when $\gamma_{dD}^* = \gamma_{bD}^* = 0$, the well test model is suitable for the conventional composite gas reservoir; when $\gamma_{dD}^* = \gamma_{bD}^* = 0.04$, the model is suitable for the composite gas reservoir considering the stress sensitivity effect.

It can be seen from Figures 6 and 7 that type curves of the composite gas reservoir well test considering the stress sensitivity effect can be divided into five basic flow regimes

including ① the wellbore storage stage, ② the transition stage dominated by the skin factor, ③ the inner zone radial flow stage, ④ the transition stage between the inner zone and outer zone radial flow stages, and ⑤ the outer zone radial flow stage. The characteristics of the wellbore storage stage are not affected by the stress sensitivity effect. The transition stage and the radial flow stage are significantly affected by the stress sensitivity effect. For the pressure drawdown well test shown in Figure 6, the pseudo-pressure curve has a significant uplift compared with the conventional well test curve indicating a larger pressure drawdown at the same dimensionless time t_D after considering the stress sensitivity effect; the pseudo-pressure derivative curve at the inner zone radial flow stage gradually deviates upward from the 0.5 horizontal line indicating the permeability decreases with the increase of the pressure drawdown at the inner zone radial flow stage; the pseudo-pressure derivative curve at the outer zone radial flow stage gradually deviates upward from the $M/2$ (mobility ratio/2) horizontal line indicating the permeability decreases with the increase of the pressure drawdown at the outer zone radial flow stage.

For the pressure buildup well test shown in Figure 7, the pseudo-pressure curve has a significant uplift compared with the conventional well test curve after considering the stress sensitivity effect; the pseudo-pressure derivative curve at the inner zone radial flow stage gradually descends towards the 0.5 horizontal line indicating the permeability increases with the increase of the formation pressure during the buildup stage, while the pseudo-pressure derivative curve at the outer zone radial flow stage gradually descends towards the $M/2$ horizontal line. The pseudo-pressure derivative curve is slightly higher than the $M/2$ horizontal line at the outer zone radial flow stage indicating that the formation pressure is close to the initial pressure.

3.3. Parameter Sensitivity Analysis

3.3.1. The Effect of Stress Sensitivity Coefficient. The type curves of pressure drawdown and buildup well tests considering the stress sensitivity effect under different stress sensitivity coefficients are shown in Figures 8 and 9. The corresponding parameter values are $C_D = 1$, $S = 1$, $r_{1D} = 500$, $M = 0.5$, $\omega = 1$, and $\gamma_{dD}^* = \gamma_{bD}^* = 0, 0.02, 0.04$, and 0.06 .

As shown in Figure 8, for the pressure drawdown well test, as stress sensitivity coefficients γ_{dD}^* and γ_{bD}^* increase, the pseudo-pressure curve gradually rises, and the pseudo-pressure derivative curve deviates upward from the 0.5 horizontal line at the inner zone radial flow stage, while it deviates upward from the $M/2$ (mobility ratio/2) horizontal line at the outer zone radial flow stage. The changes indicate that the pressure drawdown becomes larger, and the permeability becomes lower at the same dimensionless time t_D as stress sensitivity coefficients increase.

For the pressure buildup well test shown in Figure 9, as stress sensitivity coefficients γ_{dD}^* and γ_{bD}^* increase, the pseudo-pressure and pseudo-pressure derivative curves gradually rise, which means that the speed of formation pressure buildup is quicker as stress sensitivity coefficients γ_{dD}^* and γ_{bD}^* increase. The pseudo-pressure derivative

FIGURE 14: Well test curves of the pressure drawdown well test under different inner zone radii.

FIGURE 15: Well test curves of the pressure buildup well test under different inner zone radii.

curve at the inner zone radial flow stage gradually descends towards the 0.5 horizontal line, while the pseudo-pressure derivative curve at the outer zone radial flow stage gradually descends towards the $M/2$ (mobility ratio/2) horizontal line in all cases.

3.3.2. The Effect of Hysteresis Effect. The type curves of pressure drawdown and buildup well tests considering the hysteresis effect are shown in Figures 10 and 11. The corresponding parameter values are $C_D = 1$, $S = 1$, $r_{1D} = 500$, $M = 0.5$, $\omega = 1$, $\gamma_{dD}^* = 0.1$, and $\gamma_{bD}^* = 0.1, 0.05$, and 0.01 .

As shown in Figure 10, for the pressure drawdown well test with one production stage followed by one shut-in stage, the stress hysteresis effect has no effect on the curve of pressure drawdown well test due to the same stress sensitivity coefficient γ_{dD}^* at the pressure drawdown stage.

For the pressure buildup well test curve shown in Figure 11, as the stress sensitivity coefficient γ_{bD}^* of buildup stage decreases, and the pseudo-pressure derivative curve gradually rises at both inner zone and outer zone radial flow stages. Moreover, the transition stage between inner zone and outer zone radial flow stages appears later. Because the lower stress sensitivity coefficient of buildup stage γ_{bD}^* is, the slower of permeability recovery speed is. The final permeability recovery value is also smaller at the pressure buildup stage.

