

Research Article

Determination of Scale Effects on Mechanical Properties of Berea Sandstone

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The key rock mechanical parameters are strength, elastic modulus, Poisson's ratio, etc., which are important in reservoir development. The accurate determination of reservoir's mechanical properties is critical to reduce drilling risk and maximize well productivity. Precisely estimating rock mechanical properties is important in drilling and well completion design, as well as crucial for hydraulic fracturing. Rocks are heterogeneous and anisotropic materials. The mechanical properties vary not only with rock types but also with measurement methods, sample geometric dimensions (sample length to diameter ratio and size), and other factors. To investigate sample scale effects on rock mechanical behaviors, unconfined compression tests were conducted on 41 different geometric dimensions of Berea sandstones; unconfined compressive strength (UCS), Young's modulus (E), Poisson's ratio (ν), bulk modulus (K), and shear modulus (G) were obtained and compared. The results indicate that sample geometry can significantly affect rock mechanical properties: (1) UCS decreases with the increase of length to diameter ratio (LDR), and the UCS standardize factor is between 0.71 and 1.17, which means -30% to +20% variation of UCS with LDR changing from 1 to 6.7. The test results show UCS exhibits positive relationship with sample size. (2) Young's modulus slightly increases with LDR increases, while Poisson's ratio decreases with the increase of LDR. For the tested Berea sandstones, Poisson's ratio standardizing factor is between 0.57 and 1.11. (3) Bulk modulus of Berea sandstone samples decreases with the increase of LDR, while shear modulus increases with LDR increases. Both bulk modulus and shear modulus increase with the increase of sample size. (4) The principal failure modes were analyzed. The failure modes of the tested Berea sandstones are axial splitting and shear failure. Stocky samples ($LDR < 2$) tend to go axial splitting, while slender samples ($LDR > 2$) tend to show shear failure.

1. Introduction

Rock mechanical properties introduced terms such as strength, elastic modulus, and Poisson's ratio, which are important in drilling, well completion, and hydraulic fracturing [1–7]. Strength and elastic properties are important parameters in drilling, evaluations, completion, and production forecasting for conventional and unconventional resources, which are key inputs in geomechanical models (Prasad 2019). Rocks are heterogeneous materials. Therefore, rock mechanical properties vary not only with rock types but

also with sample geometry. In pure mechanics, a true material property should not vary with sample geometric dimensions. Thus, material properties will remain unchanged at any range of scales only if the material itself remains purely homogeneous. As a matter of fact, rocks are heterogeneous, and the mechanical properties obtained from different geometry of core sample are different [8]. Generally, rock mechanical properties show a dependency on LDR and sample size [9–11], which is defined as scale effects. In fact, there is no contradiction between rock mechanics and pure mechanics. The basic concepts of mechanics have to be kept, and some

practical adjustments have to be adopted for rock engineering problems. Hence, whatever the geometry of the samples used in the test, the test results for determining the material mechanical properties have to be changed accordingly. Usually, rock sample geometry can influence rock mechanical behaviors. One reason is that the friction between the sample and the load platen may influence the test results [12]. In addition, long samples may lead to bending, while short samples could cause shear fracture plane during unconfined and confined tests [13]. Therefore, it is necessary to qualify and integrate the scale effects to make sure the values are reliable. According to the test results of Forbes et al. [14], the suitable correction for unconfined compressive strength (UCS) was well established, which would be applied for apparent high strength in UCS and CCS.

The scale effects on rock mechanical properties have been well investigated by many researchers. John [15] conducted uniaxial compression tests on dry sandstones and concluded that the strength was almost unchanged when the sample's length to diameter ratio equals to 2 or greater. But when LDR was less than 2, the strength apparently decreased with the increase of LDR. Thuro et al. [11] conducted an unconfined compression test on kersantite and limestone to study the scale effects on rock mechanical properties and concluded that scale effects on elastic modulus and tensile strength were significant, while the influence on UCS was much lower. Mogi [16, 17] conducted unconfined compression tests on different rock types with different LDR and concluded that UCS decreased with the increase of LDR. According to the research of Dirige and Archibald [18], UCS of granodiorite and granite moderately decreased with the increase of LDR, while Young's modulus was almost unchanged. Tuncay and Hasancebi [19] concluded that high UCS values were obtained at LDR less than 2, a very slight difference in values between 2 and 2.5, and they were remaining effectively constant when LDR was larger than 2.5. Prasad et al. [20] concluded that a larger correction was needed on smaller L/D core plugs, and the effect of confining pressures continues to decrease until the rock behaves plastically or shows the barreling effect. The effects of sample size on rock mechanical behaviors were also well investigated. According to the research conducted by Hoek and Brown [21], the diameter approximated to 50 mm was preferred; Hawkins [22] got a similar conclusion. ASTM D2938 recommended the diameter of the core should be at least six to ten times that of the largest grain or fragment of the rock. According to the results of Thuro et al. [11], sample diameter had a marginal effect on UCS of limestone and granite and had a little effect on elastic modulus and tensile strength of limestone. Masoumi and Douglas [23] concluded that UCS first increased then slightly decreased with the increase of sample size, and the boundary between ascending and descending zones was variable, depending on the rock type and most likely crystal or grain size. Forbes et al. [14] conducted both axial and diametric point load tests of Gosford sandstone and found that the point load strength index varies increasingly with sample diameter for all investigated samples at different length to diameter ratios. Their experimental results also

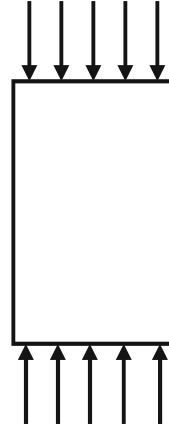


FIGURE 1: UCS test setup schematic.

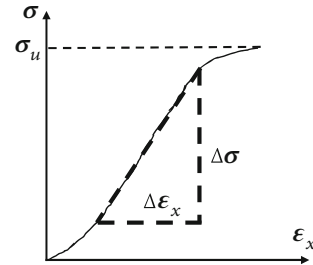


FIGURE 2: Stress-strain curve of the UCS test.

demonstrated that as the sample diameter increased, the point load strength index decreased.

