

Research Article

Frictional Pressure Drop and Liquid Holdup of Churn Flow in Vertical Pipes with Different Viscosities

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Received 27 November 2020; Revised 24 December 2020; Accepted 8 January 2021; Published 27 January 2021

Academic Editor: Shiyuan Zhan

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Churn flow commonly exists in the pipe of heavy oil, and the characteristics of churn flow should be widely understood. In this paper, we carried out air and viscous oil two-phase flow experiments, and the diameter of the test section is 60 mm. The viscosity range of the oil was 100~480 mPa·s. Based on the measured liquid holdup and pressure drop data of churn flow, it can be concluded that, due to the existence of liquid film backflow, positive and negative frictional pressure drop can be found and the change of frictional pressure drop with the superficial gas velocity is related to superficial liquid velocity. With the increase of viscosity, the change rate of frictional pressure drop increases with the increase of the superficial gas velocity. Combining our previous work and the Taitel model, we proposed a new pressure drop model for viscous oil-air two-phase churn flow in vertical pipes. By comparing the predicted values of existing models with the measured pressure drop data, the proposed model has better performance in predicting the pressure drop.

1. Introduction

Churn flow commonly exists in the petroleum industry. Heavy oil accounts for more than half of the world's total oil reserves [1]. High viscosity brings more difficulties to the development of oil and gas fields, such as more difficult to flow in the wellbore and pipelines. However, due to the complexity of churn flow, the understanding and model research of gas-liquid two-phase churn flow with different viscosity received little attention.

The liquid film and the central gas core are the two main continuous phase in churn flow. Some researchers studied the drop entrainment in churn flow. Wang et al. [2] and Zhang et al. [3] studied drop entrainment in churn flow. Wang et al. [2] proposed a new model for drop entrainment in churn flow. For the vertical pipe, Zhang et al. [3] found dif-

ferent bubble entrainment processes and droplet entrainment processes. Wang et al. [4] presented a new model of pressure gradient, liquid holdup, and wave behavior in churn flow, and he analyzed the liquid viscosity effects on the pressure gradient, liquid holdup, and wave behavior. Wang et al. [5] proposed a new function to describe the wave shape of the churn flow.

Frictional pressure drop and liquid holdup are the most important parameters. For the liquid holdup with different viscosities, Gokcal et al. [6] and Sarica et al. [7] studied the void fraction of slug flow with different viscosities (40~587 mPa·s). Foletti et al. [8] and Unander et al. [9] performed experiments with different viscosity oil-gas two-phase flow and investigated the flow pattern transition. Baba et al. [10, 11] investigated the effects of high viscosity on the frequency and liquid holdup of the slug. Al-Safran et al. [12],

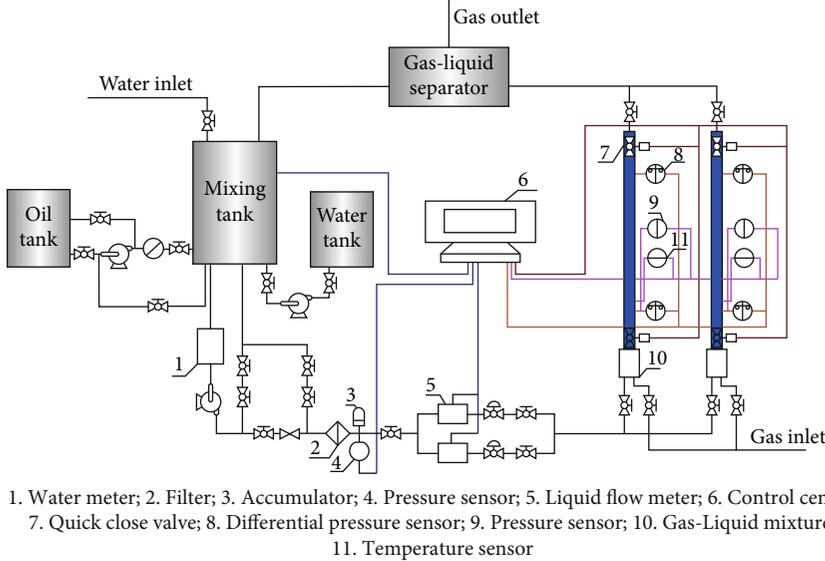


FIGURE 1: The multiphase flow test system.