3.3.3. The Effect of Skin Factor. The type curves of pressure drawdown and buildup well tests considering the stress sensitivity effect under different skin factors are shown in Figures 12 and 13. The corresponding parameter values are $C_D = 1$, $\gamma_{dD}^* = \gamma_{bD}^* = 0.04$, $M = 0.5$, $\omega = 1$, $r_{1D} = 500$, and $S = -1, 1, 3$, and 5 .

As shown in Figures 12 and 13, the pseudo-pressure curves of pressure drawdown and buildup well tests both have an uplift as the skin factor S increases indicating a larger pressure drawdown or buildup at the same dimensionless time t_D . At the inner zone or outer zone radial flow stages, the pseudo-pressure derivative curves of pressure drawdown well test obviously deviate upward as the skin factor increases, but the pseudo-pressure derivative curves at the radial flow stage of pressure buildup well test are not affected by the skin factor. This feature is different from that of the conventional composite gas reservoir well test curve. For conventional composite gas reservoir well test type curves under different skin factors, the pseudo-pressure derivative curves for both pressure drawdown and buildup well tests are 0.5 horizontal lines at inner zone radial flow stages. Those are $M/2$ horizontal lines at the outer zone radial flow stages.

3.3.4. The Effect of Inner Radius. The type curves of pressure drawdown and buildup well tests considering the stress

FIGURE 16: Well test curves of the pressure drawdown well test under different mobility ratios.

FIGURE 19: Well test curves of the pressure buildup test under different storage ratios.

FIGURE 17: Well test curves of the pressure buildup well test under different mobility ratios.

FIGURE 18: Well test curves of the pressure drawdown well test under different storage ratios.

TABLE 1: The basic parameters of gas well test in the deepwater gas reservoir.

Parameters	Values
Well type	Vertical well
Initial pressure (MPa)	72.91
Reservoir temperature ($^{\circ}\text{C}$)	167.68
Effective thickness (m)	9.1
Wellbore radius (m)	0.127
Porosity (decimal)	0.2197
Rock compressibility (MPa^{-1})	0.000501
Relative density of natural gas (decimal)	0.764

FIGURE 20: The pressure history of the well in the deepwater gas reservoir.

sensitivity effect under different inner radii are shown in Figures 14 and 15. The corresponding parameter values are $C_D = 1$, $S = 1$, $\gamma_{dD}^* = \gamma_{bD}^* = 0.04$, $M = 0.5$, $\omega = 1$, and $r_{1D} = 200, 300, 400$, and 500 .

As shown in Figures 14 and 15, the radius of inner zone mainly affects the duration of inner zone radial flow. The larger inner zone radius leads to a longer duration of inner zone radial flow and a later appearance of the transition stage between inner zone and outer zone radial flow stages.

FIGURE 21: Comparison of interpretation results with and without considering the stress sensitivity effect. (a, b) Well test interpretation results in the double logarithmic curve. (a) The interpretation result without considering the stress sensitivity effect. (b) The interpretation result considering the stress sensitivity effect. (c, d) Well test interpretation results in the semilogarithm curve. (c) The interpretation result without considering the stress sensitivity effect. (d) The interpretation result considering the stress sensitivity effect.

3.3.5. The Effect of Mobility Ratio. The type curves of pressure drawdown and buildup well tests considering the stress sensitivity effect under different mobility ratios are shown in Figures 16 and 17. The corresponding parameter values are $C_D = 1$, $S = 1$, $\gamma_{dD}^* = \gamma_{bD}^* = 0.04$, $\omega = 1$, $r_{1D} = 500$, and $M = 0.2, 0.6, 1.0, 1.5$, and 2 .

As shown in Figures 16 and 17, for the pressure drawdown well test, the mobility ratio mainly affects characteristics of the transition stage and the outer zone radial flow stage. A larger mobility ratio leads to a higher pseudo-pressure and pseudo-pressure derivative curve at the outer zone radial flow stage indicating a larger pressure drawdown and a lower permeability in the outer zone at the same dimensionless time. For the pressure buildup well test, the mobility ratio has an effect on both inner zone and outer zone radial flow stages. A larger mobility ratio leads to a higher pseudo-pressure and pseudo-pressure derivative curve in both two flow regimes. The feature is different from the pressure buildup well test curve of conventional composite gas reservoir. The mobility ratio has no effect on the pseudo-pressure derivative of both pressure drawdown and buildup well tests at the inner zone radial flow stage for the conventional well test curve.

3.3.6. The Effect of Storage Ratio. The type curves of pressure drawdown and buildup well tests considering the stress sensitivity effect under different storage ratios are shown in Figures 18 and 19. The corresponding parameter values are

$C_D = 1$, $S = 1$, $\gamma_{dD}^* = \gamma_{bD}^* = 0.04$, $r_{1D} = 500$, $M = 1$, and $\omega = 0.1, 0.5, 1, 5$, and 10 .

