Research Article

A Novel Radial-Composite Model of Pressure Transient Analysis for Multistage Fracturing Horizontal Wells with Stimulated Reservoir Volume

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In the process of stimulated reservoir volume of tight reservoir, horizontal well can form three zones, the inner zone is multistage fracturing zone, the middle zone is skin damage zone, and the outer zone is undamaged zone. In this paper, a transient well test analysis model of multistage fracturing horizontal well in three area composite reservoir is proposed. Based on Laplace transformation, point source integration, and superposition principle, the infinite conductivity multifracture model of three area composite reservoir is obtained. The linear equations of finite conductivity multifracture in Laplace space are established by using the equal conditions of flow and pressure at the fracture wall. Gauss-Newton iteration method and Stehfest number are used to obtain the solution of wellbore pressure. The accuracy of the results is verified by numerical simulation. Then, the flow characteristics of multistage fracturing horizontal wells in three area composite reservoirs are analyzed by type curves. The flow is divided into ten stages, which are the bilinear flow, the linear flow, the first radial flow stage, the inner zone linear flow, the inner zone radial flow, the middle zone linear flow, the middle zone radial flow, the outer zone linear flow, the outer zone radial flow, and the boundary dominated flow. The pressure derivative curves show different characteristics in different flow stages. The influences of fracture conductivity, fracture spacing, radius ratio of the middle zone to inner zone, radius ratio of the outer zone to the middle zone, permeability ratio of inner zone to the middle zone, permeability ratio of inner zone to outer zone, storage capacity ratio of inner zone to the middle zone, and storage capacity ratio of inner zone to outer zone on type curves are analyzed. Finally, the application and reliability of the proposed model are verified by a case example.

1. Introduction

China is rich in unconventional oil and gas resources with great development potential. The efficient development of unconventional oil and gas resources is of great significance to meeting China's energy needs [1]. With the continuous advancement of hydraulic fracturing technologies, horizontal wells are generally subjected to multistage perforating and fracturing to increase the drainage area of reservoirs, improve the development effect of reservoirs and the connectedness of fluids between reservoirs and wellbore, and then increase the production of single wells [2]. The analysis of pressure dynamic characteristics is an important method for evaluating reservoirs and obtain-

ing physical property parameters of reservoirs and is of great significance to providing guidance to the development of oil and gas fields.

Many scholars have made remarkable achievements in the simulation of pressure and the dynamic analysis of productivity of vertical fractured wells. In 1973, Gringarten et al. introduced the point source function to seepage mechanics and solved the infinite formation seepage equation by means of the point source function and Green function. In addition, they analyzed infinitely homogeneous reservoirs, double porosity media, fractures with infinite conductivity, and dynamic characteristics of well pressure [3, 4]. In 1978, Cinco-Ley et al. firstly proposed the concept of "bilinear flow." According to the research

results of Cinco-Ley et al., fractures in fractured wells have finite conductivity and the seepage in fractured wells can be divided into four stages. Cinco-Ley et al. solved the pressure solution for each flow stage [5, 6]. Later, many scholars did a lot of improvement work based on Cinco's model and applied their results in the well test analysis of multistage fractured horizontal wells [7–9].

Due to the uneven development of lithology and volume fracturing, low permeability reservoirs have planar heterogeneity and tend to show the characteristics of composite reservoirs. Ozkan et al. [10] considered the “volume fracturing” characteristics of hydraulic fractures in tight reservoirs and represented the region between fractures as a double-porosity medium. In addition, they built a new unsteady-state productivity analysis model for low permeability homogeneous medium in other parts. Brohi et al. [11] established a two-region double-porosity medium composite model, considering the influence of hydraulic fractures on regional permeability changes. Ozkan et al. [10] discussed the difference in production performance of fractured horizontal wells in conventional and unconventional reservoirs using a trilinear percolation model. Stalgorova and Mattar [12] combined with Brown et al. [13] to extend the model to a five-linear seepage model. Wang et al. [14] considered the difference in physical properties of reservoirs between stimulated zones and nonstimulated zones caused by volume fracturing in tight reservoirs and quantitatively investigated the fluid channeling between different zones. Tao et al. [15] presented a new productivity prediction hybrid based on grey relation analysis. Ahmadi et al. [16] analyzed the multifractured well production data using the dynamic drainage area concept.

