Research Article
Thermal Conductivity Model Analysis of Unsaturated Ice-Containing Soil

Qiang Han1, Zhiguo Wang1, and Rui Qin2

1Heilongjiang Key Laboratory of Disaster Prevention, College of Civil and Architecture Engineering, Northeast Petroleum University, Daqing 163318, China
2Jinchuan Nickel-Cobalt Research and Design Institute Co., Ltd, Jinchang 737100, China

Correspondence should be addressed to Qiang Han; bruise@alumni.sjtu.edu.cn

Received 19 May 2022; Revised 19 June 2022; Accepted 27 June 2022; Published 12 July 2022

Academic Editor: Hao Wu

Copyright © 2022 Qiang Han et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

In cold locales, the thermal conductivity of soil porous media varies according to their composition and the phase state of the substance contained within the pore space. During the winter, water and other media in the soil pore space freeze-thaw, resulting in their phase state, composition, distribution, and significant thermal conductivity changes. There are some shortcomings in the current research regarding the thermal conductivity change pattern of unsaturated ice-containing soils. In this paper, the representative elementary volume (REV) selection method is given for unsaturated ice-containing soil with porosity as a representative state variable. Under the condition of freeze-thaw, two thermal conductivity REV analysis models for unsaturated ice-containing soil are established: a simplified volume-weighted average REV model and a fine volume-weighted average REV model; accordingly, a macroscopic thermal conductivity analysis model is given. The computational analysis is carried out with an actual unsaturated ice-containing soil example. The influence of the application of frozen soil in China is examined for its effect on the variation law of the thermal conductivity of porous medium. The variation characteristics of thermal conductivity of permafrost soil with related parameters (porosity, water ratio, moisture percentage, ice content, and tortuosity) are discussed. The model built in this paper provides novel concepts and methods for analyzing the thermal conductivity characteristics of unsaturated soil, as well as enhancing and advancing the analysis.

1. Introduction

In industry and agriculture, the study of soil thermal conductivity has been a hot topic, and soil thermal conductivity is a critical parameter in calculation and analysis. In some cold areas, the pore water freeze-thaws as the temperature varies, the internal condition of soil changes, and the thermal conductivity is affected by the water ratio, ice content, and other factors, exhibiting some variance.

At present, domestic and international scholars have conducted some research on soil porous media. Lu et al. [1] developed a simple novel model to estimate soil thermal conductivity utilizing seven soil samples for test verification, analyzing the test value and relevant data, and calculating the error. The results demonstrated that the model can express the \( \lambda(\theta) \) curve from dry to saturated for a fixed bulk value; the new model can be used to study the thermal motion in soils with varying textures, bulk weight, and water content (\( \theta \)).

Kong and Wang [2] examined the case of icing and derived the heat and wet transfer equation based on the water content and temperature. In addition, a 321-day experiment was conducted in the Building Energy Conservation Laboratory of Harbin Institute of Technology, and the model built to predict the thermal and wet coupling transmission was validated using the actual measurement data.

In 2015, Dong et al. [3] divided the unsaturated soil thermal conductivity model into three groups: hybrid models involving series/parallel elements, empirical models using dry and saturated thermal conductivity, and mathematical model based on phase volume fraction demonstrated that none of the existing models can accurately reflect pore structure or interface properties, and proposed a conceptual
model based on soil-water retention mechanism, which can be used to establish a quantitative thermal conductivity model of variable saturated soil.

Kang and Ge [4] established a one-dimensional thermal conductivity enhancement series-parallel model, by testing the characteristics of the thermal conductivity with time and discussed the relevant parameters (porosity, the characteristics of the thermal conductivity with time) on the thermal conductivity of fly ash soil mixture, and for the experimental data and predicted value, the results are basically consistent.

In 2016, Tian et al. [5] developed a simplified De Vries-based model to estimate the thermal conductivity of thawing and frozen soil, improved the method for calculating the thermal conductivity of soil air and mineral and mineral factors, and estimated the thermal conductivity and shape factor of silt soil using the reverse simulation method. Tokoro et al. [6] proposed a new soil thermal conductivity model and empirical equation, conducted thermal conductivity test and conductivity tests using the thermal probe method, and studied the influence of water content on soil thermal conductivity. The thermal conductivity of water is closely related to the thermal conductivity and has nearly the same inflection point; the soil thermal conductivity of soil is dependent on whether the continuity of pore water contributing significantly to the thermal conductivity.

In 2018, Di Sipio and Bertermann [7], in the ITER (EU-funded project) test ground, monitored the soil water content changes of five different soil mixtures under the same climatic conditions and thermal stress, discussed the relationship between precipitation and natural/induced ground temperature changes and the water freezing point, to better understand the relationship between thermal conductivity and water content, and analyzed the change characteristics of shallow geothermal (VSG) system under saturated and unsaturated conditions.

