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Research Article

Study on Connectivity Mechanism and Robustness of Three-Dimensional Pore Network of Sandstone

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As one of the most prevalent porous media, rock contains a large number of pore throats of varying size and shape. It is essential to analyze the complex pore network structure and to define the network structural features to reveal the microscopic mechanism of the rock permeability. In this paper, based on the complex network theory and CT scanning technology, sandstone is used as an example to study the structural characteristics of the rock network with different porosities. The results show that the structural characteristics of the sandstone seepage network are consistent with BA scale-free network, whose average path length increases with the size of the network. At the same time, the porosity of the sandstone is strongly influenced by the number of throat in the rock pore network. Furthermore, our analysis concludes that a few pores with a large number of connections contribute significantly to the overall connectivity of the sandstone seepage network. Removing the 'hub' pores increased the average path length of the entire network by 27.63-37.26%, which could not be achieved by randomly removing method. While the sandstone seepage network has better fault tolerance and robustness to external random attacks, this study provides a new approach to study the mechanisms of fluid storage and migration in porous media.

1. Introduction

In recent years, with the extensive development and utilisation of nonlinear reservoirs, oil and gas exploration techniques have become the focus of current research [1–3]. However, the depth of the reservoir and the complex stresses making are it difficult to visualise and quantitatively characterise the microscopic pore network structure [4–6]. Therefore, it has become the aim of many scientists and engineers to further explore the rock pore structure, define the structural characteristics of the network, and reveal the contribution of rock microstructure to the permeability [7–10].

With the development of CT scanning and SEM techniques, the microstructure of rocks at the micro- and nanoscale can be characterised by methods such as digital cores [11, 12]. Zhao reconstructed artificial cores, sandstone, and carbonate digital cores using a process approach and simulated annealing algorithms [13]. Blunt et al. based his work on high-precision computed tomography to extract the pore structure of different porous media and introduce it to the pore scale [14]. The central axis that connects the pores of the core extracted and connected by Dong and Blunt with the central axis method can characterise the porosity of the rock accurately [15]. Kelly et al. used fibre scanning electron microscopy to extract digital cores of shales with different porosities and explored the network connectivity of shale pores using the collocation number method [16]. Based on statistical information of rock slice, porosity and two-point

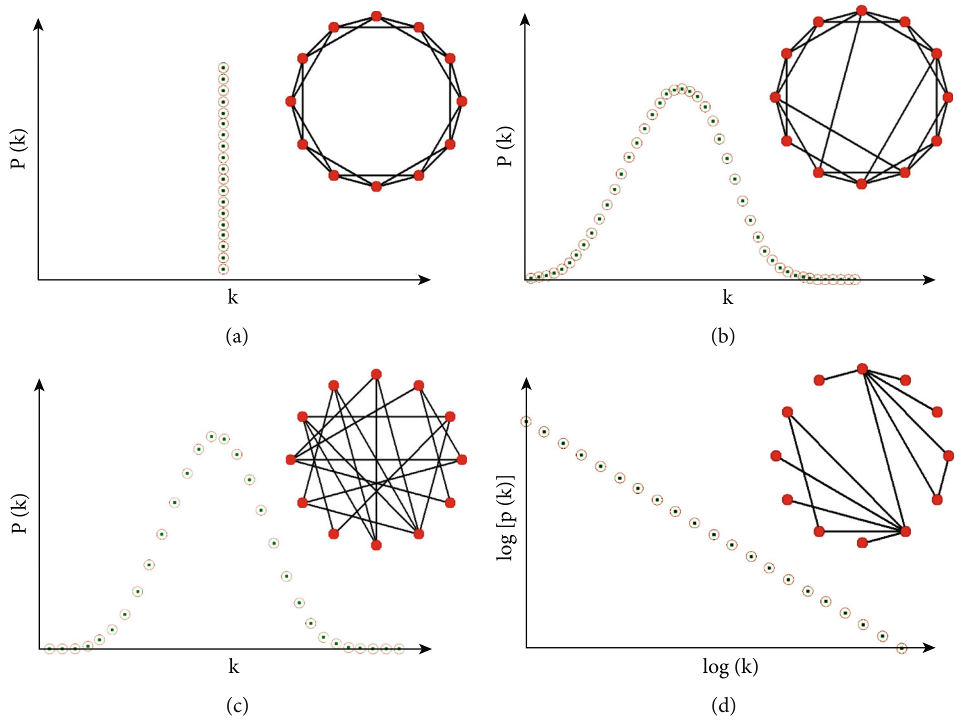

Figure 1: Network model structure: (a) the regular network [28], (b) ER random network [29], (c) WS small-world network [30], and (d) BA scale-free network [31].