The above authors all considered planar heterogeneity caused by volume fracturing. The interpretation and processing results of a large number of site well test data indicate that many oil wells often show changes in physical properties of reservoirs around their wellbore or at a certain distance away from the wellbore and are characterized by three-region or multiregion composite reservoirs. A horizontal well can be divided into three regions in the process of volume fracturing stimulation: the inner region is a multistage fracturing region, the middle region is a skin damage region, and the outer region is a nonstimulated region. Based on this, the purpose of the paper is to establish a new well test analysis model for multistage fractured horizontal wells in a three-region composite reservoir. In this paper, a point source seepage model for a three-region composite reservoir has been firstly established; moreover, an infinite conductivity multifracture solution to the three-region composite reservoir has been obtained by means of superposition principle and point source integration. Then, the linear equations of fracture flow rate and horizontal well pressure in Laplace space have been obtained by coupling the Fredholm integral equation, and a real space pressure solution has been obtained using the Stehfest numerical inversion algorithm. Finally, the typical curves and flow characteristics have been analyzed using the obtained solution, thereby, estab-

lishing a theoretical foundation for the well test analysis of multistage fractured horizontal wells in the three-region composite reservoir.

2. Establishment of New Models for Multistage Fractured Horizontal Wells in a Multiregion Composite Reservoir

2.1. Physical Model. Figures 1(a) and 1(b) are the schematic diagram of the model for the multistage fractured horizontal well in the three-region composite reservoir. After multistage fracturing of the horizontal well in the tight reservoir, multistage fractures are formed in the well (Figure 1(b)). After volume fracturing of the multistage fractured horizontal well, the reservoir is divided into three regions. The first region is the fractured region formed first near the wellbore after the forming of the stimulated region near the bottom hole through fracturing; the second region is a skin damage region; the third region is an undamaged region. For convenience, the three regions are called the inner region, middle region, and outer region. Fluids flow first from the outer region to the middle region, then from the middle region to the inner region, from the inner region to the hydraulic fractures, and finally from the hydraulic fractures to the wellbore (Figure 2). The physical model is simplified below:

- (1) The three-region reservoir is divided into inner region, middle region, and outer region, and their permeability is k_1 , k_2 , and k_3 , respectively
- (2) All rock properties such as reservoir permeability, reservoir porosity, hydraulic fracture permeability, and hydraulic fracture porosity are all a constant. The compressibility factor and viscosity of fluids and reservoir thickness are also a constant
- (3) The hydraulic fracture formed by staged fracturing of the horizontal well is assumed to run through the whole reservoir; in addition, the length of each fracture is x_f and the fracture width is w on $x-y$ plane
- (4) The reservoir belongs to the radial coordinate; the horizontal and radial outer boundaries of the reservoir are closed, and the upper and lower boundaries are impermeable
- (5) Isothermal Darcy seepage occurs in the reservoir; moreover, consider the occurrence of finite conductivity flow in the hydraulic fractures
- (6) The total production of all hydraulic fractures is a constant Q , so the pressure changes with time
- (7) The pressure is distributed uniformly in initial state and equals to the initial pressure P_i

2.2. Point Source Model for Three-Region Porous Medium Enclosed Space. According to the assumptions of the

FIGURE 1: Schematic of the multistage fractured horizontal well in the three-region heterogeneous reservoir, where the inner region is the volume fracturing stimulation region of the multistage fractured horizontal well, the middle region is the skin damage region, and the outer region is the nonstimulated and undamaged region. ((a) Multiregion heterogeneous space schematic; (b) multiregion heterogeneous plane schematic).

FIGURE 2: Schematic of flow in the multistage fractured horizontal well in the three-region reservoir.

physical model, it is known that the reservoir is divided into three regions through multistage fracturing stimulation of the horizontal wells. The inner region is the volume fracturing stimulation region; the middle region is the skin damage region, and the outer region is the nonstimulated and undamaged region. By combining the continuity equation, velocity equation, and state equation, the

obtained system of equations for seepage control in dual permeability medium is as follows (for the following symbols, see their annotations):

$$\begin{cases} \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial p_1}{\partial r} \right) = \frac{\phi_1 \mu c_{t1}}{k_1} \frac{\partial p_1}{\partial t}, \\ \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial p_2}{\partial r} \right) = \frac{\phi_2 \mu c_{t2}}{k_2} \frac{\partial p_2}{\partial t}, \\ \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial p_3}{\partial r} \right) = \frac{\phi_3 \mu c_{t3}}{k_3} \frac{\partial p_3}{\partial t}. \end{cases} \quad (1)$$

Internal boundary constant flow rate:

$$\left[2\pi r h \frac{k_1}{\mu} \frac{\partial p_1}{\partial r} \right]_{r \rightarrow 0} = Q. \quad (2)$$

There are pressure and flow connection conditions at the interfaces between the inner region and the middle region and between the middle region and the outer region:

$$\begin{cases} p_1 = p_2|_{r=r_{e1}}, \\ p_2 = p_3|_{r=r_{e1}}, \\ \frac{k_1}{\mu} \frac{\partial p_1}{\partial r} = \frac{k_2}{\mu} \frac{\partial p_2}{\partial r} \Big|_{r=r_{e1}}, \\ \frac{k_2}{\mu} \frac{\partial p_2}{\partial r} = \frac{k_3}{\mu} \frac{\partial p_3}{\partial r} \Big|_{r=r_{e2}}. \end{cases} \quad (3)$$

