Research Article

A New Approach for Determining the Control Volumes of Production Wells considering Irregular Well Distribution and Heterogeneous Reservoir Properties

Cheng Lu ^{1,2,3} **Ling Chen** ⁴ **Xiaodong Wang** ¹ **Wanjin Luo** ¹ **Yue Peng** ¹
Yudong Cui ¹ **Lin Wang** ⁵ and **Bailu Teng** ¹

¹School of Energy Resources, China University of Geosciences (Beijing), Beijing 100083, China

²Center of Oil & Natural Gas Resource Exploration, China Geological Survey, Beijing 100083, China

³Guangzhou Marine Geological Survey, China Geological Survey, Guangzhou 510075, China

⁴China National Offshore Oil Corporation Research Institute, Beijing 100028, China

⁵Research Institute of Petroleum Exploration & Development of Qinghai Oil Field, Qinghai 736202, China

Correspondence should be addressed to Xiaodong Wang; wxd_cugb@cugb.edu.cn and Bailu Teng; bailu@cugb.edu.cn

Received 20 November 2020; Revised 5 January 2021; Accepted 27 April 2021; Published 20 May 2021

Academic Editor: Stefano Lo Russo

Copyright © 2021 Cheng Lu et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

The oil and gas fields are commonly developed with a group of production wells. Therefore, it can be essential for the industries to predict the performance of the production wells in order to optimize the development strategies. In practice, it frequently happens that we only hope to study the performance of a single production well. In such cases, it can be time consuming to run the reservoir simulation with the entire reservoir model to study the well performance. Hence, it can be preferred to determine the control volume (or drainage volume) of the target well from the entire reservoir and run the simulation with the small control volume to reduce the simulation cost. However, an irregular layout of the production wells and the heterogeneity of reservoir properties, which can be commonly observed in real field cases, can induce a stringent barrier for one to determine the control volumes. At present, we are still lacking a method to determine the control volumes of the production wells considering well distribution and reservoir heterogeneities. In order to overcome such a barrier, the authors proposed a new approach to divide the entire reservoir into small control volumes on the basis of the fast marching method (FMM). This approach is validated by comparing the simulation outputs of the target well calculated only with the determined control volume to those calculated with the entire reservoir model. The calculated results show that using the control volume that is determined with the proposed method to calculate the well performance can yield results that agree well with the results that are calculated with the entire reservoir model. This indicates that this proposed method is reliable to determine the control volume of the production wells. In addition, the calculated results in this work show that changing fracture length exerts a slight influence on the control volumes if the length of all fractures is increased, whereas, if only one of the fracture lengths is increased, the control volume of the corresponding well will be significantly increased. The number of the production wells and the distribution of the production well can noticeably influence the control volumes of the production wells. The findings of this study can help for optimizing the well spacing, estimating the ultimate recovery, and reducing the computational cost.

1. Introduction

The reservoirs produce oil and gas through production wells. Knowledge of the control volumes of the production wells is crucial for the industries to estimate the ultimate recovery. Denson et al. [1] developed a new method to evaluate the

effective well control volume and effective porosity. Their method requires the results of a pressure build-up test and the approximate geometry of the target well. Kang et al. [2] proposed a rigorous approach to calculate control volumes of the production wells in tight gas reservoirs. This approach relies on an asymptotic solution of the diffusivity equation

FIGURE 1: Reservoir models that have multiple production wells: (a) regular well distribution in a homogeneous reservoir and (b) irregular well distribution in a heterogeneous reservoir.

and emulates the propagation of drainage boundaries. Zeynal and Kashikar [3] developed a new methodology to quantify the fracture intensity through a deterministic discrete fracture network model on the basis of the microseismic data, pumping parameters, and rock properties. Gherbati [4] utilized a material-balance approach for determining control volumes of multifracture unconventional oil wells. This estimated control volumes can be used to optimize the well spacing and completions. However, these aforementioned methods can normally provide a quantitative estimation of the magnitude of the control volumes rather than provide a straightforward insight into the geometries of the control volumes. In practice, knowing the geometries of the control volume can help one to reduce the simulation cost.

