Research Article

Study on Water Injection Indicator Curve Model in Fractured Vuggy Carbonate Reservoir

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Fractures and cavities are very well developed in fractured vuggy carbonate reservoirs in Tahe Oilfield, showing obvious constant volume or approximate constant volume characteristics, and the development of karst vuggy reservoirs is characterized by strong randomness, locality, heterogeneity, and discontinuous development, which has a great impact on oilfield development effect. The application of conventional reservoir engineering methods in this kind of reservoir has many limitations. The traditional water injection indication curve is mainly based on a sandstone reservoir, ignoring the elastic energy of the water body in a reservoir and the combination relationship of fracture and cavity, which can not meet the needs of the carbonate reservoir. According to the principle of volume balance of constant volume body and considering the elastic energy of original formation water, a new model of improved water injection indication curve and the original water-oil ratio of carbonate reservoir is proposed, and the chart of water injection indication curve is established. Compared with other methods, this method has the advantages of low cost and accurate identification results. The calculated reservoir parameters such as single well-controlled reserves, well-controlled water volume, and original water-oil ratio have high reliability, and the chart is easy to use. The model is applied to the Yuejin block of Tahe Oilfield, and the results show that the improved model is consistent with the results of other test methods, such as high-precision seismic combined with fracture cavity carving technology, which indicates that the model has strong reliability.

1. Introduction

There have been many discussions on the characteristics and uses of the water injection indicator curve in the carbonate reservoir, but the theoretical research on the water injection indicator curve model in this kind of reservoir is relatively rare or has not formed a unified understanding [1–5]. To fully understand and utilize the data in the process of water injection in the carbonate reservoir, analyze the oil-water relationship in the reservoir, and correctly guide the implementation of the next working system and measures, the author takes the unsaturated carbonate reservoir as an example to study the establishment of reservoir water injection indicator curve model, the drawing of curve line, and its application, to provide the selection of water injection volume in the process of oil displacement by water injection. Therefore, the water injection indicator curve will play a better role in the development of carbonate reservoir in the future.

The carbonate reservoir is different from the sandstone reservoir. The calculation error of the conventional sandstone reservoir water injection indicator curve model is large [6–10]. Therefore, it is necessary to study and analyze the carbonate reservoir and reconstruct a new model that is consistent with the characteristics of the carbonate reservoir. In the process of water injection, there is little difference between the response of porous media and karst cave medium [10–13]. Therefore, the main types of carbonate reservoir are divided into single cave constant volume, multiple cave constant volumes, and fracture cavity constant volume (dual media). What is the constant volume refers to the constant volume of the karst cave. When water injection develops, the volume of reservoir fluid decreases with the increase of injected water volume, which is also called a constant volume reservoir [14–16].

Ma and Mei et al. derived the modified water injection indication curve considering that the elastic energy of

original formation water in the fractured vuggy reservoir can not be ignored: (1) the theoretical model is not combined with the actual needs of the site; (2) the theoretical model ignores the calculation of the original water-oil ratio R in the actual use process. In this case, the relative error can be reduced to 0, which lacks practical use value [17, 18]. Therefore, based on the previous experience and the principle of volume balance in cave constant volume body, a new model of water injection indication curve is derived. The expression of the original water-oil ratio R is added into the new model to determine the volume of crude oil and formation water in the cave constant volume body more accurately.

2. Establishment of Water Injection Indication Curve Model

2.1. Water Injection Indication Curve Model of Single Karst Cave and Constant Volume Body

2.1.1. Model Derivation. Considering the elastic energy of the original formation water body, the volume change of the single cave constant volume is ignored. In other words, the volume of a single cave and constant volume V_p remains unchanged, as shown in Figure 1.

Under the original conditions, the original formation pressure is P_0 , and the volume of a single karst cave is a constant

$$V_p = V_{oi} + V_{wi}. \quad (1)$$

After W_e volume of surface water is injected, the formation pressure rises to P ,

$$\begin{aligned} V_p &= V_o + V_w + W_e \cdot B_w, \\ C_o &= \frac{1}{\Delta P} \cdot \frac{\Delta V_o}{V_o}, \\ \Delta V_o &= V_{oi} - V_o = C_o \cdot V_{oi} \cdot \Delta P, \\ C_w &= \frac{1}{\Delta P} \cdot \frac{\Delta V_w}{V_w}, \\ \Delta V_w &= V_{wi} - V_w = C_w \cdot V_{wi} \cdot \Delta P, \\ V_{oi} + V_{wi} &= V_o + V_w + W_e \cdot B_w, \\ W_e \cdot B_w &= \Delta V_o + \Delta V_w = C_o \cdot V_{oi} \cdot \Delta P + C_w \cdot V_{wi} \cdot \Delta P, \\ P &= P_0 + \frac{W_e \cdot B_w}{C_o \cdot V_{oi} + C_w \cdot V_{wi}}. \end{aligned} \quad (2)$$

