Research Article

A New Method for Research on Unsteady Pressure Dynamics and Productivity of Ultralow-Permeability Reservoirs

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In the numerous low-permeability reservoirs, knowing the real productivity of the reservoir became one of the most important steps in its exploitation. However, the value of permeability interpreted by a conventional well-test method is far lower than logging, which further leads to an inaccurate skin factor. This skin factor cannot match the real production situation and will mislead engineer to do an inappropriate development strategy of the oilfield. In order to solve this problem, key parameters affecting the skin factor need to be found. Based on the real core experiment and digital core experiment results, stress sensitivity and threshold pressure gradient are verified to be the most influential factors in the production of low-permeability reservoirs. On that basis, instead of a constant skin factor, a well-test interpretation mathematical model is established by defining and using a time-varying skin factor. The time-varying skin factor changes with the change of stress sensitivity and threshold pressure gradient. In this model, the Laplace transform is used to solve the Laplace space solution, and the Stehfest numerical inversion is used to calculate the real space solution. Then, the double logarithmic chart of dimensionless borehole wall pressure and pressure derivative changing with dimensionless time is drawn. The influences of parameters in expressions including stress sensitivity, threshold pressure, and variable skin factor on pressure and pressure derivative and productivity are analyzed, respectively. At last, the method is applied to the well-test interpretation of low-permeability oil fields in the eastern South China Sea. The interpretation results turn out to be reasonable and can truly reflect the situation of low-permeability reservoirs, which can give guidance to the rational development of low-permeability reservoirs.

1. Introduction

As the exploration of oil and gas gets deeper, the scale of proven reserves in offshore low-permeability oilfields gets larger and larger. Many topics worth study show up in offshore low-permeability reservoirs [1]. One of the most tricky issues is the precise evaluation of the physical parameters, which are variant and hard to predict in the low-permeability reservoirs. The accurate evaluation model needs to be established and can give guidance to the future development of offshore reservoirs.

The seepage mechanism of low-permeability reservoirs is very complex. Due to the stress sensitivity effect and threshold pressure gradient in low-permeability reservoirs [1–3], the current research on the theory and application of seepage flow in offshore low-permeability oil reservoirs has limitation, including two common features: (i) as the two variables, stress sensitivity effect and threshold pressure gradient, both have big influences on seepage, most models only consider one of them as factor; (ii) in order to prevent the blowout, using of the mud with high density in offshore drilling and mud pollution caused by this method is not taken into consideration in most models.

After years of exploration, researchers all over the world have done a lot of work figuring out the seepage mechanism of low-permeability reservoirs. Stress-sensitive effect and threshold pressure gradient are accepted as the most influential factors. At present, there are two main ways to consider
the influence of stress sensitivity: one is to redefine a pseudo-pressure, and the other is to derive from permeability variation function as the productivity of oil and gas well is usually affected by formation permeability [4]. Dou et al. introduced a new program to quantitatively analyze the stress sensitivity of permeability by using the traditional linear analysis method and determined the permeability modulus and other formation parameters based on the concepts of modified pseudoparameters and progressive approach [5]. Many scholars have also done a lot of research work in the stress-sensitive unstable seepage theory and the application of multiple fractured horizontal wells in unconventional oil reservoirs [2, 6–9]. When dealing with the nonlinear term of seepage differential equation, Pedrosa’s variable substitution and perturbation technique are commonly used [10–12]. Zhang and Ambastha [13] further studied the influence of stress-sensitive permeability on unstable pressure dynamics by using the subsection permeability model. Wu and Pruess have applied the integral method [13, 14]. Zeng and Zhao established an approximate equation to analyze the unstable pressure performance of reservoirs [15]. Zhang et al. put forward a well-test interpretation model of natural fractured reservoir considering stress sensitivity and threshold pressure gradient, but the model only got numerical solution, not analytical solution [16]. Qanbari and Clarkson provided a method for analyzing the transient linear flow in stress-sensitive reservoirs by using production data, but this method could not stably display all flow stages of pressure dynamic state (such as quasisteady flow stage) [17].

Several phenomena of non-Darcy flow in low-permeability reservoirs (threshold pressure gradient, low-velocity non-Darcy) can be explained theoretically from the perspective of force [18]. When the pressure is particularly high, the adsorption of liquid molecules on the rock surface is strong, and the liquid molecules adsorbed on the rock surface cannot move. The higher the pressure, the lower the liquid permeability, which is the characteristic of threshold pressure gradient. Malhasin discussed the principle of threshold pressure gradient generation from the perspective of micromolecular structure and force [18]. Miller and Low discovered the phenomenon of threshold pressure gradient when discussing the seepage of water in clay [19]. Pascal et al. considered the influence of the differential pressure threshold of low-speed flow and solved the consolidation problem in geotechnical engineering using the finite difference method in the numerical method [20]. Prada and Civan proposed that Darcy’s formula should be corrected for the threshold pressure gradient [21]. Zeng et al. measured the small threshold pressure gradient value of ultralow-permeability reservoirs and proposed a method to determine the value using the pseudothreshold pressure gradient and nonlinear coefficients [3]. Other scholars have also done a lot of work in experimental methods and new algorithms [22–26], which provides a strong theoretical basis for the study of seepage laws in low-permeability reservoirs.

The cost and risk of offshore drilling are higher than land drilling. To prevent blowouts, the pressure of the mud column must be greater than the reservoir pressure; thus, mud particles and filtrate can enter the reservoir. When the filtrate is not compatible with the minerals of the reservoir, especially the clay mineral fluid, it will cause the clay minerals to swell, disperse, precipitate, etc. If blockage of the flow channel happens, it will result in the decrease of permeability and increase of skin surface and even can lower the productivity of the reservoir. For wells in low-permeability reservoirs, when mud invasion happens, the particles in the formation gradually accumulate in the near-well zone. Thus, the skin factor will slowly increase obviously as time passes. Therefore, it is inappropriate to use constant skin factor in the models, which will come up with wrong well-test interpretation results [27]. However, in the current mathematical model of well-test interpretation, skin factor is usually regarded as a constant [28].

Therefore, we consider seepage in offshore low-permeability oilfield as a comprehensive result of threshold pressure gradient, stress sensitivity, and mud pollution. In this paper, the author established the mathematical model of unsteady seepage in offshore low-permeability oilfield by using the law of conservation of mass and Laplace transform. It is developed not only considering the influences of threshold pressure gradient and stress sensitivity but also considering the influence of mud pollution in the actual seepage process of offshore low permeability. By drawing the typical curves with different influencing factors by numerical inversion and programming, their influences are analyzed. In addition, from the result of the interpretation in actual low-permeability oilfield, this model is more reasonable than conventional methods and can give guidance to the rational development of low-permeability oilfield.

2. Establishment and Solution of Unsteady Seepage Mathematical Model for Low-Permeability Oilfield

2.1. Influencing Factors. In order to further analyze the influence of the three factors of threshold pressure gradient, stress sensitivity, and mud pollution and to study reasonably mathematical representation methods, this paper first uses the offshore low-permeability core experiment to study the expressions of stress sensitivity and threshold pressure gradient. In addition, the mud intrusion process is modeled by a numerical core simulation method, and the reasonable expression of three factors is determined.