At present, numerical reservoir simulation is one of the most widely used technique to study the well performance [5, 6]. However, a numerical reservoir model that characterizes the entire reservoir normally contains a large number of grids, and running such a numerical model can be very time consuming [7, 8]. This is especially true when one only aims to study the performance of a single well. In order to reduce the simulation cost, one can split the control volume of the target well from the entire reservoir model and conduct the simulation only with the control volume. Figure 1(a) shows a top view of the reservoir model. In this model, the production wells are regularly distributed in the reservoir and the properties of the reservoir are homogeneous; thus, one can readily figure out the control volume of each production well according to the principle of symmetry. The reservoir volume within the red box in Figure 1(a) represents the control volume of a production well. In practice, however, the distribution of the production wells can be irregular and the properties of the reservoir can be heterogeneous (see Figure 1(b)). For the scenario shown in Figures 1(b), there still does not exist a reliable method for accurately determining the control volumes.

In this work, the authors proposed a novel approach to determine the control volumes of production wells by the use of FMM. This proposed approach is applicable even if the production well is irregularly distributed and the reservoir properties are heterogeneous. FMM is developed by Sethian [9] and has been widely used to track moving boundaries. By the use of FMM, Sethian and Popovici [10] pre-

FIGURE 2: Illustration of the core idea of the proposed method.

FIGURE 3: Reservoir model #1 that is used for validation purpose.

sented a fast algorithm for solving the Eikonal equation in three dimensions. Xie et al. [11] described the growth of depth of investigation in spatially heterogeneous and fractured unconventional reservoirs with FMM. Ding et al. [12] proposed an improved FMM method for calculating the three-dimension traveltime by considering the 26 neighbors of the calculating point. Teng [13] combined the implicit finite difference method with the FMM method to improve the calculating efficiency of numerical simulation. More application of FMM can be found in Sharifi et al. [14], Zhang et al. [15], Jenkins [16], Iino et al. [17], and Ojha et al. [18], but FMM has hitherto not been used to determine the control volumes.

According to the aforementioned arguments, one can form the following conclusions: firstly, we are still lacking a method to determine the control volumes of the production

FIGURE 4: Maps of TOI that are separately calculated with the four fractured wells.

wells due to the irregular distribution of production wells and the heterogeneity of the reservoir properties; and secondly, FMM has been used to track moving boundaries but has not been utilized to determine the control volumes. In this work, the authors proposed a new approach to determine the control volumes on the basis of FMM. This new approach is applicable even if the production wells are irregularly distributed and the reservoir properties are heterogeneous.

The organization of this work can be summarized as follows: first, the authors demonstrate a new method for calculating the control volumes of production well with the fast marching method in Section 2. This proposed method is subsequently validated in Section 3. In Section 4, the effects of fracture length, the number of production wells, and well distribution on the control volumes are examined. Finally, Section 5 gives the conclusions that are formed on the basis of the calculated results in this work.

2. Methodology

In this work, we proposed a new approach to determine the control volumes of the production wells. The core idea of the proposed method is to think that a reservoir volume belongs to the control volume of the well that the reservoir volume earliest contributes to its production. In order to demonstrate the core idea of the proposed approach, we will take a two-well case as an example. Figure 2 presents a reservoir model that contains two fractured wells (the red rectan-

FIGURE 5: Geometries of the control volumes of the four fractured wells of model #1.

gles). For the reservoir volume shown in the blue box, we can calculate the time of investigation (TOI) of the reservoir volumes with well 1 and well 2 separately, and we can obtain TOI that is calculated with well 1 (t_{in1}) and TOI that is calculated with well 2 (t_{in2}). It should be noted that t_{in} indicates the smallest time that the reservoir volume starts contributing to the production. If $t_{in1} < t_{in2}$, it can be inferred that the reservoir volume contributes to the production of well 1 earlier than that of well 2, and we can think the reservoir volume belongs to the control volume of well 1. Otherwise, we can think the reservoir volumes belong to the control volumes of well 2.

FIGURE 6: Comparison between the well performance that is predicted with the control volume to that with the entire reservoir model #1: (a) results of well #1, (b) results of well #2, (c) results of well #3, and (d) results of well #4.

FIGURE 7: Reservoir model #2 that is used for the validation purpose: (a) distribution of the matrix permeability and (b) distribution of the fractured wells.