Based on the effective medium approximation of seepage, Sadeghi et al. [8] proposed a theoretical model of the thermal conductivity of porous media. Concerning the implicit function of volume water content, Sadeghi et al. enhanced and validated the display function of the GD model. The results indicate that the GD model can accurately estimate the thermal conductivity of soil.

In 2019, Gamage et al. [9] used a time domain reflection probe and a thermal pulse probe to measure soil water content, soil properties, soil texture, and total organic carbon at 80 test locations on cultivated land in QC, southwest Canada. The results revealed a high spatial association between water content and thermal properties; soil water content has the greatest impact on soil thermal conductivity.

On the basis of the multivariate distribution method, Zou et al. [10] improved the existing Cote model and Konrad model calculation model, introduced simplified binary and multivariate correlation, analyzed the related parameters (dry density, porosity, saturation, quartz content, sand content, and clay content) on unsaturated soil thermal conductivity, and find that quartz content is a more significant factor than the high quartz unsaturated soil porosity model.

Garrouch et al. [11] obtained the diffusion coefficient through the molecular diffusion and conductivity experi-

ments of sandstone, confirmed the similarity of power conduction and molecular diffusion in sandstone, and provided the tortuosity value obtained by converting the diffusion coefficient measurements with various models in order to select the most applicable detour model for a particular porous medium. Through conductance experiments, Barrande et al. [12] derived an empirical formula of the tortuosity suitable for the suspended medium and compared it to other empirical models. Due to the equivalence of electricity and heat, it is possible to anticipate the detour degree in thermal conductivity prediction is feasible.

The majority of the above studies are through experiments or characteristics of soil pore structure to construct models and conduct simulation exploration. Practice is the only criterion to test the truth. Numerous instances demonstrate that the thermodynamic features of frozen soil play a crucial role in influencing the engineering construction of the cold area, which is irreplaceable. Whether it is hot karst, icy mantle, or thawing, frost swelling and other freezing damage are related to the thermal field of permafrost, and the establishment of the thermal field must know the thermodynamic parameters of frozen soil, so the study of the thermal conductivity of frozen soil is of great significance. The influencing factors of the thermal conductivity of frozen soil as the starting point give new practical significance to the thermodynamic properties of frozen soil. This paper considers the pore water under freeze-thaw conditions, develops the thermal conductivity model for permafrost, derives the thermal conductivity formula, and analyzes the thermal conductivity variation law for permafrost.

2. REV Description Method of Soil Porous Media

2.1. REV Description Characterization Method of Porous Media. Generally speaking, porous media are multiphase structures, and a reasonable scale should be selected before their description. At present, the description, analysis, and calculation of porous media use the representative elementary volume (REV) expression method [13–30].

When the volume increases to a specific value $V$ for the REV, the structure and characteristic parameters no longer change accordingly. In general, the range of values for $V$ is $V_{\text{min}} < V < V_{\text{max}}$, as shown in Figure 1.

The definition model depicted in Figure 1 is a general method for REV selection. In practical application, the REV selection method has its own focus due to the structural differences of various porous media, the calculation, analysis background, and the application field also differ.

2.2. REV Description Characterization Method of Soil Porous Media Based on the Conception of Porosity. Similar to general porous media, soil porous media are composed of a collection of solid-phase units (spherical, flaky, needle-like particles, etc.) and the pores between each solid-phase unit. The pores are filled with fluid, typical of solid-fluid two-phase media [31]. Porosity is a critical characteristic for describing the composition and structure of this type of porous media. Since the composition structure primarily determines the thermal
conductivity of the soil porous media discussed in this paper, it is inextricably related to the porosity. Therefore, porosity will be used as a representative state variable to demonstrate a volume-weighted average REV selection method.

A porous soil medium can be equated to a single continuum. At any point in this hypothetical continuum, the state variables are the statistical average of the corresponding state variables in a volume of the porous medium centered at that point (as shown in Figure 2).

If the state variable derived by statistical averaging is a continuous function of the spatial coordinates and time of the volume center of the selected region, the volume is the characterization unit cell of corresponding state variable.

For Figure 2, the geometric region $\Omega$ (REV region) of the unsaturated ice-containing soil is assumed to be divided into matrix region $\Omega_s$, water system region $\Omega_w$, ice system $\Omega_i$, and air system $\Omega_a$. (Figure 2)

In Figure 4, the simplified series (vertical) model shows the flow paths for different media.

Figure 1: Porosity $\phi$ for different sizes of averaging volumes.

Figure 2: Average volume around a point of $x$ in porous media.