In this work, uniaxial compression tests were conducted on 41 different dimensions of Berea sandstone samples to systematically study the scale effects on mechanical properties of Berea sandstones. Correction equations and standardizing factors were obtained to minimize the scale effects and get more reliable rock mechanical properties. Meanwhile, failure modes of the tested samples were studied.

2. Experimental Parameters and Sample Description

2.1. Experimental Parameters. The mechanical parameters studied in this work mainly include unconfined compressive strength (UCS), Young's modulus (E), Poisson's ratio (ν), bulk modulus (K), and shear modulus (G). In a uniaxial compression test, a right circular cylinder of a rock is compressed between two parallel rigid plates (Figure 1). The axial stress is the controlled, independent variable, while the axial strain is the dependent variable. The strain can be measured by a strain gauge. UCS is the maximum stress that the rock can sustain under a uniaxial compression test. The most useful description of the mechanical behavior of rock is the stress-strain curve, from which UCS and Young modulus can be determined (Figure 2). Bulk modulus and shear modulus are the parameters related to Young's modulus and Poisson's

TABLE 1: Berea sandstone samples in the uniaxial compression tests.

LDR	D (in)						
	0.5	0.75	1	1.5	2	3	4
	0.75	1.5	1				
	1	2	1.3	1			
	1.2	2.4	1.6	1.2			
	1.5	3	2	1.5	1		
	1.8		2.4	1.8	1.2		
L (in)	2		2.7	2	1.3	1	
	3			3	2	1.5	1
	4			4	2.7	2	1.3
	6			6	4	3	2
	8				5.3	4	2.7
	10				6.7	5	3.3

ratio. The specific calculation of these mechanical parameters is as follows:

$$\sigma_u = \frac{P}{A}, \quad (1)$$

$$E = \frac{\Delta\sigma}{\Delta\varepsilon_x}, \quad (2)$$

$$\nu = \left| \frac{\varepsilon_y}{\varepsilon_x} \right|, \quad (3)$$

$$K = \frac{E}{3(1-2\nu)}, \quad (4)$$

$$G = \frac{E}{2(1+\nu)}, \quad (5)$$

where σ_u is the uniaxial compressive strength (UCS), P is failure load and A is the cross-sectional area, $\Delta\sigma$ is the change in stress, $\Delta\varepsilon_x$ is the change in longitudinal strain, ε_y is the lateral strain, and ε_x is the longitudinal strain.

2.2. Sample Description. Berea sandstone is a Mississippian terrestrial sandstone. The sandstone samples used in this research are finer-grained and well-sorted outcrop Berea sandstones. X-ray diffraction was conducted to analyze the mineralogy of Berea sandstone which shows 91% quartz, 2% clay, 3% feldspar, 2% plagioclase, and 2% chlorite. There are 41 different geometric dimensions of Berea sandstone samples that were tested with sample diameters (D) ranging from 0.5 inch to 4 inches, sample length (L) from 0.75 inch to 10 inches, and LDR ranging from 1 to 6.7. The detailed sample geometric dimensions are listed in Table 1.

3. Experimental Procedures

The experimental setup mainly consists of a material testing system, MTS-810 (Figure 3), strain gauges, and a data acquisition system. The test procedures are described as follows: (1) prepare 41 Berea sandstone core samples. Then, attach the strain gauge to the core sample. For a



FIGURE 3: The MTS-810 material testing system.



FIGURE 4: Core samples with the strain gauges attached.

long sample, two strain gauges were attached and the average strain was used in the calculation (Figure 4); (2) put the sample in the middle of the material testing system. Turn on the computer and set the program for testing; (3) start running the programs, and the loading force will be added to the sample gradually. When hearing breaking sound, stop the programs to end the test; (4) acquire the data and calculate UCS, E , ν , K , and G based on Equations (1)–(5).

4. Experimental Results and Analysis

The test results of Berea sandstone samples are illustrated in Table 2. The mechanical parameters were calculated according to Equations (1)–(5).

4.1. LDR and Size Effects on UCS. 41 Berea sandstone samples used in this research can be divided into 17 different LDR (1, 1.2, 1.3, 1.5, 1.6, 1.8, 2, 2.4, 2.5, 2.7, 3, 3.3, 4, 5, 5.3, 6, and 7), see Table 3. To investigate the 17 different LDR effects, normalization process was used. Equation (6) is the normalized equation used for evaluating the LDR effect on UCS.

$$\text{UCSN}_j^i = \frac{\text{UCS}_j^i}{\text{UCS}_2^i}, \quad (6)$$

TABLE 2: Test results of 41 Berea sandstone samples.