FIGURE 8: Maps of TOI that are separately calculated with the four fractured wells: (a) TOI of well #1, (b) TOI of well #2, (c) TOI of well #3, and (d) TOI of well #4.

Comparing the t_{in} that are calculated with well 1 and well 2 of all the positions through the reservoir, we can obtain the control volumes of well 1 and well 2. For the scenarios that have more than two production wells, we can determine the control volumes with a similar method. In this work, although only fractured wells are taken as examples for introducing the proposed approach and for conducting the investigation, this proposed approach is also applicable for determining the control volumes of vertical wells, horizontal wells, and multistage fractured wells. A detailed introduction of using FMM to calculate the t_{in} is provided in Appendix A. In this work, the calculations are all conducted on two-dimension scenarios, and the Eikonal equation which describes the propagation of control volumes can be expressed as

$$\left. \begin{aligned} &\max \left(\frac{\tau_{i,j} - \tau_{i-1,j}}{S_{i-}}, \frac{\tau_{i,j} - \tau_{i+1,j}}{S_{i+}}, 0 \right)^2 \\ &+ \max \left(\frac{\tau_{i,j} - \tau_{i,j-1}}{S_{j-}}, \frac{\tau_{i,j} - \tau_{i,j+1}}{S_{j+}}, 0 \right)^2 \end{aligned} \right\} = 1. \quad (1)$$

In Equation (1), τ is diffusive time of flight which is a function of t_{in} , and S is slowness. The parameters τ and S are all defined in Appendix A. The FMM can be used to

FIGURE 9: Geometries of the control volumes of the four fractured wells of model #2.

consecutively solve Equation (1); therefore, the t_{in} through the entire reservoir can be calculated.

3. Validation

It is worth noting that, for each production well, the well performance that is predicted with the control volume should be sufficiently close to the well performance that is predicted with the entire reservoir model. Therefore, we can validate

FIGURE 10: Comparison between the well performance that is predicted with the control volume to that with the entire reservoir model #2: (a) results of well #1, (b) results of well #2, (c) results of well #3, and (d) results of well #4.

the proposed approach by comparing the simulation output of each well from the control volume to that from the entire reservoir model. In this work, the well performance is predicted with the implicit finite difference method (IMFD). A detailed introduction of the IMFD method can be found in Ertekin et al. [19]. The validation is conducted on two reservoir models, including a homogeneous reservoir model (i.e., model #1) and heterogeneous reservoir model (i.e., model #2). The reservoir model #1 that is used for validating the proposed method is shown in Figure 3. As one can see in Figure 3, this model contains 4 fractured wells and these wells are irregularly distributed. The properties of the fluid and the rock matrix are as follows: the reservoir dimension

is $1000 \text{ m} \times 600 \text{ m} \times 50 \text{ m}$ which is discretized into $100 \times 100 \times 1$ grids, the matrix permeability is 0.01 mD , the porosity of the matrix is 0.2 , the total compressibility of the matrix is $1.2 \times 10^{-3} \text{ MPa}$, the initial reservoir pressure is 30 MPa , the viscosity of the oil is $1 \text{ mPa}\cdot\text{s}$, the radius of the wellbore is $5 \times 10^{-2} \text{ m}$, the permeability of the fracture is $1 \times 10^6 \text{ mD}$, the fracture width is $1 \times 10^{-3} \text{ m}$, the fracture length is 90 m , the bottomhole pressure is a constant of 5 MPa , and the studied time is 3000 day that is divided into 100 timesteps.

Figure 4 presents the maps of TOI that are separately calculated with the four fractured wells. As one can see in this figure, a smaller value of TOI can be found near the wellbore.

FIGURE 11: Geometries of the control volumes of the production wells with different fracture lengths.