Ruhaimani et al. [13], and Abdul-Majeed and Al-Mashat [14, 15] presented empirical closure relationship for liquid holdup with different viscosities, respectively.

In churn flow, due to the existence of negative frictional pressure drop, the prediction of the frictional pressure drop is the complicated. Lockhart [16], Shannak [17], Zhang et al. [18], and Pamitran et al. [19] developed the correlations for the calculation of frictional pressure drop. Yang et al. [20] found that the flow patterns affect the frictional pressure drop. Shaahid et al. [21] analyzed the effects of different inclination angles on the two-phase frictional pressure drop. In addition, Ganat et al. [22] proposed a new method of the frictional pressure drop for the oil-water-gas three-phase flow.

Churn flow is the transitional flow pattern of slug flow and annular flow, which is characterized by up and down oscillation, and the gas velocity range of churn flow is relatively wide. However, the research of its characteristics is the inadequate. In this paper, we carried out the oil-gas two-phase flow experiments, and the churn flow was observed and the data of liquid holdup and the total pressure drop was measured in the experiments. In addition, we proposed a combined model for churn flow and gave a comparison of performance with other models.

2. The Experiment of Gas-Liquid Two-Phase Churn Flow

2.1. Experimental Test System. The gas and different viscosity liquid two-phase churn flow experiment was carried out in a large multiphase flow test system. In the experiment, the gas is pressurized by the air compressor, and the liquid is pressurized by the plunger pump. The liquid phase and the gas phase together enter the gas-liquid mixer through pipes. Then, the mixture passes through the 2.5 m flow development section and enters the test pipe section. The test section can measure pressure, temperature, flow pattern, pressure difference, and the liquid holdup. The test section provides

TABLE 1: Viscosity vs. temperature of the liquid phase.

Liquid phase	Viscosity (mPa·s)
White oil with tackifier	$\mu_1 = -0.0033T^3 + 0.5599T^2 - 33.597T + 765.36$

a 7 m pipe to observe the flow dynamics and flow pattern, and a high speed camera was used to record the flow patterns. In the end, the liquid phase flows into the liquid tank. The experiment system is shown in Figure 1.

For this experiment, the gas and the while oil with a tackifier are the gas and liquid phases, respectively. The oil viscosities sated in the experiment were 100 mPa·s, 290 mPa·s, and 480 mPa·s. The range of the superficial liquid velocity was 0.02-0.52 m/s, and the range of the superficial gas velocity was 1.6~11 m/s. The change of oil viscosity with temperature (T) is shown in Table 1.

2.2. Experimental Measurement. In the experiment, when the flow system is stable, liquid holdup and total pressure were recorded. Liquid holdup refers to the average liquid holdup measured between two quick-closing valves. The liquid holdup was to test the volume of residual liquid in the test section for three times, and the ratio of the volume of the residual liquid to the volume of the whole pipe was used as the average liquid holdup of the experiment. Table 2 shows the measurement parameters of different sensors.

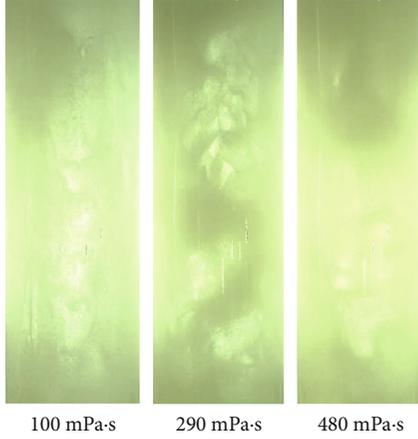
3. Experimental Results

3.1. Uncertainty Analysis. The total pressure drop (dP/dL)_{*t*} is expressed as

$$-\left(\frac{dP}{dL}\right)_t = \left(\frac{dP}{dL}\right)_h + \left(\frac{dP}{dL}\right)_f. \quad (1)$$

TABLE 2: Measurement parameters of different sensors.

