Research Article

The Mathematical Model of Integrated Coupling of Gas Reservoir-Wellbore-Choke in Sulige Gas Field, China

Zizi Han,1 Yongsheng An,2 Ping Guo,1 Chengyin Jiang,1 Yinghai Feng,1 and Qiang Zhang1

1No. 3 Gas Production Plant PetroChina Changqing Oilfield Company, Ordos 017300, China
2MOE Key Laboratory of Petroleum Engineering in China University of Petroleum, Beijing 102249, China

Correspondence should be addressed to Yongsheng An; 64580752@qq.com

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1. Introduction

With the global energy transition, the development and utilization of clean energy has gradually become the focus of research in various countries. To achieve clean and low-carbon fossil energy, the natural gas is a realistic choice. And the natural gas will play a bridge and supporting role in the energy transition. Tight sandstone gas has massive objective reserves in China and has become an important part of China’s natural gas industry with great potential for development [1–3]. The proven reserves in Sulige gas field have reached $5.16 \times 10^{12} \text{ m}^3$, with $3.24 \times 10^{12} \text{ m}^3$ for the geological reserves for development [3]. The Sulige gas field is a typical tight sandstone gas reservoir [4]. The gas well in Sulige gas field has a short stable production period and poor liquid carrying capacity, and the wellbore is prone to liquid accumulation. To prevent hydrate formation and reduce surface pipeline pressure, gas wells are generally equipped with throttles underground [5–8].

There are 929 sets of downhole throttles in Sulige Gas field, which mainly adopts the multiwell gas collection production mode of “downhole throttling, wellhead no heating, normal temperature separation, and centralized treatment.” Installation and use of downhole throttles in the field exhibit the following characteristics [9, 10]: (1) the working depth of the downhole throttle is generally 1500 m–2500 m underground. The flow cooled by throttling is heated by formation temperature to restore it to its prethrottling temperature, so hydrate formation can be avoided. (2) The application of downhole throttling technology improves the liquid carrying capacity of gas wells. (3) After downhole throttling, the wellhead pressure decreases, which reduces the pressure of gathering system and the methanol injection amount. There is little need for methanol injection in summer and only
2 Integrated Coupling Prediction Model

2.1. Mathematical Model of Gas Reservoir Simulation. The black oil model was selected as the study object, ignoring the influence of capillary force [13]. The equation for the conservation of mass in the flow of the gas and water phases is as follows:

\[
\frac{\partial}{\partial t} \left( \frac{\phi \rho_w S_w}{B_w} \right) = -\nabla \cdot \left( \frac{\rho_w u_w}{B_w} \right) + q_w, \tag{1}
\]

\[
\frac{\partial}{\partial t} \left( \frac{\phi \rho_g S_g}{B_g} \right) = -\nabla \cdot \left( \frac{\rho_g u_g}{B_g} \right) + q_g,
\]

where \( \rho_w \) and \( \rho_g \) are the density of the gas phase and the water phase under gas reservoir conditions (kg/m\(^3\)), respectively, \( S_g \) and \( S_w \) are saturation of the gas phase and the water phase, respectively, and \( \phi \) is the porosity of the rock under gas reservoir conditions.

\[
u_w = \frac{k_{rw}}{\mu_w} k (\nabla p_w - \rho_w g \nabla z),
\]

\[
u_g = \frac{k_{rg}}{\mu_g} k (\nabla p_g - \rho_g g \nabla z),
\]

where \( \mu_g \) and \( \mu_w \) are the viscosity of the gas phase and the water phase under gas reservoir conditions (mPa\(s\)), \( k_{rg} \) and \( k_{rw} \) are the relative permeability of the gas phase and the water phase under gas reservoir conditions, \( P_g \) and \( P_w \) are the pressure of the gas phase and the water phase, respectively (Pa), \( k \) is the absolute permeability \( (10^{-13} \mu m^2) \), \( g \) is gravitational acceleration \((m^2/s)\), and \( z \) is the reservoir depth (m).

\[
q_w = WI \cdot \frac{k_{rw}}{\mu_w B_w} (P_w - P_{wφ}),
\]

\[
q_g = WI \cdot \frac{k_{rg}}{\mu_g B_g} (P_g - P_{wφ}),
\]

where \( WI \) is the well index \((m^3/MPa)\) and \( P_{wφ} \) is the well bottom flow pressure (MPa).

The time and space terms of the partial differential equations are discretized using the finite difference method, and the Newton-Raphson method is used to solve them [14].