Figure 3: Simplified volume-weighted average REV model.

Figure 4: Simplified series (vertical) model.
region \( \Omega_I \), and air region \( \Omega_A \), namely, soil matrix medium, water medium, ice medium, and air medium, which can be seemed as four continuous media superimposed on each other in space [31].

If the \( x \) is a point in the geometric region \( \Omega \) of the unsaturated ice-containing soil, the media region with the point \( x \) as the geometric center is \( \Omega(x) \). In this region, the geometric volume is \( V \), and the epitaxial volume of the region \( \Omega(x) \) is \( E(x) \).

When the epitope volume on the region \( \Omega_n(x) \) is \( E_n(x) \) and \( F(x) \) is the macroscopic intrascale content (structure, seepage, pressure, etc.) corresponding to the epitope \( E \) at the point \( x \), then

\[
\Omega = \sum_{n} \Omega_n, n \in \{S, W, I, A\},
\]

\[
\Omega_n(x) = \Omega(x) \cap \Omega_n, n \in \{S, W, I, A\},
\]

\[
E(x) = \sum_{n} E_n(x),
\]

\[
F_n(x) = \lim_{V_n \to V_0} \frac{E_n(x)}{V_0},
\]

\[
F_n(x) = \lim_{y \to x} F_n(y), y \in \{\Omega(x)\}.
\]

When the selection of the medium area \( \Omega(x) \) satisfies both equations (4) and (5), the corresponding \( V \) is the REV of unsaturated ice-containing soil. This paper selects the REV (i.e., volume-weighted average REV) of the heat transfer process in the porous soil medium according to the above method.

3. Heat Transfer Model for Unsaturated Ice-Containing Soils

Due to the freezing and thawing, seepage, and fissure, the thermal distribution within the soil varies, making it challenging to describe the internal heat transfer process. The thermal conductivity of a porous media in soil is influenced by its composition and structure, and the analysis of its thermal mass transfer has some restrictions. According to the distribution of solid, liquid, and gas in soil and related parameters (porosity, water content, ice content, torosity,
etc.), a thermal conductivity analysis model of ice-containing unsaturated soil was established and a thermal conductivity calculation formula was derived for each model.

### 3.1. Simplified Volume-Weighted Average REV Model

The simplified volume-weighted average REV model is a simplified model that can only be analyzed briefly when the internal structure and parameters of the characterized unit are few. The porous media REV is regarded as an “opaque” whole. It is mainly applicable to when the porous medium-related characteristics are few or only one is known (such as the porosity of the porous media), and the heat mass transfer process can be evaluated generally, as illustrated in Figure 3.

Based on a two-dimensional structure, Figure 3 characterizes the cell as a pore structure with a cross-sectional area \( x \times x \), so the relationship between the porosity \( \phi \) and the pore length \( x \) can be obtained from the definition of porosity as follows:

\[
x = \sqrt{\phi}.
\]

For the simplified volume-weighted average model in Figure 3, the thermal conductivity model of the lacunal soil medium can be represented as a simplified series (vertical) model and a simplified parallel (horizontal) model according to the relationship between the heat flow direction and the pore direction.

Figure 4 depicts the simplified series (vertical) model. In this model, the characterization unit channel is perpendicular to the direction of heat flow, and the skeleton inside the soil porous medium is vertically aligned with each component within the pore space.

As shown in Figure 5, the heat transfer analysis for the model in Figure 4 is performed by representing the model as several thermal resistances in series.

Based on the series resistance electrical principle formula, the simplified series (vertical) model thermal conductivity \( \lambda_c \) is

\[
\frac{1}{\lambda_c} = \frac{1}{\lambda_s} + \sqrt{\frac{x}{\phi}} \frac{\lambda_k}{\lambda_s},
\]

where

\[
\frac{1}{\lambda_{kc}} = \frac{1 - \phi}{\lambda_{air}} + \phi \left( \frac{\alpha}{\lambda_i} + \frac{\beta}{\lambda_w} \right).
\]

The derivation of the above equation leads to

\[
\lambda_i = \frac{\lambda_s \lambda_{air} \lambda_{wc}}{(1 - \sqrt{\phi}) \lambda_{air} \lambda_i + \sqrt{(1 - \phi)} \lambda_i \lambda_{iz} + \sqrt{\phi} \lambda_i \lambda_{iw} + \phi \lambda_{air} (\alpha \lambda_{iw} + \beta \lambda_i)},
\]

where \( \phi \) is the porosity of porous media, %, \( \varphi \) is the moisture percentage, \( \beta \) is the water ratio, \( \partial \) is the ice content, \( \lambda_i \) is the thermal conductivity of soil skeleton, \( \lambda_{air} \) is the thermal conductivity of air, \( \lambda_i \) is the thermal conductivity of solid ice, and \( \lambda_{iw} \) is the thermal conductivity of liquid water, W/(m·K).

