Research Article

Modelling of Transient CO₂/Water Flow in Wellbore considering Multiple Mass and Heat Transfer

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A transient fully coupled model is proposed to investigate the two-phase flow of CO₂ and water-based fluid in a wellbore, considering the complex mass and heat transfer in different flow patterns and dynamic coupling between the wellbore and reservoir. Based on mass conservation, momentum, and energy balance, the model employs a state-of-the-art equation of state and transport models to analyze the variations of multiphase flow behaviors and CO₂ properties in a wellbore. Applied in the scenario of a drilled gas kick, the proposed model is used to simulate the processes of gas migration and two-phase flow in the wellbore. The results indicate that the CO₂ solubility increases gradually with the increment of depth, the trend of which shows an abrupt change in 500-1000 m due to the phase transition of CO₂. During kick development, the fronts of free gas and dissolved gas increase almost linearly with time. Through a comparison of CO₂ and CH₄ kicks, gas dissolution is found to significantly suppress the development process of CO₂ kick. The error in kick prediction can reach 42% if the effect of gas dissolution is neglected. However, it can be neglected for CH₄ kick.

1. Introduction

As an important greenhouse gas, carbon dioxide has aroused wide concern in the field of energy and environment. Its use in energy development is one of the hot researches recently. Compared to the conventional working fluids, liquid or supercritical CO₂ has good properties (large density and heat capacity, low viscosity, and surface tension, etc.) for heat transfer and fluid flow and is widely used in the operations of drilling, fracturing, enhanced oil recovery (EOR), and geothermal exploitation [1–5]. Generally, affected by variations of the temperature and pressure in the wellbore, the thermophysical properties of CO₂ change significantly in the temporal and spatial scales, as shown in Figure 1. Particularly, there exist complicated flow patterns and mass and heat transfer processes while CO₂ and water-based fluid coexisting in the wellbore. Therefore, it is necessary to develop a reliable model for transient CO₂/water flow, which can be of important significance for accurate prediction and control of multiphase flow parameters in the actual wellbore/reservoir systems.

At present, a lot of research works have been done to model the single CO₂ flow in the wellbore during the processes of CO₂ drilling, injection, and production [6–8]. In actual operations, the wellbore two-phase flow of CO₂ and water can be generated due to the influx of formation fluids (such as a drilled gas kick). As the flow of different fluid components, the phase interface may exhibit various flow patterns, accompanied with complicated mass and heat transfer phenomena. Zha et al. [9] proposed an equivalent single-phase flow model to interpret the oil-water two-phase flow in a wellbore during the well test period. Shang et al. [10] developed a mathematical model coupling fluid flow in the horizontal wellbore and reservoir, based on the principle of mirror reflection and mass conservation. Regarding the nonisothermal multiphase flow
Subsequently, the proposed model is applied to analyze the thermodynamic coupling between wellbore and reservoir. Properties of CO₂ can vary significantly.

process in the wellbore, Pan and Oldenburg [11] developed an integrated simulator based on the drift-flux model, which can be applied in the process of geothermal exploitation using supercritical CO₂. Considering the migration characteristics of gas kick in the wellbore, He et al. [12] built a kick simulation model and analyzed the effect of sour gas dissolution on well control operations. Lu and Connell [13] developed a quasisteady wellbore two-phase flow model to simulate the nonisothermal flow of carbon dioxide in injection wells during geological storage. Recently, Wang et al. [14–16] proposed a series of calculation models of CO₂ density, friction coefficient, Joule–Thomson coefficient, and so on, which can significantly improve the simulation accuracy of CO₂ thermophysical parameters in the wellbore in the drilling scenario. Based on that, they developed the wellbore temperature and pressure models considering the phase transition of sour gases. These models can perform more accurate estimations of wellbore temperature and pressure fields [17–19]. Their simulation results indicated that the gas kicks can be “hidden” and “abrupt” successively, affected by the phase transition of the fluids in the dynamically changing temperature and pressure environment. Furthermore, the critical wellhead back pressures for suppressing the abrupt expansion of sour gases were obtained [20]. These research and findings can provide a solid theoretical basis for early kick detection and wellbore pressure control.

In this study, we developed a transient fully coupled model for wellbore CO₂/water flow, which considers the complicated mass and heat transfer mechanisms in different flow patterns and the dynamical coupling between wellbore and reservoir. Subsequently, the proposed model is applied to analyze the multiphase flow process during a drilled CO₂ kick.

