

## Research Article

# Determination of the Fractal Dimension of CO<sub>2</sub> Adsorption Isotherms on Shale Samples

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Received 19 January 2022; Accepted 31 January 2022; Published 22 February 2022

Academic Editor: Kouqi Liu

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The fractal theory has been widely applied to the analysis of gas adsorption isotherms, which are used for the pore structure characterization in unconventional reservoirs. Fractal dimension is a key parameter that can indicate the complexity of the pore structures. So far, most fractal models for gas adsorption are for N<sub>2</sub> adsorption, while fractal models for CO<sub>2</sub> adsorption are rarely reported. In this paper, we built a fractal model for CO<sub>2</sub> adsorption by combining a thermodynamic model and the Dubinin–Astakhov model. We then applied the new model to three CO<sub>2</sub> adsorption isotherms measured on shale samples. The results show that the fractal dimension from the new model lies between 2 and 3, which agrees with the fractal geometry. The new model presented in this paper can be used for the CO<sub>2</sub> adsorption analysis, which allows characterizing micropore structures in shales.

## 1. Introduction

Knowing the pore structures of shale rocks is an essential part for the reservoir characterization which could assist in understanding the original oil/gas in place and the flow characteristics of the shale rocks [1–3]. The gas adsorption method is now a standard method for pore structure correction. The gas adsorption method involves bringing the gas/vapor into contact with the solid surface [4–6]. For shale rocks, N<sub>2</sub> and CO<sub>2</sub> are the two gases that are typically used for gas adsorption. N<sub>2</sub> adsorption (at 77 k) can be used to derive the specific surface area (using the Brunauer, Emmett, and Teller equation) and pore size distribution (using the Barrett–Joyner–Halenda model or density functional theory) [7, 8]. N<sub>2</sub> adsorption can mainly get the meso-macropore information (pore size >2 nm) and cannot provide the micropore information. Under low temperature, the N<sub>2</sub> molecule is kinetically restricted from accessing the micropores [9]. In order to overcome the limitations of N<sub>2</sub> adsorption, CO<sub>2</sub> adsorption is commonly performed. The critical dimensions of the CO<sub>2</sub> molecule and the N<sub>2</sub> molecule are very similar (0.28 nm for CO<sub>2</sub> and 0.30 nm for N<sub>2</sub>), but the higher working temperature for CO<sub>2</sub> adsorption (273 k for CO<sub>2</sub> adsorption) helps the CO<sub>2</sub> molecule to enter into the

micropores [9]. CO<sub>2</sub> adsorption (273 k) is usually complemented by the N<sub>2</sub> adsorption (77 K) to get a wider pore size information in shale reservoir characterization.

The pore structure of shale samples is very complicated, which has been shown by many researchers using the scanning electron microscope [10–15]. In order to understand the complexity of the pore structures, the fractal theory can be applied. Avnir et al. [16] found that at the molecular level, the surface of most materials has a fractal behavior with fractal dimension varying from 2 to 3, where 2 means a perfectly smooth surface and 3 denotes significantly rough and a disordered surface. Several fractal models have been developed for N<sub>2</sub> adsorption, such as the Frenkel–Halsey–Hill (FHH) theory [17] or the thermodynamic model by Neimark [18]. These models for N<sub>2</sub> adsorption are mainly focused on the meso-macro pore (>2 μm) capillary condensation by using Kelvin's equation. However, Kelvin's equation is not valid for the pores with sizes smaller than 7.5 nm [19], which indicates that these current fractal models cannot be used for CO<sub>2</sub> adsorption. Most researchers only analyzed the fractal dimension from the N<sub>2</sub> adsorption, even when they performed both N<sub>2</sub> adsorption and CO<sub>2</sub> adsorption experiments [3, 20, 21]. To the best of the authors' knowledge, the fractal model for the CO<sub>2</sub> adsorption in shale

studies has not been yet reported. In this paper, we present a fractal model for CO<sub>2</sub> adsorption built by combining the thermodynamic model and the Dubinin–Astakhov analysis model.

## 2. Model Description

From the thermodynamic viewpoint, the differential of the interface area  $ds$  can be calculated from the balance between the work of forming the interface and the work from the adsorption of CO<sub>2</sub> [18]:

$$\sigma ds = \Delta\mu dN, \quad (1)$$

where  $\sigma$  is the surface tension,  $\mu$  is the differential chemical potential of CO<sub>2</sub>, and  $N$  is the adsorption amount of CO<sub>2</sub>.