The simplified parallel (horizontal) model is shown in Figure 6. In this model, the characterization unit channels are parallel to the heat flow direction, and the skeleton inside the soil porous medium is aligned in parallel to the components contained within the pores.

As shown in Figure 7, the heat transfer analysis for the model in Figure 6 is performed by representing the model as several thermal resistances in parallel.

Based on the parallel coupling resistance electrical principle formula, the simplified parallel (horizontal) model model
Table 2: Medium thermal conductivity.

<table>
<thead>
<tr>
<th>Media</th>
<th>Thermal conductivity W/(m-K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sandy soil</td>
<td>0.59</td>
</tr>
<tr>
<td>Ice</td>
<td>2.26</td>
</tr>
<tr>
<td>Dry air</td>
<td>0.0244</td>
</tr>
<tr>
<td>Liquid water</td>
<td>0.0551</td>
</tr>
</tbody>
</table>

Figure 6 Geo...of the pore channels can be further introduced to undertake characteristics relating to the distribution and composition of the pore channels, as illustrated in Figure 8.

Figure 8 reveals the heat transfer process, based on the simplified volume-weighted average REV model, considers the degree of curvature of the pores, and introduces the tortuosity \( \tau \), i.e., a parameter reflecting the degree of curvature of the pore channels, defined by the equation

\[
\tau_i = \frac{L_i}{L},
\]

where \( L_i \) is the true length of the bent pore channel and \( L \) is the length of the straight line connecting the two ends of the bent pore channel.

The average tortuosity \( \tau \) is defined by

\[
\tau = \sum_{i=1}^{N} \tau_i / N,
\]

where \( N \) is the total number of pore channels within the characterization cell.

This leads to the fine volume-weighted average REV model thermal conductivity \( \lambda_w \) as

\[
\lambda_w = \tau_b (1 - \eta) \lambda_b + \tau_s \eta \lambda_s,
\]

\[
= \tau_b (1 - \eta) \left\{ \left( 1 - \sqrt{\phi} \right) \lambda_b + \sqrt{\phi} (1 - \phi) \lambda_{w}, \phi + \phi (\alpha \lambda_b + \beta \lambda_{w}) \right\}
+ \tau_s \eta \lambda_{w} \lambda_{w} \left( 1 - \sqrt{\phi} \right) \lambda_{s} \lambda_{s} + \sqrt{\phi} (1 - \phi) \lambda_{s} \lambda_{s} + \phi \lambda_{s} \lambda_{s} (\alpha \lambda_{w} + \beta \lambda_{w}) \right\},
\]

(15)

when \( \tau_b = \tau_s = \tau \), the above equation can be simplified as

\[
\lambda_w = \tau (1 - \eta) \lambda_b + \tau \eta \lambda_s,
\]

\[
= \tau_b (1 - \eta) \left\{ \left( 1 - \sqrt{\phi} \right) \lambda_b + \sqrt{\phi} (1 - \phi) \lambda_{w}, \phi + \phi (\alpha \lambda_b + \beta \lambda_{w}) \right\}
+ \tau_s \eta \lambda_{w} \lambda_{w} \left( 1 - \sqrt{\phi} \right) \lambda_{s} \lambda_{s} + \sqrt{\phi} (1 - \phi) \lambda_{s} \lambda_{s} + \phi \lambda_{s} \lambda_{s} (\alpha \lambda_{w} + \beta \lambda_{w}) \right\},
\]

(16)

where \( \eta \) is the pore channel distribution coefficient, \( \eta = N_c / N \times 100\% \), where \( N_c \) is the number of pore channels in series.

3.3. Macroscopic Unsaturated Ice-Containing Soil Thermal Conductivity Analysis Model. According to the so-called thermal conductivity analysis model of the macroscopic containing unsaturated soil, the control body of the porous medium REV is infinitely close and entirely transparent. As depicted in Figure 9, the model takes top of the thin volume-weighted average REV model when the great majority of the porous medium is known or by introducing more accurate correction parameters. Compared with the fine volume-weighted average REV model, the thermal conductivity analysis model of macroscopic ice-containing unsaturated soil is closer to the real model. The specific analysis
procedure divides the actual macroscopic unsaturated ice-containing soil into $n \times n$ REVs.

For the heat transfer analysis of Figure 9, according to the different heat transfer paths, an integrated model of thermal conductivity of soil lacunal media can be communicated as the series-parallel integrated model and the parallel-series integrated model.