As shown in Figures 18 and 19, when the storage ratio ω is greater than 1, the curve shows a small hump at the transition stage between inner zone and outer zone radial flow stages indicating that the inner zone storage capacity is greater than that of the outer zone. When the storage ratio ω is less than 1, the curve shows a small concave.

4. Case Study

Taking a case from a deepwater gas reservoir with high temperature and pressure as an example, the basic parameter of the well is shown in Table 1. The pressure history of the well is shown in Figure 20.

Taking the second buildup stage to conduct the well test interpretation, interpretation results are shown in Figure 21 and Table 2. It can be seen that there is no obvious boundary response in the well test curve; thus, the well test model is chosen as the infinite model with a constant well storage coefficient and skin factor. The pressure derivative curve of the pressure buildup well test has obvious downward trend indicating the influence of the stress sensitivity effect. Therefore, a well test model considering the stress sensitivity effect is used in the well test interpretation of this case. Besides, the results of well test interpretation with and without considering the stress sensitivity effect are compared shown in Figure 21 and Table 2.

TABLE 2: Comparison of interpretation results with and without considering the stress sensitivity effect.

Parameters	Considering the stress sensitivity effect	Without considering the stress sensitivity effect	Variation amplitude of the interpretation parameters (%)
Well storage coefficient (m^3/MPa)	0.280	0.283	1.06
Skin factor	3.50	5.46	56.0
Permeability (mD)	5.54	6.50	17.3
Stress sensitivity coefficient (MPa^{-1})	0.012	–	–
Mobility ratio	1.0	1.0	–
Storage ratio	1.0	1.0	–

It can be seen from Table 2 and Figure 21 that better fitting results of pressure curves can be obtained after considering the stress sensitivity effect. The skin factor will be exaggerated without considering the stress sensitivity effect. Therefore, the accuracy of the well test interpretation for the deepwater gas reservoir with high temperature and pressure is improved after considering the stress sensitivity effect.

5. Conclusion

The paper establishes a composite gas reservoir pressure buildup well test model considering the stress sensitivity and the hysteresis effect to guide well test interpretations for high temperature and pressure composite gas reservoirs in deepwater. Based on the research, the following conclusions are obtained:

- (1) The solutions to composite gas reservoir pressure drawdown and buildup well test models considering the stress sensitivity and the hysteresis effect are obtained by the numerical differentiation method. Then, the solution to pressure drawdown well test model is verified by comparing with the analytical solution obtained by the perturbation method and the analytical solution to the conventional well test model. The solution to the pressure buildup well test model is verified by comparing with the homogeneous gas reservoir well test solution from the research of Zhang and He after setting $M = 1$, $\omega = 1$, and $\gamma_{bD}^* = \gamma_{dD}^*$
- (2) The differences between pressure drawdown and buildup well test curves considering the stress sensitivity effect and the hysteresis effect are compared. The main flow regimes affected by the stress sensitivity effect are the inner zone and outer zone radial flow stages. The pressure derivative curve of pressure drawdown well test considering the stress sensitivity effect deviates upward from the 0.5 horizontal line at the inner zone radial flow stage, while it deviates upward from the $M/2$ (mobility ratio/2) horizontal line at the outer zone radial flow stage. However, for the pressure buildup well test curve considering the stress sensitivity effect, the pressure derivative

curve gradually descends to the 0.5 horizontal line at the inner zone radial flow stage, while it descends to the $M/2$ (mobility ratio/2) horizontal line at the outer zone radial flow stage. The pressure derivative curve considering the hysteresis effect is higher than the curve without considering the hysteresis effect for the pressure buildup well test, because the permeability cannot be recovered to its original value in the buildup stage after considering the hysteresis effect

- (3) The effects of different parameters such as stress sensitivity coefficient, hysteresis effect coefficient, skin factor, inner radius, mobility ratio, and storage ratio on type curves are analyzed. The stress sensitivity coefficient, skin factor, and mobility ratio have different effects on pressure drawdown and buildup well test curves after considering the stress sensitivity effect, which are different from conventional well test curves without considering the stress sensitivity effect
- (4) A well test interpretation case from the deepwater gas reservoir with high temperature and pressure is studied. The result indicates that the accuracy of interpretation is improved after considering the stress sensitivity effect. The skin factor will be exaggerated without considering the stress sensitivity effect

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no conflict of interest.

Authors' Contributions

Conceptualization was done by Yihua Gao; methodology was done by Yihua Gao, Ruizhong Jiang, and Zhaobo Sun; validation was done by Yihua Gao, Xiangdong Xu, Zhaobo Sun, and Zhiwang Yuan; the original draft preparation was done by Yihua Gao; the review and editing were done by

Xiangdong Xu, Kang Ma, Zhiwang Yuan, Bin Jiang, Botao Kang, Guoning Chen, and Chenxi Li.

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