The internal boundary is assumed to be closed:

$$\frac{k_3}{\mu} \frac{\partial p_3}{\partial r} \Big|_{r=r_{e3}} = 0. \quad (4)$$

The following dimensionless quantities are defined for convenience of the solution:

$$r_D = \frac{r}{x_f} \lambda_{21} = \frac{k_1}{k_2} \lambda_{31} = \frac{k_1}{k_3} \omega_{21} = \frac{(\phi \mu c_t)_1}{(\phi \mu c_t)_3} \omega_{31} = \frac{(\phi \mu c_t)_1}{(\phi \mu c_t)_3}, \quad (5)$$

$$p_{1D} = \frac{2\pi k_1 h}{Q\mu} (p_i - p_1) \quad p_{2D} = \frac{2\pi k_1 h}{Q\mu} (p_i - p_{22}) \quad p_{3D} = \frac{2\pi k_1 h}{Q\mu} (p_i - p_3), \quad (6)$$

$$r_{eD1} = \frac{r_{e1}}{x_f} \quad r_{eD2} = \frac{r_{e2}}{k_2} \quad r_{eD3} = \frac{r_{e3}}{x_f} \quad \Delta_{21} = \omega_{21}/\lambda_{21} \quad \Delta_{31} = \omega_{31}/\lambda_{31}, \quad t_D = \frac{k_1 t}{(\phi c_t)_1 \mu x_f^2}. \quad (7)$$

According to the above dimensionless definitions, equations (1) to (4) can be transformed into the following dimensionless form:

$$\begin{cases} \frac{\partial^2 p_{1D}}{\partial r_D^2} + \frac{1}{r} \frac{\partial p_{1D}}{\partial r_D} = \frac{\partial p_{1D}}{\partial t_D}, \\ \frac{\partial^2 p_{2D}}{\partial r_D^2} + \frac{1}{r} \frac{\partial p_{2D}}{\partial r_D} = \frac{1}{\Delta_{21}} \frac{\partial p_{2D}}{\partial t_D}, \\ \frac{\partial^2 p_{3D}}{\partial r_D^2} + \frac{1}{r} \frac{\partial p_{3D}}{\partial r_D} = \frac{1}{\Delta_{31}} \frac{\partial p_{3D}}{\partial t_D}. \end{cases} \quad (8)$$

The internal and external boundary conditions are transformed into:

$$\begin{cases} r_D \frac{\partial p_{1D}}{\partial r_D} \Big|_{r_D=0} = -q_D(s), \\ p_{1D} = p_{2D} \Big|_{r_D=r_{eD1}}, \\ p_{2D} = p_{3D} \Big|_{r_D=r_{eD2}}, \\ \frac{\partial p_{1D}}{\partial r_D} = \frac{1}{\lambda_{21}} \frac{\partial p_{2D}}{\partial r_D}, \\ \frac{1}{\lambda_{21}} \frac{\partial p_{2D}}{\partial r_D} = \frac{1}{\lambda_{31}} \frac{\partial p_{3D}}{\partial r_D}, \\ \frac{\partial p_{3D}}{\partial r_D} \Big|_{r_D=r_{eD3}} = 0. \end{cases} \quad (9)$$

Through Laplace transformation, the general solution of the above equations is

$$\begin{aligned} \tilde{p}_{1D} &= \tilde{q}_D(s) \cdot [K_0(r_D \sqrt{s}) + B_5 I_0(r_D \sqrt{s})], \\ \tilde{p}_{2D} &= A_3 K_0(r_D \sqrt{s/\Delta_{21}}) + A_4 I_0(r_D \sqrt{s/\Delta_{21}}), \\ \tilde{p}_{3D} &= A_5 K_0(r_D \sqrt{s/\Delta_{31}}) + A_6 I_0(r_D \sqrt{s/\Delta_{31}}). \end{aligned} \quad (10)$$