Sensor	Range	Uncertainty
Pressure	0~1 MPa	±0.1%
Liquid flow rate	1~500 m ³ /d	±0.3%
Gas flow rate	2500~50000 m ³ /d	±1%

FIGURE 2: Pictures of churn flow in the air-oil experiment ($v_{sl} = 0.15$ m/s, $v_{sg} = 6.5$ m/s).

The frictional pressure drop $(dP/dL)_f$ is expressed as

$$\left(\frac{dP}{dL}\right)_f = -\left(\frac{dP}{dL}\right)_t - \left(\frac{dP}{dL}\right)_h. \quad (2)$$

The gravity pressure drop $(dP/dL)_h$ is expressed as

$$\left(\frac{dP}{dL}\right)_h = \rho_m g \sin \theta, \quad (3)$$

$$\rho_m = \rho_l H_l + \rho_g (1 - H_l). \quad (4)$$

In our previous work [23], the uncertainty analysis of data is the basis to ensure the accuracy of experimental data; the uncertainty for total pressure drop, liquid holdup, and the frictional pressure drop has been provided; they are within ±2%, ±5%, and ±5.4%, respectively.

3.2. Flow Pattern. The pictures of churn flow with different viscosities are shown in Figure 2. Compared with the flow pattern pictures under different viscosities, it can be seen that the gas phase is distributed in the middle of the pipe. Under the same superficial gas and liquid velocity, more liquid will adhere to the wall, which makes the liquid film on the pipe wall thicker, and it will change the transition boundary between different flow patterns.

3.3. Effect of Viscosity on Frictional Pressure Drop and Liquid Holdup. Figure 3 shows the variation of liquid holdup with the oil viscosity. There is a layer of liquid film flowing along the pipe wall in the churn flow. Figure 2 shows that the thickness of the liquid film increases with the increase of the liquid (oil) viscosity. At a constant gas velocity, the energy of gas

carrying liquid is insufficient, which makes more liquid stay in the pipe, resulting in the increase of liquid holdup.

Figure 4 shows the variation in frictional pressure drop with superficial gas velocity. The figure shows that when the superficial liquid velocity is large ($v_{sl} = 0.52$ m/s), the friction pressure drop is positive, and the value of positive frictional pressure drop will increase with the increase of superficial gas velocity. However, when the superficial liquid velocity is small ($v_{sl} = 0.02$ m/s), the frictional pressure drop is negative, and the value of negative frictional pressure drop will decrease with the increase of superficial gas velocity. In a certain range of superficial liquid velocity, the frictional pressure drop changes from negative to positive with the increase of superficial gas velocity. Therefore, due to the existence of liquid film backflow, there are positive and negative frictional pressure drops in the churn flow, and the change of frictional pressure drop with increasing superficial gas velocity is related to superficial liquid velocity.

Figure 5 shows the variation in frictional pressure drop with viscosity. It can be seen from the figure that the value of frictional pressure drop increases with the increase of viscosity. If the frictional pressure drop is positive, the value of positive frictional pressure drop increases with the increase of viscosity, and when the frictional pressure drop is negative, the negative value of frictional pressure drop increases with the increase of viscosity. Therefore, it can be proved that with the increase of viscosity, the change rate of frictional pressure drop increases with the increase of the superficial gas velocity.

4. Verification of Liquid Holdup and Pressure Drop Models

4.1. Verification of Liquid Holdup Models. Numbers of liquid holdup models for gas-liquid two-phase flow have been developed. They can be divided into two categories: empirical model and mechanism model [24–29]. However, most studies neglected the viscosity influence on liquid holdup of different flow patterns.

The correlation provided by Xiao et al. [26] for the whole slug unit liquid holdup can be expressed as follows:

$$H_l = \frac{v_t H_{ls} + v_b (1 - H_{ls}) - v_{sg}}{v_t}. \quad (5)$$

In our previous work [30], we choose correlations of v_t , v_b , and H_{ls} .