The value of TOI is increased as the distance from the well-bore is increased. As introduced in Methodology, comparing the value of TOIs that are shown in Figure 4, one can obtain the control volumes of the four fractured wells (see Figure 5). It can be found in Figure 5 that an irregular distribution of the fractured well leads to irregular shapes of the control volumes. With the aid of these control volumes, we can conduct reservoir simulation of each fractured well. Figure 6 compares the well performance that is predicted with the control volume to that with the entire reservoir model. It should be noted that, if the well performance is predicted with the control volume, the reservoir volume that out of the control volume will be neglected. Since the number of the grids in the control volumes is much smaller than that of the entire reservoir, using the control volume to predict the performance of a single well can be much more computationally efficient than using the entire reservoir. In Figure 6, it can be observed that the well performance that is predicted with the control volume undergoes excellent agreement to the well performance that is predicted with the entire reservoir model.

Furthermore, we validated the proposed method with a heterogeneous reservoir model. The only differences between the properties of model #2 and that of model #1 are the distribution of matrix permeability and the distribution of the fractured wells. Figure 7(a) shows the top view of the reservoir permeability of model #2, and Figure 7(b) shows the distribution of the fractured wells. On the basis of the reservoir model shown in Figure 7, we calculated the TOI with FMM.

FIGURE 12: Comparison of the control volumes of the production wells with different fracture lengths.

Figure 8 shows the maps of TOI that are calculated with the four wells. As one can see in Figure 8, the heterogeneity of the reservoir permeability renders the contours of TOI not as smooth as those shown in Figure 4. Figure 9 illustrates the control volumes of the four fractured well of model #2. Based on the control volumes shown in Figure 9, we can predict the performance of each fractured well. Figure 10 compares the well performances that are predicted with the control volumes to those with the entire reservoir model #2. As shown in Figure 10, the well performances that are predicted with the control volumes agree well with the well

FIGURE 13: Geometries of the control volumes of the production wells with different fracture lengths of well 2.

performances that are predicted with the entire reservoir model #2. The calculated results shown in Figures 6 and 10 indicate that the proposed approach is reliable to determine the control volumes of the production wells although the wells are irregularly distributed and the reservoir properties are heterogeneous.

4. Results and Discussion

With the aid of the proposed approach, we investigated the effects of fracture length, the number of production wells, well distribution, and fracture conductivity on the control volumes. The reservoir properties and the well distribution in this section are the same to those of reservoir model #2 that is used for validation.

4.1. Fracture Length. The length of the four fractures is varied from 90 m to 330 m in order to explore the effect of fracture length on the control volumes. Figure 11 shows the control volumes of the production wells that are calculated with the proposed approach. As one can see in Figure 11, the control volumes of the production wells are all irregularly shaped. Figure 12 compares the control volumes of the production wells with different fracture lengths. It can be found that the control volumes show slight changes as the fracture length is varied. The calculated results shown in Figures 11 and 12 indicate that the fracture length will

FIGURE 14: Control volumes of the production wells with different fracture lengths of well 2.

not significantly influence the control volumes of the production wells. However, it should be noted that in Figures 11 and 12, the fracture length are all changed for each study. Furthermore, we investigate the effect of fracture length on the control volumes by only varying the fracture length of well 2. Figures 13 and 14 present the geometries and magnitude, respectively, of the control volumes of the production wells with difference fracture lengths of well 2 (i.e., 90 m, 170 m, 250 m, and 330 m). It can be found in Figures 11 and 12 that the control volume of well 2 is

FIGURE 15: Distribution of the production wells: (a) 4 production wells, (b) 5 production wells, (c) 6 production wells, and (d) 7 production wells.

increased as the fracture length of well 2 is increased. The calculated results in Figures 11 through 14 indicate that the fracture length will not significantly influence the control volumes of the production well if the length of the fractures is all changed, whereas, for each single well, increasing its fracture length will increase its control volume if the length of the other fractures remains unchanged.

4.2. Well Distribution and Number of Wells. Figure 15 shows the distribution of the production wells with different numbers of wells. The positions of the production wells are randomly generated. The number of the production wells is increased from 4 to 7 in Figures 15(a)–15(d). With the aid of the proposed method, we calculated the control volumes of these fractured wells. The geometries of the calculated control volumes are shown in Figure 16. As one can see in Figure 16, the control volumes become smaller as the number of the production well is increased. In addition, one can find that the distribution of the production wells can significantly influence the geometries of the control volume.