In the above equations: $A_1 = 1/s$; $A_2 = A_1 \cdot B_5$;

$$B_5 = \frac{B_4 K_1(r_{eD1} \sqrt{s}) - K_0(r_{eD1} \sqrt{s})}{B_4 I_1(r_{eD1} \sqrt{s}) - I_0(r_{eD1} \sqrt{s})}, \quad (11)$$

$$B_4 = \frac{K_0(r_{eD1} \sqrt{s/\Delta_{31}}) + B_3 I_0(r_{eD1} \sqrt{s/\Delta_{31}})}{\left(1/\lambda_{21} \sqrt{\Delta_{21}}\right) \left(K_1(r_{eD1} \sqrt{s/\Delta_{31}}) - B_3 I_0(r_{eD1} \sqrt{s/\Delta_{31}})\right)}, \quad (12)$$

$$B_3 = \frac{B_2 K_1(r_{eD2} \sqrt{s/\Delta_{21}}) - K_0(r_{eD2} \sqrt{s/\Delta_{21}})}{I_0(r_{eD2} \sqrt{s/\Delta_{21}}) + B_2 I_1(r_{eD2} \sqrt{s/\Delta_{21}})}, \quad (13)$$

$$B_2 = \frac{K_0(r_{eD2} \sqrt{s/\Delta_{31}}) + B_1 I_0(r_{eD2} \sqrt{s/\Delta_{31}})}{(\lambda_{21}/\lambda_{31}) \sqrt{(\Delta_{21}/\Delta_{31})} \left(K_1(r_{eD2} \sqrt{s/\Delta_{31}}) - B_1 I_1(r_{eD2} \sqrt{s/\Delta_{31}})\right)}, \quad (14)$$

$$B_1 = \frac{K_1(r_{eD3} \sqrt{s/\Delta_{31}})}{I_1(r_{eD3} \sqrt{s/\Delta_{31}})}. \quad (15)$$

Taking fracture half-length as the reference length, the dimensionless length of the fracture is 1. The position of any point of the reservoir and the point source is, respectively (x_D, y_D) and (x_D', y_D') , and the distance of the point source to any point is r_D . Then, the following geometrical relationship is established:

$$r_D^2 = (x_D - x_D')^2 + (y_D - y_D')^2. \quad (16)$$

The hydraulic fracture is located in the stimulation region, so the formula used for point source integration is the inner region pressure formula. Substitute equation (16) into the inner region pressure formula in equation (10) and then obtain:

$$\tilde{p}_{1D} = \tilde{q}_D(s) \cdot \xi(x_D, y_D, x'_D, y'_D, r_{eD1}, r_{eD2}, r_{eD3}, s), \quad (17)$$

where

$$\xi(x_D, y_D, x'_D, y'_D, r_{eD1}, r_{eD2}, r_{eD3}, s) = [K_0(r_D \sqrt{s}) + B_5 I_0(r_D \sqrt{s})], \quad (18)$$

where s is Laplace space time variable. Assuming that the number of fractures in the multistage fractured horizontal well is N_f , the pressure drop of each section of fractures can be obtained as follows by means of point source integration and superposition principle:

$$\tilde{p}_{Dj} = \sum_{j=1}^{N_f} \int_0^1 \tilde{q}_{fDj}(\gamma_{Dj}, s) \xi(x_{Dj}, y_{Dj}, x'_D, y'_D, r_{eD1}, r_{eD2}, r_{eD3}, s) dL, \quad (19)$$

where $dL = \sqrt{dx^2 + dy^2} = dx \sqrt{1 + \tan^2 \theta}$ is the integration path, and θ is the included angle with the y -axis. (x_{Dj}, y_{Dj}) is the coordinate of the center point of the j^{th} fracture, and q_{fDj} is the flow rate of the j^{th} fracture. If each fracture is divided into N_{sj} sections, the pressure drop formula for the i^{th} section of the j^{th} fracture is as follows:

$$\tilde{p}_{Dji} = \sum_{m=1}^{N_f} \sum_{n=1}^{N_{sm}} \tilde{q}_{fDmn} \int_{(n-1)\Delta x}^{n\Delta x} \xi(x_{Dji}, y_{Dji}, x'_D, y'_D, r_{eD1}, r_{eD2}, r_{eD3}, s) dL, \quad (20)$$

where $\Delta x = 1/N_{sm}$ is the hydraulic fracture section length. (x_{Dji}, y_{Dji}) is the coordinate of the center point of the i^{th} section of the j^{th} fracture. Where k_1 is the permeability of the inner region, m^2 ; k_2 is the permeability of the middle region, m^2 ; k_3 is the permeability of the outer region, m^2 ; r_{e1} is the radius of the inner region, m; r_{e2} is the radius of the middle region, m; r_{e3} is the radius of the outer region, m; φ_1 , φ_2 , and φ_3 are the porosity of the inner region, middle region, and outer region, respectively; c_{t1} , c_{t2} , and c_{t3} are the total compressibility factor of the inner region, middle region, and outer region, respectively, $1/\text{Pa}$; μ is oil viscosity, cp; t is time, s; r is radial radius, m; λ_{21} and λ_{31} are the permeability ratio of the inner region to the middle region and the permeability ratio of the inner region to the outer region, respectively; ω_{21} and ω_{31} are the storage ratio of the inner region to the middle region and the storage ratio of the inner region to the outer region, respectively; p_1 , p_2 , and p_3 are the pressure of the inner region, middle region, and outer region, respectively, Pa; x_f is fracture half-length, m; K_0

is the zero-order modified Bessel function; Q is horizontal well production, m^3/s ; D is a dimensionless subscript.