The original water-oil ratio $R = V_{wi}/V_{oi}$, $V_{oi} = N \cdot B_{oi}$:

$$P = P_0 + \frac{W_e \cdot B_w}{N \cdot B_{oi}(C_o + R \cdot C_w)}. \quad (3)$$

From this, the water injection model of single volume is obtained.

FIGURE 1: The injection indication curve model in single karst-cave constant volume.

Intercept $b = P_0$, reflecting the formation pressure before water injection:

Slope $K = B_w/(N \cdot B_{oi}(C_o + R \cdot C_w))$; the smaller the slope, the larger the swept volume.

When the formation water body and injected water elastic energy is not taken into account, $R = 0$, $B_w = 1$, substituting into equation (3), we can get

$$P = P_0 + \frac{W_e}{N \cdot B_{oi} \cdot C_o}. \quad (4)$$

According to equations (3) and (4), the water injection indication curve under different driving energy is drawn, as shown in Figure 2.

2.1.2. Parameter Calculation. According to the water injection indicator curve model of single cave constant volume body and its characteristic analysis, the following parameters can be determined:

(1) Geological reserves:

$$N = \frac{V_{oi}}{B_{oi}} = \frac{B_w}{K \cdot B_{oi}(C_o + R \cdot C_w)}. \quad (5)$$

(2) Original water volume:

$$V_{wi} = R \cdot V_{oi} = \frac{R \cdot B_w}{K(C_o + R \cdot C_w)}. \quad (6)$$

(3) Cave volume:

$$V_p = V_{oi} + R \cdot V_{oi} = (1 + R) V_{oi} = \frac{B_w(1 + R)}{K(C_o + R \cdot C_w)}. \quad (7)$$

FIGURE 2: The injection indication curve in single karst-cave constant volume.

2.2. Water Injection indicator Curve Model of Multikarst Caves and Constant Volume. The multicave constant volume model is also one of the typical reservoir types of carbonate reservoirs. The author takes the double cave-constant volume model, as an example, and the multicave constant volume model is the same. Considering the energy of the water body, it is assumed that the fracture between the two caves only plays a role of diversion, and the cavern is the main reservoir, ignoring the reservoir performance of fractures, as shown in Figure 3.

2.2.1. Model Derivation. Set the volume of cave 1 and cave 2 as V_{p1} and V_{p2} , respectively, and the water injection volume is W_e . When the water injection volume is w_{e1} , cave 1 is full, cave 2 starts to inject, and the corresponding pressure when cave 1 is full is P_1 . The results are as follows:

$$\begin{aligned} V_{p1} &= V_{oi1} + V_{wi1}, \\ V_{p2} &= V_{oi2} + V_{wi2}, \\ R_1 &= \frac{V_{oi1}}{V_{wi}}, \\ R_2 &= \frac{V_{oi2}}{V_{wi2}}, \end{aligned} \quad (8)$$

When $W_e < W_{e1}$, it is a single cave constant volume model:

$$P = P_o + \frac{W_e \cdot B_w}{C_o \cdot V_{oi1} + C_w \cdot V_{wi1}} = P_o + \frac{W_e \cdot B_w}{N_1 \cdot B_{oi}(C_o + C_w \cdot R)} \quad (9)$$

When $W_e > W_{e1}$, it is a double cave constant volume model:

At this time, the water injection can be divided into two stages:

(1) When the pressure is P_1 ,

$$P_1 = P_o + \frac{W_{e1} \cdot B_w}{N_1 \cdot B_{oi}(C_o + C_w \cdot R_1)} \quad (10)$$