For CO<sub>2</sub>, the differential chemical potential under pressure  $p$  can be calculated using the following equation [22, 23]:

$$\Delta\mu = RT \ln \frac{p}{p_0}, \quad (2)$$

where  $R$  is the universal gas constant, 8.314 Jmol<sup>-1</sup>K<sup>-1</sup>;  $T$  is the temperature, 273 K;  $p$  is the working pressure; and  $p_0$  is the CO<sub>2</sub> saturation pressure under 273 K.

By combing equations (1) and (2), we can obtain the following equation:

$$S = \frac{1}{\sigma} \int_{N_{(p/p_0)}}^{N_{\max}} RT \ln \frac{p}{p_0} dN, \quad (3)$$

where  $N_{\max}$  is the maximum cumulative adsorption quantity and  $N_{(p/p_0)}$  is the cumulative adsorption quantity under the relative pressure ( $p/p_0$ ).

The correlation of the area for a fractal surface and the volume circumscribed by the surface obeys the following equation [24]:

$$S^{1/D} \sim V^{1/3}, \quad (4)$$

where  $D$  is the fractal dimension.

If the fractal surface is measured on a Euclidean area, equation (4) can be changed to the following equation by the dimensional analysis [25]:

$$S = k^D r^{2-D} V^{D/3}, \quad (5)$$

where  $k$  is a correlation factor between the surface and the volume,  $r$  is the radius, and  $V$  is the volume.

Assuming that the gas molecules cannot be compressed, the volume can be calculated using the following equation:

$$V = \left( N_{\max} - N_{N_{(p/p_0)}} \right) V_L, \quad (6)$$

where  $V_L$  is the molecular volume of CO<sub>2</sub>.

By combining equations (3) and (6), we obtain the following expression:

$$\frac{1}{\sigma} \int_{N_{(p/p_0)}}^{N_{\max}} RT \ln \frac{p}{p_0} dN = k^D r_{p/p_0}^{2-D} \left( N_{\max} - N_{p/p_0} \right)^{D/3} V_L^{D/3}, \quad (7)$$

which can be further rewritten as

$$\frac{\int_{N_{(p/p_0)}}^{N_{\max}} \ln (p/p_0) dN}{r_{p/p_0}^2} = \frac{\sigma}{RT} k^D V_L^{D/3} \left[ \frac{N_{\max} - N(p/p_0)}{r_{p/p_0}} \right]^D, \quad (8)$$

where  $r_{p/p_0}$  is the pore radius under the relative pressure  $p/p_0$ .

The form of equation (8) is similar to the equation which was provided by Wang and Li [25] for the N<sub>2</sub> adsorption analysis. However, in their model, they applied the Kelvin equation to obtain the pore radius for the mesopore capillary condensation stage, which is not suitable for the micropores. For the micropores,  $r_{p/p_0}$  can be derived from the Dubinin–Astakhov model [26]:

$$\frac{W}{W_0} = \exp \left[ - \left( \frac{-RT \ln (p/p_0)}{E} \right)^n \right], \quad (9)$$

where  $w_0$  is the limiting adsorption volume,  $w$  is the occupied adsorption volume,  $E$  is the characteristic energy of the system, and  $n$  is an empirical constant.

Then, the pore size of the sample can be calculated using the following equation:

$$\frac{d(W/W_0)}{dr_{p/p_0}} = 3n \left( \frac{D_0}{E} \right)^n r_{p/p_0}^{-(3n+1)} \exp \left[ - \left( \frac{D_0}{E} \right)^n r_{p/p_0}^{-3n} \right], \quad (10)$$

where  $D_0$  is the dispersion interaction energy.

If we combine equations (9) and (10), we can express the radius:

$$r_{p/p_0} = \left( \frac{-RT \ln (p/p_0)}{D_0} \right)^{-1/3}. \quad (11)$$

For equation (8), let  $A_{p/p_0} = \ln \left( \int_{N_{(p/p_0)}}^{N_{\max}} \ln (p/p_0) dN / r_{p/p_0}^2 \right)$  and  $B_{p/p_0} = \ln \left( (N_{\max} - N(p/p_0)) / r_{p/p_0} \right)$ ; then, equation (8) can be written in the following form:

$$A_{p/p_0} = C + DB_{p/p_0}. \quad (12)$$

Thus, if  $A$  and  $B$  values are calculated under different relative pressure for a CO<sub>2</sub> adsorption isotherm, then the fractal dimension  $D$  can be easily determined from the slope of the function in equation (12).  $C$  is a constant which can be derived from curve fitting.