D (in)	L (in)	UCS (MPa)	E (GPa)	Poisson's ratio	K (GPa)	G (GPa)
0.5	0.75	30.2	8.2	0.406	14.5	2.9
0.5	1	28.3	8.2	0.336	8.3	3.1
0.5	1.2	33.7	9.8	0.328	9.5	3.7
0.5	1.5	21.2	9.4	0.261	6.6	3.7
0.75	0.75	33.6	7.9	0.315	7.1	3.0
0.75	1	34.1	7.3	0.372	9.5	2.7
0.75	1.2	33.6	8.3	0.367	10.4	3.0
0.75	1.5	31.8	8.7	0.331	8.6	3.3
0.75	1.8	28.8	8.2	0.286	6.4	3.2
0.75	2	34.0	9.2	0.314	8.2	3.5
1	1	43.9	8.6	0.326	8.2	3.2
1	1.2	36.2	8.1	0.334	8.1	3.0
1	1.5	33.6	12.8	0.399	21.1	4.6
1	1.8	33.5	9.3	0.362	11.2	3.4
1	2	33.2	9.5	0.376	12.8	3.5
1	3	34.0	10.7	0.224	6.5	4.4
1	4	30.2	9.3	0.378	12.7	3.4
1	6	24.1	11.7	0.308	10.2	4.5
1.5	1.5	43.2	10.2	0.318	9.3	3.9
1.5	1.8	39.4	10.2	0.328	9.9	3.8
1.5	2	43.4	10.2	0.376	13.7	3.7
1.5	3	43.5	9.7	0.389	14.6	3.5
1.5	4	38.3	12.0	0.389	18.0	4.3
1.5	6	41.3	12.2	0.269	8.8	4.8
1.5	8	33.6	10.4	0.231	6.4	4.2
1.5	10	30.8	11.1	0.222	6.7	4.5
2	2	51.3	11.6	0.326	11.1	4.4
2	3	39.5	11.9	0.367	14.9	4.4
2	4	39.7	10.1	0.365	12.5	3.7
2	6	37.8	11.7	0.378	16.0	4.2
2	8	36.0	14.0	0.245	9.2	5.6
2	10	28.8	11.6	0.322	10.9	4.4
3	3	49.7	13.2	0.324	12.5	5.0
3	4	45.8	13.3	0.357	15.5	4.9
3	6	42.6	13.0	0.351	14.5	4.8
3	8	43.1	12.3	0.208	7.0	5.1
3	10	38.7	12.6	0.375	16.8	4.6
4	4	56.2	14.9	0.332	14.8	5.6
4	6	45.8	12.1	0.355	13.9	4.5
4	8	46.5	15.3	0.364	18.8	5.6
4	10	41.8	15.8	0.368	19.9	5.8

TABLE 3: Effects of LDR on UCS of Berea sandstone samples.

LDR	D (in)	L (in)	UCS (MPa)	UCSN	SF _{UCS}
1	0.75	0.75	33.6	1.06	
1	1	1	43.9	1.32	
1	1.5	1.5	43.2	0.99	1.17
1	2	2	51.3	1.29	
1	3	3	49.7	1.17	
1	4	4	56.2	1.21	
1.2	1	1.2	36.2	1.09	1.00
1.2	1.5	1.8	39.4	0.91	
1.3	0.75	1	34.1	1.07	
1.3	1.5	2	43.4	1.00	1.05
1.3	3	4	45.8	1.08	
1.5	0.5	0.75	30.2	1.07	
1.5	1	1.5	33.6	1.01	1.01
1.5	2	3	39.5	0.99	
1.5	4	6	45.8	0.98	
1.6	0.75	1.2	33.6	1.06	1.06
1.8	1	1.8	33.5	1.01	1.01
2	0.5	1	28.3	1	
2	0.75	1.5	31.8	1	
2	1	2	33.2	1	
2	1.5	3	43.5	1	1
2	2	4	39.7	1	
2	3	6	42.6	1	
2	4	8	46.5	1	
2.4	0.5	1.2	33.7	1.19	1.05
2.4	0.75	1.8	28.8	0.91	
2.5	4	10	41.8	0.90	0.90
2.7	0.75	2	34	1.07	
2.7	1.5	4	38.3	0.88	0.99
2.7	3	8	43.1	1.01	
3	0.5	1.5	21.2	0.75	
3	1	3	34	1.02	0.91
3	2	6	37.8	0.95	
3.3	3	10	38.7	0.91	0.91
4	1	4	30.2	0.91	
4	1.5	6	41.3	0.95	0.92
4	2	8	36	0.91	
5	2	10	28.8	0.73	0.73
5.3	1.5	8	33.6	0.77	0.77
6	1	6	24.1	0.73	0.73
6.7	1.5	10	30.8	0.71	0.71

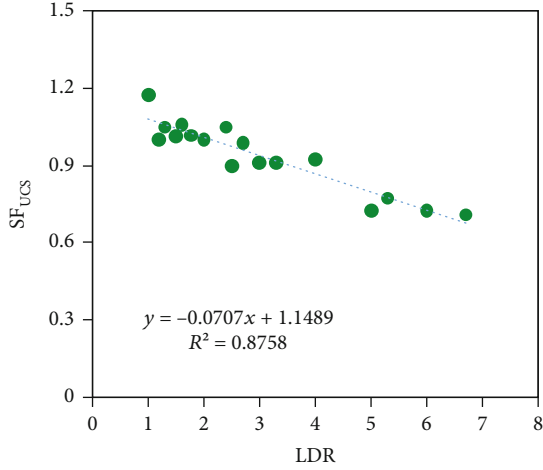


FIGURE 5: The relationship of LDR and SF_{UCS}.

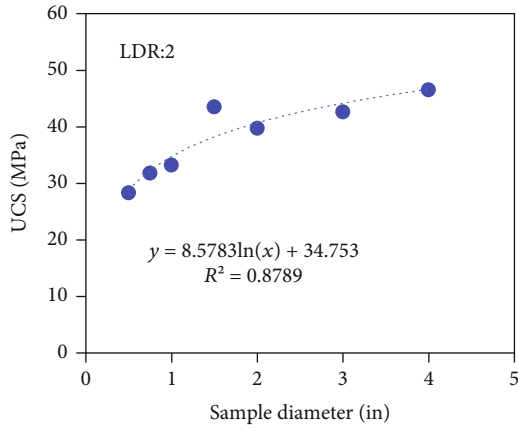


FIGURE 6: Effect of sample size on UCS.

where $UCSN_j^i$ is defined as the normalized uniaxial compressive strength indicator of the sample with diameter i and LDR j . UCS_j^i is the UCS of the sample with diameter i and LDR j . UCS_2^i is the UCS of the sample with diameter i and LDR at 2. The reason why LDR at 2 was chosen as the reference UCS is based on the ASTM standard (the desirable specimen length to diameter ratio of ASTM standard is 2 to 2.5). In order to get more clear results of the LDR on UCS, Equation (7) was used to calculate the average value of normalized compressive strength indicator, which is defined as UCS standardizing factor.

$$SF_{UCS_j} = \frac{1}{n} \sum_{i=1}^n UCS_j^i, \quad (7)$$

where SF_{UCS_j} is the UCS standardizing factor with LDR j . n is the number of the tested samples with the same LDR. Table 3 provides a summary of the results of UCS for the tested 41 Berea sandstone samples. Figure 5 is the relationship of UCS standardizing factor and LDR,

TABLE 4: Effects of LDR on Young's modulus of Berea sandstones.