(2) When the pressure is greater than P_1 , $\Delta P = P - P_1$

$$W_e - W_{e1} = \Delta V_{oi1} + \Delta V_{wi1} + \Delta V_{oi2} + \Delta V_{wi2}, \quad (11)$$

$$\begin{aligned} W_e - W_{e1} &= C_o \cdot V_{oi1} \cdot \Delta P + C_w \cdot V_{wi1} \cdot \Delta P + C_o \cdot V_{oi2} \cdot \Delta P \\ &\quad + C_w \cdot V_{wi2} \cdot \Delta P, \end{aligned} \quad (12)$$

$$P = P_1 + \frac{(W_e - W_{e1}) \cdot B_w}{C_o \cdot V_{oi1} + C_w \cdot V_{wi1} + C_o \cdot V_{oi2} + C_w \cdot V_{wi2}}, \quad (13)$$

$$P = P_1 + \frac{(W_e - W_{e1}) \cdot B_w}{N_1 \cdot B_{oi}(C_o + C_w \cdot R_1) + N_2 \cdot B_{oi}(C_o + C_w \cdot R_2)} \quad (14)$$

By substituting equation (9) into equation (13), the

$$\begin{aligned} P &= P_o + \frac{W_{e1} \cdot B_w}{N_1 \cdot B_{oi}(C_o + C_w \cdot R_1)} \\ &\quad + \frac{(W_e - W_{e1}) \cdot B_w}{N_1 \cdot B_{oi}(C_o + C_w \cdot R_1) + N_2 \cdot B_{oi}(C_o + C_w \cdot R_2)}. \end{aligned} \quad (15)$$

Thus, the water injection indicator curve model of multikarst caves and constant volume body is obtained.

Intercept $b = P_o$, reflecting the formation pressure before water injection:

(1) Cave 1 slope:

FIGURE 3: The injection indication curve model in multiple karst-cave constant volume.

$$K_1 = \frac{B_w}{N_1 \cdot B_{oi}(C_o + R_1 \cdot C_w)} \cdot \quad (16)$$

$$N_1 = \frac{V_{oi1}}{B_{oi}} = \frac{B_w}{K_1 \cdot B_{oi}(C_o + R_1 \cdot C_w)} \quad (20)$$

(2) Cave 2 slope:

$$K_2 = \frac{B_w}{N_1 \cdot B_{oi}(C_o + C_w \cdot R_1) + N_2 \cdot B_{oi}(C_o + C_w \cdot R_2)} \cdot \quad (17)$$

The smaller the slope is, the larger the swept volume is. Obviously, $K_2 < K_1$, the water injection indicator curve should be shifted downward, and the migration amplitude is related to the volume of cave 2.

When the formation water body and injected water elastic energy are not taken into account, substituting $R_1 = R_2 = 0, B_w = 1$ into equation (9) into (15), we can get the following:

When $W_e < W_{e1}$, it is a single cave constant volume model:

$$P = P_o + \frac{W_e}{N_1 \cdot B_{oi} \cdot C_o} \cdot \quad (18)$$

When $W_e > W_{e1}$, it is a multicave constant volume model:

$$P = P_o + \frac{W_{e1} \cdot B_w}{N_1 \cdot B_{oi} \cdot C_o} + \frac{W_e - W_{e1}}{N_1 \cdot B_{oi} \cdot C_o + N_2 \cdot B_{oi} \cdot C_o} \cdot \quad (19)$$

According to equations (18) and (19), the water injection indicator curve of multikarst cave constant volume body is drawn, as shown in Figure 4.

2.2.2. Parameter Calculation. According to the water injection indicator curve model of single cave constant volume body and its characteristic analysis, the following parameters can be determined:

(1) The geological reserves of cave 1 are

(2) The original water volume is

$$V_{wi1} = R_1 \cdot V_{oi1} = \frac{B_w}{K_1(C_o + R_1 \cdot C_w)} \quad (21)$$

(3) The total volume of cave 1 is

$$V_{p1} = V_{oi1} + R_1 \cdot V_{oi1} = (1 + R_1) V_{oi1} = \frac{B_w(1 + R_1)}{K_1(C_o + R_1 \cdot C_w)} \quad (22)$$

(4) The geological reserves of karst cave 2 are

$$N_2 = \frac{V_{oi2}}{B_{oi}} = \frac{B_w - K_2 \cdot N_1 \cdot B_{oi}(C_o + C_w \cdot R_1)}{K_2 \cdot B_{oi}(C_o + C_w \cdot R_2)} \quad (23)$$