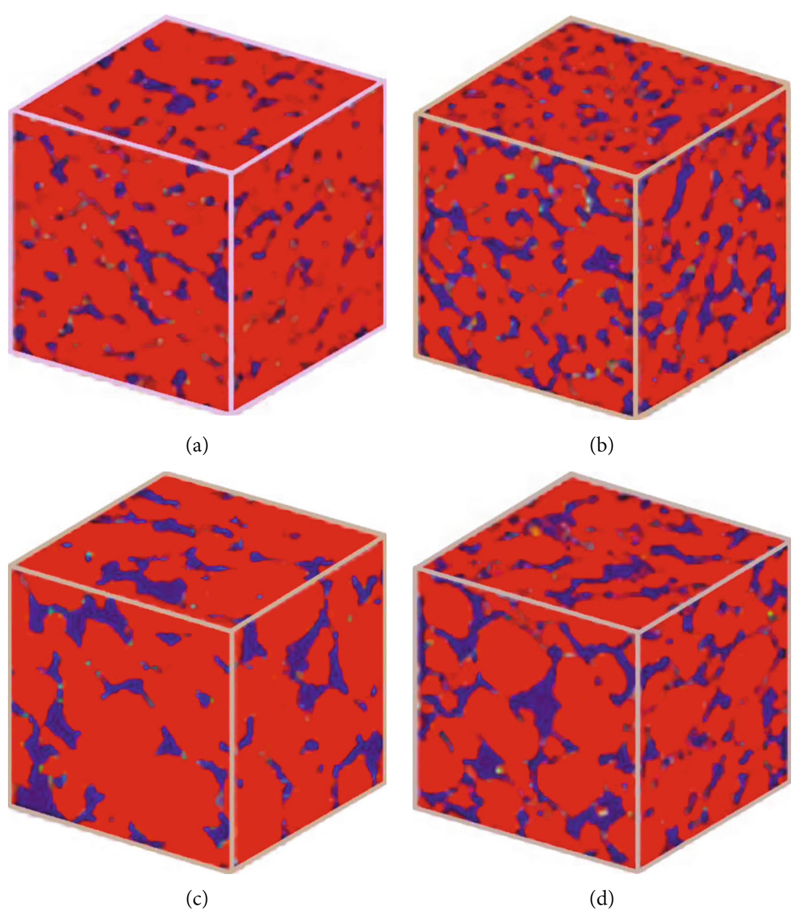

Figure 2: Three-dimensional sandstone pore network. The porosities are (a) 14.1%, (b) 21.1%, (c) 24.6%, and (d) 25.1%.