LDR	D (in)	L (in)	E (GPa)	EN	SF _E
1	0.75	0.75	7.9	0.91	
1	1	1	8.6	0.91	
1	1.5	1.5	10.2	1.05	1
1	2	2	11.6	1.15	
1	3	3	13.2	1.02	
1	4	4	14.9	0.97	
1.2	1	1.2	8.1	0.85	0.95
1.2	1.5	1.8	10.2	1.05	
1.3	0.75	1	7.3	0.84	
1.3	1.5	2	10.2	1.05	0.97
1.3	3	4	13.3	1.02	
1.5	0.5	0.75	8.2	1.00	
1.5	1	1.5	12.8	1.35	1.08
1.5	2	3	11.9	1.18	
1.5	4	6	12.1	0.79	
1.6	0.75	1.2	8.3	0.95	0.95
1.8	1	1.8	9.3	0.98	0.98
2	0.5	1	8.2	1	
2	0.75	1.5	8.7	1	
2	1	2	9.5	1	
2	1.5	3	9.7	1	1
2	2	4	10.1	1	
2	3	6	13	1	
2	4	8	15.3	1	
2.4	0.5	1.2	9.8	1.20	1.07
2.4	0.75	1.8	8.2	0.94	
2.5	4	10	15.8	1.03	1.03
2.7	0.75	2	9.2	1.06	
2.7	1.5	4	12	1.24	1.08
2.7	3	8	12.3	0.95	
3	0.5	1.5	9.4	1.15	
3	1	3	10.7	1.13	1.14
3	2	6	11.7	1.16	
3.3	3	10	12.6	0.97	0.97
4	1	4	9.3	0.98	
4	1.5	6	12.2	1.26	1.21
4	2	8	14	1.39	
5	2	10	11.6	1.15	1.15
5.3	1.5	8	10.4	1.07	1.07
6	1	6	11.7	1.23	1.23
6.7	1.5	10	11.1	1.14	1.14

TABLE 5: Correlation Equations of Elastic Parameters and LDR.

Normalized elastic parameters	Standardizing factor	Correction equation	R^2
$EN_j^i = \frac{E_j^i}{E_2^i}$	$SF_{Ej} = \frac{1}{n} \sum_{i=1}^n EN_j^i$	$SF_E = 0.114 \ln(\text{LDR}) + 0.952$	0.57
$vN_j^i = \frac{v_j^i}{v_2^i}$	$SF_{vj} = \frac{1}{n} \sum_{i=1}^n vN_j^i$	$SF_v = -0.17 \times \ln(\text{LDR}) + 1.06$	0.42
$KN_j^i = \frac{K_j^i}{K_2^i}$	$SF_{Kj} = \frac{1}{n} \sum_{i=1}^n KN_j^i$	$SF_K = 1.17e^{-0.103x}$	0.37
$GN_j^i = \frac{G_j^i}{G_2^i}$	$SF_{Gj} = \frac{1}{n} \sum_{i=1}^n GN_j^i$	$SF_G = 0.176 \ln(\text{LDR}) + 0.932$	0.65

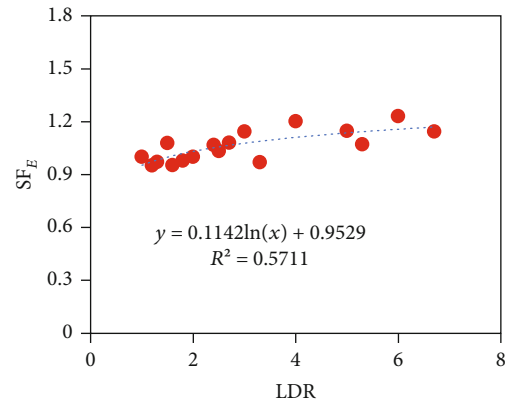
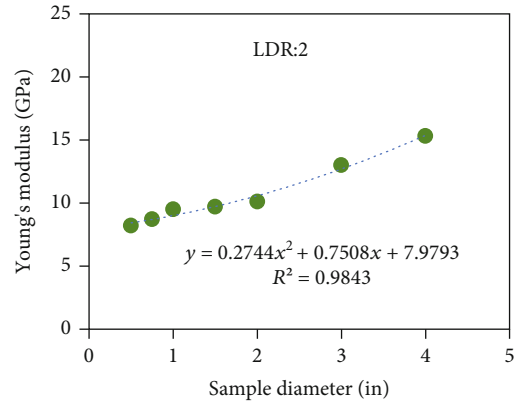
showing SF_{UCS} of Berea Sandstone cores decreases as LDR increases, which illustrates that UCS decreases with LDR increases. This is probably caused by bending effects. It is easier for a rock sample to bend if it is long. In UCS tests, the force was added at both ends of the sample, so it is relatively easier to break down if a sample is with a larger LDR. Equation (8) is the correction equation of SF_{UCS} and LDR for Berea sandstones, which indicates the negative linear relationships between SF_{UCS} and LDR; the correlation coefficient is 0.88, which is relatively high. SF_{UCS} is between 0.71 and 1.17, which means -30% to +20% variation of UCS with LDR changing from 1 to 6.7.

$$SF_{UCS} = -0.071(\text{LDR}) + 1.149. \quad (8)$$

From Table 3, we can observe that UCS increases with sample diameter increases at the same LDR, except for LDR at 2.4 and 4. Because there are only 2 sample sizes at LDR of 2.4 and only 3 sample sizes at LDR of 4, these data are not representative. Figure 6 shows the UCS increases with sample size increases with LDR at 2, which is representative because there are 7 different sample sizes. From Figure 6, we can observe the positive relationship with UCS and sample diameter.