2. Wellbore Multiphase Flow Model

2.1. Mass Conservation Equation. Considering the one-dimensional unsteady flow [19], the equations expressing the mass conservation laws among the free CO₂ phase, continuous water phase, and dissolved CO₂ phase can be presented as follows:

\[
\frac{\partial}{\partial t} (A\alpha_{CO2}\rho_{CO2}v_{CO2}) + \frac{\partial}{\partial s} (A\alpha_{CO2}\rho_{CO2}v_{CO2}) = -\dot{m}_L + q_g, \tag{1}
\]

where \( t \) is the time, \( s \) is the distance, \( m \) is the cross-sectional area, \( \alpha \) is the void fraction of continuous water phase; \( \rho_{CO2} \) is the CO₂ density, kg/m³; \( v_{CO2} \) is the velocity of free CO₂ phase, m/s; \( \dot{m}_L \) is the mass transfer rate of CO₂ dissolution, kg/(m·s); and \( q_g \) is the mass transfer rate of CO₂ between the wellbore and reservoir, kg/(m·s). The rate of CO₂ influx from reservoir to wellbore is mainly dominated by the pressure underbalance and reservoir properties, which can be estimated using the model of Sun et al. [21].

\[
\frac{\partial}{\partial t} (A\alpha_{x_{sw}}\rho_{w}) + \frac{\partial}{\partial s} (A\alpha_{x_{sw}}\rho_{w}) v_l = \dot{m}_L, \tag{2}
\]

where \( \alpha_{x_{sw}} \) is the mass fraction of dissolved gas, kg/kg.

In equations (1)–(3), the relation for velocities of different phases can be described by the drift-flux model:

\[
v_{CO2} = C_0 v_l + v_{\infty}, \tag{4}
\]

where \( C_0 \) is distribution coefficient, dimensionless and \( v_{\infty} \) is the drift velocity of gas, m/s. Commonly, the distribution coefficient and drift velocity are closely related to the flow pattern, phase distributions, pipeline configuration, etc. Here, the slip relation developed by Bhagwat and Ghatjar [22] is employed.

2.2. Momentum Conservation Equation.

\[
\frac{\partial}{\partial t} (A\alpha_{CO2}\rho_{CO2}v_{CO2}^2 + A\alpha_{w}\rho_{w}v_l^2) + \frac{\partial}{\partial s} (A\alpha_{CO2}\rho_{CO2}v_{CO2}^2 + A\alpha_{w}\rho_{w}v_l^2) \tag{5}
\]

\[
+ \frac{\partial}{\partial s} (Ap) = -Af \frac{\rho_m v_m^2}{2d_e^2} + A\rho_m g \cos \theta,
\]

where \( \rho_m \) is the density of fluid mixture, kg/m³; \( v_m \) is the velocity of fluid mixture, m/s; \( p \) is pressure, Pa; \( d_e \) is the hydraulic diameter, m; \( \theta \) is the inclination angle, rad; and \( g \) is gravitational acceleration, m/s². In equation (5), \( f \) is the...
friction coefficient, which can be described using the model of Wang et al. [15].

\[
\begin{align*}
\frac{f}{Re} = 64, \\
\frac{f}{Re} = 0.06539 \times \exp \left( -\left( \frac{Re - 3516}{1248} \right)^2 \right), \\
\frac{1}{\sqrt{f}} = -2.34 \times \log \left( \frac{\varepsilon}{1.72d_e} \right) - 9.26 \times \log \left( \left( \frac{\varepsilon}{29.36d_e} \right)^{0.95} + \left( \frac{18.35}{Re} \right)^{1.108} \right),
\end{align*}
\]

where \( Re \) is the Reynolds number, dimensionless and \( \varepsilon \) is roughness, m.

### 2.3. Energy Conservation Equation

Considering the phase transition, Joule–Thomson cooling, heat transfer between the wellbore and formations, and reservoir coupling, the temperature model for wellbore CO₂/water flow can be obtained based on energy conservation [23]:

\[
A(a_{CO2}P_{CO2}C_{p,CO2} + \alpha_1 \rho_1 C_{p,l}) \frac{\partial T}{\partial t} + A(a_{CO2}P_{CO2}C_{p,CO2}^v + \alpha_2 \rho_2 C_{p,v}) \frac{\partial T}{\partial s} - Aa_{CO2}P_{CO2}C_{p,CO2} \frac{\partial p}{\partial t} + \frac{1}{2} \left[ A(a_{CO2}P_{CO2}C_{p,CO2}^v + \alpha_2 \rho_2 C_{p,v}) \right] + \frac{1}{2} \left[ A(a_{CO2}P_{CO2}C_{p,CO2}^v + \alpha_2 \rho_2 C_{p,v}) \right] - Aa_{CO2}P_{CO2}C_{p,CO2} \frac{\partial p}{\partial s} - \frac{\partial (Ap)}{\partial t} = -A(a_{CO2}P_{CO2}C_{p,CO2} + \alpha_1 \rho_1 C_{p,l}) \frac{\partial p}{\partial s} + a_1 \rho_1 \alpha_1 g \cos \theta + q_g(h_v - h_g) + Q_{\text{transfer}} + A \frac{\rho_1 \varepsilon \alpha_1}{\ln \left( \frac{Re}{18} \right)} + m_1 \Delta H_{\text{sol}},
\]

where \( T \) is the fluid temperature, °C; \( C_{p,CO2} \) is specific heat capacity of CO₂ at constant pressure, J/(kg°C); \( C_{p,l} \) is specific heat capacity of the liquid phase at constant pressure, J/(kg°C); \( C_j \) is the Joule–Thomson coefficient of gas, °C/Pa; \( h_v \) is the enthalpy of the influx gas at reservoir condition, J/kg; \( h_g \) is the enthalpy of CO₂ in the wellbore, J/kg; \( \Delta H_{\text{sol}} \) is the dissolution heat of gas in water, J/kg; \( Q_{\text{transfer}} \) is the rate of heat exchange between the wellbore and formations, W/m²; \( \varepsilon \) is the convective heat transfer coefficient, W/(m²°C); \( \Delta T \) is the temperature difference, °C; \( r_i \) represents the geometric size of wellbore system, m; and \( \lambda_1 \) is the thermal conductivity of wellbore system, W/(m°C).

Generally, the Joule–Thomson effect of water can be negligible compared to that of CO₂. According to the PVT properties of gas, the Joule–Thomson coefficient of CO₂ is defined as follows:

\[
\mu_{JT} = \lim_{AP \to 0} \frac{\Delta T}{AP} = \frac{\partial T}{\partial P}
\]

Wang et al. [16] revealed that the absolute average errors of the CO₂ Joule–Thomson coefficient predicted by the state equations are relatively low in vapor and supercritical states, but larger errors appear near the CO₂ critical point and liquid state. Therefore, they built a religious empirical model, in which the absolute average errors at the vapor, liquid, and supercritical states are 1.52%, 4.59%, and 3.08%, respectively.

\[
C_j = \frac{(\eta - \eta_1)(\eta - \eta_2)(1 - \eta)}{\eta_1 \eta_2} \mu_{JT-0} + \frac{\eta_1(\eta_1 - \eta_2)(\eta_1 - 1)}{\eta_2(\eta_1 - \eta_2)} \mu_{JT-1} + \frac{\eta_2(\eta_2 - \eta_1)(\eta_2 - 1)}{\eta_1(\eta_2 - \eta_1)} \mu_{JT-2}
\]

where \( \eta, \eta_1, \eta_2, \mu_{JT-0}, \mu_{JT-1}, \) and \( \mu_{JT-2} \) are the functions of temperature and pressure.

### 3. Mass and Heat Transfer for Different Flow Patterns

#### 3.1. Flow Pattern Transition

The mass and heat transfer characteristics in the two-phase flow are significantly governed by the flow patterns. In this study, the model developed by Hasan and Kabir [24] is used to flow pattern identification, as shown in Table 1.

#### 3.2. Interphase Mass Transfer Model

The dissolution of sour gas in the wellbore two-phase flow is a diffusion process governed by concentration difference. Therefore, the gas dissolution rate is a function of gas concentration and mass transfer.
coefficient:

\[ \dot{m}_L = M_p S_{int} k_M (c_{sat} - c), \]  

where \( c \) is the gas concentration, mol/m\(^3\); \( c_{sat} \) is the gas concentration at saturation, mol/m\(^3\); \( M_p \) is the molecular mass of CO\(_2\), kg/mol; \( k_M \) is the interphase mass transfer coefficient between CO\(_2\) and liquid phase, m/s; and \( S_{int} \) is the contact area of CO\(_2\) and liquid phase, m\(^2\). In this study, the contact area in different flow patterns is estimated using the model proposed by Sun et al. [23].