2.1.1. Research on Stress-Sensitive Influence of Offshore Low Permeability. 13 cores (interval 4158.6–4171.9 m) were taken from a low-permeability offshore block, and stress sensitivity tests were carried out on 7 of them. The specific experimental process is as follows: preheat the equipment and check the air tightness of the device; test experiment of variable confining pressure of pseudocore; and test experiment of stress sensitivity of matrix core under variable confining pressure. Then, six confining pressure test points were set with confining pressures of 35, 40, 45, 50, 55, and 60 MPa. During the test, the confining pressure changed continuously, and the single point lasted for 40 minutes. The stress sensitivity curves of matrix core was drawn, and the stress-sensitive damage rate was analyzed (Table 1).
It can be seen from the above figure (Figures 1–3) the exponent type has a wide range of changes in the early and middle stages. The change rates of power formula are larger in the early stage and smaller in the later stage. The overall change of the SS method is relatively smooth and uniform, the rate of change in the previous stage is smaller than that in the former two, and there is no smooth section in the two later stage’s curves. Through the above analysis, it is concluded that the exponential form fitting is the best, so the exponential form method is used to fit the experimental data.

Figure 4 shows the change of permeability of 4 cores during pressure increasing and pressure decreasing stages and proves that permeability is sensitive to effective stress. It can be seen from the figure that the lower the permeability, the greater the permeability decline rate when the effective stress increases: sample No.6 has the largest permeability decline rate, while sample No.10 has the smallest permeability decline rate. Therefore, the hysteresis effect is obvious, and the lower the permeability is, the more obvious the nonlinear effect is. The confining pressure range 38-60 MPa is measured, and 60 MPa is used as reference point to fit the experimental data.

The exponential model is the most common mathematical model of permeability variation. Assuming that the deformation of reservoir fluid viscosity and permeability is changing due to stress sensitivity accords with the Hooke elastic rheological law, similar to the definition of compressibility, Pedrosa defines the permeability modulus, which is used to describe the relationship between reservoir permeability and pressure and can be written as homogeneous medium.

\[ K = K_0 e^{-\delta(P_i - P_f)}. \] (1)

The results of stress sensitivity test got the relationship between stress sensitivity coefficient and reference point permeability. As shown in the figures, the stress sensitivity coefficient and the reference point permeability satisfy the exponential relationship, and the correlation is good. The weaker the nonlinear effect, the apparent permeability changes linearly with the pressure. The smaller the permeability is, the stronger the nonlinear effect is, and the more obvious the influence of the pore compression on the flow. The nonlinear effect in the boosting process is much greater than that in the depressurization process.

<table>
<thead>
<tr>
<th>Table 1: Permeability characterization method.</th>
<th>$\frac{K}{K_0} = ae^{-bx+c}$</th>
<th>$\frac{K}{K_0} = a\sigma_{eff}^{-b}$</th>
<th>$(\frac{K}{K_0})^2 = 1 - Ss \log \left( \frac{\sigma_{eff}}{\sigma_{ref}} \right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential form</td>
<td>Power formula</td>
<td>Stress-sensitive coefficient method</td>
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**Figure 1:** Stress-sensitive index.

**Figure 2:** Stress-sensitive power function.

**Figure 3:** Stress-sensitive coefficient method.
2.1.2. Characterization of the Influence of Low-Permeability Offshore Pressure Gradient. Through the derivation of the threshold pressure gradient characterization method (Appendix B), the constants $c_1$ and $c_2$ have rich connotations. Through the derivation process, it can be seen that $c_1$ reflects the existence of the yield stress of the fluid and the influence of the boundary layer on seepage, and $c_2$ mainly reflects the influence of the boundary layer on seepage. Because of the microscale flow effect, the influence of surface force on seepage cannot be ignored. The greater the surface force, the greater the binding force of boundary layer on fluid, which makes the fluid show stronger non-Newtonian and finally leads to the more deviation of seepage law from Darcy’s law. At the same time, the greater the binding force, the greater the offset displacement force, making the fluid more easily adsorbed on the boundary layer, making the boundary layer thicker and further aggravating the microscale flow effect, which is a mutual coupling effect.

13 cores of WC oil group in LF were taken, and 7 of them were used in the threshold pressure gradient experiment of oil phase. In the flow experiment of low-permeability cores, the “pressure difference-flow method” is used. In the Cartesian coordinate system of flow and pressure gradient, not only a straight line is presented but also an upturned curve and a straight line are formed, so as determining the threshold pressure gradient. As shown in Figure 5, the experimental results of each core are fitted based on the nonlinear seepage model equation, the slope of the straight line segment is the permeability, and the intercept is the threshold pressure gradient.

The permeability obtained by experimental fitting (slope of linear segment) is different from that measured by gas. It is generally smaller because of the use of crude oil as medium. The permeability measured by liquid is generally lower than that measured by gas (absolute permeability). The lower the absolute permeability is, the greater the decrease of liquid permeability will be. This reflects that the denser the reservoir, the greater the additional resistance to liquid. Therefore, for low-permeability tight reservoirs, the relationship between fluid seepage velocity and pressure gradient is no longer a straight line passing through the origin. The velocity equation of low-speed non-Darcy seepage considering the

![Figure 4: Permeability variation and fitting of core during step-up and down-pressure stage.](image)
threshold pressure gradient can be described by the following pseudothreshold pressure gradient model:

\[
v = \begin{cases} 
  \frac{k}{\mu} (\nabla P - G), & \nabla P > G \\
  0, & \nabla P \leq G 
\end{cases}.
\]  

(2)

2.1.3. Study on Pollution Impact Caused by Offshore Mud Intrusion. Figure 6 shows the core numerical simulation. In order to clarify the law of core pollution by drilling fluid, a core numerical model is established, the experimental results of core pollution by drilling fluid are fitted, and the correlation between core damage degree and drilling fluid flux is obtained.

The basic experimental information is as follows: inject polymer drilling fluid into the core and measure the core permeability at the same time. After injecting the polymer drilling fluid, the core permeability decreased from 13.0 mD to 5.4 mD, and the permeability loss was 58.3%. The core-damaged model obtained by fitting is as follows:

\[
\frac{K_{\text{si}}}{K_s} = 1 - \alpha \psi.
\]

(3)

The radial grid model of single well is established to simulate the invasion of drilling filtrate into formation, and the
formation mechanism of pollution zone and its influence on productivity are studied. Combined with the basic geological characteristics of the target area, the theoretical model is established. Figure 7 shows the recovery under different conditions. The following two situations are compared: polluting reservoir and nonpolluting reservoir after mud filtrate immersion in reservoir.

1) If the pollution of drilling fluid around the well is not considered, the invasion of drilling fluid will be little further, but this shows very tiny difference. (2) In all reservoirs with different original permeability value, the recovery degree will decrease when pollution happens.

Figure 8 shows the influence of soaking time on pollution. Through simulation, it can be found that the higher the permeability, the smaller the skin caused by stress sensitivity; the larger the skin caused by pollution, the larger the pollution radius; the more mud invasion, and the higher the recovery loss.

As shown in Figures 9 and 10, reservoir pollution caused by mud invasion during drilling is usually the result of physical and chemical effects. If the average permeability of polluted area is regarded as a function of time, it will change obeying the rule as follows: (i) in the early stage, because the fine particles in the mud fluid can easily accumulate near the well zone, the average permeability in the polluted area will change rapidly; (ii) as time passes by, the number of this kind of particles is getting smaller, so the change of permeability will get slower; and (iii) the average permeability of the polluted area will become a constant. According to the results of numerical core fitting and the shape of pollution invasion curve, there is a certain exponential relationship between soaking time and permeability damage; the law is summarized as follows:

\[ \frac{K_r}{K_i} = 1 - a \left( 1 - e^{-u_i (t + T_0)} \right). \]  

2.2. Establishment and Solution of Mathematical Model under the Influence of Three Factors

2.2.1. Assumptions. Suppose the plane is infinite. A production well in the homogeneous layer of thickness \( h \) is produced at the rate of \( q \). The original formation pressure is \( p_i \). The bottom-hole flow pressure is \( p_{wf} \). The permeability of the reservoir is \( K \). The porosity is \( \phi \). The radius of the wellbore is \( r_w \). The fluid viscosity is \( \mu \). The comprehensive compression factor is \( c_r \). The skin factor is \( S \). The wellbore storage coefficient
Non-Darcy radial seepage occurs in low-permeability reservoirs.

