

Research Article

Optimal Inflow Performance Relationship Equation for Horizontal and Deviated Wells in Low-Permeability Reservoirs

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After Vogel proposed a dimensionless inflow performance equation, with the rise of the horizontal well production mode, a large number of inflow performance relationship (IPR) equations have emerged. In the productivity analysis of deviated and horizontal wells, the IPR equation proposed by Cheng is mainly used. However, it is still unclear whether these inflow performance models (such as the Cheng, Klins-Majcher, Bendakhlia-Aziz, and Wiggins-Russell-Jennings types) are suitable for productivity evaluations of horizontal and deviated wells in low-permeability reservoirs. In-depth comparisons and analyses have not been carried out, which hinders improvements in the accuracy of the productivity evaluations of horizontal wells in low-permeability reservoirs. In this study, exploratory work was conducted in two areas. First, the linear flow function relationship used in previous studies was improved. Based on the experimental pressure-volume-temperature results, a power exponential flow function model was established according to different intervals greater or less than the bubble point pressure, which was introduced into the subsequent derivation of the inflow performance equation. Second, given the particularity of low-permeability reservoir percolation, considering that the reservoir is a deformation medium, and because of the existence of a threshold pressure gradient in fluid flow, the relationship between permeability and pressure was changed. The starting pressure gradient was introduced into the subsequent establishment of the inflow performance equation. Based on the above two aspects of this work, the dimensionless IPR of single-phase and oil-gas two-phase horizontal wells in a deformed medium reservoir was established by using the equivalent seepage resistance method and complex potential superposition principle. Furthermore, through regression and error analyses of the standard inflow performance data, the correlation coefficients and error distributions of six types of IPR equations applicable to deviated and horizontal wells at different inclination angles were compared. The results show that the IPR equation established in this study features good stability and accuracy and that it can fully reflect the particularity of low-permeability reservoir seepage. It provides the best choice of the IPR between inclined wells and horizontal wells in low-permeability reservoirs. The other types of IPR equations are the Wiggins-Russell-Jennings, Klins-Majcher, Vogel, Fetkovich, Bendakhlia-Aziz, and Harrison equations, listed here in order from good to poor in accuracy.

1. Introduction

Research on horizontal well productivity began in the 1950s. First, people studied horizontal wells using electrical simulation in the laboratory. In 1958, Merkulov [1] first proposed a mathematical analysis method for calculating the production of horizontal wells. Then, in 1964, Borisov [2] systematically summarized the development process and production prin-

ciple of horizontal wells and inclined wells and proposed an equivalent seepage resistance method. Since the 1980s, Giger et al. [3, 4] and Joshi [5], researchers in the United States [6–8], and Xu [9] and Li et al. [10], have derived various horizontal well productivity formulas.

Many methods are used to study the productivity of horizontal wells. Two representative methods are the (semi)-analytical method and the numerical simulation method.

Analytical and semianalytical methods include applying integral transformation to solve the partial differential equation of seepage, removing the time variable through a Laplace transform, and obtaining a solution of a specific problem in the space variable using the classical Fourier method [11, 12]. For example, the equivalent seepage resistance method is used to establish the inflow performance relationship (IPR) of horizontal wells. The flow resistance of a horizontal well can be divided into two parts: the external resistance when flowing from the elliptic boundary to an imaginary vertical fracture of the horizontal well and the internal resistance of the oil well itself [5, 10]. The numerical simulation method generally uses a numerical simulator to calculate and predict the productivity of horizontal wells based on the established geological and numerical models. To verify the correctness of the numerical simulation results, it is necessary to maintain consistency with the analytical solution. The advantage is that the flexible grid system can be used to consider reservoir heterogeneity and overcome some limitations of the analytical solution [13–15].

At present, the productivity analysis of deviated and horizontal wells, the determination of a reasonable working system, and the design of lifting technology [16–20] are mainly based on a series of Vogel IPR equations under different well inclination angles proposed by Cheng in 1990 [21] on the basis of reservoir numerical simulations. It is not clear whether other commonly used IPR equations, such as the Vogel equation (derived in 1968, 2012) [22–24], the Klins-Majcher equation (derived in 1989) [25], the Bendakhlia-Aziz equation (derived in 1989) [26], and the Wiggins-Russell-Jennings equation (derived in 1992) [27] (listed in Table 1), can be used for the productivity evaluations of inclined or horizontal wells.

The purpose of this study was to determine the optimal IPR equations for horizontal and inclined wells and to provide a better evaluation tool for field productivity analysis. In this study, the productivity analysis solutions of single-phase and oil-gas two-phase horizontal wells were established by using equivalent seepage resistance and were transformed into the IPR equation in a dimensionless manner. Using the standardized inflow performance data of Cheng in 1990 [21], which is suitable for low-permeability reservoirs, the calculation errors of six IPR equations and the IPR equations established in this study under different well inclination angles were comprehensively compared.

2. Inflow Performance Relationship of Horizontal Wells in Low-Permeability Deformation Medium Reservoir

2.1. Establishment of Single-Phase Flow Stability Model. A schematic diagram of the horizontal well is shown in Figure 1. The basic assumptions used in the derivation are as follows:

- (1) The reservoir is horizontal, has equal thickness, and has a top and bottom impermeable barrier
- (2) There is a horizontal well parallel to the top and bottom in the middle of the reservoir

- (3) The vertical plane flow is equivalent to a sink point in a parallel plate channel
- (4) The formation is homogeneous and isotropic, ignoring the influence of gravity and capillary force
- (5) The starting pressure gradient in the horizontal plane is the same as that in the vertical plane

The flow resistance of a horizontal well can be divided into two parts: the external resistance when the flow is from the elliptic boundary to the imaginary vertical fracture of the horizontal well and the internal resistance of the oil well itself.

2.1.1. External Resistance of the Horizontal Well. The conformal transformation diagram used to derive the external resistance of horizontal wells under elliptic boundary conditions is shown in Figure 2. It is supposed that the semimajor axis of the drainage ellipse is a' , the semiminor axis is b' , the focal length is c , and the length of the horizontal well is L .