The determination of the mass transfer coefficient, which is related to the fluid properties (such as density, viscosity, and diffusivity), flow velocity, and annulus size, is challenging. Considering the laminar flow and turbulent flow conditions, the expression presented by Cussler [25] is employed.

\[ k_M = \begin{cases} 
1.62 \frac{D_g}{D_c} \left( \frac{d\nu_{CO2}}{LD_g} \right)^{1/3}, & \text{Laminar flow,} \\
0.026 \frac{D_g}{D_c} \left( \frac{d\nu_{CO2}}{v} \right)^{0.8} \left( \frac{v}{D_g} \right)^{1/3}, & \text{Turbulent flow,} 
\end{cases} \]

where \( L \) is the pipe length, m; \( v \) is the kinematic viscosity, m\(^2\)/s; and \( D_g \) is the gas diffusivity coefficient, m\(^2\)/s.

### Table 1: Criteria of gas/liquid flow pattern transition.

<table>
<thead>
<tr>
<th>Flow pattern</th>
<th>Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bubble</td>
<td>( v_{ps} &gt; (0.429v_{LL} + 0.357v_{co}) \cos \theta ) ( \sigma ) &lt; 0.52, and ( \nu_{L0}^{1.22} &gt; 4.68k_{0.48}^0 \left( \rho_L - \rho_g \right) \sigma^{0.3} \left( \sigma / \rho_L \right)^0.6 (\rho_m / \rho_L)^{0.08} )</td>
</tr>
<tr>
<td>Slug</td>
<td>( v_{ps} &gt; (0.429v_{LL} + 0.357v_{co}) \cos \theta ) ( \sqrt{v_{ps} \rho_g} &lt; 171.1 \log_{10} (\nu_{LL} \rho_L) - 23.2 ) if ( \nu_{ps} \rho_L &lt; 50\nu_{LL} \rho_L &lt; 0.00673 (\nu_{LL} \rho_L)^{1.7} ) if ( \nu_{LL} \rho_L \geq 50 )</td>
</tr>
<tr>
<td>Churn</td>
<td>( v_{ps} &lt; 3.1 \left[ \sigma \left( \rho_L - \rho_g \right) / \rho_g^2 \right]^{0.25} ) and ( \nu_{ps} \rho_g &lt; 171.1 \log_{10} (\nu_{LL} \rho_L) - 23.2 ) if ( \nu_{ps} \rho_L &lt; 50\nu_{LL} \rho_L &lt; 0.00673 (\nu_{LL} \rho_L)^{1.7} ) if ( \nu_{LL} \rho_L \geq 50 )</td>
</tr>
<tr>
<td>Annular</td>
<td>( v_{ps} &gt; 3.1 \left[ \sigma \left( \rho_L - \rho_g \right) / \rho_g^2 \right]^{0.25} )</td>
</tr>
</tbody>
</table>

Note that \( v_{ps} \) is the superficial velocity of gas phase, m/s and \( v_{ps} \) is the superficial velocity of liquid phase, m/s.

### Table 2: Main parameters of the kicking well.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Well depth</td>
<td>3718.84 m</td>
<td>Displacement</td>
<td>30 L/s</td>
</tr>
<tr>
<td>Density of drilling fluid</td>
<td>1.43 g/cm(^3)</td>
<td>Plastic viscosity</td>
<td>30 mPa•s</td>
</tr>
<tr>
<td>Yield value</td>
<td>15 Pa</td>
<td>Permeability</td>
<td>480 mD</td>
</tr>
<tr>
<td>Geothermal gradient</td>
<td>1.9 °C/100 m</td>
<td>Surface temperature</td>
<td>26 °C</td>
</tr>
<tr>
<td>Shut-in standpipe pressure</td>
<td>1.9 MPa</td>
<td>Shut-in casing pressure</td>
<td>2.0 MPa</td>
</tr>
</tbody>
</table>

Wellbore configuration: 20″ casing × 70 m + 17 – 1/2″ casing × 900 m + φ 320 mm open hole × 3718.84 m

Drilling assembly: EH1317φ319 mm PDC × 0.41 m + 8″ drill collar × 80m + 5″ heavy weight drill pipe × 120.53 m + 5″ drill pipe × 1392.56 m + 5 – 1/2″ drill pipe

### Figure 2: Comparison between measured and simulated pit gains during CO\(_2\) kick.

Furthermore, we use the model of Duan and Sun [26] to calculate the CO\(_2\) solubility in water-based fluid in this study.

\[ \ln \frac{y_{CO2} / c_{sat}}{\varphi_{CO2}} = \ln \frac{\mu_{CO2}}{RT} - \ln \varphi_{CO2} + \sum_{a} \lambda_{CO2-a} m_a \]

where \( \varphi_{CO2} \) is the fugacity coefficient of CO\(_2\) and \( y_{CO2} \) is the
mole fraction of CO₂ in vapor phase, \( \lambda_{CO_2-v} \) is the binary interaction parameter of CO₂ and cation, \( \lambda_{CO_2-c} \) is the binary interaction parameter of CO₂ and anion, and \( \lambda_{CO_2-c-a} \) is the ternary interaction parameter of CO₂, cation, and anion.