(5) The original water volume is

$$V_{wi2} = R_2 \cdot V_{oi2} = \frac{B_w - K_2 \cdot N_1 \cdot B_{oi}(C_o + C_w \cdot R_1)}{K_2(C_o + C_w \cdot R_2)} \quad (24)$$

(6) The total volume of cave 2 is

FIGURE 4: The injection indication curve in multiple karst-cave constant volume.

$$V_{p2} = (1 + R_2)V_{oi2} = (1 + R_2) \frac{B_w - K_2 \cdot N_1 \cdot B_{oi}(C_o + C_w \cdot R_1)}{K_2(C_o + C_w \cdot R_2)} \quad (25)$$

2.3. Water Injection Indication Curve Model of Fracture Cavity Constant Volume Body

2.3.1. Model Derivation. For fractured vuggy reservoirs with constant volume, fractures can be used as both seepage channels and reservoirs to store oil and gas. Therefore, it is divided into two parts: the fractured reservoir and karst cave reservoir. At this time, after a certain amount of water is injected into the reservoir, the reservoir pressure rises from the original formation pressure P_o to the current formation pressure P , and the increasing value of reservoir pressure is $\Delta P = P - P_o$, as shown in Figure 5.

(1) *Fractured Reservoir.* Define the volume of the fracture unit is V_{pf} , the volume of crude oil is V_{of} and the volume of formation water is V_{wf} ; then $V_{pf} = V_{of} + V_{wf}$:

The original fluid in the fracture is compressed due to the increase of pressure, and the compression amount is the volume of injected water in the fracture, viz W_{if} .

$$\Delta V_{of} + \Delta V_{wf} = W_{if} \times B_w, \quad (26)$$

$$\Delta V_{of} = C_o \times V_{of} \times \Delta P, \quad (27)$$

$$\Delta V_{wf} = C_w \times V_{wf} \times \Delta P. \quad (28)$$

By combining formulas (26), (27), and (28), we can get

$$C_o \times V_{of} \times \Delta P + C_w \times V_{wf} \times \Delta P = W_{if} \times B_w. \quad (29)$$

(2) *Cavernous Reservoir.* The volume of cave unit is V_{pr} , the volume of crude oil is V_{or} , and the volume of formation water is V_{wr} ; then $V_{pr} = V_{or} + V_{wr}$:

The original fluid in the cave is compressed due to the increase of pressure, and the compression amount is the volume of injected water in the cavern in the reservoir, i.e. W_{ir} .

$$\Delta V_{or} + \Delta V_{wr} = W_{ir} \times B_w, \quad (30)$$

$$\Delta V_{or} = C_o \times V_{or} \times \Delta P, \quad (31)$$

$$\Delta V_{wr} = C_w \times V_{wr} \times \Delta P. \quad (32)$$

By combining equations (30), (31), and (32), we can get

$$C_o \times V_{or} \times \Delta P + C_w \times V_{wr} \times \Delta P = W_{ir} \times B_w. \quad (33)$$

(3) *Fracture Cavity Type Constant Volume Body.* For fractured vuggy reservoir with constant volume, because fractures not only are seepage channels but also have certain reservoir performance, it is considered that the pressure in fracture vuggy constant volume reservoir is the synchronous response. If the ratio of fracture volume to cave volume is set as α , the water-oil ratio in the fracture is R_1 , and the water-oil ratio in karst cave is R_2 , then

$$\alpha = \frac{V_{pf}}{V_{pr}}, \quad (34)$$

$$R_1 = \frac{V_{wf}}{V_{of}}, \quad (35)$$

$$R_2 = \frac{V_{wr}}{V_{or}}, \quad (36)$$

$$V_{of} = \frac{1}{(1 + R_1)V_{pf}} = \frac{\alpha}{(1 + R_1)V_{pr}}, \quad (37)$$

FIGURE 5: The injection indication curve model in fractured-vuggy karst-cave constant volume.

$$V_{wf} = \frac{R_1}{(1 + R_1)V_{pf}} = \frac{\alpha \cdot R_1}{(1 + R_1)V_{pr}}, \quad (38)$$

$$V_{or} = \frac{1}{(1 + R_2)V_{pr}}, \quad (39)$$

$$V_{wr} = \frac{R_2}{(1 + R_2)V_{pr}}. \quad (40)$$

By combining equations (29) and (33), we can get

$$C_o \times V_{of} \times \Delta P + C_w \times V_{wf} \times \Delta P + C_o \times V_{or} \times \Delta P + C_w \times V_{wr} \times \Delta P = W_{if} \times B_w + W_{ir} \times B_w, \quad (41)$$

$$C_o \times (V_{of} + V_{or}) \times \Delta P + C_w \times (V_{wf} + V_{wr}) \times \Delta P = W_i \times B_w. \quad (42)$$