The Zhukovsky transform is taken as follows:

$$z = \frac{c}{2} \left(w + \frac{1}{w} \right). \quad (1)$$

In this function, the circumference of an ellipse on plane z is transformed into a circle of radius $R = (2(a' + b'))/L$ on plane w , and the length of a horizontal well in plane z is transformed into a circle of unit radius on plane w . Furthermore, the difference in the potential function is obtained as follows [10]:

$$\phi_R - \phi_1 = \frac{q}{2\pi} \ln \frac{a' + \sqrt{a'^2 - L^2/4}}{L/2}, \quad (2)$$

because

$$\frac{d\phi}{dr} = -\frac{K}{\mu_o} \left(\frac{dp}{dr} - G_o \right). \quad (3)$$

If the permeability changes with pressure in accordance with the power exponent law [13]

$$\frac{K}{K_0} = e^{-\alpha_k(p_r - p)}. \quad (4)$$

The integral of equation (3) over (r_w, \sqrt{ab}) is as follows:

$$\frac{q_h}{2\pi} \ln \frac{a' + \sqrt{(a'^2 - L^2)/4}}{L/2} = \frac{K_0}{\mu_o \alpha_k} \left\{ 1 - \exp \left[-\alpha_k (p_r - p_{wf}) + \alpha_k G_o (\sqrt{a' b'} - r_w) \right] \right\}. \quad (5)$$

TABLE 1: Types and equations of inflow performance of oil wells.

Type	Inflow performance equation	Comments on a form
Vogel type (type 1)	$q_o/q_{o \max} = 1 - V_v \left(\frac{p_{wf}}{p_r} \right) - (1 - V_v) \left(\frac{p_{wf}}{p_r} \right)^2$	$V_v = (1 + 0.5bp_r/a)^{-1}$
Klins-Majcher type (type 2)	$q_o/q_{o \max} = 1 - V_k \left(\frac{p_{wf}}{p_r} \right) - (1 - V_k) \left(\frac{p_{wf}}{p_r} \right)^n$	$V_k = [1 + bp_r^m / (1 + m)a]^{-1}$
Fetkovich type (type 3)	$q_o/q_{o \max} = \left[1 - \left(\frac{p_{wf}}{p_r} \right)^2 \right]^n$	Change range of n is $[0.5, 1]$
Bendakhlia-Aziz type (type 4)	$q_o/q_{o \max} = \left[1 - V_v \left(\frac{p_{wf}}{p_r} \right) - (1 - V_v) \left(\frac{p_{wf}}{p_r} \right)^2 \right]^n$	
Wiggins-Russell type (type 5)	$q_o/q_{o \max} = 1 - V_{w1} \left(\frac{p_{wf}}{p_r} \right) - V_{w2} \left(\frac{p_{wf}}{p_r} \right)^2 - V_{w3} \left(\frac{p_{wf}}{p_r} \right)^3 - V_{w4} \left(\frac{p_{wf}}{p_r} \right)^4$ $k_{r0}/\mu_o B_o = a_0 + a_1 p + a_2 p^2 + a_3 p^3$	$V_1 = a_0 p_r / (a_0 p_r + (a_1 p_r^2 / 2) + (a_2 p_r^3 / 3) + (a_3 p_r^4 / 4))$ $V_2 = a_1 p_r^2 / (2a_0 p_r + a_1 p_r^2 + (2a_2 p_r^3 / 3) + (a_3 p_r^4 / 2))$ $V_3 = a_2 p_r^3 / (3a_0 p_r + (3a_1 p_r^2 / 2) + a_2 p_r^3 + (3a_3 p_r^4 / 4))$ $V_4 = a_3 p_r^4 / (4a_0 p_r + 2a_1 p_r^2 + (4a_2 p_r^3 / 3) + a_3 p_r^4)$
Harrison type (type 6)	$q_o/q_{o \max} = (1 + V_h) - V_h \exp \left(C_h \left(\frac{p_{wf}}{p_r} \right) \right)$	$V_h = \exp(-bp_e) / [1 - \exp(-bp_r)]$ $C_h = bp_r$
The IPR proposed (type 7)	$q/q_{\max} = E' g \left[1 - M_1 \left(\frac{p_{wf}}{p_r} \right) - M_2 \left(\frac{p_{wf}}{p_r} \right)^2 - M_3 \left(\frac{p_{wf}}{p_r} \right)^3 - M_4 \left(\frac{p_{wf}}{p_r} \right)^{m+1} - M_5 \left(\frac{p_{wf}}{p_r} \right)^{m+2} - M_6 \left(\frac{p_{wf}}{p_r} \right)^{m+3} \right]$	

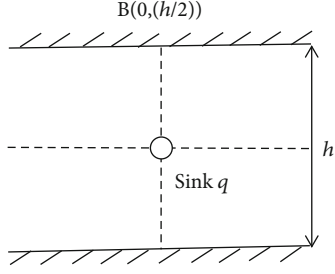


FIGURE 1: Schematic diagram of a horizontal well in top-bottom closed boundary reservoir.

Considering the influence of pollution, the deformation results are as follows:

$$q_h = \frac{2\pi K_0}{\mu_o} \frac{1 - \exp \left\{ -\alpha_k \left[(p_r - p_{wf}) - G_o \left(\sqrt{a'b'} - r_w \right) \right] \right\}}{\alpha_k \left(\ln \left(\left(a' + \sqrt{(a'^2 - L^2)/4} \right) / (L/2) \right) + s_h \right)}. \quad (6)$$

Therefore, the external resistance is

$$R_h = \frac{\mu_o}{2\pi K_0 h} \frac{\alpha_k \left(\ln \left(\left(a' + \sqrt{(a'^2 - L^2)/4} \right) / (L/2) \right) + s_h \right) \Delta p}{1 - \exp \left\{ -\alpha_k \left[(p_r - p_{wf}) - G_o \left(\sqrt{a'b'} - r_w \right) \right] \right\}}. \quad (7)$$

2.1.2. Internal Resistance System. The conformal transformation w is taken, and its corresponding relationship is shown in Figure 3.