2.2.2. Establishment and Solution of Mathematical Model. Since the threshold pressure gradient and stress sensitivity of low-permeability fluid cannot be ignored, and porous media and fluid are both compressible, it is necessary to consider them as porous media and elastic fluid. The basic differential equation of unstable seepage in low-permeability reservoir considering two factors at the same time can be obtained as follows (Appendix A):

$$\frac{\partial^2 P}{\partial R^2} + \frac{1}{R} \frac{\partial P}{\partial R} + \delta \frac{G}{R} \frac{\partial^2 P}{\partial T^2} = \frac{\phi_i H}{K_i} \left( \epsilon_i + \epsilon_m \right) e^{(P_i - P)} \frac{\partial P}{\partial T}. \tag{5}$$

The above formula is a partial differential equation with strong nonlinearity. Since the simplified partial differential equations are still relatively nonlinear, in order to facilitate the production of the plates and form a unified pressure and pressure derivative plate, a dimensionless conversion is required. The following dimensionless definition is introduced (Table 2).

After dimensionless transformation, the simplified differential equation of dimensionless seepage flow considering both the threshold pressure gradient and stress sensitivity is obtained:

$$\frac{\partial^2 P_D}{\partial P_D^2} + \frac{1}{R_D} \frac{\partial P_D}{\partial R_D} - \delta_D \left( \frac{\partial P_D}{\partial R_D} \right)^2 + \frac{G_D}{R_D^2} = \delta_D^e P_D D \frac{\partial P_D}{\partial T_D}. \tag{6}$$
The table 2: Dimensionless definition.

<table>
<thead>
<tr>
<th>Dimensionless radius</th>
<th>$R_D = \frac{k}{K_s}$</th>
<th>Dimensionless time</th>
<th>$T_D = \frac{kT}{\mu_s(C_w + C_m)R_s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimensionless pressure</td>
<td>$P_D = \frac{2\pi hK_s}{Q\mu}(P_i - P)$</td>
<td>Dimensionless pseudostress sensitivity coefficient</td>
<td>$\delta_D = \frac{Q\mu}{2\pi hK_s}$</td>
</tr>
<tr>
<td>Defining pseudopressure gradient</td>
<td>$G_D = \frac{2\pi hR_w}{Q\mu}e^{-p_0\delta_D}G$</td>
<td>Dimensionless pollution radius</td>
<td>$R_{D_D} = \frac{R_s}{R_w}$</td>
</tr>
</tbody>
</table>

There is a gradient square term on the left side of the above equation, which is a nonlinear differential equation and cannot be solved directly. Make the following transformation:

$$m_D = -\frac{1}{R_D} \ln \left[ 1 - \delta_D \xi \left( \delta_D, T_D \right) \right].$$  

(7)

According to the canonical perturbation theory, considering that the dimensionless permeability modulus is usually very small ($\delta_D \ll 1$), the zero-derivative perturbation solution can meet the engineering accuracy requirements, so it can be simplified:

$$\frac{\partial^2 \xi}{\partial R_D^2} + \frac{1}{R_D} \frac{\partial \xi}{\partial R_D} + \frac{G_D}{R_D} = \frac{\partial \xi}{\partial T_D}. $$

(8)

After conversion, the corresponding definite conditions under the conditions of skin factor and well storage are

Internal boundary conditions: $\frac{\partial \xi}{\partial R_D} |_{R_D = 1} = - (G_D + 1) + C_D \frac{\partial \xi}{\partial T_D}.$

External interface condition: $\lim_{R_D \to \infty} [\xi(R_D, T_D)] = 0.$

(9)

The influence of mud contamination is not considered in the current low-permeability mathematical model, and the corresponding initial conditions are $\xi_{u_D}(T_D) = \xi(R_D, T_D) = S(\partial \xi / \partial R_D)|_{R_D = 1}$. When the mud invasion is not considered, the skin $s$ is usually regarded as a fixed value. In this paper, considering the influence of mud pollution, the difference between the initial time of mud threshold and the pressure recovery test is $T_D$, and the dimensionless immersion time is introduced: $T_{u_D} = K_{u_D}/\phi(C_i + C_m)R_s^2$. In the mud invasion process, the dimensionless permeability initial rate of change is $u_{D} = u_0\phi(C_i + C_m)R_s^2/K_s$.

Above the partial differential equation is a nonhomogeneous second-order linear ordinary differential equation, where $-\left(\frac{\lambda_{BD}}{sr_D}\right)$ is a nonhomogeneous term, and the general solution of the inhomogeneous equation is the sum of the general solution and the particular solution of the inhomogeneous equation. In this paper, the constant variation method is used to find the particular solution (Appendix A).

In order to describe the permeability recovery characteristics of polluted areas, the skin factor is defined as

$$S = \left( \frac{K}{K_s} - 1 \right) \ln \frac{R_s}{R_w}. $$

(10)

Assuming that the radius of the polluted area is constant, that is, $R_s$ is a fixed value, defining the skin factor $S_i$ at the initial time ($t = 0$) as a constant, and $S$ is a function of time (because $K_s$ is a function of time), the relationship between $S$ and $S_i$ can be established by using the relationship between $K_{u_D}$ and $K_{u_D}$; then, there is a relational expression:

$$\frac{K_{u_D}}{K_s} = S + \ln \frac{R_s}{R_w}. $$

(11)

According to the relationship between formula (10) and formula (11), the following relationship can be obtained:

$$\frac{K_{u_D}}{K_s} = -\alpha \left( 1 - e^{-(r_i/r_w)^p} \right) + 1 = \frac{S + \ln (r_s/r_w)}{S_i + \ln (r_s/r_w)}. $$

(12)

Thus, the relationship between the skin and time can be obtained:

$$S = \left[ 1 - \alpha \left( 1 - e^{-(r_i/r_w)^p} \right) \right] \left( S_i + \ln \frac{r_s}{r_w} \right) - \ln \frac{r_s}{r_w}. $$

(13)

A partial differential equation can be obtained. Based on the partial differential equation, Laplace transform is applied to obtain the partial differential equations and boundary conditions:

$$\frac{\partial^2 \tilde{\xi}}{\partial R_D^2} + \frac{1}{R_D} \frac{\partial \tilde{\xi}}{\partial R_D} + \frac{G_D}{Z* R_D} = \tilde{\xi};$$

$$\left. \frac{\partial \tilde{\xi}}{\partial R_D} \right|_{R_D = 1} = \frac{-(G_D + 1)}{Z} + C_D \tilde{\xi}_{u_D}$$

$$\left. \tilde{\xi} \right|_{R_D \to \infty} = 0.$$  

(15)

(16)

Based on the initial condition and formula (15), we can get:

$$\tilde{\xi}_{u_D}(z) = \tilde{\xi}(R_D, z)|_{R_D = 1} - La \left( S_i - \alpha (S_i + \ln R_{D_D}) \right) + \alpha (S_i + \ln R_{D_D})e^{-\nu_1(T_D + T_{u_D})} \left. \frac{\partial \xi}{\partial R_D} \right|_{R_D = 1}.$$  

(17)


La is the symbol of laplace transformation, and the dimensionless bottom-hole flowing pressure in the Laplace space can be obtained, taking into account the threshold pressure gradient.