### 3.3. Convection Heat Transfer Model in Different Flow Patterns

As wellbore multiphase flow, convection heat transfer mainly occurs in the heat transfer boundary layer. Therefore, the flow pattern can have an important influence on the heat transfer mechanisms. Generally, the convection heat transfer coefficient can be written as follows:

\[
h = \frac{Nu \lambda}{\mu},
\]

where \( \lambda \) is the thermal conductivity of CO₂, W/(m·°C).

Through the experiments of heat transfer in wellbore multiphase flow, Gao et al. [27] proposed the model of convection heat transfer coefficient in different flow patterns.

\[
Nu = a \text{Re}^{0.7922} \text{Pr}^{0.3} \left( \frac{\mu_f}{\mu_w} \right)^{0.25} \left( 1 - b \alpha_d^c \right),
\]

where \( \text{Pr} \) is the Prandtl number, \( \mu_f \) is the fluid viscosity at characteristic temperature, \( \text{Pa} \); \( \mu_w \) is the fluid viscosity at surface temperature, \( \text{Pa} \); \( a, b, \) and \( c \) are constants related to flow patterns: \( a = 0.01215, \ b = 0.30577, \) and \( c = -0.16578 \) for bubble flow; \( a = 0.46359, \ b = 0.97599, \) and \( c = -0.01314 \) for slug flow; and \( a = 0.50861, \ b = 0.93808, \) and \( c = -0.15418 \) for churn flow and annular flow.

Furthermore, the thermophysical properties of CO₂ vary greatly accompanied with the complicated mass and heat transfer processes. The detailed calculation models for thermophysical parameters of CO₂ are presented in Appendix A.

### 4. Model Simulation and Verification

#### 4.1. Model Simulation

The integrated model is solved using the simulation method proposed by Sun et al. [23], which employs a fully implicit scheme, constant space steps, and varying time steps. The overall simulation process is consisting of three layers of iterations. At first, the phase velocities and fractions at different space blocks are calculated based on a drift-flux model. Subsequently, the pressure field is estimated using a predictor-corrector shooting technique. With updating the fluid properties and multiphase flow parameters, the temperature distribution in the drilling pipe and annulus are iteratively simulated until a desired convergence tolerance is achieved.

#### 4.2. Model Verification

The proposed model is validated using the measured data of a field well, which comes across a CO₂ kick accident. The pit gain approaches 1.9m³ at 12:00 when the alarm sounded. Then, the well was shut to measure the standpipe pressure and casing pressure. At 13:05, the gas influx was circulated gradually while a constant casing pressure (2 MPa) is maintained. At 15:30, a large amount of CO₂ blows out from the wellhead. The main parameters of the kicking well are presented in Table 2.

Figure 2 shows the simulated and measured pit gains during CO₂ kick. At point A, the gas kick was detected with a pit gain of 1.9 m³. The multiphase flow process of gas circulation is simulated from point A to point B, in which the pressure underbalance and gas influx rate at the bottom hole is low. CO₂ dissolves into the drilling fluid gradually, which leads to the nonobvious variations in pit gain. After point B, the

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**Table 3: Basic parameters for kick simulation.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Well type</td>
<td>Vertical</td>
</tr>
<tr>
<td>Reservoir temperature</td>
<td>125°C</td>
</tr>
<tr>
<td>Rock permeability</td>
<td>50 mD</td>
</tr>
<tr>
<td>Temperature gradient</td>
<td>0.03°C/m</td>
</tr>
<tr>
<td>Density of drilling fluid</td>
<td>1050 kg/m³</td>
</tr>
<tr>
<td>Surface temperature</td>
<td>20°C</td>
</tr>
<tr>
<td>Rate of penetration</td>
<td>10 m/h</td>
</tr>
<tr>
<td>Well depth</td>
<td>3500 m</td>
</tr>
<tr>
<td>Reservoir pressure</td>
<td>42 MPa</td>
</tr>
<tr>
<td>Porosity</td>
<td>0.15</td>
</tr>
<tr>
<td>Total compressibility</td>
<td>0.0002 (1/MPa)</td>
</tr>
<tr>
<td>Displacement</td>
<td>0.02 m³/s</td>
</tr>
<tr>
<td>Viscosity of drilling fluid</td>
<td>20 cp</td>
</tr>
</tbody>
</table>

---

**Figure 3:** The distribution of fluid temperature and CO₂ density and solubility in the wellbore at 500 s.
dissolved CO₂ rapidly releases out from the drilling fluid due to the decrease of gas solubility near the surface.