2.3. Hydraulic Fracture Flow Model. The flow in fractures can be assumed to be one-dimensional linear flow. According to the idea of Cinco-Ley (1988), the dimensionless governing equation of flow in the single wing of a one-dimensional fracture can be written as follows:

$$-\frac{\partial^2 p_{fD}}{\partial r_D^2} + \frac{2\pi}{C_{fD}} = 0. \quad (21)$$

The initial condition of a hydraulic fracture can be written as:

$$p_{fD}(r_D, t_D = 0) = 0, 0 \leq r_D \leq 1. \quad (22)$$

The internal boundary condition can be written as:

$$\left(\frac{\partial p_{fD}}{\partial r_D} \right)_{r_D=0} = -\frac{2\pi}{C_{fD}} q_{fwD}, \quad (23)$$

$$\left(\frac{\partial p_{fD}}{\partial r_D} \right)_{r_D=1} = 0. \quad (24)$$

Perform double integration of equation (21) from 0 to v and from 0 to r_D while using boundary equations (24) and (25) to obtain:

$$p_{fD}(r_D) = p_{wD} + \frac{2\pi}{C_{fD}} \int_0^{r_D} \int_0^v q_{fD}(u) du dv - \frac{2\pi}{C_{fD}} q_{fwD}. \quad (25)$$

Carry out Laplace transformation to obtain:

$$\tilde{p}_{fD}(r_D) = \tilde{p}_{wD} + \frac{2\pi}{C_{fD}} \int_0^{r_D} \int_0^v \tilde{q}_{fD}(u) du dv - \frac{2\pi}{C_{fD}} \tilde{q}_{fwD}. \quad (26)$$

Assume that a multistage fractured horizontal well consists of multiple single-wing fractures. If the j^{th} fracture wing is divided into n_j sections, the pressure of the i^{th} section of fracture can be obtained from the discrete equation (30):

$$\begin{aligned} \tilde{p}_{fDi}(r_D) = \tilde{p}_{wD} - \frac{2\pi}{C_{fD}} \cdot \left\{ \left(i\Delta r_D - \frac{\Delta r_D}{2} \right) \cdot \sum_{k=1}^{n_j} q_{fDk} - \frac{\Delta r_D}{8} \right. \\ \left. \cdot \left(q_{fDi} \right) - \sum_{k=1}^{i-1} \left(q_{fDi} \right) \cdot \left[\frac{\Delta r_D}{2} + \left(i\Delta r_D - \left(k + \frac{1}{2} \right) \Delta r_D \right) \right] \right\}, \end{aligned} \quad (27)$$

where $\Delta r_D = 1/n_j$; $\tilde{p}_{fDi}$ and $\tilde{q}_{fDi}$ are the pressure and flow rate of the i^{th} section of the j^{th} fracture wing. C_{fD} is dimensionless fracture conductivity.

2.4. Coupling Solution to the Multiregion Reservoir Model and Fracture Model. Assume that N fracture wings are formed during horizontal well fracturing. If the j^{th} fracture wing is divided into n_j sections, the total number of fracture sections is

$$m = \sum_{j=1}^N n_j. \quad (28)$$

Based on the superposition principle, the pressure on the fracture surface shall be equal to that on the hydraulic fracture section, then,

$$\tilde{p}_{fDji} = \sum_{k=1}^N \sum_{m=1}^{n_k} \tilde{q}_{fDkm} \cdot \tilde{s}_{pD(ji,km)}, \quad (29)$$

where the subscript ji is the i^{th} section of the j^{th} fracture wing; $\tilde{p}_{D(ji,km)}$ is the influence of the m^{th} section of the k^{th} fracture on the pressure drop in the i^{th} section of the j^{th} fracture. Now m equations can be listed, but there are $m+1$ unknowns. Assuming that the horizontal well is produced at a fixed flow rate, the sum of the flow rates of all fracture sections shall be equal to the fixed flow rate of the horizontal well; then,

$$\sum_{j=1}^N \sum_i^{n_j} \tilde{q}_{fDji} = \frac{1}{s}. \quad (30)$$

By solving equations (29) and (30), m flow values and the pressure in the horizontal well can be obtained.