\[ Q_\text{D} = \frac{(G_D + 1/2z)I_1(\sqrt{2}) (1 + C_D S M_1) - (\pi G_D C_D / 2\sqrt{2}) I_0(\sqrt{2})}{(1 + C_D S M_2) \cdot K_1(\sqrt{2}) \cdot \sqrt{z} + C_D z K_0(\sqrt{2})} \]

\[ z_{wD}(s) = \frac{(G_D + 1/2z)I_1(\sqrt{2}) (1 + C_D S M_1) - (\pi G_D C_D / 2\sqrt{2}) I_0(\sqrt{2})}{(1 + C_D S M_2) \cdot K_1(\sqrt{2}) \cdot \sqrt{z} + C_D z K_0(\sqrt{2})} \]

\[ K_0(\sqrt{2}) + \frac{G_D}{z} I_0(\sqrt{2}) + \frac{\pi}{2\sqrt{2}} + |S_i - \alpha(S_i + \ln R_\text{D})| \]

\[ \cdot \left[ (G_D + 1/2z)I_1(\sqrt{2}) (1 + C_D S M_1) - (\pi G_D C_D / 2\sqrt{2}) I_0(\sqrt{2}) \right] K_1(\sqrt{2}) \sqrt{z} - \frac{\pi G_D}{2\sqrt{2}} I_1(\sqrt{2}) - \alpha(S_i + \ln R_\text{D}) \]

\[ \cdot \left[ (G_D + 1/2z)I_1(\sqrt{2}) (1 + C_D S M_1) - (\pi G_D C_D / 2\sqrt{2}) I_0(\sqrt{2}) \right] K_1(\sqrt{2} + u_\text{ID}) \sqrt{z} + u_\text{ID} - \frac{\pi G_D}{2(\sqrt{z} + u_\text{ID})} I_1(\sqrt{z} + u_\text{ID}) \right] \]

\[ (18) \]

Among them are

\[ M_1 = S_i - \alpha(S_i + \ln R_\text{D}) \left( 1 + \frac{z}{z + u_\text{ID}} I_1(\sqrt{z} + u_\text{ID}) I_1(\sqrt{2}) e^{-u_\text{D}T_\text{D}} \right), \]

\[ M_2 = S_i - \alpha(S_i + \ln R_\text{D}) \left( 1 + \frac{\sqrt{z} + u_\text{ID}}{\sqrt{z}} K_1(\sqrt{z} + u_\text{ID}) K_1(\sqrt{2}) e^{-u_\text{D}T_\text{D}} \right). \]

\[ (19) \]

In order to further illustrate the scalability of the model, the model is simplified by using an asymptotic solution. When \( u_\text{ID} \) and \( G_D \) approach 0, only stress-sensitive models are considered. The simplified result is the same as the textbook [29]; \( u_\text{ID} \) and \( \alpha \) approach what value? When it is 0, it is a model that only considers the threshold pressure gradient. The simplified result is in consistent with the textbook [29], which confirms the scalability of the paper.

3. Analysis and Application of Well-Test/Productivity Curve Considering Stress Sensitivity, Threshold Pressure, and Reservoir Damage

3.1. Typical Curve Analysis. By using the Stehfest numerical inversion method, the dimensionless bottom-hole pseudopressure in the Laplace space is inverted, and the double logarithmic typical curve of unsteady seepage bottom-hole pseudopressure in low-permeability reservoir considering stress sensitivity, threshold pressure, and reservoir damage can be obtained by programming (Figures 11–16). Moreover, the threshold pressure gradient, the threshold pressure gradient, stress sensitivity, the initial gradient rate of permeability, and the influence of well storage:

![Figure 11: Dimensionless productivity curve under different threshold pressure gradients.](image)

![Figure 12: Typical curves under different threshold pressure gradients.](image)
gradient, the faster the productivity decreases. Figure 12 shows that the pressure and pressure derivative curves increase with the increase of the threshold pressure gradient ($G$), and the pressure derivative curves are close to 0.5 horizontal line when not considering the influence of the threshold pressure gradient. The existence of the threshold pressure gradient makes the pressure derivative curves of low-permeability reservoirs no longer tend to the 0.5 horizontal line. When the threshold pressure gradient gets larger, the upward warping becomes bigger. Therefore, the influence of threshold pressure gradient in low-permeability reservoirs cannot be ignored.

From the productivity curve in Figure 13, the stress sensitivity mainly affects productivity in the middle and late periods. The greater the stress sensitivity, the earlier the impact and the faster the productivity decreases. Figure 14 shows that in low-permeability reservoirs, the pressure and pressure derivative curves rise with the increase of threshold pressure gradient ($G$) and stress sensitivity ($\delta$). The larger threshold pressure gradient ($G$) and stress sensitivity ($\delta$) indicate a more difficult condition of the fluid flow. As the curve upward warping gets higher, the bigger influence of threshold pressure gradient and stress sensitivity in low-permeability reservoirs should be considered.

It can be seen from the productivity curve in Figure 15 that the mud pollution coefficient has great influence on productivity. Mud invasion has an impact on productivity from the beginning, and the more serious the invasion, the faster the productivity declines. Figure 16 shows that in low-permeability reservoirs, the greater the mud pollution ($\alpha$), the greater its influence on permeability will be. This result indicates that more serious mud pollution and larger skin
show similar effect as the skin mainly affects the early hump stage of the pressure derivative curve.

All three factors have certain influence on productivity and should not be ignored in well-test interpretation. Moreover, since the influence stages of each factor on productivity and pressure are different, chart analysis can have certain guiding significance for low-permeability oilfield development.

3.2. Application

3.2.1. Comparison of Interpretation Results of Examples. In order to illustrate the universality of the model, the model is applied in the conventional oil reservoir. Taking the offshore HZ reservoir as an example, the permeability of the commercial well-test interpretation software is 395.8 mD. The result explained in this paper model is 400 mD and the explained skin is -1.4. The explained results are very close to reality. This demonstrates the rationality of the model derived in the article to be interpreted in conventional oil reservoirs.

Through the analysis of multiple low-permeability wells offshore, the upturn at the end of the pressure derivative curve is typical in low-permeability reservoirs. However, when fitting the curve, the most conventional well-test interpretation software adopts the radial composite model without considering the influence of the special threshold pressure gradient, stress sensitivity, and mud pollution in the low-permeability formation. It results in a serious discrepancy between the well-test interpretation values and the actual situation, which is mainly manifested in two aspects: as shown in Table 3: (1) when there is no stimulation measures, the skin interpreted of some wells are negative; (2) the permeability of well-test interpretation is far lower than that of logging.

To solve the problems, the offshore low-permeability wells are reinterpreted using our model. Taking an offshore low-permeability reservoir as an example, the basic parameters of DST well-test interpretation are shown in Table 4.

According to the basic parameter (Table 4), the parameter fitting range is determined, and the history fitting is finished. Figure 17 shows the good fitting results. Table 5 is the basic formation parameter information of low-permeability reservoir after fitting.

Table 6 is a comparison of the interpretation results of the method in this paper and the conventional method. First of all, the permeability interpreted by this method is closer to that interpreted by logging than the conventional method. In addition, the skin interpreted by the conventional method is -1.01, which is inconsistent with the actual situation because the well has no stimulation measures and mud pollution. The method in this paper is more in line with the percolation theory of low-permeability reservoir. The skin after pollution is explained to be 5.5, which is more reasonable than the commercial well-test software. Permeability and skin affect oil reservoir production allocation, which is a key part in the development and production process. Skin factor in near-well zone is an important parameter to evaluate oil well productivity and completion efficiency. Through skin factor, formation conditions can be understood more deeply.

3.2.2. Skin Decomposition. Taking the XJ Oilfield in this paper as an example, considering the influence of threshold pressure gradient, stress sensitivity, and mud invasion, the interpreted skin factor is bigger than the conventional method, which can provide a more accurate basis for reservoir protection and stimulation measures. It makes the calculation of pollution range and degree, and the prediction of stimulation rate and output more accurate. By applying good reservoir protection measures, the output of single well and oilfield can be increased, which will be helpful to the reasonable development evaluation of low-permeability oilfield and promote the development of low-permeability oilfield.