$$w = e^{(\pi/h)z} = e^{(\pi/h)x} \left(\cos \frac{\pi y}{h} + i \sin \frac{\pi y}{h} \right) = \zeta + i\eta. \quad (8)$$

In this transformation, the strip field $-(h/2) < y < (h/2)$ on the z -plane becomes the right half plane on the w -plane, and the origin on the z -plane becomes the point $B(1, 0)$ on the w -plane. The complex potential of a point can be obtained by mirror image reflection in the w -plane:

$$\begin{aligned} W &= \frac{q}{2\pi} \ln (w-1)(w+1) + C_1, \\ W &= \frac{q}{4\pi} \ln \left[(\zeta^2 + \eta^2 + 1)^2 - 4\zeta^2 \right] \\ &\quad + i \frac{q}{2\pi} \left(\arctan \frac{\eta}{\zeta-1} + \arctan \frac{\eta}{\zeta+1} \right) + C_2. \end{aligned} \quad (9)$$

Therefore, the potential function is

$$\phi = \frac{q}{4\pi} \ln \left[\left(e^{2\pi x/h} + 1 \right)^2 - 4e^{2\pi x/h} \cos^2 \frac{\pi y}{h} \right] + C_2. \quad (10)$$

When $x=0, y=(h/2)$, the potential function at the boundary is

$$\phi_{h/2} = \frac{q}{2\pi} \ln 2 + C_2. \quad (11)$$

The potential on the well wall of point sink B can be considered the superposition of point sink A and point sink B , and the radius of point sink B is

$$\begin{aligned} \rho &= \left| \frac{d(w-1)(w+1)}{dz} \right|_{(0,0)}, \\ r_w &= \frac{2\pi r_w}{h}. \end{aligned} \quad (12)$$

Then, the potential on the well wall of point sink B is as follows:

$$\phi_{wf} = \frac{q}{2\pi} \left(\ln 2 + \ln \frac{2\pi r_w}{h} \right) + C_2. \quad (13)$$

If equation (3) is integrated within $(r_w, (h/2))$, the following results are obtained:

$$\frac{K}{\mu_o \alpha_k} \left\{ 1 - \exp \left[-\alpha_k (p_{h/2} - p_{wf}) + \alpha_k G_o \left(\frac{h}{2} - r_w \right) \right] \right\} = \phi_{h/2} - \phi_{wf}. \quad (14)$$

By substituting equations (11) and (13) into equation (14) and considering the impact of pollution, it is concluded that

$$q_v = q = \frac{2\pi K}{\mu} \frac{1 - \exp \left\{ -\alpha_k \left[(p_{h/2} - p_{wf}) - G_o \left((h/2) - r_w \right) \right] \right\}}{\alpha_k \left(\ln (h/2\pi r_w) + s_v \right)}. \quad (15)$$

According to equation (15), the internal resistance is as follows:

$$R_v = \frac{\mu_o}{2\pi K_0 L} \frac{\Delta p \alpha_k \left(\ln (h/2\pi r_w) + s_v \right)}{1 - \exp \left\{ -\alpha_k \left[(p_{h/2} - p_{wf}) - G_o \left((h/2) - r_w \right) \right] \right\}}. \quad (16)$$

Here, $p_{h/2} = p_r$ is taken, because

$$R_v + R_H = \frac{\Delta p}{Q}. \quad (17)$$

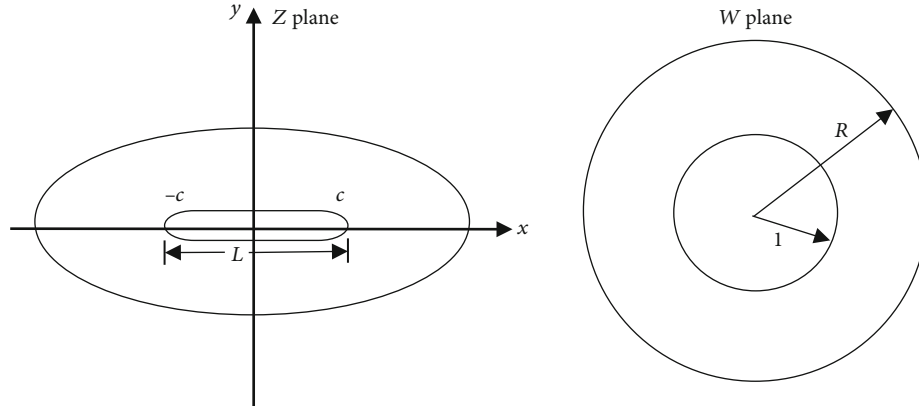


FIGURE 2: Conformal mapping of horizontal well with elliptic boundary.

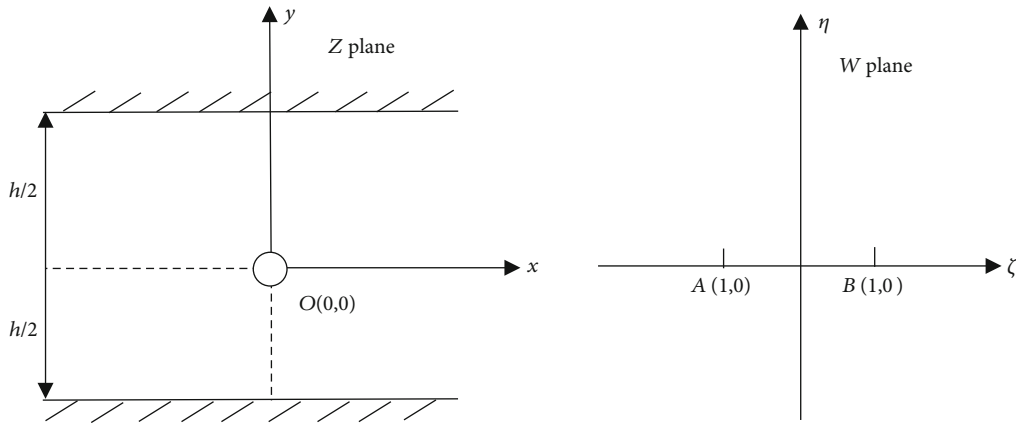


FIGURE 3: Correspondence diagram of conformal transformation.