2.2. Wellbore Simulation Mathematical Model. The Hasan-Kabir model was selected as the calculation model for the pressure drop between the wellbore gas-liquid two-phase flow [15]. In the Hasan-Kabir method, the gas-water two-phase flow is divided into the following four forms: bubble flow, segment plug flow, agitation flow, and annular flow. When the liquid content in the gas-liquid two-phase flow is high, the gas phase exists in the liquid phase in the form of dispersed bubbles and moves forward along the liquid flow in an irregular path. This flow pattern is called bubble flow. With the increase of gas flow rate, small bubbles begin to collide and gather in the pipe. When the bubble gathers to a certain extent, its diameter is close to the pipe diameter, and the bubble flow changes into slug flow. With the continuous increase of gas flow, the interaction between the rising Taylor bubble and the falling liquid film near the pipe wall will also increase. When this effect is enough to destroy the Taylor bubble, the slug flow begins to transform to the stirred flow. When the pressure is high enough and the gas phase density is large enough, annular flow can be formed at low gas phase velocity, which is difficult to achieve in the actual wellbore.

The pressure drop is calculated as follows:

\[
\frac{dp}{dz} = \rho_m g + \frac{2f_{m}\nu_m p_m}{d} + \rho_m \nu_m \frac{dv_m}{dz}, \tag{4}
\]

where \( \rho_m \) is the gas-water mixing density \((kg/m^3)\), \( f_m \) is the coefficient of friction, \( \nu_m \) is the gas-water mixed flow rate \((m/s)\), and \( d \) is the borehole inner diameter \((m)\).

In the calculation of gravity pressure gradient, friction pressure gradient, and acceleration pressure gradient, the mixture density is an indispensable parameter, and the porosity needs to be solved first. Based on the drift model, Hassan and Kabir obtained the relationship between porosity \( \phi \) and diffusion coefficient \( C_0 \) and drift velocity \( v_d \), as shown in Equation (5) [16].
throttling is g Master
gas phase apparent velocity (m/s),
transition of stirred
\( \rho \) is the density of liquid phase (kg/m\(^3\)), \( \rho_g \) is the density of the gas phase (kg/m\(^3\)), \( g \) is the acceleration of gravity (m/s\(^2\)), \( v_{sg} \) is the gas phase apparent velocity (m/s), \( v_{bg} \) is the boundary of transition of bubble flow to slug flow (m/s) \( (v_{bg} = 0.429v_{ms} + 0.357v_{ms}) \), \( v_{ms} \) is the boundary of transition of bubble flow to diffuse bubble flow (m/s) \( (2v_{ms}^2(0.5\phi/D)^{0.4}(\rho_g/\sigma)^{0.6} \sqrt{0.4\sigma/g(\rho_l - \rho_g)}) = 0.725 + 4.15\sqrt{v_{bg}/v_{ms}} \).

\[ \phi = \frac{v_{bg}}{C_0 + v_d} \]  \( \text{(5)} \)

The diffusion coefficient \( C_0 \) and drift velocity \( v_d \) corresponding to different flow patterns given by Hassan and Kabir are shown in Table 1.

### 2.3. Throttle Simulates a Mathematical Model.

The throttle is equivalent to a nozzle device, which satisfies the law of nozzle flow, as shown in Figure 1. When the fluid passes through a fixed-size throttle, there is a maximum flow rate of the gas through the throttle in the critical flow state. As the pressure ratio of the upper end of the throttle (outlet) and the lower end of the throttle (inlet) increases in \( P_2/P_1 \), the gas flow rate will decrease accordingly, and the gas will be in a noncritical flow state [14, 17]. Under the condition of critical flow, the pressure fluctuation downstream of the throttle does not affect the upstream pressure of the throttle, so the flow of gas-liquid mixture through the throttle is usually designed as critical flow [18].

In the critical flow state, the gas flow through the throttle is [19]

\[ Q_g = 4.066 \times 10^3 P_1 d^2 \frac{2^{2K}}{\sqrt{T_1 Z_1}} \sqrt{\frac{K}{K - 1}} \left[ 2 \frac{K - 1}{K + 1} - \frac{K + 1}{K + 1} \right]. \]  \( \text{(6)} \)

In a noncritical flow state, the gas flow through the throttling is

\[ Q_g = 4.066 \times 10^3 P_1 d^2 \frac{P_2^{2K}}{\sqrt{T_1 Z_1}} \sqrt{\frac{K}{K - 1}} \left[ \frac{P_2}{P_1} \right]^{\frac{K}{K - 1}} - \frac{P_2}{P_1}, \]  \( \text{(7)} \)

where \( d \) is the diameter of the nozzle (mm), \( T_1 \) is the temperature at the lower end of the nozzle (K), \( P_1 \) is the pressure at the lower end of the nozzle (MPa), \( Z_1 \) is the gas compression coefficient at the lower end of the nozzle, \( T_2 \) is the temperature of the upper end of the nozzle (K), \( P_2 \) is the pressure at the lower end of the nozzle (MPa), and \( K \) is the natural gas insulation index.