4.2. LDR and Size Effects on E , Poisson's Ratio, K , and G . Young's modulus is a parameter used to describe the stiffness of elastic materials. It is the ratio of the stress to the strain along the same axis in the range of stress in which Hooke's law holds [24, 25]. Table 4 is the summary of Young's modulus and the calculated EN and SF_E of the tested Berea sandstone samples. Table 5 shows the correlation equation of Young's modulus standardizing factor (SF_E) and LDR with correlation coefficient which is 0.57. According to the normalized results of this research, Young's modulus of Berea sandstone samples would slightly increase with LDR increases, as shown in Figure 7. Figure 8 shows Young's modulus increases with the increase of sample diameter with LDR at 2, meaning Young's modulus increases with sample size increases.

E_j^i is Young's modulus of the sample with diameter i and LDR j ; E_2^i is Young's modulus of the sample with diameter i

FIGURE 7: The relationship of LDR and SF_E .FIGURE 8: Effect of sample size on E .

and LDR 2.0; EN_j^i is normalized Young's modulus indicator of the sample with diameter i and LDR j ; SF_E is Young's modulus standardizing factor; v_j^i is Poisson's ratio of the sample with diameter i and LDR j ; v_2^i is Poisson's ratio of the sample with diameter i and LDR 2.0; vN_j^i is normalized Poisson's ratio indicator of the samples with diameter i and LDR j ; SF_v is Poisson's ratio standardizing factor; K_j^i is the bulk modulus of the sample with diameter i and LDR j ; K_2^i

TABLE 6: Effects of LDR on Poisson’s ratio of Berea sandstones.

LDR	D (in)	L (in)	ν	ν_N	SF_ν
1	0.75	0.75	0.315	0.95	
1	1	1	0.326	0.87	
1	1.5	1.5	0.318	0.82	0.89
1	2	2	0.326	0.89	
1	3	3	0.324	0.92	
1	4	4	0.332	0.91	
1.2	1	1.2	0.334	0.89	
1.2	1.5	1.8	0.328	0.84	0.87
1.3	0.75	1	0.372	1.12	
1.3	1.5	2	0.376	0.97	1.04
1.3	3	4	0.357	1.02	
1.5	0.5	0.75	0.406	1.21	
1.5	1	1.5	0.399	1.06	
1.5	2	3	0.367	1.01	1.06
1.5	4	6	0.355	0.98	
1.6	0.75	1.2	0.367	1.11	1.11
1.8	1	1.8	0.362	0.96	0.96
2	0.5	1	0.336	1	
2	0.75	1.5	0.331	1	
2	1	2	0.376	1	
2	1.5	3	0.389	1	1
2	2	4	0.365	1	
2	3	6	0.351	1	
2	4	8	0.364	1	
2.4	0.5	1.2	0.328	0.98	
2.4	0.75	1.8	0.286	0.86	0.92
2.5	4	10	0.368	1.01	1.01
2.7	0.75	2	0.314	0.95	
2.7	1.5	4	0.389	1.00	0.85
2.7	3	8	0.208	0.59	
3	0.5	1.5	0.261	0.78	
3	1	3	0.224	0.60	0.80
3	2	6	0.378	1.04	
3.3	3	10	0.375	1.07	1.07
4	1	4	0.378	1.01	
4	1.5	6	0.269	0.69	0.79
4	2	8	0.245	0.67	
5	2	10	0.322	0.88	0.88
5.3	1.5	8	0.231	0.59	0.59
6	1	6	0.308	0.82	0.82
6.7	1.5	10	0.222	0.57	0.57

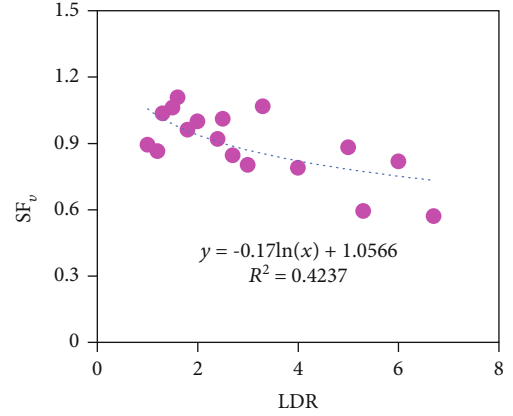


FIGURE 9: The relationship of LDR and SF_ν .

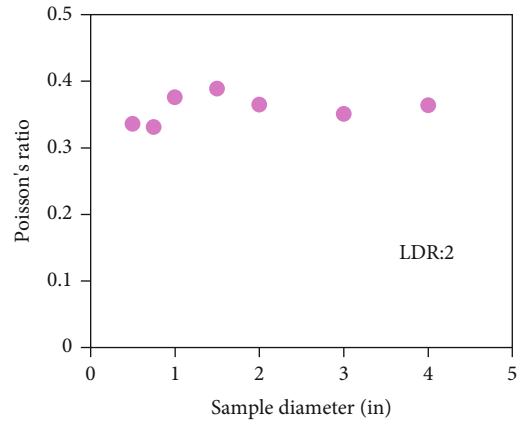


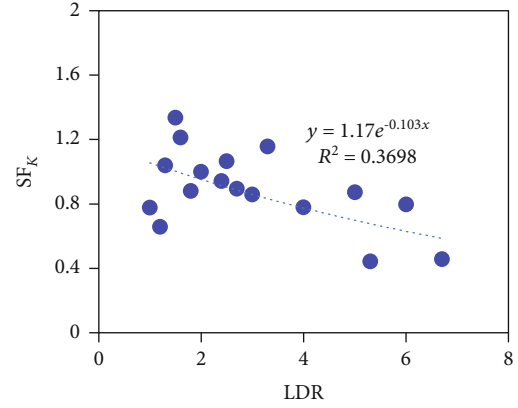
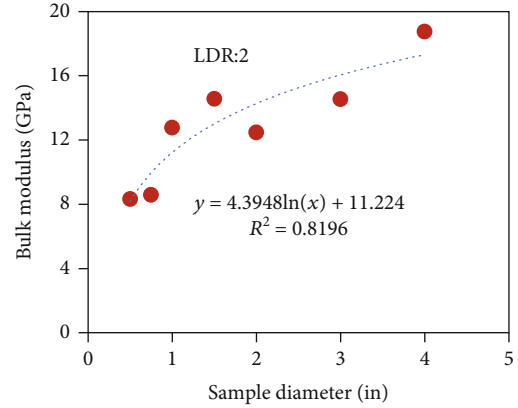
FIGURE 10: Effect of sample size on ν .