Substituting equations (34)–(40) into equation (42), we can get

$$P = P_o + \frac{W_e \cdot B_w}{[C_o \cdot ((\alpha/(1 + R_1)) + (1/(1 + R_2))) + C_w \cdot (((\alpha \cdot R_1)/(1 + R_1)) + (R_2/(1 + R_2)))] \cdot V_{pr}}. \quad (43)$$

Substituting $V_{pr} = V_{or}(1 + R_2)$ into equation (43), we get

$$P = P_o + \frac{W_i \cdot B_w}{\{C_o \cdot [((\alpha(1 + R_2))/(1 + R_1)) + 1] + C_w \cdot [((\alpha R_1 \cdot (1 + R_2))/(1 + R_1)) + R_2]\} \cdot V_{or}}. \quad (44)$$

Thus, the water injection indicator curve model of fracture cavity constant volume body is obtained.

Intercept $b = P_o$, reflecting the formation pressure before water injection:
The slope is

$$K = \frac{B_w}{\{C_o \cdot [((\alpha(1 + R_2))/(1 + R_1)) + 1] + C_w \cdot [((\alpha R_1 \cdot (1 + R_2))/(1 + R_1)) + R_2]\} \cdot V_{or}}. \quad (45)$$

The smaller the slope, the larger the swept volume.

In the process of reservoir development, fracture as a seepage channel, once water is cut, it will inevitably cause rapid water breakthrough and even violent water flooding of the well. Therefore, $R_1 = 0$ is substituted into equation (44), and

$$P = P_o + \frac{W_i \cdot B_w}{\{C_o[\alpha(1 + R_2) + 1] + C_w \cdot R_2\} \cdot V_{or}}. \quad (46)$$

When the fracture is not developed and can not be used as a reservoir, $\alpha = 0$, it is a single karst cave constant volume model, which can be substituted into equation (46)

$$P = P_o + \frac{W_i \cdot B_w}{(C_o + C_w \cdot R_2) \cdot V_{or}}. \quad (47)$$

Obviously, because fractures contribute part of the reservoir, the slope decreases; that is, the larger α is, the smaller the slope of the water injection indicator curve is.

Thus, the fracture cavity type constant volume injection indicator curve is drawn, as shown in Figure 6.

2.3.2. Parameter Calculation. The following are the parameter calculations:

(1) Geological reserves in fractures:

$$N_f = \frac{V_{of}}{B_{oi}} = \frac{1}{1 + R_1} \cdot V_{pf} = \frac{\alpha \cdot B_w}{\{C_o \cdot (\alpha + ((1 + R_1)/(1 + R_2))) + C_w \cdot [\alpha \cdot R_1 + ((R_2 \cdot (1 + R_1))/(1 + R_2))]\} \cdot K \cdot B_{oi}}. \quad (48)$$

(2) Volume of water in fracture:

$$V_{wf} = \frac{R_1}{1 + R_1} \cdot V_{pf} = \frac{\alpha \cdot R_1 \cdot B_w}{\{C_o \cdot (\alpha + ((1 + R_1)/(1 + R_2))) + C_w \cdot [\alpha \cdot R_1 + ((R_2 \cdot (1 + R_1))/(1 + R_2))]\} \cdot K}. \quad (49)$$

(3) Crack volume:

$$V_{pf} = \alpha \cdot V_{pr} = \frac{\alpha \cdot B_w}{[C_o \cdot ((\alpha/(1 + R_1)) + (1/(1 + R_2))) + C_w \cdot (((\alpha \cdot R_1)/(1 + R_1)) + (R_2/(1 + R_2)))] \cdot K}. \quad (50)$$

(4) Geological reserves in karst cave:

$$N_r = \frac{V_{or}}{B_{oi}} = \frac{B_w}{\{C_o \cdot [((\alpha(1 + R_2))/(1 + R_1)) + 1] + C_w \cdot [((\alpha \cdot R_1 \cdot (1 + R_2))/(1 + R_1)) + R_2]\} \cdot K \cdot B_{oi}}. \quad (51)$$