Therefore, the horizontal well production formula is as follows:

$$Q = \frac{2\pi K_0}{(\mu_o \alpha_k / L) \left(\ln(h/2\pi r_w) + s_w/1 - \exp \left\{ -\alpha_k \left[(p_r - p_{wf}) - G_o(h/2 - r_w) \right] \right\} \right) + (\mu_o \alpha_k / h) \left(\ln \left(a' + \sqrt{(a'^2 - L^2)/4} / (L/2) \right) + s_h/1 - \exp \left\{ -\alpha_k \left[(p_r - p_{wf}) - G_o(\sqrt{a'b'} - r_w) \right] \right\} \right)} \quad (18)$$

2.2. Establishment of Oil-Gas Two-Phase Flow Model

2.2.1. *Generalization of the Mobility Function Relation.* The commonly used empirical expression of mobility function ($K_{ro}/\mu_o B_o$) is $(K_{ro}/\mu_o B_o) = f(p) = a + bp$, but the mobility function relationship in an actual oilfield is not approximately a straight line, but rather a convex or concave curve.

In Figure 4, the mobility curve of a well in a tight reservoir is shown. At $p \leq p_b$, comparing the curve and the linear fitting formula shows that the linear fitting relationship is not in accordance with the experimental data, and it cannot accurately reflect the relationship between $(K_{ro}/\mu_o B_o)$ and pressure. Therefore, the mobility function can be generalized as follows [27]:

$$\frac{K_{ro}}{\mu_o B_o} = f(p) = \begin{cases} a + bp^m, & p \leq p_b, & (a), \\ a + bp, & p > p_b, & (b). \end{cases} \quad (19)$$

2.2.2. *External Resistance System.* We have the following hypothesis:

$$dH = \frac{k_{ro}}{\mu B_o} dp. \quad (20)$$

Equation (3) is arranged as follows:

$$d\phi dp = K dH dp - KG_o dH dL. \quad (21)$$

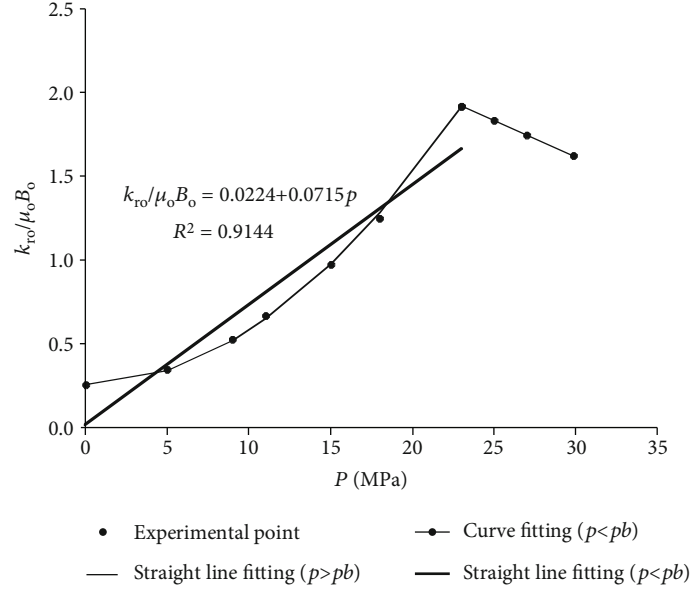


FIGURE 4: Mobility function curve.

By using a double integral and considering the influence of pollution, the horizontal production is obtained as follows:

$$q_h = \frac{2\pi K_0}{\alpha_k \left(\ln \left(a' + \sqrt{(a'^2 - L^2)/4/(L/2)} \right) + s_h \right)} \left\{ 1 - \exp \left[-\alpha_k (p_r - p_{wf}) + \alpha_k G_o \left(\sqrt{a'b'} - r_w \right) \right] \right\} \times \left(ap_r + \frac{b}{m+1} p_r^{m+1} - ap_{wf} - \frac{b}{m+1} p_{wf}^{m+1} \right) (p_r - p_{wf})^{-1}. \quad (22)$$

Then, the external resistance is

$$R_h = \frac{\Delta p^2 \alpha_k \left(\ln \left(a' + \sqrt{(a'^2 - L^2)/4/(L/2)} \right) + s_h \right)}{2\pi K_0 h \left\{ 1 - \exp \left[-\alpha_k (p_r - p_{wf}) + \alpha_k G_o \left(\sqrt{a'b'} - r_w \right) \right] \right\} \left(ap_r + (b/(m+1)) p_r^{m+1} - ap_{wf} - (b/(m+1)) p_{wf}^{m+1} \right)}. \quad (23)$$

2.2.3. Internal Resistance System. By double integration of equation (21) and considering the influence of pollution, the yield of the vertical plane can be obtained as follows:

$$q_v = \frac{2\pi K_0}{\alpha_k \left(\ln (h/2\pi r_w) + s_v \right)} \left\{ 1 - \exp \left[-\alpha_k (p_{h/2} - p_{wf}) + \alpha_k G_o \left(\frac{h}{2} - r_w \right) \right] \right\} \times \left(ap_{h/2} + \frac{b}{m+1} p_{h/2}^{m+1} - ap_{wf} - \frac{b}{m+1} p_{wf}^{m+1} \right) (p_{h/2} - p_{wf})^{-1}. \quad (24)$$