3. Model Verification

A semianalytical model proposed by Wang et al. [14] is compared with our model to validate. Some important parameters are as follows: homogeneous reservoir area $2500 \text{ m} \times 2500 \text{ m}$, reservoir permeability $k = 0.01 \text{ md}$, fracture half-length $x_f = 100 \text{ m}$, fracture permeability $k_f = 10 \text{ D}$, number of fractures formed by fracturing $N = 5$, fracture spacing 200 m , and horizontal well production $40 \text{ m}^3/\text{D}$. For the purpose of comparison with the homogeneous model, the permeability ratio of the inner region to the middle region and the permeability ratio of the inner region to the outer region in the proposed model are $\lambda_{21} = \lambda_{31} = 1$; the storage ratio of the inner region to the middle region and the storage ratio of the inner region to the outer region are $\omega_{21} = \omega_{31} = 1$; fracture conductivity $C_{fD} = 100$; the dimensionless drainage radius of the inner region, middle region, and outer region is $r_{eD1} = 5$, $r_{eD2} = 10$, and $r_{eD3} = 14.01$, respectively. The simulation validation result is shown in Figure 3. As shown in the figure, the result from the semianalytical model is in good agreement with the presented model, thus, verifying the reliability of the model result.

FIGURE 3: Comparative verification of the proposed model with the analytical model proposed by Wang et al. [14].

4. Analysis and Application Example of Typical Curves

4.1. Analysis of Flow Characteristics of a Multistage Fractured Horizontal Well in a Three-Region Composite Reservoir. The parameters in the proposed model for the multistage fractured horizontal well in the three-region reservoir are below: fracture conductivity $C_{fD} = 20$, dimensionless fracture spacing $L_D = 8$, number of fractures $N = 5$, permeability ratio of the inner region to the middle region $\lambda_{21} = 2$, permeability ratio of the inner region to the outer region $\lambda_{31} = 4$, storage ratio of the inner region to the middle region $\omega_{21} = 2$, storage ratio of the inner region to the outer region $\omega_{31} = 3$, included angle between the fracture and the x -axis $\theta = 90^\circ$, and the dimensionless drainage radius of the inner region, middle region, and outer region $r_{eD1} = 100$, $r_{eD2} = 1000$, and $r_{eD3} = 5000$, respectively. The calculation result is shown in Figure 4. As shown in the figure, the flow in the multistage fractured horizontal well in the three-region composite reservoir can be divided into 10 stages based on the pressure derivative curves in Figure 4.

Stage 1. Bilinear flow stage: the pressure and pressure derivative curves show a straight line segment of $1/4$ slope, respectively; the fluids near the reservoir flow perpendicularly to the fractures, while the fluids in the fractures flow perpendicularly to the horizontal wellbore.

Stage 2. Linear flow stage: the pressure and pressure derivative curves show a straight line segment of $1/2$ slope, respectively; the fluids flowing linearly near the reservoir mainly flow towards the hydraulic fractures in this stage.

Stage 3. The first radial flow stage: in this stage, a straight line of a constant value $1/2 N$ is shown on the pressure derivative curve. The number of fractures $N = 5$, so a straight line of a constant value 0.1 is shown on the derivative curve.

Stage 4. Linear flow stage in the inner region: the pressure derivative curve shows a straight line segment of $1/2$ slope;

FIGURE 4: Verification of the model for the multistage fractured horizontal well in the three-region composite reservoir.

the fluids in the inner region flow perpendicularly to all fractures in this stage.

Stage 5. Radial flow stage in the inner region: a straight line segment of a constant value 0.5 is shown on the pressure derivative curve. As the pressure wave is propagated farther and farther, multiple hydraulic fractures appear to be a whole. From the perspective of the entire reservoir, radial flow in all hydraulic fractures occurs, and the distant fluids begin to converge towards the direction of multiple hydraulic fractures.

Stage 6. Linear flow stage in the middle region: the pressure derivative curve shows a straight line segment of 1/2 slope; the fluids in the middle region flow linearly in this stage.

Stage 7. Radial flow stage in the middle region: in this stage, a straight line of a constant value $\lambda_{21}/2$ is shown on the pressure derivative curve. The permeability ratio λ_{21} of the inner region to the middle region is equal to 2, so a straight line of a constant value 1 is shown on the derivative curve.

Stage 8. Linear flow stage in the outer region: the pressure derivative curve shows a straight line segment of 1/2 slope; the fluids in the outer region flow linearly in this stage.

Stage 9. Radial flow stage in the outer region: in this stage, a straight line of a constant value $\lambda_{31}/2$ is shown on the pressure derivative curve. The permeability ratio λ_{31} of the inner region to the outer region is equal to 4, so a straight line of a constant value 2 is shown on the derivative curve.

Stage 10. Boundary dominated flow stage: in this stage, the pressure derivative curve shows a straight line segment of unit slope; this stage symbolizes that the reservoir energy will be gradually depleted.