Figure 18 shows the relationship between skin and invasion time during mud invasion process. The ultimate goal of formation testing is to obtain reservoir seepage parameters, formation pressure, and production information. Engineers can use skin factor to see the damage degree of reservoir. Based on that, the effect of increase measures can be evaluated by predicting the increase oil rate and giving evidence of the implementation. The skin factor obtained by formation test is usually the sum of the real skin damage factor and various pseudo-skin factors caused by various factors during drilling and completion. If the sum of the skins is used in the evaluation of reservoir damage degree, it will inevitably lead to deviation to the real value and poor effect of the stimulation measures. The total skin factor of reservoirs changes with different physical properties. Only through the fine decomposition of skin factor can the skin effect produced by each link be clearer, and the main reasons and links that affect the total skin factor can be found.

Based on the method provided in this paper, the relationship between skin and mud invasion time can be obtained by curve fitting. The initial skin is 5.5, and the total skin after the final mud invasion is 6.87. According to the relationship curve, the mud invasion skin (s) is equal to 1.37, and the ratio of mud invasion skin is 20.0%.

Figure 19 shows the factors affecting the XJX-2Sa well. In order to further decompose the threshold pressure skin and stress-sensitive skin, so as to evaluate the influence of various factors, it is necessary to decompose the initial skin 5.5, which can be realized by the following steps:

(1) Based on the above fitting parameters, when the skin is 5.5, the total pressure drop ΔP considering the threshold pressure and stress sensitivity is obtained

(2) When the threshold pressure skin is S1 (stress sensitivity is 0), the pressure drop only considering the threshold pressure gradient (ΔP_{11}) is calculated. At the same time, when the stress-sensitive skin is 5.5-S1 (the threshold pressure gradient is 0), the pressure drop only considering stress sensitivity (ΔP_{12}) is obtained

(3) When the threshold pressure skin is S2 (stress sensitivity is 0), the pressure drop only considering the threshold pressure gradient (ΔP_{21}) is calculated. At
the same time, when the stress-sensitive skin is 5.5-S2 (the threshold pressure gradient is 0), the pressure drop only considering stress sensitivity ($\Delta P_{22}$) is obtained

(4) In this way, different $\Delta P_{n1}$ and $\Delta P_{n2}$ can be obtained through a large number of different $S_{n}$. By calculating $\sqrt{[(\Delta P^2 - (\Delta P_{n1} + \Delta P_{n2})^2)]}$, when the standard deviation ($S_{n}$) is the smallest, $S_{n}$ is the $S^*$ that we need to get, so that we can get the decomposed threshold pressure skin $S^*$ and stress-sensitive skin (5.5 – $S^*$). In this paper, the threshold pressure gradient is 4, and the stress-sensitive skin is 1.5. The threshold pressure skin accounts for 58.2%, and the stress-sensitive skin accounts for 21.8%.

Table 3: Well-test interpretation statistics for offshore low-permeability wells.

<table>
<thead>
<tr>
<th>Well name</th>
<th>Logging permeability (mD)</th>
<th>Well-test permeability (mD)</th>
<th>Skin factor</th>
<th>Well-test interpretation model</th>
<th>Stratigraphic model</th>
<th>Boundary</th>
<th>Drilling fluid density (g/cm³)</th>
<th>Drilling fluid immersion time (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.6</td>
<td>4.4</td>
<td>21.4</td>
<td>Vertical</td>
<td>Composite</td>
<td>Fault</td>
<td>1.3</td>
<td>&lt;1200</td>
</tr>
<tr>
<td>2Sa</td>
<td>4.5</td>
<td>1.1</td>
<td>-1.01</td>
<td>Vertical</td>
<td>Composite</td>
<td>Infinity</td>
<td>1.08</td>
<td>48</td>
</tr>
<tr>
<td>2</td>
<td>10.2</td>
<td>3.5</td>
<td>2.08</td>
<td>Vertical</td>
<td>Composite</td>
<td>Infinity</td>
<td>1.25</td>
<td>600</td>
</tr>
<tr>
<td>3d</td>
<td>16</td>
<td>0.9</td>
<td>-0.28</td>
<td>Vertical</td>
<td>Composite</td>
<td>Infinity</td>
<td>1.3</td>
<td>672</td>
</tr>
</tbody>
</table>

Table 4: Basic parameters of DST well-test interpretation in well XJX-2Sa.

<table>
<thead>
<tr>
<th>Parameter category</th>
<th>Parameter name</th>
<th>Symbol</th>
<th>Unit</th>
<th>Parameter value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Well parameters</td>
<td>Well diameter</td>
<td>$R_w$</td>
<td>m</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>Volume factor</td>
<td>$B_o$</td>
<td>1</td>
<td>1.29</td>
</tr>
<tr>
<td></td>
<td>Viscosity of crude oil</td>
<td>$\mu_o$</td>
<td>mPa s</td>
<td>0.58</td>
</tr>
<tr>
<td></td>
<td>Compressibility of crude oil</td>
<td>$C_o$</td>
<td>1/MPa</td>
<td>$1.39E-3$</td>
</tr>
<tr>
<td>Formation fluid parameters</td>
<td>Average porosity</td>
<td>$\varphi$</td>
<td>%</td>
<td>11.2</td>
</tr>
<tr>
<td></td>
<td>Effective thickness</td>
<td>$h$</td>
<td>m</td>
<td>17.80</td>
</tr>
<tr>
<td></td>
<td>Formation compressibility</td>
<td>$C_f$</td>
<td>1/MPa</td>
<td>$1.39E-3$</td>
</tr>
</tbody>
</table>

Figure 17: Fitting curve of double logarithmic well-test in DST pressure recovery interpretation section of XJX-2Sa.

Table 5: Result interpretation table of the XJX-2Sa well.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wellbore storage (m³/MPa)</td>
<td>0.011</td>
</tr>
<tr>
<td>Skin factor</td>
<td>5.5</td>
</tr>
<tr>
<td>Permeability (mD)</td>
<td>5.33</td>
</tr>
<tr>
<td>Stress sensitivity coefficient (1/MPa)</td>
<td>0.002162</td>
</tr>
<tr>
<td>Threshold pressure gradient (MPa/m)</td>
<td>0.06813</td>
</tr>
<tr>
<td>Mud immersion rate</td>
<td>0.0005</td>
</tr>
<tr>
<td>Pollution damage coefficient</td>
<td>-0.12</td>
</tr>
<tr>
<td>Polluted well diameter (m)</td>
<td>30</td>
</tr>
<tr>
<td>Pressure recovery test lag time (h)</td>
<td>48</td>
</tr>
</tbody>
</table>
As shown in Figure 20, in order to further confirm the proportion of the various factors and verify the rationality of the method, this paper is verified by the numerical simulation method. On the basis of a low-permeability geological model of an oil reservoir in the South China Sea, the stress sensitivity and threshold pressure gradient laws of the target area are added to the numerical simulator, and the process of drilling filtrate invading the formation is simulated for 11 times, with permeability values ranging from 1 mD to 40 mD. Combined with the pressure curve under the condition of pollution, the skin factor under the condition of pollution is calculated and decomposed, and the pollution is quantitatively evaluated, so that the skin maps corresponding to reservoirs with different permeability can be obtained. Then, the skin factor obtained under different permeability is normalized by the total skin, and the contribution graph of different factors to the skin can be obtained.

As shown in Figure 21, we can see that the permeability 5.33 mD calculated by this method is about 19%, the stress-sensitive skin is about 26%, and the threshold pressure is about 55%. When the permeability is 5.33 mD, the threshold pressure skin accounts for the largest proportion, the stress-sensitive skin is the second, and the mud pollution skin is the smallest. The results of simulation match with the result using mathematic model in this paper, which proves that the method in this paper has wide applicability.