In our work, the experimental liquid holdup data was used to evaluate the performance of four correlations (Beggs [24], Xiao [26], Choi [28], and Liu [30]). Figure 6 shows the comparison of the predicted values with the measured data of liquid holdup in the experiment. The errors of Beggs [24], Xiao [26], and Choi [28] models are large, and the predicted values of Beggs [24] and Xiao [26] models are smaller than the experimental values. However, most of the predicted values of the Chio [28] model are larger than the experimental values. Compared with these three models, the Liu [30] model has the smallest errors, and most of the errors are within 20%.

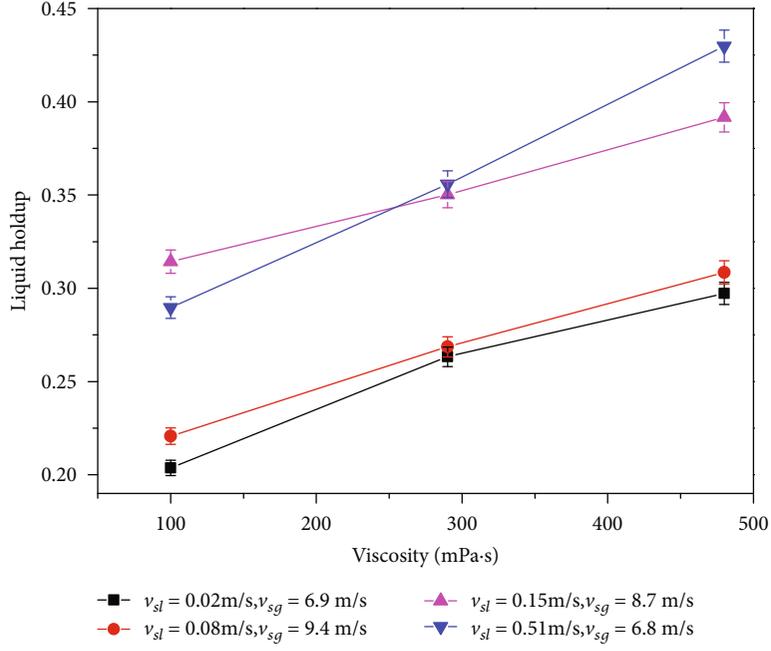


FIGURE 3: Liquid holdup vs. liquid viscosity in the air-oil experiment.

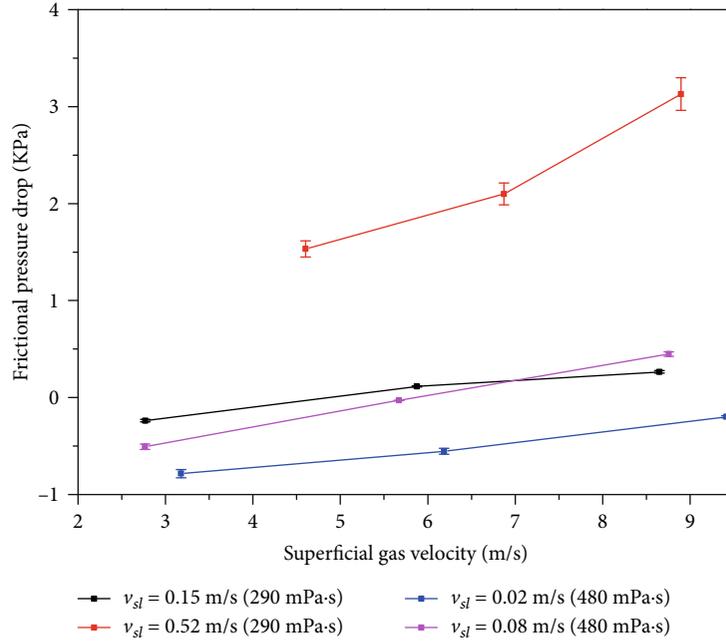


FIGURE 4: Frictional pressure drop vs. superficial gas velocity.

The average percent error and the average absolute percent error are selected for the evaluation the performance of different models.