The series-parallel integrated model involves connecting the $j$ columns of thermal resistance in series and considering them as a whole and then connecting the $i$ rows of thermal resistance in parallel. The series-parallel integrated model thermal conductivity $\lambda_{gc-i}$ is

$$\lambda_{gc-i} = \frac{\sum_{j=1}^{i} \lambda_{gc-j}}{i} = \frac{1}{i} \sum_{j=1}^{i} \frac{\lambda_{kc} \lambda_{sc}}{\lambda_{kc} + (1 - \varepsilon_{j}) \lambda_{sc}}, \quad (17)$$

where $\varepsilon_{j}$ is the channel composition coefficient of column (row) $j$, $\varepsilon_{j} = N_{j}/N_{z} \times 100\%$, where $N_{j}$ is the number of pore channels in parallel (series) of column (row) $j$ and $N_{z}$ is the total of pores of column (row) $j$.

The parallel-series integrated model involves connecting the $i$ rows of thermal resistance in parallel and considering them as a whole and then connecting the $j$ columns of thermal resistance in series. The parallel-series integrated model thermal conductivity $\lambda_{gb-j}$ is

$$\frac{1}{\lambda_{gb-j}} = \frac{1}{j} \sum_{i=1}^{j} \frac{1}{\lambda_{gb-i}} = \frac{1}{j} \sum_{i=1}^{j} \frac{1}{\varepsilon_{i} \lambda_{kb} + (1 - \varepsilon_{i}) \lambda_{kb}}, \quad (18)$$

4. Model Verification and Comparison and Numerical Simulation Analysis

4.1. Model Validation Method

4.1.1. Steady-State Heat Flow Meter Method. After the temperature distribution on the sample has stabilized, the steady-state approach involves measuring the heat and temperature gradient flowing through the sample to determine its thermal conductivity. The steady-state approach is comprised of the flat plate method, the heat flow meter method, the heat box method, and the comparison method. According to Fourier's law, the heat flow meter method calculates the thermal conductivity of the soil based on the heat flow and temperature gradient of the soil sample per unit time. According to Fourier’s law, the principle is simple and repetitive, but the test time and soil sample size are large. The calculation formula of thermal conductivity by heat flow meter method is as follows:

$$\lambda = \frac{ckh}{\Delta T}. \quad (19)$$

In the formula, $c$ is the heat flow meter calibration coefficient; $k$ is the heat flow meter reading, $\mu V$; $h$ is the soil sample thickness, cm; $\Delta T$ is the soil sample two surface temperature difference, °C.

4.1.2. Transient Hotline Method. The transient method, also known as the unsteady-state method, does not have to wait for the temperature distribution on the test sample to be stable before measuring. It determines the temperature field change and calculates the thermal conductivity using the
differential equation for unsteady thermal conductivity. Transient method includes the following: hot line method, hot plate method, tropical method, and laser method. Hotline method, also called probe method, is a commonly used measurement method of soil thermal conductivity. A metal heating wire with constant heating power is inserted into a soil sample to be tested to continuously heat it up, and the relevant parameters are measured to calculate the thermal conductivity of the soil. Hotline method is easy to measure, it is fast, the size of the soil sample is not strict requirements, and it can also determine the thermal conductivity of wet materials, but the determination of thermal disturbance cannot avoid and have a certain impact. The hotline method thermal conductivity calculation formula is as follows:

\[
\lambda = \frac{I^2R}{4\pi L} \times \ln \left(\frac{t_2/t_1}{T_2 - T_1}\right).
\]

In the formula, \(I\) is the metal hotline heating current, \(A\); \(R\) is the metal hotline resistance at test temperature, \(\Omega\); \(L\) is the length of the metal hotline, \(m\); \(t_1\) is the time of heating time, \(s\); \(t_2\) is the time of measurement time, \(s\); \(T_1\) is the temperature at \(t_1\), °C; \(T_2\) is the temperature at \(t_2\), °C.

4.2 Modelling Verification. Wang [32], through the hotline method and heat flow meter method, studied the standard
tortuosity of 101.325 kPa (1.4 g/cm³ in dry density) at -15°C. The results demonstrate that the conclusion measured by the hotline approach is close to the actual one, which can provide a reference for similar experiments. This experiment and related data are used to verify the established model, to determine the correctness of the new model. The soil structural property parameters and medium thermal conductivity characteristics included in the experiment [32] are shown in Table 1.

Considering that the integrated REV model is the most realistic heat transfer model to the real situation in the established model, the model is compared to experimental data. This is seen in Figure 10.