(5) Volume of water in karst cave:

$$V_{wr} = R_2 \cdot V_{or} = \frac{R_2 \cdot B_w}{\{C_o \cdot [((\alpha(1 + R_2))/(1 + R_1)) + 1] + C_w \cdot [((\alpha \cdot R_1 \cdot (1 + R_2))/(1 + R_1)) + R_2]\} \cdot K}. \quad (52)$$

(6) Cave volume:

$$V_{pr} = (1 + R_2) \cdot V_{or} = \frac{B_w}{[C_o \cdot ((\alpha/(1 + R_1)) + (1/(1 + R_2))) + C_w \cdot (((\alpha \cdot R_2)/(1 + R_2)) + (R_2/(1 + R_2)))] \cdot K}. \quad (53)$$

FIGURE 6: Water injection indication curve of fracture cavity-type constant volume body.

FIGURE 7: Acid fracturing construction curve.

FIGURE 8: Seismic profile of well Y1-x.

FIGURE 9: Y1-x energy indicating curve.

FIGURE 10: Y1-x First and fourth round injection indicating curve.

2.4. Derivation of Original Water Oil Ratio R . The original water-oil ratio R is not only the core of the model but also the key parameter to solve the model. In general, it is necessary to make a relationship curve fitting through field simulation to get R . To facilitate the field application, the author made a simplified model and solved the expression of R .

2.4.1. Solving R by Multiple Water Injection ($n \geq 2$). In a constant volume reservoir, the well control reserves of the multicave constant volume reservoir can be obtained from the first round of water injection indicator curve, and then, the difference between the remaining oil of the well and the remaining oil of the previous round is calculated as the volume of crude oil produced in this round.

$$\Delta V_{o^n} = V_{o^{n-1}} - V_{o^n} = \frac{B_w}{K_{n-1}(C_o + R \cdot C_w)} - \frac{B_w}{K_n(C_o + R \cdot C_w)}, \quad (54)$$

where n is the water injection round ($n \geq 2$).

For the convenience of field application, let $\Delta V_{o^{n-1}} = N_{p^{n-1}}$ and $N_{p^{n-1}}$ be the actual volume of crude oil produced between two water injection cycles.

$$\frac{B_w}{K_{n-1}(C_o + R \cdot C_w)} - \frac{B_w}{K_n(C_o + R \cdot C_w)} = N_{p^{n-1}}, \quad (55)$$

$$R = \left[\frac{B_w \cdot (K_n - K_{n-1})}{N_{p^{n-1}} \cdot K_{n-1} \cdot K_n} - C_o \right] \cdot \frac{1}{C_w}. \quad (56)$$

2.4.2. Using Energy Indicating Curve to Solve R . In the process of field application, it often occurs that the water injection rounds are not enough or some water injection indicating curves cannot be used. Therefore, the energy index curve is used to replace the water injection indication

FIGURE 11: Energy indicating curve of well Y2-x.

curve. The slope of the energy indicating curve is K_0 , which is taken as the slope of the first water injection indicating curve and can be substituted into formula (56) to obtain R .

$$R = \left[\frac{B_w \cdot (K_n - K_0)}{N_{p^{n-1}} \cdot K_0 \cdot K_n} - C_o \right] \cdot \frac{1}{C_w}. \quad (57)$$

At this time, the $B_{oi}V_{oi}$ obtained is the well-controlled reserves of the cavern constant volume body, and $R \cdot V_{oi}$ is the original water volume of the reservoir.

The following are the instructions:

V_{o^n} , V_{w^n} are the volume of remaining oil and bottom water in the formation during the n th round of water injection.

$V_{o^{n-1}}V_{o^{n-1}}$ is the volume of oil produced between two rounds.

$V_{w^{n-1}} - V_{w^n}$ is the volume of injected water to volume of produced water between two rounds.

Therefore, R is a variable quantity in the production process, but it can be regarded as a constant value on the premise that there are not many water injection rounds and the difference between the volume of oil produced and the volume of injected water and produced water is small. When

FIGURE 12: Water injection indication curve of well Y2-x in the first four rounds.

FIGURE 13: Fifth round water injection indicator curve of well Y2-x.

TABLE 1: Comparisons of the calculated results for the reserves.