As seen, the simulated results of pit gain agree well with the measured data, and the average error is 3.253%. It indicates that the proposed model can accurately describe the effect of phase transition on gas migration in wellbore.

5. Case Analysis

The formation fluids can enter the wellbore driven by pressure underbalance, if the bottom hole pressure at open hole section is less than the pore pressure during drilling. The understanding of phase transition and kick migration is important for early kick detection and wellbore control procedure. Using the proposed model, we simulate and analyze the flow behaviors of CO₂ and water-based fluid in the scenario of a drilled CO₂ kick. The basic parameters for kick simulation [19] are shown in Table 3.

5.1. Analysis of the Gas Migration Process. Figure 3 shows the profiles of fluid temperature and CO₂ density and solubility in the wellbore at 500 s. As seen, the wellbore temperatures increase gradually with the well depth increasing, affected by variations of the formation temperatures. At a given depth, the fluid temperature in the annulus is larger than that in the drill pipe.

Due to the dynamically changing temperature and pressure fields, the thermophysical properties of CO₂ vary significantly along the wellbore. Furthermore, the variation trend can be abrupt at 500 m to 1000 m because of phase transition. There results indicate that the free gas and dissolved gas can rapidly expand and separate out, which will pose a challenge to kick management and well control.

Figures 4(a) and 4(b) show the dynamical distributions of the free gas and dissolved gas in the wellbore at different times. As shown in Figure 4, the volume of gas influx in the wellbore increases gradually with time. And the front of gas

![Figure 4: (a) The distribution of free gas in the wellbore at different times. (b) The distribution of dissolved gas in the wellbore at different times.](image-url)
profiles rises nearly linearly with time. In general, the void fraction of free gas increases as the increment of well depth.

As shown in Figure 4, the gas dissolves into the drilling fluids gradually as it migrates upward in the wellbore. The void fraction of dissolved gas reaches saturation state with time increasing, which can be a dynamical process. For example, the void fraction of dissolved gas at 3400 m increases rapidly at first and then decreases slightly due to variation of the gas solubility. It can be expected that the gas dissolution process will significantly suppress the migration and expansion of gas kick in the wellbore. After the gas kick initializes 500 s, the void fraction of dissolved gas is 42 percent of that of free gas near the bottom hole.

In this section, we study the migration process of a gas bubble which is released at the bottom hole as the gas kick initializes, as shown in Figure 5.

The gas influx rate at the open hole section is mainly governed by the wellbore pressure distribution, which consists of hydrostatic pressure and friction pressure. As the gas kick enters the wellbore, the flow velocity and friction pressure of fluid mixture will increase abruptly, which can lead to the sudden increment of bottom hole pressure. Subsequently, the decrease of hydrostatic pressure plays a dominant role, and the bubble pressure decreases gradually.

As seen, the bubble radius and mass decrease gradually with time. However, the bubble shrinkage rate decreases gradually due to the decrease of gas/liquid contact area and mass transfer rate. Furthermore, the volume change of gas bubble is also affected by the pressure change. At the early stage, the increment of bubble pressure will increase the bubble shrinkage rate. Then the decrease of bubble pressure will decrease the rate of bubble shrinkage.

Figure 5: Variations of the pressure, radius, and mass of a gas bubble released at the initial time of gas kick (initial radius: 5 mm).

Figure 6: The profiles of free gas along the wellbore during methane and CO₂ kicks.

Figure 7: The profiles of dissolved gas along the wellbore during methane and CO₂ kicks.
5.2. Effect of Gas Dissolution. A comparison of CO$_2$ and CH$_4$ kicks is conducted to analyze the effect of gas dissolution and phase transition on kick migration.

Figure 6 shows the distributions of free gas along the wellbore during methane and CO$_2$ kicks. For a given depth, the void fraction of methane is significantly larger than that of CO$_2$. Because the gas influx rate for methane kick is larger under the same pressure underbalance, since the viscosity and percolation resistance of methane in reservoir are much smaller than that of CO$_2$. Affected by the large gas influx rate, the velocity of the fluid mixture is larger and gas front rises faster during the methane kick.