According to Figure 10, the test, standard [33], and calculated values all increase. The literature [32] test values compared to the specification showed a maximum deviation of +26.4% and a minimum deviation of +12.5% as compared to the specification. The computed value of the constructed integrated REV model resulted in the maximum deviation that is -8.5% and -2.3% compared to the specification. Therefore, it can be seen that the deviation of the integrated REV model relative to the standard value is smaller than the reference [32] test value. The thermal conductivity computed by the developed model corresponds to the observed value. In addition, it can be seen from Figure 10 which demonstrates that the test results deviate greatly from the simulation calculation and specification, mostly as a result of the heterogeneous composition and porosity of the soil porous media during the test. In the meantime, the simplified volume-weighted REV model and the fine volume-weighted REV model are also confirmed, which is similar to the comprehensive REV model and will not be repeated.

4.3. Numerical Calculation and Analysis. Taking sandy soil as an example, we analyzed the influence of each parameter factor (porosity, water ratio, moisture percentage, ice content, and tortuosity) on the thermal conductivity after numerical calculations on the basis of the developed models and derived formulas. The relevant parameters [33] are recorded in Table 2.

4.3.1. Simplified Volume-Weighted Average REV Model Thermal Conductivity Calculation. The simplified volume-weighted average REV model thermal conductivity is calculated numerically according to equations (9) and (12).

When the water ratio $\beta = 0.3$ and ice content $\delta = 0.5$, the thermal conductivity $\lambda$ varies with the porosity $\phi$ when the moisture percentage $\varphi$ is 0.1, 0.3, and 0.5, respectively, as displayed in Figure 11. From the figure, when the moisture percentage is specified, the thermal conductivity of both series and parallel models decreases with the porosity; when the porosity is specified, the thermal conductivity increases with the increment of the moisture percentage.

When the moisture percentage $\varphi$ is 0.3 and the porosity $\Phi$ is 0.5, the thermal conductivity $\lambda$ varies with the porosity $\phi$ when the water ratio $\beta$ is 0.2, 0.5, and 0.8, respectively, as displayed in Figure 12. From the figure, when the water ratio is specified, the thermal conductivity of both series and parallel models decreases with the porosity; when the porosity is specified, the parallel model thermal conductivity increases with the increment of the water ratio, and the series model thermal conductivity decreases with the increase of the water ratio.
When the water ratio $\beta$ is 0.5 and the porosity $\phi$ is 0.5, the thermal conductivity $\lambda$ of series model is 0.044 W/(m·K); that of parallel model is 0.43 W/(m·K); that is, the curve change of series model is more gentle.

When the moisture percentage $\varphi = 0.3$ and water ratio $\beta = 0.5$, the thermal conductivity $\lambda$ varies with the porosity $\phi$ when the ice content $\partial$ is 0.3, 0.5, and 0.7, respectively, as displayed in Figure 13. Because the change cannot be determined in Figure 13(a), consider the porosity and the simplified volume-weighted average REV model ice content (0.3, 0.5, and 0.7) to determine the change trend separately in Figure 13(b). From the figure, when the ice content is specified, the thermal conductivity of both series and parallel models decreases with the porosity; when the porosity is specified, the parallel model thermal conductivity increases with the increment of the ice content, and the series model thermal conductivity decreases with the increase of the ice content.

When the ice content $\partial$ is 0.5 and the porosity $\phi$ is 0.5, the thermal conductivity $\lambda$ of series model is 0.04395 W/(m·K); that of parallel model is 0.43044 W/(m·K); the curve change of series model is more gentle.

Figures 11–13 show that the thermal conductivity of the series model is less than that of the parallel model. This is because the thermal conductivity of solid ice water is about four times that of liquid water and is much greater than that of the gas.

When the water content, ice content, and moisture content are constant, the thermal conductivity falls progressively with increasing porosity. This is due to the fact that when the porosity increases, the density of soil particles reduces and the contact area between soil particles decreases, making heat transmission between soil particles more difficult and decreasing the thermal conductivity.

4.3.2. Fine Volume-Weighted Average REV Model Thermal Conductivity Calculation. The fine volume-weighted average REV model thermal conductivity is calculated numerically according to equations (15) and (16).

When the porosity $\phi = 0.5$, water ratio $\beta = 0.3$, ice content $\partial = 0.5$, moisture percentage $\varphi = 0.3$, and pore channel distribution coefficient $\eta$ are 0.1, 0.3, and 0.5, respectively, the thermal conductivity $\lambda$ varies with the tortuosity $\tau$, as shown in Figure 14. From the figure, when the pore channel distribution coefficient is specified, the thermal conductivity of the fine volume-weighted average REV model increases with the tortuosity; when the tortuosity is specified, the thermal conductivity decreases with the increment of the pore...
channel composition coefficient. This is because, as the distribution coefficient of pore channels increases, the number of series channels in the characterization unit body increases and the number of parallel channels decreases. Since the parallel model has a higher thermal conductivity than the series model, the thermal conductivity decreases.