If $p_{h/2} = p_r$, the internal resistance is

$$R_v = \frac{\Delta p^2 \alpha_k (\ln (h/2\pi r_w) + s_v)}{2\pi K_0 L \left\{ 1 - \exp \left[-\alpha_k (p_r - p_{wf}) + \alpha_k G_o ((h/2) - r_w) \right] \right\} \left(ap_r + (b/(m+1))p_r^{m+1} - ap_{wf} - (b/(m+1))p_{wf}^{m+1} \right)}, \quad (25)$$

because

$$R_v + R_H = \frac{\Delta p}{Q}. \quad (26)$$

Therefore, the corresponding two-phase flow rate formula for the deformation medium reservoir is as follows:

$$\begin{aligned} \frac{1}{Q} = & \frac{\ln (h/2\pi r_w) + s_v}{\left\{ (p_r - p_{wf}) - G_o ((h/2) - r_w) + (1/2)\alpha_k \left[(p_r - p_{wf}) - G_o ((h/2) - r_w) \right]^2 + (1/6)\alpha_k^2 \left[(p_r - p_{wf}) - G_o ((h/2) - r_w) \right]^3 \right\}} \\ & \times \frac{(p_r - p_{wf})}{2\pi K_0 L \left(ap_r + (b/(m+1))p_r^{m+1} - ap_{wf} - (b/(m+1))p_{wf}^{m+1} \right)} \\ & + \frac{(p_r - p_{wf})}{2\pi K_0 h \left(ap_r + (b/(m+1))p_r^{m+1} - ap_{wf} - (b/(m+1))p_{wf}^{m+1} \right)} \\ & + \frac{\ln \left(a' + \sqrt{(a'^2 - L^2)/4} / (L/2) \right) + s_h}{\left\{ (p_r - p_{wf}) - G_o (\sqrt{a'b'} - r_w) + (1/2)\alpha_k \left[(p_r - p_{wf}) - G_o (\sqrt{a'b'} - r_w) \right]^2 + (1/6)\alpha_k^2 \left[(p_r - p_{wf}) - G_o (\sqrt{a'b'} - r_w) \right]^3 \right\}}. \end{aligned} \quad (27)$$

2.3. Inflow Performance of Oil and Gas Phases. Owing to the complexity of the two-phase flow production formula for horizontal wells in low-permeability and deformable media reservoirs, it is difficult to obtain the inflow performance equation with a simple form and strong generality by the usual mathematical means. In this study, the scaling method

is used to deduce the inflow dynamic equation with a simple form and strong universality.

2.3.1. The Reservoir Is Vertically Magnified. If $(h/2) = \sqrt{ab}$, then equation (27) shows that

$$\begin{aligned} \frac{1}{Q} = & \frac{(p_r - p_{wf})}{2\pi K_0 L \left(ap_r + (b/(m+1))p_r^{m+1} - ap_{wf} - (b/(m+1))p_{wf}^{m+1} \right)} \\ & \cdot \left[\left(\ln \frac{h}{2\pi r_w} + s_v \right) + \left(\ln \frac{a' + \sqrt{(a'^2 - L^2)/4}}{L/2} + s_h \right) \right] \\ & \times \frac{(p_r - p_{wf})}{\left\{ (p_r - p_{wf}) - G_o (\sqrt{a'b'} - r_w) + (1/2)\alpha_k \left[(p_r - p_{wf}) - G_o (\sqrt{a'b'} - r_w) \right]^2 + (1/6)\alpha_k^2 \left[(p_r - p_{wf}) - G_o (\sqrt{a'b'} - r_w) \right]^3 \right\}}. \end{aligned} \quad (28)$$

TABLE 2: Average normalized dimensionless production of different well inclination angles [10].

p_{wf}/p_e	0°	15°	30°	45°	60°	75°	85°	88.56°	90°
0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.1	0.972	0.970	0.975	0.982	0.986	0.989	0.990	0.990	0.995
0.2	0.927	0.924	0.935	0.947	0.957	0.962	0.964	0.964	0.974
0.3	0.868	0.864	0.879	0.897	0.910	0.918	0.921	0.921	0.935
0.4	0.791	0.786	0.806	0.828	0.844	0.854	0.856	0.857	0.875
0.5	0.700	0.695	0.717	0.742	0.760	0.771	0.774	0.775	0.796
0.6	0.592	0.587	0.611	0.636	0.655	0.666	0.669	0.669	0.692
0.7	0.468	0.464	0.486	0.510	0.527	0.537	0.540	0.540	0.562
0.8	0.328	0.325	0.343	0.362	0.377	0.385	0.387	0.387	0.405
0.9	0.172	0.171	0.182	0.194	0.202	0.207	0.209	0.209	0.219
1.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

The dimensionless equation of vertical enlargement is obtained as follows:

$$\frac{Q}{Q_{\max}} = E'_f g \left[1 - M_1 \frac{p_{wf}}{p_r} - M_2 \left(\frac{p_{wf}}{p_r} \right)^2 - M_3 \left(\frac{p_{wf}}{p_r} \right)^3 - M_4 \left(\frac{p_{wf}}{p_r} \right)^{m+1} - M_5 \left(\frac{p_{wf}}{p_r} \right)^{m+2} - M_6 \left(\frac{p_{wf}}{p_r} \right)^{m+3} \right]. \quad (29)$$

In that formula,

$$\begin{aligned} E'_f &= 1 - \frac{hs_v + Ls_h}{h(\ln(h/2\pi r_w) + s_v) + L(\ln(a' + \sqrt{(a'^2 - L^2)/4/(L/2)} + s_h))}, \\ g &= \left[V + (1 - V) \frac{1}{1 - (p_{wf}/p_r)} \right], \\ V &= \frac{p_r}{p_r - G_0(r_e - r_w)}, \\ K_1 &= \frac{3\alpha_k p_r + 2\alpha_k^2 p_r [p_r - G_0(\sqrt{ab} - r_w)]}{6K_3}, \\ K_2 &= -\frac{\alpha_k^2 p_r^2}{K_3}, \\ K_3 &= 1 + \frac{1}{2}\alpha_k [p_r - G_0(\sqrt{a'b'} - r_w)] + \frac{1}{6}\alpha_k^2 [p_r - G_0(\sqrt{a'b'} - r_w)]^2, \\ U_1 &= \frac{a}{a + (b/(m+1))p_r^{m+1}}, \\ M_1 &= U_1 + K_1, \\ M_2 &= -K_1 U_1 + K_2, \\ M_3 &= -K_2 U_1, \\ M_4 &= (1 - U_1), \\ M_5 &= -K_1(1 - U_1), \\ M_6 &= K_2(1 - U_1). \end{aligned} \quad (30)$$