Figure 7 shows the distributions of dissolved gas along the wellbore during methane and CO$_2$ kicks. Under the same temperature and pressure condition, the solubility of carbon dioxide in water is much larger than that of methane. At 3000 m, we find that the void fraction of dissolved CO$_2$ is 20 times larger than that of dissolved methane.

As shown in Figure 8, the effect of gas dissolution on kick migration can be negligible in the methane kick, because the solubility of methane in water is low. As for the CO$_2$ kick, it can be found that gas dissolution has an important influence on the process of kick development. In the figure, the simulation error of pit gain can reach 50% if the effect of gas dissolution is neglected at 500 s.

6. Conclusions

(1) A transient fully coupled model for the two-phase flow of CO$_2$ and water-based fluid in wellbore is developed, considering the mass and heat transfer in different flow patterns and the dynamic coupling between wellbore and reservoir. Model validation against the measured data of a kicking well indicates that it can produce an average error of 3.253%.

(2) Affected by variations of the wellbore temperature and pressure fields, the solubility of CO$_2$ increases with depth, which shows an abrupt increasing trend at 500 m to 1000 m due to phase transitions.

(3) A methane kick can develop more rapidly than a CO$_2$ kick, i.e., the velocity of gas migration and void fraction of gas in a methane kick are much larger.

(4) The simulation error of pit gain can reach 50% if the effect of gas dissolution is neglected in the CO$_2$ kick. However, it is negligible in the methane kick.

Appendix

A. Appendix: Calculation Models for the Thermophysical Parameters of CO$_2$

At different temperature and pressure conditions, the PVT relation of gas can be described using the PR equation:

$$P = \frac{RT}{V - b} - \frac{a}{V(V + b) + b(V - b)}.$$  \hspace{1cm} (A.1)

The relation of enthalpies at different states can be written as follows:

$$H_2 - H_1 = \int_{V_1}^{V_2} T \left( \frac{\partial P}{\partial V} \right)_T dV + (PV)_2 - (PV)_1.$$ \hspace{1cm} (A.2)

By substituting equation (A.2) into equation (A.1), we obtain

$$H = RT(Z - 1) - \frac{\sqrt{2a}}{4b} \left[ 1 + \frac{\kappa T}{\sqrt{2} \alpha T} \right] \ln \left( \frac{Z + (1 + \sqrt{2})b}{Z + (1 - \sqrt{2})b} \right) + H_{\text{ideal}}.$$ \hspace{1cm} (A.3)

where $R$ is the gas constant; $b, a$, and $\kappa$ are parameters in the PR equation, $a = a(b, \alpha, \kappa)$, $B = bP/RT$, and $Z$ is the gas compression factor:

$$Z^3 - (1 - B)Z^2 + (A - 2B - 3B^2)Z - (AB - B^2 - B^3) = 0,$$ \hspace{1cm} (A.4)

where $A = aRT^2$.

According to the S-W model [28], the enthalpy of ideal gas is

$$H_{\text{ideal}} = RT(1 + \tau \times \phi^0),$$ \hspace{1cm} (A.5)

where $\tau$ is the reduced temperature, $\tau = T_c/T$; $\phi^0$ is the Helmholtz free energy of ideal gas.
Combining equations (A.3) and (A.5), the specific enthalpy of carbon dioxide can be obtained:

\[ h_i = \frac{1}{M_i} \left( Z + \frac{T_c \phi^0}{T} \right) - \frac{\sqrt{2}a}{4bM_c} \left[ 1 + \frac{\kappa T}{\sqrt{\alpha T_i}} \right] \ln \left( \frac{Z + (1 + \sqrt{2})B}{Z + (1 - \sqrt{2})B} \right) \]

\[ C_p = \left( \frac{\partial h}{\partial T} \right)_p \]

\[ = \frac{d^2a}{dT^2} \times \frac{T}{2\sqrt{2bM_i}} \times \ln \left( \frac{Z + (1 + \sqrt{2})B}{Z + (1 - \sqrt{2})B} \right) + \frac{R(M - N)^2M_{c-1}}{M^2 - 2A(Z + B)} + \frac{R}{M} + C_{p,\text{ideal}} \frac{M_c}{M} \]  

\[ (A.6) \]

In which, the parameters are defined as follows:

\[ M = \frac{(Z^2 + 2 \times B \times Z - B^2)}{(Z - B)}, \quad (A.7) \]

\[ N = \frac{(da/dT)}{(B/bR)}. \]

Additionally, the viscosity and thermal conductivity of CO\textsubscript{2} can be estimated using the model of Vesovic et al. [29].