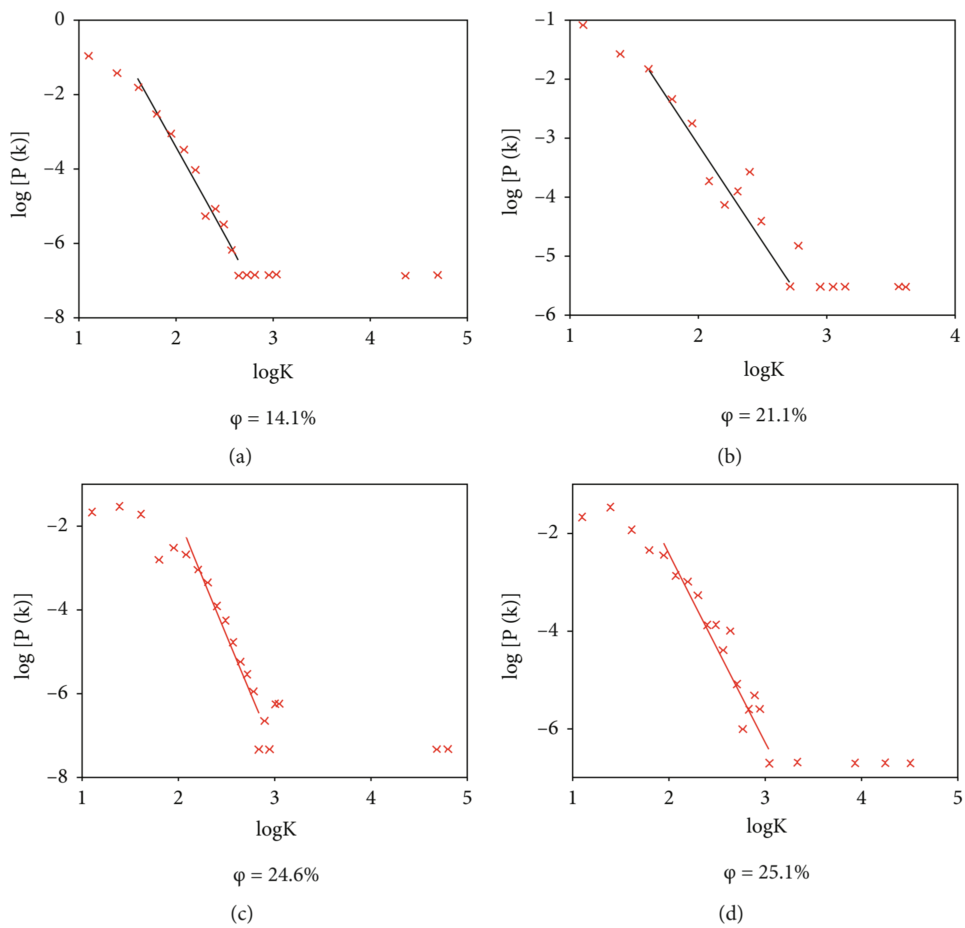

Figure 3: Double logarithm curve of degree distribution of different porosity sandstone seepage networks.

correlation function were used as the constraint condition, and the model of the two-dimensional numerical core was obtained. A sandstone structure model that was reconstructed by Øren and Bakke with the process method was found can better reflect the geometric structure and permeability of the real rock by comparing with the digital image of the core slice [17].

Additionally, the rock pore network reconstruction method based on digital cores has been widely adopted. The Gaussian field algorithm proposed by Joshi can be used to reconstruct porous media [18]. Hu and Li used the theory of directional seepage to develop a stochastic network model which can characterise the microscopic pore structure and wettability of rocks [19]. Zhao et al. searched the pore space in different directions by the multidirectional scanning method and established a pore structure model [20]. However, the pores of the rock network are interconnected, and they are changed unceasing by the action of the unavoidable external force so that the entire network changes irreversibly, which will cause great interference for the seepage process. Under the impact of multiple factors, different kinds of changes in the structure and macroscopic properties of rocks occur, leading to uncertainty in the applicability of the original results. All of which make it exceptionally difficult to study the seepage properties of rocks under multiple factors.

As a new interdisciplinary theory, a complex network theory has been applied for numerous fields from biology to sociology [21–23]. To address the uncertainty in the microstructural behaviour of rocks under multiple factors, we apply the complex network theory to characterise the microtopology of the rock pore network. Simultaneously, the robustness of rock pore space and the impact on macroscopic permeability are analyzed based on the contribution of variations in stratigraphic factors to the rock-ore network.

2. The Type of Rock Network Structure

The network structure is divided into regular networks and complex networks. And among the networks, the regular network, the ER random network, and the WS small-world network are three of the most classically recognised models. In the network structure, $P(k)$ is considered the probability of a node whose degree is k, where k is the number of edges connected to the node. In the regular networks, the number of connected edges among nodes is identical. Any two nodes in the network are directly connected and with an average path length of 1. As for complex network, each pair of nodes