4.2. Analysis of Influential Factors. The parameters in the proposed model for the multistage fractured horizontal well in the three-region reservoir are below: dimensionless fracture spacing $L_D = 8$, number of fractures $N = 5$, permeability ratio of the inner region to the middle region $\lambda_{21} = 2$, permeability ratio of the inner region to the outer region $\lambda_{31} = 4$, storage ratio of the inner region to the middle region $\omega_{21} = 2$, storage ratio of the inner region to the outer region $\omega_{31} = 3$, fracture conductivity $C_{fD} = 20$, the dimensionless drainage radius of the inner region, middle region, and outer region $r_{eD1} = 100$, $r_{eD2} = 1000$, and $r_{eD3} = 5000$, respectively, and the included angle between the fracture and the x-axis $\theta = 90^\circ$.

Figure 5(a) shows the influence of fracture conductivity on the pressure and pressure derivative curves, where the conductivity C_{fD} is set to 20, 50, and 300, respectively. As shown in the figure, the fracture conductivity mainly affects the early bilinear flow stage of fractures. The larger the conductivity, the lower the dimensionless pressure, the smaller the pressure drop, and the smaller the pressure loss. As the conductivity increases, the early bilinear flow time becomes short. When fracture conductivity is ≥ 300 , the bilinear flow stage is replaced by the linear flow stage.

Figure 5(b) shows the storage coefficient of fracture spacing on the pressure and pressure derivative curves, where the fracture spacing L_D is set to 8, 4, and 2, respectively. As shown in the figure, the larger the fracture spacing, the smaller the pressure loss. The fracture spacing mainly affects the radial flow in the early stage of fractures and the radial flow in the inner region. The smaller the fracture spacing, the longer the time of the radial flow in fractures and the inner region and the shorter the time of the linear flow in the reservoir in the early stage.

Figure 5(c) shows the radius ratio of the middle region to the inner region on the pressure and pressure derivative curves, where the radius ratio r_{eD2}/r_{eD1} is set to 10, 5, and 2, respectively. As shown in the figure, the smaller the radius ratio of the middle region to the inner region, the smaller the dimensionless pressure and the smaller the pressure loss. As can be seen in the pressure derivative curve, the radius ratio of the middle region to the inner region mainly affects the duration of the radial flow in the middle region. As the radius ratio of the middle region to the inner region decreases, the time of the radial flow in the middle region gradually decreases until disappearance, and the time of the radial flow in the inner region is extended accordingly.

Figure 5(d) shows the radius ratio of the outer region to the middle region on the pressure and pressure derivative curves, where the radius ratio r_{eD3}/r_{eD2} is set to 8, 5, and 2, respectively. As shown in the figure, the smaller the radius ratio of the outer region to the middle region, the larger the late dimensionless pressure and the larger the pressure loss. As can be seen in the pressure derivative curve, the radius ratio of the outer region to the middle region mainly affects the duration of the radial flow in the outer region. As the radius ratio of the outer region to the middle region decreases, the time of the radial flow in the outer region gradually decreases until disappearance, and the time of pseudosteady state flow is reduced accordingly.

FIGURE 5: Continued.

FIGURE 5: Continued.

FIGURE 5: Continued.

FIGURE 5: Analysis of influential factors for the multistage fractured horizontal well in the three-region reservoir: (a) influence of fracture conductivity C_{fD} on the typical curves; (b) influence of fracture spacing L_D on the typical curves; (c) influence of the dimensionless radius ratio of the middle region to the inner region on the typical curves; (d) influence of the dimensionless radius ratio of the outer region to the middle region on the typical curves; (e) influence of the permeability ratio λ_{21} of the inner region to the middle region on the typical curves; (f) influence of the permeability ratio λ_{31} of the inner region to the outer region on the typical curves; (g) influence of the storage ratio ω_{21} of the inner region to the middle region on the typical curves; (h) influence of the storage ratio ω_{31} of the inner region to the outer region on the typical curves.

FIGURE 6: Example of fitting between the model for the multistage fractured horizontal well in the three-region reservoir and the production data.

Figure 5(e) shows the permeability ratio of the inner region to the middle region on the pressure and pressure derivative curves, where the permeability ratio λ_{21} is set to 2, 6, and 10, respectively. As shown in the figure, the larger the permeability ratio of the inner region to the middle region, the larger the late dimensionless pressure and the larger the pressure loss. As can be seen in the pressure derivative curve, the permeability ratio of the inner region to the middle region mainly affects the flow characteristics of the middle region. When the permeability ratio λ_{21} of the inner region to the middle region is set to 2, 6, and 10, respectively, a constant value of 1, 3, and 5 is, respectively, shown on the dimensionless pressure derivative curve. As the permeability ratio of the inner region to the middle region increases, the duration of the radial flow in the outer region is reduced.