is the bulk modulus of the sample with diameter i and LDR 2.0; KN_j^i is the normalized bulk modulus indicator of the sample with diameter i and LDR j ; SF_K is the bulk modulus standardizing factor; G_j^i is the shear modulus of the sample with diameter i and LDR j ; G_2^i is the shear modulus of the sample with diameter i and LDR 2.0; GN_j^i is the normalized shear modulus indicator of the sample with diameter i and LDR j ; SF_G is the shear modulus standardizing factor.

Poisson’s ratio is a measure of the effect that when a material is compressed in one direction, the direction perpendicular to the compressive direction would usually expand. Poisson’s ratio is dependent on the material and is valid in the material’s elastic deformation region [26, 27]. Table 6 is the summary of Poisson’s ratio of the tested Berea sandstone samples. According to the normalized results of this work, Poisson’s ratio standardizing factor (SF_ν) has negative relationship with LDR (Figure 9), which means that Poisson’s ratio of the tested samples decreases with the increase of LDR. The relationship between LDR and Poisson’s ratio is not linear (Table 5). Poisson’s ratio standardizing factor for the tested Berea sandstone samples is between 0.57 and 1.11, which means

TABLE 7: Effects of LDR on bulk modulus of Berea sandstones.

LDR	D (in)	L (in)	K (GPa)	K_N	SF_K
1	0.75	0.75	7.1	0.83	
1	1	1	8.2	0.65	
1	1.5	1.5	9.3	0.64	0.78
1	2	2	11.1	0.89	
1	3	3	12.5	0.86	
1	4	4	14.8	0.79	
1.2	1	1.2	8.1	0.64	0.66
1.2	1.5	1.8	9.9	0.68	
1.3	0.75	1	9.5	1.11	
1.3	1.5	2	13.7	0.94	1.04
1.3	3	4	15.5	1.07	
1.5	0.5	0.75	14.5	1.74	
1.5	1	1.5	21.1	1.65	1.33
1.5	2	3	14.9	1.20	
1.5	4	6	13.9	0.74	
1.6	0.75	1.2	10.4	1.21	1.21
1.8	1	1.8	11.2	0.88	0.88
2	0.5	1	8.3	1	
2	0.75	1.5	8.6	1	
2	1	2	12.8	1	
2	1.5	3	14.6	1	1
2	2	4	12.5	1	
2	3	6	14.5	1	
2	4	8	18.8	1	
2.4	0.5	1.2	9.5	1.14	0.94
2.4	0.75	1.8	6.4	0.74	
2.5	4	10	19.9	1.06	1.06
2.7	0.75	2	8.2	0.96	
2.7	1.5	4	18.0	1.24	0.89
2.7	3	8	7.0	0.48	
3	0.5	1.5	6.6	0.79	
3	1	3	6.5	0.51	0.86
3	2	6	16.0	1.28	
3.3	3	10	16.8	1.16	1.16
4	1	4	12.7	0.99	
4	1.5	6	8.8	0.60	0.78
4	2	8	9.2	0.73	
5	2	10	10.9	0.87	0.87
5.3	1.5	8	6.4	0.44	0.44
6	1	6	10.2	0.80	0.80
6.7	1.5	10	6.7	0.46	0.46

FIGURE 11: The relationship of LDR and SF_K .FIGURE 12: Effect of sample size on K .

the dramatic variation of Poisson's ratio caused by LDR changing from 1 to 6.7. From Figure 10, we can observe that there is no consistent positive or negative relationship between sample size and Poisson's ratio for LDR at 2. Therefore, the relationship of sample size and Poisson's ratio is not apparent for the tested Berea sandstone samples in this study.

Table 7 is the summary of bulk modulus and the calculated K_N and SF_K of the tested Berea sandstone samples. In order to study the LDR effect on bulk modulus, data normalization process was performed. According to the normalized results of this research, bulk modulus of Berea sandstone samples decreases with LDR increases, as shown in Figure 11. Because the correlation coefficient of the regression curve is only 0.37 (Table 5), the relationship between LDR and bulk modulus is not strong. But we can observe that bulk modulus standardizing factor (SF_K) varies between 0.44 and 1.33, which means that LDR changing from 1 to 6.7 can result in great variation of bulk modulus of the tested Berea sandstones. Figure 12 shows the bulk modulus increases with sample size increases with LDR at 2, which illustrates bulk modulus increases with the increase of sample size.

Table 8 is the summary of shear modulus and the calculated G_N and SF_G of the tested Berea sandstone samples.

TABLE 8: Effects of LDR on shear modulus of Berea sandstones.