4.3. Fracture Conductivity. In order to explore the effect of fracture conductivity on the control volumes, the fracture

permeability is varied from 1×10^3 md to 1×10^6 md, such that the fracture conductivity can be varied from 1 md-m to 1×10^3 md-m. Figure 17 presents the control volumes of the production wells with different fracture conductivities. It can be observed in this figure that the control volumes of the four production wells undergo negligible change with different fracture conductivities.

5. Summary and Conclusions

In this work, we proposed a new method to determine the control volumes of the production wells. This proposed method is developed on the basis of the FMM method. With the aid of this developed method, one can determine the control volumes of production wells considering the irregular well distribution and reservoir heterogeneity. The geometries of the control volumes can also be described. Therefore, one can use this method to optimize the well spacing and estimate the ultimate recovery. However, this proposed method bears the following inherent deficiencies: firstly, the method requires a good knowledge of the reservoir properties; secondly, the wells should be put into production at a similar time; and finally, multiphase flow cannot be considered.

FIGURE 16: Geometries of the control volumes: (a) 4 production wells, (b) 5 production wells, (c) 6 production wells, and (d) 7 production wells.

With the aid of the proposed method, we investigated the effects of the fracture length, number of production wells, and well distribution on the control volumes. The calculated results help us draw the following conclusions:

- (1) This proposed method is validated by comparing the simulation outputs with the control volume to those with the entire reservoir model. The calculated results show that the simulation outputs with the control volume illustrate excellent agreement to those with the entire reservoir model, implying that the proposed method is reliable to determine the control volumes
- (2) If the lengths of the fractures are all increased, the change of the fracture length exerts a slight influence on the control volumes, whereas, if only one of the fracture length is increased, the control volume of the corresponding well will be increased
- (3) The number of the production wells and the distribution of the production well can significantly influence the control volumes of the production wells, whereas the fracture conductivity exerts negligible influence on the control volumes

Appendix

Fast Marching Method

In practice, the moving boundary can be normally characterized with the Eikonal equations, and the FMM is an extremely efficient method to solve the Eikonal equations. Equation (A.1) is the Eikonal equation that describes the growth of volume investigation (VOI):

$$\sqrt{\eta}|\nabla\tau| = 1, \quad (\text{A.1})$$

where τ is diffusive time of flight (DTOF) and η is diffusivity whose definition can be expressed as

$$\eta = \frac{\beta k}{\mu \phi c_i}. \quad (\text{A.2})$$

In Equation (A.2), β is the unit conversion factor, k is the permeability, μ is the viscosity, ϕ is the porosity, and c_i is the

FIGURE 17: Geometries of the control volumes with different fracture conductivities: (a) 1 md-m, (b) 10md-m, (c) 100md-m, and (d) 1000md-m.

total compressibility. In addition, the DTOF τ has the following relationship with the physical time t_{in} :

$$t_{in} = \frac{\tau^2}{c}, \quad (\text{A.3})$$

where t_{in} is the time of investigation (TOI), c is a constant which equals to 2, 4, and 6 for one-dimension, two-dimension, and three-dimension flows, respectively. Since the dimension of a reservoir along the horizontal direction is normally much larger than that along the vertical direction, the fluid flow mainly occurs in two-dimension; thus, $c=4$ is used in this work. The volume of investigation indicates the reservoir volume that contributes to the production, and the time of investigation indicates the shortest physical time that the reservoir volume at the corresponding position starts contributing to the production. Figure 18 schematically presents the change of the boundary of VOI as a function of time, and in this figure $t_{in3} > t_{in2} > t_{in1}$. As one can see in Figure 18, the VOI expands from the wellbore to a further position as time proceeds.

In order to characterize the expansion of the VOI, we can solve Equation (A.1) with an upwind scheme. Discretizing the reservoir into small grids and applying the upwind

FIGURE 18: Boundary of VOI as a function of time ($t_{in3} > t_{in2} > t_{in1}$).