## 3. Results and Discussion

**3.1. Model Verification.** In order to verify this model, we performed the CO<sub>2</sub> adsorption experiment on three shale samples and calculated their fractal dimension using equation (12). Figure 1 shows the adsorption isotherms of the three samples (the relative pressure is from 0.001 to 0.03).

**3.2. Impact of  $D_0$  on the Results.** We applied the new model to calculate the  $A$  and  $B$  values of each sample under different relative pressure values and then plot  $A$  as a function of  $B$  (Figure 2) (here, we assume that  $D_0$  is 1500 Jnm<sup>3</sup>mol<sup>-1</sup>, from Hiden Isochema). Very strong linear relations exist between  $A$  and  $B$  for all three samples, indicating that the CO<sub>2</sub> adsorption isotherm shows the fractal behavior. The fractal

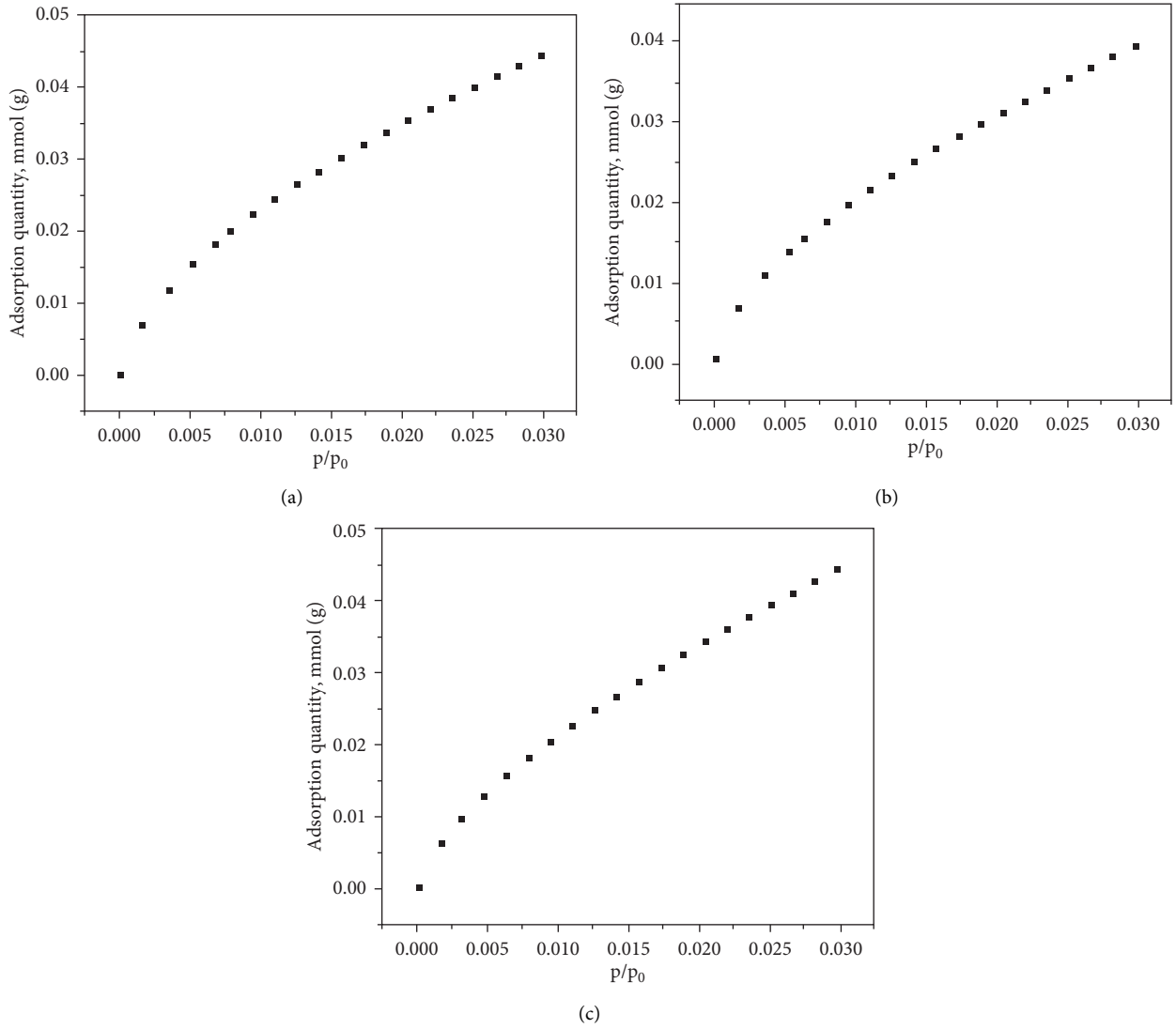


FIGURE 1: Measured CO<sub>2</sub> adsorption isotherms of three shale samples. (a) Sample 1. (b) Sample 2. (c) Sample 3.