The following is the average percent error:

$$APE = \left(\frac{1}{n} \sum_{i=1}^n \frac{X_{cal} - X_{exp}}{X_{exp}} \right) \times 100. \quad (6)$$

The following is the average absolute percent error:

$$AAPE = \left(\frac{1}{n} \sum_{i=1}^n \left| \frac{X_{cal} - X_{exp}}{X_{exp}} \right| \right) \times 100. \quad (7)$$

Among them, X_{cal} is the model predicted value and X_{exp} is the measured value in the experiment.

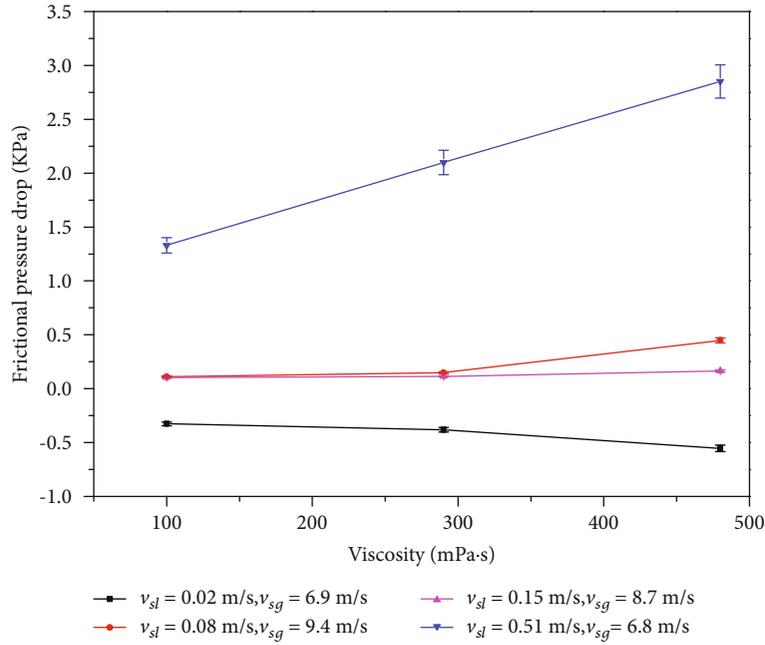


FIGURE 5: Frictional pressure drop vs. viscosity.

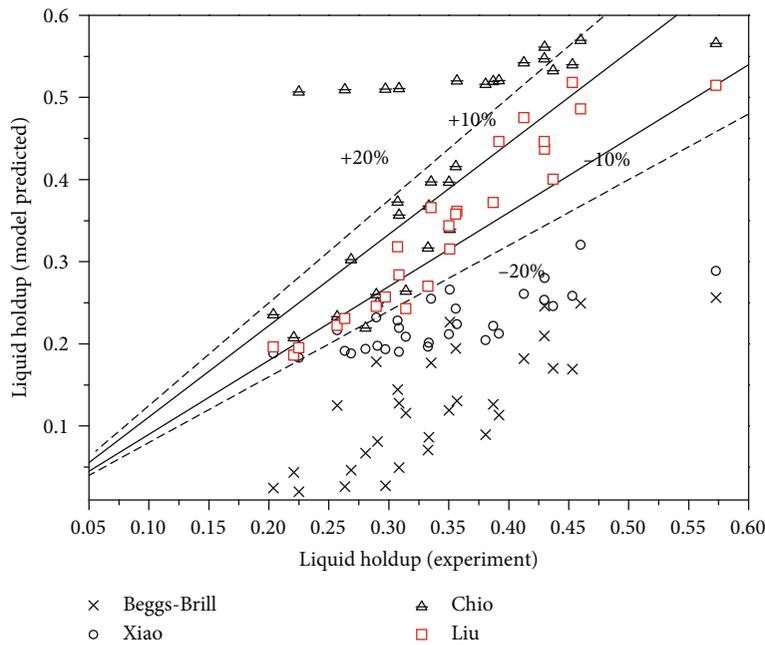


FIGURE 6: Measured data vs. predicted of four models (liquid holdup).