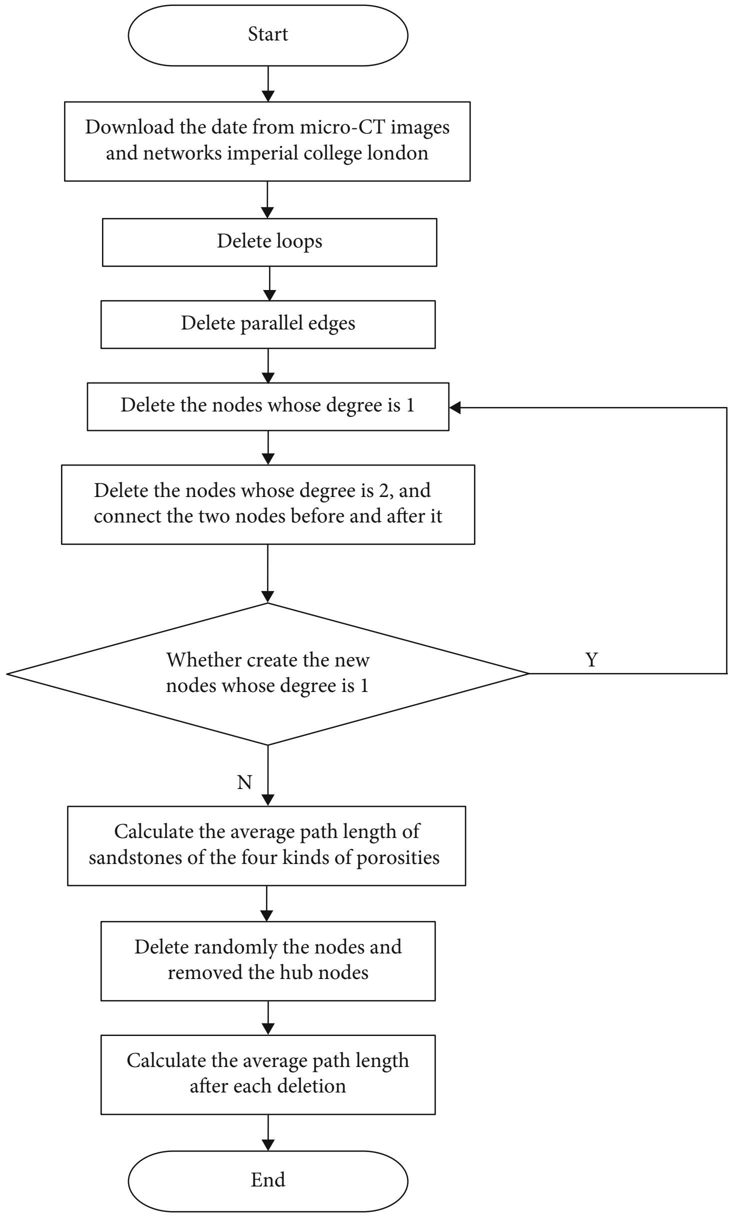

Figure 4: The process of simplification.

in ER random network is connected at the probability of P, and degree of the great majority of nodes is average $\langle k \rangle$. Most of the nodes of the WS small-world network are not connected to each other, but different nodes can be reached with each other through the few sides. ER random networks and WS small-world networks satisfy the Poisson distribution. The widespread real networks that have been demonstrated cannot be effectively explained by teh above homogeneous networks, such as mobile Internet [24], network of cell metabolism [25, 26], Internet networks [27], and rock internal structures. The BA scale-free network is one of the nonuniformity networks, and few nodes with a large degree play a leading role in connectivity of the network. Besides, the growth and mechanism of preferential connection in the BA scale-free network are similar to the characteristics in natural rock. The degree distribution satisfies the power-law distribution $(P(k) \approx k^{-\gamma})$. And BA scale-free has better fault tolerance and robustness to random node failures since there is no distinct eigenvalues in

Table 1: The sides of seepage network is expressed by d, the power exponent is expressed by γ, the mean of nodal degree is expressed by k, and $P(k) \approx k^{-\gamma}$.

ϕ	d	γ	$\langle k \rangle$	D
14.1%	2241	4.7694	4.4833	2.378
21.1%	674	3.2638	5.1538	2.272
24.6%	4490	5.5606	5.7786	2.468
25.1%	2552	3.8674	6.1541	2.539