### 2.4. Integrated Mathematical Model.

Due to the installation of the throttle underground, the production dynamics and liquid accumulation dynamics of the Sulige gas well are more complex than those of conventional gas wells. The liquid accumulation process must be analyzed first, and then the gas reservoir model, the wellbore model, and the throttle model are coupled to solve.

#### 2.4.1. Analysis of the Liquid Accumulation Process.

The actual operation process of the gas well is: install the throttle in the downhole at the beginning of production, and pull out

![Figure 1: Flow characteristics through nozzle.](image-url)
the throttle after the pressure drops in the middle and late stages of production. The production process of the throttle gas well can be divided into the following five stages (Figure 2).

1) Initial Gas Well Production. The production characteristics of this stage are that the output and the bottom pressure of the well are continuously decreasing with the production time. The output is higher than the critical liquid carrying flow, and the two-phase flow type of gas and liquid in the wellbore is a circular flow [20]. The wellbore does not produce a liquid accumulation, and the flow of gas through the throttle is a critical flow state.

2) The Lower End Liquid Accumulation of the Throttle. As production decreases further, the flow rate of wellbore gas at the lower end of the throttle is lower than the critical carrying flow rate due to higher pressures. The stage creates a fluid build-up in the wellbore, and some of the flow patterns in the wellbore are transformed into stirred flows, or even segment plug flows.

3) The Upper End Liquid Accumulation of the Throttle. With the continuous development of the wellbore liquid accumulation at the lower end of the throttle, the flow pressure at the bottom of the well continues to rise, and the pressure at the lower end of the throttle continues to decrease, resulting in a further decrease in gas production. The flow of the gas over-throttle may become a noncritical flow state during this process. The flow rate of the wellbore gas at the upper end of the throttle is lower than the critical liquid carrying rate, and the liquid accumulation begins to occur, and the production of the gas well is further accelerated.

4) The Production Stage after Salvaging the Throttle. Due to the serious liquid accumulation of wellbore, the throttle will be salvaged to further take measures to drain the liquid and gas in the later stage. The production dynamics after salvaging the throttle can be divided into two situations:

Under the better formation conditions, the production of gas wells is higher than the critical liquid carrying flow, and the wellbore liquid accumulation is discharged, and the output has rebounded significantly. Under the poor formation conditions, the production of gas wells is still lower than the critical liquid carrying flow, and the liquid accumulation of the wellbore cannot be fully discharged, and the yield does not change significantly.

5) Reliquid Accumulation. As production continues, the wellbore will again or continue to liquid accumulation, and the gas production will further decline, and the follow-up will be transferred to the discharge stage [21].

2.4.2. Establishment and Solution of Integrated Coupling Model. The numerical reservoir model is solved by finite difference method, and this paper adopts the explicit method; that is, in the calculation process of each time step of the gas reservoir numerical simulation model, the parameters of the previous time step are used to calculate the wellbore and nozzle models. The specific solution steps of the gas reservoir-wellbore-throttle integrated coupling model are as follows and shown in Figure 3.

1) Enter the basic parameters, including reservoir parameters, wellbore parameters, throttle parameters, fluid properties parameters, yield data, and thermodynamic parameters, and the initial assignments are made to the temperature and pressure distribution in reservoirs and wellbores.

2) When \( t = t_0 \), set the initial value of the well bottom flow pressure \( P_{wf} \); using the dichotomy, take \( P_{wf} = 1/2(P_{wf1} + P_{wf2}) \) (the first calculation can be set to 0 and \( P_e \))

3) Put \( P_{wf} \) into the numerical simulation model of the gas reservoir to obtain the gas production \( Q_{g1} \) under the current well bottom flow pressure.
(4) The bottom flow pressure \(P_{wf}\) and \(Q_{g1}\) are substituted into the wellbore model, and the multiphase pipe flow is calculated from the bottom up to obtain the nozzle front pressure \(P_1\).

(5) The wellhead oil pressure \(P_t\) and \(Q_{g1}\) are substituted into the wellbore model, and the multiphase pipe flow can obtain the nozzle post pressure \(P_2\) from top to bottom.

(6) Under the premise of a given nozzle diameter \(d\), according to the pressure \(P_1\) in front of the nozzle and the pressure \(P_2\) behind the nozzle, the nozzle flow rate \(Q_{g2}\) can be calculated.

(7) Compare the size of \(Q_{g1}\) and \(Q_{g2}\); if \(|Q_{g1} - Q_{g2}| < \epsilon\), then enter the next time step; \(\epsilon\) is the maximum allowable error, which is equal to 3%.