When the pore channel distribution coefficient $\eta = 0.5$, moisture percentage $\varphi = 0.3$, water ratio $\beta = 0.3$, ice content $\delta = 0.5$, and porosity $\phi$ are 0.1, 0.3, and 0.5, respectively, the thermal conductivity $\lambda$ varies with the tortuosity $\tau$, as shown in Figure 15. From the figure, when the porosity is specified, the thermal conductivity of the fine volume-weighted average REV model increases with the tortuosity; when the tortuosity is specified, the thermal conductivity decreases with the increment of the porosity.

When the pore channel distribution coefficient $\eta = 0.5$, the porosity $\phi = 0.3$, the moisture percentage $\varphi = 0.3$, the ice content $\delta = 0.5$, and the water ratio $\beta$ are 0.1, 0.3, and 0.5, respectively, the thermal conductivity $\lambda$ varies with the tortuosity $\tau$, as shown in Figure 16. Because the fine volume-weighted average REV model cannot determine the change trend in Figure 16(a), consider the tortuosity $\tau = 1$ and the fine volume-weighted average REV model water ratio $\beta (0.1, 0.3, 0.5)$ to determine the change trend separately in Figure 16(b). From the figure, when the water ratio is specified, the thermal conductivity of the fine volume-weighted average REV model increases with the increase of the tortuosity; when the tortuosity is specified, the thermal conductivity decreases with the increment of the water ratio.

When the pore channel distribution coefficient $\eta = 0.5$, porosity $\phi = 0.3$, water ratio $\beta = 0.3$, moisture percentage $\varphi = 0.3$, and ice content $\delta = 0.5$, respectively, the thermal conductivity $\lambda$ varies with the tortuosity $\tau$, as shown in Figure 17. From the figure, when the ice content rate is specified, the thermal conductivity of the fine volume-weighted average REV model increases with the increase of the tortuosity; when the tortuosity is specified, the thermal conductivity all increases with the increase of the ice content rate.

When the pore channel distribution coefficient $\eta = 0.5$, the porosity $\phi = 0.3$, the water ratio $\beta = 0.3$, the ice content $\delta = 0.5$, and the moisture percentage $\varphi$ are 0.1, 0.3, and 0.5, respectively, the thermal conductivity $\lambda$ varies with the tortuosity $\tau$, as shown in Figure 18. From the figure, when the moisture content is specified, the thermal conductivity of the fine volume-weighted average REV model increases with the increase of the tortuosity; when the tortuosity is specified, the thermal conductivity increases with the increment of the moisture percentage.

4.3.3. Integrated REV Model Thermal Conductivity Calculation. According to equations (9) and (10), the thermal conductivity of the integrated model is calculated numerically.

When the water ratio $\beta = 0.3$, ice content $\delta = 0.5$, moisture percentage $\varphi = 0.3$, and porosity $\phi$ are 0.3, 0.5, and 0.7, respectively, the thermal conductivity $\lambda$ varies with the channel composition coefficient $\epsilon$ as shown in Figure 19. From the figure, it can be seen that the thermal conductivity of both the series-parallel integrated model and the parallel-series integrated model decreases with the increment of the channel composition coefficient when the porosity is specified; when the channel composition coefficient is specified, the thermal conductivity of both the series-parallel integrated model and the parallel-series integrated model decreases with the increment of the porosity.

When the porosity $\phi$ is 0.5 and the channel composition coefficient $\epsilon$ is 0.5, the thermal conductivity $\lambda$ of the series-parallel integrated model is $0.08446 \text{ W/(m-K)}$; the thermal conductivity decreases with the increment of the porosity.
conductivity $\lambda$ of the parallel-series integrated model is 0.50905 W/(m·K); namely, the curve of the parallel-series integrated model changes more gently.

When the porosity $\phi$ = 0.5, the ice content $\partial$ = 0.5, the moisture percentage $\varphi$ = 0.3, and the water ratio $\beta$ are 0.3, 0.5, and 0.7, respectively, the thermal conductivity $\lambda$ of the series-parallel integrated model changes with the channel composition coefficient $\varepsilon$ as shown in Figure 20(a), and the thermal conductivity $\lambda$ of the parallel-series integrated model changes with the channel composition coefficient $\varepsilon$ as shown in Figure 20(b). From the figure, when the water ratio is certain, the thermal conductivity of both the series-parallel integrated model and the parallel-series integrated model decreases with the increase of the channel composition coefficient; when the channel composition coefficient is specified, the thermal conductivity of the series-parallel integrated model decreases with the increase of the water ratio, and the thermal conductivity of the parallel-series integrated model increases with increasing water ratio. This is because the change of water ratio affects less on the thermal conductivity than its structural distribution.