The analysis of the application effect of skin factor decomposition shows that the causes of various skin effects and the proportion of injuries caused by various factors can be clarified through the decomposition of skin factor. Taking the XJX-2 well in this paper as an example, we can see the proportion of skin-influencing factors in this 4low-

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Table 6: Comparison table for result interpretation of different wells in the XJX-2Sa well.

<table>
<thead>
<tr>
<th>Case</th>
<th>Well name</th>
<th>Test horizon</th>
<th>Logging permeability mD</th>
<th>Well-test permeability mD</th>
<th>Explain skin</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional method</td>
<td>XJX-2Sa</td>
<td>WC</td>
<td>4.5</td>
<td>1.1</td>
<td>-1.01</td>
</tr>
<tr>
<td>Methods in this paper</td>
<td>XJX-2Sa</td>
<td>WC</td>
<td>4.5</td>
<td>5.33</td>
<td>5.5</td>
</tr>
</tbody>
</table>

---

![Figure 18: Relationship between skin and invasion time in mud invasion process.](image1.png)

![Figure 19: Factors affecting the XJX-2Sa well.](image2.png)

![Figure 20: Skin corresponding to reservoirs with different permeability.](image3.png)

![Figure 21: Factors affecting the XJX-2Sa well.](image4.png)
permeability well. In order to ensure the full release of production capacity of low-permeability well, it is suggested that the threshold pressure gradient should be appropriately reduced by water injection and fracturing. Therefore, the skin factor decomposition technology has practical value in reservoir engineering.

4. Conclusion

(1) Since the effects of threshold pressure gradient, stress sensitivity, and reservoir pollution on seepage flow in low-permeability reservoirs cannot be ignored, to more precisely explain and predict its flow mechanism, in this paper, a mathematical model of unsteady pressure seepage flow is derived and established.

(2) In this paper, the numerical model is solved by the Laplace transform, and the double logarithmic chart of dimensionless borehole wall pressure and pressure derivative changing with dimensionless time is drawn by programming. After studying the sensitivity of the factors, it is found that threshold pressure gradient, stress sensitivity, and reservoir pollution in low-permeability reservoirs have different effects on different stages of pressure performance and should be considered, respectively.

(3) If not considering the threshold pressure gradient, stress sensitivity reservoir pollution, and the permeability, the skin factor calculated by the model will be lower than reality. Considering these factors, the interpretation results will be more reasonable. The results are applied to the well-test interpretation of the low-permeability oilfield in the eastern South China Sea, which successfully reflect the actual situation of low-permeability reservoirs. It has a certain guiding significance for the rational development of low-permeability reservoirs.

(4) The fine decomposition of skin factor can give more accurate evidence for applying reservoir protection and stimulation measures. It can also help the prediction of the effect of the measures, including the calculation of pollution range and degree and the prediction of stimulation rate and output. By separately evaluating the influence of each factor, the real effect and level each factor contributes to the change of the total skin factor can be clearly found.

Appendix

A. Model Derivation

According to the seepage mechanics of oil and gas layers, the continuity equation is established based on the principle of conservation of matter in stratigraphic units. No matter the formation, fluid nature and movement state, continuity equation has the same form in mathematical expressions [29].

$$\frac{\partial (\rho \phi)}{\partial T} + \nabla \cdot (\rho \vec{v}) = 0. \quad (A.1)$$

The flow threshold pressure gradient and stress sensitivity of low-permeability fluids cannot be ignored. Taking into account these two factors, the equation of motion can be obtained from the formula (1) and (2):

$$v = -\frac{K_i}{\mu} e^{-\delta i(\rho_i - P)} (\nabla P - G). \quad (A.2)$$

Because porous medium and fluid are compressible, the state equation of porous medium and elastic fluid is needed.

$$\Phi = \Phi_i e^{-C_m(\rho_i - P)} \rho = \rho_i e^{-C_i(\rho_i - P)}. \quad (A.3)$$

The equation of motion (A.2) and equation of state (A.3) can be substituted by the continuous equation (A.1), which can be used to obtain two basic factors.

$$\frac{1}{R} \frac{\partial}{\partial R} \left[ R \Phi_i e^{-C_i(\rho_i - P)} \frac{K_i}{\mu} e^{-\delta i(\rho_i - P)} \left( \frac{\partial P}{\partial R} - G \right) \right] = \frac{\partial}{\partial T} \left[ \Phi_i e^{-C_i(\rho_i - P)} \Phi_i e^{-C_m(\rho_i - P)} \right]. \quad (A.4)$$

Consider that the compression coefficient is much smaller than the stress sensitivity coefficient ($C_m \ll \gamma$). To further simplify the seepage differential equation, we can get

$$\frac{\partial^2 P}{\partial R^2} + \frac{1}{R} \frac{\partial P}{\partial R} + \delta \left( \frac{\partial P}{\partial R} \right)^2 - \frac{G}{R} = \frac{\Phi_i \mu}{K_i} (\gamma_i + \gamma_m) e^{-\delta i(\rho_i - P)} \frac{\partial P}{\partial T}. \quad (A.5)$$

According to the Laplace formula (14), initial condition (15), boundary condition formula (16), and formula (17), the partial differential equation and the condition of definite solution in the Laplace space can be obtained.

$$+ \frac{1}{R^D} \frac{\partial \tilde{\xi}}{\partial R^D} - \tilde{z} \tilde{\xi} = - \frac{G_D}{zR_D},$$

$$\frac{\partial \tilde{\xi}}{\partial r^D} \left|_{r^D=1}^{r^D=\infty} \right. = - \frac{(G_D + 1)}{z},$$

$$\tilde{\xi} \left|_{r^D=\infty} = 0. \quad (A.6)$$

The upper partial differential equation is a nonhomogeneous two-order linear ordinary differential equation. $-G_D/zR_D$. For nonhomogeneous terms, the general solution of the nonhomogeneous equation is the sum of the general solution of the corresponding homogeneous equation and the special solution of the nonhomogeneous equation. The corresponding homogeneous linear ordinary differential equation of order two is

$$\frac{\partial^2 \tilde{\xi}}{\partial R^2} + \frac{1}{R^D} \frac{\partial \tilde{\xi}}{\partial K^D} - z \tilde{\xi} = 0. \quad (A.7)$$
First, the general solution of the corresponding homogeneous two-order linear ordinary differential equation is first solved:

\[
\tilde{\xi} = a I_0\left(\sqrt{zR_D}\right) + b K_0\left(\sqrt{zR_D}\right). \tag{A.8}
\]

Then, the special solution of the nonhomogeneous equation is obtained, and the special solution is obtained by means of the constant variation method:

\[
\tilde{\xi}_* = a_{(RD, z)} I_0\left(\sqrt{zR_D}\right) + b_{(RD, z)} K_0\left(\sqrt{zR_D}\right). \tag{A.9}
\]

By undetermined functions \(a_{(RD, z)}\) and \(b_{(RD, z)}\), the following equations are satisfied [30]:

\[
\begin{cases}
 a_{(RD, z)}' I_0\left(\sqrt{zR_D}\right) + b_{(RD, z)}' K_0\left(\sqrt{zR_D}\right) = 0 \\
 a_{(RD, z)}' K_0\left(\sqrt{zR_D}\right) + b_{(RD, z)}' I_0\left(\sqrt{zR_D}\right) = -\frac{G_D}{R_D} \cdot z.
\end{cases} \tag{A.10}
\]

The equations are obtained:

\[
a_{(RD, z)}' = -\frac{G_D}{z} K_0\left(\sqrt{zR_D}\right), \tag{A.11}
\]

\[
b_{(RD, z)}' = \frac{G_D}{z} I_0\left(\sqrt{zR_D}\right).
\]