(8) If \(|Q_{g1} - Q_{g2}| > \epsilon\), when \(Q_{g1} > Q_{g2}\), set \(P_{wf} = P_{w1}\) and return to step (2); when \(Q_{g1} < Q_{g2}\), make \(P_{wf} = P_{w2}\) return to step (2).

Among them, the initial assignment of temperature and pressure distribution in reservoir and wellbore is determined by the following methods: according to the geological report of Sulige gas field, the bottom hole flow pressure and reservoir pressure of the gas well are obtained. Assuming the average distribution of the pressure from the wellhead to the bottom hole, the pressure values at various points of the wellbore are calculated as the initial assignment of the wellbore pressure distribution. According to the wellhead temperature, the ground temperature gradient is used to calculate the temperatures at various parts of the wellbore and the reservoir, which is used as the initial value of the temperature.

### 3. Field Application

Taking a throttle gas well in Sulige YPT1 as an example, the reservoir model mesh is divided into \(21 \times 21 \times 1\), the grid steps in the three directions are 20 m, 20 m, and 3.6 m, and the gas reservoir depth is 3400 m. After mesh independence analysis, the grid number can meet the requirements of calculation accuracy and ensure the calculation accuracy. The gas reservoir pressure is 37.8 MPa, and the gas reservoir temperature is 90°C. The porosity is 12%, and the permeability is \(7.23 \times 10^{-6}\) \(\mu m^2\). The rock compression coefficient is \(7.23 \times 10^{-6}\) MPa\(^{-1}\). The throttle size is 2 mm, and the throttle entry depth is 1800 m. The inner diameter and depth of the wellbore are 0.062 m and 3000 m, respectively, and wellhead temperature is 20°C. The ground temperature gradient is 3.1°C/100 m, and the minimum wellhead oil pressure is 2.5 MPa. The relative permeability is shown in Table 2.

The parameters required for the gas reservoir, wellbore, and throttle are input into the integrated coupling model to simulate the production dynamic curve of a typical gas well production for 1500 days. The throttle valve is removed.
606 days after commissioning, and the simulation results of the integrated coupling model are paired with the actual production data as shown in Figure 4.

As can be seen from Figure 4, the gas production of the well YPT1 has been declining since it was put into production. After 400 days of commissioning, the wellbore at the lower end of the throttle accumulated fluid. After 470 days of commissioning, the wellbore at the upper end of the throttle accumulated fluid. After 606 days of production, it began to salvage the throttle. Due to the strong energy of the formation, the wellbore liquid accumulation was discharged, and the production of the gas wells rose rapidly, and the liquid was accumulated again after continuing to produce for a period of time. As the flow pattern in the wellbore switches back and forth between the annular flow and the segment plug flow, the gas production curve oscillates significantly and eventually accumulates again.

Compared with conventional numerical simulation methods, the simulation results of the integrated coupling model can more accurately reflect the production dynamics of gas wells, which is more reliable and practical. The software is used to calculate the effect of the throttle salvage time on the cumulative gas production volume of 1500 days of production, assuming that the 0 day of the salvage throttle starts from the upper end of the throttle liquid accumulation as the salvage throttle, and the 300th day after the throttle upper end liquid accumulation is calculated at 25-day intervals, and the results are shown in Figure 5.

As can be seen from Figure 5, the earlier the choke is salvaged, the greater the cumulative gas production increases, which is more beneficial to the later production. When the choke is salvaged 100 days after the upper end of the choke accumulates, the cumulative gas production of the gas well (well YPT1) shows a more rapid downward trend. Due to the large number of gas wells in Sulige gas field and heavy workover operation, it is recommended to salvage the choke within 100 days after the upper end of the choke accumulates according to the local workover system.

4. Summary and Conclusions

On the basis of establishing the gas reservoir model, wellbore pipe flow model, and throttle nozzle flow model, this paper proposes the mathematical model of integrated coupling gas reservoir-wellbore-throttle and its solution method and carries out the case calculation of throttle gas well in Sulige gas field. The conclusions reached are as follows:

(1) The influence of wellbore throttling on the liquid accumulation of the Sulige gas field was analyzed, and the dynamic process of liquid accumulation of throttling gas well before and after the salvage of the throttle was proposed.

(2) Based on the reservoir seepage model, wellbore pipe flow model, and throttle nozzle flow model, the mathematical model of integrated coupling of gas reservoir-wellbore-throttle is established, and a solution method is proposed.

(3) The integrated coupling model of gas reservoir-wellbore-throttle is used for example calculation, and the results show that the model is effectively used for dynamic simulation of throttle gas well production in Sulige gas field and optimization of throttle salvage timing.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.
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References