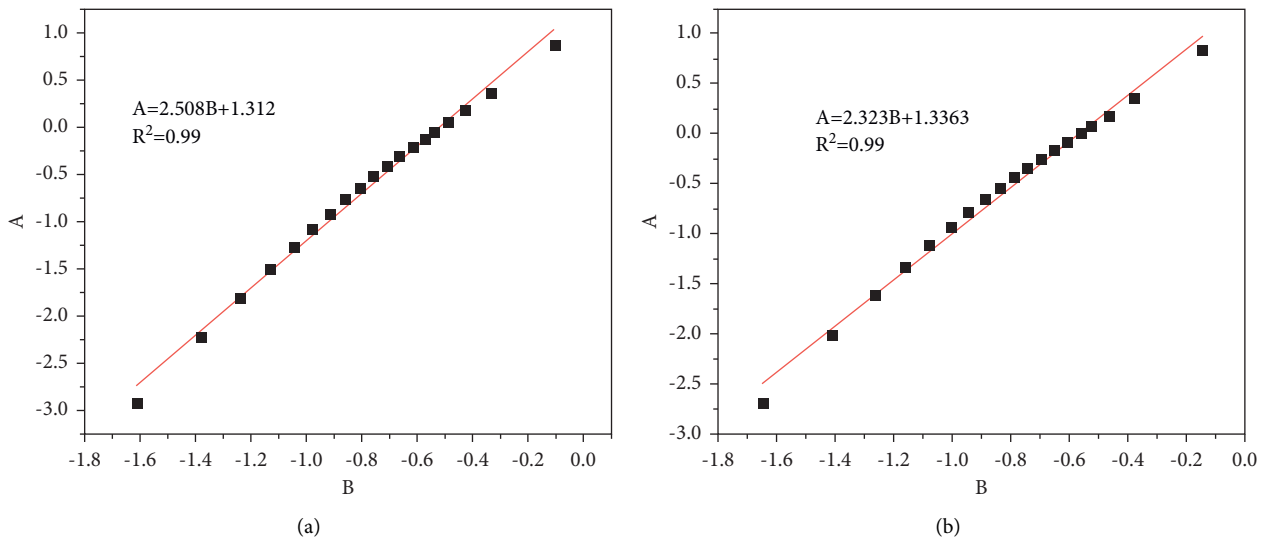


FIGURE 2: Continued.

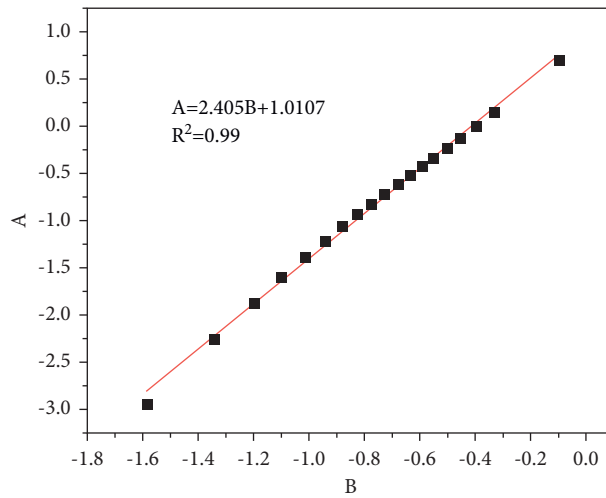


FIGURE 2: Fractal analysis of the CO<sub>2</sub> adsorption isotherms of the three shale samples. (a) Sample 1. (b) Sample 2. (c) Sample 3.