The Bessel function relation is used here:

\[
I_0(x)K_1(x) + I_1(x)K_0(x) = \frac{1}{x}. \tag{A.12}
\]

For the upper integral for infinite strata, the special solution of the nonhomogeneous equation can be obtained:

\[
\tilde{\xi}_* = -\frac{G_D}{z} \int_0^{R_D} K_0\left(\sqrt{zrD}\right) d\sqrt{zrD} \cdot I_0\left(\sqrt{zrD}\right) + \frac{G_D}{z} \int_{R_D}^\infty I_0\left(\sqrt{zrD}\right) d\sqrt{zrD} \cdot K_0\left(\sqrt{zrD}\right). \tag{A.13}
\]

Consider the range of the special solution of the nonhomogeneous equation and give the upper and lower limits:

\[
\tilde{\xi}_* = -\frac{G_D}{z} \int_0^{R_D} K_0\left(\sqrt{zrD}\right) d\sqrt{zrD} \cdot I_0\left(\sqrt{zrD}\right) + \frac{G_D}{z} \int_{R_D}^\infty I_0\left(\sqrt{zrD}\right) d\sqrt{zrD} \cdot K_0\left(\sqrt{zrD}\right). \tag{A.14}
\]

The general solution of the corresponding homogeneous equation is discussed again.

From boundary conditions

\[
: \quad \tilde{\xi}_* \bigg|_{RD} = 0 \quad \Rightarrow \quad a = 0. \tag{A.15}
\]

The general solution of the nonhomogeneous equation is that the sum of the general solution of the homogeneous equation and the special solution of the nonhomogeneous equation can be further reduced to

\[
\eta = bK_0\left(\sqrt{zrD}\right) - \frac{G_D}{z} \int_0^{R_D} K_0\left(\sqrt{zrD}\right) d\sqrt{zrD} \cdot I_0\left(\sqrt{zrD}\right) + \frac{G_D}{z} \int_{R_D}^\infty I_0\left(\sqrt{zrD}\right) d\sqrt{zrD} \cdot K_0\left(\sqrt{zrD}\right). \tag{A.16}
\]

Consider the internal boundary conditions of nonhomogeneous equations:

\[
\left. \frac{\partial \tilde{\xi}}{\partial R_D} \right|_{RD=1} = -(G_D + 1) \frac{1}{z}. \tag{A.17}
\]

Based on the special solution of the nonhomogeneous equation after simplification, in order to obtain \(B\) by using the inner boundary condition of the nonhomogeneous equation, the special solution is derived.

\[
\left. \frac{\partial \tilde{\xi}}{\partial R_D} \right|_{RD=1} = -bK_1\left(\sqrt{z}\right) \sqrt{z} - \frac{G_D}{z} \int_0^{R_D} K_0\left(\sqrt{zrD}\right) d\sqrt{zrD} \cdot I_1\left(\sqrt{zrD}\right) \sqrt{z} + \frac{G_D}{z} \int_{R_D}^\infty I_0\left(\sqrt{zrD}\right) d\sqrt{zrD} \cdot K_1\left(\sqrt{zrD}\right) \sqrt{z}. \tag{A.18}
\]

The closed property of the Bessel function is

\[
\int_0^\infty \cos(bx)K_0(ax) dx = \frac{1}{2} \frac{\pi}{\sqrt{a^2 + b^2}}. \tag{A.19}
\]

It can be further obtained:

\[
\left. \frac{\partial \tilde{\xi}}{\partial R_D} \right|_{RD=1} = -\frac{G_D + 1}{z} = -bK_1\left(\sqrt{z}\right) \sqrt{z} \tag{A.20} + \frac{G_D}{z} \frac{\pi}{2} I_1\left(\sqrt{z}\right) \sqrt{z}.
\]

Then \(b\) can be expressed as

\[
b = \frac{(G_D + 1/z) + (G_D \pi / 2z) I_1(\sqrt{z})}{\sqrt{z}K_1(\sqrt{z})}. \tag{A.21}
\]

We can get the general solution of the Laplace space considering both the threshold pressure gradient and the stress-sensitive dimensionless seepage differential equation:
\[ \tilde{\xi} = \frac{(G_D + 1/\epsilon) + (G_D n/2\epsilon) I_1(\sqrt{\epsilon})}{\sqrt{\epsilon} K_1(\sqrt{\epsilon})} K_0(\sqrt{\epsilon} R_D) \]
\[ = \frac{G_D}{\epsilon} \int_{\epsilon}^{\infty} K_0(\sqrt{\epsilon} R_D) dR_D \cdot I_0(\sqrt{\epsilon} R_D) \]
\[ + \frac{G_D}{\epsilon} \int_{R_D}^{\infty} I_0(\sqrt{\epsilon} R_D) dR_D \cdot K_0(\sqrt{\epsilon} R_D). \]

(A.22)

Considering the influence of mud invasion, combined with the above study, pollution is regarded as a function of time. The process of slurry pollution is often changing with time. Referring to the definite conditions, the corresponding conditions for the corresponding conversion of pollution are considered as follows:

Initial condition \( \tilde{\xi}_{\omega D}(T_D) \)
\[ = \left[ \tilde{\xi}(R_D, T_D) - \left( \left[ 1 - \alpha \left( 1 - e^{-\omega(T+T_\omega)} \right) \right] \cdot (S_i + \ln R_D, -\ln R_D) \right) \right] \frac{\partial \tilde{\xi}}{\partial R_D} \bigg|_{R_D=1}, \]

External interface condition \( \lim_{R_D \to \infty} [\tilde{\xi}(R_D, T_D)] = 0. \) (A.23)

Referring to the above variation coefficient method, the general solution of the nonhomogeneous equation can be obtained, and the bottom-hole pressure can be obtained when the Laplace space takes into account the threshold pressure gradient, stress sensitivity, mud pollution (changing skin), and the dimensionless change of well storage influence:

\[ \tilde{\xi}_{\omega D}(z) = \frac{(G_D + 1/\epsilon) + (\pi G_D/2\epsilon) I_1(\sqrt{\epsilon}) (1 + C_0 SM_1) - (\pi G_D C_D/2\epsilon) I_0(\sqrt{\epsilon}) K_0(\sqrt{\epsilon})}{(1 + C_0 SM_2) \cdot K_1(\sqrt{\epsilon}) \cdot \sqrt{\epsilon} + C_0 z K_0(\sqrt{\epsilon})} \]
\[ 
\[ = -\alpha(S_i + \ln R_D) \left. \left[ \frac{(G_D + 1/\epsilon) + (\pi G_D/2\epsilon) I_1(\sqrt{\epsilon}) (1 + C_0 SM_1) - (\pi G_D C_D/2\epsilon) I_0(\sqrt{\epsilon}) K_0(\sqrt{\epsilon})}{(1 + C_0 SM_2) \cdot K_1(\sqrt{\epsilon}) \cdot \sqrt{\epsilon} + C_0 z K_0(\sqrt{\epsilon})} \right] \right] \cdot \left( \sqrt{\epsilon} + u_{\omega D} \right) \left( \sqrt{\epsilon} + u_{\omega D} \right) I_1(\sqrt{\epsilon} + u_{\omega D}). \]

(A.24)

Among them are

\[ M_1 = \alpha(S_i + \ln R_D) \left( 1 + \frac{z}{\sqrt{\epsilon} + u_{\omega D}} I_1(\sqrt{\epsilon} + u_{\omega D}) \right), \]
\[ M_2 = \alpha(S_i + \ln R_D) \left( 1 + \frac{\sqrt{\epsilon} + u_{\omega D}}{\sqrt{\epsilon} + u_{\omega D}} K_1(\sqrt{\epsilon} + u_{\omega D}) \right) \cdot e^{u_{\omega D}}. \]

(A.25)

B. Derivation of Threshold Pressure Gradient

The basic assumptions are as follows: the model is derived from the reference capillary model and the boundary layer theory. There is yield stress in the fluid \( \tau \), laminar flow in the pipe, pressure difference at the ends of the capillary \( \Delta P \), length \( L \), radius of the tube \( r_0 \), and boundary layer thickness \( \Omega \). The movement of fluid in capillary tube can be regarded as the steady seepage of the fluid with yield stress in the radius of the capillary tube.