2.3.2. *The Reservoir Shrinks Vertically.* It is assumed that the oil layer is thin enough and that the vertical plane has no

contribution to the production, i.e., $R_v = \infty$. The dimensionless equation of the vertical shrinkage of the reservoir can be obtained as follows:

$$\frac{Q}{Q_{\max}} = E''_f g \left[1 - M_1 \frac{p_{wf}}{p_r} - M_2 \left(\frac{p_{wf}}{p_r} \right)^2 - M_3 \left(\frac{p_{wf}}{p_r} \right)^3 - M_4 \left(\frac{p_{wf}}{p_r} \right)^{m+1} - M_5 \left(\frac{p_{wf}}{p_r} \right)^{m+2} - M_6 \left(\frac{p_{wf}}{p_r} \right)^{m+3} \right]. \quad (31)$$

In the formula, we have

$$E''_f = \frac{\ln(a' + \sqrt{(a'^2 - L^2)/4/(L/2)})}{\ln(a' + \sqrt{(a'^2 - L^2)/4/(L/2)} + S_h)}. \quad (32)$$

Comparing equations (29) and (31) reveals that the form of the inflow performance equation does not change when the liquid supply radius of the vertical plane changes from 0 to the maximum; however, the difference lies in the difference in flow efficiency. Because the flow efficiency in equation (31) is a special case result of equation (29), the IPR equation of the two-phase flow in horizontal wells of the deformation medium reservoir can be determined using equation (29).

The first two parts of equation (29) are the flow efficiency terms considering the pollution degree and the additional term of seepage resistance considering the starting pressure gradient. When pollution and the starting pressure gradient are not considered, $E_f = 1$, $g = 1$. An exceptional case is when $p_{wf} = p_r$, because there is no pressure difference and no fluid flow, and the starting pressure gradient does not work, so G is equal to 0, then $g = 1$. The third part, when $m = 0$, can be transformed into the Wiggins-Russell-Jennings equation, and the deformation capacity of the medium is reflected in $M_1 - M_6$. Therefore, the factors considered in the general formula are more comprehensive and maintain the continuity of theoretical research with the Wiggins-Russell-Jennings equation, which can be extended to more general cases.

TABLE 3: Fitting values of undetermined parameters of inflow performance equations for different types of inclined wells.

Type	Parameter	Inclination angle								
		0°	15°	30°	45°	60°	75°	85°	88.56°	90°
1	Equation	$q_o/q_{o \max} = V_0 - V_1 p_{wf}/p_r - V_2 (p_{wf}/p_r)^2$								
	V_0	0.99981	0.99980	0.99691	0.99455	0.99257	0.99152	0.99151	0.99141	0.98845
	V_1	0.20080	0.22095	0.12536	0.02214	-0.05487	-0.100230	-0.11199	0.11411	-0.20545
	V_2	0.79883	0.77832	0.86818	0.96632	1.03951	1.08287	1.09417	1.09639	1.18182
	R^2	1.00000	1.00000	0.99996	0.99986	0.99976	0.99970	0.99969	0.99970	0.99943
2	Equation	$q_o/q_{o \max} = V_0 - V_1 p_{wf}/p_r - V_2 (p_{wf}/p_r)^n$								
	V_0	0.99998	1.00006	1.00040	1.00079	1.00087	1.00084	1.00095	1.00086	1.00136
	V_1	0.20447	0.22677	0.19835	0.14755	0.10807	0.08101	0.07164	0.06974	0.03726
	V_2	0.79546	0.77296	0.80146	0.85221	0.89170	0.91867	0.92781	0.92983	0.96297
	n	2.00612	2.01001	2.12511	2.20695	2.26093	2.28354	2.28453	2.28418	2.37119
R^2	1.00000	1.00000	1.00000	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	
3	Equation	$q_o/q_{o \max} = V_0 \left[1 - (p_{wf}/p_r)^2 \right]^n$								
	V_0	0.97383	0.97125	0.97901	0.98918	0.99729	1.00252	1.00412	1.00440	1.01443
	n	1.07623	1.08297	1.03468	0.98651	0.95291	0.93416	0.92877	0.92869	0.88890
	R^2	0.99873	0.99845	0.99926	0.99983	0.99996	0.99989	0.99986	0.99984	0.99928
4	Equation	$q_o/q_{o \max} = \left[V_0 - V_1 p_{wf}/p_r - V_2 (p_{wf}/p_r)^2 \right]^n$								
	V_0	0.99983	1.00005	0.99893	0.99812	0.99722	0.99668	0.99719	0.99727	0.99537
	V_1	0.20105	0.22471	0.15410	0.06921	0.00311	-0.03811	-0.04439	-0.04432	-0.12852
	V_2	0.79864	0.77528	0.84536	0.92983	0.99547	1.03633	1.04371	1.04430	1.12610
	n	1.00021	1.00311	1.02495	1.04322	1.05571	1.06141	1.06741	1.06974	1.08173
R^2	1.00000	1.00000	0.99999	0.99998	0.99996	0.99995	0.99995	0.99995	0.99990	
5	Equation	$q_o/q_{o \max} = V_0 - V_1 p_{wf}/p_r - V_2 (p_{wf}/p_r)^2 - V_3 (p_{wf}/p_r)^3 - V_4 (p_{wf}/p_r)^4$								
	V_0	1.00010	0.99997	0.99994	1.00005	0.99986	0.99990	0.99982	0.99983	1.00001
	V_1	0.20626	0.22125	0.17161	0.10591	0.05577	0.03011	0.01329	0.01690	-0.02625
	V_2	0.78106	0.78794	0.75557	0.76221	0.77188	0.75318	0.79508	0.76495	0.73091
	V_3	0.01865	-0.02681	0.05808	0.10548	0.13423	0.19600	0.14161	0.19716	0.26321
	V_4	-0.00583	0.01748	0.01457	0.02622	0.03788	0.02040	0.04953	0.02040	0.03205
R^2	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	
6	Equation	$q_o/q_{o \max} = V_0 - V_1 \exp(C \cdot p_{wf}/p_r)$								
	V_0	1.28731	1.29955	1.25344	1.19596	1.17106	1.15712	1.15336	1.13851	1.13049
	V_1	0.26947	0.28265	0.23318	0.17532	0.14796	0.13282	0.12868	0.11810	0.10358
	C	1.57272	1.53367	1.68847	1.92981	2.07920	2.17630	2.20507	2.28252	2.40374
R^2	0.99911	0.99920	0.99909	0.99888	0.99864	0.99844	0.99838	0.99824	0.99806	
7	Equation	$q_o/q_{o \max} = V_0 - V_1 p_{wf}/p_r - V_2 (p_{wf}/p_r)^2 - V_3 (p_{wf}/p_r)^3 - V_4 (p_{wf}/p_r)^{m+1} - V_5 (p_{wf}/p_r)^{m+2} - V_6 (p_{wf}/p_r)^{m+3}$								
	V_0	1.00002	0.99999	0.99998	1.00000	0.99997	0.99998	0.99998	0.99998	0.99998
V_1	0.19009	0.27680	0.17409	0.07000	0.07535	0.02221	0.02088	-0.00095	-0.03362	