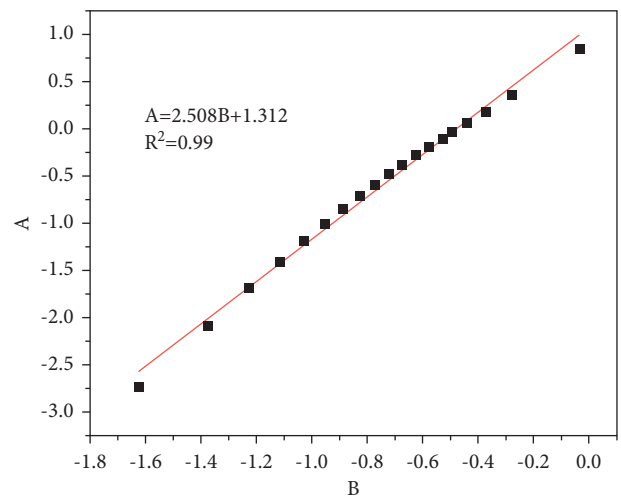
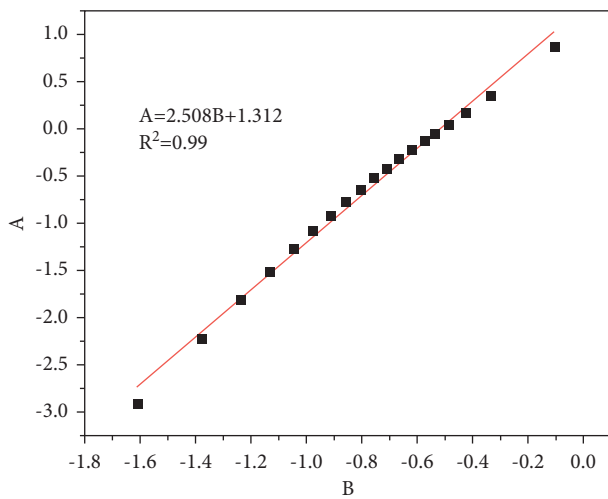
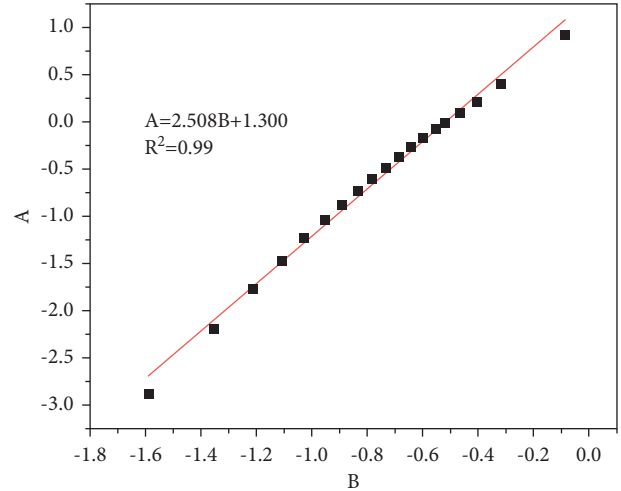
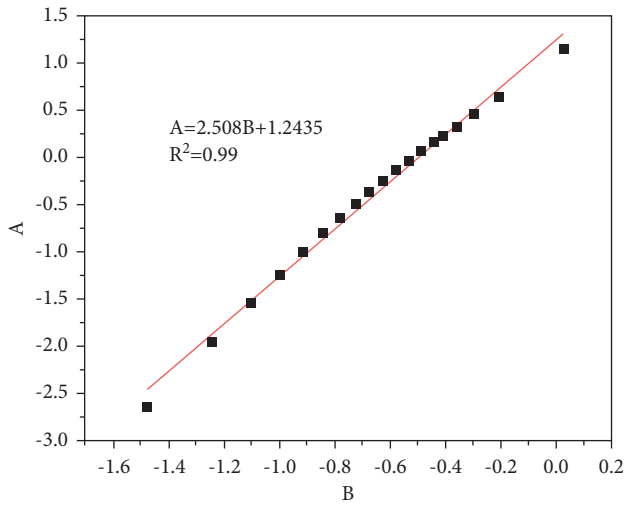


FIGURE 3: Continued.