scheme to Equation (A.1), the Eikonal equation in three-dimension can be written as

$$\left. \begin{aligned} & \max \left(\frac{\tau_{i,j,k} - \tau_{i-1,j,k}}{S_{i-}}, \frac{\tau_{i,j,k} - \tau_{i+1,j,k}}{S_{i+}}, 0 \right)^2 \\ & + \max \left(\frac{\tau_{i,j,k} - \tau_{i,j-1,k}}{S_{j-}}, \frac{\tau_{i,j,k} - \tau_{i,j+1,k}}{S_{j+}}, 0 \right)^2 \\ & + \max \left(\frac{\tau_{i,j,k} - \tau_{i,j,k-1}}{S_{k-}}, \frac{\tau_{i,j,k} - \tau_{i,j,k+1}}{S_{k+}}, 0 \right)^2 \end{aligned} \right\} = 1, \quad (\text{A.4})$$

FIGURE 19: Calculating process of FMM.

where (i, j, k) indicates the grid number and S is slowness which is defined as

$$\begin{aligned}
 S_{i\pm} &= \frac{\Delta x_{i,j,k}}{2\sqrt{\eta_{x(i,j,k)}}} + \frac{\Delta x_{i\pm 1,j,k}}{2\sqrt{\eta_{x(i\pm 1,j,k)}}}, \\
 S_{j\pm} &= \frac{\Delta x_{i,j,k}}{2\sqrt{\eta_{y(i,j,k)}}} + \frac{\Delta x_{i,j\pm 1,k}}{2\sqrt{\eta_{y(i,j\pm 1,k)}}}, \\
 S_{k\pm} &= \frac{\Delta x_{i,j,k}}{2\sqrt{\eta_{z(i,j,k)}}} + \frac{\Delta x_{i,j\pm 1,k}}{2\sqrt{\eta_{z(i,j\pm 1,k)}}}.
 \end{aligned} \quad (A.5)$$

Equation (A.4) is a quadratic equation, and the only unknown is $\tau_{i,j,k}$. If any values of $\tau_{i\pm 1,j,k}$ or $\tau_{i,j\pm 1,k}$ is unknown, the term containing the unknown should be dropped. FMM can be used to track moving boundaries by consecutively solving the Eikonal equation with Equation (A.4). The detailed introduction of the FMM is given as follows:

- (1) Assigning a grid with an initial value of DTOF. This grid usually represents the wellbore position, and the initial value of DTOF can be set as 0. Labeling the initial grid as frozen (see the red grid in Figure 19(a))
- (2) Calculating the DTOF of the neighbor grids (see the green grids in Figure 19(b)) of the frozen grid with Equation (A.4)
- (3) Finding out the smallest DTOF of the neighbor grids and labeling this grid as frozen (see Figure 19(c))

- (4) Calculating the DTOF of the neighbor grids of the new frozen grid (see Figure 19(d))
- (5) Repeating steps 2 through 4 until all the grids are labeled as frozen

After obtaining the DTOFs of all the grids, we can convert the DTOFs into TOI with Equation (A.3), thus, we can know the time that the reservoir at different positions starts contributing to the production.

Nomenclature

- k : Permeability, md
 μ : Viscosity, mPa·s
 ϕ : Porosity
 c_i : Total compressibility, MPa⁻¹
 t_{in} : Time of investigation, day
 β : Unit conversion factor, 0.0853
 τ : Diffusive time of flight, day^{0.5}
 S : Slowness, day^{0.5}
 c : 2, 4, and 6 for one-dimension, two-dimension, and three-dimension flows
 η : Diffusivity, m²/day.

Data Availability

Some or all data, models, or code generated or used during the study are available from the corresponding author by reasonable request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

The authors would like to acknowledge the financial support provided by “the Fundamental Research Funds for the Central Universities” (No. 99959/53200659042) and National Natural Science Foundation of China (grant number 51674227).