TABLE 3: Continued.

Type	Parameter	Inclination angle								
		0°	15°	30°	45°	60°	75°	85°	88.56°	90°
	V_2	1.05611	-0.37791	0.74178	1.55253	0.44541	1.04504	0.80697	1.37156	0.86277
	V_3	14.00211	-58.99156	0.82784	49.58385	-16.13077	23.18914	8.04119	46.75339	7.29149
	V_4	-13.5795	57.24579	-0.67021	-47.55929	15.81206	-21.89913	-7.28745	-44.3826	-6.80887
	V_5	-0.82033	3.54701	-0.11652	-3.40044	0.96540	-1.79647	-0.83814	-3.61157	-0.39675
	V_6	0.15156	-0.70018	0.04292	0.75325	-0.16753	0.43915	0.25641	0.87007	0.08486
	R^2	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

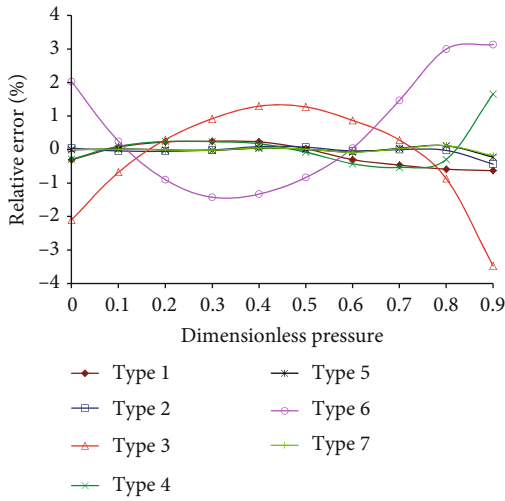


FIGURE 5: Relative error analysis of inflow performance equation fitting for seven types of 30° inclined wells.

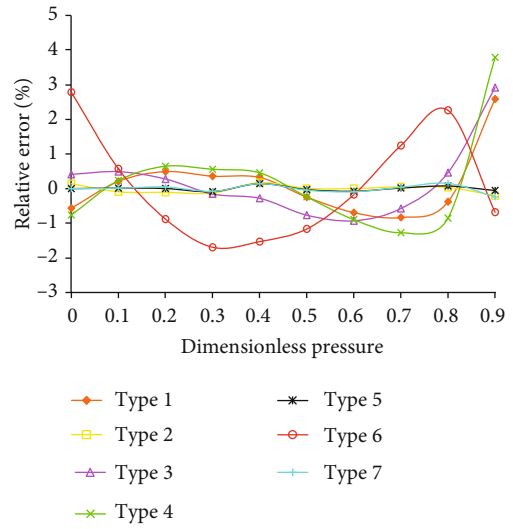


FIGURE 7: Relative error analysis of inflow performance equation fitting for seven types of 85° inclined wells.

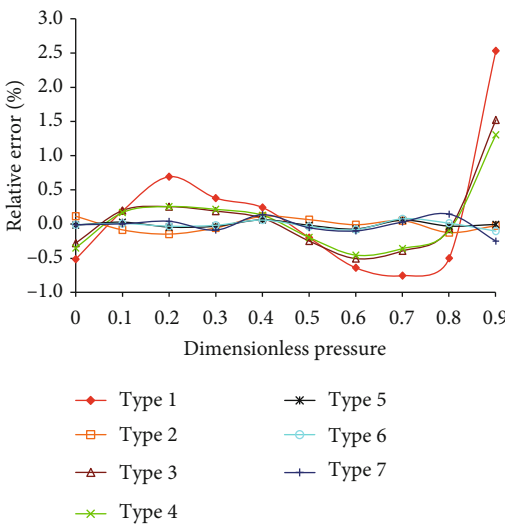


FIGURE 6: Relative error analysis of inflow performance equation fitting for seven types of 60° inclined wells.

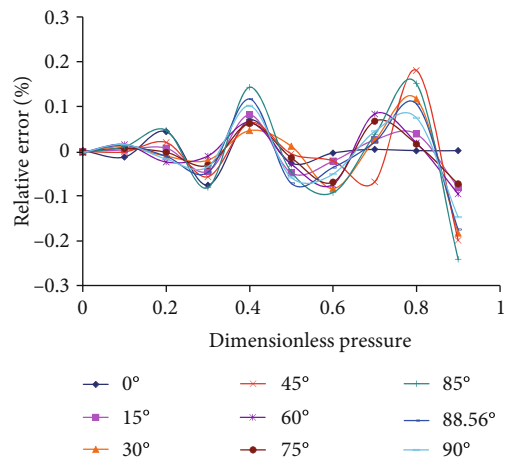


FIGURE 8: Relative error of normalized dimensionless yield fitting of type 7 IPR equation.