The liquid holdup errors of different models are shown in Table 3. It can be seen from the table that the AAPE and APE of the Beggs [24] model are large. The AAPE and APE of the Xiao [26] and Choi [28] models are smaller than those of the Beggs [24] model. The errors of the Liu model [30] are smaller than those of the other three models, and the errors of AAPE and APE are 9.6% and 5.9%, respectively.

4.2. Verification of Pressure Drop Models. From the previous analysis, we can find that there are both negative and positive

TABLE 3: Errors of liquid holdup for four different models.

Error	Beggs [24]	Xiao [26]	Choi [28]	Liu [30]
AAPE (%)	65.1	32.6	28.1	9.6
APE (%)	-65.1	-32.6	22.5	5.9

frictional pressure drop, and when the liquid velocity is small, most of the frictional pressure drop is negative. Therefore, it is necessary to consider the model which can calculate the

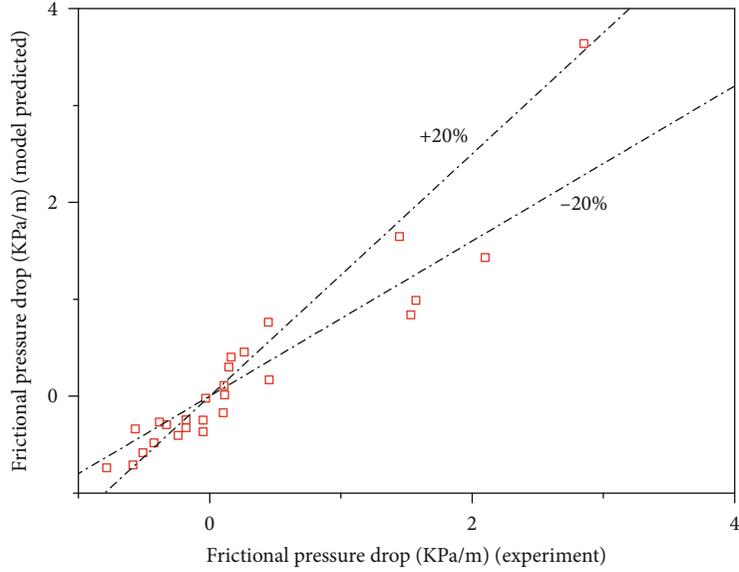


FIGURE 7: Measured data vs. predicted of Taitel [32] model (frictional pressure drop).

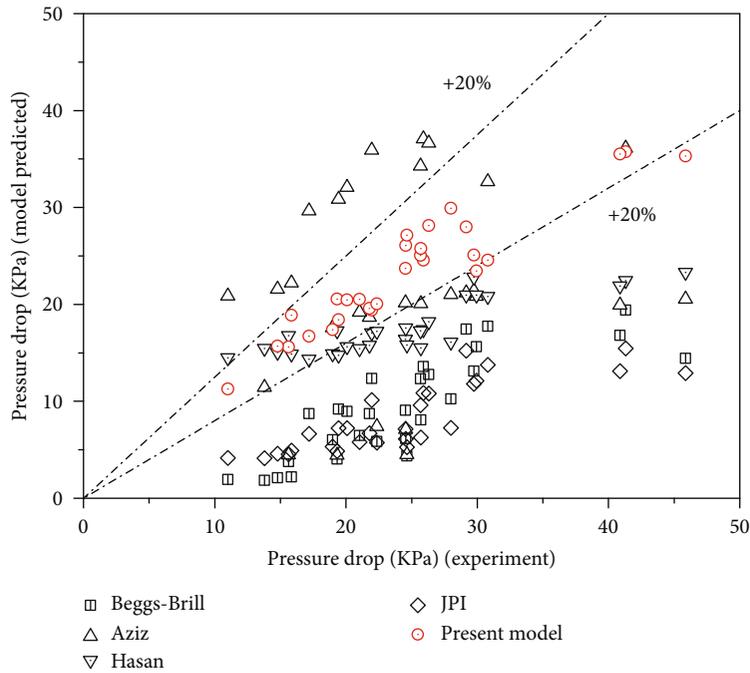


FIGURE 8: Measured data vs. predicted of different models (total pressure drop).