LDR	D (in)	L (in)	G (GPa)	GN	SFG
1	0.75	0.75	3.0	0.92	
1	1	1	3.2	0.94	
1	1.5	1.5	3.9	1.11	1.03
1	2	2	4.4	1.18	
1	3	3	5.0	1.04	
1	4	4	5.6	1.00	
1.2	1	1.2	3.0	0.88	
1.2	1.5	1.8	3.8	1.10	0.99
1.3	0.75	1	2.7	0.81	
1.3	1.5	2	3.7	1.06	0.96
1.3	3	4	4.9	1.02	
1.5	0.5	0.75	2.9	0.95	
1.5	1	1.5	4.6	1.33	1.06
1.5	2	3	4.4	1.18	
1.5	4	6	4.5	0.80	
1.6	0.75	1.2	3.0	0.93	0.93
1.8	1	1.8	3.4	0.99	0.99
2	0.5	1	3.1	1	
2	0.75	1.5	3.3	1	
2	1	2	3.5	1	
2	1.5	3	3.5	1	1
2	2	4	3.7	1	
2	3	6	4.8	1	
2	4	8	5.6	1	
2.4	0.5	1.2	3.7	1.20	
2.4	0.75	1.8	3.2	0.98	1.09
2.5	4	10	5.8	1.03	1.03
2.7	0.75	2	3.5	1.07	
2.7	1.5	4	4.3	1.24	1.12
2.7	3	8	5.1	1.06	
3	0.5	1.5	3.7	1.21	
3	1	3	4.4	1.27	1.21
3	2	6	4.2	1.15	
3.3	3	10	4.6	0.95	0.95
4	1	4	3.4	0.98	
4	1.5	6	4.8	1.38	1.29
4	2	8	5.6	1.52	
5	2	10	4.4	1.19	1.19
5.3	1.5	8	4.2	1.21	1.21
6	1	6	4.5	1.30	1.30
6.7	1.5	10	4.5	1.30	1.30

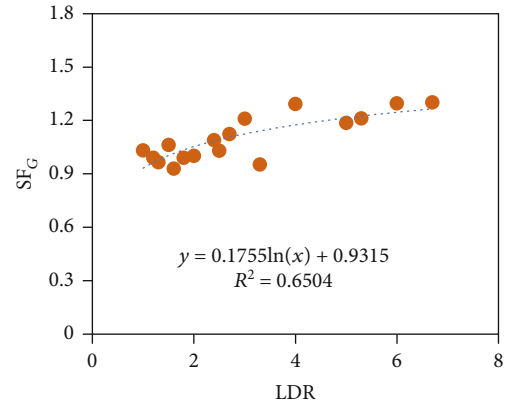


FIGURE 13: The relationship of LDR and SF_G .

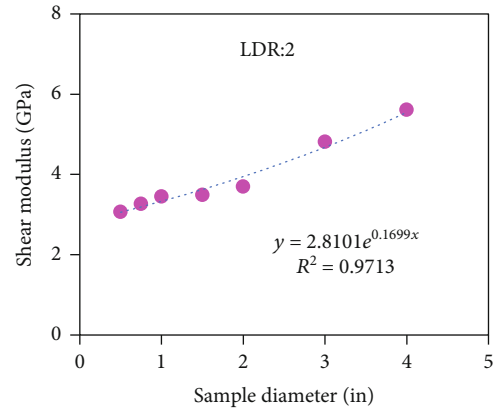


FIGURE 14: Effect of sample size on G.

According to the normalized test results, shear modulus of Berea sandstone samples increases with LDR increases (Figure 13). From Figure 13, we can observe that shear modulus standardizing factor (SF_G) varies between 0.93 and 1.3, which means that LDR change from 1 to 6.7 can result in -10% to +30% variation of shear modulus of the tested Berea sandstone samples. Figure 14 shows the shear modulus increases with sample size increases with LDR at 2, which illustrates the positive relationship of shear modulus and sample size.

4.3. Failure Mode Analysis. In this study, failure modes of 41 Berea sandstone samples under uniaxial compression tests were investigated. Based on the test results, the principal failure modes of the Berea sandstone samples are axial splitting and shear failure. We can also observe that the failure modes are affected by LDR. When LDR is less than 2, samples exhibited axial splitting. When LDR is larger than 2, samples showed shear failure. For samples with LDR equals to 2, some samples show shear failure, while some samples exhibit axial splitting (Figure 15). The test results of Berea sandstone samples illustrated that under uniaxial compression tests, stocky samples (LDR < 2) tend to show axial splitting, while slender samples (LDR > 2) tend to go shear failure. Figures 16 and 17 show

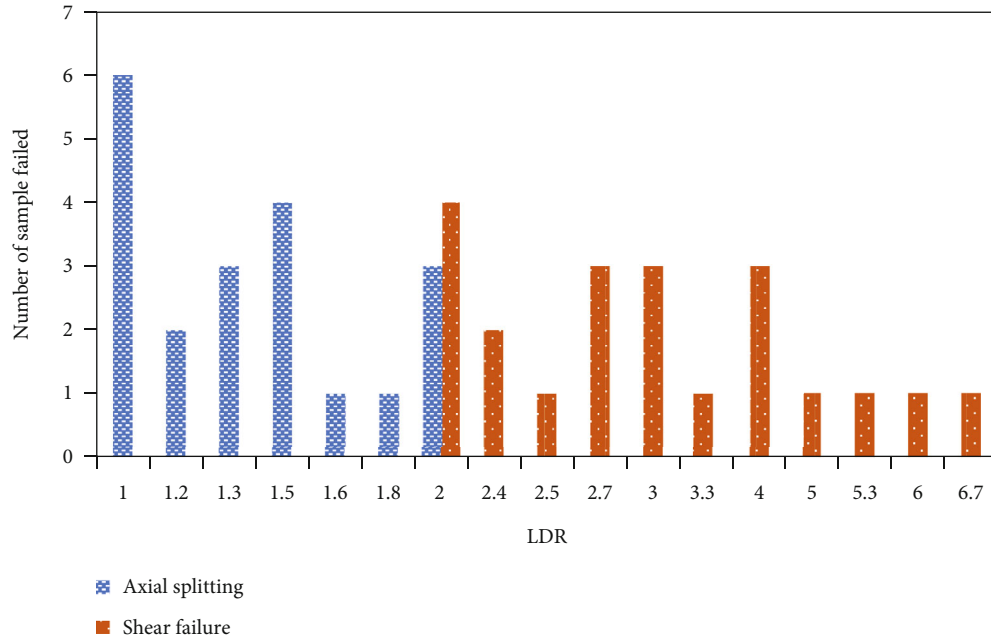


FIGURE 15: The failure modes of Berea sandstone samples.