The driving force of fluid flow is \( \Delta m \pi r^2 \). The viscous resistance is \( 2\pi r L \). In the case of fluid steady seepage, the driving force is equal to the viscous resistance:

\[ \Delta m \pi r^2 = 2\pi r L. \] (B.1)

The constitutive equation with yield stress is

\[ \tau = \tau_0 - \frac{d\nu}{dr}. \] (B.2)

To get in and collate

\[ \nu = \frac{\Delta P}{4\mu L} \left[ (r_0 - \Omega)^2 - r^2 \right] + \frac{\tau_0}{\mu} r - (r_0 - \Omega). \] (B.3)
The flow rate of single capillary tube is
\[ q = \int_0^1 \frac{\pi r^4}{8\mu} \nabla P \left( 1 - \frac{\Omega}{r_0} \right)^4 \left( 1 - \frac{8r_0}{3r_0(1 - \Omega r_0)} \Delta P \right). \]  
(B.4)

The real rock can be equivalent to a capillary bundle composed of capillary tubes with a radius of \( r_p \) per unit area. The flow rate of the rock surface passing through the area of \( A \) is
\[ Q = nA \frac{\pi r^4}{8\mu} \nabla P \left( 1 - \frac{\Omega}{r_0} \right)^4 \left( 1 - \frac{8r_0}{3r_0(1 - \Omega r_0)} \Delta P \right). \]  
(B.5)

Then, we can get:
\[ Q = \frac{k}{\mu} A \nabla P \left( 1 - \frac{\Omega}{r_0} \right)^4 \left( 1 - \frac{8r_0}{3r_0(1 - \Omega r_0)} \Delta P \right). \]  
(B.6)

The upper form is more practical than Darcy’s law, but its form is too complicated. Previous researchers have studied the effect of boundary layer fluid on the seepage characteristics. The thickness of the boundary layer changes with the threshold pressure gradient, and the thickness of the boundary layer decreases exponentially with the increase of the driving pressure gradient.

Based on previous studies, the larger the pressure gradient is, the thinner the boundary layer is for the same capillary, \( \Omega/r_0 \). The smaller it is, that is \( \Omega/r_0 \), it is inversely proportional to the pressure gradient.

\[ \frac{\Omega}{r_0} = \frac{d_1}{\nabla P}. \]  
(B.7)

Assuming that the yield stress values of the same fluid remain unchanged, \( 8r_0/3r_0 \) it can be regarded as a constant, \( 8r_0/3r_0 = d_2 \). Formula (B.6) is changed to
\[ Q = \frac{k}{\mu} A \nabla P \left[ 1 - \frac{d_1}{\nabla P} + \frac{d_2}{\nabla P - d_1} \right] \left( \frac{6d_1^2}{(\nabla P)^3} + \frac{4d_1^2d_2}{(\nabla P)^4} \right) \]
\[ - \left( \frac{4d_1^2}{(\nabla P)^3} + \frac{6d_1^2d_2}{(\nabla P)^4} \right) + \left( \frac{d_1^2d_2}{(\nabla P)^4} + \frac{4d_1^2d_2}{(\nabla P)^5} \right) \left( \frac{\nabla P}{\nabla P - d_1} \right), \]  
(B.8)

because
\[ \frac{\Omega}{r_0} = \frac{d_1}{\nabla P} - \frac{d_2}{\nabla P - d_1} = \frac{8r_0}{3r_0(1 - \Omega r_0) \Delta P} < 1. \]  
(B.9)

\[ Q = \frac{k}{\mu} A \nabla P \left[ 1 - \frac{4d_1 + d_2}{\nabla P} + \frac{6d_1^2 + 3d_1d_2}{\nabla P(\nabla P - d_1)} \right]. \]  
(B.10)

In fact, in the upper forms \( d_1, d_2 \), there is no specific value that needs to be fitted by experiments, so we can get the new model we need.
\[ Q = \frac{k}{\mu} A \nabla P \left[ 1 - \frac{c_1}{\nabla P} + \frac{c_3}{\nabla P(\nabla P - c_2)} \right]. \]  
(B.11)

To establish a new model, the parameters introduced are independent variables. Therefore, there is a correlation between the three parameters mentioned above. In (B.11), deformation is obtained.
\[ Q = \frac{k}{\mu} A \nabla P - c_1 + \frac{c_3}{\nabla P - c_2}. \]  
(B.12)

Obviously, when the displacement pressure gradient is 0, the flow rate should be zero. \( \nabla_m = 0 \). You can get it in the upper form, \( c_3 = -c_1c_2 \). So the new model we get will turn into
\[ Q = \frac{k}{\mu} A \nabla P \left[ 1 - \frac{c_1}{\nabla P - c_2} \right]. \]  
(B.13)

The model has particularity and generality, \( c_1 = 0 \), simplified to the Darcy model, \( c_2 = 0 \). The new nonlinear seepage model is suitable not only for low-permeability reservoirs but also for medium high-permeability reservoirs.

**Abbreviations**

- **P:** Pressure, MPa
- **\( \delta \):** Stress sensitivity coefficient, dimensionless quantity
- **\( K_i \):** Average permeability of initial time \( t = 0 \) of pollution area, \( \mu m^2 \)
- **\( T \):** Test time, d
- **\( T_0 \):** Mud invasion time and threshold pressure recovery test time difference, d
- **\( P_i \):** Fluid density under pressure \( P_i \)
- **\( r_w \):** Radius
- **\( D \):** Dimensionless radius
- **\( \delta_D \):** Dimensionless pseudostress sensitivity coefficient
- **\( G \):** Pseudopressure gradient
- **\( z \):** Laplace space time
- **\( C \):** Dimensionless well storage
- **\( \psi \):** Drilling fluid flux
- **\( K \):** Average permeability of polluted area, \( \mu m^2 \)
- **\( \delta \):** Characterization of the influence of pollution on permeability (multiple \( \delta < 1 \))
- **\( v \):** Liquid velocity, m/h
- **\( K_0 \):** Original permeability, \( \mu m^2 \)
- **\( \varphi \):** Porosity of rock under pressure \( P \)
- **\( \rho \):** Fluid density under pressure \( P \)
Radius of contamination
Dimensionless intrusion time
The initial rate of nondimensional permeability change in mud invasion process
The pressure behind the Laplace space transformation
The skin factor of the regression
t scale factor
Permeability, μm²
The original permeability of core
Mud invasion time and threshold pressure recovery test time difference, d⁻¹
Liquid viscosity, mPa s
Initiation pressure gradient, MPa/m
Rock compressibility, MPa⁻¹
Fluid compressibility, MPa⁻¹
Dimensionless radius
Dimensionless pressure
Dimensionless radius of pollution
The 0- and 1-order imaginary quantities of Bessel’s function
The initial time of skin factor.

Data Availability
Because the oilfield involved in the paper is still under development and the oilfield data is still in a confidential state, the oilfield data used in the paper cannot be made public now.

Conflicts of Interest
We declare that we have no financial and personal relationships with other people or organizations that can inappropriately influence our work; there is no professional or other personal interest of any nature or kind in any product, service, and/or company that could be construed as influencing the position presented in, or the review of, the manuscript entitled “A New Method for Research on Unsteady Pressure Dynamics and Productivity of Ultra-low-Permeability Reservoirs.”

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References