TABLE 4: Pressure drop errors of different models.

Error	Beggs-Brill [24]	Aziz [33]	Hasan [34]	JPI [35]	Present model
AAPE (%)	61.0	46.7	26.8	65.2	9.5
APE (%)	-61.0	2.6	-21.8	-65.2	-1.7

negative frictional pressure drop in order to calculate the frictional pressure drop of churn flow. As the intermediate flow pattern of transition from slug flow to annular flow, churn

flow has some characteristics of slug flow, in the calculation of slug flow frictional pressure drop in the mechanism model, the negative frictional pressure drop can be calculated and

Liu [31] has given the normal explanation for negative frictional pressure drop in slug flow. Therefore, in this paper, the frictional pressure drop calculation method of slug flow in the Taitel [32] model is used to calculate the frictional pressure drop of churn flow.

In slug flow, the liquid film in the liquid slug region flows upward because the liquid phase in the liquid film region flows downward. In the model of Taitel [32], the frictional pressure drop can be expressed as follows:

$$-dp = \frac{\tau_s \pi d}{A_p} L_s + \frac{\tau_F S_F}{A_p} L_F + \frac{\tau_g S_g}{A_p} L_F. \quad (8)$$

Then, the frictional pressure gradients can be expressed

$$\left(\frac{dP}{dL}\right)_f = \frac{-((\tau_s \pi d / A_p) L_s + (\tau_F S_F / A_p) L_F + (\tau_g S_g / A_p) L_F)}{L_U}, \quad (9)$$

$$L_U = L_F + L_s, \quad (10)$$

where L_U is the length of slug unit, L_F is the length of liquid film, and L_s is the length of liquid slug.

Figure 7 shows the comparison of the predicted values from the Taitel [32] model with the measured data. It can be seen from the figure that the Taitel [32] model can calculate both negative and positive frictional pressure drop, and the calculated value conforms to the variation law of frictional pressure drop. Most of errors with the Taitel [32] model are within 20%.

Combined with Equations (1), (3), (4), and (9), we can get the calculation method of pressure drop:

$$-\left(\frac{dP}{dL}\right)_t = \left[\rho_l H_l + \rho_g (1 - H_l) \right] \cdot g \sin \theta \frac{-((\tau_s \pi d / A_p) L_s + (\tau_F S_F / A_p) L_F + (\tau_g S_g / A_p) L_F)}{L_U}. \quad (11)$$

In this paper, we use the total pressure drop measured in the experiment to verify one empirical model (Beggs-Brill [24]), three mechanism models (Aziz [33], Hasan [34], and JPI [35]), and the present model. Figure 8 shows the comparison of the predicted values from different models with the experimental data of pressure drop. The pressure errors are shown in Table 4. It can be seen from the figure and the table that the errors of the Beggs-Brill [24] and JPI [35] models are large, and the predicted values of Beggs-Brill [24] and JPI [35] are smaller than the experimental values. The errors of the Aziz [33] and Hasan [34] models are relatively small. Compared with these four models, the present model has the smallest errors, and most of the errors are within 20%. The values of the APE and AAPE for the present model are 9.5% and -1.7%, respectively, and both of them are the smallest.

5. Conclusion

In this paper, liquid holdup and pressure drop data of churn flow were obtained from the oil-air two-phase flow experiment. The experimental results reveal that there are positive and negative frictional pressure drops in the churn flow, and the change of frictional pressure drop with increasing superficial gas velocity is related to superficial liquid velocity. With the increase of viscosity, the change rate of frictional pressure drop increases with the increase in the superficial gas velocity. Combining our previous work [30] and the Taitel [32] model, a combined pressure drop model of churn flow for viscous oil-air two-phase flow was proposed. By comparing the predicted values of different models with the measured pressure drop data, the proposed model has the highest prediction accuracy.

Data Availability

The liquid holdup and pressure drop data used during the study are available from the corresponding author by request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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