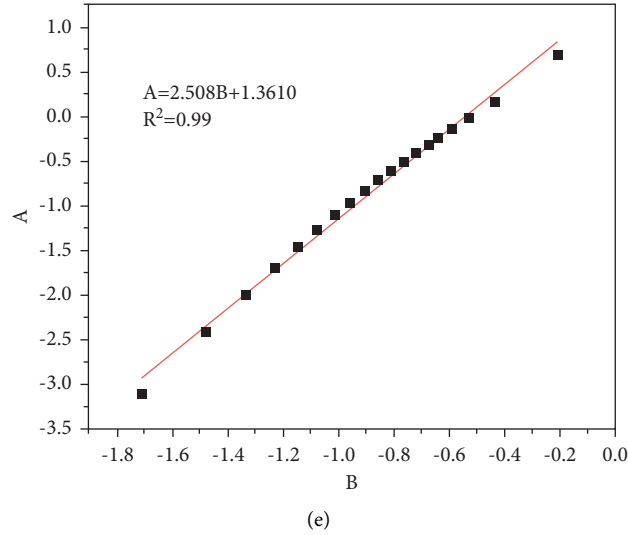


FIGURE 3: The impact of  $D_0$  on the fractal dimension (sample 1). (a)  $D_0 = 1000 \text{ Jnm}^3 \text{ mol}^{-1}$ . (b)  $D_0 = 1400 \text{ Jnm}^3 \text{ mol}^{-1}$ . (c)  $D_0 = 1500 \text{ Jnm}^3 \text{ mol}^{-1}$ . (d)  $D_0 = 1600 \text{ Jnm}^3 \text{ mol}^{-1}$ . (e)  $D_0 = 2000 \text{ Jnm}^3 \text{ mol}^{-1}$ .

dimensions of these three samples are 2.508, 2.323, and 2.405, respectively. The fractal dimension falls within the expected range of  $2 < D < 3$ , predicted by the fractal geometry [25, 27]. Thus, the model yields robust results and can be used to calculate the fractal dimension of  $\text{CO}_2$  adsorption on shale samples.

In the previous examples, we had to assume a value for  $D_0$  in order to calculate  $r_{p/p_0}$ .  $D_0$  is the dispersion interaction energy of  $\text{CO}_2$ , which is not well constrained but is usually set to around  $1500 \text{ Jnm}^3 \text{ mol}^{-1}$  [28]. In this part, we further studied the effect of  $D_0$  value on the fractal dimension. We set three  $D_0$  values (1000, 1400, 1500, 1600, and  $2000 \text{ Jnm}^3 \text{ mol}^{-1}$ ) and then calculated the fractal dimension of sample 1 for all three cases. Figure 3 shows that the absolute values of the  $A$  and  $B$  values do vary for different  $D_0$ , but the slope of the linear regression of  $A$  to  $B$  remains the same. Therefore, the choice of  $D_0$  value does not affect the fractal dimension calculation. The fractal dimension value of the  $\text{CO}_2$  adsorption isotherm from shale samples can be derived even when the exact  $D_0$  value is not well constrained.

**3.3. Future Research.** Clay bound water is an important factor that could affect the fractal dimensions of the gas adsorption which has been studied by many researchers [29–31]. Under different moisture content, the fractal dimension changes. However, in this study, we preheated the samples under  $105^\circ\text{C}$  for over 12 hours and we believe that the effect of the clay bound water effect can be neglected. In this study, our focus is to derive a model to describe the fractal model for the analyzing the fractal dimensions of the  $\text{CO}_2$  gas adsorption. Thus, the samples we choose are from a single basin and very simple. More samples from the different shale basins will be collected and analyzed to verify the applicability of this model. In addition, based on the studies by other researchers, the fractal dimension from  $\text{N}_2$  gas adsorption could be correlated with the pore structures [32]. Whether the fractal dimension from  $\text{CO}_2$

gas adsorption is correlated with the microstructures of the samples and how the microstructures affect the fractal dimension will be the target for the next step research.

## 4. Conclusions

- (1) Based on the Dubinin–Astakhov analysis model and the thermodynamic model, we built the fractal analysis model for  $\text{CO}_2$  adsorption on shale samples.
- (2) We applied the new model to calculate the fractal dimensions of the  $\text{CO}_2$  adsorption isotherms for three shale samples. We found that the  $\text{CO}_2$  adsorption isotherms had the fractal behavior, and the fractal dimension value was between 2 and 3. This agrees with the fractal geometry and indicates the robust performance of the new model.
- (3) We conducted a sensitivity analysis to investigate the effect of the dispersion interaction energy  $D_0$  on the fractal dimension calculation and demonstrated that the choice of the  $D_0$  value does not affect the model outcomes.

## Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

## Acknowledgments

The authors appreciate PetroChina Xinjiang Oilfield Company and shale oil research team. This study was supported by the Forward-Looking Basic Projects of CNPC (grant no. 2021DJ1803).

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