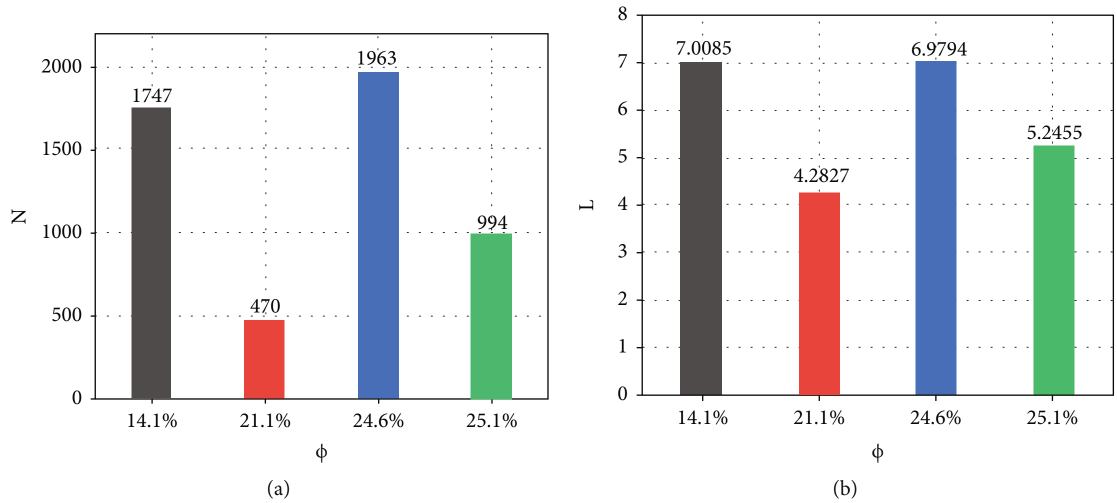

Figure 5: (a) The network magnitude. (b) The average path length (L).

the network. The network model structures are as shown in Figure 1.

3. Acquirement and Simplification of Network Structure

Binarization models of the sandstone pore structure were constructed by Imperial College London. And the images were analyzed by the micro-CT produced by Phoenix, Germany, with the 1 μm focus system and field of view 512×512 pixel 8-inch 16-bit detectors. In experiments, nine different porosities of sandstone samples were scanned by CT with different image resolutions (from 4.892 to 8.96 μm), and nine different pore structure models with different image accuracies were obtained after binarization processing. For the purpose of testing and comparison, four sandstone models with porosity (see Figure 2) of 14.1%, 21.1%, 24.6%, and 25.1% are selected in this paper [32].

In order to develop the model of the pore network, the data of pore structure is extracted based on the skeletonization algorithm. The pores are numbered as nodes and the throats as edges, and the connection relationships are extracted and then analyzed [33, 34]. In the process of studying the network node, the degree distribution of the nodes is described by the distribution function $P(k)$, and $P(k)$ means the probability of the degree of an arbitrarily selected node is exactly k. The result demonstrates that the sandstone seepage network belongs to the scale-free network, and the degree of sandstone seepage networks satisfies the power-law distribution (see Figure 3). In the network, some nodes named 'hub' with a large degree play a leading role in the seepage process of network [35, 36].

Porosity is defined as the percentage of the volume of pores and the total volume of the rock:

$$\phi = \frac{V_P}{V} \times 100\%, \tag{1}$$

where V_P is the pore volume and V is the rock volume.

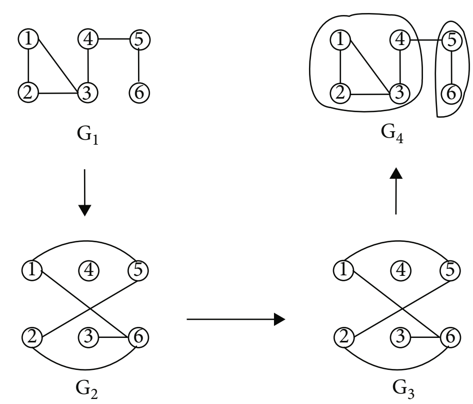

Figure 6: Schematic diagram of Greedy coloring box cover algorithm ($l_B = 3$).

In the network, the degree of a pore is the number of edges connected to the pore. During the analysis, the pore network is considered to be undirected. The average of the pores degree ki is defined as the degree of the network. The average path length of the sandstone seepage network is defined as the average of the distance between any two pores, and it is also the most efficient seepage path:

$$L = \frac{1}{1/2N(N+1)} \sum_{i \geq j} d_{ij}, \tag{2}$$

where N is the total number of voids in the seepage network, and d_{ij} is the minimum number of halves or the distance between pore i and pore j. The shorter average path length results in a reduced distance between the two pores, leading to easier overall network seepage.