When the water ratio $\beta$ is 0.5 and the channel composition coefficient $\varepsilon$ is 0.5, the thermal conductivity $\lambda$ of the series-parallel integrated model is 0.0818 W/(m·K); the thermal conductivity $\lambda$ of the parallel-series integrated model is 0.51022 W/(m·K); that is, the curve of the parallel-series integrated model changes more gently.

When the porosity $\phi$ = 0.5, water ratio $\beta$ = 0.3, moisture percentage $\varphi$ = 0.3, and ice content $\partial$ are 0.3, 0.5, and 0.7, respectively, the thermal conductivity $\lambda$ varies with the channel composition coefficient $\varepsilon$, as appeared in Figure 21. Since the changing trend of the series-parallel integrated model cannot be derived from Figure 21(a), the channel composition coefficient $\varepsilon = 0.2$ is used and Figure 21(b) is taken separately when the parallel-series integrated model ice content $\partial$ is 0.3, 0.5, and 0.7 to determine the change trend. The graph below shows that the thermal conductivity of both the series-parallel integrated model and the parallel-series integrated model decreases with the increment of the channel composition coefficient when the ice content is specified; when the channel composition coefficient is specified, the thermal conductivity of the series-parallel integrated model decreases with the increment of the ice content, and the thermal conductivity of the parallel-series integrated model increases with the increment of the channel composition coefficient. This is due to the fact that the change in ice content affects less on the thermal conductivity than its structural distribution.

When the ice content $\partial$ = 0.5 and the channel composition coefficient $\varepsilon$ is 0.5, the thermal conductivity $\lambda$ of the series-parallel integrated model is 0.08446 W/(m·K); the thermal conductivity $\lambda$ of the parallel-series integrated model is 0.50905 W/(m·K); namely, the curve of the series-parallel integrated model changes more gently.

When the porosity $\phi$ = 0.5, water ratio $\beta$ = 0.3, ice content $\partial$ = 0.5, and moisture percentage $\varphi$ are 0.1, 0.3, and 0.5, respectively, the thermal conductivity $\lambda$ varies with the channel composition coefficient $\varepsilon$, as appeared in Figure 22. From the figure, the thermal conductivity of both the series-parallel integrated model and the parallel-series integrated model decreases with the increment of the channel composition coefficient when the moisture percentage is specified; the thermal conductivity increases with the increase of the moisture percentage when the channel composition coefficient is specified. This is because the change in moisture percentage has a less effect on the thermal conductivity than the distribution of its structure.

When the moisture percentage $\varphi$ is 0.5 and the channel composition coefficient $\varepsilon$ is 0.5, the thermal conductivity $\lambda$
of the series-parallel integrated model is 0.08446 W/(m·K); the thermal conductivity $\lambda$ of the parallel-series integrated model is 0.50905 W/(m·K); that is, the curve of the parallel-series integrated model changes more gently.

5. Conclusion

In the winter, the water and other media in the pores of soil porous media produce freeze-thawing phenomenon, and its phase, composition, and distribution change, resulting in a large change in thermal conductivity. In this paper, based on the composition structure and material distribution of unsaturated soil under the condition of freeze and thaw, we establish the thermal conductivity analysis model of unsaturated soil with ice, derive the thermal conductivity expression under various models, use the existing tests for comparative verification and calculation analysis, and draw the following conclusions:

(1) Assuming that the actual soil porous medium is a continuum, the properties are described and analyzed according to the composition and structure of ice-containing unsaturated soil, to verify the description and feasibility of employing porosity and using porosity as a representative state variable. It offers a new path for the development of frozen soil resources and the construction of engineering structures in the frozen soil area. It contributes to the exploration and analysis of the fundamental characteristics and development rules of permafrost and the prevention and management of freezing damage, as well as the development and economic construction of the cold area.

(2) Comparative of the existing experiments, pertinent normative data, and the comprehensive REV model, the results demonstrate that the deviation from the calculated and gauge values of the integrated REV model is less than the test value, and that the thermal
conductivity of the constructed model agrees with the actual condition; thus, the model developed in this work is applicable

(3) The established simplified series (vertical) model, simplified parallel (parallel) model, fine volume-weighted REV model, and macroscopic thermal conductivity analysis model are calculated and analyzed in sand under the conditions of standard atmospheric pressure of 101.325 kPa and temperature of 0°C. It is concluded that the thermal conductivity of unsaturated ice-containing soil is controlled not only affected by pore size but also restricted by soil structure distribution, material composition, pore channel bending degree, phase shift, and heat transfer direction, which must be examined for particular scenarios.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

No potential conflict of interest was reported by the authors.

References