3. Optimal IPR Equation for Horizontal and Deviated Wells

3.1. Regression Analysis. The types of inflow performance for the comparative analysis are shown in Table 1, and standardized inflow performance data [10] are selected for the regression analysis (see Table 2). The data in Table 2 are the average values of the 16 reservoir conditions. The permeability value under the third reservoir condition does not meet the upper limit standard of a low-permeability reservoir (more than $200 \times 10^{-3} \mu\text{m}^2$). The permeability in the simulation of the other 15 reservoirs is 20, which belongs to the category of low-permeability reservoirs. Therefore, it has strong confidence and is ideal for comparative analyses of various IPR equations.

The seven inflow performance equations in Table 1 are used for the regression analysis of dimensionless production under the different well inclination angles in Table 2. The undetermined parameter values are listed in Table 3. The overall fitting degrees of different IPR equations for well deviation data are quite different. Among them, the complex correlation coefficient of the type 2 and type 7 inflow performance equations is 1; the fitting degree is the highest, and the complex correlation coefficient of type 2 is close to 1, which also shows good applicability for inclined wells with different angles. The second is type 1, and the applicability of types 3, 4, and 6 is relatively poor.

3.2. Error Analysis

3.2.1. Error Analysis of Inflow Performance Fitting in Inclined Wells. The normalized dimensionless data with inclination angles of 30° , 60° , and 85° were selected to analyze the relative errors of the seven types of IPR equations. The statistical analysis of the relative error is shown in Figures 5–8. In the entire dimensionless pressure range, the third, fourth, and sixth IPR equations have the most obvious changes. For example, when the inclination angle of type 6 is 30° , the arch shape is large at both ends and small in the middle. When the deviation angle is 60° , the variation range is reduced. A well deviation angle of 85° results in the shape of the sinusoidal curve with large fluctuation amplitude and wave crest and trough.

3.2.2. Overall Evaluation of Fitting Error. According to the analysis of fitting errors of the seven types of IPR equations, the seventh type of IPR equation established in this study has the highest fitting accuracy with a maximum relative error of less than 0.25%. The fifth type of IPR equation has the second highest accuracy with a maximum relative error of less than 0.80%. The second type of IPR equation has a maximum relative error of less than 0.80%. The first type of IPR equation performs poorly, with a maximum relative error of no more than 3.00%. Types 3, 4, and 6 had the poorest performances, with maximum relative errors of no more than 5.00%.

Using the inflow performance equation established in this study, the relative error of each well inclination angle fitted by the data in Table 1 is shown in Figure 8. The fitting

accuracy is very high, and it has good adaptability for wells with different well inclination angles; therefore, it can be applied to the performance analysis of oil wells on site.

4. Conclusions

- (1) In past studies, the linear flow function relationship has often been used to characterize the fluid pressure-volume-temperature model. Based on the results of the pressure-volume-temperature analysis of real reservoirs, it is considered that the improvement of the linear flow function to the power exponential flow function is helpful to describe the pressure-volume-temperature relationship of the fluid accurately under different pressure ranges and to improve the prediction accuracy of the IPR
- (2) To reflect the special percolation law of a low-permeability reservoirs fully, the threshold pressure gradient effect and the pressure sensitivity of reservoir permeability were considered in the establishment of the IPR equation. This was different from previous research and resulted in some progress. In the dimensionless inflow dynamic relationship of the oil-gas two-phase model established by the analytical method, the influence of the starting pressure gradient is reflected in G , and the deformation ability of the medium is reflected in M_1-M_6 . When $m = 0$, it can be transformed into the Wiggins-Russell-Jennings equation. The factors considered in the equation are more comprehensive and maintain the continuity of theoretical research with the Wiggins-Russell-Jennings equation, which can be extended to more general cases
- (3) Through regression and error analyses of standardized inflow dynamic data, the correlation coefficients and error distribution of six IPR equations for inclined and horizontal wells and the model established in this study were compared under different well inclination angles. The results show that the IPR equation established in this study is the best choice for the inflow dynamic relationship of inclined and horizontal wells in a low-permeability reservoir. The other types of IPR equations can be listed in order from good to poor as follows: Wiggins-Russell-Jennings, Klins-Majcher, Vogel, Fetkovich, Bendakhlia-Aziz, and Harrison.

Nomenclature

IPR:	Inflow performance relationship
ϕ_R :	Potential with radius equal to R
ϕ_1 :	Potential with radius equal to 1
L :	Horizontal well length (m)
K :	Formation permeability (md)
p :	Formation pressure (MPa)
G :	Starting pressure gradient (MPa/m)
μ_O :	Viscosity of crude oil (MPa•s)
α_k :	Deformation coefficient (MPa^{-1})

p_r :	Original formation pressure of reservoir (MPa)
K_0 :	Initial formation permeability corresponding to original formation pressure (md)
q_h :	Production per unit reservoir thickness on horizontal plane (m^3/d)
p_{wf} :	Bottom hole flowing pressure (MPa)
$p_{h/2}$:	Pressure at the boundary (MPa)
r_e :	Drainage radius (m)
r_w :	Wellbore radius (m)
s_h :	Skin factor on the horizontal plane
Δp :	Production pressure difference (MPa)
C_2, C_2 :	Undetermined constant
q :	Production per unit reservoir thickness (m^3/d)
h :	Reservoir thickness (m)
q_v :	Production per unit reservoir thickness in vertical plane (m^3/d)
s_v :	Skin factor in vertical plane
a, b, m, V :	Undetermined parameters
p_b :	Saturation pressure (MPa)
K_{ro} :	Relative permeability of oil phase
B_o :	Volume coefficient of crude oil at pressure p (m^3/m^3)
Q :	Horizontal well production (m^3/d)
Q_{\max} :	Maximum production of horizontal wells (m^3/d)
H :	Pseudopotential function
E'_f, E''_f :	Flow efficiency
$K_1, K_2, K_3, U_1, M_1, M_2, M_3, M_4, M_5, M_6$:	Undetermined coefficient.

Data Availability

The raw/processed data required to reproduce these findings cannot be shared at this time as the data also forms part of an ongoing study.

Conflicts of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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