In order to simplify the analysis, nodes of the network are managed through these steps (see Figure 4) to obtain valuable nodes for permeability and obtain the data (Table 1 and Figure 5(a)). The average path length of the

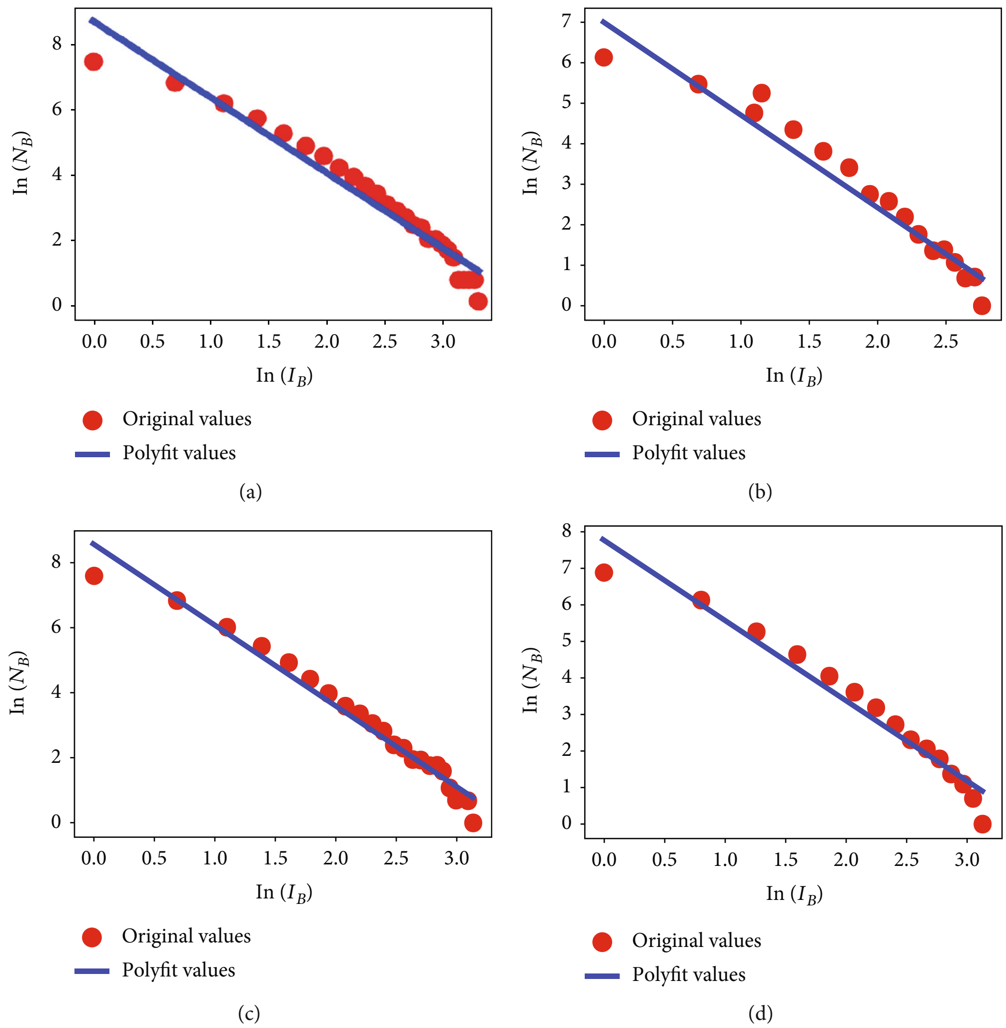

Figure 7: Curve of ln $(N_B(l_B)) \sim \ln(l_B)$: (a) 14.1%, (b) 21.1%, (c) 24.6%, and (d) 25.1%.

four sandstone networks (Table 1) is evaluated by the breadth-first search algorithm $O(MN)$ (Figure 5(b)).

4. The Analysis of the Network

In rock pore networks, changes in the pores may lead to a complex variety of changes in the network. Therefore, the study of network properties, such as the distribution of pore networks, mean path lengths, shortest edges in pores, and robustness, enables a better approach to analyze the formation and evolution of seepage networks [37, 38].

4.1. The Analysis on Network Statistic. In the four sandstone networks, the average degree of the network increases sequentially with increasing porosity. And the average length of sandstone is generally proportional to the magnitude of the network. The more the number of pores is, the larger the sandstone network is and the longer the average path length is. Porosity is related to the network magnitude and the average of node degree, and the average of node degrees plays a dominant role. It is presumed that there are few nodes with a large degree have a great influence on the average path length among nodes and the seepage process. At the same time, the value of the power exponent is between 3 and 6, and its order is similar to the average path length and the network magnitude (see Table 1 and Figure 5).