Well name	Volume of solid carving			Volume factor	Crude oil density (t/m ³)	Oil saturation			Single well reserves (t)		Error/%
	Cave	Pore	Fracture			Cave	Pore	Fracture	Carving	New	
YJ1-X	186136	38467	9531	2.171	0.795	0.814	0.664	0.9	67978	62275	8.39
YJ2-X	118089	41126	7282	2.171	0.795	0.814	0.664	0.9	47600	41994	11.78

the output and water injection increase in the middle and later stages, R will also increase correspondingly, but the R value of two adjacent times has little change, so this formula can also be used.

3. Field Application and Analysis of New Model

The well control reserves and water volume of key wells in the Yuejin block are predicted by using the new model of water injection indication curve and the original water-oil ratio formula established, and the predicted results are compared with the actual production results [19, 20].

3.1. Well Y1-x. The completion depth of Y1-x well is 7205 m (slant)/7185.68 m (vertical), and the bushing elevation is 82.5 m (slant)/82.2 m (vertical), and acid fracturing is completed. It can be clearly seen from the acid fracturing operation curve (Figure 7) that the acidizing effectively communicated with the reservoir. The pressure drop was measured after the pump was stopped for 20 min. The oil pressure rose from 15.7 to 15.9 MPa, and the acidizing effect was good. Combined with seismic data (Figure 8) and energy indicating curve (Figure 9), the seismic time migration section shows that Y1-x well has “string bead” reflection characteristics under T_7^4 seismic reflection wave, and the energy indicator curve shows that the well has obvious constant volume characteristics, so it can be judged that the well is a karst cave constant volume body.

The well has accumulated 4 rounds of water injection, and the first and fourth rounds of water injection indication curves are as follows (Figure 10). During this period, the cumulative oil production is 13305 t, and the surface crude oil density of the Yuejin block is 0.795 g/cm^3 , the volume coefficient is 1.617, the compressibility coefficient of formation crude oil is $15 \times 10^{-4} \text{ MPa}^{-1}$, and the compressibility coefficient of formation water is $4 \times 10^{-4} \text{ MPa}^{-1}$ ($K_0 = 0.0048$, $K_1 = 0.0073$). Substituting the above data into formula (56), $R = 2.9$ is obtained. The original oil-bearing volume of the well is 78333 m^3 , the remaining reserves are 61597 m^3 , the total well control reserves are 62275 t, and the remaining reserves are 48970 t.

3.2. Well Y2-x. The drilling depth of well Y2-x is 7091.73 m, and the bushing elevation is 23.73 m. The lost circulation during drilling is 3160.9 m^3 , showing a cavernous reservoir. The energy indicator curve (Figure 11) shows that the well has obvious constant volume characteristics, so it belongs to a typical karst cave constant volume reservoir. During the first four rounds of water injection, the water injection indicator curve (Figure 12) shows that the data correlation is poor, and the data is scattered, reflecting the obvious sand burial characteristics. At the same time, there is sand burial history in the early production process of the well, so the sand washing acidification is carried out, and the fifth round water injection indicator curve (Figure 13) after acidification is available.

After the fifth round of water injection, the produced crude oil is 4900 t, the surface crude oil density is 0.795 g/cm^3 , the volume coefficient is 1.617, the formation

crude oil compression coefficient is $15 \times 10^{-4} \text{ MPa}^{-1}$, and the formation water compression coefficient is $4 \times 10^{-4} \text{ MPa}^{-1}$ ($K_0 = 0.0104$, $K_1 = 0.0128$). By substituting the above data into formula (57), $R = 0.8$ is obtained. The original oil-bearing volume of the well is 52822 m^3 , the remaining reserves are 42918 m^3 , the total well control reserves are 41994 t, and the remaining reserves are 37094 t.

3.3. New Model Validation. Dong Xianzhang and others used the volume method to check the calculation results when calculating the well-controlled reserves [21–25]. The essence of high-precision seismic combined with fracture cavity carving technology is also a volume method, and it is more accurate than other methods in the calculation of oil well control area and reservoir height [26–29]. Therefore, the author uses this method to test the calculation results of the new model. The calculation results are shown in Table 1. Obviously, the geological reserves calculated by high-precision earthquake combined with fracture cavity carving technology are very close to the results calculated by the author, and the error is controlled at about 10%.