**Nomenclature**

**Variables**

- \( A \): Cross-sectional area (m\(^2\))
- \( C_{0}\text{p} \): Distribution coefficient, dimensionless
- \( C_{p,\text{CO2}} \): Specific heat capacity at constant pressure of CO\textsubscript{2} (J/(kg\textsuperscript{\circ}C))
- \( C_{p,f} \): Specific heat capacity at constant pressure of the liquid phase (J/(kg\textsuperscript{\circ}C))
- \( C_{f} \): Joule–Thomson coefficient of gas (°C/Pa)
- \( c \): Gas concentration (mol/m\(^3\))
- \( c_{\text{sat}} \): Gas concentration at saturation (mol/m\(^3\))
- \( d_{s} \): Hydraulic diameter (m)
- \( D_{g} \): Gas diffusivity coefficient (m\(^2\)/s)
- \( f \): Friction coefficient
- \( g \): Gravitational acceleration (m/s\(^2\))
- \( h_i \): Enthalpy of the influx gas at reservoir condition (J/kg)
- \( h_{\text{w}} \): Enthalpy of CO\textsubscript{2} in the wellbore (J/kg)
- \( \Delta H_{\text{sol}} \): Dissolution heat of gas in water (J/kg)
- \( h \): Convective heat transfer coefficient (W/(m\(^2\)°C))
- \( k_{M} \): Interphase mass transfer coefficient between CO\textsubscript{2} and liquid phase (m/s)
- \( L \): Pipe length (m)
- \( m_{L} \): Mass transfer rate of CO\textsubscript{2} dissolution (kg/(m\textsuperscript{\circ}s))
- \( M_{g} \): Molecular mass of CO\textsubscript{2} (kg/mol)
- \( p \): Pressure (Pa)
- \( Pr \): Prandtl number
- \( q_{g} \): Mass transfer rate of CO\textsubscript{2} between the wellbore and reservoir (kg/(m\textsuperscript{\circ}s))
- \( Q_{\text{transfer}} \): Rate of heat exchange between the wellbore and formations
- \( R \): Gas constant
- \( Re \): Reynolds number, dimensionless
- \( r_{i} \): Geometric size of wellbore system (m)
- \( s \): Distance (m)
- \( S_{int} \): Contact area of CO\textsubscript{2} and liquid phase (m\(^2\))
- \( T \): Fluid temperature (°C)
- \( t \): Time (s)
- \( \Delta T \): Temperature difference (°C)
- \( v_{\text{CO2}} \): Velocity of free CO\textsubscript{2} phase (m/s)
- \( v_{co} \): Drift velocity of gas (m/s)
- \( v_{i} \): Velocity of water (m/s)
- \( v_{m} \): Velocity of fluid mixture (m/s)
- \( v_{gs} \): Superficial velocity of gas phase (m/s)
- \( v_{Ls} \): Superficial velocity of liquid phase (m/s)
- \( x_{sol} \): Mass fraction of dissolved gas (kg/kg)
- \( \gamma_{\text{CO2}} \): Mole fraction of CO\textsubscript{2} in vapor phase.

**Greek letters**

- \( \alpha_{\text{CO2}} \): Void fraction of free CO\textsubscript{2} phase
- \( \alpha_{i} \): Void fraction of continuous water phase
- \( P_{\text{CO2}} \): CO\textsubscript{2} density (kg/m\(^3\))
- \( \rho \): Density of water (kg/m\(^3\))
- \( \rho_{m} \): Density of fluid mixture (kg/m\(^3\))
- \( \theta \): Inclination angle (rad)
- \( \varepsilon \): Roughness (m)
- \( \lambda \): Thermal conductivity of CO\textsubscript{2} (W/(m·°C))
- \( \lambda_{i} \): Thermal conductivity of wellbore system (W/(m·°C))
- \( \nu \): Kinematic viscosity (m\(^2\)/s)
- \( \phi_{\text{CO2}} \): Fugacity coefficient of CO\textsubscript{2}
- \( \mu_{f} \): Fluid viscosity at characteristic temperature (Pa·s)
- \( \mu_{w} \): Fluid viscosity at surface temperature (Pa·s).

**Data Availability**

The data in this paper used to support the findings of this study are currently under embargo while the research findings are commercialized. Requests for data, 6 months after publication of this article, will be considered by the corresponding author.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

**References**


[2] J. Zhou, N. Hu, X. Xian et al., “Supercritical CO\textsubscript{2} fracking for enhanced shale gas recovery and CO\textsubscript{2} sequestration: results,