References

- [1] A. H. Denson, J. T. Smith, and W. M. Cobb, “Determining well drainage pore volume and porosity from pressure buildup tests,” *Society of Petroleum Engineers Journal*, vol. 16, no. 4, pp. 209–216, 1976.
- [2] S. Kang, A. Datta-Gupta, and W. J. Lee, “Impact of natural fractures in drainage volume calculations and optimal well placement in tight gas reservoirs,” in *Paper SPE-144338-MS presented at North American Unconventional Gas Conference and Exhibition*, The Woodlands, Texas, USA, June 2011.
- [3] A. R. Zeynal and S. Kashikar, “Understanding and quantifying variable drainage volume for unconventional wells,” in *Paper URTEC-2459223-MS presented at SPE/AAPG/SEG Unconventional Resources Technology Conference*, San Antonio, Texas, USA, August 2016.
- [4] S. A. Gherbati, “Material-balance approach for determining drainage volume of multifracture unconventional oil wells,” in *Proceedings of the 6th Unconventional Resources Technology Conference*, Houston, Texas, USA, July 2018.
- [5] P. Jia, L. Cheng, S. Huang, and H. Liu, “Pressure-transient analysis of a finite-conductivity inclined fracture connected to a slanted wellbore,” *SPE Journal*, vol. 21, no. 2, pp. 522–537, 2016.
- [6] S. Liu and P. P. Valkó, “Optimization of spacing and penetration ratio for infinite-conductivity fractures in unconventional reservoirs: a section-based approach,” *SPE Journal*, vol. 22, no. 6, pp. 1877–1892, 2017.
- [7] L. Deng and Y. Pan, “Machine-learning-assisted closed-loop reservoir management using echo state network for mature fields under waterflood,” *SPE Reservoir Evaluation & Engineering*, vol. 23, no. 4, pp. 1298–1313, 2020.
- [8] J. Wang, A. Jia, and Y. Wei, “An alternative BEM modeling of transient pressure response of fractured reservoir by use of a semi-analytical approach,” in *Paper URTEC-198305-MS presented at SPE/AAPG/SEG Asia Pacific Unconventional Resources Technology Conference*, Brisbane, Australia, November 2019.
- [9] J. A. Sethian, *Level Set Methods and Fast Marching Methods: Evolving Interfaces in Computational Geometry, Fluid Mechanics, Computer Vision, and Materials Science*, Cambridge University Press, New York City, 1999.
- [10] J. A. Sethian and A. M. Popovici, “3-D traveltimes computation using the fast marching method,” *Geophysics*, vol. 64, no. 2, pp. 516–523, 1999.
- [11] J. Xie, C. Yang, N. Gupta, M. J. King, and A. Datta-Gupta, “Depth of investigation and depletion in unconventional reservoirs with fast-marching methods,” *SPE Journal*, vol. 20, no. 4, pp. 831–841, 2015.
- [12] P. Ding, Y. Guoquan, K. Zhang, and Z. Li, “3D traveltimes calculation and raytracing based on multistencils fast marching method,” in *Paper SEG-2016-13849097 presented at International Exposition and Annual Meeting*, Dallas, Texas, October 2016.
- [13] B. Teng, “Simulation model size reduction using volume of investigation by fast marching method,” *Journal of Petroleum Science and Engineering*, vol. 191, p. 107183, 2020.
- [14] M. Sharifi, M. Kelkar, A. Bahar, and T. Slettebo, “Dynamic ranking of multiple realizations by use of the fast-marching method,” *SPE Journal*, vol. 19, no. 6, pp. 1069–1082, 2014.
- [15] Y. Zhang, N. Bansal, Y. Fujita, A. Datta-Gupta, M. J. King, and S. Sankaran, “From streamlines to fast marching: rapid simulation and performance assessment of shale-gas reservoirs by use of diffusive time of flight as a spatial coordinate,” *SPE Journal*, vol. 21, no. 5, pp. 1883–1898, 2016.
- [16] C. J. Jenkins, “Fast-marching methods to model the acoustic responses of biological seafloors,” *Journal of the Acoustical Society of America*, vol. 142, no. 4, pp. 2666–2666, 2017.
- [17] A. Iino, T. Onishi, and A. Datta-Gupta, “Optimizing CO₂- and field-gas-injection EOR in unconventional reservoirs using the fast-marching method,” *SPE Reservoir Evaluation & Engineering*, vol. 23, no. 1, pp. 261–281, 2020.
- [18] S. P. Ojha, S. Yoon, and S. Misra, “Pressure-transient responses of naturally fractured reservoirs modeled using the multistencils fast-marching method,” *SPE Journal*, vol. 23, no. 1, pp. 112–131, 2020.
- [19] T. Ertekin, J. H. Abou-Kassem, and G. R. King, *Basic Applied Reservoir Simulation*, SPE Textbook Series, Society of Petroleum Engineers, 2001.