Figure 5(f) shows the permeability ratio of the inner region to the outer region on the pressure and pressure derivative curves, where the permeability ratio λ_{31} is set to 2, 4, and 8, respectively. As shown in the figure, the larger the permeability ratio of the inner region to the outer region, the larger the late dimensionless pressure and the larger the pressure loss. As can be seen in the pressure derivative curve, the permeability ratio of the inner region to the outer region mainly affects the flow characteristics of the outer region. As the permeability ratio of the inner region to the outer region decreases, the constant value of the radial flow in the outer region is reduced. When the permeability of the middle region is equal to that of the outer region ($\lambda_{31} = \lambda_{21} = 2$), the inner region and the outer region are equivalent to a region, and the derivative curve shows a unit constant value.

Figure 5(g) shows the storage ratio of the inner region to the middle region on the pressure and pressure derivative

curves, where the storage ratio ω_{21} is set to 1, 2, and 4, respectively. As shown in the figure, the larger the storage ratio of the inner region to the middle region, the larger the late dimensionless pressure and the larger the pressure loss, but the variation is not very remarkable. As can be seen in the pressure derivative curve, the storage ratio of the inner region to the middle region mainly affects the characteristics of medium-term linear flow and medium-term radial flow; as the storage ratio increases, the medium-term linear flow deviates from the straight line segment of 1/2 slope.

Figure 5(h) shows the storage ratio of the inner region to the outer region on the pressure and pressure derivative curves, where the storage ratio ω_{31} is set to 3, 6, and 9, respectively. As shown in the figure, the larger the storage ratio of the inner region to the outer region, the larger the late dimensionless pressure and the larger the pressure loss. As can be seen in the pressure derivative curve, the storage ratio of the inner region to the outer region mainly affects the characteristics of late linear flow, radial flow, and pseudosteady state flow; as the storage ratio increases, the linear flow in the outer region deviates from the straight line segment of 1/2 slope, and the radial flow deviates from the constant and enters the pseudosteady state flow stage ahead of time.

4.3. Example Analysis. Through fitting of the site-measured data of pressure and pressure derivative based on the model in the paper, the parameters of a reservoir after multistage fracturing of a horizontal well can be obtained as follows: inner region permeability $84.8 \times 10^{-3} \mu\text{m}^2$, middle region permeability $6.1 \times 10^{-3} \mu\text{m}^2$, outer region permeability $13.05 \times 10^{-3} \mu\text{m}^2$, fracture half-length 120 m, fracture number 5, oil viscosity 9.7 mPa·s, inner region radius 1524 m,

middle region radius 1869 m, outer region radius 6081.6 m, fracture spacing 216 m, initial formation pressure 30 MPa, the storage ratio of the inner region to the middle region 10.67, and the storage ratio of the inner region to the outer region 5.54. Figure 6 shows the result of the comparison between the actual production data and the model for the multistage fractured horizontal well in the three-region reservoir. According to the result, they tally with each other well, and the accuracy of the unsteady-state model presented in this paper for the multistage fractured horizontal well in the three-region reservoir has been verified.

5. Conclusion

- (1) This paper has presented the model for the multistage fractured horizontal well in a three-region composite reservoir. The model takes account of the flow in both three-region composite media and hydraulic fractures. The semianalytical solution of the model has been obtained from Laplace transformation and point source integration
- (2) The accuracy of the proposed model has been verified through the comparison with the numerical simulation result
- (3) According to the analysis of flow characteristics, the flow stage is divided into ten stages below: bilinear flow stage: a straight line of 1/4 slope is shown on the pressure derivative curve; linear flow stage: a straight line of 1/2 slope is shown on the pressure derivative curve; the first radial flow stage: in this flow stage, a straight line segment of a constant value $1/2 N$ is shown on the pressure derivative curve; linear flow stage in the inner region: a straight line of 1/2 slope is shown on the pressure derivative curve; radial flow stage in the inner region: a straight line of a constant value 0.5 is shown on the pressure derivative curve; linear flow stage in the middle region: a straight line of 1/2 slope is shown on the pressure derivative curve; radial flow stage in the middle region: a straight line of a constant value $\lambda_{21}/2$ is shown on the pressure derivative curve; linear flow stage in the outer region: a straight line of 1/2 slope is shown on the pressure derivative curve; radial flow stage in the outer region: a straight line of a constant value $\lambda_{31}/2$ is shown on the pressure derivative curve; boundary control flow stage: a straight line segment of unit slope is shown on the pressure derivative curve
- (4) The influence of some reservoir parameters and fracture sensitivity parameters on the typical curves has been analyzed, mainly including fracture conductivity, fracture spacing, the radius ratio of the middle region to the inner region, the radius ratio of the outer region to the middle region, the permeability ratio of the inner region to the middle region, the permeability ratio of the inner region to the outer region, the storage ratio of the inner region to the middle

region, and the storage ratio of the inner region to the outer region

- (5) Finally, the accuracy of the presented model has been verified through the analysis of an example

Data Availability

All data generated or used during the study appear in the submitted article are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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