Besides, the fractal dimension of networks is mapped to the fractal dimension to the color problem by the Greedy coloring box cover algorithm proposed by Song et al. in 2007.

The specific steps of the coloring problem are as follows:

(1) The nodes of the network G are assigned orderly

(2) A box size l_B is selected, and a network G' is reconstructed: when the distance l of two nodes in G is equal or greater than the l_B, they are connected in G'

(3) G' is put color: nodes in G' are filled with fewest number of color, and the color of the connected nodes in G' is different between each other. In the original network, the distance of nodes with the

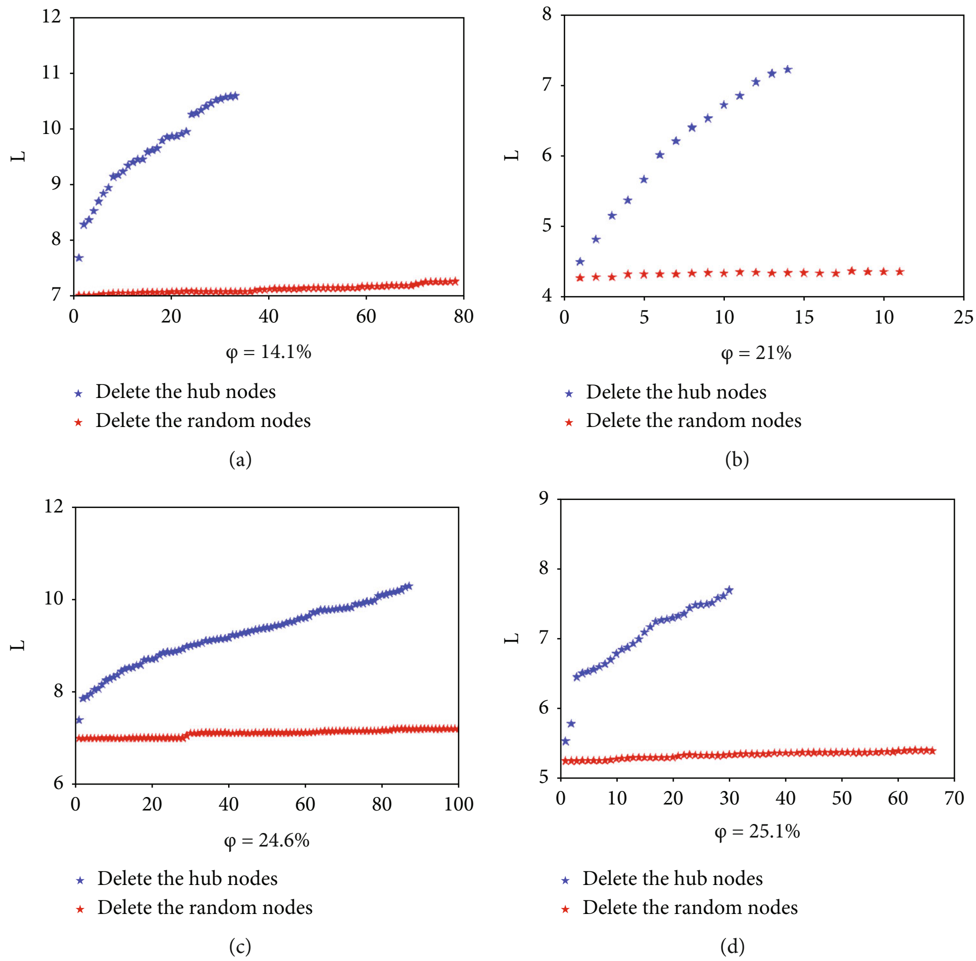

Figure 8: The alteration of the average path length of the four kinds of porosity sandstones after random nodes (nodes that are chose randomly) and hub nodes are removed separately.

same color in the reconstructed network is less than the size of the box; thus, they can be overlaid by the same box in the original network. Therefore, the number of the box overlaying network is equal to the number of colors in G'

In order to achieve the Greedy coloring box cover algorithm, a two-dimensional matrix $c_{il}(N \times l_B^{\max})$ needs to be established; here, N is the number of nodes in network, and $l_B^{\max}$ is the biggest box size, which is equal to the network diameter plus 1. The color values of box that belongs to node whose order is i in box size whose order is l is expressed by row i and column l of c_{il}.