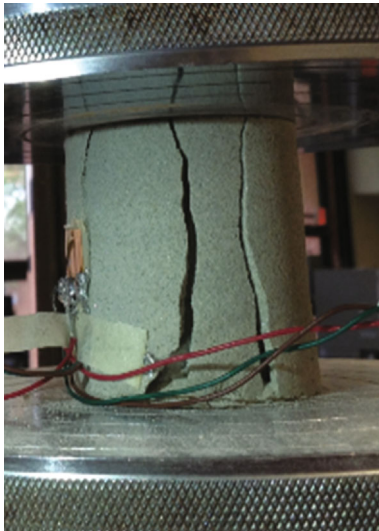


FIGURE 16: Axial splitting failure with 2 inches in diameter and 2 inches in length.

the axial splitting failure mode and shear broken failure mode of the tested Berea sandstone samples.

5. Conclusions

Experimental investigation has been conducted on 41 Berea sandstone samples to study the scale effects on the mechanical behaviors. According to the test results, the geometry of core samples has dramatic impacts on rock mechanical properties. The specific conclusions are listed below.

- (1) UCS of Berea sandstone decreases with LDR increases. The UCS standardizing factor is between 0.71 and 1.17, which means -30% to +20% variation of UCS with LDR changing from 1 to 6.7. UCS of the tested samples exhibits positive relationship with sample size
- (2) Young's modulus slightly increases with the increase of LDR. When LDR changes from 1 to 6.7, Young's modulus is in -5% to +23% variation. At the same LDR, Young's modulus of Berea sandstone increases with sample size increases
- (3) Poisson's ratio decreases significantly with LDR increases. The standardizing factor of Poisson's ratio is between 0.57 and 1.11, which means LDR has a dramatic influence on Poisson's ratio. Sample size effect on Poisson's ratio is not apparent in this study
- (4) Bulk modulus of Berea sandstone samples decreases with LDR increases, while shear modulus increases with LDR increases. Both bulk modulus and shear modulus increase with the increase of sample size
- (5) Failure modes of the tested 41 Berea sandstone samples are analyzed. The principal failure modes are axial splitting and shear failure, which are affected by LDR. Stocky samples ($LDR < 2$) tend to show axial splitting, while slender samples ($LDR > 2$) tend to show shear failure

Nomenclature

UCS: Uniaxial compressive strength, MPa
E: Young's modulus, GPa

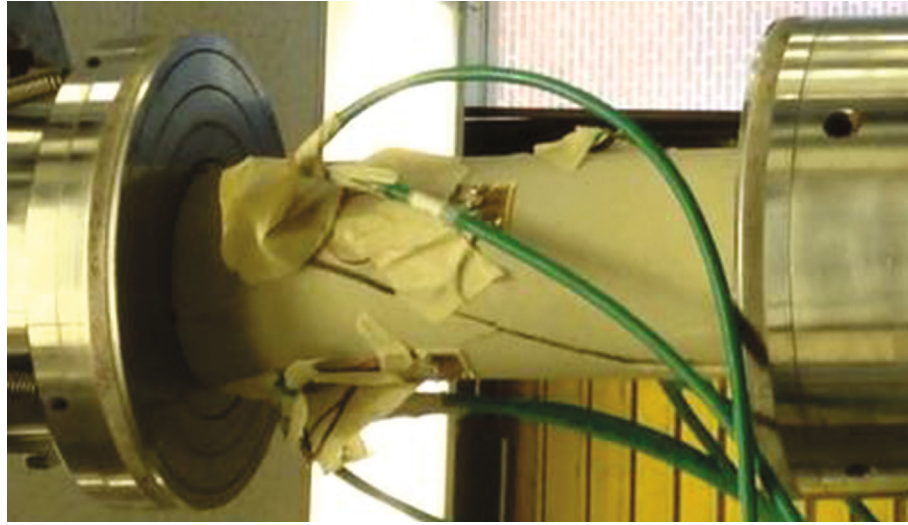


FIGURE 17: Shear broken failure with 3 inches in diameter and 8 inches in length.

ν : Poisson's ratio
 D : Diameter, inch
 L : Length, inch
 LDR: Length to diameter ratio
 ASTM: American Society for Testing and Materials
 ISRM: International Society of Rock Mechanics
 σ_u : Uniaxial compressive strength (UCS), MPa
 P : Failure load, MPa
 A : Cross-sectional area, cm^2
 ϵ_y : Lateral strain
 ϵ_x : Longitudinal strain
 UCS_j^i : UCS of sample with diameter i and LDR j
 UCS_2^i : UCS of the test samples with diameter i and LDR 2
 UCSN_j^i : Normalized UCS indicator of samples with diameter i and LDR j
 SF_{UCS} : UCS standardizing factor
 $\text{SF}_{\text{UCS}_j^i}$: UCS standardizing factor with LDR j
 E_j^i : Young's modulus of the sample with diameter i and LDR j
 E_2^i : Young's modulus of the sample with diameter i and LDR 2
 EN_j^i : Normalized Young's modulus indicator of the sample with diameter i and LDR j
 SF_E : Young's modulus standardizing factor
 ν_j^i : Poisson's ratio of the sample with diameter i and LDR j
 ν_2^i : Poisson's ratio of the sample with diameter i and LDR 2
 νN_j^i : Normalized Poisson's ratio indicator of the samples with diameter i and LDR j
 SF_ν : Poisson's ratio standardizing factor
 K_j^i : Bulk modulus of the sample with diameter i and LDR j
 K_2^i : Bulk modulus of the sample with diameter i and LDR 2
 KN_j^i : Normalized bulk modulus indicator of the sample with diameter i and LDR j
 SF_K : Bulk modulus standardizing factor

G_j^i : Shear modulus of the sample with diameter i and LDR j
 G_2^i : Shear modulus of the sample with diameter i and LDR 2
 GN_j^i : Normalized shear modulus indicator of the sample with diameter i and LDR j
 SF_G : Shear modulus standardizing factor.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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