4. Conclusions

- (1) Through the study of carbonate reservoir, it can be divided into single cave constant volume, multicave constant volume, and fracture cavity constant volume (double porosity medium). On this basis, the water injection indication curve model under different reservoir conditions is given. The water injection indicator curve is drawn by using the established new model, and the corresponding geological reserves, original water volume, and reservoir volume corresponding to the model are solved, which provides the basic theoretical basis for the formulation of the working system and measures of carbonate reservoir and water injection for oil displacement in the next step
- (2) For the convenience of field application, the original water-oil ratio and its solution formula are derived by using multiple rounds of water injection and energy indicating curve, which can more accurately calculate the well control reserves and original water volume of karst cave constant volume reservoir. Based on the field data, this paper demonstrates the accuracy of the new model and the original water-oil ratio formula and, on this basis, rechecks and verifies the well control reserves of the internal wells in the block, which provides the basis for the determination of the next working system, the formulation of water injection, and other related measures. This method has been applied to the Tahe Oilfield of China
- (3) This model is derived from the principle of volume balance of constant volume body. It applies to the cavern constant volume reservoir with edge and bottom water and without edge and bottom water and the cave constant volume body with water content

rising in the reservoir in the middle and late stage of water injection development. It has good adaptability to approximate constant volume reservoir and low permeability carbonate reservoir

Abbreviations

P :	Formation pressure (MPa)	V_{wf} :	The volume of formation water in fractured vuggy reservoir with constant volume (m^3)
P_0 :	Original formation pressure (MPa)	V_{or} :	The volume of crude oil in fractured vuggy cavern reservoir with constant volume (m^3)
P_1, P_2, P_3 :	Different injection pressures (MPa)	V_{wr} :	The volume of formation water in fractured vuggy reservoir with constant volume (m^3)
ΔP :	Production pressure difference (MPa)	V_o :	When the pressure is P , the volume of formation crude oil in a single cave constant volume body 0 (m^3)
V_p :	Total volume of single cave and constant volume (m^3)	V_w :	When the pressure is P , the volume of formation water in a single cave constant volume body (m^3)
V_{p1} :	Total volume of cave 1 with multiple caves and constant volume (m^3)	W_{if} :	When the pressure increases from P_0 to P , the amount of water compression in the fracture increases (m^3)
V_{p2} :	Total volume of cave 2 with multiple caves and constant volume (m^3)	W_{ir} :	When the pressure increases from P_0 to P , the amount of water compression in the cave will decrease (m^3)
V_{pf} :	Total volume of fractures in fracture cavity type constant volume body (m^3)	α :	The ratio of fracture volume to cave volume
V_{pr} :	The total volume of fracture cavern with constant volume (m^3)	C_o :	When the pressure is P , the compressibility coefficient of formation crude oil is (MPa^{-1})
W_e :	Surface water injection volume (m^3)	C_w :	When the pressure is P , the compressibility coefficient of formation water is (MPa^{-1})
V_{oi} :	The volume of crude oil in single cave constant volume formation (m^3)	$\Delta V_o, \Delta V_{o1}, \Delta V_{o2}, \Delta V_{of}, \Delta V_{or}$:	Pressure rise ΔP , the amount of formation crude oil compression (m^3)
V_{wi} :	The volume of formation water in a single karst cave with constant volume (m^3)	$\Delta V_w, \Delta V_{w1}, \Delta V_{w2}, \Delta V_{wf}, \Delta V_{wr}$:	Pressure rise ΔP , formation water compression (m^3)
V_{oi1} :	The volume of crude oil in karst cave 1 with multiple caves and constant volume (m^3)	B_w :	When the pressure is P , the volume coefficient of formation water is (m^3/m^3)
V_{wi1} :	The volume of layer water in cave 1 with multiple caves and constant volume (m^3)	B_o :	When the pressure is P , the volume coefficient of formation crude oil is (m^3/m^3)
V_{oi2} :	The volume of crude oil in the inner layer of cave 2 with multiple caves and constant volume (m^3)	B_{oi} :	When the pressure is P_0 , the volume coefficient of formation crude oil is 0 (m^3/m^3)
V_{wi2} :	The volume of formation water in cave 2 with multiple caves and constant volume (m^3)	R, R_1, R_2 :	Water oil ratio (m^3/m^3)
V_{of} :	The volume of crude oil in fractured vuggy reservoir with constant volume (m^3)	N, N_1, N_2, N_f, N_r :	Geological reserves (m^3)

b : Intercept of water injection indication curve (f)
 K, K_1, K_2 : Slope of water injection indication curve (f).

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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