The specific steps are as follows:

(1) Nodes of the network are numbered from 1 to N

(2) For node 1, regardless of the size of the box, its color value is assigned as 0

(3) From node 2, the color values of the node i in different box sizes are assigned until N

 (a) The distance l_{ij} between node i and node j whose number is less than i is figure up

 (b) The size of the box is set, l_B=1

 (c) The node j selected whose distance with node i satisfies the condition of $l_{ij} \geq l_B$, and its color value is assigned with untapped number as the color value of node i in the current size l_B

 (d) The size of the box is increased and step (c) is repeated until $l_B = \max\,(l_B)$

(4) The total number of color values used in each column in c_{il} is calculated, namely, the total number of boxes $N_B(l_B)$ that covers the network when box sizes are different is needed. The slope of ln $(N_B(l_B))$ and ln (l_B) is fitted, and its opposite number is the fractal dimension of the network D:

$$D = -\frac{d(\ln\,(N_B(l_B)))}{d(\ln\,(l_B))} \quad (3)$$

4.2. Robustness Analysis of Seepage Networks under External Attacks. The sandstone pore network can be affected by a variety of factors (such as crustal movement and reservoir mining), and these external effects will lead to the complex changes in the network. During the extraction of oil and gas, the ability to control network seepage within a certain range is required. And it is also necessary to analyze the characteristics of pore structure changes under crustal stress. Therefore, we analyzed the robustness of the rock network structure [39, 40]. The attack on the network is fundamentally on the pores, and the targets can be divided into 'hub' nodes and noncentral nodes. In order to simulate the attacks under natural conditions to the maximum extent, random nodes (nodes that are chose randomly) and hub nodes are removed separately in a simplified sandstone pore network. The effect of two attacks on the average path length is demonstrated in Figure 6. The average path length of the seepage network nodes is increased in general when random nodes are removed. The average path length tends to grow extremely slowly with the random removal of nodes from the network, which indicates that the sandstone seepage network has strong robustness in the face of random attack. Meanwhile, the conclusion that the sandstone seepage networks with different porosities have a similar fault tolerance mechanism can be made from four experiments, as shown in Figure 7. In contrast, as shown in Figure 8, removing the 'hub' pores increased the average path length of the entire network by 27.63-37.26%, which could not be achieved by randomly removing pores from the rock pore network. With the removal of nodes, the entire seepage network is decomposed into several relatively independent subnetworks of similar size. The network connectivity is very different from that of the initial ones, which indicates that the pores with a large degree play a critical role in the percolation network. It can be inferred that the total seepage capacity of the sandstone pore network does not change greatly when a small amount of pores of sandstone is blocked by the solid particles or the action of ground pressure. While when the quantity of pores with a large degree increases, even if few, the connectivity of sandstone pores may improve greatly, which will significantly increase the seepage rate. The conclusion has a great reference value for solving the problems such as the low rate of return in natural resources of oil and gas.

5. Conclusion

Based on the complex network theory, we have explored the structural characteristics of the sandstone pore network. The interaction between the microstructure and macroscopic properties of the rock under multiple factors was analyzed, and the main conclusions are as follows:

(1) The topological evolution of the sandstone seepage network is the result of the physicochemical properties and the external environment. The scale-free network can reasonably describe the topology of the sandstone seepage network, and all sandstone seepage networks with different porosities satisfy the power-law distribution

(2) Removing the 'hub' pores increased the average path length of the entire network by 27.63-37.26%, which could not be achieved by randomly removing pores from the rock-pore network. Consequently, these 'hub' pores are essential to the connectivity of the rock seepage network

(3) Sandstone seepage networks are highly robust in the face of external attacks, and this result provides a basis for developing a reasonable transport model for complex networks of sandstone seepage and for modelling sandstone seepage processes using the lattice Bertzmann method. The quantitative relationship between the average path length and permeability has not yet been understood, and there is still a long way to go to accurately quantify the rock seepage network

Data Availability

All data included in this study are available upon request by contacting with the corresponding author. The digital cores of Blunt can be found in the website of Imperial College London: http://www.imperial.ac.uk/earth-science/research/research-groups/perm/research/pore-scale-modelling/micro-ct-images-and-networks/.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

Acknowledgments

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